

**Laporan Tugas Kecil 3**  
**IF2211 Strategi Algoritma**  
**Penyelesaian Persoalan 15-Puzzle dengan Algoritma *Branch and Bound***



**oleh**

**Raden Rifqi Rahman (13520166)**

**PROGRAM STUDI TEKNIK INFORMATIKA**  
**SEKOLAH TEKNIK ELEKTRO DAN INFORMATIKA**  
**INSTITUT TEKNOLOGI BANDUNG**  
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## Penyelesaian Persoalan 15-Puzzle dengan Algoritma *Branch and Bound*

### Cara kerja aplikasi dalam menyelesaikan 15-puzzle menggunakan algoritma *branch and bound*

Untuk menyelesaikan 15-puzzle, aplikasi melakukan langkah-langkah sebagai berikut.

Dari masukan yang diberikan, bangun sebuah matriks bilangan bulat  $4 \times 4$  yang berisi nilai-nilai masukan. Cek apakah nilai-nilai masukan yang diberikan dapat membangun matriks 15-puzzle yang valid, yaitu matriks yang berisi nilai 1 hingga 15 dan nilai kosong yang masing-masing muncul **tepat 1 kali**. Jika masukan tidak valid, tampilkan pesan kesalahan.

Untuk setiap elemen matriks  $i$ , hitung nilai  $l_i$  (\*) dari elemen tersebut dengan nilai kosong dianggap 16. Nilai  $l$  dikalkulasi sebagai berikut.

$$l_i = \text{banyak elemen } e \text{ sehingga } e < i \text{ dan posisi}(e) > \text{posisi}(i) \\ \text{dengan} \\ \text{posisi}(e) = \text{indeks baris } e * 4 + \text{indeks kolom } e$$

Selanjutnya, hitung nilai  $x$  berdasarkan posisi elemen kosong yang dikalkulasi sebagai berikut.

$$x = (\text{indeks baris elemen kosong} + \text{indeks kolom elemen kosong}) \bmod 2$$

Jika nilai dari  $\sum_{i=1}^{16} l_i + x$  adalah ganjil, maka puzzle tidak memiliki solusi dan tampilkan pesan kesalahan.

Selanjutnya, misalkan sebuah *priority queue*  $Q$  menyimpan *state-state* puzzle dengan prioritas nilai *cost* terendah dari *state*. Nilai *cost* dari sebuah *state* puzzle dihitung sebagai berikut.

$$\text{cost}(S) = \text{kedalaman state parent } S + \text{banyak sel tak kosong pada } S \text{ yang tidak sesuai state goal}$$

Masukan matriks puzzle  $M$  ke dalam  $Q$ . Selama  $M$  bukan merupakan *goal* dari 15-puzzle, *expand*  $M$  dengan cara menggerakkan sel kosong pada  $M$  ke bawah, kanan, kiri, dan atas—jika memungkinkan—sehingga didapatkan *state-state* baru. Simpan *state* yang dibangkitkan oleh  $M$  dan belum pernah dibangkitkan sebelumnya ke dalam  $Q$ . Kemudian ambil matriks dari *state* yang berada pada elemen terdepan  $Q$  dan simpan sebagai  $M$ .

Jika  $M$  adalah *goal*, telusuri *parent* dari  $M$  hingga mencapai *state* awal puzzle yang diberikan. Dengan demikian, akan diperoleh jalur '*path*'  $P$  dari *state goal* menuju *state* awal puzzle. Selanjutnya, balik *path*  $P$  sehingga diperoleh *path*  $P'$  dari *state* awal puzzle menuju *state goal*. *Path*  $P'$  inilah yang merupakan solusi menuju *state goal* dari 15-puzzle.

\* Nilai  $l_i$  adalah nilai dari fungsi  $KURANG(i)$ .

### Source code aplikasi

Berikut adalah source code aplikasi dalam bahasa Java yang telah diminifikasi untuk menyelesaikan 15-puzzle dengan algoritma *branch and bound*.

## Application.java

```
package id.ac.itb.stei.informatika.fifteenp;
import javafx.fxml.FXMLLoader;
import javafx.scene.Scene;
import javafx.scene.image.Image;
import javafx.stage.Stage;
import java.io.IOException;
public class Application extends javafx.application.Application {
    @Override
    public void start(Stage stage) throws IOException {
        FXMLLoader fxmlLoader = new
FXMLLoader(Application.class.getResource("main.fxml"));
        Scene scene = new Scene(fxmlLoader.load(), 1080, 540);
        stage.setTitle("15-Puzzle Solver");
        stage.setScene(scene);
        stage.getIcons().add(new
Image(getClass().getResourceAsStream("fifteenp.png")));
        stage.show();
    }
    public static void main(String[] args) {
        launch();
    }
}
```

## MainController.java

```
package id.ac.itb.stei.informatika.fifteenp;
import id.ac.itb.stei.informatika.fifteenp.io.FifteenMatrixParser;
import id.ac.itb.stei.informatika.fifteenp.io.FileReader;
import id.ac.itb.stei.informatika.fifteenp.util.Direction;
import id.ac.itb.stei.informatika.fifteenp.util.FifteenMatrix;
import javafx.fxml.FXML;
import javafx.geometry.Pos;
import javafx.scene.control.Alert;
import javafx.scene.control.Button;
import javafx.scene.control.Label;
import javafx.scene.layout.*;
import javafx.scene.paint.Color;
import javafx.scene.text.Font;
import javafx.scene.text.FontWeight;
import javafx.scene.text.TextAlignment;
import javafx.stage.FileChooser;
import java.io.File;
import java.util.ArrayList;
import java.util.Random;
public class MainController {
    @FXML
    private FifteenMatrixController matrixController;
    @FXML
    private GridPane mainContainer;
    @FXML
    private Label labelCell0;
    @FXML
    private Label labelCell1;
    @FXML
    private Label labelCell2;
    @FXML
    private Label labelCell3;
```

```

@FXML
private Label labelCell4;
@FXML
private Label labelCell5;
@FXML
private Label labelCell6;
@FXML
private Label labelCell7;
@FXML
private Label labelCell8;
@FXML
private Label labelCell9;
@FXML
private Label labelCell10;
@FXML
private Label labelCell11;
@FXML
private Label labelCell12;
@FXML
private Label labelCell13;
@FXML
private Label labelCell14;
@FXML
private Label labelCell15;
@FXML
private Button prevButton;
@FXML
private Button nextButton;
@FXML
private Button chooseFileButton;
@FXML
private Button solveButton;
@FXML
private Button randomButton;
@FXML
private Label depthLabel;
@FXML
private Label stateLabel;
@FXML
private Label sumLowerLabel;
@FXML
private Label lowerLabel;
@FXML
private Label execTimeLabel;
private Label[] labelCells;
private ArrayList<FifteenMatrix> solutionPath;
private ArrayList<Direction> solutionDir;
private int currentDepth;
private final Font defaultFont = Font.font("Arial", FontWeight.BOLD, 16);
private final Font defaultFontSmall = Font.font("Arial", FontWeight.BOLD, 12);
private final Font defaultFontBig = Font.font("Arial", FontWeight.BOLD, 20);
private final String[] postfix = {"ns", "us", "ms", "s"};
@FXML
private void initialize() {
    this.mainContainer.setBackground(new Background(
        new BackgroundFill(Color.valueOf("#37474f"), null, null)
    ));
    Border border = new Border(

```

```

        new BorderStroke(
            Color.WHITESMOKE,
            BorderStrokeStyle.SOLID,
            CornerRadii.EMPTY,
            BorderStroke.DEFAULT_WIDTHS
        )
    );
    this.labelCells = new Label[]{
        labelCell0, labelCell1, labelCell2, labelCell3,
        labelCell4, labelCell5, labelCell6, labelCell7,
        labelCell8, labelCell9, labelCell10, labelCell11,
        labelCell12, labelCell13, labelCell14, labelCell15,
    };
    this.depthLabel.setFont(this.defaultFontBig);
    this.nextButton.setFont(this.defaultFont);
    this.prevButton.setFont(this.defaultFont);
    for (Label cell: this.labelCells) {
        cell.setFont(this.defaultFont);
        cell.getStyleClass().add("white");
        cell.setBorder(border);
        cell.getParent().getParent().prefHeight(Double.POSITIVE_INFINITY);
        cell.getParent().getParent().prefWidth(Double.POSITIVE_INFINITY);
        HBox parent = (HBox) cell.getParent();
        VBox grandParent = (VBox) parent.getParent();
        parent.prefHeightProperty().bind(grandParent.heightProperty());
        parent.prefWidthProperty().bind(grandParent.widthProperty());
        cell.prefHeightProperty().bind(parent.heightProperty());
        cell.prefWidthProperty().bind(parent.widthProperty());
        cell.setTextAlignment(TextAlignment.CENTER);
        cell.setAlignment(Pos.CENTER);
    }
    this.execTimeLabel.setFont(this.defaultFontSmall);
    this.stateLabel.setFont(this.defaultFontSmall);
    this.sumLowerLabel.setFont(this.defaultFontBig);
    this.solveButton.setFont(this.defaultFont);
    this.randomButton.setFont(this.defaultFont);
    this.chooseFileButton.setFont(this.defaultFont);
    this.lowerLabel.setFont(this.defaultFontSmall);
    this.randomButton.setStyle("-fx-background-color: transparent;" +
        "-fx-text-fill: white; -fx-cursor: hand;");
    this.randomButton.setBorder(border);
    this.chooseFileButton.setStyle("-fx-background-color: transparent;" +
        "-fx-text-fill: white; -fx-cursor: hand;");
    this.chooseFileButton.setBorder(border);
    this.solveButton.setStyle("-fx-background-color: transparent;" +
        "-fx-text-fill: white; -fx-cursor: hand;");
    this.solveButton.setBorder(border);
    this.nextButton.setStyle("-fx-background-color: transparent;" +
        "-fx-text-fill: white; -fx-cursor: hand;");
    this.nextButton.setBorder(border);
    this.prevButton.setStyle("-fx-background-color: transparent;" +
        "-fx-text-fill: white; -fx-cursor: hand;");
    this.prevButton.setBorder(border);
}
@FXML
protected void onSolve() {
    FifteenMatrix matrix;
    try {

```

```

        matrix = matrixController.parse();
    } catch (IllegalArgumentException ignored) {
        this.alert("The given matrix is invalid.");
        return;
    }
    try {
        this.currentDepth = 0;
        this.displayLowerValues(matrix);
        this.displayLowerValuesSum(matrix.lowerSum());
        FifteenPuzzle solver = new FifteenPuzzle(matrix);
        long start = System.nanoTime();
        solver.solve();
        long end = System.nanoTime();
        this.displayExecutionTime(end - start);
        this.solutionPath = solver.getSolutionPathMatrix();
        this.solutionDir = solver.getSolutionPathDir();
        this.matrixController.setMatrix(this.solutionPath.get(0));
        this.disablePrevButton();
        this.setGeneratedStatesText(solver.generatedStates());
        if (this.solutionDir.size() != 0) {
            this.matrixController.setDirection(this.solutionDir.get(0));
            this.enableNextButton();
        }
        this.setLabelText(1, this.solutionPath.size());
    } catch (IllegalArgumentException ignored) {
        this.alert("This puzzle is unsolvable.");
    }
}

@FXML
protected void onNext() {
    this.enablePrevButton();
    this.currentDepth++;
    this.matrixController.setMatrix(this.solutionPath.get(
        this.currentDepth));
    if (this.currentDepth == this.solutionDir.size()) {
        this.matrixController.setDirection(null);
    } else {
        this.matrixController.setDirection(this.solutionDir.get(
            this.currentDepth));
    }
    this.setLabelText(this.currentDepth + 1, this.solutionPath.size());
    if (this.currentDepth + 1 == this.solutionPath.size()) {
        this.disableNextButton();
    } else {
        this.enableNextButton();
    }
}

@FXML
protected void onPrev() {
    this.enableNextButton();
    this.currentDepth--;
    this.matrixController.setMatrix(this.solutionPath.get(
        this.currentDepth));
    this.matrixController.setDirection(this.solutionDir.get(
        this.currentDepth));
    this.setLabelText(this.currentDepth + 1, this.solutionPath.size());
    if (this.currentDepth == 0) {
        this.disablePrevButton();
    }
}

