1 Liar, Liar

Take a look at these questions and determine what these statements really mean.

1. Suppose a liar claims that his friend is telling the truth. What does this mean?

   He is a liar too, or he is not his friend, or his friend is not him, etc.

2. If a liar says that a marble is blue, what does this mean?

   The marble is not blue.

3. If a liar says that it is raining, what does this mean?

   It is not raining.

Now, for a harder question: if someone says they are a liar, are they telling the truth or lying?

Impossible. It's a paradox.

2 Knights and Liars

There is an island far away, whose inhabitants are quite unusual. Some of them never tell a lie, and others never utter a true statement. Those that who never lie call themselves Knights. Those who never tell the truth are known as Liars. Even though they have different personalities, knights and liars cannot be told apart by appearance: they wear the same types of clothes, and have the same haircuts. Sometimes tourists visit this island. The Tourists are regular people: sometimes they lie, and sometimes they tell the truth. A few of the following problems are about the inhabitants of this island.

1. While visiting this island, you meet a boy who says he a liar. Does he live on this island?

   He does not live on the island because neither a knight nor a liar would say they are a liar.

2. Two island boys, Sam and Kurt, are introducing themselves to you. Sam says, "At least one of us is a liar." Who is/are the liar(s)?

   Sam is a knight and Kurt is a liar. If Sam were a liar then his statement would be true which would contradict the assumption that he was a liar. Therefore, he must be a knight, his statement must be true, and Kurt must be the liar

3. While visiting this island, I had a conversation with a local knight. I asked him the same question twice, and he gave me two different answers. What was my question?

   A variety of answers are possible, for example, What time is it?, How many questions have I asked? etc.

4. At the island, you meet two islanders, Ben and Jerry. Ben says at least one of the two is a liar. Is Ben a knight or a liar? What about Jerry?

   Ben is a knight and Jerry is a liar. If Ben were a liar, then his statement would be true, which would contradict the assumption that he was a liar. Therefore, he must be a knight, his statement must be true, and Jerry must be the liar.

5. While visiting the island, you pass a beautiful garden where you meet three islanders, Kevin, James, and Kyle. You ask Kevin, "Are you a Knight or a Liar?" Kevin ignores you. You then ask James, "Is Kevin a Knight or a Liar?" James says, "Kevin is a liar." Then, Kylie screams, "Don't listen to James! He's a liar!" Is James a liar? Is Kylie a liar?

   First, we know that no liar would say he is a liar, since that would be the truth. Therefore, James must be a liar. Kylie, then, is telling the truth and must be a knight.

3 A Couple Logic Puzzles

1. Twenty kids came to a math circle. Out of 13 of them, there was at least one boy. How many girls are there?
   When you choose thirteen kids, there must be seven leftover people. In order to guarantee that there is a boy in the group of 13, there must be eight boys. (This is Pigeonhole)

2. The weight of a gold bar is three-fourths of itself plus three pounds. How heavy is the bar?
   The 3 pounds must be 1/4 of the weight of the bar, thus, the whole bar must be twelve pounds.

3. If you had a 5-liter bowl and a 3-liter bowl, and an unlimited access to water, how would you measure exactly 4 liters?
   First, you pour water into the 5-liter bowl. Then, you pour three liters into the 3-liter bowl. You pour those three liters out, and pour the two liters from the remaining water of the 5-liter bowl into the 3-liter bowl. You then pour water once again into the 5-liter bowl, and you pour one liter (because there are already two liters in the 3-liter bowl) into the 3-liter bowl. Now, only four liters remain in the five liter bowl

4. Peter says, "The day before yesterday, I was 10, but next year I will turn 13." How is this possible, if we are sure that Peter is not lying?

   Peter's birthday is on December 31.

5. Julia is walking home from school. She left the school 5 minutes earlier than her next-door neighbor, Josh. However, Josh is in a hurry because he wants to give Julia the cell phone that she left at school. Josh is walking 1.5 times faster than Julia. How soon will Julia get her cell phone back?

   Julia will get her cell phone back after 10 minutes.

4 Logic Challenge

Three types of magic fruit apples of wisdom, pears of bravery, and plums of kindness grow on the Magic Tree in the center of the Far Away Kingdom. From time to time, some of the fruits are harvested for the benefit of the Kingdom. The Magic Tree immediately regrows the picked fruit according to the following set of rules:

1. If a single fruit is picked from the tree, another of the same kind grows in its place.
2. If 2 apples are picked, 4 pears grow back.
3. If 2 pears are picked, 4 plums grow back.

4. If 2 plums are picked, 4 apples grow back.

5. If 2 fruits of different kinds are picked, nothing else happens.

Currently, the tree has 11 apples, 10 pears and 8 plums. The wicked witch planes to weaken the Kingdom by stealing all the fruit. She intends to sneak to the tree several mornings in a row and pick one or two fruits every time. Is there a way for her to pick all the fruits of the tree? Either show how or explain why not.

Originally, the tree has 29 fruits (an odd number). This number either remains the same, or goes up or down by 2. Therefore, whatever, the witch does, the number of fruits will still remain odd. Therefore, 0 is not possible. (It is possible to reduce the tree to a single fruit of one type though. For example, pick a pear and a plum 4 times, then pick an apple and a plum 4 times, then pick a pear and an apple six times. This leaves a single apple that will be replaced by a single apple every time it is picked.)