تمرین سری 5 درس سیستم های مخابراتی - رادین خیام - 99101579

سوال 1)

$$S_X(f) = \lim_{T \to \infty} \frac{1}{T} E\{X_T(f)^2\}$$

$$x(t) = A_C\left(x_i(t)\cos(2\pi f_c t + \theta) - x_q(t)\sin(2\pi f_c t + \theta)\right)$$

$$\to x_T(t) = A_C\left(x_i(t)\cos(2\pi f_c t + \theta) - x_q(t)\sin(2\pi f_c t + \theta)\right)\Pi\left(\frac{t}{T}\right)$$

$$= A_C\left(x_i(t)\cos(2\pi f_c t + \theta)\Pi\left(\frac{t}{T}\right) - x_q(t)\sin(2\pi f_c t + \theta)\Pi\left(\frac{t}{T}\right)\right)$$

$$= A_C\left(x_{iT}(t)\cos(2\pi f_c t + \theta) - x_{qT}(t)\sin(2\pi f_c t + \theta)\right)$$

$$\to S_X(f) = \lim_{T \to \infty} \frac{1}{T} E\left\{\left|F\left\{A_C\left(x_{iT}(t)\cos(2\pi f_c t + \theta) - x_{qT}(t)\sin(2\pi f_c t + \theta)\right)\right\}\right|^2\right\}$$

$$= \lim_{T \to \infty} \frac{1}{T} E\left\{\left|\frac{A_C}{2}\left(e^{j\theta}X_{iT}(f-f_c) + e^{-j\theta}X_{iT}(f+f_c)\right) + \frac{A_C}{2j}\left(e^{j\theta}X_{qT}(f-f_c) - e^{-j\theta}X_{qT}(f+f_c)\right)\right|^2\right\}$$

$$= \lim_{T \to \infty} \frac{1}{T} E\left\{\frac{A_C^2}{4}\left(\left|e^{j\theta}X_{iT}(f-f_c) + e^{-j\theta}X_{iT}(f+f_c)\right|^2 + \left|e^{j\theta}X_{qT}(f-f_c) - e^{-j\theta}X_{qT}(f+f_c)\right|^2 + 2\left|\left(e^{j\theta}X_{iT}(f-f_c) - e^{-j\theta}X_{iT}(f+f_c)\right)\left(e^{j\theta}X_{qT}(f-f_c) - e^{-j\theta}X_{qT}(f+f_c)\right)\right|\right)\right\}$$

$$= \lim_{T \to \infty} \frac{1}{T} \frac{A_C^2}{4}\left|E\left\{\left|e^{j\theta}X_{iT}(f-f_c) + e^{-j\theta}X_{iT}(f+f_c)\right|^2\right\} + E\left\{\left|e^{j\theta}X_{qT}(f-f_c) - e^{-j\theta}X_{qT}(f+f_c)\right|^2\right\} + 2E\left\{\left|\left(e^{j\theta}X_{iT}(f-f_c) - e^{-j\theta}X_{iT}(f+f_c)\right)\left(e^{j\theta}X_{qT}(f-f_c) - e^{-j\theta}X_{qT}(f+f_c)\right)\right|\right\}\right|$$

همانطور که در صورت سوال گفته شده است x_i و x_q مستقل هستند و حداقل یکی از آن ها میانگین صفر دارد بنابراین:

ادامه سؤال ۱)

$$E\{X_{iT}(f\pm f_c)X_{qT}(f\pm f_c)\} = E\{X_{iT}(f\pm f_c)\}E\{X_{qT}(f\pm f_c)\} = 0$$

چون E هر کدام صفر است

$$\longrightarrow S_X(f) = \lim_{T\to\infty}\frac{1}{T}\frac{A_c^2}{4}\Big(E\{|X_{iT}(f-f_c)|^2+|X_{iT}(f+f_c)|^2+2|X_{iT}(f-f_c)X_{iT}(f+f_c)|\}+$$

$$E\{|X_{qT}(f-f_c)|^2+|X_{qT}(f+f_c)|^2-2|X_{qT}(f-f_c)X_{qT}(f+f_c)|\}\Big)$$

$$\xrightarrow{f_c \gg w} S_X(f) = \frac{A_c^2}{4}\Big(\lim_{T\to\infty}\frac{1}{T}E\{|X_{iT}(f-f_c)|^2\}+\lim_{T\to\infty}\frac{1}{T}E\{|X_{iT}(f+f_c)|^2\}$$

$$+\lim_{T\to\infty}\frac{1}{T}E\{|X_{qT}(f-f_c)|^2\}+\lim_{T\to\infty}\frac{1}{T}E\{|X_{qT}(f+f_c)|^2\}\Big)$$

$$\longrightarrow \boxed{S_X(f) = \frac{A_c^2}{4}\Big(S_{X_i}(f-f_c)+S_{X_i}(f+f_c)+S_{X_q}(f-f_c)+S_{X_q}(f+f_c)\Big)}$$