

# RLC Circuits and Laplace Transforms

## Problem Set + Notes

- Find the Laplace transforms of the given functions
  - $f(t) = (2t + 1)(t - 5)$
  - $g(t) = 4t \cos(t) \sin(t)$
  - $h(t) = \frac{t^2 + 2t}{\sqrt{t}}$
  - $a(t) = t^2 2^t + 2$
  - $b(t) = \ddot{x}(t) + 8\dot{x}(t) + 15x(t)$
- Show that the laplace transform of 1 is  $\frac{1}{s}$  using the derivative rule for laplace transforms
- Use a Laplace transform to solve the following ODEs
  - $y' + 5y = 3 \cos(2t) \quad y(0) = 2$
  - $3y' - 14y = \sqrt{t} \quad y(0) = 1$
  - $y'' + 8y' + 15 = 0 \quad y(0) = 1 \quad y'(0) = 0$
  - $2y'' + 7y' = 2t \cos(t) \sin(t) \quad y(0) = 0 \quad y'(0) = 0$
- A simple single loop circuit is constructed by wiring a  $9V$  battery, a  $3\Omega$  resistor, a  $1H$  inductor, and a  $2F$  capacitor, all wired in series. Find the current in the circuit as a function of time given that  $I_0 = 1A$  and  $I'_0 = 0 \frac{A}{s}$ . Assume wire resistivity is negligible and the circuit exhibits Ohmic behavior

5. A potentiometer is a type of resistor whose resistance can be changed. A simple loop circuit is constructed by wiring a potentiometer, a  $3H$  inductor, and a  $1F$  capacitor, all in series. A programmer hooks up a digital device to the potentiometer that adjusts the resistance of the potentiometer according to a function  $R(t)$ . Given that  $I_0 = 10mA$  and  $I'_0 = 0\frac{A}{s}$ , what must the programmer program  $R(t)$  to be if he wants to keep the current in the circuit at a constant  $10mA$ ? Assume wire resistivity is negligible and the circuit exhibits Ohmic behavior
  
  
  
  
  
  
  
  
  
  
6. An AC power supply is similar to a battery, except its voltage changes according to a sinusoidal function. What would the current in the circuit in problem #3 be if the  $9V$  battery were replaced instead with an AC power supply whose voltage was described by the function  $V = 9\sin(2\pi t)$ ? Assume the same initial conditions, that wire resistivity is negligible and the circuit exhibits Ohmic behavior