16-720A Computer Vision: Homework 6 Photometric Stereo

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1 Calibrated photometric Stereo

Q1.a

As the object is Lambertian it follows Lambert's cosine law that says that the radiant intensity or luminous intensity observed from an ideal diffusely reflecting surface or ideal diffuse radiator is directly proportional to the cosine of the angle θ between the direction of the incident light and the surface normal.

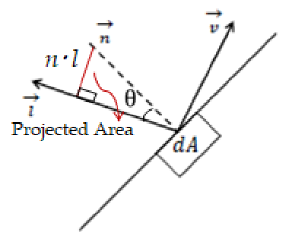


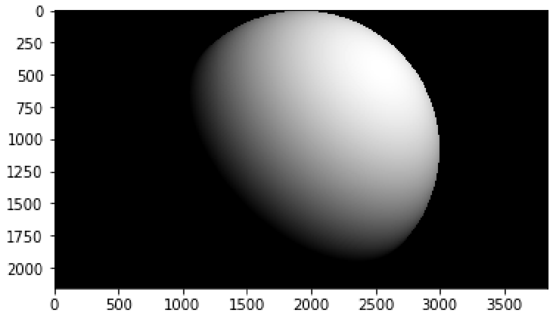
Figure 1: Projected Area

$$dA_{projected} = cos\theta = (n) \cdot (l) \tag{1}$$

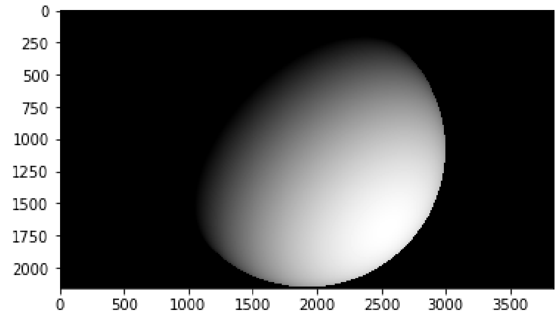
When an area element is radiating as a result of being illuminated by an external source, the irradiance (energy or photons/time/area) landing on that area element will be proportional to the cosine of the angle between the illuminating source and the normal. A Lambertian scatterer will then scatter this light according to the same cosine law as a Lambertian emitter. This means that although the radiance of the surface depends on the angle from the normal to the illuminating source, it will not depend on the angle from the normal to the observer.

Q1.b

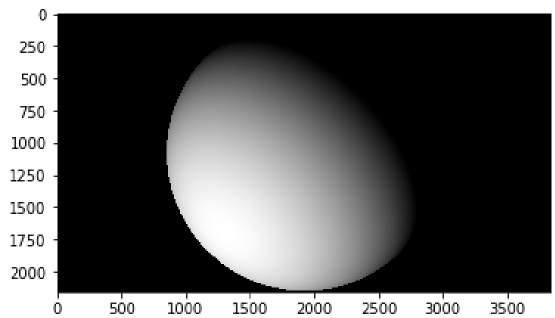
The rendering of n-dot-l lighting is as follows:



light direction - $(1, 1, 1)/\sqrt{3}$



light direction - $(1, -1, 1)/\sqrt{3}$



light direction - $(-1, -1, 1)/\sqrt{3}$

Figure 2: Appearance of sphere under n-dot-l model

In the above images the sphere appears bright near the lighting direction, which is the required outcome.

Q1.c

Code has been implemented such that the images loaded are 16-bit.tifs

Q1.d

In the equation $I = L^T B$, the size of L is $3X7$, B is $3XP$ and I is $7XP$
The rank of matrix I is expected to be 3. After performing singular value decomposition, the rank of the matrix I comes out to be 7. This is because the image capture is not ideal, the image is capturing all inter-reflected lights and more noise from the surroundings due to which we have more independent measurements (7 per pixel) than variables (3 per pixel) leading to a rank of 7 rather than 3.

