LeCHe I (January 2025)

Elementary Integrals

{ Leandros's Calculus Heaven }

The following is a practice exam designed to gauge your knowledge of elementary integrals. Each question is worth 4 points and are sorted into different stages of difficulty (named "levels"). Level 2s are the simplest questions, consisting of simple computational and conceptual challenges. Level 3s are of more intermediate difficulty, consisting of advanced computational and indermediate conceptual challenges. Level 4s are the hardest type of questions, consisting of advanced conceptual challenges which demand a mastery and thorough understanding of all concepts involved.
This exam will test you on the following concepts:

- Left, right, midpoint, and trapezoidal sums
- Definition of integral as limit of riemman sum
- Fundamental theorem of calculus
- Basic rules of integrals
- Reverse power rules
- U-substitution
- Reverse trig identities
- Mean value theorem
- Integration by dividing each theorem
- Integration by dividing by polynomial

You have 60 minutes to complete the entire test. Show **all** work, read each question carefully, follow all instructions

Name: ____

For grader use only

Final score: ________ / 184 Comments: ____________________________

1. For each of the following equations, compute the riemann sum in the provided form and range, given that there are n subintervals
   (a) (4 points) (Level 2) compute the right riemann sum: $f(x) = 3x$; $[0, 4]$; $n = 4$ subintervals

   (b) (4 points) (Level 2) compute the left riemann sum: $f(x) = x^2$; $[2, 8]$; $n = 3$ subintervals

   (c) (4 points) (Level 3) compute the midpoint riemann sum: $f(x) = \sin(x)$; $[0, 2\pi]$; $n = 4$ subintervals

   (d) (4 points) (Level 3) compute the trapezoidal riemann sum: $f(x) = x^3 - 2x^2 + x + 8$; $[-5, 3]$; $n = 4$ subintervals

   (e) (4 points) (Level 3) compute the trapezoidal riemann sum: $f(x) = \sqrt{3x + 2}$; $[0, 6]$; $n = 3$ subintervals

2. Use the Fundamental Theorem of Calculus to evaluate each of the following expressions

   (a) (4 points) (Level 2) $\frac{d}{dx}\int_5^x x^3\,dx$

   (b) (4 points) (Level 2) $\frac{d}{dx}\int_0^x e^x\,dx$

   (c) (4 points) (Level 3) $\frac{d}{dx}\int_{-\pi}^{\sin(x)} \cos(x)\,dx$

   (d) (4 points) (Level 3) $\frac{d}{dx}\int_{-x^3}^{4x^2} (\ln(\sqrt{x}))^2\,dx$

3. Use the given values and rules for integrals to solve each integral

$$\int_1^4 f(x)\,dx = 5; \quad \int_4^9 f(x)\,dx = -3; \quad \int_1^9 g(x)\,dx = 2; \quad \int_4^9 g(x)\,dx = 3; \quad \int_9^{15} g(x)\,dx = 5$$

   (a) (4 points) (Level 2) $\int_7^7 f(x)\,dx$

(b) (4 points) (Level 3) $\int_4^1 f(x) + g(x)\,dx$

(c) (4 points) (Level 3) $\int_1^9 f(x) - g(x)\,dx$

(d) (4 points) (Level 3) $\int_{15}^1 g(x)\,dx$

4. Use geometry to find the value of each integral

(a) (4 points) (Level 2) $\int_0^5 x\,dx$

(b) (4 points) (Level 2) $\int_2^6 3x\,dx$

(c) (4 points) (Level 3) $\int_{-1}^{4} |2x - 6| \, dx$

(d) (4 points) (Level 3) $\int_{-4}^{4} \sqrt{16 - x^2} \, dx$

(e) (4 points) (Level 3) $\int_{-5}^{5} 5 - \sqrt{25 - x^2} \, dx$

(f) (4 points) (Level 3) $\int_{-2}^{3} 5 - |x| \, dx$

5. Convert each integral into Newton's notation and then evaluate the integral using a summation and a limit.

   (a) (4 points) (Level 3) $\int_{0}^{5} 5x + 1 \, dx$

(b) (4 points) (Level 3) $\int_{-2}^{5} 3x^2 - 2x + 1\,dx$

(c) (4 points) (Level 3) $\int_{1}^{9} \frac{x^2}{4} - 4\,dx$

6. Evaluate

(a) (4 points) (Level 2) $\int 3x^2 + 7x - 4\,dx$

(b) (4 points) (Level 3) $\int \frac{3x^3+5x^2+2}{\sqrt{x}}\,dx$

(c) (4 points) (Level 3) $\int \frac{5x^4+5x+2}{x+1}\,dx$

(d) (4 points) (Level 3) $\int 3\sec^3 x \tan(x)\, dx$

(e) (4 points) (Level 3) $\int x^5 \sqrt{x^3}\, dx$

(f) (4 points) (Level 3) $\int \frac{1}{4+x^2}\, dx$

(g) (4 points) (Level 3) $\int \frac{8x^3-3x+1}{\sqrt[3]{x}}\, dx$

(h) (4 points) (Level 3) $\int \frac{\sin x}{\cos^2 x}\, dx$

(i) (4 points) (Level 3) $\int \frac{2x^3-3x^2+1}{x-1}\, dx$

(j) (4 points) (Level 3) $\int \sqrt{4x+3}\,dx$

(k) (4 points) (Level 3) $\int x \ln(3x^2)\,dx$

(l) (4 points) (Level 3) $\int x^2 \ln(3x^3)\,dx$

(m) (4 points) (Level 3) $\int \cos x \sqrt{\sin x}\,dx$

(n) (4 points) (Level 3) $\int \frac{1}{\sqrt{16-x^2}}\,dx$

(o) (4 points) (Level 3) $\int (1+\frac{1}{x})^2 \frac{1}{x^2}\,dx$

7. Find the average value of each function on the given interval, if MVT holds for the function, find the x value of the point in the interval whose y value corresponds to the average value
   (a) (4 points) (Level 3) e^{2x} on $[1, 5]$

   (b) (4 points) (Level 3) $\frac{x^3+1}{x+1}$ on $[3, 7]$

   (c) (4 points) (Level 3) $(x-5)^6$ on $[0, 2]$

   (d) (4 points) (Level 3) $\sqrt{2x+3}$ on $[2, 9]$

8. For each of the following derivatives, find the original function
   (a) (4 points) (Level 2) $f'(x) = 3x^2; \quad f(x) = ?$

   (b) (4 points) (Level 4) $f'(x) = 2f(x); \quad f(x) = ?$

   (c) (4 points) (Level 4) $f'(x) = x^2 f(x); \quad f(x) = ?$

9. (4 points) (Level 4) In an AC circuit, current I is given by $I = I_M \sin \omega t$, where t is the time and I_M is the maximum current. The rate P at which heat is being produced in the resistor of R ohms is given by $P = I^2 R$. Compute the average rate of production of heat over one complete cycle (from $t = 0$ to $t = \frac{2\pi}{\omega}$).

10. (4 points) (Level 4) Evaluate $\int \frac{1}{x^2 - x + 1}\, dx$