تمرین سری 2 درس BSP - رادین خیام - 9910579

سئوال 1 -

$$X(t) = 3\cos(2t+\varphi_1) - 2\cos(7t+\varphi_2) + 5\alpha(t)$$

$$E[X(t)] = 3E[\underbrace{\cancel{\cos(2t+\varphi_1)}}_{0}] - 2E[\underbrace{\cancel{\cos(7t+\varphi_2)}}_{0}] + 5E[\underbrace{\cancel{\alpha(t)}}_{0}] = 0$$

$$\rightarrow \boxed{\bar{X}(t) = 0}$$

$$R_X(t_1, t_2) = E[X(t_1)X^*(t_2)]$$

$$= E\Big[\big(3\cos(2t_1+\varphi_1) - 2\cos(7t_1+\varphi_2) + 5\alpha(t_1)\big)\big(3\cos(2t_2+\varphi_1) - 2\cos(7t_2+\varphi_2) + 5\alpha(t_2)\big)\Big]$$

$$= E\Big[9\cos(2t_1+\varphi_1)\cos(2t_2+\varphi_1) + 4\cos(7t_1+\varphi_2)\cos(7t_2+\varphi_2) + 25\alpha(t_1)\alpha(t_2)$$

$$\underbrace{\cancel{-6\cos(2t_1+\varphi_1)\cos(7t_2+\varphi_2)}}_{0} + \underbrace{\cancel{15\cos(2t_1+\varphi_1)\alpha(t_2)}}_{0} \underbrace{\cancel{-6\cos(7t_1+\varphi_2)\cos(2t_2+\varphi_1)}}_{0}$$

$$\underbrace{\cancel{-10\cos(7t_1+\varphi_2)\alpha(t_2)}}_{0} + \underbrace{\cancel{15\alpha(t_1)\cos(2t_2+\varphi_1)}}_{0} \underbrace{\cancel{-10\alpha(t_1)\cos(7t_2+\varphi_2)}}_{0}\Big]$$

$$\rightarrow R_X(t_1, t_2) = E\big[9\cos(2t_1+\varphi_1)\cos(2t_2+\varphi_1) + 4\cos(7t_1+\varphi_2)\cos(7t_2+\varphi_2) + 25\alpha(t_1)\alpha(t_2)\big]$$

$$= 9E[\cos(2t_1+\varphi_1)\cos(2t_2+\varphi_1)] + 4E[\cos(7t_1+\varphi_2)\cos(7t_2+\varphi_2)] + 25E[\alpha(t_1)\alpha(t_2)]$$

$$= \frac{9}{2}\cos(2(t_1-t_2)) + 2\cos(7(t_1-t_2)) + 25(4\delta(t_2-t_1))$$

$$\rightarrow \boxed{R_X(\tau) = \frac{9}{2}\cos(2\tau) + 2\cos(7\tau) + 100\delta(\tau)}$$

$$\boxed{S_X(f) = F\{R_X(\tau)\} = \frac{9}{4}\Big(\delta(f-\frac{1}{\pi}) + \delta(f+\frac{1}{\pi})\Big) + \delta(f-\frac{7}{2\pi}) + \delta(f+\frac{7}{2\pi}) + 100}$$

سؤال 2-

الف)

$$Z[n] = aX[n] + bY[n] + cn^2 + n$$

$$E[Z[n]] = \overset{0}{\cancel{E[a]}}\,E[X[n]] + \overset{0}{\cancel{E[b]}}\,E[Y[n]] + \overset{0}{\cancel{E[c]}}\,E[n^2] + E[n] = \boxed{n}$$

$$R_Z[n_1,n_2] = E[Z[n_1]Z^*[n_2]] = E[(aX[n_1]+bY[n_1]+cn_1^2+n_1)(aX[n_2]+bY[n_2]+cn_2^2+n_2)]$$