Q1.e

The equation to solve is as follows:

$$\begin{aligned} I &= L^T B \\ {L^T}^{-1} I &= B \end{aligned} \tag{2}$$

Which can be written as $Ax = y$
where,

$$\begin{aligned} A &= {L^T}^{-1} \\ x &= I \\ y &= B \end{aligned} \tag{3}$$

As mentioned in the question that the pseudonormals are to be estimated in least square sense, we use the numpy function numpy.linalg.lstsq for calculating B. In numpy.linalg.lstsq we need not perform any additional construction of matrix A , but just use in the form as,

$$B = numpy.linalg.lstsq(L.T, I, rcond = None)[0] \tag{4}$$

Q1.f

The images are as below:

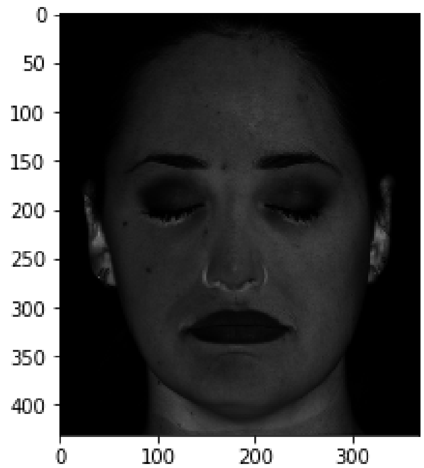


Figure 3: Albedo Image

In the above image the regions which is dark in the actual image (near nose and ears) have become brighter, this is due to the fact that the image capture process violates the n-dot-l model as it is capturing inter-reflections from the corners as well.

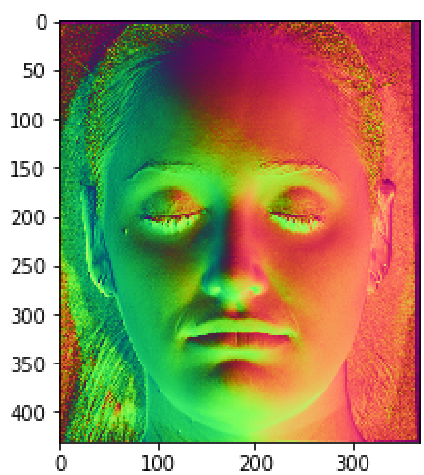


Figure 4: Normals Image

In this image the normals match the curvature of the face.

Q1.g

The shape of the face is represented as a 3D depth map given by $z = f(x, y)$
The components of surface gradient at point (x, y) is,

$$p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y} \tag{5}$$

We know that if we move a small distance ∂x in x, then the change in height is $\partial z = p\partial x$ (since p is the slope of the surface in the x direction). Thus $(1, 0, p)^T$ is a tangent to the surface. If we move a small distance ∂y in y, then the change in height is $\partial z = q\partial y$ (since q is the slope of the surface in the y direction). Thus $(0, 1, q)^T$ is also a tangent to the surface.. The normal is perpendicular to all tangents, thus parallel to the cross-product of these particular tangents, that is parallel to $(-p, -q, 1)^T$.Hence a unit normal can be written in the form,

$$n = \frac{(-p, -q, 1)^T}{\sqrt{1 + p^2 + q^2}} \tag{6}$$

It is given in the question that $n = (n_1, n_2, n_3)$, which gives:

$$\begin{aligned} n_1 &= \frac{-p}{\sqrt{1 + p^2 + q^2}} \\ n_2 &= \frac{-q}{\sqrt{1 + p^2 + q^2}} \\ n_3 &= \frac{1}{\sqrt{1 + p^2 + q^2}} \end{aligned} \tag{7}$$

On dividing $-n_1$ by n_3 and $-n_2$ by n_3 we get:

$$\begin{aligned} p &= \frac{-n_1}{n_3} = \frac{\partial z}{\partial x} \\ q &= \frac{-n_2}{n_3} = \frac{\partial z}{\partial y} \end{aligned} \tag{8}$$

In this way n is related to partial derivatives of z

Q1.h

The 2D discrete function given is,

$$g = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \tag{9}$$

The x gradient for the function is given as $g_x(x_i, y_i) = g(x_{i+1}, y_i) - g(x_i, y_i)$ and y gradient is given as $g_y(x_i, y_i) = g(x_i, y_{i+1}) - g(x_i, y_i)$, g_x and g_y matrices can be written as,

$$g_x = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, g_y = \begin{bmatrix} 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix} \tag{10}$$

