Optimisation Assignment

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FWC22066 IITH Future Wireless Communication (FWC) ASSIGN-6

Contents

1 Problem

Find a point on the curve $x^2+2y^2=6$ whose distance from the line x+y=7 is minimum

2 Construction

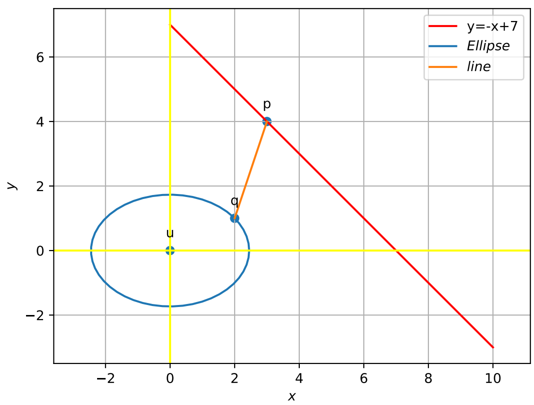

Figure of construction

given equation

$$x^2+2y^2=6 \tag{1}$$
$$\mathbf{x}+\mathbf{y}=7 \tag{2}$$

we can write as

$$\frac{x^2}{6}+\frac{y^2}{3}=1 \tag{3}$$

The dimensions of the figure is taken as below

Symbol	Value
a	$\sqrt{6}$
b	$\sqrt{3}$

3 Solution

Ellipse equation :

$$\frac{x^2}{6}+\frac{y^2}{3}=1 \tag{4}$$

The standard equation of the conics is given as :

$$\mathbf{x}^\top\mathbf{V}\mathbf{x}+2\mathbf{u}^\top\mathbf{x}+f=0 \tag{5}$$

For ellipse given that

$$\mathbf{u}=\begin{pmatrix}0\\0\end{pmatrix} \tag{6}$$

From major axes equation of ellipse

$$a=\sqrt{\frac{\mathbf{u^TV^{-1}u}-f}{\lambda_1}} \tag{7}$$
$$a=\sqrt{\frac{-f}{\lambda_1}} \tag{8}$$
$$a^2=\frac{-f}{\lambda_1} \tag{9}$$

$$\therefore \lambda_1=\frac{-f}{a^2} \tag{10}$$

From minor axes equation of ellipse

$$b=\sqrt{\frac{\mathbf{u^TV^{-1}u}-f}{\lambda_2}} \tag{11}$$
$$b=\sqrt{\frac{-f}{\lambda_2}} \tag{12}$$
$$b^2=\frac{-f}{\lambda_2} \tag{13}$$

$$\therefore \lambda_2=\frac{-f}{b^2} \tag{14}$$
$$\therefore \lambda_1=-f/a^2 \;\; and \;\; \lambda_2=-f/b^2 \tag{15}$$

$$\mathbf{V}=\begin{pmatrix}\lambda_1 & 0\\0 & \lambda_2\end{pmatrix} \tag{16}$$

$$\mathbf{V}=\begin{pmatrix}-f/a^2 & 0\\0 & -f/b^2\end{pmatrix} \tag{17}$$

By substituting (16) in (4) we will get

$$\begin{pmatrix}x & y\end{pmatrix}\begin{pmatrix}-f/a^2 & 0\\0 & -f/b^2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+f=0 \tag{18}$$

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} -f/a^2 & 0 \\ 0 & -f/b^2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -f \tag{19}$$

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1 \tag{20}$$

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1/6 & 0 \\ 0 & 1/3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1 \tag{21}$$

by using eq-14
$\therefore \lambda_1 = 1/a^2 \Rightarrow (1/6) \;\; and \;\; \lambda_2 = -f/b^2 \Rightarrow (1/3)$

$$\mathbf{V} = \begin{pmatrix} 1/6 & 0 \\ 0 & 1/3 \end{pmatrix} andf = -1 \tag{22}$$

For finding the point

$$\mathbf{q} = \mathbf{V}^{-1}(k\mathbf{n} - \mathbf{u}) \tag{23}$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{24}$$

And the intermediate parameters are given by

$$k = \pm\sqrt{\frac{\mathbf{u}^T\mathbf{v}^{-1}\mathbf{u} - f}{\mathbf{n}^T\mathbf{v}^{-1}\mathbf{n}}} \tag{25}$$

substitute eq 5 and 21 in eq-23
we get $k = \pm\frac{1}{3}$
substitute k in eq-22
yielding

$$\mathbf{q} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \mathbf{m} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} -7 \\ 0 \end{pmatrix}, c = 7 \tag{26}$$

equation of the line
$\mathbf{n^T x} = c$

For finding the distance

$$d^2 = \|\mathbf{x} - \mathbf{q}\|^2 \tag{27}$$

The parametric equation of the line is

$$\mathbf{x} = \mathbf{A} + \lambda\mathbf{m} \tag{28}$$

by 27 and 28 we can write as

$$d^2 = \|(\mathbf{A} + \lambda\mathbf{m}) - \mathbf{q}\|^2 \tag{29}$$

yielding,

$$\lambda_{min} = -\frac{\mathbf{m^T}(\mathbf{A} - \mathbf{q})}{\|\mathbf{m}\|^2} \tag{30}$$

substitute 26 in eq 30
yielding $\lambda_{min} = -4$

$$\mathbf{x} = \begin{pmatrix} -7 \\ 0 \end{pmatrix} + -4\begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{31}$$

yielding
$\mathbf{x} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

substitute in eq 29
yielding,
$\therefore d = \sqrt{10} = 3.16$

$\therefore$The equation of ellipse is

$$\frac{x^2}{6} + \frac{y^2}{3} = 1 \tag{32}$$

4 Execution

Verify the above problem in the following code.
https://github.com/Radhikarkv/fwcproject.git