```

```

    } else {
        this.enablePrevButton();
    }
}
@FXML
protected void onRandom() {
    Random random = new Random();
    int steps = (random.nextInt() % 28 + 28) % 28 + 12;
    FifteenMatrix matrix = FifteenPuzzle.SOLUTION.copy();
    for (int i = 0; i < steps; i++) {
        while (true) {
            int dirIndex = (random.nextInt() % 4 + 4) % 4;
            try {
                matrix = matrix.moveBlankTile(
                    Direction.DIRECTIONS[dirIndex]
                );
                break;
            } catch (IndexOutOfBoundsException ignored) {}
        }
    }
    this.matrixController.setMatrix(matrix);
}
@FXML
private void onChooseFile() {
    FileChooser dialog = new FileChooser();
    File input = dialog.showOpenDialog(null);
    if (input != null) {
        String filename = input.toString();

        FileReader reader = new FileReader();
        try {
            reader.readFile(filename);
        } catch (Throwable ignored) {
            this.alert("There was an error reading the file.");
        }

        try {
            String fileContent = reader.result();
            FifteenMatrixParser parser = new FifteenMatrixParser();
            parser.parse(fileContent);
            this.matrixController.setMatrix(parser.result());
        } catch (Throwable ignored) {
            this.alert("The file has an invalid format.");
        }
    }
}
private void setLabelText(int depth, int maxDepth) {
    this.depthLabel.setText(depth + "/" + maxDepth);
}
private void setGeneratedStatesText(Integer states) {
    this.stateLabel.setText("Generated "
        + states.toString() + " states");
}
private void disablePrevButton() {
    prevButton.setDisable(true);
}
private void enablePrevButton() {

```

```

        prevButton.setDisable(false);
    }
    private void disableNextButton() {
        nextButton.setDisable(true);
    }
    private void enableNextButton() {
        nextButton.setDisable(false);
    }
    private void displayLowerValues(FifteenMatrix matrix) {
        for (int i = 1; i < 16; i++) {
            Integer l = matrix.lower(i);
            this.labelCells[i - 1].setText(l.toString());
        }
        Integer l = matrix.lowerNull();
        this.labelCells[15].setText(l.toString());
    }
    private void displayExecutionTime(double duration) {
        int i = 0;
        double factor = 1;
        while (i < 3 && duration * factor > 100) {
            factor *= 1e-3;
            i++;
        }
        String durationString = String.format("%.3f", duration * factor);
        String execTime = "Solved in " + durationString
            + " " + this.postfix[i];
        this.execTimeLabel.setText(execTime);
    }
    private void alert(String message) {
        Alert alert = new Alert(Alert.AlertType.ERROR);
        alert.setContentText(message);
        alert.show();
    }
    private void displayLowerValuesSum(Integer sum) {
        this.sumLowerLabel.setText(sum.toString());
    }
}

```

#### FifteenMatrixController.java

```

package id.ac.itb.stei.informatika.fifteenp;
import id.ac.itb.stei.informatika.fifteenp.util.Direction;
import id.ac.itb.stei.informatika.fifteenp.util.FifteenMatrix;
import id.ac.itb.stei.informatika.fifteenp.util.FifteenMatrixBuilder;
import javafx.fxml.FXML;
import javafx.scene.control.TextArea;
import javafx.scene.layout.*;
import javafx.scene.paint.Color;
import javafx.scene.text.Font;
import javafx.scene.text.FontWeight;
public class FifteenMatrixController {
    @FXML
    private TextArea cell0;
    @FXML
    private TextArea cell1;
    @FXML
    private TextArea cell2;
    @FXML

```



```

private TextArea cell3;
@FXML
private TextArea cell4;
@FXML
private TextArea cell5;
@FXML
private TextArea cell6;
@FXML
private TextArea cell7;
@FXML
private TextArea cell8;
@FXML
private TextArea cell9;
@FXML
private TextArea cell10;
@FXML
private TextArea cell11;
@FXML
private TextArea cell12;
@FXML
private TextArea cell13;
@FXML
private TextArea cell14;
@FXML
private TextArea cell15;
private TextArea[] cells;
private FifteenMatrix currentMatrix;
private Border swapBorderStyle;
private Border defaultBorderStyle;
@FXML
public void initialize() {
    this.swapBorderStyle = new Border(new BorderStroke(
        Color.valueOf("#ffb300"),
        BorderStrokeStyle.SOLID,
        CornerRadii.EMPTY,
        BorderStroke.THICK
    ));
    this.cells = new TextArea[]{
        cell0, cell1, cell2, cell3,
        cell4, cell5, cell6, cell7,
        cell8, cell9, cell10, cell11,
        cell12, cell13, cell14, cell15,
    };
    this.defaultBorderStyle = new Border(
        new BorderStroke(
            Color.WHITESMOKE,
            BorderStrokeStyle.SOLID,
            CornerRadii.EMPTY,
            BorderStroke.DEFAULT_WIDTHS
        )
    );
    for (TextArea cell: this.cells) {
        cell.setFont(
            Font.font("Arial", FontWeight.BOLD, 24)
        );
        cell.setStyle("-fx-background-color: transparent;" +
            "-fx-text-fill: whitesmoke;" +
            "-fx-line-spacing: 0;");
    }
}

```

```

        cell.setBorder(defaultBorderStyle);
        cell.textProperty().addListener((observable, oldValue, newValue) -> {
            if (!newValue.matches("\\d*")) {
                cell.setText(oldValue);
            } else if (!newValue.equals("")) {
                Integer value = Integer.valueOf(newValue);
                if (value > 15) {
                    cell.setText(oldValue);
                }
            }
        });
    }
}

public void setMatrix(FifteenMatrix matrix) {
    this.currentMatrix = matrix;
    for (int i = 0; i < 4; i++) {
        for (int j = 0; j < 4; j++) {
            TextArea cell = this.cells[i * 4 + j];
            Integer value = matrix.get(i, j);
            if (value == null) {
                cell.setText("");
            } else {
                cell.setText(value.toString());
            }
        }
    }
}

public void setDirection(Direction dir) {
    for (TextArea cell: this.cells) {
        cell.setBorder(defaultBorderStyle);
    }
    if (dir != null) {
        int blankTileIndex = this.currentMatrix.blankTileIndex();
        int swapIndex = switch (dir) {
            case UP -> blankTileIndex - 4;
            case DOWN -> blankTileIndex + 4;
            case RIGHT -> blankTileIndex + 1;
            case LEFT -> blankTileIndex - 1;
        };
        TextArea blankTileCell = this.cells[blankTileIndex];
        TextArea swapCell = this.cells[swapIndex];
        blankTileCell.setBorder(this.swapBorderStyle);
        swapCell.setBorder(this.swapBorderStyle);
    }
}

public FifteenMatrix parse() {
    FifteenMatrixBuilder builder = new FifteenMatrixBuilder();
    for (TextArea cell: cells) {
        Integer value;
        if (cell.getText().equals("")) {
            value = null;
        } else {
            value = Integer.valueOf(cell.getText());
        }
        builder.append(value);
    }
    return builder.build();
}

```

```
}
}
```

## FifteenPuzzle.java

```

package id.ac.itb.stei.informatika.fifteenp;
import id.ac.itb.stei.informatika.fifteenp.util.Direction;
import id.ac.itb.stei.informatika.fifteenp.util.FifteenMatrix;
import id.ac.itb.stei.informatika.fifteenp.util.FifteenMatrixBuilder;
import java.util.ArrayList;
import java.util.Collections;
import java.util.Comparator;
import java.util.PriorityQueue;
public class FifteenPuzzle {
    private FifteenMatrix puzzle;
    private PriorityQueue<FifteenMatrixNode> queue;
    private ArrayList<FifteenMatrixNode> evaluatedStates;
    private ArrayList<FifteenMatrix> solutionPathMatrix;
    private ArrayList<Direction> solutionPathDir;
    public static final FifteenMatrix SOLUTION
        = new FifteenMatrixBuilder()
            .append(1).append(2).append(3).append(4)
            .append(5).append(6).append(7).append(8)
            .append(9).append(10).append(11).append(12)
            .append(13).append(14).append(15).append(null)
            .build();
    public static final long SOLUTION_ID
        = FifteenPuzzle.SOLUTION.identity();
    public FifteenPuzzle(FifteenMatrix puzzle) {
        this.puzzle = puzzle;
        this.queue = new
PriorityQueue<>(Comparator.comparingInt(FifteenMatrixNode::cost));
        this.evaluatedStates = new ArrayList<>();
        this.solutionPathMatrix = new ArrayList<>();
        this.solutionPathDir = new ArrayList<>();
    }
    public void solve() {
        this.checkSolvability();
        this.setup();
        FifteenMatrixNode state = this.queue.poll();
        while (state.matrixId() != FifteenPuzzle.SOLUTION_ID) {
            this.expand(state);
            state = this.queue.poll();
        }
        this.evaluatedStates.add(state);
        this.buildSolution();
    }
    private void checkSolvability() throws IllegalArgumentException {
        boolean solvable = this.puzzle.lowerSum() % 2 == 0;
        if (!solvable) {
            throw new IllegalArgumentException();
        }
    }
    private void setup() {
        this.queue.add(new FifteenMatrixNode(
            this.puzzle.identity(),
            null,
            null,

```

```

        1,
        1
    ));
}
private boolean has(long stateMatrixId) {
    for (FifteenMatrixNode node: this.evaluatedStates) {
        if (node.matrixId() == stateMatrixId) {
            return true;
        }
    }
    for (FifteenMatrixNode node: this.queue) {
        if (node.matrixId() == stateMatrixId) {
            return true;
        }
    }
    return false;
}
private void expand(FifteenMatrixNode state) {
    for (Direction dir: Direction.DIRECTIONS) {
        if (dir.flip() == state.dir()) {
            continue;
        }
        try {
            FifteenMatrix newMatrix = FifteenMatrix.from(state.matrixId())
                .moveBlankTile(dir);
            long newMatrixId = newMatrix.identity();
            if (!this.has(newMatrixId)) {
                FifteenMatrixNode node = new FifteenMatrixNode(
                    newMatrixId,
                    state,
                    dir,
                    state.depth() + 1,
                    state.depth() + newMatrix.mismatchedTiles()
                );
                this.queue.offer(
                    node
                );
            }
        } catch (IndexOutOfBoundsException ignored) {}
    }
    this.evaluatedStates.add(state);
}
private void buildSolution() {
    FifteenMatrixNode node = this.evaluatedStates.get(
        this.evaluatedStates.size() - 1
    );
    while (node.parent() != null) {
        this.solutionPathMatrix.add(FifteenMatrix.from(
            node.matrixId()
        ));
        this.solutionPathDir.add(node.dir());
        node = node.parent();
    }
    this.solutionPathMatrix.add(FifteenMatrix.from(
        node.matrixId()
    ));
    Collections.reverse(this.solutionPathMatrix);
}

```

```

        Collections.reverse(this.solutionPathDir);
    }
    public ArrayList<FifteenMatrix> getSolutionPathMatrix() {
        return this.solutionPathMatrix;
    }
    public ArrayList<Direction> getSolutionPathDir() {
        return this.solutionPathDir;
    }
    public int generatedStates() {
        return this.queue.size() + this.evaluatedStates.size();
    }
}
record FifteenMatrixNode(long matrixId,
                        FifteenMatrixNode parent,
                        Direction dir,
                        int depth,
                        int cost) {
}

```

#### util/FifteenMatrix.java

```

package id.ac.itb.stei.informatika.fifteenp.util;
import java.util.ArrayList;
import java.util.Collections;
import java.util.Objects;
public class FifteenMatrix extends Matrix<Integer> {
    public FifteenMatrix() {
        super(4, 4);
    }
    public FifteenMatrix copy() {
        FifteenMatrixBuilder builder = new FifteenMatrixBuilder();
        for (int i = 0; i < 4; i++) {
            for (int j = 0; j < 4; j++) {
                Integer elem = this.values.get(i).get(j);
                builder.append(elem);
            }
        }
        return builder.build();
    }
    public FifteenMatrix moveBlankTile(Direction dir) {
        switch (dir) {
            case UP:
                return this.up();
            case DOWN:
                return this.down();
            case RIGHT:
                return this.right();
            case LEFT:
                return this.left();
            default:
                return this;
        }
    }
    public FifteenMatrix up() throws IndexOutOfBoundsException {
        int blankTileIndex = this.blankTileIndex();
        if (blankTileIndex < 4) {
            throw new IndexOutOfBoundsException();
        }
        FifteenMatrix newMatrix = this.copy();
    }
}

