1 What is a random number?

When someone asks for a random number in a certain range, they are usually looking for a number in that range that is being selected without any method, where every number has an equal probability of occurring. If you have ever been asked to provide a random number, you most likely have not given a random number. This is because the first numbers that often come to mind are numbers we have heard very often. When asked to give a number between $1 - 100$ inclusive, common answers are 1, 42, 50, and 100. In fact, people also tend to choose prime numbers with a high probability. People only start to become more random as they are given more time.

2 What is the modulo operation?

You have used the modulo operation many times without actually knowing it. It is a fancy way of saying remainder. The modulo operation looks like %, and the statement $a\%b$ asks for the remainder when a is divided by b. For example, $5\%3 = 2$ because 2 is the remainder. Practice modulo on these practice problems.

1. $17\%3 =$

2. $99\%10 =$

3. $42121\%1000 =$

4. $(438912 + 10234)\%10 =$

5. $(8 * 7)\%6 =$

6. (CHALLENGE) $6^{10}\%7 =$

3 Creating a Random Algorithm

So how does one create a random number generator? A function is used to generate these random numbers, but it cannot be any function. To have a true number generator, it must generate a different number every **time** someone asks for a number, and it shouldn't follow a fixed cycle. For example, to generate a random number from $1 - 10$, what would be wrong with doing $(x^2 + 1)\%10$ where x increases every **time** someone tries making a new random number. List out the first 15 "random" numbers that would be generated through this method.

Clearly, we need to involve more variables, specifically ones that can allow for the user to get a different random number at a different **time**. In fact, you may have noticed that the word **time** is bolded. This is because we can make our function time dependent so the user gets a different random number no matter what time they try and produce a random number. We could make our function just dependent on the time of day, but even this will cycle every 12 hours (24 if you use a 24-hour clock). Clearly, we must use all the time variables we have at our disposal. Using the space below, brainstorm every time variable that you can come up with.

Now that you have brainstormed all of this, create your random function by manipulating every variable.

4   Monte Carlo Algorithm

A Monte Carlo Algorithm occurs where we want to approximate something with a small margin of error. We repeat a trial over and over again, to figure out what a value is. This algorithm uses random numbers to best approximate the values in question.

4.1   Examples

1. Approximating π with a circle (Buffon and Laplace).

2. Approximating the fairness of a coin.

3. Estimating where you are in a room (Robotics application)

5   Las Vegas Algorithm

A Las Vegas Algorithm always produces the right answer using random numbers to generate all possible solutions to a problem. The random element makes this slightly more efficient that simply calculating all possible solutions in a predetermined order.

5.1   Examples

1. Quicksort

2. Eight Queens Problem