تمرین سری 5 سیستم های مخابراتی - رادین خیام - 99101579

سوال 4)

همانطور که در اسلایدهای درس اشاره شده است:

$$x_i(t) = \sum_k \cos(\phi_k) P(t-kD)$$

$$x_q(t) = \sum_k \sin(\phi_k) P(t-kD) \quad , \quad \phi_k = \frac{\pi(2a_k+1)}{M} \quad , \quad a_k = 0,1,\dots,M-1$$

$$\rightarrow S_{x_i} = \frac{1}{D}|P(f)|^2 \sum_{-\infty}^{+\infty} R_{a_i}[n] e^{-j2\pi nfD}$$

$$S_{x_q} = \frac{1}{D}|P(f)|^2 \sum_{-\infty}^{+\infty} R_{a_q}[n] e^{-j2\pi nfD}$$

$$\rightarrow S_{lp}(f) = S_{x_i} + S_{x_q} = \frac{1}{D}|P(f)|^2 \left(\sum_{-\infty}^{+\infty} (R_{a_i}[n] + R_{a_q}[n]) e^{-j2\pi nfD} \right)$$

$$R_{a_i}[0] + R_{a_q}[0] = \frac{1}{M} \sum_{i=0}^{M-1} \cos^2\left(\frac{(2i+1)\pi}{M}\right) + \frac{1}{M} \sum_{i=0}^{M-1} \sin^2\left(\frac{(2i+1)\pi}{M}\right)$$

$$= \frac{1}{M} \sum_{i=0}^{M-1} \cos^2\left(\frac{(2i+1)\pi}{M}\right) + \sin^2\left(\frac{(2i+1)\pi}{M}\right) = \frac{1}{M}\sum_{i=0} = \boxed{1}$$

$$\rightarrow R_{a_i}[n] + R_{a_q}[n] = \frac{1}{M} \sum_{i=0}^{M-1} \cos\left(\frac{(2i+1)\pi}{M}\right) \cos\left(\frac{(2(i+n)+1)\pi}{M}\right)$$

$$+ \frac{1}{M} \sum_{i=0}^{M-1} \sin\left(\frac{(2i+1)\pi}{M}\right) \sin\left(\frac{(2(i+n)+1)\pi}{M}\right)$$

همانطور که در تمرین قبلی اشاره شد:

$$\sum_{i=0}^{M-1} \cos\left(\frac{2\pi i}{M}\right) = 0$$

$$\sum_{i=1}^{M-1} \sin\left(\frac{2\pi i}{M}\right) = 0 \quad \rightarrow R_{a_i}[n] + R_{a_q}[n] = 0$$

$$\rightarrow S_{lp} = \frac{1}{D}|P(f)|^2 = \frac{1}{D} \times \frac{1}{r^2} \operatorname{sinc}^2\left(\frac{f}{r}\right) \overset{D=\frac{1}{r}}{=} \frac{1}{r} \operatorname{sinc}^2\left(\frac{f}{r}\right)$$

$$\rightarrow S_x(f) = \frac{A_c^2}{4}\left(S_{lp}(f-f_c) + S_{lp}(f+f_c)\right) = \boxed{\frac{A_c^2}{4r}\left(\operatorname{sinc}^2\left(\frac{f-f_c}{r}\right) + \operatorname{sinc}^2\left(\frac{f+f_c}{r}\right)\right)}$$