تمرین سری و درس BSP - رادین خیاط - 99101579

سوال ۱-

الف)

$$y[n] = \alpha\underbrace{\left((0.5)^n u[n]\right)}_{z[n]} * x[n]$$

$$\rightarrow Z(z) = \frac{1}{1-0.5z^{-1}} \rightarrow Y(z) = \alpha Z(z) X(z)$$

$$\rightarrow \hat{Y}(z) = \ln\alpha - \ln(1-0.5z^{-1}) + \hat{X}(z)$$

$$\hat{Z}(z) = -\ln(1-0.5z^{-1}) = \sum_{n=1}^{\infty} \frac{0.5^n}{n} z^{-n} \quad |z|>|\alpha|$$

$$\rightarrow \hat{z}[n] = \frac{0.5^n}{n} u[n-1] \rightarrow \boxed{\hat{y}[n] = \ln\alpha\,\delta[n] + \frac{0.5^n}{n} u[n-1] + \hat{x}[n]}$$

ب)

$$\hat{y}[n] = \alpha\,\delta[n] + \hat{x}[n]$$

$$\rightarrow \hat{Y}(z) = \alpha + \hat{X}(z) \rightarrow \ln(Y(z)) = \alpha + \ln(X(z)) \rightarrow \frac{\ln Y(z)}{\ln X(z)} = \alpha$$

$$\rightarrow \frac{Y(z)}{X(z)} = e^{\alpha} \rightarrow Y(z) = e^{\alpha} X(z) \rightarrow \boxed{y[n] = e^{\alpha} x[n]}$$

سؤال 2 -

$y_K[n] = x_K[n] * y_{K-1}[n]$

$x_K[n] = \alpha_K \, 0.5^{Kn} u[n]$

$y_0[n] = \alpha_0 \delta[n]$

$x_0[n] = \alpha_0 u[n]$

$x_1[n] = \alpha_1 \, 0.5^n u[n]$

$Y_K(z) = X_K(z) Y_{K-1}(z) \rightarrow \ln Y_K(z) = \ln X_K(z) + \ln Y_{K-1}(z)$

$\rightarrow \hat{Y}_K(z) = \hat{X}_K(z) + \hat{Y}_{K-1}(z)$

$\rightarrow \hat{y}_K[n] = \hat{x}_K[n] + \hat{y}_{K-1}[n]$

$X_K(z) = \dfrac{\alpha_K}{1 - 0.5^K z^{-1}} = \dfrac{\alpha_K}{1 - (2^K z)^{-1}}$

$\rightarrow \hat{X}_K(z) = \ln X_K(z) = \ln \alpha_K + \ln(1 - (2^K z)^{-1})$

$\rightarrow \hat{x}_K[n] = \ln \alpha_K \delta[n] - \dfrac{0.5^{Kn}}{n} u[n-1]$

$\hat{y}_3[n] = \hat{x}_3[n] + \hat{y}_2[n] = \hat{x}_3[n] + \hat{x}_2[n] + \hat{y}_1[n] = \hat{x}_3[n] + \hat{x}_2[n] + \hat{x}_1[n] + \hat{y}_0[n]$

$\rightarrow \hat{y}_3[n] = \delta[n] (\ln(\alpha_1 \alpha_2 \alpha_3)) - \left(\dfrac{0.5^n}{n} + \dfrac{0.25^n}{n} + \dfrac{0.125^n}{n}\right) u[n-1]$

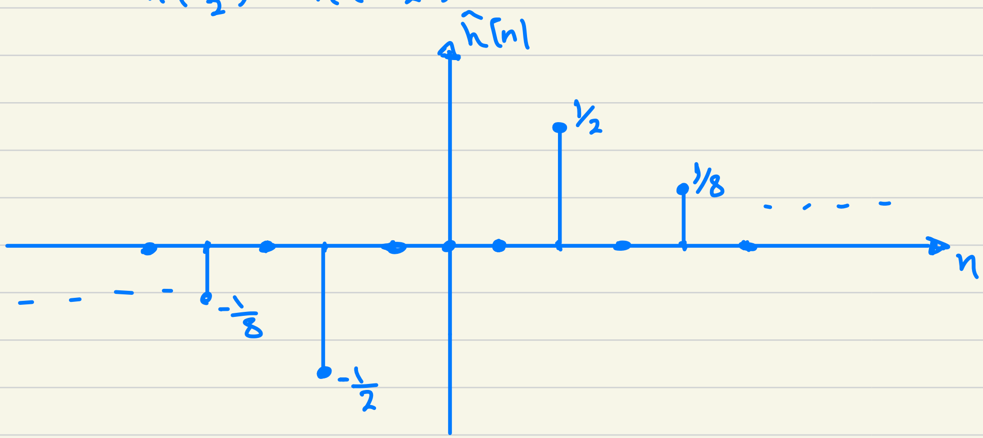
$y_0[n] = \alpha_0 \delta[n] \rightarrow Y_0(z) = \alpha_0 \rightarrow \hat{Y}_0(z) = \ln \alpha_0 \rightarrow \hat{y}_0[n] = \ln \alpha_0 \delta[n]$

