

$$1) C(X, Y, Z) = \alpha_1 P_1(x_1, y_1, z_1) + \alpha_2 P_2(x_2, y_2, z_2) + \alpha_3 P_3(x_3, y_3, z_3)$$

i) Normalized chromaticity coordinates of  $P_1, P_2, P_3$

$$P_1 \Rightarrow x_1 = \frac{x_1}{x_1 + y_1 + z_1} \quad y_1 = \frac{y_1}{x_1 + y_1 + z_1} \quad z_1 = \frac{z_1}{x_1 + y_1 + z_1}$$

$$P_2 \Rightarrow x_2 = \frac{x_2}{x_2 + y_2 + z_2} \quad y_2 = \frac{y_2}{x_2 + y_2 + z_2} \quad z_2 = \frac{z_2}{x_2 + y_2 + z_2}$$

$$P_3 \Rightarrow x_3 = \frac{x_3}{x_3 + y_3 + z_3} \quad y_3 = \frac{y_3}{x_3 + y_3 + z_3} \quad z_3 = \frac{z_3}{x_3 + y_3 + z_3}$$

Normalized chromaticity coordinates of colour C

$$C \Rightarrow x = \frac{x}{x + y + z} \quad y = \frac{y}{x + y + z} \quad z = \frac{z}{x + y + z}$$

ii) Normalized chromaticity coordinates of the color C in terms of normalized chromaticity coordinates of  $P_1, P_2, P_3$  is given by

$$\left. \begin{aligned} X &= \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 \\ Y &= \alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3 \\ Z &= \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3 \end{aligned} \right\} \text{Individual components}$$

$$x = \frac{\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3}{\alpha_1(x_1 + y_1 + z_1) + \alpha_2(x_2 + y_2 + z_2) + \alpha_3(x_3 + y_3 + z_3)}$$

$$y = \frac{\alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3}{\alpha_1(x_1 + y_1 + z_1) + \alpha_2(x_2 + y_2 + z_2) + \alpha_3(x_3 + y_3 + z_3)}$$

$$z = \frac{\alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3}{\alpha_1(x_1 + y_1 + z_1) + \alpha_2(x_2 + y_2 + z_2) + \alpha_3(x_3 + y_3 + z_3)}$$

iii) Chromaticity coordinates of any color C can also be represented as a linear combination of the chromaticity coordinates of the respective primaries

$$x = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

$$y = \beta_1 y_1 + \beta_2 y_2 + \beta_3 y_3$$

$$z = \beta_1 z_1 + \beta_2 z_2 + \beta_3 z_3$$

$$x_1 = \frac{x_1}{x_1 + y_1 + z_1}$$

$$x = \frac{x}{x + y + z}$$

$$\text{and } x = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3$$

$$\therefore x(x + y + z) = \alpha_1 x_1(x_1 + y_1 + z_1) + \alpha_2 x_2(x_2 + y_2 + z_2) + \alpha_3 x_3(x_3 + y_3 + z_3) \rightarrow (2)$$

$$\text{But } (x + y + z) = \alpha_1(x_1 + y_1 + z_1) + \alpha_2(x_2 + y_2 + z_2) + \alpha_3(x_3 + y_3 + z_3)$$

$$\therefore x = \frac{\alpha_1 x_1(x_1 + y_1 + z_1)}{(x + y + z)} + \frac{\alpha_2 x_2(x_2 + y_2 + z_2)}{(x + y + z)} + \frac{\alpha_3 x_3(x_3 + y_3 + z_3)}{(x + y + z)}$$

$$\therefore \beta_1 = \frac{\alpha_1(x_1 + y_1 + z_1)}{x + y + z}$$

$$\therefore \beta_2 = \frac{\alpha_2(x_2 + y_2 + z_2)}{x + y + z}$$

$$\therefore \beta_3 = \frac{\alpha_3(x_3 + y_3 + z_3)}{x + y + z}$$

$$\therefore \beta_1 = \frac{\alpha_1(x_1 + y_1 + z_1)}{\alpha_1(x_1 + y_1 + z_1) + \alpha_2(x_2 + y_2 + z_2) + \alpha_3(x_3 + y_3 + z_3)}$$

$$\therefore \beta_2 = \frac{\alpha_2(x_2 + y_2 + z_2)}{\alpha_1(x_1 + y_1 + z_1) + \alpha_2(x_2 + y_2 + z_2) + \alpha_3(x_3 + y_3 + z_3)}$$

$$\therefore \beta_3 = \frac{\alpha_3(x_3 + y_3 + z_3)}{\alpha_1(x_1 + y_1 + z_1) + \alpha_2(x_2 + y_2 + z_2) + \alpha_3(x_3 + y_3 + z_3)}$$