```

```

        newMatrix.swap(blankTileIndex, blankTileIndex - 4);
        return newMatrix;
    }
    public FifteenMatrix down() throws IndexOutOfBoundsException {
        int blankTileIndex = this.blankTileIndex();
        if (blankTileIndex > 12) {
            throw new IndexOutOfBoundsException();
        }
        FifteenMatrix newMatrix = this.copy();
        newMatrix.swap(blankTileIndex, blankTileIndex + 4);
        return newMatrix;
    }
    public FifteenMatrix right() throws IndexOutOfBoundsException {
        int blankTileIndex = this.blankTileIndex();
        if (blankTileIndex % 4 == 3) {
            throw new IndexOutOfBoundsException();
        }
        FifteenMatrix newMatrix = this.copy();
        newMatrix.swap(blankTileIndex, blankTileIndex + 1);
        return newMatrix;
    }
    public FifteenMatrix left() throws IndexOutOfBoundsException {
        int blankTileIndex = this.blankTileIndex();
        if (blankTileIndex % 4 == 0) {
            throw new IndexOutOfBoundsException();
        }
        FifteenMatrix newMatrix = this.copy();
        newMatrix.swap(blankTileIndex, blankTileIndex - 1);
        return newMatrix;
    }
    @Deprecated
    public boolean equals(FifteenMatrix other) {
        for (int i = 0; i < 4; i++) {
            for (int j = 0; j < 4; j++) {
                if (!Objects.equals(this.values.get(i).get(j),
                    other.values.get(i).get(j))) {
                    return false;
                }
            }
        }
        return true;
    }
    public int lowerSum() {
        int blankTileIndex = this.blankTileIndex();
        int factor = (blankTileIndex / 4 + blankTileIndex % 4) % 2;
        int sum = 0;
        for (int i = 1; i < 16; i++) {
            sum += this.lower(i);
        }
        return sum + this.lowerNull() + factor;
    }
    public int lower(Integer value) throws IllegalArgumentException {
        if (value == null) {
            return this.lowerNull();
        }
        if (value < 1 || value > 15) {
            throw new IllegalArgumentException();
        }
    }

```

```

        int opposite = 0;
        iteration:
        for (int i = 0; i < 4; i++) {
            for (int j = 0; j < 4; j++) {
                Integer elem = this.values.get(i).get(j);
                if (elem != null && elem < value) {
                    opposite++;
                }
                if (elem != null && elem == value) {
                    break iteration;
                }
            }
        }
        return value - 1 - opposite;
    }

    public int lowerNull() {
        int opposite = 0;
        iteration:
        for (int i = 0; i < 4; i++) {
            for (int j = 0; j < 4; j++) {
                Integer elem = this.values.get(i).get(j);
                if (elem != null) {
                    opposite++;
                } else {
                    break iteration;
                }
            }
        }
        return 15 - opposite;
    }

    public int mismatchedTiles() {
        int mismatches = 0;
        for (int i = 0; i < 4; i++) {
            for (int j = 0; j < 4; j++) {
                Integer elem = this.values.get(i).get(j);
                if (elem != null && elem != 4 * i + j + 1) {
                    mismatches++;
                }
            }
        }
        return mismatches;
    }

    public int blankTileIndex() {
        for (int i = 0; i < 4; i++) {
            for (int j = 0; j < 4; j++) {
                if (this.values.get(i).get(j) == null) {
                    return 4 * i + j;
                }
            }
        }
        return -1;
    }

    private void swap(int firstIndex, int secondIndex)
        throws IndexOutOfBoundsException {
        if (firstIndex < 0 || firstIndex > 15
            || secondIndex < 0 || secondIndex > 15) {
            throw new IndexOutOfBoundsException();
        }
    }

```

```

        Integer a = this.values.get(firstIndex / 4)
            .get(firstIndex % 4);
        Integer b = this.values.get(secondIndex / 4)
            .get(secondIndex % 4);
        this.values.get(firstIndex / 4)
            .set(firstIndex % 4, b);
        this.values.get(secondIndex / 4)
            .set(secondIndex % 4, a);
    }
    public static FifteenMatrix from(long id) {
        ArrayList<Integer> values = new ArrayList<>();
        for (int i = 0; i < 4; i++) {
            for (int j = 0; j < 4; j++) {
                int value = (int) (((id % 16) + 16) % 16);
                if (value == 0) {
                    values.add(null);
                } else {
                    values.add(value);
                }
                id >>= 4;
            }
        }
        Collections.reverse(values);
        FifteenMatrixBuilder builder = new FifteenMatrixBuilder();
        for (Integer value: values) {
            builder.append(value);
        }
        return builder.build();
    }
    public long identity() {
        long id = 0;
        for (int i = 0; i < 4; i++) {
            for (int j = 0; j < 4; j++) {
                id <= 4;
                Integer elem = this.values.get(i).get(j);
                if (elem != null) {
                    id += elem;
                }
            }
        }
        return id;
    }
}

```

util/Matrix.java

```

package id.ac.itb.stei.informatika.fifteenp.util;
import java.util.ArrayList;
public class Matrix<T> {
    protected final int rows;
    protected final int cols;
    ArrayList<ArrayList<T>> values;
    public Matrix(int rows, int cols) {
        this.rows = rows;
        this.cols = cols;
        this.values = new ArrayList<>(this.rows);
    }
}

```



```

        for (int i = 0; i < this.rows; i++) {
            ArrayList<T> row = new ArrayList<>(this.cols);
            for (int j = 0; j < this.cols; j++) {
                row.add(null);
            }
            this.values.add(row);
        }
    }
    @Override
    public String toString() {
        String res = "";
        for (int i = 0; i < this.rows; i++) {
            for (int j = 0; j < this.cols; j++) {
                if (j != 0) {
                    res += " ";
                }
                res += this.get(i, j).toString();
            }
            res += "\n";
        }
        return res;
    }
    public int rows() {
        return this.rows;
    }
    public int cols() {
        return this.cols;
    }
    public boolean hasIndices(int rowIndex, int colIndex) {
        return rowIndex >= 0 && rowIndex < this.rows &&
            colIndex >= 0 && colIndex < this.cols;
    }
    public void set(int rowIndex, int colIndex, T element) {
        this.values.get(rowIndex).set(colIndex, element);
    }
    public T get(int rowIndex, int colIndex) {
        return this.values.get(rowIndex).get(colIndex);
    }
}

```

util/FifteenMatrixBuilder.java

```

package id.ac.itb.stei.informatika.fifteenp.util;
import java.util.ArrayList;
public class FifteenMatrixBuilder {
    private Integer[] values;
    private int cursor;
    public FifteenMatrixBuilder() {
        this.values = new Integer[16];
        for (int i = 0; i < 16; i++) {
            this.values[i] = null;
        }
        this.cursor = 0;
    }
    public FifteenMatrixBuilder append(Integer value)
        throws IllegalArgumentException, IndexOutOfBoundsException {
        if (value != null && (value < 0 || value > 15)) {
            throw new IllegalArgumentException();
        }
    }
}

```

```

    }
    if (this.cursor > 16) {
        throw new IndexOutOfBoundsException();
    }
    this.values[this.cursor] = value;
    this.cursor++;
    return this;
}
public FifteenMatrix build() throws IllegalArgumentException {
    FifteenMatrix puzzle = new FifteenMatrix();
    ArrayList<Boolean> flags = new ArrayList<>();
    int count = 16;
    while (count-- != 0) {
        flags.add(false);
    }
    for (int i = 0; i < 16; i++) {
        int flagIndex;
        if (this.values[i] == null) {
            flagIndex = 15;
        } else {
            flagIndex = this.values[i] - 1;
        }
        int row = i / 4;
        int col = i % 4;
        puzzle.set(row, col, this.values[i]);
        if (flags.get(flagIndex)) {
            throw new IllegalArgumentException();
        } else {
            flags.set(flagIndex, true);
        }
    }
    return puzzle;
}
}

```

#### util/Direction.java

```

package id.ac.itb.stei.informatika.fifteenp.util;
public enum Direction {
    UP,
    DOWN,
    RIGHT,
    LEFT;
    public static final Direction[] DIRECTIONS = {
        Direction.DOWN,
        Direction.RIGHT,
        Direction.LEFT,
        Direction.UP,
    };
    public Direction flip() {
        return switch (this) {
            case UP -> Direction.DOWN;
            case DOWN -> Direction.UP;
            case RIGHT -> Direction.LEFT;
            case LEFT -> Direction.RIGHT;
        };
    }
}
}

```

io/FileReader.java

```
package id.ac.itb.stei.informatika.fifteenp.io;
import java.io.File;
import java.io.FileNotFoundException;
import java.util.Scanner;
public class FileReader {
    private String result = "";
    public FileReader() {}
    public boolean readFile(String path) {
        this.result = "";
        try {
            File file = new File(path);
            Scanner reader = new Scanner(file);
            if (reader.hasNextLine()) {
                this.result += reader.nextLine();
            }
            while (reader.hasNextLine()) {
                this.result += "\n" + reader.nextLine();
            }
        } catch (FileNotFoundException e) {
            return false;
        }
        return true;
    }
    public String result() {
        return this.result;
    }
}
```

io/FifteenMatrixParser.java

```
package id.ac.itb.stei.informatika.fifteenp.io;
import id.ac.itb.stei.informatika.fifteenp.util.FifteenMatrix;
import id.ac.itb.stei.informatika.fifteenp.util.FifteenMatrixBuilder;
public class FifteenMatrixParser {
    private FifteenMatrix result;
    private FifteenMatrixBuilder builder;
    public FifteenMatrixParser() {
        this.builder = new FifteenMatrixBuilder();
    }
    public void parse(String input) {
        String[] lines = input.split("\n");
        if (lines.length != 4) {
            throw new IllegalArgumentException();
        }
        for (String line: lines) {
            this.parseLine(line);
        }
        this.result = builder.build();
    }
    public FifteenMatrix result() {
        return this.result;
    }
    private void parseLine(String line) throws IllegalArgumentException {
        String[] values = line.split(" ");
    }
}
```

```

        if (values.length != 4) {
            throw new IllegalArgumentException();
        }
        for (String value: values) {
            this.parseValue(value);
        }
    }
    private void parseValue(String valueString) throws IllegalArgumentException {
        if (valueString.equals("")) {
            throw new IllegalArgumentException();
        }

        if (valueString.equals("-")) {
            builder.append(null);
        } else {
            Integer value = Integer.valueOf(valueString);
            builder.append(value);
        }
    }
}

```

### Instansi-instansi persoalan 15-puzzle untuk pengujian

Berikut adalah instansi-instansi 15-puzzle yang digunakan dalam melakukan pengujian dalam bentuk file teks dan gambar.

(Catatan : Seluruh file teks yang digunakan dalam melakukan pengujian dapat dilihat pada [laman test.](#))

13.txt

```

1 2 3 4
5 10 6 8
13 11 9 12
14 - 7 15

```

1	2	3	4
5	10	6	8
13	11	9	12
14		7	15

Gambar 1. Instansi 15-puzzle yang direpresentasikan file 13.txt.

16.txt

```

2 - 3 4
1 5 6 8

```

```
14 13 7 12
9 11 10 15
```

2		3	4
1	5	6	8
14	13	7	12
9	11	10	15

Gambar 2. Instansi 15-puzzle yang direpresentasikan file 16.txt.

20.txt

```
2 - 3 4
1 5 6 8
9 14 12 15
10 13 11 7
```

2		3	4
1	5	6	8
9	14	12	15
10	13	11	7

Gambar 3. Instansi 15-puzzle yang direpresentasikan file 20.txt.