Reconstruction of g given $g(0,0) = 1$,

1. Using g_x to construct the first row of g and g_y to construct the rest of g
   First row of g

$$\begin{aligned} g(0,0) &= 1, \\ g(1,0) &= g(0,0) + g_x(0,0) = 1 + 1 = 2, \\ g(2,0) &= g(1,0) + g_x(1,0) = 2 + 1 = 3, \\ g(3,0) &= g(2,0) + g_x(2,0) = 3 + 1 = 4 \\ g(row1) &= \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \end{aligned} \tag{11}$$

   First column of g constructed by g_y

$$\begin{aligned} g(0,0) &= 1, \\ g(0,1) &= g(0,0) + g_y(0,0) = 1 + 4 = 5, \\ g(0,2) &= g(0,1) + g_y(0,1) = 5 + 4 = 9, \\ g(0,3) &= g(0,2) + g_y(0,2) = 9 + 4 = 13 \\ g(column1) &= \begin{bmatrix} 1 \\ 5 \\ 9 \\ 13 \end{bmatrix} \end{aligned} \tag{12}$$

   Following the above process we construct g as,

$$g = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \tag{13}$$

2. Using g_y to construct the first row of g and g_x to construct the rest of g
   First column of g constructed by g_y

$$\begin{aligned} g(0,0) &= 1, \\ g(0,1) &= g(0,0) + g_y(0,0) = 1 + 4 = 5, \\ g(0,2) &= g(0,1) + g_y(0,1) = 5 + 4 = 9, \\ g(0,3) &= g(0,2) + g_y(0,2) = 9 + 4 = 13 \\ g(column1) &= \begin{bmatrix} 1 \\ 5 \\ 9 \\ 13 \end{bmatrix} \end{aligned} \tag{14}$$

   First row of g

$$\begin{aligned} g(0,0) &= 1, \\ g(1,0) &= g(0,0) + g_x(0,0) = 1 + 1 = 2, \\ g(2,0) &= g(1,0) + g_x(1,0) = 2 + 1 = 3, \\ g(3,0) &= g(2,0) + g_x(2,0) = 3 + 1 = 4 \\ g(row1) &= \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \end{aligned} \tag{15}$$

   second row of g

$$\begin{aligned} g(0,1) &= 5, \\ g(1,1) &= g(0,1) + g_x(0,1) = 5 + 1 = 6, \\ g(2,1) &= g(1,1) + g_x(1,1) = 6 + 1 = 7, \\ g(3,1) &= g(2,1) + g_x(2,1) = 7 + 1 = 8 \\ g(row2) &= \begin{bmatrix} 5 & 6 & 7 & 8 \end{bmatrix} \end{aligned} \tag{16}$$

   Following the above process we construct g as,

$$g = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \tag{17}$$

g obtained by both the above methods is same.
Modifications in the gradients for making g_x and g_y non-integrable is:

1. If the gradient elements are negative then the addition of the gradient in x and y direction will be different leading to different g
2. If the values of the elements in the gradient matrix are not equal then it can lead to different g .
3. The gradients estimated in g can be also be non-integrable because of the presence of noise.

Q1.i

The 3D Surface is as below:

