Math Riddle of the Day: If 9999 = 4, 8888 = 8, 1816 = 6, 1212 = 0, then what does 1919 equal?

1 Theory

What is Induction? Induction is a method to prove statements that are usually true for all natural numbers. Induction works by first, proving that P(1) is true, and then proving the statement, "If P(n) is true, P(n+1) is also true."

To understand why this works, we can use the analogy of dominoes. To prove that a line of dominoes will all fall when we push the first one, we just have to prove that:

1. The first domino falls down (base case)
2. The dominoes are close enough that each domino will knock over the next one when it falls (step)

2 Examples:

Example 1: Using induction, prove that $1 + 2 + 3 + 4 + ... + n = \frac{n(n+1)}{2}$

Example 2: Given n circles in a plane, prove that we can color the regions determined by them with two colors s.t. any two neighboring regions have different colors.

Example 3: Prove Fermat's Little Theorem, or $a^p \equiv a \mod p$. Prove the base case for a = 1: $1^p \equiv 1 \mod p$ is true because $1^d = 1$ for any positive integer d.

Example 4 In the game Survivor, people have pebbles. They can add either 1,2,3, or 4 pebbles gto the pile. The person places the 21st pebble in the pile loses. Prove by induction that all multiples of 5 are P-positions. *A P-position is a position in which the previous player will win (who moved to that position) and a N-position is a position where the next player will win (who moves away from that position.

3 Problems:

Problem 1: Using induction, prove that $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{(n)(n+1)(2n+1)}{6}$ for all natural numbers n.

Problem 2: $5^n - 1$ is divisible by 4.

Problem 3: Show that an isosceles triangle triangle with one angle of 120 can be partitioned into n triangles similar to it.

Problem 4: Prove that every positive integer can be written in infinitely many ways in the form:
$n = \pm 1^2 \pm 2^2 \pm ... \pm m^2$

Problem 5: $2n$ dots are placed around the outside of the circle. n of them are colored red and the remaining n are colored blue. Going around the circle clockwise, you keep a count of how many red and blue dots you have passed. If at all times the number of red dots you have passed is at least the number of blue dots, you consider it a successful trip around the circle. Prove that no matter how the dots are colored red and blue, it is possible to have a successful trip around the circle if you start at the correct point.

Problem 6: On a circular route, there are n identical cars. Together, they have enough gas to make a complete tour. Prove that there is a car that can make a complete tour by taking the gas from all the cars that it encounters.

Problem 7: Prove, by Mathematical Induction, that n(n + 1)(n + 2)(n + 3) is divisible by 24, for all natural numbers n.

4 Challenge Problems

Problem 1: Prove, by Mathematical Induction, that n(n + 1)(n + 2)(n + 3) (n + r 1) is divisible by r! , for all natural numbers n, where r = 1, 2, . . .

Problem 2: A circular loop of wire has a radius of 0.025 m and a resistance of 3.0 . It is placed in a 1.6 T magnetic field which is directed in through the loop as shown and then turned off uniformly over a period of 0.10 s. What is the current in the wire during the time that the magnetic field changes from 1.6 T to zero? (Note: this requires a special type of induction: magnetic induction)