1 IMA1 - Domácí úkol 3

1. úkol

$\sqrt{1-x^3} = (1-x^3)^{\frac{1}{2}} = \left((1-x^3)^{\frac{1}{2}}\right)' = \frac{1}{2} \cdot (1 - x^3)^{-\frac{1}{2}} (-3x^2) = -\frac{3}{2} \cdot \frac{x^2}{\sqrt{1-x^3}}$

2. úkol

$\left(\frac{2}{\sqrt{1-x^3}}\right)' = 2 \cdot \left((1-x^3)^{-\frac{1}{2}}\right)' = 2 \cdot \left(-\frac{1}{2} \cdot (1-x^3)^{-\frac{3}{2}} \cdot (-3x^2)\right) = \frac{3x^2}{(1-x^3)^{\frac{3}{2}}}$

3. úkol

$$\begin{aligned}\left(\sqrt[3]{(4-x)(2-x^3)}\right)' &= \left((4-x)(2-x^3)\right)^{\frac{1}{3}\prime} \\ &= \frac{1}{3} \cdot \left((4-x)(2-x^3)\right)^{-\frac{2}{3}} \cdot \left((4-x)(-3x^2) + (2-x^3)(-1)\right) \\ &= \frac{1}{3} \cdot \frac{-3x^2(4-x) - (2-x^3)}{((4-x)(2-x^3))^{\frac{2}{3}}} \\ &= \frac{1}{3} \cdot \frac{-3x^2(4-x) - (2-x^3)}{((4-x)(2-x^3))^{\frac{2}{3}}}\end{aligned}$$

4. úkol

$$\begin{aligned}\left(\sqrt[3]{(1-x)\cdot(2-x)^2}\right)' &= \left((1-x)(2-x)^2\right)^{\frac{1}{3}\prime} \\ &= \frac{1}{3} \cdot \left((1-x)(2-x)^2\right)^{-\frac{2}{3}} \cdot \left((1-x)' \cdot (2-x)^2 + (1-x) \cdot \left((2-x)^2\right)'\right) \\ &= \frac{1}{3} \cdot \frac{-1 \cdot (2-x)^2 + (1-x) \cdot 2 \cdot (2-x) \cdot (-1)}{((1-x)(2-x)^2)^{\frac{2}{3}}}\end{aligned}$$

5. úkol

$\ln\left(\frac{x}{x^2-1}\right) = \ln x - \ln(x^2-1) \quad \Rightarrow \quad \frac{1}{x} + \frac{1}{x+1} = \frac{x+1+x}{x(x+1)} = \frac{2x+1}{x(x+1)}$

6. úkol

$$\left(\ln\left(\sqrt{1-x^2}\right)\right)' = \left(\frac{1}{2}\ln(1-x^2)\right)' = \frac{1}{2} \cdot \frac{(1-x^2)'}{1-x^2} = \frac{1}{2} \cdot \frac{-2x}{1-x^2} = \frac{-x}{1-x^2}$$

7. úkol

$$\begin{aligned}\left(\frac{\sqrt{1-x-x^2}}{\sqrt{2x-x^2}}\right)' &= \frac{1}{2} \cdot \left(\frac{(1-x-x^2)' \cdot \sqrt{2x-x^2} - (2x-x^2)' \cdot \sqrt{1-x-x^2}}{(2x-x^2) \cdot \sqrt{1-x-x^2} \cdot \sqrt{2x-x^2}}\right) \\ &= \frac{1}{2} \cdot \frac{(-1-2x) \cdot \sqrt{2x-x^2} - (2-2x) \cdot \sqrt{1-x-x^2}}{(2x-x^2) \cdot \sqrt{1-x-x^2} \cdot \sqrt{2x-x^2}}\end{aligned}$$