Similarly

$$y = \alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3 \quad \text{and} \quad y_1 = \frac{y_1}{x_1 + y_1 + z_1} \quad y = \frac{y}{x + y + z}$$

$$y(x + y + z) = \alpha_1(x_1 + y_1 + z_1) + \alpha_2(x_2 + y_2 + z_2) + \alpha_3(x_3 + y_3 + z_3)$$

$$\therefore y = \beta_1 y_1 + \beta_2 y_2 + \beta_3 y_3 \quad (\text{Same as above})$$

$$\therefore z = \beta_1 z_1 + \beta_2 z_2 + \beta_3 z_3 \quad (\text{Same as above})$$

2) Given sequence obtained by sampling a signal  
 $\{5.8, 6.2, 6.2, 7.2, 7.3, 7.3, 6.5, 6.8, 6.8, 6.8, 5.5, 5.0, 5.2, 5.2, 5.8, 6.2, 6.2, 6.2, 5.9, 6.3, 5.2, 4.2, 2.8, 2.8, 2.3, 2.9, 1.8, 2.5, 2.5, 3.3, 4.1, 4.9\}$   
 The sequence is quantized by assuming level 0 at 0.25, level 1 at 0.5, level 2 at 0.75, level 3 at 1.0 and so on

(i) Thus the quantized sequence is  
 $\{22, 24, 24, 28, 28, 28, 25, 26, 26, 26, 21, 19, 20, 20, 22, 24, 24, 24, 23, 24, 20, 16, 10, 10, 8, 11, 6, 9, 9, 12, 15, 19\}$

(ii) How many bits do need to transmit it  
 Number of uniformly distributed levels =  $32 = 2^5$   
 Thus Number of bits =  $32 \times 5 = 160$  bits

(iii) To encode using DPCM, we need to compute the difference between the successive values (consider the levels not the values and first difference value  $22 - 0 = 22$  is ignored)

$\{2, 0, 4, 0, 0, -3, 1, 0, 0, -5, -2, 1, 0, 2, 2, 0, 0, -1, 1, -4, -4, -6, 0, -2, 3, -5, 3, 0, 3, 3, 4\}$

Maximum difference is 4

Minimum difference is -6

$[-6, 4]$

$-6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$

No of levels =  $11 = 2^{3.31} \approx 2^4$

So we need 4 bits per signal

Number of bits required to encode sequence =  $31 \times 4 = 124$  bits (because we are not considering first difference value, is 31 instead of 32)