25.txt

```
2 3 15 4
1 5 8 -
13 7 6 14
10 9 12 11
```

2	3	15	4
1	5	8	
13	7	6	14
10	9	12	11

Gambar 4. Instansi 15-puzzle yang direpresentasikan file 25.txt.

26.txt

```
2 3 7 4
10 15 6 8
9 - 14 11
1 5 13 12
```

2	3	7	4
10	15	6	8
9		14	11
1	5	13	12

Gambar 5. Instansi 15-puzzle yang direpresentasikan file 26.txt.

unsolvable1.txt

```
6 2 15 3
1 9 5 4
10 14 - 7
13 12 8 11
```

6	2	15	3
1	9	5	4
10	14		7
13	12	8	11

Gambar 6. Instansi 15-puzzle yang direpresentasikan file unsolvable1.txt.

unsolvable2.txt

```
2 5 4 7
1 11 15 3
9 6 - 8
13 10 12 14
```

2	5	4	7
1	11	15	3
9	6		8
13	10	12	14

Gambar 7. Instansi 15-puzzle yang direpresentasikan file unsolvable2.txt.

### Contoh penggunaan aplikasi

Aplikasi tersebut telah diuji dengan 9 kasus uji, yaitu:

- 1) 5 kasus uji dari file masukan dengan masing-masing memiliki solusi pada kedalaman 13, 16, 20, 25, dan 26;
- 2) 1 kasus uji acak '*randomized*' dengan puzzle yang dibangkitkan '*generated*' oleh aplikasi;
- 3) 2 kasus uji *puzzle* yang tidak memiliki solusi; dan
- 4) 1 kasus uji matriks *puzzle* yang tidak valid.

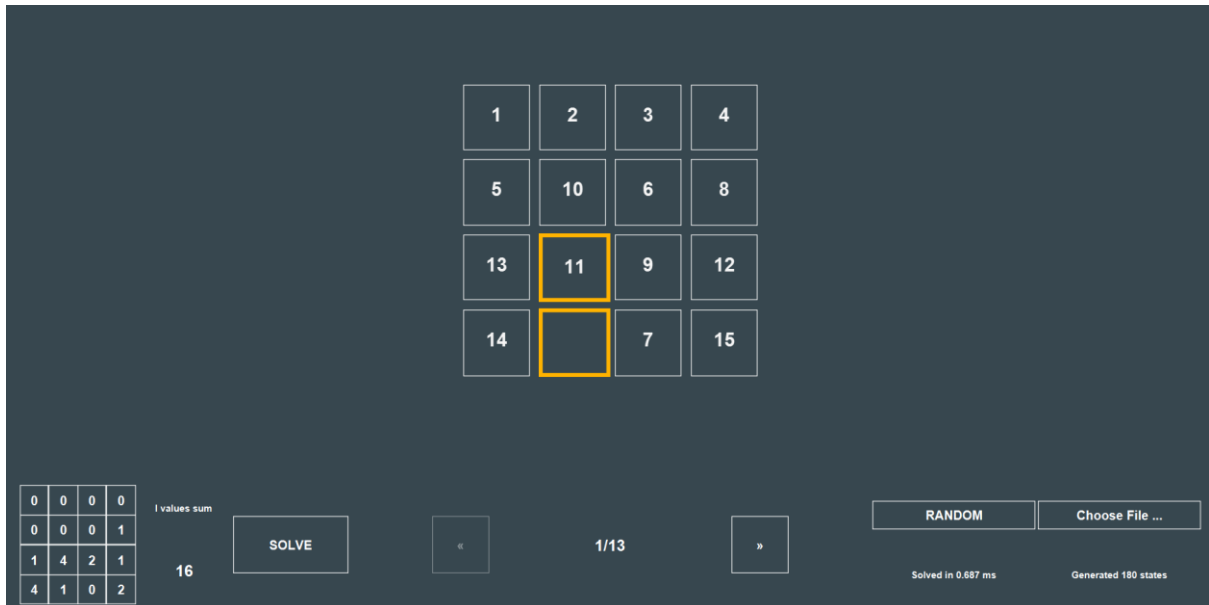
Berikut adalah *input* dan *output* dalam menjalankan aplikasi.

(Catatan : Seluruh tampilan matriks keluaran aplikasi dalam melakukan pengujian dapat dilihat pada laman [demo](#) dalam bentuk animasi GIF.)

1. *Puzzle yang solvable* dengan kedalaman solusi 13

1	2	3	4
5	10	6	8
13	11	9	12
14	-	7	15

Gambar 8. *Input puzzle yang solvable* dengan kedalaman solusi 13.



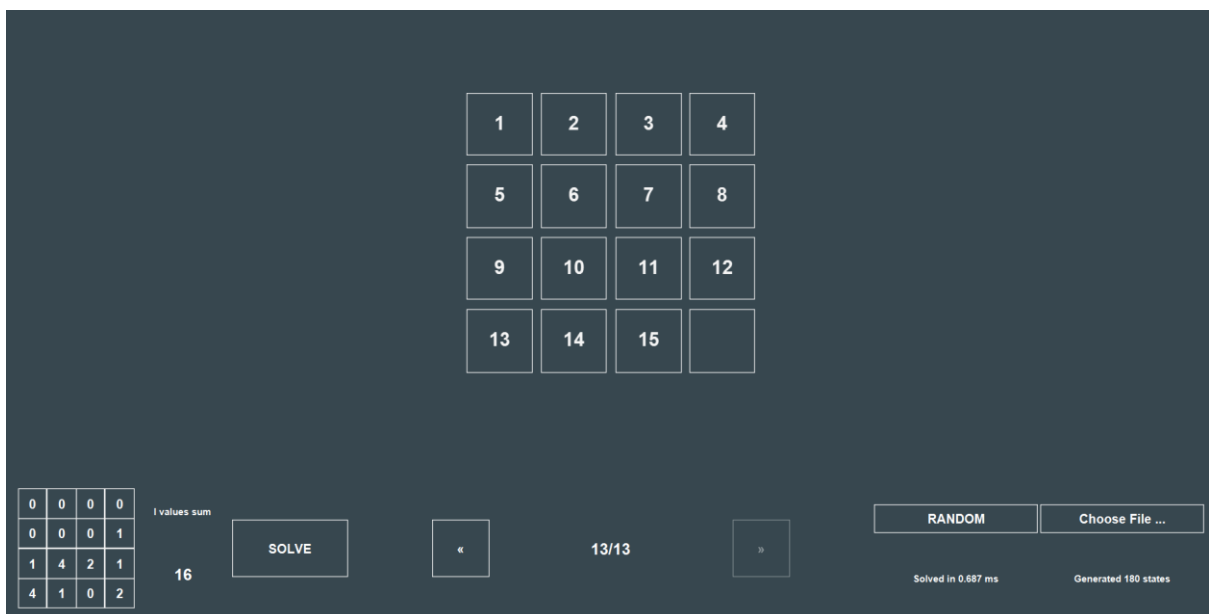
Gambar 9. *State awal puzzle yang solvable* dengan kedalaman solusi 13.







Gambar 10. *State* antara *puzzle* yang *solvable* dengan kedalaman solusi 13.



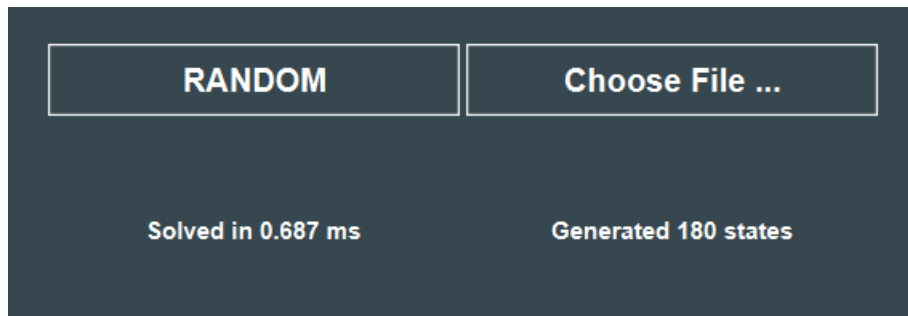
Gambar 11. *State* akhir *puzzle* yang *solvable* dengan kedalaman solusi 13.

0	0	0	0
0	0	0	1
1	4	2	1
4	1	0	2

I values sum

16

Gambar 12. *Output* nilai  $l$  untuk setiap elemen matriks dan jumlah seluruh nilai  $l$  dengan nilai  $x$ .

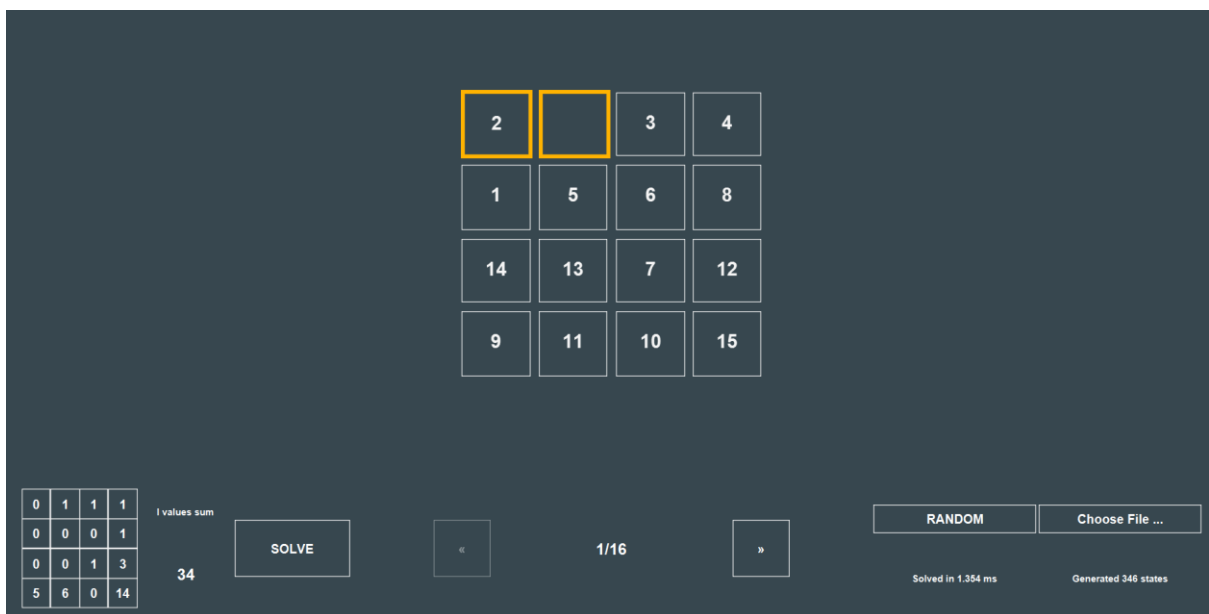


Gambar 13. *Output* lama eksekusi dan banyak *state* yang dibangkitkan pada *puzzle*.

## 2. *Puzzle* yang *solvable* dengan kedalaman solusi 16

1	2	-	3	4
2	1	5	6	8
3	14	13	7	12
4	9	11	10	15

Gambar 14. *Input puzzle* yang *solvable* dengan kedalaman solusi 16.



Gambar 15. *State awal puzzle* yang *solvable* dengan kedalaman solusi 16.