$$= E\Big[a^2X[n_1]X[n_2] + b^2Y[n_1]Y[n_2] + c^2n_1^2n_2^2 + n_1n_2 + ab\,\overset{0}{\cancel{X[n_1]}}Y[n_2] + ac\,\overset{0}{\cancel{X[n_1]}}n_2^2 + a\,\overset{0}{\cancel{X[n_1]}}n_2$$
$$+ ba\,\overset{0}{\cancel{Y[n_1]}}X[n_2] + bc\,\overset{0}{\cancel{Y[n_1]}}n_2^2 + b\,\overset{0}{\cancel{Y[n_1]}}n_2 + can_1^2\,\overset{0}{\cancel{X[n_2]}} + cb\,n_1^2\,\overset{0}{\cancel{Y[n_2]}} + \overset{0}{\cancel{c}}n_1^2n_2 + n_1\,\overset{0}{\cancel{aX[n_2]}}$$
$$+ n_1\,\overset{0}{\cancel{bY[n_2]}} + n_1\,\overset{0}{\cancel{c}}n_2^2\Big]$$

$$\rightarrow R_Z[n_1,n_2] = E[a^2X[n_1]X[n_2]] + E[b^2Y[n_1]Y[n_2]] + E[c^2n_1^2n_2^2] + E[n_1n_2]$$

$$E[a^2] - \overset{0}{\cancel{E[a]}} = \sigma_a^2 = 1$$

$$\rightarrow E[a^2] = E[b^2] = E[c^2] = 1 \rightarrow R_Z[n_1,n_2] = E[X[n_1]X[n_2]] + E[Y[n_1]Y[n_2]] + n_1^2n_2^2 + n_1n_2$$

$$\rightarrow \boxed{R_Z[n_1,n_2] = \frac{6}{1+(n_1-n_2)^2} + 4(0.5)^{|n_1-n_2|} + n_1^2n_2^2 + n_1n_2}$$

هم میانگین ثابت نیست هم اینکه R_Z نیز تنها تابع اختلاف زمانی نیست پس فرآیند

ایستا نیست.

ب) لزومی ندارد که نرمال باشد چون ضرب دو متغیر تصادفی نرمال لزوماً نرمال نیست

بنابراین نمی‌توان مثلاً درباره نرمال بودن $aX[n]$ نظری داد.

پ)

$n_1 = 1 \longrightarrow E[Z[n_1]] = n_1 = 1 \quad , \quad R_Z(n_1, n_1) = n_1^4 + n_1^2 + 4 + 6 = 12$

$\longrightarrow \sigma_{n_1}^2 = 12 - 1 = 11 \longrightarrow \boxed{f_{Z_{n_1}}(z) = \frac{1}{\sqrt{2\pi \times 11}} \exp\left(\frac{-(z-1)^2}{22}\right)}$

$n_2 = 2 \longrightarrow E[Z[n_2]] = n_2 = 2 \quad , \quad R_Z(n_2, n_2) = n_2^4 + n_2^2 + 4 + 6 = 16 + 4 + 10 = 30$

$\longrightarrow \sigma_{n_2}^2 = 30 - 2^2 = 26 \longrightarrow \boxed{f_{Z_{n_2}}(z) = \frac{1}{\sqrt{2\pi \times 26}} \exp\left(\frac{-(z-2)^2}{52}\right)}$

$\sigma_{n_1 n_2} = E[Z[n_1] Z[n_2]] - E[Z[n_1]] E[Z[n_2]]$

$= R_Z(1,2) - 1 \times 2 = 3 + 2 + 4 + \cancel{2} - \cancel{2} = 9$

$$\longrightarrow \boxed{f_{Z_{n_1}, Z_{n_2}}(z_1, z_2) = \frac{1}{\sqrt{(2\pi)^2 \begin{vmatrix} 11 & 9 \\ 9 & 26 \end{vmatrix}}} \exp\left(-\frac{1}{2} \begin{bmatrix} z_1 - 1 & z_2 - 2 \end{bmatrix} \begin{bmatrix} 11 & 9 \\ 9 & 26 \end{bmatrix}^{-1} \begin{bmatrix} z_1 - 1 \\ z_2 - 2 \end{bmatrix}\right)}$$

$$f_{Z_{n_2}}(z_2 | z_1) = \frac{f_{Z_{n_1}, Z_{n_2}}(z_1, z_2)}{f_{Z_{n_1}}(z_1)}$$

