Math 1700 - Project 2

Introduction

This project explores the use of Series in safe and effective dosage of drugs calculations. When a single dose of drug is administered, the drug concentration in the bloodstream can be modeled as a series. The dosage is lost slowly over time, and the concentration can be modeled as a series of decreasing terms.

Model for Effect of Repeated Doses

$$R_n = C_0 e^{-kt_0} + C_0 e^{-2kt_0} + \ldots + C_0 e^{-nkt_0}$$

Where:

- C_0 is the change in concentration achievable by a single dose (mg/mL)
- k is the elimination constant (h^{-1})
- t_0 is the time between doses (h)

The graph below shows the concentration of the drug in the bloodstream over time. The x-axis represents time in hours, and the y-axis represents the concentration of the drug in mg/mL. The graph shows how the concentration of the drug decreases over time after each dose.

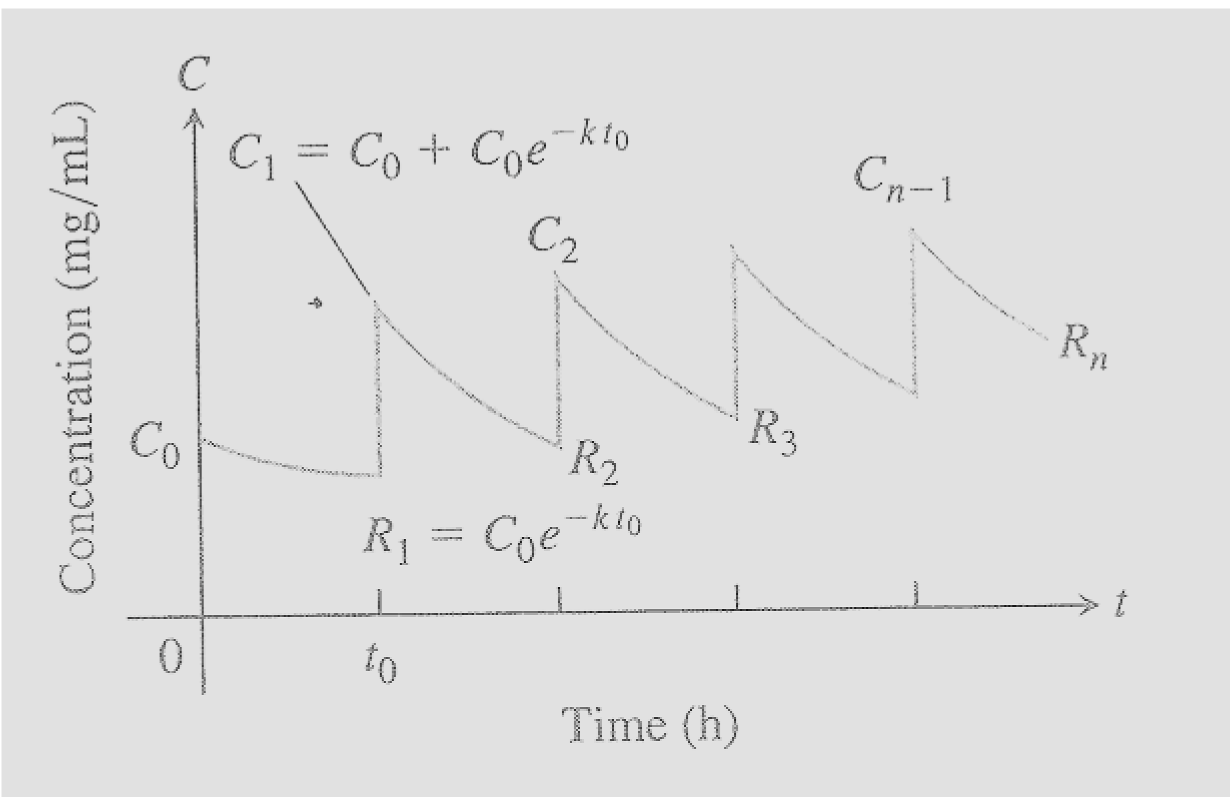

Figure 1: Concentration VS Time Graph

Questions to Answer

A) **Write R_n in closed form as single fraction, and find** $R = \lim_{n\to\infty} R_n$

The given expression for R_n is:

$$R_n = C_0e^{-kt_0} + C_0e^{-2kt_0} + \ldots + C_0e^{-nkt_0}$$

This expression can be simplified using by taking the common factor C_0 out of the summation:

$$R_n = C_0\left(e^{-kt_0} + e^{-2kt_0} + \ldots + e^{-nkt_0}\right)$$

For simplicity, let $x = e^{-kt_0}$. Then we can rewrite the expression as:

$$R_n = C_0\left(x + x^2 + \ldots + x^n\right)$$

The terms in the parentheses form a geometric series with first term x and common ratio (i.e. r) x.

