Sorbonne Université

Calcul haute performance : programmation et algorithmique avancées

Projet : Batch merge and merge path sort

Code source et présentation du travail le 25/11/2021

Le code source doit être lisible et suffisamment commenté

La durée des présentations est de ∼ 15 minutes par binôme

Based on the merging sort presented in [1], this project has three parts. The first part deals with sorting an array on GPU. The second part deals with batch merging various small arrays at the same time. The last part is left to students to propose applications involving merging arrays. In the following, given a set A, the cardinal $|A|$ denotes the number of its elements.

1 Merge path and sort

We start with the merge path algorithm. Let A and B be two ordered arrays (increasing order), we want to merge them in an M sorted array. The merge of A and B is based on a path that starts at the top-left corner of the $|A| \times |B|$ grid and arrives at the down-right corner. The Sequential Merge Path is given by Algorithm 1 and an example is provided in Figure 1.

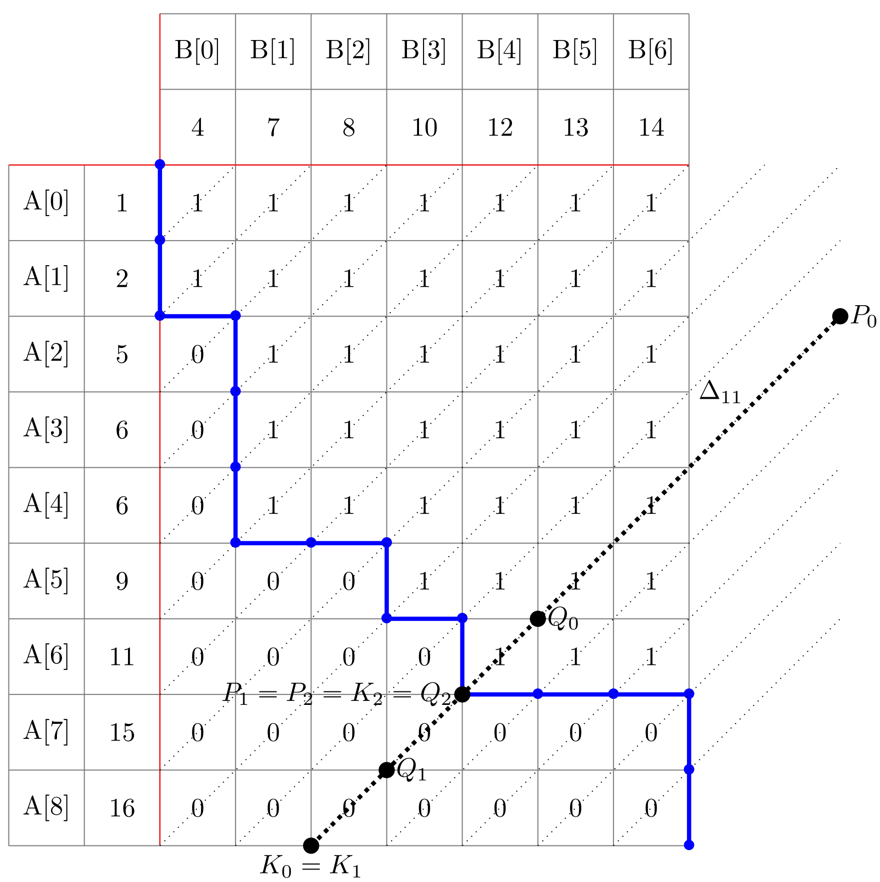

Figure 1: An example of Merge Path procedure

Algorithm 1 Sequential Merge Path

Require: A and B are two sorted arrays
Ensure: M is the merged array of A and B with $|M| = |A| + |B|$

```
procedure MergePath (A, B, M)
    j = 0 and i = 0
    while i + j < |M| do
        if i ≥ |A| then
            M[i+j]=B[j]
            j = j + 1                                  ▷ The path goes right
        else if j ≥ |B| or A[i] < B[j] then
            M[i+j]=A[i]                                ▷ The path goes down
            i = i + 1
        else
            M[i+j]=B[j]
            j = j + 1                                  ▷ The path goes right
        end if
    end while
end procedure
```

Algorithm 2 Merge Path (Indexes of n threads are 0 to $n-1$)

