1 Introduction

Probability is a tool which we use to find how likely it is that an event will occur. A classic example is flipping a coin. There are two possible outcomes: the coin could come up heads or tails. Since we don't know which one will happen, we can say that there is a probability for each of the two events. Probabilities are written as numbers between 0 and 1, with 0 meaning the outcome definitely will not occur and 1 meaning it definitely will occur. You expect that if you flip a coin it is equally likely to come up heads or tails, so the probability is 1/2 for each outcome. Counting refers to counting the number of outcomes (there are two when you flip a coin).

2 Coin Games

1. Flip a fair coin twice. What is the probability that you get two heads (HH)?

   What is the probability that you get heads followed by tails (HT)?

   Are these probabilities the same?

2. Flip a fair coin repeatedly until you get two heads in a row (HH). On average, how many flips should this take?

   What if we flip until we get heads followed by tails (HT)?

   Are the answers the same?

3. Let's play a game! Pick a partner, and decide whether you are HH (heads-heads) or HT (heads-tails). Now flip a coin repeatedly until you get either HH or HT. The person whose sequence is flipped first wins.

   Who is more likely to win, and by how much?

4. Now, Between you and your partner, decide whether you are HHT or THH. Now flip a fair coin repeatedly until you guys get HHT in a row or THH. Who is more likely to win, and by how much?

3 Penney's Game

Let's keep playing! Pick a partner, and decide whether you are player A or B. Each of you selects a sequence of heads and tails of length 3, and shows this sequence to the other player. Now flip a penny in sets of 3. The winner is the one whose sequence is flipped first!

	Player A:	Player B:
Sequence	__ __ __	__ __ __
Winner		
Probability		

Now, calculate the probability of your sequence. Compare it to your partner's probability. Who was more likely to win? Who actually won? How do you maximize your chance at winning this game? Is the first or second person more likely to win?

¡¡IMPORTANT!!

This is an example of how probability can PREDICT the likelihood of an event but DOES NOT GUARANTEE that the more likely event will happen. We can see this if the person who had the higher probability still lost. However, if we repeated this game many, many times, we would EXPECT that the person with the higher probability of winning would win.

4 Boxes, Treats, and Tricks

It's Halloween night, and you're trick-or-treating at the scariest haunted house. The door opens, and you are shown three closed boxes (A B C), and you have to choose one. One box is full of candy, and the others are filled with spiders. But the haunted boss wants to confuse you (because she doesn't want you to take her candy). If you chose a box, for example, A, the boss might open box C and show you that it has a hairy tarantula. She will then say, "Do you want to change your box to box B?" In order to have a better chance of getting the candy, should you switch to the other box?

5 Counting

For more difficult probability questions, we often need to count the number of outcomes and the number of outcomes that we want. For example, how do we find the probability of getting three heads in a row when flipping a coin? We would first have to count the total number of sequences that are possible (8): HHH,HHT, HTH, THH, HTT, THT, TTH, TTT. And how many of these possibilities do we want? Only one (HHH). So our probability of getting HHH is 1/8. Let's practice counting:

How many odd numbers are there from 1 to 50?

Pretty simple right? Now let's try a more challenging question:

How many two digit numbers are NOT multiples of 7?

Did you count ALL the numbers that weren't multiples of 7? Is there a faster way???

This question is an example of where we would use complementary counting, where we count the number of items that we don't want and subtract that from the total number of items (TOTAL - DON'T WANT = WANT). We often use this technique when it is easier to count the number of items that we don't want (it is easier to count multiples of 7 than count all the numbers that aren't multiples of7).

6 Counting AND Probability (Challenge)

Now let's use what we know about counting AND probability!

1. How many non-multiples of 7 from 1-100 are odd?
2. Suppose we order a surprise pizza with two topping. The pizza toppings can be pepperoni, sausage, green peppers, onions, mushrooms, and pineapple. What is the probability that we get a pizza with onions and pineapple?

7 Coin Prediction

Tired of coins yet? I hope not, because this last game is about coins too. I have 100 coins. Half of them are face up, half of them are face down. I will keep showing each coin one by one. If you say stop, and the next coin is a head, I win. If the next coin is a tail, you loose. If you pick the second to last coin, the last coin will decide the winner or looser. What should you do to win?