$\rightarrow \boxed{\hat{y}_3[n] = \delta[n] . \ln(\alpha_0 \alpha_1 \alpha_2) - u[n-1] \left(\dfrac{0.5^n + 0.25^n + 0.125^n}{n}\right)}$

سؤال 3 -

$$H(z) = \frac{1-0.5z^{2}}{1-0.5z^{-2}} = \frac{(1-\sqrt{0.5}\,z)(1+\sqrt{0.5}\,z)}{(1-\sqrt{0.5}\,z^{-1})(1+\sqrt{0.5}\,z^{-1})}$$

$$\rightarrow \hat{h}[n] \begin{cases} 0 & n=0 \\ \frac{1}{n}\left(\frac{\sqrt{2}}{2}\right)^{n} + \frac{1}{n}\left(-\frac{\sqrt{2}}{2}\right)^{n} & n>0 \\ \frac{1}{n}\left(\frac{\sqrt{2}}{2}\right)^{-n} + \frac{1}{n}\left(-\frac{\sqrt{2}}{2}\right)^{-n} & n<0 \end{cases}$$



- سؤال 4

$$\hat{x}[n] = -\hat{x}[-n] \longrightarrow \hat{X}(z) = -\hat{X}(z^{-1}) \longrightarrow \hat{X}(z) + \hat{X}\left(\frac{1}{z}\right) = 0$$

$$\longrightarrow \ln X(z) + \ln X\left(\frac{1}{z}\right) = 0 \longrightarrow X(z)\, X\left(\frac{1}{z}\right) = 1$$

$$\longrightarrow X(e^{j\omega})\, X(e^{-j\omega}) = 1$$

$$E = \sum_{n=-\infty}^{+\infty} x^2[n] = \sum_{n=-\infty}^{+\infty} x[n]\left(x^*[n]\right)^*$$

$$\longrightarrow \tilde{E} = \frac{1}{2\pi} \int X(e^{j\omega})\, X(e^{-j\omega})\, d\omega = 1$$

سؤال ۴-

$$\hat{y}[n] = (\hat{x}[n] - \hat{x}[-n])u[n-1] \to \hat{Y}(z) = \hat{X}(z) - \hat{X}(z^{-1})$$

$$\to \ln Y(z) = \ln X(z) - \ln X(z^{-1}) \to Y(z) = \frac{X(z)}{X(z^{-1})}$$

الف)

چون کپستروم مختلط برای سیستم‌های مینیمم فاز سببی می‌باشد و همچنین

$\hat{y}[n]=0,\ n<0$ ← $y[n]$ مینیمم فاز است.

ب)

$$\hat{Y}(z) = \ln Y(z) = \ln|Y(z)| + j\angle Y(z)$$

$$Y(e^{j\omega}) = \frac{X(e^{j\omega})}{X(e^{-j\omega})} \to \angle Y(e^{j\omega}) = \angle \frac{X(e^{j\omega})}{X(e^{-j\omega})}$$

$$\to \angle Y(e^{j\omega}) = \angle X(e^{j\omega}) - \angle X(e^{-j\omega}) = 2\angle X(e^{j\omega})$$

پ)

$$\mathrm{Im}\{Y(e^{j\omega})\} = -\frac{1}{\pi}\int_{-\pi}^{+\pi} \mathrm{Re}\{\hat{Y}(e^{j\theta})\} \cot\left(\frac{\omega-\theta}{2}\right) d\theta$$

$$\rightarrow \arg\left(Y(e^{j\omega})\right) = -\frac{1}{\pi}\int_{-\pi}^{+\pi} \ln|Y(e^{j\omega})| \cot\left(\frac{\omega-\theta}{2}\right) d\theta$$

ت)

$x[n]$ minimum phase $\rightarrow \hat{x}[n] = 0$ for $n < 0$

$$\rightarrow \hat{y}[n] = \begin{cases} 0 & n < 1 \\ \hat{x}[n] & n \geq 1 \end{cases} \rightarrow \hat{y}[n] = \hat{x}[n] u[n-1]$$

$$\hat{Y}(z) = \sum_{n=1}^{\infty} \hat{x}[n] z^{-n} = \sum_{n=-\infty}^{+\infty} \hat{x}[n] z^{-n} - \sum_{n=-\infty}^{0} \hat{x}[n] z^{-n}$$

$$\rightarrow \hat{Y}(z) = \hat{X}(z) - \hat{x}[0] \rightarrow \ln Y(z) = \ln X(z) - \hat{x}[0]$$

$$\rightarrow \frac{Y(z)}{X(z)} = e^{-\hat{x}[0]} \rightarrow Y(z) = X(z) e^{-\hat{x}[0]} \; x[n]$$

$$e^{-\hat{x}[0]} = e^{-\ln \lim_{z\to\infty} X(z)} = \frac{1}{\underbrace{\lim_{z\to\infty} X(z)}_{x[0]}} \rightarrow \boxed{y[n] = \frac{x[n]}{x[0]}}$$