Require: A and B are two sorted arrays
Ensure: M is the merged array of A and B with $|M| = |A| + |B|$

```
for each thread in parallel do
    i=index of the thread
    if i > |A| then
        K = (i − |A|, |A|)                             ▷ Low point of diagonal
        P = (|A|, i − |A|)                             ▷ High point of diagonal
    else
        K = (0, i)
        P = (i, 0)
    end if
    while True do
        offset = abs(K_y − P_y)/2
        Q = (K_x + offset, K_y − offset)
        if Q_y ≥ 0 and Q_x ≤ B and
          (Q_y = |A| or Q_x = 0 or A[Q_y] > B[Q_x − 1]) then
            if Q_x = |B| or Q_y = 0 or A[Q_y − 1] ≤ B[Q_x] then
                if Q_y < |A| and (Q_x = |B| or A[Q_y] ≤ B[Q_x]) then
                    M[i] = A[Q_y]                      ▷ Merge in M
                else
                    M[i] = B[Q_x]
                end if
                Break
            else
                K = (Q_x + 1, Q_y − 1)
            end if
        else
            P = (Q_x − 1, Q_y + 1)
        end if
    end while
end for
```

Each point of the grid has a coordinate $(i, j) \in [\![0, |A|]\!] \times [\![0, |B|]\!]$. The merge path starts from the point $(i,j) = (0,0)$ on the left top corner of the grid. If $A[i] < B[j]$ the path goes down else it goes right. The array $[\![0, |A|-1]\!] \times [\![0, |B|-1]\!]$ of boolean values $A[i] < B[j]$ is not important in the algorithm. However, it shows clearly that the merge path is a frontier between ones and zeros.

To parallelize the algorithm, the grid has to be extended to the maximum size equal to $\max(|A|, |B|) \times \max(|A|, |B|)$. We denote K_0 and P_0 respectively the low point and the high point of the ascending diagonals Δ_k. On GPU, each thread $k \in [\![0, |A| + |B| - 1]\!]$ is responsible of one diagonal. It finds the intersection of the merge path and the diagonal Δ_k with a binary search described in Algorithm 2.

1. For $|A| + |B| \leq 1024$, write a kernel `mergeSmall_k` that merges A and B using only one block of threads.

As mentioned in [1], merge path algorithm is divided into 2 stages: partitioning stage and merging stage. The partitioning stage is important to propose an algorithm that involves various blocks.

2. For any size $|A| + |B| = d$ sufficiently smaller than the global memory, write a solution that merges A and B using various blocks.

3. Looping on appropriate calls of the solution proposed in question 2, write a function that sorts any array M of size d sufficiently smaller than the global memory. Give the execution time with respect to d.

2 Batch merge

In this part, we assume that we have a large number $N(\geq 1e3)$ of arrays $\{A_i\}_{1\leq i\leq N}$ and $\{B_i\}_{1\leq i\leq N}$ with $|A_i| + |B_i| = d \leq 1024$ for each i. Using some changes on `mergeSmall_k`, we would like to write `mergeSmallBatch_k` that merges two by two, for each i, A_i and B_i.

Given a fixed common size $d \leq 1024$, `mergeSmallBatch_k` is launched using the syntax

```
mergeSmallBatch_k<<<numBlocks, threadsPerBlock>>>(...);
```

with `threadsPerBlock` is multiple of d but smaller than 1024 and `numBlocks` is an arbitrary sufficiently big number.

4. Explain why the indices
```
int tidx = threadIdx.x%d;
int Qt = (threadIdx.x-tidx)/d;
int gbx = Qt + blockIdx.x*(blockDim.x/d);
```
are important in the definition of `mergeSmallBatch_k`.

5. Write the kernel `mergeSmallBatch_k` that batch merges two by two $\{A_i\}_{1\leq i\leq N}$ and $\{B_i\}_{1\leq i\leq N}$. Give the execution time with respect to $d = 4, 8, ..., 1024$.

3 Merge sort applications

You are free to give a specific application of either question 3 or question 5, like establishing histograms or finding the q-quantiles.

References

[1] O. Green, R. McColl and D. A. Bader GPU Merge Path - A GPU Merging Algorithm. *26th ACM International Conference on Supercomputing (ICS),* San Servolo Island, Venice, Italy, June 25-29, 2012.