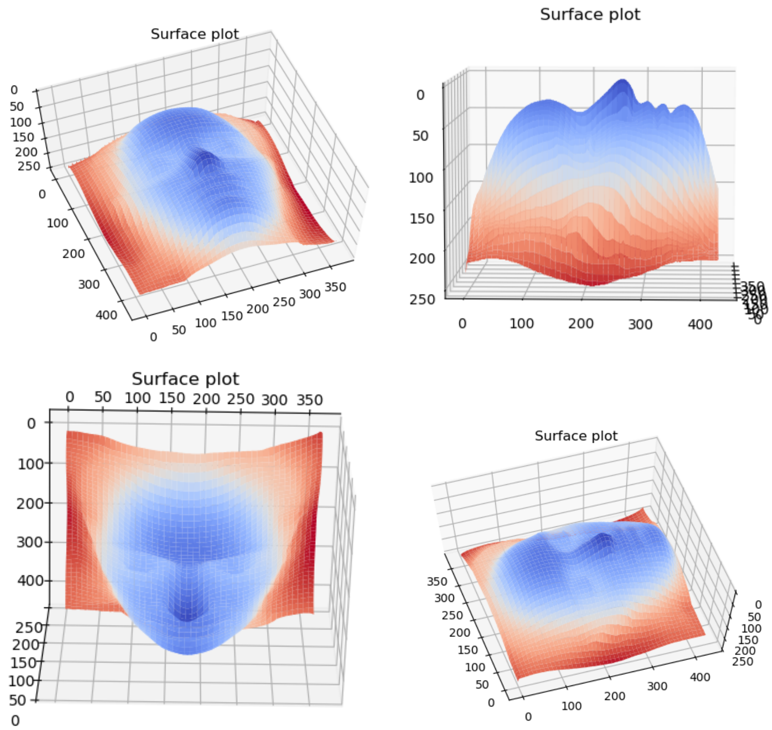


Figure 5: 3D Surface

From the above images we can see that the algorithm can detect the shape well.

Q2.a

In the relation $I = L^T B$ both L and B are unknown. The rank of the matrix I should be 3, but as seen in $Q1.d$ the rank of the matrix is 7
To reduce the rank to 3, we need to set all the singular values to 0 except for the first 3.

1. Decompose I by computing the SVD of I: $I = U\Sigma V^T$.
2. Create U_3 by taking the first 3 columns of U
3. Create V_3 by taking the first 3 columns of V
4. Create Σ_3 by taking the upper left 3x3 block of Σ
5. Create L^T and B matrices as follows:

$$L^T = U_3 {\Sigma_3}^{1/2}, B = {\Sigma_3}^{1/2} V_3^T \tag{18}$$

In this way we can factorize I to obtain L^T and B

Q2.b

The visualization of albedos and normals are as below:

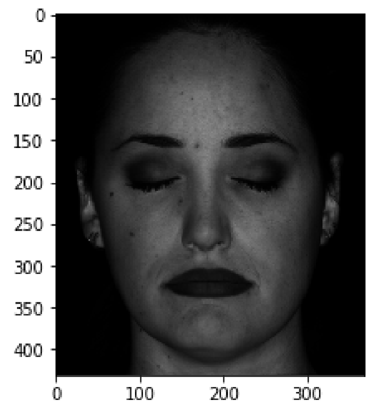


Figure 6: Albedo Image

This image looks similar to the gray colormap of the original image without any bright regions as in calibrated photometric stereo.

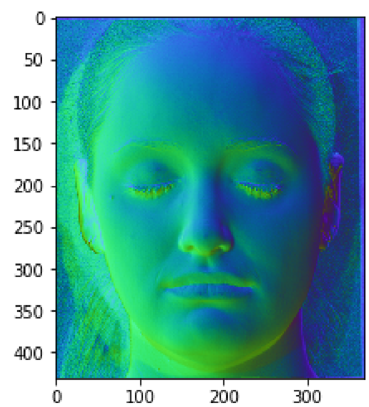


Figure 7: Normals Image

The rainbow color map of normals image appears blue but we can still identify the face curvatures.

Q2.c

L_0 ground truth lighting direction is:

```
[[-0.1418  0.1215 -0.069   0.067  -0.1627  0.      0.1478]
 [-0.1804 -0.2026 -0.0345 -0.0402  0.122   0.1194  0.1209]
 [-0.9267 -0.9717 -0.838  -0.9772 -0.979  -0.9648 -0.9713]]
```

$\hat{L}$ obtained by factorization is:

```
[[-2.85201979 -3.70224968 -2.29991163 -3.58159342 -3.42762044 -3.23473591 -3.19937559]
 [ 0.89993439 -2.2191511   0.47629996 -0.59379374  2.22099759 0.45043184 -0.74679075]
 [ 1.79851522  0.96528268  0.410125   -0.0210451  -0.28691153 -0.87230035 -1.80219843]]
```

Therefore, L_0 and $\hat{L}$ are not similar.
I can be factorized as below also:

$$L^T = U_3, B = \Sigma_3 V_3^T \tag{19}$$

The above equation will keep the images rendered.