طبق اسلاید 14 از فصل دو، این توزیع نیز گوسی می شود با میانگین و واریانس زیر:

$$\bar{v} = E[Z[n_2]] + \frac{\sigma_{n_1 n_2}}{\sigma_{n_1}^2}(z_1 - E[Z[n_1]]) = 2 + \frac{9}{11}(z_1 - 1)$$

$$\sigma_v^2 = \sigma_{n_2}^2 - \frac{\sigma_{n_1 n_2}^2}{\sigma_{n_1}^2} = 26 - \frac{81}{11} = 18.64$$

$$\longrightarrow f_{Z_{n_2}}(z_2 | z_1) \sim N(\bar{v}, \sigma_v^2)$$

ت)

$$R_Y[n_1, n_1] = R_Y[0] = 4 \rightarrow \overline{Y_1^2} = 4$$

$$R_Y[n_1, n_2] = R_Y[1, 2] = 2 \rightarrow \overline{Y_1 Y_2} = 2$$

$$\hat{Y_2} = a Y_1 \xrightarrow{MMSE} a = \frac{\overline{Y_1 Y_2}}{\overline{Y_1^2}} = \frac{2}{4} = \frac{1}{2} \rightarrow \hat{Y_2} = \frac{1}{2} Y_1 = 2.5$$

$$\hat{Y_2} = a Y_1 + b, \quad a = \frac{\sigma_{Y_2 Y_1}}{\sigma_{Y_1}^2} = \frac{2}{4} = \frac{1}{2}, \quad b = \overline{Y_2} - a\overline{Y_1} = 0$$

$$\rightarrow \hat{Y_2} = \frac{1}{2} Y_1 = 2.5$$

$$\varepsilon_{MMSE} = \frac{\overline{Y_1^2}\,\overline{Y_2^2} - (\overline{Y_1 Y_2})^2}{\overline{Y_1^2}} = \frac{16 - 4}{4} = 3$$

$$\varepsilon_{\text{آفین}} = \sigma_{Y_2}^2 - \frac{\sigma_{Y_2 Y_1}^2}{\sigma_{Y_1}^2} = 4 - \frac{4}{4} = 3$$

بله تخمین آفین بهینه هست چون Y_1 و Y_2 تواماً گوسی هستند.

ث)

$$R_{XZ}[n_1, n_2] = E\left[X[n_1] Z[n_2]\right] = E\left[X[n_1]\left(aX[n_2] + bY[n_2] + cn_2^2 + n_2\right)\right]$$

$$= E\left[a\overset{0}{\cancel{X[n_1]}}X[n_2]\right] + E\left[b\overset{0}{\cancel{X[n_1]}}Y[n_2]\right] + E\left[c\overset{0}{\cancel{X[n_1]}}n_2^2\right] + E\left[\overset{0}{\cancel{X[n_1]}}n_2\right] = \boxed{0}$$

سوال 3)

الف)

$$d[n] \longrightarrow \boxed{1+0.5z^{-1}} \xrightarrow{s[n]} \boxed{\frac{1}{1-0.5z^{-1}}} \longrightarrow X[n]$$

