Math 1700 - Project 1

Introduction

The project explores the Normal Probability Distribution and acts as a proof/argument of the theory that the area under the curve of the Normal Probability Distribution is equal to 1.

Normal Probability Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

Simplifying the function:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, as \mu = 0 and \sigma = 1$$

The function above is called the Normal probability density function with the mean μ and the standard deviation σ. The number μ tells where the distribution is centered, and σ measures the "scatter" around the mean.

From the theory of probability, it is known that:

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

Graphical Representation

For this project $\mu = 0$ and $\sigma = 1$.

$A)$ **Graph of f. Find the interval on which the f is increasing, the interval on which f is decreasing, and any extreme values and where they occur.**

To determin the intervals on which f is increasing and decreasing, we need to find the derivative of f.

$$f'(x) = \frac{d}{dx}\left(\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\right)$$

$$f'(x) = \frac{d}{dx}\left(\frac{1}{\sqrt{2\pi}}\right) e^{-\frac{x^2}{2}} + \frac{1}{\sqrt{2\pi}} \frac{d}{dx}\left(e^{-\frac{x^2}{2}}\right) = -\frac{x}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

To find the critical points, we need to solve the equation $f'(x) = 0$.

$$-\frac{x}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2} = 0$$

$$x = 0$$

The critical point is at $x = 0$. To determine the intervals on which f is increasing and decreasing, we need to find the sign of $f'(x)$.

$$f'(x) = -\frac{x}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$$

$$f'(x) = \begin{cases} > 0 & x < 0 \\ < 0 & x > 0 \end{cases}$$

Therefore, f is increasing on the interval $(-\infty, 0)$ and decreasing on the interval $(0, \infty)$.

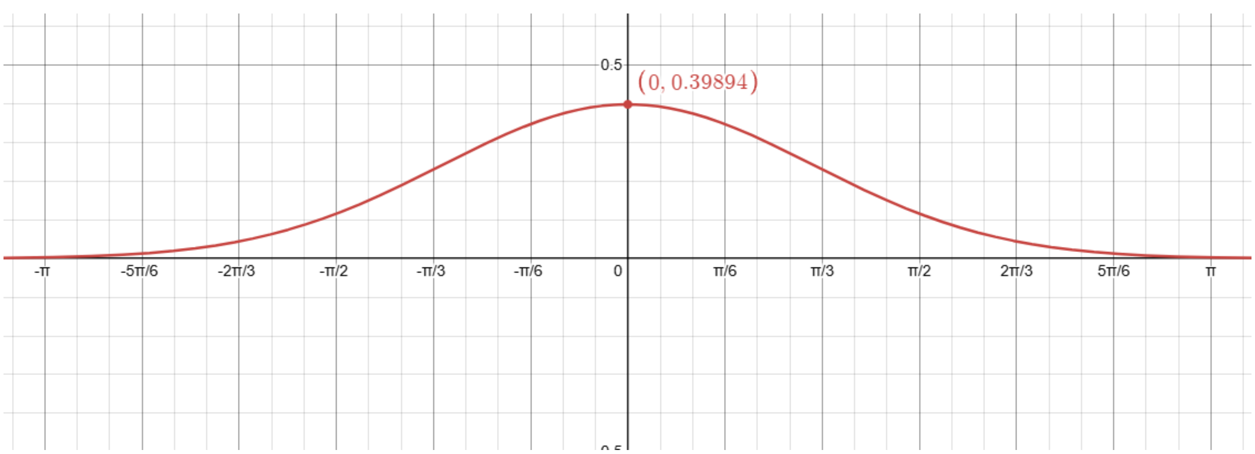
Finding Local Maximums and Minimums

Typically you'd go on to find the second derivative of f and use the second derivative test to find the local maximums and minimums.

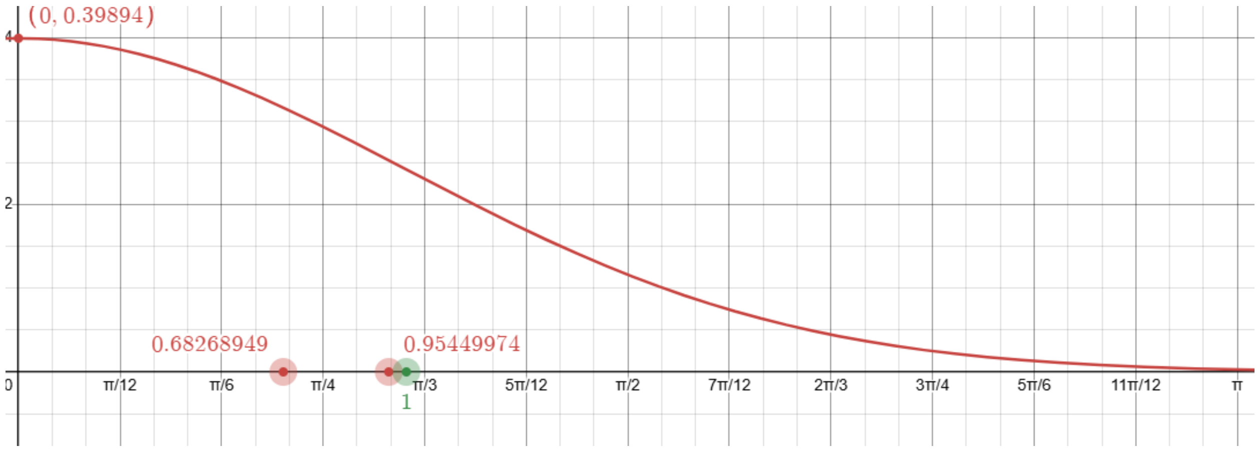
However, since we know that f has an increasing interval at $(-\infty, 0)$ and a decreasing interval at $(0, \infty)$, we can conclude that f has a local maximum at $x = 0$, this would also act as the Absolute Maximum. We can also conclude that there is no local minimum nor is there an Absolute Minimum.

$B)$ **Graph**

The graph of f is shown below:



Graph of $\int_{-n}^{n} f(x)dx$ when $n = 1, 2, and 3$:



In the image above you'll see the graph of f and the graph of $\int_{-n}^{n} f(x)dx$ when $n = 1, 2, and 3$. As n increases, the area under the curve of f increases and approaches 1.

$C)$ **Give a convincing argument that** $\int_{-\infty}^{\infty} f(x)dx = 1$

Information provided (hint):

$$0 < f(x) < e^{-\frac{x}{2}} for x > 1 and b > 1$$

$$\int_{1}^{\infty} e^{-\frac{x}{2}} dx \to 0 as b \to \infty$$

From the information provided, we can see that $f(x)$ is always less than $e^{-\frac{x}{2}}$ for $x > 1$. We also know that the integral of $e^{-\frac{x}{2}}$ from 1 to ∞ approaches 0 as b approaches ∞.

Therefore, we can conclude that the area under the curve of f approaches 1 as n approaches ∞. Meaning that $\int_{-\infty}^{\infty} f(x)dx = 1$.