Simplicial Geometry

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Ahoj, já mám malý penis.

Definition 1 (Simplectic manifold)**.** Let M be a smooth manifold of even dimension $2m$ and let $\omega \in \Omega^2(M)$ be a closed non degenerate 2-form i.e.

$$d\omega = 0 \text{ and } \omega^m = \omega \wedge \omega \wedge \cdots \wedge \omega \neq 0,$$

Then ω is called a *simplectic form* and the pair (M, ω) is called a *simplectic manifold*.

ekvivalentni definice nedegenerovanosti.

Narozdil od riemannovske geometrie nelze pouzit partitions of unity na konstrukci metriky.

Example 2 (Canonical symplectic structure)**.** Let $M = \mathbb{R}^2 m$ with the global coordinates $q_1, \ldots, q_m, p_1, \ldots, p_m$. and let ω be a form s.t.,

$$\omega = \sum_{i=1}^{m} dp_i \wedge dq_i.$$

Then

$$\omega^m = m! \cdot (-1)^{m(m-1)/2} \cdot dp_1 \wedge \cdots \wedge dp_m \wedge dq_1 \wedge \cdots \wedge dq_m$$

We call $R^2 m$ with the form ω the canonical symplectic structure.

Example 3 (Cotangent bundle is a symplectic manifold.)**.** Let M be a manifold of dimension m, let $\eta \in T^*M$ be a tangent covector and $\nu \in T_\eta(T^*M)$ be a tangent vector at η. Represent ν as a curve $\nu : (-\epsilon, \epsilon) \to T^*M$ s.t.

$$\nu(0) = \eta, \;\; \dot{\nu}(0) = \nu$$

Project this curve by the projection $\pi : T^*M \to M$ and apply η to the tangent vector of the projected curve

$$\theta(\nu) := \eta\Big(\frac{d}{dt}\big(\pi \circ \nu(t)|_{t=0}\big)\Big) \tag{1}$$

Then

$$\omega = d\theta$$

is a simlectic form on T^*M. Any system of coordinates $\{q_1, \ldots, q_m\}$ in M determines coordinates $\{q_1, \ldots, q_m, p_1, \ldots, p_m\}$ in T^*M by the relation $\eta = \sum p_i \cdot dq_i$. From (1) we have

TOHLE SPOCITAT!!!!!!!!!!!!!!

$$\theta = \sum_{i=1}^{m} p_i \cdot dq_i$$

and the 2-form

$$\omega = d\theta = \sum_{i=1}^{m} p_i \wedge dq_i$$

is non-degenerate.

Lemma 4 (Lemmatko)**.** $2 + 2 = 4 - 1 = 3$ *quick maffs.*

A ted si rekneme dulezitou vetu.

Theorem 5 (Hlavni veta o gaystvi)**.** *Jsi gay.*

Corollary 6. *Vlastne dusledek tohoto kratkeho textu je, ze bych se mel jit zabit. Jdu na to!*