

1 IMA1 - Domáci úkol 3

Úkol 1

$$\sqrt{1-x^3} = (1-x^3)^{\frac{1}{2}}$$
$$\left((1-x^3)^{\frac{1}{2}}\right)' = \frac{1}{2}(1-x^3)^{-\frac{1}{2}} \cdot (-3x^2) = -\frac{3}{2} \cdot \frac{x^2}{\sqrt{1-x^3}}$$

Úkol 2

$$\left(\frac{2}{\sqrt{1-x^3}}\right)' = \left(2 \cdot (1-x^3)^{-\frac{1}{2}}\right)' = 2 \cdot \left[(1-x^3)^{-\frac{1}{2}}\right]' = 2 \cdot \left(-\frac{1}{2}\right) \cdot (1-x^3)^{-\frac{3}{2}} \cdot (-3x^2) = -\left(1-x^3\right)^{-\frac{3}{2}} \cdot (-3x^2) = \frac{3x^2}{(1-x^3)^{\frac{3}{2}}}$$

Úkol 3

$$\sqrt[3]{(1-x) \cdot (2-x^3)} = [(1-x) \cdot (2-x^3)]^{\frac{1}{3}}$$
$$\left[\left[(1-x) \cdot (2-x^3)\right]^{\frac{1}{3}}\right]' = \frac{1}{3} \left((1-x) \cdot (2-x^3)\right)^{-\frac{2}{3}} \cdot \left((1-x)' \cdot (2-x^3) + (1-x) \cdot (2-x^3)'\right) =$$
$$= \frac{1}{3} \cdot \frac{1}{[(1-x)(2-x^3)]^{\frac{2}{3}}} \cdot \left((-1) \cdot (2-x^3) + (1-x) \cdot (-3x^2)\right) =$$
$$= \frac{1}{3} \cdot \frac{x^3 - 2 + 3x^3 - 3x^2}{((1-x)(2-x^3))^{\frac{2}{3}}} = \frac{1}{3} \cdot \frac{4x^3 - 3x^2 - 2}{((1-x) \cdot (2-x^3))^{\frac{2}{3}}}$$

Úkol 4

$$\sqrt[3]{(1-x) \cdot (2-x^3)^2} = \left[(1-x) \cdot (2-x^3)^2\right]^{\frac{1}{3}}$$
$$\left(\left[(1-x) \cdot (2-x^3)^2\right]^{\frac{1}{3}}\right)' = \frac{1}{3} \left((1-x) \cdot (2-x^3)^2\right)^{-\frac{2}{3}} \cdot \left((1-x)' \cdot (2-x^3)^2 + (1-x) \cdot ((2-x^3)^2)'\right) =$$
$$= \frac{1}{3} \cdot \left((1-x) \cdot (2-x^3)^2\right)^{-\frac{2}{3}} \cdot \left((-1) \cdot (2-x^3)^2 + (1-x) \cdot 2 \cdot (2-x^3) \cdot (-3x^2)\right)$$

Úkol 5

$$(5) \ln\left(\frac{x}{x^2-1}\right) = \ln x - \ln(x^2-1)$$

$$\left(\ln x - \ln(x^2-1)\right)' = \frac{1}{x} - \frac{1}{x^2-1} \cdot (2x) = \frac{1}{x} - \frac{2x}{x^2-1}$$

Úkol 6

$$\left(\sin \sqrt{1-x^3}\right)' = \cos \sqrt{1-x^3} \cdot \left((1-x^3)^{\frac{1}{2}}\right)' \leftarrow 1.pr.$$

Úkol 7

$$\left(\sqrt{\frac{1-x^2}{2x-x^2}}\right)' = \frac{1}{2} \left(\frac{1-x^2}{2x-x^2}\right)^{-\frac{1}{2}} \cdot \left(\frac{1-x^2}{2x-x^2}\right)' = \frac{1}{2} \cdot \sqrt{\frac{2x-x^2}{1-x^2}} \cdot \left(\frac{-2x(2x-x^2) - (1-x^2) \cdot (2-2x)}{(2x-x^2)^2}\right) =$$
$$= \frac{1}{2} \cdot \sqrt{\frac{2x-x^2}{1-x^2}} \cdot \frac{-4x^2 + 2x^3 - 2 + 2x + 2x^2 - 2x^3}{(2x-x^2)^2} =$$
$$= \frac{1}{2} \cdot \sqrt{\frac{2x-x^2}{1-x^2}} \cdot \frac{-2x^2 + 2x - 2}{(2x-x^2)^2} = \sqrt{\frac{2x-x^2}{1-x^2}} \cdot \frac{-x^2 + x - 1}{(2x-x^2)^2}$$