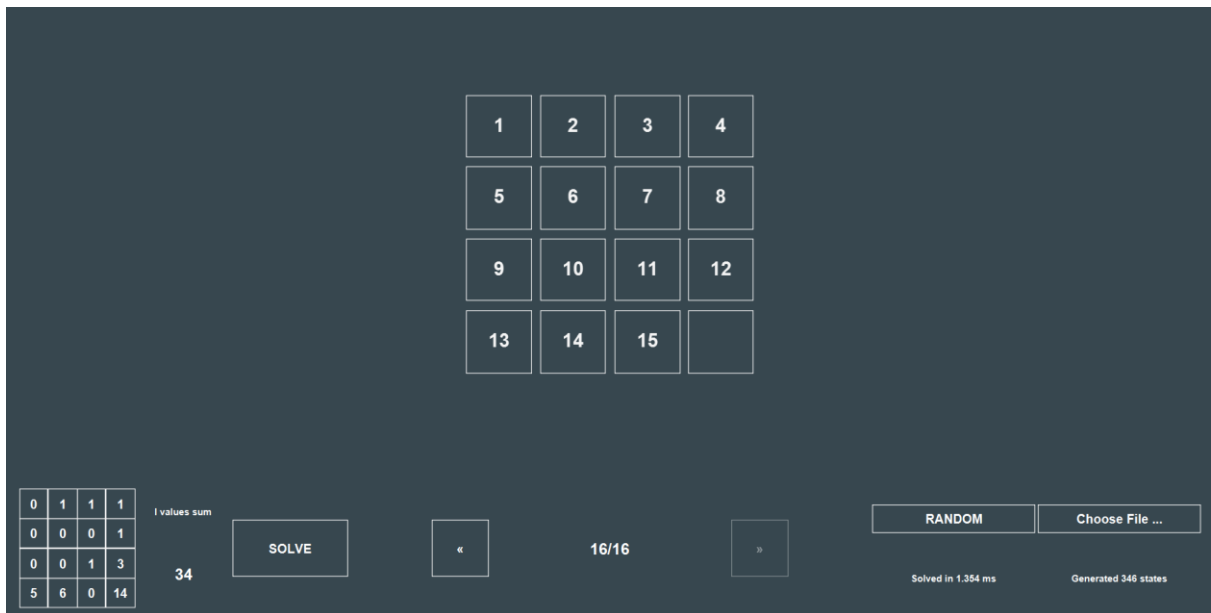
	2	3	4
1	5	6	8
14	13	7	12
9	11	10	15

1	2	3	4
	5	6	8
14	13	7	12
9	11	10	15

1	2	3	4
5		6	8
14	13	7	12
9	11	10	15



Gambar 16. *State* antara *puzzle* yang *solvable* dengan kedalaman solusi 16.

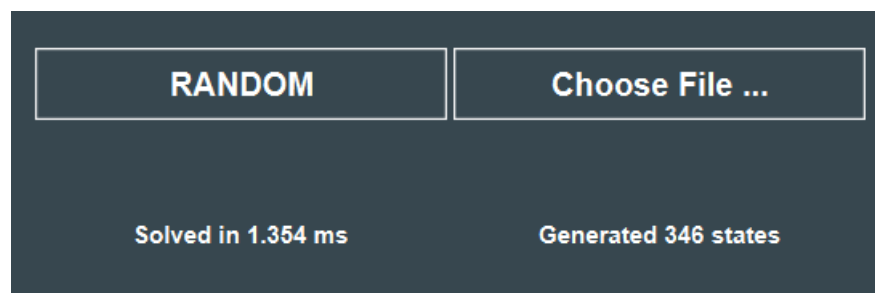


Gambar 17. State akhir puzzle yang solvable dengan kedalaman solusi 16.

0	1	1	1
0	0	0	1
0	0	1	3
5	6	0	14

l values sum  
  
34

Gambar 18. Output nilai  $l$  untuk setiap elemen matriks dan jumlah seluruh nilai  $l$  dengan nilai  $x$ .

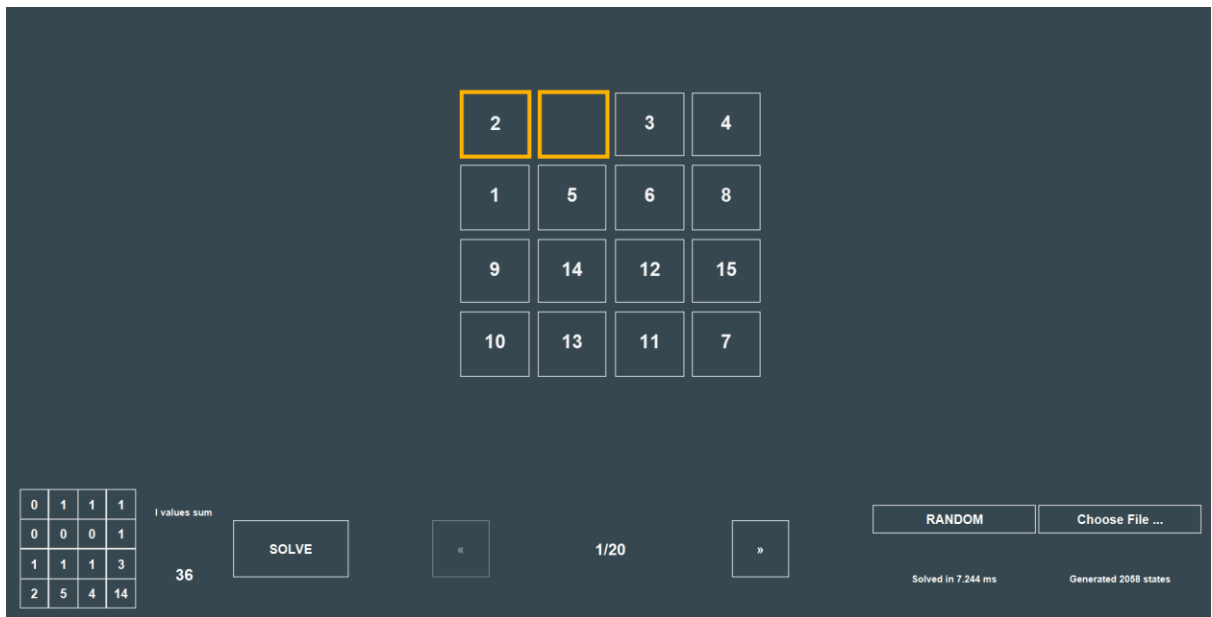


Gambar 19. Output lama eksekusi dan banyak state yang dibangkitkan pada puzzle.

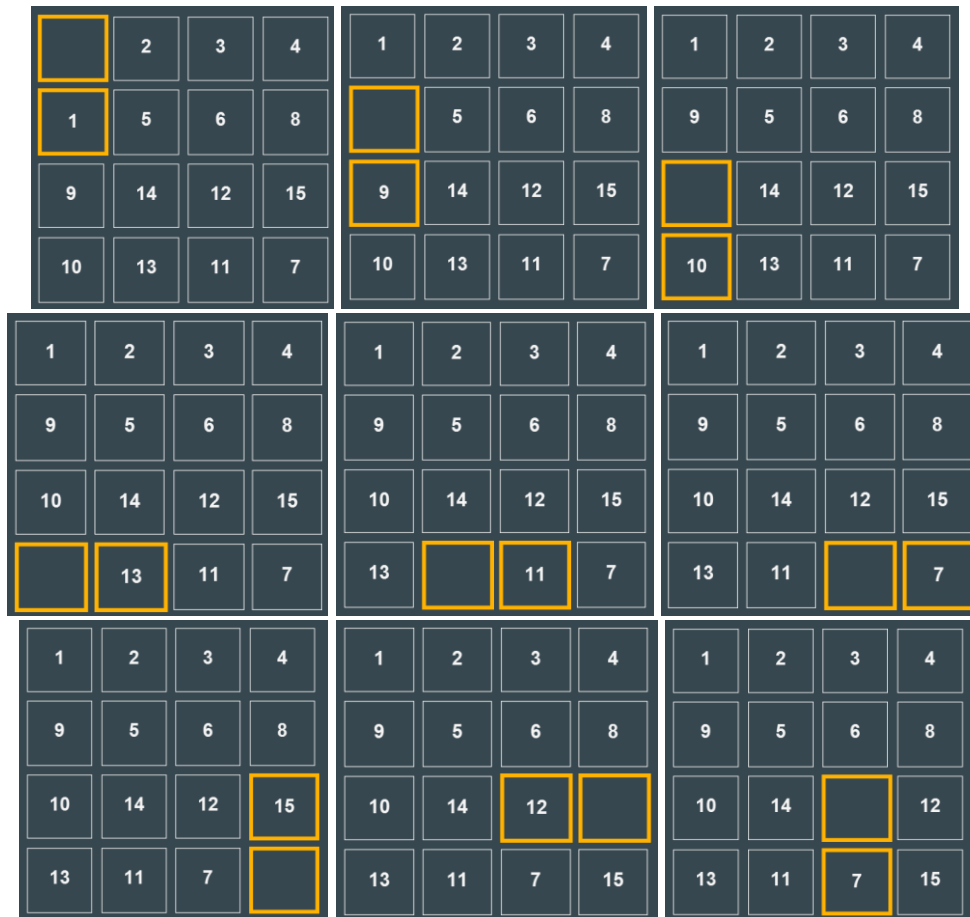
### 3. Puzzle yang solvable dengan kedalaman solusi 20

1	2	3	4
2	1	5	6
3	9	14	12
4	10	13	11

Gambar 20. Input puzzle yang solvable dengan kedalaman solusi 20.

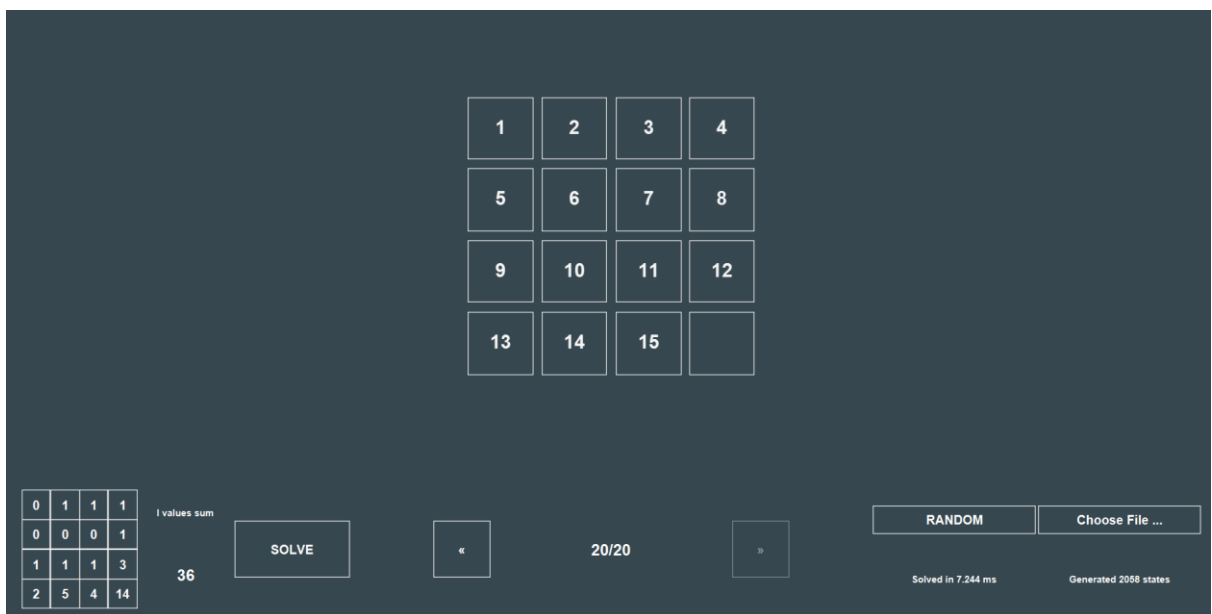


Gambar 21. *State awal puzzle yang solvable dengan kedalaman solusi 20.*





Gambar 22. *State* antara *puzzle* yang *solvable* dengan kedalaman solusi 20.



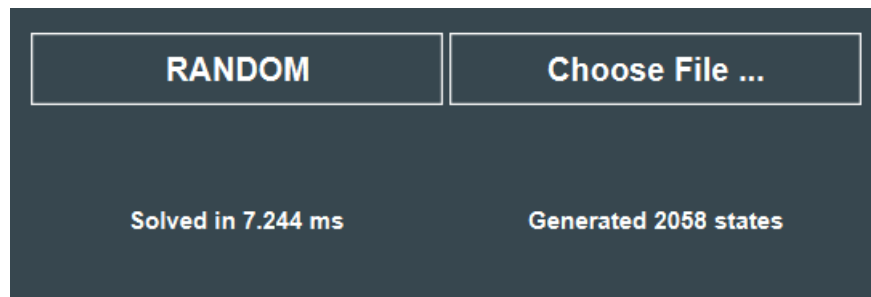
Gambar 23. *State* akhir *puzzle* yang *solvable* dengan kedalaman solusi 20.