(iv) Compression Ratio =  $\frac{160}{124} = 1.29$

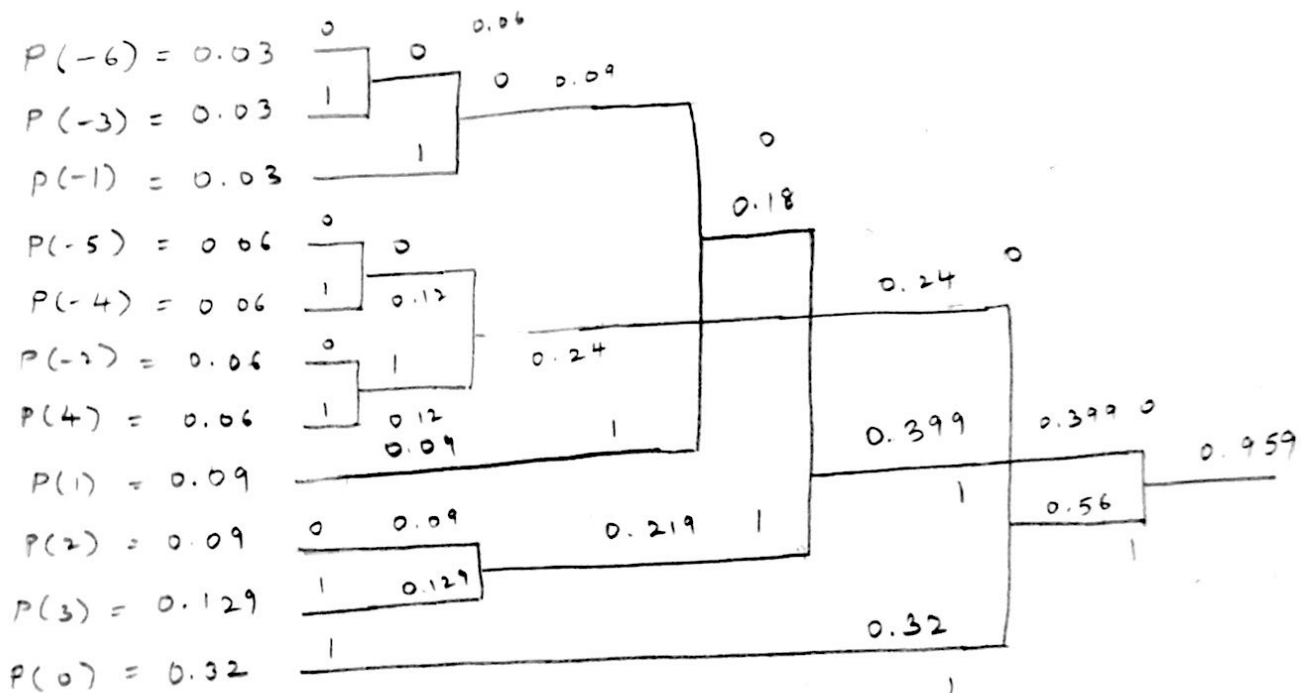
(v) Instead of transmitting differences use Huffman coded values for the differences. How many bits do you need now to encode the sequence

$\{2, 0, 4, 0, 0, -3, 1, 0, 0, -5, -2, 1, 0, 2, 2, 0, 0, -1, -4, -4, -6, 0, -2, 3, -5, 3, 0, 3, 3, 4\}$

$$\begin{aligned} P(-6) &= 1/31 = 0.0322 \\ P(-5) &= 2/31 = 0.0625 \\ P(-4) &= 2/31 = 0.0625 \\ P(-3) &= 1/31 = 0.0322 \\ P(-2) &= 2/31 = 0.0625 \\ P(-1) &= 1/31 = 0.0322 \\ P(0) &= 10/31 = 0.3226 \\ P(1) &= 3/31 = 0.0967 \\ P(2) &= 3/31 = 0.0967 \\ P(3) &= 4/31 = 0.129 \\ P(4) &= 2/31 = 0.0625 \end{aligned}$$

$$\begin{aligned} P(-6) &= 0.03 \\ P(-5) &= 0.06 \\ P(-4) &= 0.06 \\ P(-3) &= 0.03 \\ P(-2) &= 0.06 \\ P(-1) &= 0.03 \\ P(0) &= 0.32 \\ P(1) &= 0.09 \\ P(2) &= 0.09 \\ P(3) &= 0.129 \\ P(4) &= 0.062 \end{aligned}$$

$$\begin{aligned} P(-6) &= 0.03 \\ P(-3) &= 0.03 \\ P(-1) &= 0.03 \\ P(-5) &= 0.06 \\ P(-4) &= 0.06 \\ P(-2) &= 0.06 \\ P(4) &= 0.06 \\ P(1) &= 0.09 \\ P(2) &= 0.09 \\ P(3) &= 0.129 \\ P(0) &= 0.32 \end{aligned}$$



Symbol	Probability	Huffman Code	Code Length
-6 (1)	0.03	00000	5
-3 (1)	0.03	10000	5
-1 (1)	0.03	1000	4
-5 (2)	0.06	0001	4
-4 (2)	0.06	1000	4
-2 (2)	0.06	0100	4
4 (2)	0.06	1101	4
1 (3)	0.09	1000	4
2 (3)	0.09	010	3
3 (4)	0.129	110	3
0 (10)	0.32	11	2