Q2.d

On using the implementation of Frankot-Chellappa algorithm from previous question we get the following 3D depth map

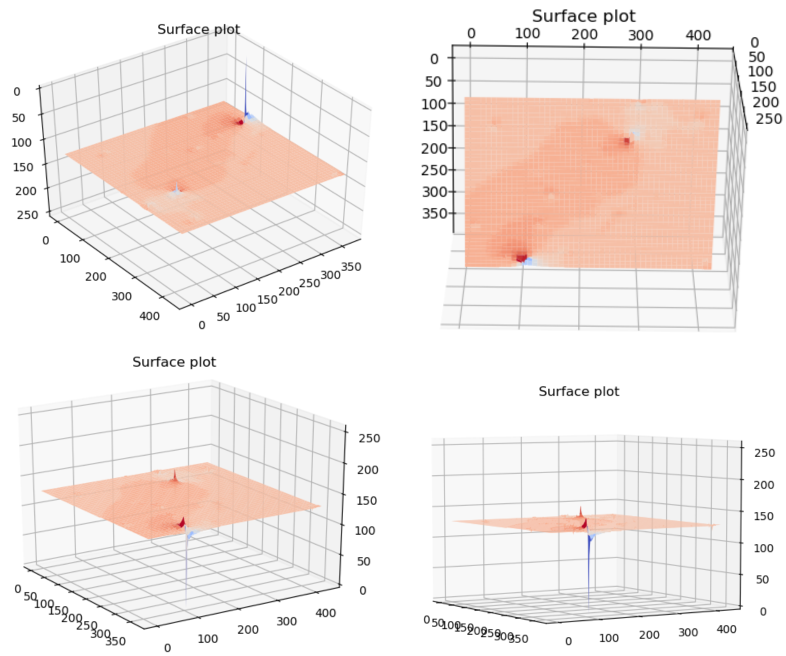


Figure 8: 3D Surface

From the above plots we see that the reconstruction does not look looks like a face from the calibrated photometric stereo of Q1.i

Q2.e

On inputting the pseudonormal to enforceIntegrability function we get the following plot:

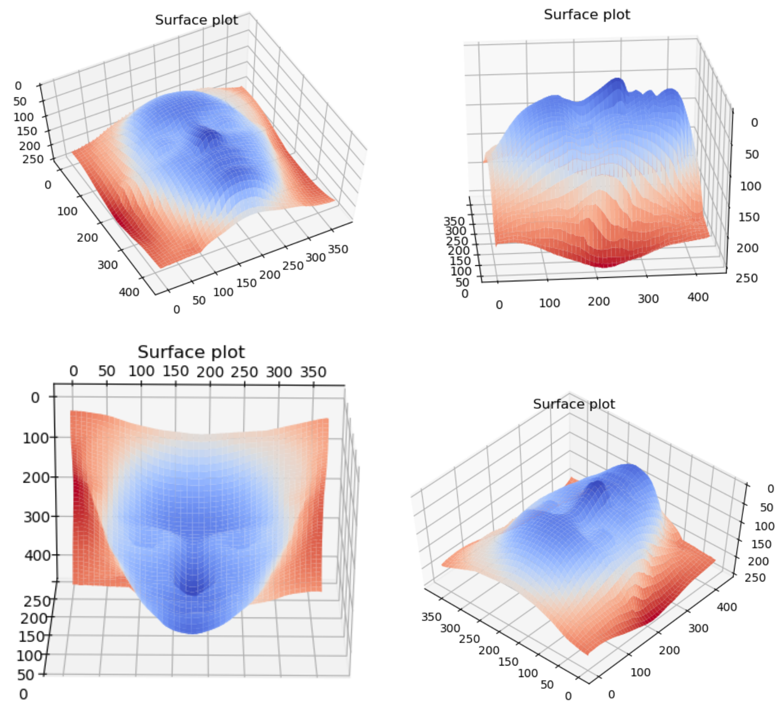


Figure 9: 3D Surface

From the above plots we see that it looks like a face similar to calibrated photometric stereo from Q1.i

Q2.f

Initially the value of $\mu = 0$, $\nu = 0$ and $\lambda = 0$

1. Changing the value of μ

   (a) When $\mu > 0$:

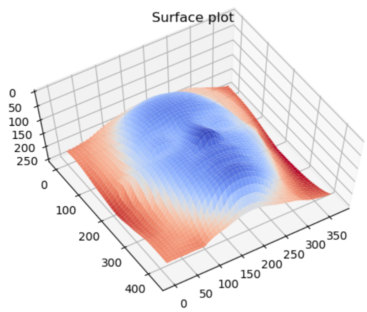


Figure 10: $\mu = 0.5$

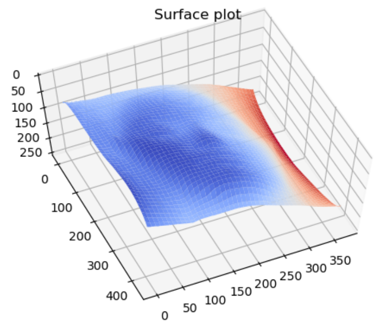


Figure 11: $\mu = 5$

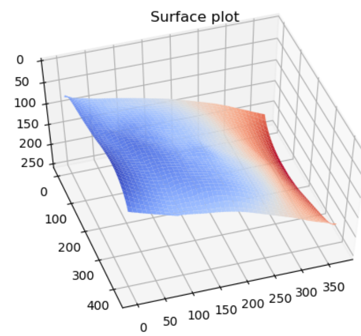


Figure 12: $\mu = 10$

   From the above images it can be seen that as the value of μ increases from zero the flatness of the surface increases

   (b) When $\mu < 0$:

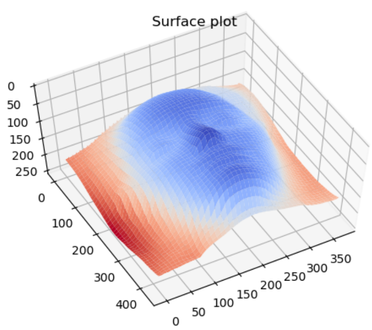


Figure 13: $\mu = -0.5$

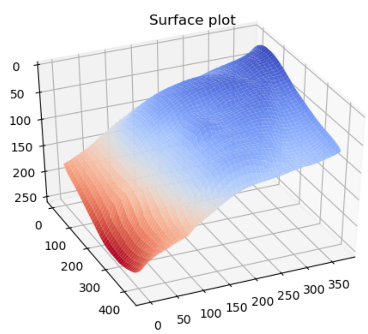


Figure 14: $\mu = -5$

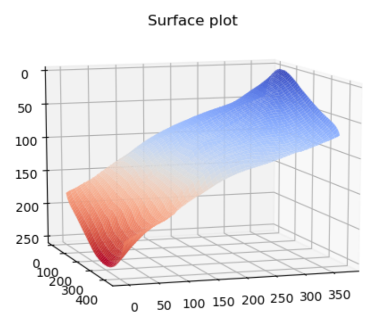


Figure 15: $\mu = -10$

   From the above images it can be seen that as the value of μ decreases from zero the flatness of the surface increases but in reverse direction.

2. Changing the value of ν

   (a) When $\nu > 0$:

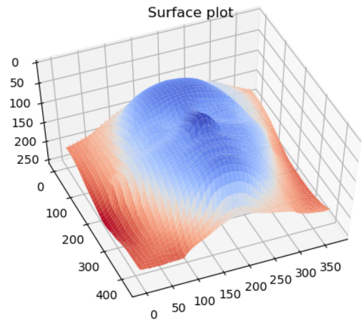


Figure 16: $\nu = 0.5$

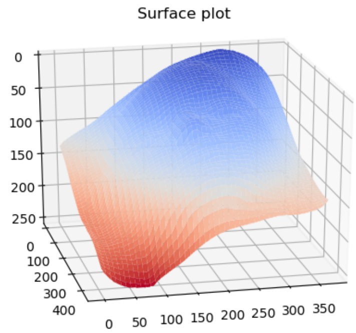


Figure 17: $\nu = 5$

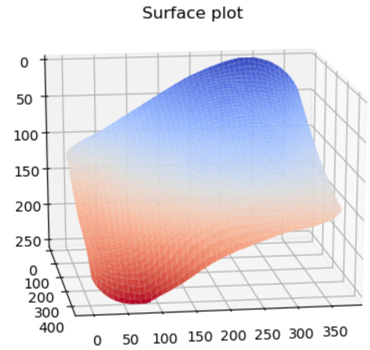


Figure 18: $\nu = 10$

From the above images it can be seen that similar to μ as the value of ν increases from zero the flatness of the surface increases

