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Design and Analysis of Algorithms

TUTORIAL - 1

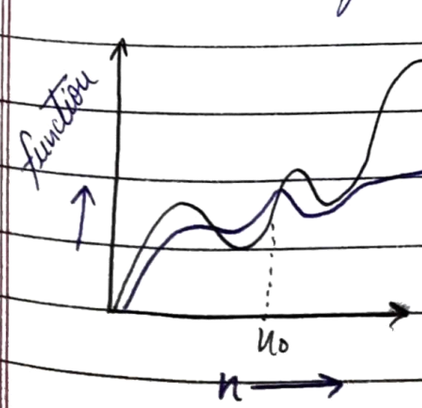
Ques (1.) What do you understand by Asymptotic notations. Define different Asymptotic notation with Example

→ Asymptotic Notation (tending to infinity)

↳ These notations are used to tell the Complexity of an algorithm when input is very large

1) Big O (O)

$f(n) = O(g(n))$



g(n) is "tight" upper bound

$f(n) = O(g(n))$

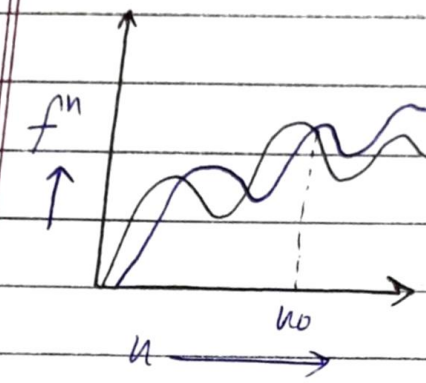
iff

$f(n) \leq cg(n)$

$\left(\forall\ n \geq n_0 \text{ and some Const, } c > 0\right)$

(2) Big Omega (Ω)

$$f(n) = \Omega(g(n))$$



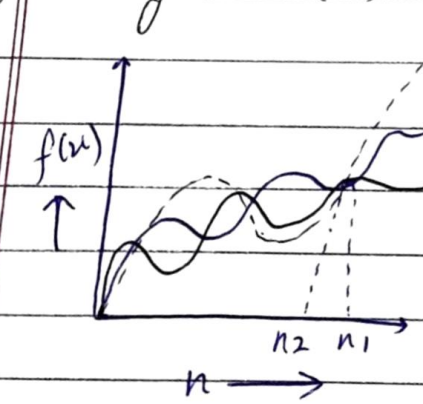
$g(n)$ is "tight" lower bound

$f(n) = \Omega(g(n))$

$(f(n) \geq cg(n))$

$\left(\forall \; n \geq n_0 \;\; \& \text{ some Const, } c>0\right)$

(3) Big theta (Θ)



$g(n)$ is both "tight" upper & lower bound of $f(n)$

$$f(n) = \Theta(g(n))$$

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

$\forall \; n \geq \max(n_1, n_2)$

& some Const, $c_1>0, c_2>0$

Ques (2) What should be time Complexity of

```
for (i=1 to n) { i = i+2 }
```

→

```
for (i=1 to n)          // i = 1, 2, 4, 8 ---- , n
{ i = i x 2 }           // O(1)
```

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$\Rightarrow \sum_{i=1}^{n} 1+2+4+8+\cdots\cdots+n$

GP of k^{th} Value $\longrightarrow T_k = ar^{k-1}$

$= 1 \times 2^{k-1}$

$n = 2^{k-1}$

$2n = 2^k$

$\Rightarrow \log 2n = k \log 2$

$\log 2 + \log n = k \log 2$

$\log n + 1 = k$

$O(k) = O(1 + \log n)$

$= O(\log n)$

Ques (3) $T(n) = \begin{cases} 3T(n-1) & \text{if } n>0 \text{ otherwise } 1 \end{cases}$

$T(n) = 3T(n-1)$ — (1)

put $n = (n-1)$

$T(n-1) = 3T(n-2)$ — (2)

from (1) and (2)

$\Rightarrow T(n) = 3(3T(n-2))$

$= 9T(n-2)$ — (3)

putting $n = n-2$ in (1)

$T(n-2) = 3T(n-3)$ — (4)

putting $T(n-2)$ in (3)

$T(n) = 3^3 T(n-3)$

$\therefore$ General $\Rightarrow T(n) = 3^k (T(n-k))$

putting $n-k = 0$

$k = n$

$T(n) = 3^n T(0)$

$= 3^n \times 1$

$\boxed{T(n) = O(3^n)}$

Ques 4 $T(n) = \begin{cases} 2T(n-1)-1 & \text{if } n>0, \\ 1 & \text{otherwise} \end{cases}$

$T(n) = 2T(n-1) - 1 \quad -(1)$

Let $n = n-1$

$\Rightarrow T(n-1) = 2T(n-2) - 1 \quad -(2)$

from (1) and (2)

$T(n) = 4T(n-2) - 1 \quad -(3)$

put $n = n-2$

$T(n-2) = 2T(n-3) - 1 \quad -(4)$

from (3) and (4)