The sum of a geometric series can be calculated using the formula:

$$S_n = \frac{a(1-r^n)}{1-r}$$

Where:

- $a = x = e^{-kt_0}$ is the first term
- $r = x = e^{-kt_0}$ is the common ratio
- n is the number of terms.

From here, we can start substituting our values into the formula:

$$R_n = C_0\left(\frac{x(1-x^n)}{1-x}\right)$$

Now, substituting back for x:

$$R_n = C_0\left(\frac{e^{-kt_0}(1-e^{-nkt_0})}{1-e^{-kt_0}}\right)$$

Or you can write it as a single fraction:

$$R_n = \frac{C_0e^{-kt_0}(1-e^{-nkt_0})}{1-e^{-kt_0}}$$

That is the closed form of R_n.

Now, we need to find the limit of R_n as $n \to \infty$:

$$R = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{C_0e^{-kt_0}(1-e^{-nkt_0})}{1-e^{-kt_0}}$$

Sense $k > 0$ and $t_0 > 0$, the term e^{-kt_0} is a positive number. Meaning it will be in the following interval: $0 < e^{-kt_0} < 1$.

As $n \to \infty$, the term $e^{-nkt_0} \to 0$, because it's a number between 0 and 1 raised to a large positive power, meaning it gets smaller the larger n becomes.

Therefore, we can conclude that:

$$\lim_{n \to \infty} e^{-nkt_0} = 0$$

Sense we've established that, we can now substitute it into our limit:

$$R = \frac{C_0 e^{-kt_0}(1-0)}{1 - e^{-kt_0}}$$

$$R = \frac{C_0 e^{-kt_0}}{1 - e^{-kt_0}}$$

In this case, R represents the total concentration of the drug in the bloodstream after many doses.

B) **Calculate R_1 and R_10 for $C_0 = 1(mg/mL)$, $k = 0.1h^{-1}$, and $t_0 = 10h$. How good an estimate of R is R_10?**

What we are given:

- $C_0 = 1(mg/mL)$
- $k = 0.1h^{-1}$
- $t_0 = 10h$

We can simplify some of our solving for some of the variables:

- $-kt_0 = -0.1(10) = -1$
- $e^{-kt_0} = e^{-1} \approx 0.36788$

To calculate R_1, we can use the fact that R_1 is just the first term of the series:

$$R_1 = C_0 e^{-kt_0} = 1 \cdot e^{-1} \approx 0.36788\,(mg/mL)$$

We could have also just used the closed form of R_n to calculate R_1:

$$R_1 = \frac{C_0 e^{-kt_0}(1 - e^{-nkt_0})}{1 - e^{-kt_0}} = \frac{1 \cdot e^{-1}(1 - e^{-1})}{1 - e^{-1}} = 0.36788\,(mg/mL)$$

This shows that the closed form of R_n is correct.

To calculate R_{10}, we can use the closed form of R_n:

$$R_{10} = \frac{C_0 e^{-kt_0}(1 - e^{-nkt_0})}{1 - e^{-kt_0}}$$

$$R_{10} = \frac{1 \cdot e^{-1}(1 - e^{-10})}{1 - e^{-1}}$$

$$R_{10} = \frac{1 \cdot 0.36788(1 - 0.0000454)}{1 - 0.36788}$$

$$R_{10} = \frac{0.36788 \cdot 0.99995}{0.63212} \approx 0.58195\,(mg/mL)$$

Therefore, $R_{10} \approx 0.58195\,(mg/mL)$.

This is a good estimate of R, because the difference between R and R_{10} is small.