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Compression Ratio of Quantized output using DPCM to quantized output using Huffman code is  $\frac{124}{96}$

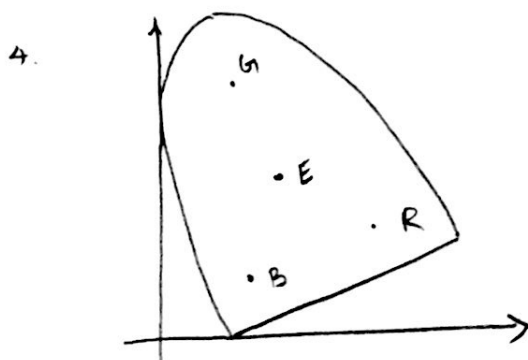
Compression Ratio of initial sequence to quantized output using Huffman code is  $\frac{160}{96}$

And when we don't consider the first level, the compression Ratio is  $\frac{155}{96} (31 \times 5)$



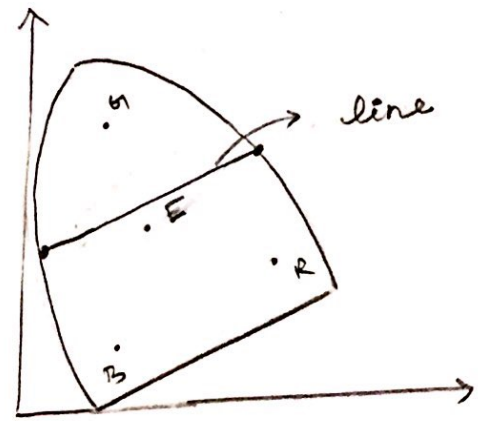
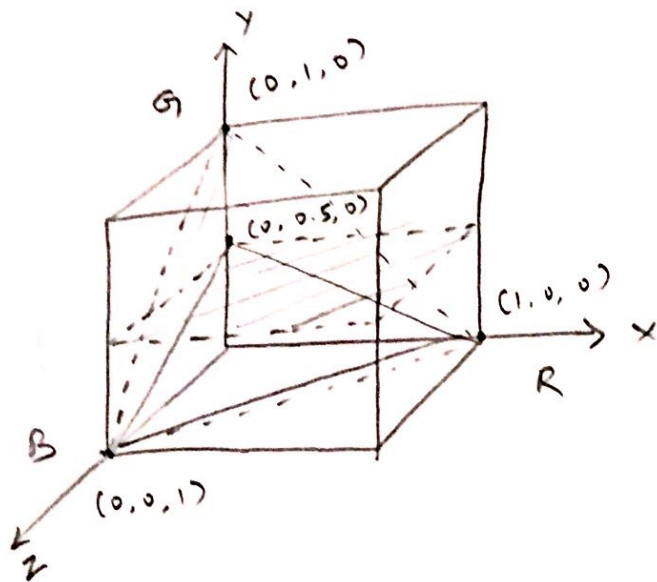
> Spectral color which is complementary to color C is C' which roughly has a wavelength of 680 nm. C' is represented by red color.

- The dominant wavelength of color D is roughly 600 nm. The color is represented by yellow
- No, all colors do not have a dominant wavelength. For some colors, the stimuli may intersect with purple line rather than the spectrum focus. From the figure, this happens when the chromaticity coordinates of a stimulus lie in the triangle whose apex is the achromatic stimulus. For such stimuli, we speak of complementary wavelength rather than dominant wavelength.
- Spectral color which is complementary to color C is C' from figure, which roughly has a wavelength of 680 nm. C' is represented by red color.



R, G, B are primaries around the equiluminous point E

Find all points which have a value of  $G=0.5$



So when we keep  $G$  at constant value of  $0.5$  and with varying values of  $R$  and  $B$ , we will get a plane, as shown in the diagram which will contain all points with  $G=0.5$

When we obtain chromaticity diagram for it, it will show a line as shown in the diagram above. So different points on the line represent different mixtures of 2 colors (Red and Blue) at end points of line.

v) Take  $G=0.5$  locus above, how does this map in the R-G-B space

Since it is a 2D plane in the R-G-B color space, the spectral locus which we will now obtain will be a line with different combinations of Red and Blue and constant value of Green =  $0.5$ .