(b) When $\nu < 0$:

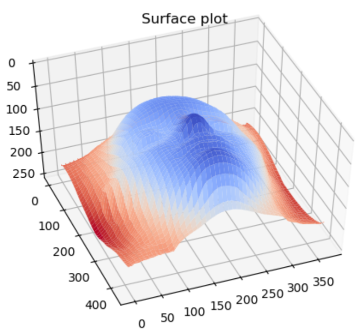


Figure 19: $\nu = -0.5$

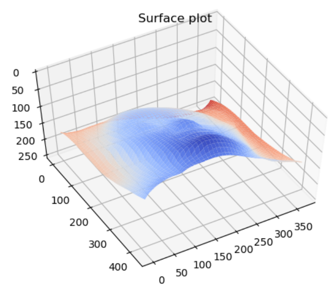


Figure 20: $\nu = -5$

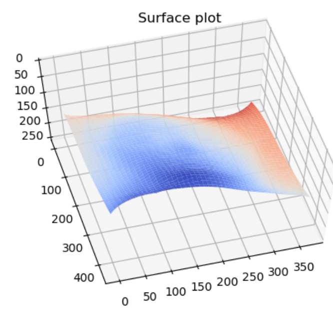


Figure 21: $\nu = -10$

From the above images it can be seen that as similar to μ as the value of ν decreases from zero the surface flatness increases but in reverse direction.

3. Changing the value of λ

(a) When $\lambda > 0$:

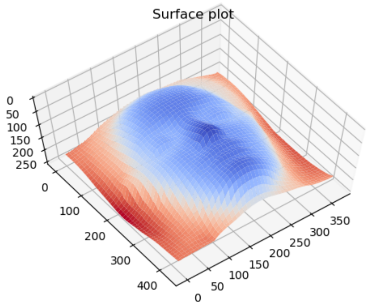


Figure 22: $\lambda = 0.5$

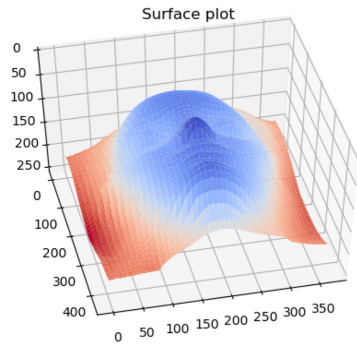


Figure 23: $\lambda = 5$

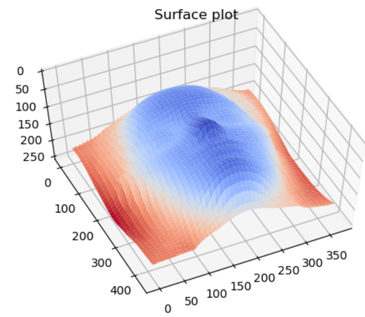


Figure 24: $\lambda = 10$

With the increase in the value of λ the surface ability to capture face curvatures improves.

(b) When $\lambda < 0$:

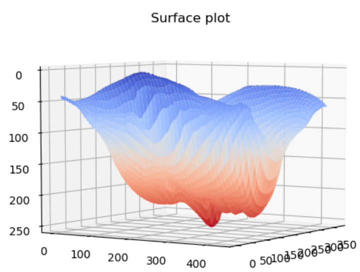


Figure 25: $\lambda = -0.1$

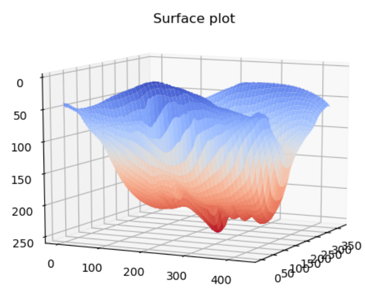


Figure 26: $\lambda = -0.5$

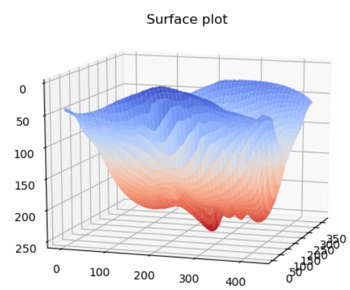


Figure 27: $\lambda = -10$

With the decrease in the value of λ the surface ability to capture face curvatures improves but in reverse direction.

