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SECTION - D
ROLL NUMBER - 31

# Tutorial - 4

Qol (1)

$$T(n) = 3T\left(\frac{n}{2}\right) + n^2$$

$a = 3$, $b = 2$, $f(n) = n^2$

∵ a, b are Constant and $f(n)$ is the function
∵ Master's theorem is applicable

$$c = \log_b a$$
$$= \log_2 3 = 1.58$$

$$n^c = n^{1.58}$$

which is $n^2> n^{1.58}$

Case 3 is applied here

$$\boxed{T(n) = \Theta(n^2)}$$

Qol (2)

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2$$

$a = 4$, $b = 2$, $f(n) = n^2$

∵ a & b are Constant and $f(n)$ is the function
∵ Master's theorem is applicable

$$c = \log_b a$$
$$= \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$$

$\therefore \quad n^c = n^2$

$n^c = f(n)$

$\therefore$ Case 2 is applied

$[T(n) = \theta(n^2 \log n)]$

**Q.No (3)** $T(n) = T\left(\frac{n}{2}\right) + 2^n$

$a = 1, \quad b = 2, \quad f(n) = 2^n$

a and b are constant & $f(n)$ is +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 1$

$n^c = n^0 = 1$

$f(n) \geq n^c$

$\therefore$ Case 3 is applied

$[T(n) = \theta(2^n)]$

**Q.No (4)** $T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$

$a = 2^n, \quad b = 2, \quad f(n) = n^n$

$\because$ a is not constant, its value depends on n

$\therefore$ Master's theorem is not applicable here.

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Sol (5)

$$T(n) = 16T\left(\frac{n}{4}\right) + n$$

$a = 16$, $b = 4$, $f(n) = n$

∵ a & b are Constant and $f(n)$ is the function

$$c = \log_b a$$

$$= \log_4 16 = \log_4 4^2 = 2\log_4 4 = 2$$

$$n^c = n^2$$

∵ $f(n) < n^c$

Case 1 is applied here

$$\boxed{T(n) = \theta(n^2)}$$

Sol (6)

$$T(n) = 2T\left(\frac{n}{2}\right) + n\log n$$

$a = 2$, $b = 2$, $f(n) = n\log n$

∵ a, b are Constant and $f(n)$ is the function

$$c = \log_b a = \log_2 2 = 1$$

$$n^c = n^1 = n$$

∵ $f(n) > n^c$

Case 3 is applied

$$\boxed{T(n) = \theta(n\log n)}$$

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Sol (1)

$$T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$$

$a = 2, b = 2, \quad f(n) = n/\log n$

∵ a & b are constant and $f(n)$ is the function

$$c = \log_b a = \log_2 2 = 1$$

$$n^c = n^1 = n$$

∵ non-polynomial difference b/w $f(n)$ and $n^c$

∴ Master's theorem is not applicable.

Sol (2)

$$T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$$

$a = 2, b = 4, \quad f(n) = n^{0.51}$

∴ a & b are constant and $f(n)$ is the function

∴ Master's theorem is applicable

$$c = \log_b a = \log_4 2 = 0.50$$

$$n^c = n^{0.50}$$

$$f(n) > n^c$$

Case 3 is applied

$$\boxed{T(n) = \Theta(n^{0.5})}$$

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Sol (9)

$$T(n) = 0.5\,T\left(\frac{n}{2}\right) + \frac{1}{n}$$

$a = 0.5$, $b = 2$, $f(n) = \frac{1}{n}$

$\because a < 1$

Masters theorem is not applicable

Sol (10)

$$T(n) = 16\,T\left(\frac{n}{4}\right) + n!$$

$a = 16$, $b = 4$, $f(n) = n!$

$\because$ a and b are Constant and $f(n)$ is the function

Masters theorem is applicable

$$c = \log_b a = \log_4 16 = 2\log_4 4 = 2$$

$$n^c = n^2$$

$$f(n) > n^c$$

Case (3) applied $\boxed{T(n) = \Theta(n!)}$

Sol (11)

$$T(n) = 4T\left(\frac{n}{2}\right) + \log n$$

$a = 4$, $b = 2$, $f(n) = \log n$

a & b are Constant, $f(n)$ is the function

$$c = \log_b a = \log_2 4 = 2$$

$$n^c = n^2$$

$$f(n) < n^c$$

Case 1 applied

$\boxed{T(n) = \Theta(n^2)}$

Sol (12)

$T(n) = T(n/2) + \log n$

$a = T_n$, $b = 2$, $f(n) = \log n$

$\therefore$ a is not Constant

Master's theorem is not applicable

Sol (13)

$T(n) = 3T\left(\frac{n}{2}\right) + n$

$a = 3$, $b = 2$, $f(n) = n$

a and b are Constant and $f(n)$ is +ve.

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 3 = 0.158$

$n^c = n^{0.158}$

$f(n) < n^2$

Case 1 is applied

$\boxed{T(n) = \Theta(n^{1.58})}$

Sol (14)

$T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

$a = 3$, $b = 3$, $f(n) = \sqrt{n}$

a and b are Constant, $f(n)$ is +ve function

Master's theorem is applicable

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$f(n) < n^c$

Case 1 applied

$\boxed{T(n) = \Theta(n)}$

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Q.No (15)

$T(n) = 4T(n/2) + c \cdot n$

$a = 4, b = 2, f(n) = c \cdot n$

a and b are Constants, $f(n)$ is the function

Master's theorem is applicable

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = n^c$

$\boxed{T(n) = \Theta(n^2)}$

Q.No (16)

$T(n) = 3T(n/4) + n \log n$

$a = 3, b = 4, f(n) = n \log n$

a and b are Constants, $f(n)$ is the function

Master's Theorem is applicable

$c = \log_b a = \log_4 3 = 0.79$

$n^c = n^{0.79}$

$f(n) > n^c$

$\boxed{T(n) = \Theta(n \log n)}$

Q.No (17)

$T(n) = 3T(n/3) + n/2$

$a = 3, b = 3, f(n) = n/2$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$f(n) = n^c$

$\boxed{T(n) = \Theta(n \log n)}$

Sol (18)

$$T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$$

$a = b$, $b = 3$, $f(n) = n^2 \log n$

$$c = \log_b a = \log_3 6 = 1.63$$

$$n^c = n^{1.63}$$

$$f(n) \succ n^c$$

Case 3

$$\boxed{T(n) = \Theta(n^2 \log n)}$$

Sol (19)

$$T(n) = 4T\left(\frac{n}{2}\right) + \frac{n}{\log n}$$

$a = 4$, $b = 2$, $f(n) = n/\log n$

$$c = \log_b a = \log_2 4 = 2$$

$$f(n) \prec n^c$$

$$\boxed{T(n) = \Theta(n^2)}$$

Sol (20)

$$T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$$

∵ a and b are Constant but function is -ive

Master's theorem is not applied

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Sol (21) $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7, \; b = 3, \; f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77}$

$f(n) > n^c$

$\boxed{T(n) = \Theta(n^2)}$

Sol (22) $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

∴ $f(n)$ is not regular function

∴ Master's theorem is not applicable