ROC: $|z|>0.5$

ب)

$$m_s[n] = \cancel{m_d}^{\,0} \sum_n h_2[n] = \boxed{0}$$

$|z|>0.5$

$|z|<2$

$$R_s[m] = R_d[m] * h_2[m] * h_2^*[-m]$$

$$R_d[m] = \delta[m] \quad , \quad h_2[m] = \delta[m] + 0.5\,\delta[m-1]$$

$$\rightarrow R_s[m] = \delta[m] * \left(\delta[m] + \tfrac{1}{2}\delta[m-1]\right) * \left(\delta[-m] + \tfrac{1}{2}\delta[-m-1]\right)$$

$$= \left(\delta[m] + \tfrac{1}{2}\delta[m-1]\right) * \left(\delta[-m] + \tfrac{1}{2}\delta[-m-1]\right)$$

$$= \boxed{1.25\,\delta[m] + 0.5\,\delta[m-1] + 0.5\,\delta[m+1]}$$

پ)

$$R_x[m] = R_s[m] * \underbrace{h_1[m] * h_1^*[-m]}_{f[m]}$$

$$F(z) = H_1(z)H_1(z^{-1}) = \frac{1}{1-0.5z^{-1}} \times \frac{1}{1-0.5z} = \frac{1}{1.25-0.5z^{-1}-0.5z}$$

$$= \frac{z^{-1}}{-0.5+1.25z^{-1}-0.5z^{-2}} = \frac{-2z^{-1}}{1-2.5z^{-1}+z^{-2}} = \frac{A}{0.5-z^{-1}} + \frac{B}{2-z^{-1}}$$

ROC: $0.5<|z|<2$

$$\rightarrow 2A - Az^{-1} + 0.5B - Bz^{-1} = -2z^{-1}$$

$$2A + 0.5B = 0$$

$$-A - B = -2$$

$$\rightarrow B = \frac{8}{3},\ A = -\frac{2}{3} \rightarrow F(z) = -\frac{2}{3}\,\frac{1}{0.5-z^{-1}} + \frac{8}{3}\,\frac{1}{2-z^{-1}} = -\frac{4}{3}\,\frac{1}{1-2z^{-1}} + \frac{4}{3}\,\frac{1}{1-\frac{1}{2}z^{-1}}$$

ادامه پ)

$$\xrightarrow{Z^{-1}} f[m] = \frac{4}{3}(0.5)^n u[n] + \frac{4}{3} 2^n u[-n-1]$$

$$R_X[m] = R_S[m] * f[m] = \left(1.25\,\delta[m] + 0.5\,\delta[m-1] + 0.5\,\delta[m+1]\right) * f[m]$$

$$= \frac{5}{3}(0.5)^n u[n] + \frac{5}{3} 2^n u[-n-1] + \frac{2}{3}(0.5)^{n-1} u[n-1] + \frac{2}{3} 2^{n-1} u[-n]$$

$$+ \frac{2}{3}(0.5)^{n+1} u[n+1] + \frac{2}{3} 2^{n+1} u[-n-2]$$

$$\rightarrow R_X[0] = \frac{5}{3} + \frac{1}{3} + \frac{1}{3} = \frac{7}{3} \rightarrow \sigma_X^2 + m_X^2 = \frac{7}{3}$$

$$m_X = \overset{0}{\cancel{m_S}} \sum_n h_1[n] = 0 \longrightarrow \boxed{\sigma_X^2 = \frac{7}{3}}$$

ت)

$$R_S[n] = 1.25\,\delta[n] + 0.5\,\delta[n-1] + 0.5\,\delta[n+1]$$

$$\longrightarrow S_S(\omega) = 1.25 + 0.5e^{-j\omega} + 0.5e^{j\omega} = \boxed{1.25 + \cos\omega}$$

$$S_X(\omega) = \left|H_1(e^{j\omega})\right|^2 S_S(\omega)$$

$$\left|H_1(e^{j\omega})\right|^2 = \left|\frac{1}{1-0.5e^{-j\omega}}\right|^2 = \frac{1}{\underbrace{\left|1-0.5\cos(\omega)+0.5j\sin(\omega)\right|^2}_{1+\frac{1}{4}\cos^2(\omega)-\cos(\omega)+\frac{1}{4}\sin^2(\omega)}} = \frac{1}{\frac{5}{4}-\cos(\omega)}$$

$$\rightarrow \boxed{S_X(\omega) = \frac{\frac{5}{4}+\cos\omega}{\frac{5}{4}-\cos\omega}}$$

