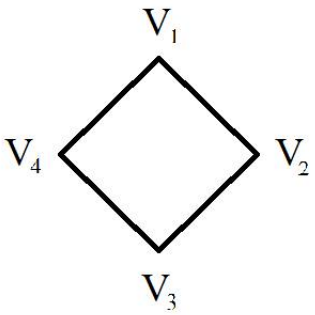
1 Abelian Sandpile Model

What is it?

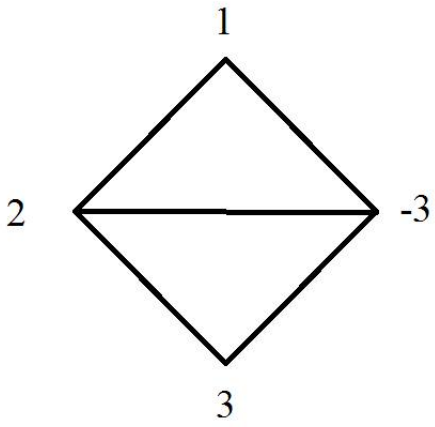
We can think of the Abelian Sandpile Model as a lending and borrowing game. In this game, we can call the figure a bank. The sum of all the money in the bank is the degree. The vertices of the figure are people. When two vertices are connected, this means they are friends. Each person is assigned a number which is the amount of money they have. This can be positive or negative(in debt).

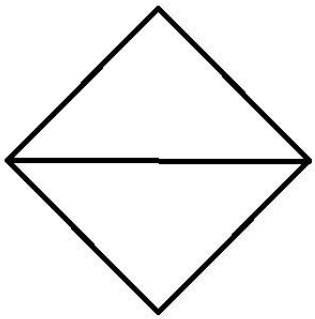
The aim of this game is to get all of the people, who have a negative number, non-negative. To do this, any vertex/person may give 1 dollar to each of its friends. If someone lends a dollar to a friend, it must lend a dollar to each of its friends (the other friends would become jelous!) Sometimes this game is unsolvable when someone can never get out of debt.

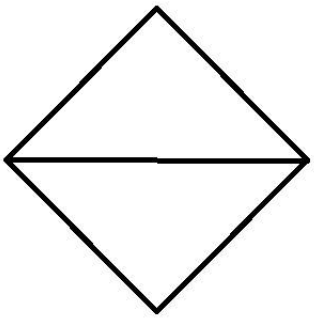
To show the different vertices, we usually label them with v_n where n is any positive integer.

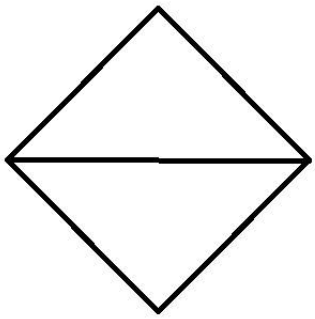


Now let's try a problem. We will give you a hint: there are three lending processes (four diagrams for you to work on).





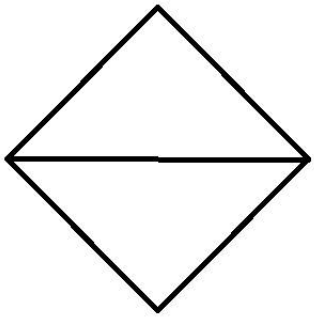




2 Strategies

1. If the degree is negative, the problem is unsolvable. Can anyone explain why?

- Does the degree ever change throughout the lending?
- If the degree is positive, does it mean the problem is solvable?

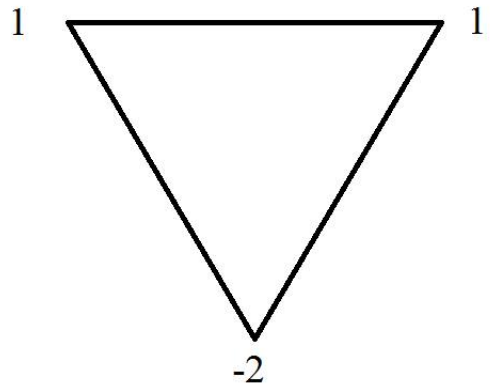


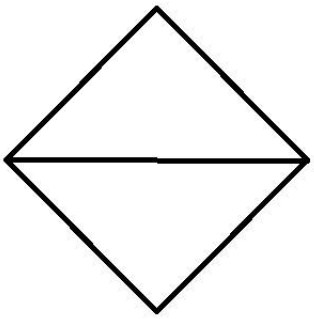
- Is this problem above solvable?

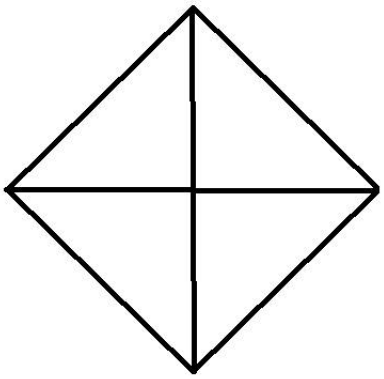
2. When you are lending, you will be more successful at winning when you lend with numbers that are positive and bigger. For example, in the first problem, if you try lending with the person with the largest amount of money, you can solve the problem.

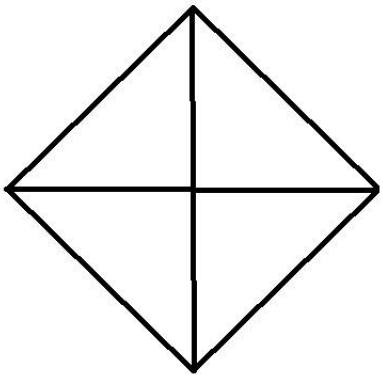
3 Problems to Try

Try some of these problems with different shapes! If there is not enough space, use scratch paper.









4 Challenge Problem