0	1	1	1
0	0	0	1
1	1	1	3
2	5	4	14

I values sum

36

Gambar 24. *Output* nilai  $l$  untuk setiap elemen matriks dan jumlah seluruh nilai  $l$  dengan nilai  $x$ .

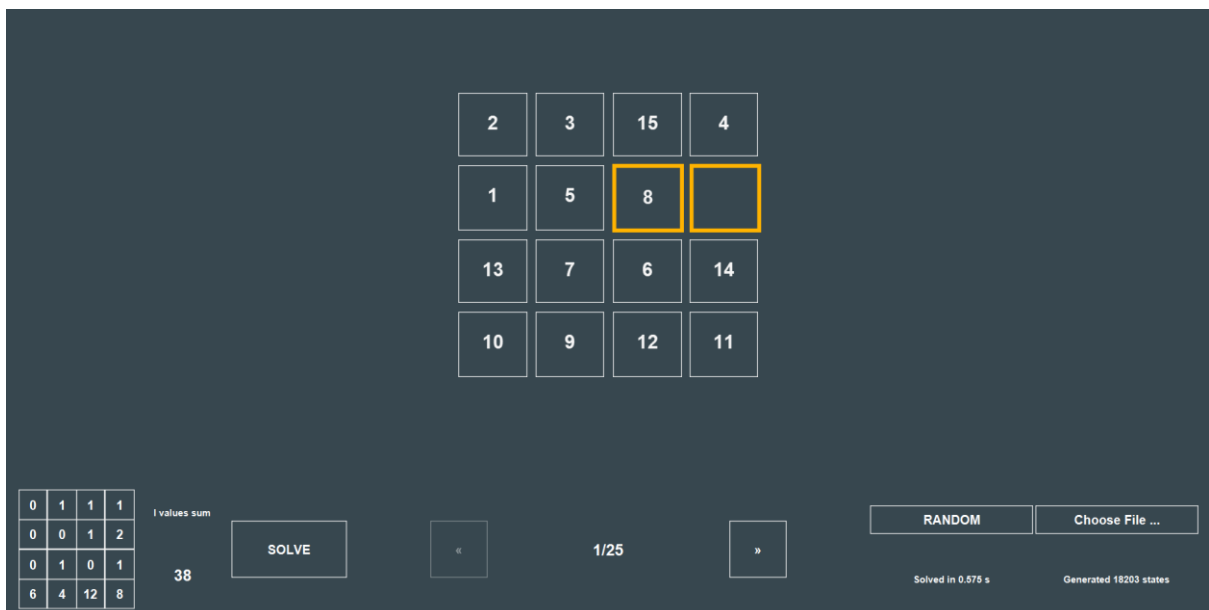


Gambar 25. *Output* lama eksekusi dan banyak *state* yang dibangkitkan pada *puzzle*.

#### 4. *Puzzle* yang *solvable* dengan kedalaman solusi 25

1	2	3	15	4
2	1	5	8	-
3	13	7	6	14
4	10	9	12	11

Gambar 26. *Input puzzle* yang *solvable* dengan kedalaman solusi 25.



Gambar 27. *State* awal *puzzle* yang *solvable* dengan kedalaman solusi 25.

2	3	15	4
1	5		8
13	7	6	14
10	9	12	11

2	3		4
1	5	15	8
13	7	6	14
10	9	12	11

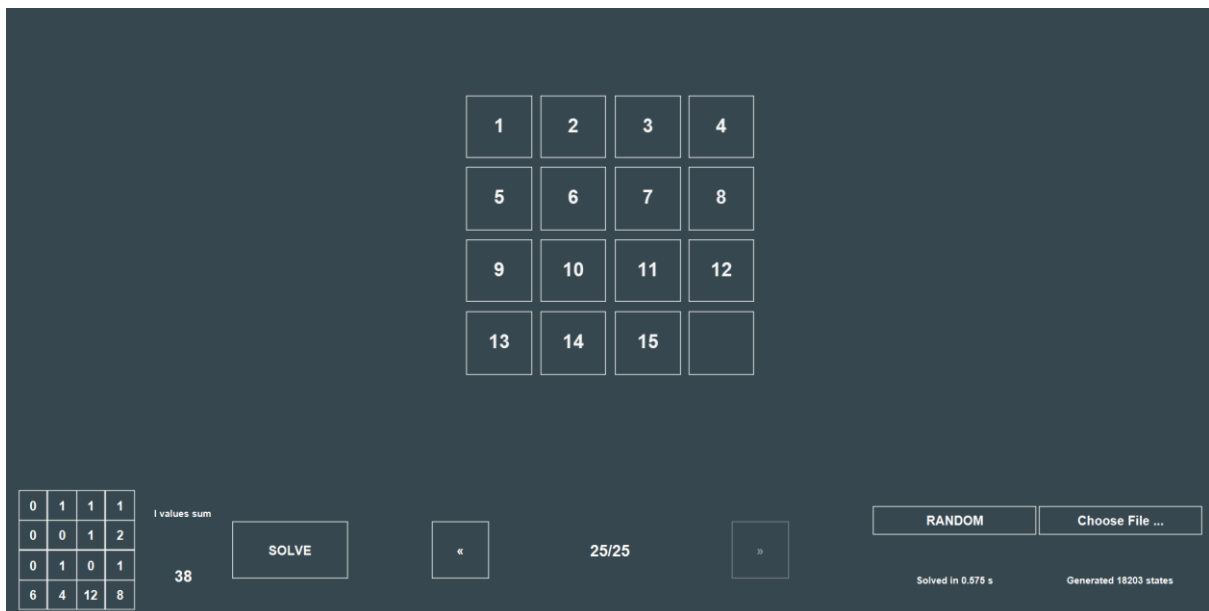
2		3	4
1	5	15	8
13	7	6	14
10	9	12	11

| |    |   |    |    | |----|---|----|----| |    | 2 | 3  | 4  | | 1  | 5 | 15 | 8  | | 13 | 7 | 6  | 14 | | 10 | 9 | 12 | 11 |  |    |   |    |    | |----|---|----|----| | 1  | 2 | 3  | 4  | |    | 5 | 15 | 8  | | 13 | 7 | 6  | 14 | | 10 | 9 | 12 | 11 |  |    |   |    |    | |----|---|----|----| | 1  | 2 | 3  | 4  | | 5  |   | 15 | 8  | | 13 | 7 | 6  | 14 | | 10 | 9 | 12 | 11 | | |    |   |    |    | |----|---|----|----| | 1  | 2 | 3  | 4  | | 5  | 7 | 15 | 8  | | 13 |   | 6  | 14 | | 10 | 9 | 12 | 11 |  |    |   |    |    | |----|---|----|----| | 1  | 2 | 3  | 4  | | 5  | 7 | 15 | 8  | | 13 | 6 |    | 14 | | 10 | 9 | 12 | 11 |  |    |   |    |    | |----|---|----|----| | 1  | 2 | 3  | 4  | | 5  | 7 | 15 | 8  | | 13 | 6 | 14 |    | | 10 | 9 | 12 | 11 | | |    |   |    |    | |----|---|----|----| | 1  | 2 | 3  | 4  | | 5  | 7 | 15 | 8  | | 13 | 6 | 14 | 11 | | 10 | 9 | 12 |    |  |    |   |    |    | |----|---|----|----| | 1  | 2 | 3  | 4  | | 5  | 7 | 15 | 8  | | 13 | 6 | 14 | 11 | | 10 | 9 |    | 12 |  |    |   |    |    | |----|---|----|----| | 1  | 2 | 3  | 4  | | 5  | 7 | 15 | 8  | | 13 | 6 |    | 11 | | 10 | 9 | 14 | 12 | | |    |   |    |    | |----|---|----|----| | 1  | 2 | 3  | 4  | | 5  | 7 |    | 8  | | 13 | 6 | 15 | 11 | | 10 | 9 | 14 | 12 |  |    |   |    |    | |----|---|----|----| | 1  | 2 | 3  | 4  | | 5  |   | 7  | 8  | | 13 | 6 | 15 | 11 | | 10 | 9 | 14 | 12 |  |    |   |    |    | |----|---|----|----| | 1  | 2 | 3  | 4  | | 5  | 6 | 7  | 8  | | 13 |   | 15 | 11 | | 10 | 9 | 14 | 12 | | |    |   |    |    | |----|---|----|----| | 1  | 2 | 3  | 4  | | 5  | 6 | 7  | 8  | | 13 | 9 | 15 | 11 | | 10 |   | 14 | 12 |  |    |    |    |    | |----|----|----|----| | 1  | 2  | 3  | 4  | | 5  | 6  | 7  | 8  | | 13 | 9  | 15 | 11 | |    | 10 | 14 | 12 |  |    |    |    |    | |----|----|----|----| | 1  | 2  | 3  | 4  | | 5  | 6  | 7  | 8  | |    | 9  | 15 | 11 | | 13 | 10 | 14 | 12 | |

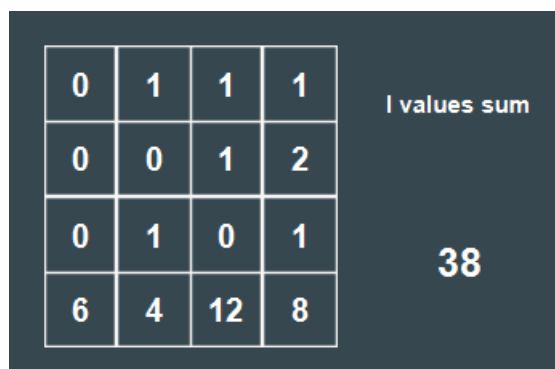




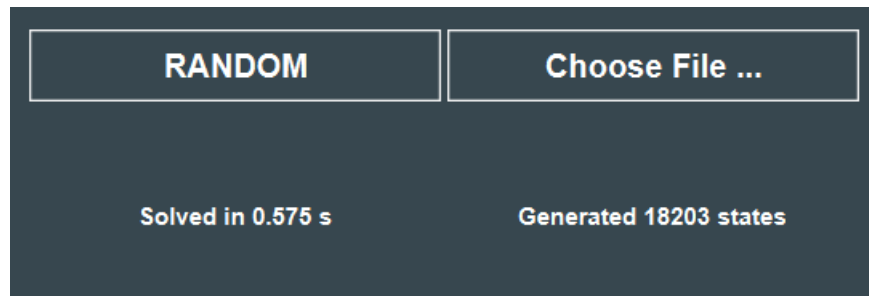
Gambar 28. *State* antara *puzzle* yang *solvable* dengan kedalaman solusi 25.



Gambar 29. *State* akhir *puzzle* yang *solvable* dengan kedalaman solusi 25.



Gambar 30. *Output* nilai  $l$  untuk setiap elemen matriks dan jumlah seluruh nilai  $l$  dengan nilai  $x$ .

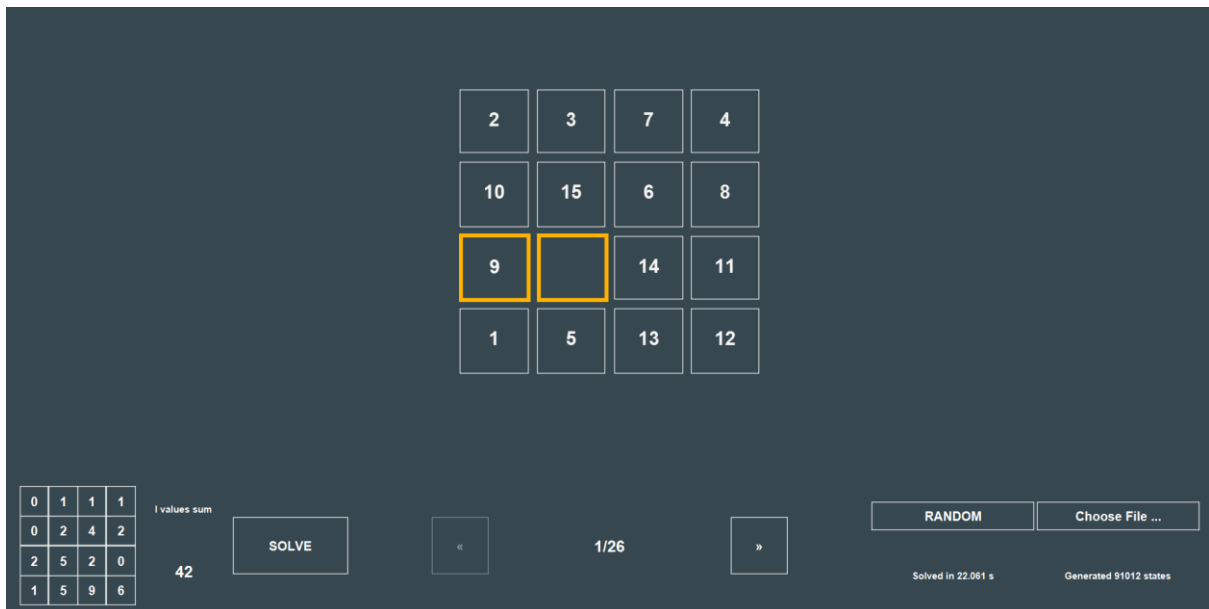


Gambar 31. *Output* lama eksekusi dan banyak *state* yang dibangkitkan pada *puzzle*.