سؤال 4 -

الف)

$$E[X[n]] = E[U\cos(w_0 n)] + E[V\sin(w_0 n)]$$

$$= \cos(w_0 n)\cancel{E[U]}^{0} + \sin(w_0 n)\cancel{E[V]}^{0} = \boxed{0}$$

$$E[Y[n]] = \cos(w_0 n)\cancel{E[V]}^{0} + \sin(w_0 n)\cancel{E[U]}^{0} = \boxed{0}$$

$$R_X[n_1,n_2] = E[(U\cos(w_0 n_1) + V\sin(w_0 n_1))(U\cos(w_0 n_2) + V\sin(w_0 n_2))]$$

$$= E[U^2\cos(w_0 n_1)\cos(w_0 n_2)] + E[\cancel{UV\cos(w_0 n_1)}^{0}\sin(w_0 n_2)]$$

$$+ E[\cancel{VU\sin(w_0 n_1)}^{0}\cos(w_0 n_2)] + E[V^2\sin(w_0 n_1)\sin(w_0 n_2)]$$

$$\rightarrow R_X[n_1,n_2] = \cos(w_0 n_1)\cos(w_0 n_2) + \sin(w_0 n_1)\sin(w_0 n_2)$$

$$= \cos(w_0(n_1-n_2)) \xrightarrow[m=n_1-n_2]{} \boxed{R_X[m] = \cos(w_0 m)}$$

به همین شکل برای فرآیند Y خواهیم داشت:

$$R_Y[n_1,n_2] = \cos(w_0(n_1-n_2)) \xrightarrow[m=n_1-n_2]{} \boxed{R_Y[m] = \cos(w_0 m)}$$

بنابراین هر دو فرآیند ایستا هستند.

ب)

$$R_{XY}[n_1,n_2] = E\left[(U\cos(w_0 n_1)+V\sin(w_0 n_1))(V\cos(w_0 n_2)+U\sin(w_0 n_2))\right]$$

$$= E[U^2\cos(w_0 n_1)\sin(w_0 n_2)] + E[V^2\sin(w_0 n_1)\cos(w_0 n_2)]$$

$$= \cos(w_0 n_1)\sin(w_0 n_2) + \sin(w_0 n_1)\cos(w_0 n_2)$$

$= \sin(w_0(n_1+n_2)) \longrightarrow$ بنابراین توأماً ایستا نیستند چون به مجموع

دو لحظه بستگی دارد نه اختلاف دو لحظه

سؤال 5)

الف)

$$f(n)=1 \longrightarrow X[n]-0.5X[n-1]=U[n]$$

$$\longrightarrow X(z)-0.5X(z)z^{-1}=U(z) \longrightarrow \frac{X(z)}{U(z)} = \frac{1}{1-0.5z^{-1}} = H(z)$$

$$U[n] \longrightarrow \boxed{\frac{1}{1-0.5z^{-1}}} \longrightarrow X[n] \qquad m_U=0 \longrightarrow m_X=0$$

$$R_U[m]=5\delta[m] \longrightarrow R_X[m]=R_U[m]*\underbrace{h[m]*h^*[-m]}_{f[m]}$$

$$h[m]=(0.5)^m u[m] \longrightarrow F(z)=H(z)H(\tfrac{1}{z})$$

طبق سؤال 4

$$\longrightarrow F(z)=\frac{4}{3}\frac{1}{1-0.5z^{-1}}-\frac{4}{3}\frac{1}{1-2z^{-1}}, \quad ROC: 0.5<|z|<2$$

$$\longrightarrow f[m]=\frac{4}{3}(0.5)^m u[m]+\frac{4}{3}2^m u[-m-1]$$

$$\longrightarrow \boxed{R_X[m] = \frac{20}{3}(0.5)^m u[m] + \frac{20}{3} 2^m u[-m-1]}$$

$$Y[n] = X[n] + N[n] \longrightarrow R_Y[m] = E\left[(X[n]+N[n])(X^*[n+m]+N^*[n+m])\right]$$

