RLC Circuits and Laplace Transforms Problem Set + Notes

1. Find the Laplace transforms of the given functions
   (a) $f(t) = (2t+1)(t-5)$
   (b) $g(t) = 4t\cos(t)\sin(t)$
   (c) $h(t) = \frac{t^2+2t}{\sqrt{t}}$
   (d) $a(t) = t^2 2^t + 2$
   (e) $b(t) = \ddot{x}(t) + 8\dot{x}(t) + 15x(t)$
2. Show that the laplace transform of 1 is $\frac{1}{s}$ using the derivative rule for laplace transforms
3. Use a Laplace transform to solve the following ODEs
   (a) $y' + 5y = 3\cos(2t) \qquad y(0) = 2$
   (b) $3y' - 14y = \sqrt{t} \qquad y(0) = 1$
   (c) $y'' + 8y' + 15 = 0 \qquad y(0) = 1 \quad y'(0) = 0$
   (d) $2y'' + 7y' = 2t\cos(t)\sin(t) \qquad y(0) = 0 \quad y'(0) = 0$
4. A simple single loop circuit is constructed by wiring a $9V$ battery, a 3Ω resistor, a $1H$ inductor, and a $2F$ capacitor, all wired in series. Find the current in the circuit as a function of time given that $I_0 = 1A$ and $I_0' = 0\frac{A}{s}$. Assume wire resistivity is negligible and the circuit exhibits Ohmic behavior

5. A potentiometer is a type of resistor whose resistance can be changed. A simple loop circuit is constructed by wiring a potentiometer, a $3H$ inductor, and a $1F$ capacitor, all in series. A programmer hooks up a digital device to the potentiometer that adjusts the resistance of the potentiometer according to a function $R(t)$. Given that $I_0 = 10mA$ and $I_0' = 0\frac{A}{s}$, what must the prorammer program $R(t)$ to be if he wants to keep the current in the circuit at a constant $10mA$? Assume wire resistivity is negligible and the circuit exhibits Ohmic behavior

6. An AC power supply is similar to a battery, except its voltage changes according to a sinusoidal function. What would the current in the circuit in problem #3 be if the $9V$ battery were replaced instead with an AC power supply whose voltage was described by the function $V = 9\sin(2\pi t)$? Assume the same initial conditions, that wire resistivity is negligible and the circuit exhibits Ohmic behavior