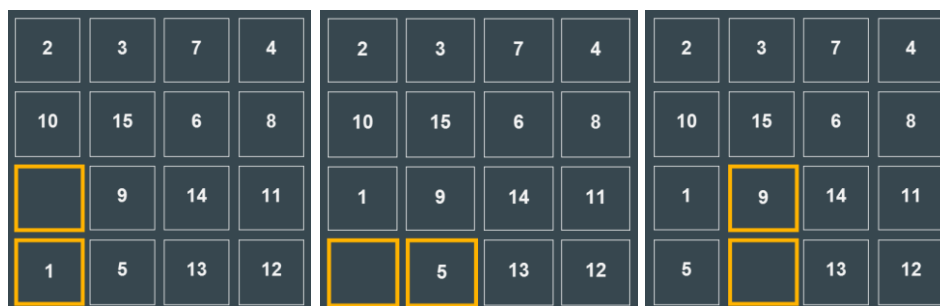
##### 5. *Puzzle* yang *solvable* dengan kedalaman solusi 26

1	2	3	7	4
2	10	15	6	8
3	9	-	14	11
4	1	5	13	12

Gambar 32. *Input puzzle* yang *solvable* dengan kedalaman solusi 26.



Gambar 33. *State awal puzzle* yang *solvable* dengan kedalaman solusi 26.



2	3	7	4
10	15	6	8
1		14	11
5	9	13	12

2	3	7	4
10		6	8
1	15	14	11
5	9	13	12

2	3	7	4
	10	6	8
1	15	14	11
5	9	13	12

2	3	7	4
1	10	6	8
	15	14	11
5	9	13	12

2	3	7	4
1	10	6	8
5	15	14	11
	9	13	12

2	3	7	4
1	10	6	8
5	15	14	11
9		13	12

2	3	7	4
1	10	6	8
5	15	14	11
9	13		12

2	3	7	4
1	10	6	8
5	15		11
9	13	14	12

2	3	7	4
1	10	6	8
5		15	11
9	13	14	12

2	3	7	4
1		6	8
5	10	15	11
9	13	14	12

2	3	7	4
1	6		8
5	10	15	11
9	13	14	12

2	3		4
1	6	7	8
5	10	15	11
9	13	14	12

2		3	4
1	6	7	8
5	10	15	11
9	13	14	12

	2	3	4
1	6	7	8
5	10	15	11
9	13	14	12

1	2	3	4
	6	7	8
5	10	15	11
9	13	14	12

1	2	3	4
5	6	7	8
	10	15	11
9	13	14	12

1	2	3	4
5	6	7	8
9	10	15	11
	13	14	12

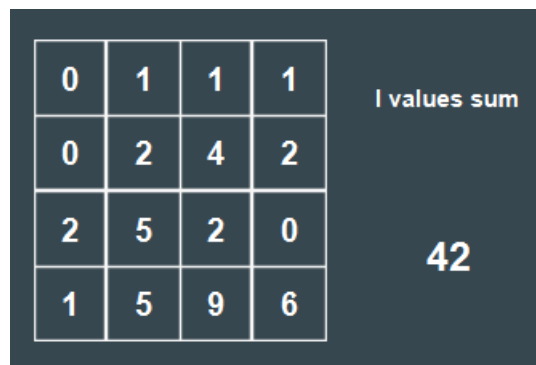
1	2	3	4
5	6	7	8
9	10	15	11
13		14	12



Gambar 34. *State* antara *puzzle* yang *solvable* dengan kedalaman solusi 26.



Gambar 35. *State* akhir *puzzle* yang *solvable* dengan kedalaman solusi 26.

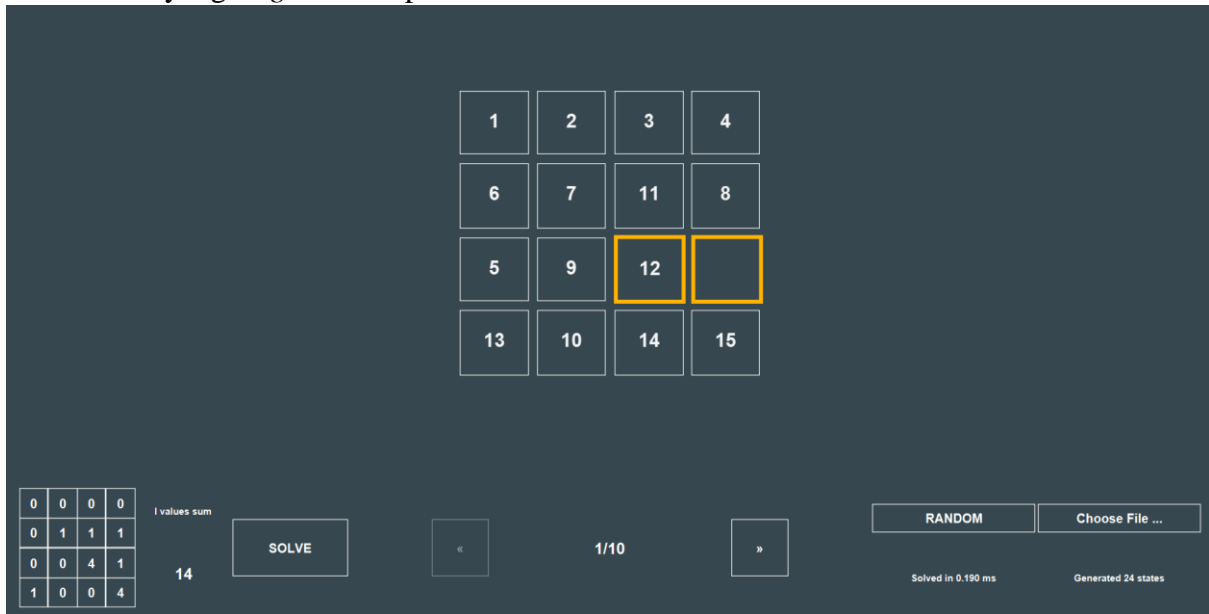


Gambar 36. *Output* nilai  $l$  untuk setiap elemen matriks dan jumlah seluruh nilai  $l$  dengan nilai  $x$ .

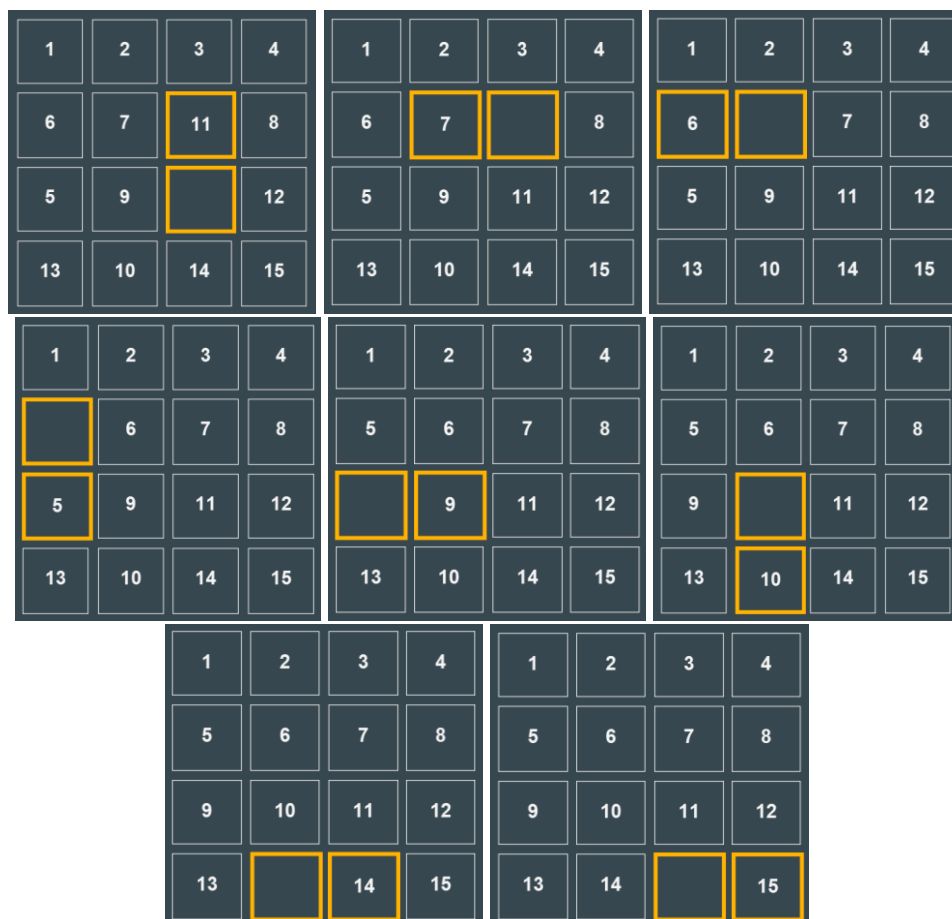


Gambar 37. *Output* lama eksekusi dan banyak *state* yang dibangkitkan pada *puzzle*.

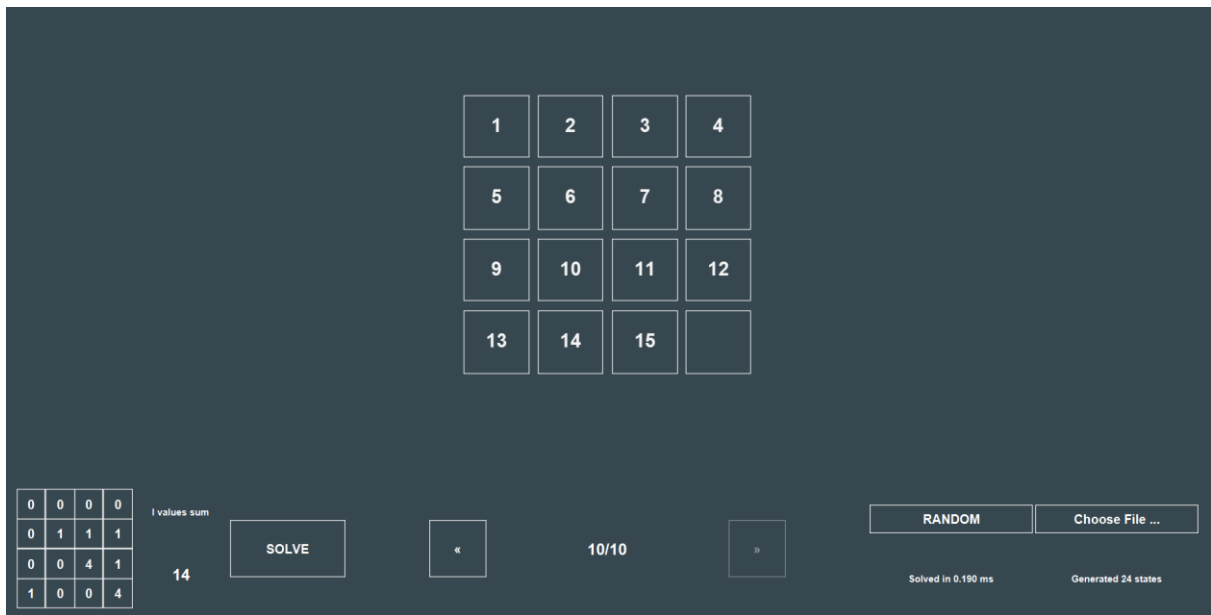
6. *Puzzle* acak yang di-generate aplikasi



Gambar 38. *State awal puzzle acak.*



Gambar 39. *State antara puzzle acak.*

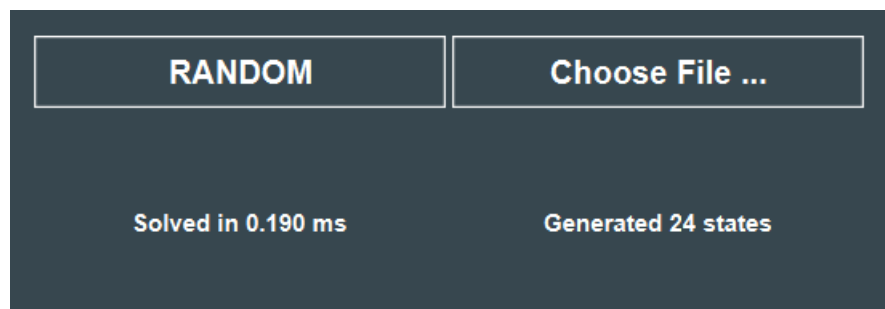


Gambar 40. *State* akhir *puzzle* acak.