$$= E\left[X[n]X^*[n+m]\right] + E\left[N[n]N^*[n+m]\right] = R_X[m] + R_N[m]$$

$$\longrightarrow \boxed{R_Y[m] = \frac{20}{3}(0.5)^m u[m] + \frac{20}{3} 2^m u[-m-1] + 2\delta[m]}$$

$$R_{XY}[m] = E\left[X[n]\left(X^*[n+m]+N^*[n+m]\right)\right] = E\left[X[n]X^*[n+m]\right]$$

$$+ E\left[X[n]\overset{0}{\cancel{N^*[n+m]}}\right] = R_X[m]$$

$$\longrightarrow \boxed{R_{XY}[m] = \frac{20}{3}(0.5)^m u[m] + \frac{20}{3} 2^m u[-m-1]}$$

$$m_Y = m_X + m_N = 0$$

$$R_X[0] = \frac{20}{3} \longrightarrow \sigma_X^2 = \frac{20}{3}$$

$$R_Y[0] = \frac{26}{3} \longrightarrow \sigma_Y^2 = \frac{26}{3}$$

$$S_X(w) = \frac{20}{3}\frac{1}{1-0.5e^{-jw}} - \frac{20}{3}\frac{1}{1-2e^{-jw}}$$

$$S_Y(w) = \frac{20}{3}\frac{1}{1-0.5e^{-jw}} - \frac{20}{3}\frac{1}{1-2e^{-jw}} + 2$$

ب)

$$m_X[n] = \begin{cases} 0, & n<0 \\ 0, & n\geq 0 \end{cases}$$

قبل $n=0$ ، $X[n]$ برابر صفر هست و بعد صفر برابر با همان $X[n]$ قبلی.

$$R_X[n_1,n_2] = E[X[n_1]X^*[n_2]]$$

اگر $n_1<0$ یا $n_2<0$ باشد که چون $X[n]$ صفر می‌شود پس R_X هم صفر می‌شود:

$$R_X[n_1,n_2] = \begin{cases} \frac{20}{3}(0.5)^m u[m] + \frac{20}{3}2^m u[-m-1], & n_1,n_2 \geq 0 \\ 0, & o.w \end{cases}$$

$$R_X[0] = \frac{20}{3} \rightarrow \sigma_X^2[n] = \begin{cases} 0, & n<0 \\ \frac{20}{3}, & n\geq 0 \end{cases}$$

Made with Goodnotes

سوال 6)

$$E[X[n]] = E[\cos(\omega_0 n + \varphi)] = \int \cos(\omega_0 n + \varphi) f_\phi(\varphi) d\varphi$$

$$= \int_{-\infty}^{+\infty} \cos(\omega_0 n + \varphi) \times \frac{1}{4}\left(\delta(\varphi) + \delta(\varphi - \frac{\pi}{2}) + \delta(\varphi - \pi) + \delta(\varphi - \frac{3\pi}{2})\right)$$

$$= \frac{1}{4}\left(\int_{-\infty}^{+\infty} \cos(\omega_0 n + \varphi)\delta(\varphi)d\varphi + \int_{-\infty}^{+\infty} \cos(\omega_0 n + \varphi)\delta(\varphi - \frac{\pi}{2})d\varphi + \int_{-\infty}^{+\infty} \cos(\omega_0 n + \varphi)\delta(\varphi - \pi)d\varphi + \int_{-\infty}^{+\infty} \cos(\omega_0 n + \varphi)\delta(\varphi - \frac{3\pi}{2})d\varphi\right.$$

$$= \frac{1}{4}\Big(\cancel{\cos(\omega_0 n)} + \underbrace{\cancel{\cos(\omega_0 n + \frac{\pi}{2})}}_{\cancel{-\sin(\omega_0 n)}} + \underbrace{\cancel{\cos(\omega_0 n + \pi)}}_{\cancel{-\cos(\omega_0 n)}} + \underbrace{\cancel{\cos(\omega_0 n + \frac{3\pi}{2})}}_{\cancel{\sin(\omega_0 n)}}\Big) = \boxed{0}$$