To check the difference between R and R_{10}, we can calculate R using the R formula we derived in part A:

$$R = \frac{C_0 e^{-kt_0}}{1 - e^{-kt_0}} = \frac{1 \cdot e^{-1}}{1 - e^{-1}} = \frac{0.36788}{1 - 0.36788}$$

$$R = \frac{0.36788}{0.63212} \approx 0.58198\,(mg/mL)$$

The difference between R and R_{10} is:

$$R - R_{10} = 0.58198 - 0.58195 \approx 0.00003\,(mg/mL)$$

This is a very small difference, which shows that R_{10} is a good estimate of R.

Percentage difference is calculated as:

$$\frac{R - R_{10}}{R} \cdot 100 = \frac{0.58198 - 0.58195}{0.58198} \cdot 100 \approx 0.0052\%$$

Conclusion

Using the calculations above, I was able to conclude that the difference between R and R_{10} is very small, which means that R_{10} is an extremely good estimate of R. Meaning that after 10 doses, the concentration of the drug in the bloodstream is very close to the long-term concentration/steady state value.

$C)$ **If $k = 0.01h^{-1}$, $t_0 = 10h$, find the smallest n such that $R_n > (\frac{1}{2})R$. Use $C_0 = 1(mg/mL)$**

What we are given:

- $C_0 = 1(mg/mL)$
- $k = 0.01h^{-1}$
- $t_0 = 10h$

We can simplify some of our solving for some of the variables:

- $-kt_0 = -0.01(10) = -0.1$
- $e^{-kt_0} = e^{-0.1} \approx 0.90484$

We can now use this to calculate R:

$$R = \frac{C_0 e^{-kt_0}}{1 - e^{-kt_0}}$$

$$R = \frac{1 \cdot e^{-0.1}}{1 - e^{-0.1}}$$

$$R = \frac{0.90484}{1 - 0.90484}$$
$$R = \frac{0.90484}{0.09516}$$
$$R = 9.5083\,(mg/mL)$$

$$\frac{1}{2}R = \frac{9.5083}{2} \approx 4.75415\,(mg/mL)$$

The inequality we need to solve is:

$$R_n = \frac{C_0 e^{-kt_0}(1 - e^{-nkt_0})}{1 - e^{-kt_0}} > \frac{1}{2}R$$

$$\frac{C_0 e^{-kt_0}(1 - e^{-nkt_0})}{1 - e^{-kt_0}} > \frac{9.5083}{2}$$

To solve this inequality, we can start by substituting in the values we know:

$$\frac{1 \cdot e^{-0.1}(1 - e^{-n(0.01)(10)})}{1 - e^{-0.1}} > 4.75415$$
$$\frac{e^{-0.1}(1 - e^{-0.1n})}{1 - e^{-0.1}} > 4.75415$$
$$\frac{0.90484(1 - e^{-0.1n})}{0.09516} > 4.75415$$

Multiply both sides by 0.09516 to eliminate the fraction:

$$0.90484(1 - e^{-0.1n}) > 4.75415 \cdot 0.09516$$

$$0.90484(1 - e^{-0.1n}) > 0.4514$$

Now, divide both sides by 0.90484 to isolate the term with n:

$$1 - e^{-0.1n} > \frac{0.4514}{0.90484}$$

$$1 - e^{-0.1n} > 0.4984$$

Now, we can isolate the term with n:

$$-e^{-0.1n} > 0.4984 - 1$$

$$-e^{-0.1n} > -0.5016$$

$$e^{-0.1n} < 0.5016$$

Now, we can take the natural logarithm of both sides to solve for n:

$$\ln(e^{-0.1n}) < \ln(0.5016)$$

Now, we can divide both sides by -0.1:

$$-0.1n < \ln(0.5016)$$

$$n > \frac{\ln(0.5016)}{-0.1}$$

$$n > \frac{-0.6931}{-0.1}$$

$$n > 6.931$$

Since n must be a whole number, we can round up to the next whole number:

$$n = 6.931 \approx 7$$

Therefore, the smallest n such that $R_n > \frac{1}{2}R$ is $n = 7$. This means that after 8 doses ($n = 7$ corresponds to the concentration before does $n + 1 = 8$), the concentration of the drug in the bloodstream will be greater than half of the long-term concentration/steady state value.