0	0	0	0
0	1	1	1
0	0	4	1
1	0	0	4

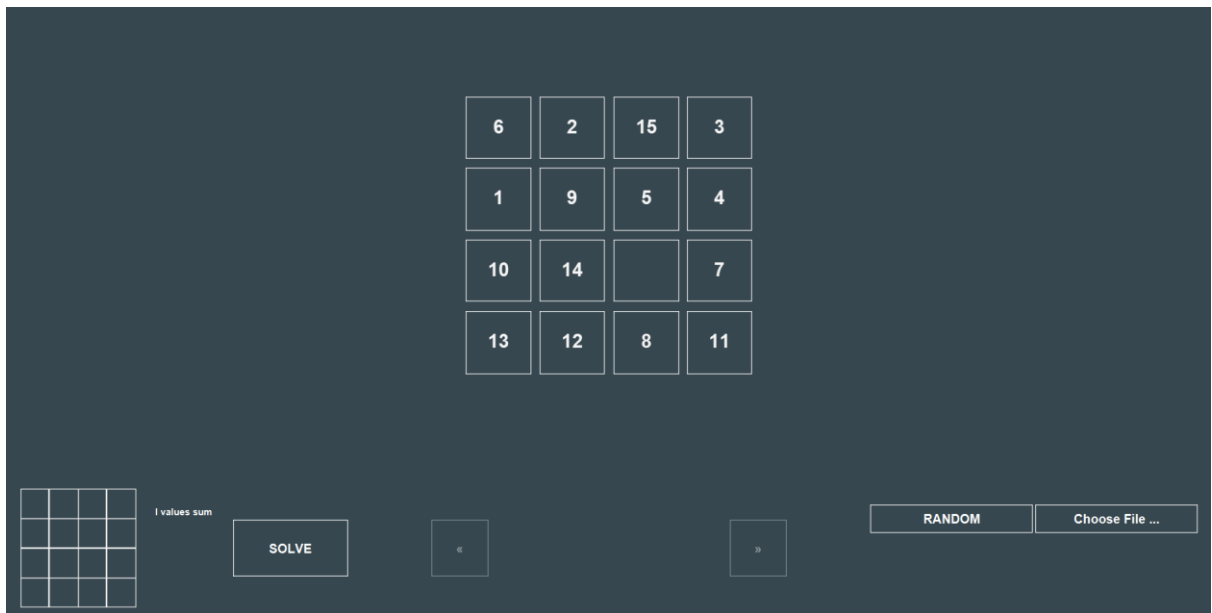
l values sum  
  
14

Gambar 41. *Output* nilai  $l$  untuk setiap elemen matriks dan jumlah seluruh nilai  $l$  dengan nilai  $x$ .



Gambar 42. *Output* lama eksekusi dan banyak *state* yang dibangkitkan pada *puzzle*.

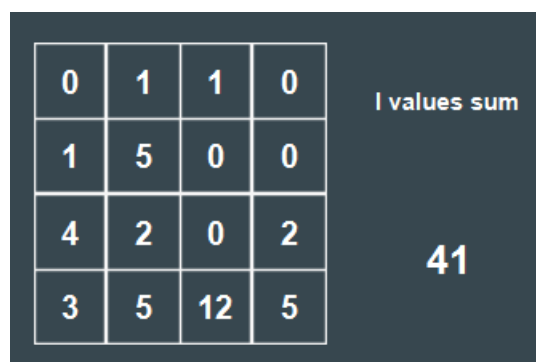
7. *Puzzle* yang tidak memiliki solusi 1



Gambar 43. *Input puzzle yang unsolvable 1.*

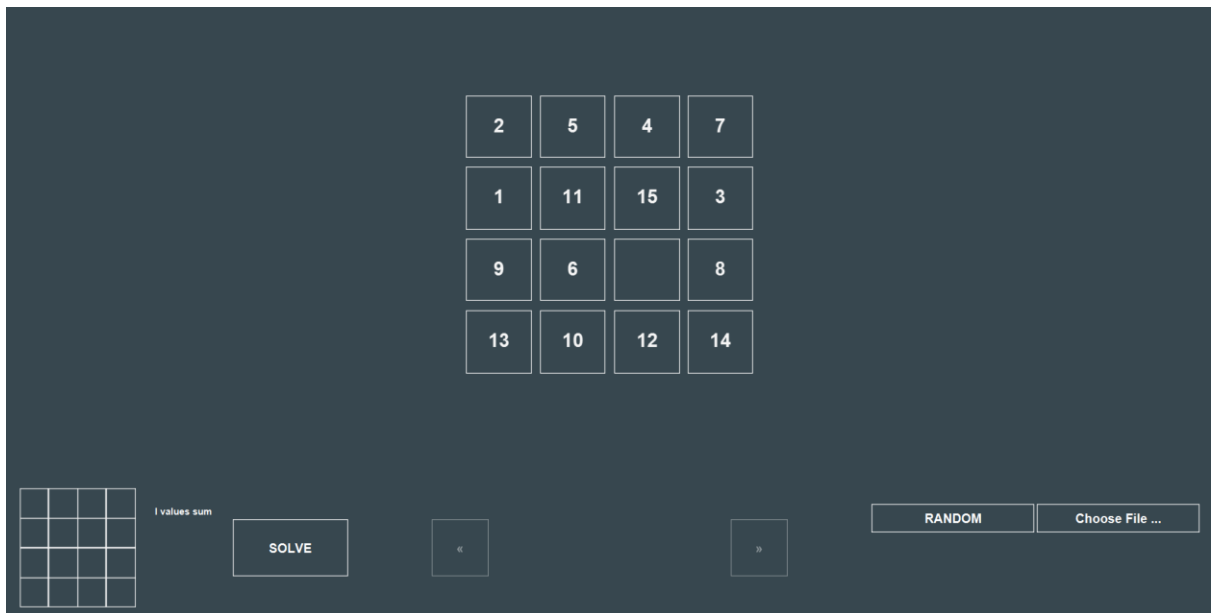


Gambar 44. *Output puzzle yang unsolvable 1.*

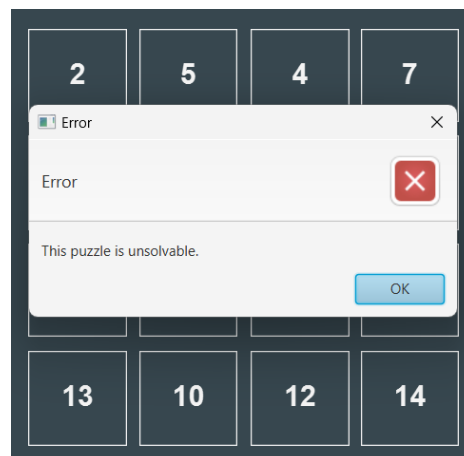


Gambar 45. *Output nilai  $l$  untuk setiap elemen matriks dan jumlah seluruh nilai  $l$  dengan nilai  $x$ .*

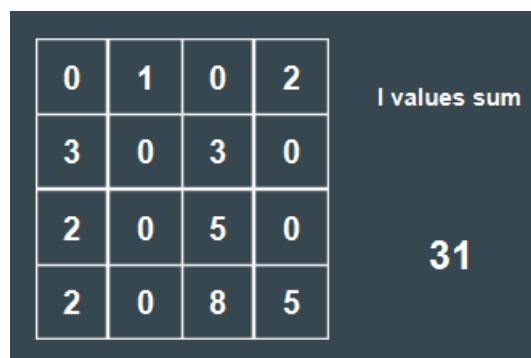
## 8. *Puzzle yang tidak memiliki solusi 2*



Gambar 46. *Input puzzle yang unsolvable 2.*



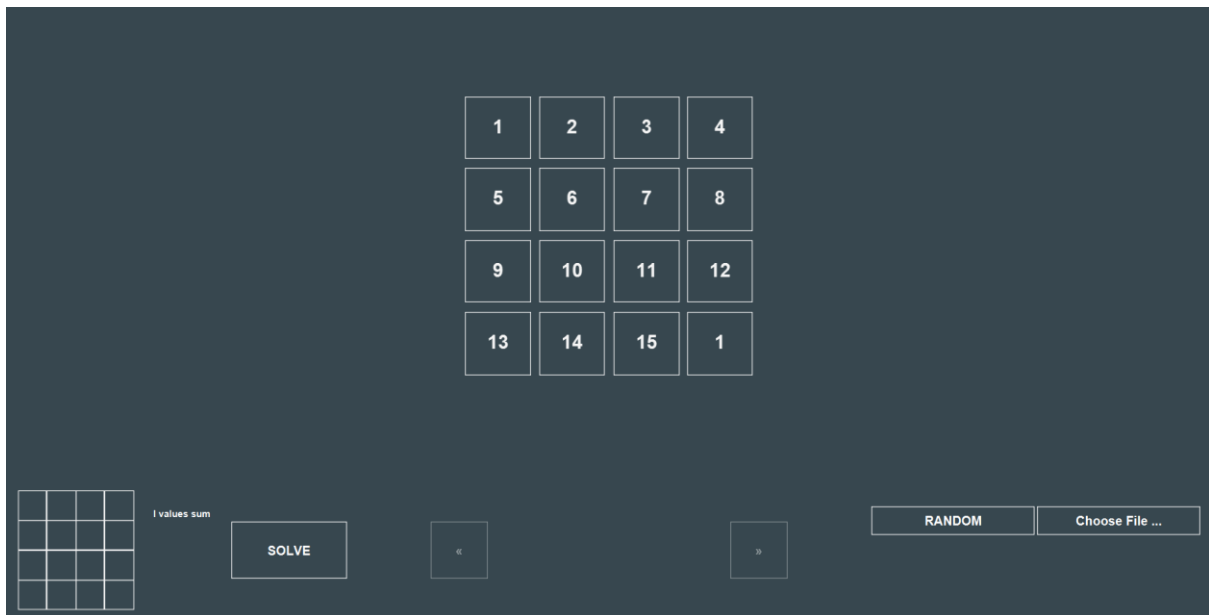
Gambar 47. *Output puzzle yang unsolvable 2.*



Gambar 48. *Output nilai l untuk setiap elemen matriks dan jumlah seluruh nilai l dengan nilai x.*

## 9. Matriks *puzzle* yang tidak valid





Gambar 49. *Input puzzle* yang tidak valid.



Gambar 50. *Output puzzle* yang tidak valid.

### Alamat *repository source code* aplikasi

*Source code* aplikasi yang telah dibuat dapat diakses melalui [tautan ini](#).

### *Check list*

Poin	Ya	Tidak
1. Program berhasil dikompilasi	✓	
2. Program berhasil <i>running</i>	✓	
3. Program dapat menerima <i>input</i> dan menuliskan <i>output</i> .	✓	
4. Luaran sudah benar untuk semua data uji	✓	

5. Bonus dibuat	✓	
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