$$R_X[n_1, n_2] = E[X[n_1]X^*[n_2]] = E[\cos(\omega_0 n_1 + \varphi)\cos(\omega_0 n_2 + \varphi)]$$

$$= E\left[\frac{1}{2}\left(\cos(\omega_0(n_1 + n_2) + 2\varphi) + \cos(\omega_0(n_1 - n_2))\right)\right]$$

$$= \frac{1}{2}\Big(\underbrace{\cancel{E[\cos(\omega_0(n_1 + n_2) + 2\varphi)]}}^{0} + E[\cos(\omega_0(n_1 - n_2))]\Big) = \frac{1}{2}\cos(\omega_0(n_1 - n_2))$$

$$\xrightarrow{m = n_1 - n_2} \boxed{R_X[m] = \frac{1}{2}\cos(\omega_0 m)}$$

* بنابراین فرآیند ایستا هست.

سؤال 7)

الف)

$$X[n] = S[n] + V[n]$$

$$R_X[n_1, n_2] = E[X[n_1]X^*[n_2]] = E[(S[n_1] + V[n_1])(S^*[n_2] + V^*[n_2])]$$

$$= E[S[n_1]S^*[n_2]] + \overset{0}{\cancel{E[S[n_1]V^*[n_2]]}} + \overset{0}{\cancel{E[V[n_1]S^*[n_2]]}} + E[V[n_1]V^*[n_2]]$$

$$= R_S[m] + R_V[m] \longrightarrow R_X[m] = R_S[m] + \delta[m]$$

$$S_S(\omega) = \frac{9}{41 - 40\cos\omega} = \frac{9}{41 - 20(e^{j\omega} + e^{-j\omega})} = \frac{1}{1 - 0.8e^{-j\omega}} - \frac{1}{1 - \frac{5}{4}e^{-j\omega}}$$

$$\longrightarrow R_S[n] = (0.8)^n u[n] + \left(\frac{5}{4}\right)^n u[-n-1] = 0.8^{|n|}$$

$$\longrightarrow \boxed{R_X[m] = 0.8^{|m|} + \delta[m]}$$

ب)

$$S_X(z) = 1 + \frac{1}{1 - \frac{4}{5}z^{-1}} - \frac{1}{1 - \frac{5}{4}z^{-1}} = \frac{(1 - \frac{4}{5}z^{-1})(1 - \frac{5}{4}z^{-1}) + \cancel{1} - \frac{5}{4}z^{-1} - \cancel{1} + \frac{4}{5}z^{-1}}{(1 - \frac{4}{5}z^{-1})(1 - \frac{5}{4}z^{-1})}$$

$$= \frac{1 - \cancel{\frac{4}{5}z^{-1}} - \frac{5}{4}z^{-1} + z^{-2} - \frac{5}{4}z^{-1} + \cancel{\frac{4}{5}z^{-1}}}{(1 - \frac{4}{5}z^{-1})(1 - \frac{5}{4}z^{-1})} = \frac{z^{-2} - \frac{5}{2}z^{-1} + 1}{(1 - \frac{4}{5}z^{-1})(1 - \frac{5}{4}z^{-1})}$$

$$= \frac{(z^{-1} - 2)(z^{-1} - \frac{1}{2})}{(1 - \frac{4}{5}z^{-1})(1 - \frac{5}{4}z^{-1})}$$