$T(n) = 4[2T(n-3) - 1] - 2 - 1$

$= 2T(n-3) - 4 - 2 - 1$

General $\Rightarrow 2^k T(n-k) - (2^k - 1)$

$n-k = 0$

$n = k$

$T(n) = 2^n T(0) - 2^n + 1$

$= 2^n \times 1 - 2^n + 1 = 1$

$T(n) = O(1)$

Ques 5 What should be the time Complexity of

```
int i=1, s=1
while (s<=n)
{   i++; s = s+i;
    printf ("#");
}
```

→ Sum of S = 1+3+6+10+ - - - + Tn -(1)

also S = 1+3+8+10+ - - - + Tn-1 + Tn -(2)

from (1) and (2)

0 = 1+2+3+4+ - - - + n - Tn

Tk = 1+2+3+4+ - - - + k

$T_k = \frac{1}{2}k(k+1)$

for k iterations

$\Rightarrow \frac{k(k+1)}{2} <= n$

$\frac{k^2+k}{2} <= n$

$O(k^2) <= n$

$k = O(\sqrt{n})$

$T(n) = O(\sqrt{n})$

Ques (6) Time Complexity of : Void f(n) {int w) //O(n)

{ int i, count = 0; //O(1)

for (i=1; i*i <= n; i++)

Count ++ //O(1)

}

→ i*i = $\underbrace{1^2, 2^2, 3^2, 4^2, - - - - -, k}_{k \text{ terms}}$

⇒ k^{th} term :

$t_k = k^2$

$\Rightarrow k^2 = n$

$k = n^{1/2}$

$T(n) = O(1+1+1+n^{1/2}+1)$

$= O(n^{1/2})$

$T(n) = O(\sqrt{n})$

Ques (19) Time Complexity of

```
Void fn (int n)
{ int i,j,k, count = 0;
  for (i = n/2) ; i<=n ; i++)
    { for (i=1 ; j<=n ; j = j*2)
      { for (k=1 ; k<=n ; k = k*2)
          count ++;
      }
    }
}
```

→ for $k = k*2$

$k = 1, 2, 4, 8 \dots n$

GP $\Rightarrow a = 1, r = 2$

$$\text{Sum} = \frac{a(r^n - 1)}{r-1} = \frac{1(2^k - 1)}{1}$$

$$n = 2^k$$

$$\log n = k$$

$i \rightarrow 1, 2, \dots, n$

$j \rightarrow \log n, \log n, \dots, \log n$

$k \rightarrow \log n * \log n, \dots, \log n * \log n$

$\Rightarrow O(n * \log n * \log n)$

$\Rightarrow O(n \log^2 n)$

Ques (8) Time Complexity of :

```
function (int n)
{  if (n == 1) return ;                    // O(1)
   for (i=1 to n)                          // O(n)
   {  for (j=1 to n)                       // O(n)
      {  printf ("* ");                    // O(1)
      }
   }
   function (n-3) ;
}
```

→ for function Call,

$\underbrace{n, n-3, n-6, \dots, 1}_{K \text{ Terms}}$

⇒ AP with $d = -3$

⇒ $l = a + (k-1)d$

$1 = n + (k-1)(-3)$

$\frac{1-n}{-3} = k-1$

⇒ $k-1 = \frac{n-1}{3}$

$\left(K = \frac{n+2}{3}\right)$

⇒ function gives a recursive call $\frac{n+2}{3}$ times

⇒ Time Complexity = $\frac{(n+2)}{3}(n)(n)$

$= n^3$

$\boxed{T(n) = O(n^3)}$

Ques (9) Time Complexity of Void function (int n)

```
{
  for (i=1 to n)
  {
    for (j=1; j<=n; j=j+i)
      print ("*");
  }
}
```

→

for $i=1 \rightarrow j = 1,2,3,4,\dots,n \quad = n$

for $i=2 \rightarrow j = 1,3,5,\dots,n \quad = n/2$

for $i=3 \rightarrow j = 1,4,7\dots,n \quad = n/3$

⋮

for $i=n \rightarrow j = 1,\dots,n \quad = 1$

$$\Rightarrow \sum_{j=n}^{1} n + n/2 + n/3 + n/4 + \dots + 1$$

$$\Rightarrow \sum_{j=n}^{1} n\,[1 + 1/2 + 1/3 + \dots + 1/n]$$

$$\Rightarrow \sum_{j=n}^{1} n \log n \Rightarrow T(n) = [n \log n]$$

$$\boxed{T(n) = O(n \log n)}$$

Ques (10) → As Given, n^k & c^n

relation b/w n^k & c^n is $\boxed{n^k = O(c^n)}$

as $n^k \leq ac^n \quad \forall\ n \geq n_0$ for (a) Constant ($a>0$)

for $n_0 = 1$

$c = 2$

$\Rightarrow 1^k \leq a2^1$

$\therefore \boxed{n_0 = 1 \text{ & } c = 2}$