4. After looking at these images, the bas-relief ambiguity is so named because when the relief is low and there is only single viewpoint and no given light direction, there is an ambiguity in the recovery of the surface. No information in either the shadowing or shading can resolve this. Furthermore, neither small motion of the object, nor of the viewer will resolve the ambiguity.

Q2.g

From Q2.f we can infer that, with increase or decrease in μ and ν from zero the surface flatness increases and for λ the surface curvature improves. Therefore, in order to obtain a flat surface we increase the value of μ or ν (not together as they cancel out each other's effect if increased together) and decrease the value of λ.
When $\mu = 10$ and $\nu = 0$ and $\lambda = 0.1$ we get the flattest surface which is as follows:

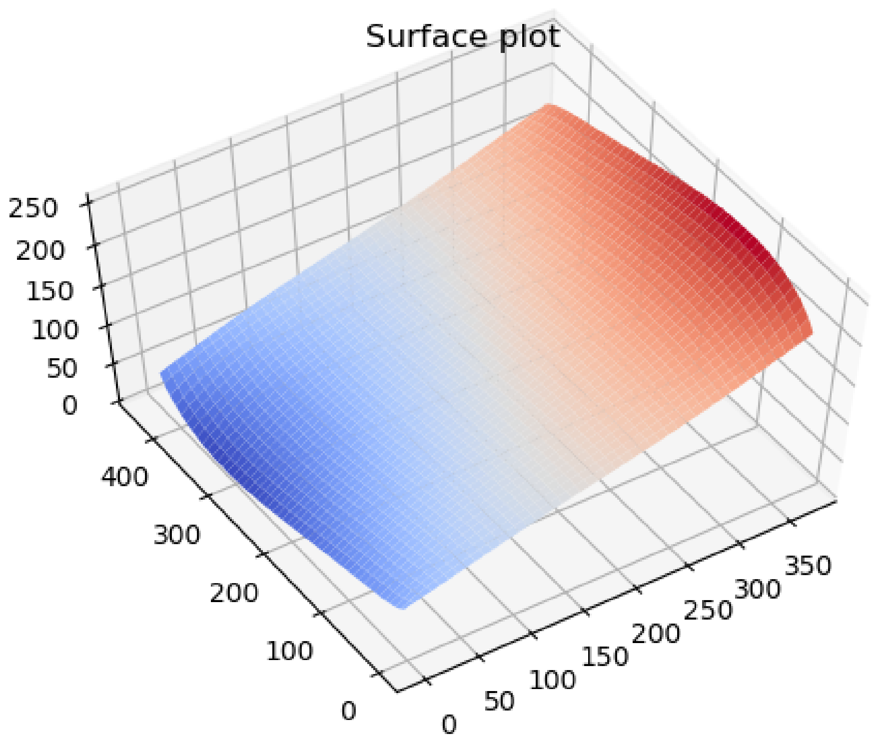


Figure 28: Flat Surface

Q2.h

No, acquiring more pictures from more lighting directions will not help to solve the ambiguity because, even if the dimension of I from $I = L^T B$ matrix is increased (Currently it is $7XP$) to NXP (where $N > 7$), there will again be an uncertainty about the method in which I should be factorized to L^T and B. This is similar to the current situation and hence will not help in the reduction of ambiguity.

3 Extra Credit

3a

The homework could be improved in the following ways:

1. Like the previous assignments output images of the expected results could be included in the write up, so the students can do an individual check of the results obtained and also perform a sanity check of the code
2. More clarity could be provided about the expected inclusion in the write-up.(eg. 1e and 1h it was unclear as to what to be included in the write-up)
3. More references could be added to go through which could help with the assignment.