ادامه ب)

$$S_X(z)\,|H_w(z)|^2 = 1 \longrightarrow H_w(z)H_w\left(\frac{1}{z}\right) = \frac{1}{S_X(z)} = \frac{(1-\frac{4}{5}z^{-1})(1-\frac{5}{4}z^{-1})}{(z^{-1}-2)(z^{-1}-\frac{1}{2})}$$

$$= \frac{(1-\frac{4}{5}z^{-1})}{(z^{-1}-2)} \times \frac{(1-\frac{4}{5}z)}{(z-2)} \longrightarrow H_w(z) = \frac{(1-\frac{4}{5}z^{-1})}{z^{-1}-2} = \frac{-\frac{1}{2}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{2}{5}z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$\longrightarrow \boxed{h_w[n] = -\frac{1}{2}\left(\frac{1}{2}\right)^n u[n] + \frac{2}{5}\left(\frac{1}{2}\right)^{n-1} u[n-1]}$$

$$H_{\text{ابداع}}(z) = \frac{1}{H_w(z)} = \frac{z^{-1}-2}{1-\frac{4}{5}z^{-1}} = \frac{z^{-1}}{1-\frac{4}{5}z^{-1}} - \frac{2}{1-\frac{4}{5}z^{-1}}$$

$$\longrightarrow \boxed{h_{\text{ابداع}}[n] = -2\left(\frac{4}{5}\right)^n u[n] + \left(\frac{4}{5}\right)^{n-1} u[n-1]}$$

سؤال 18)

الف)

$$E[X(t)] = \int_{-\infty}^{+\infty} 1\times\frac{1}{2} - 1\times\frac{1}{2}\, dx(t) = \boxed{0}$$

$$R_X(t_1, t_2) = E[X(t_1)X(t_2)]$$

چنانچه t_1 و t_2 در یک بازه نباشند:

$$R_X(t_1,t_2) = \frac{1}{4}\times 1 + \frac{1}{4}\times -1 + \frac{1}{4}\times 1 + \frac{1}{4}\times -1 = \boxed{0}$$

اگر در یک بازه باشند:

$$R_X(t_1,t_2) = \frac{1}{2}\times 1 + \frac{1}{2}\times 1 = \boxed{1}$$

بنابراین فرآیند WSS نیست چون R_X به هر دوی t_1 و t_2 وابسته است.

ب)

$$Y(t) = X(t-\theta)$$

$$E[Y(t)] = E[X(t-\theta)] = \int_0^T m_X(t)\times\frac{1}{T}\, d\theta = \boxed{0}$$

$$R_Y(t_1, t_2) = E[X(t_1-\theta)X(t_2-\theta)]$$

اگر $|t_1 - t_2|$ بزرگتر از T باشد معادل این است که به ازای تمام مقادیر θ و $t_1-\theta$ و $t_2-\theta$ در دو بازه مختلف قرار می‌گیرند و $R_Y=0$ است.

حال اگر $|t_1-t_2|$ کمتر از T باشد به ازای برخی مقادیر θ $t_1-\theta$, $t_2-\theta$ در یک بازه قرار می‌گیرند.

Made with Goodnotes

ادامه ب)

مقادیری از θ که به ازای آن $t_1-\theta$ و $t_2-\theta$ در یک بازه قرار می‌گیرند عبارت است از:

$$(n-1)T < t_1-\theta < nT \rightarrow t_1-(n-1)T > \theta > t_1-nT$$

$$(n-1)T < t_2-\theta < nT \rightarrow t_2-(n-1)T > \theta > t_2-nT$$

اشتراک بین دو بازه:

$$\min(t_1,t_2)-nT+T > \theta > \max(t_1,t_2)-nT$$

$$R_Y(t_1,t_2) = \int_{\max(t_1,t_2)-nT}^{\min(t_1,t_2)-nT+T} \frac{1}{T}\times 1 \, d\theta = \frac{1}{T}\left(\min(t_1,t_2)-\cancel{nT}+T-\max(t_1,t_2)+\cancel{nT}\right)$$

$$= \frac{1}{T}\left(T-|t_1-t_2|\right) = 1-\frac{|t_1-t_2|}{T}$$

$$\rightarrow R_Y(t_1,t_2) = \begin{cases} 1-\dfrac{|t_1-t_2|}{T}, & |t_1-t_2| \leq T \\ 0, & |t_1-t_2| > T \end{cases}$$

بنا بر این این فرآیند WSS است. ✓