EEG signal processing - Radin Khayyam - 99101579

1-

a)

$f_K = \frac{K}{N} f_s$. we assume power line noise at 50 HZ

$\longrightarrow 50 = \frac{K}{512} \times 256 \longrightarrow \boxed{K = 100}$

so if we have peak on $K = 100, 200, 300, \dots$

it means we have 50 HZ, 100 HZ, 150 Hz, ...

components in signal, which means we have power

line noise.

b)

alpha Band: 8 - 13 HZ

delta Band: 0.5 - 4 HZ

8 HZ ~ K = 16 , 13 HZ ~ K = 26 , 0.5 HZ ~ K = 1

4 HZ ~ K = 8

$$\longrightarrow E_{alpha} = \sum_{k=16}^{26} |x[k]|^2 = \sum_{k=16}^{26} 2^8 \left(\frac{1}{2}\right)^k$$

$$= 2^8 \sum_{k=16}^{26} \left(\frac{1}{2}\right)^k = 2^8 \times \frac{2^{-16}(2^{-11}-1)}{-\frac{1}{2}}$$

$$E_{delta} = \sum_{k=1}^{8} 2^8 \left(\frac{1}{2}\right)^k = 2^8 \times \frac{2^{-1}(2^{-8}-1)}{-\frac{1}{2}}$$

$$\longrightarrow \frac{E_{alpha}}{E_{delta}} = \frac{2^{-16}(2^{-12}-1)}{2^{-1}(2^{-9}-1)} \simeq \boxed{2^{-15}}$$

c)

$$f_k = \frac{k}{N} f_s \longrightarrow \text{alpha band} = 8\,Hz - 13\,Hz \sim k = 32-52$$

$$\text{delta band} = 0.5\,Hz - 4\,Hz \sim k = 2-8$$

$$E_{alph} = 2^8 \times \frac{2^{-32}(2^{-21}-1)}{-\frac{1}{2}}$$

$$E_{delta} = 2^8 \times \frac{2^{-2}(2^{-7}-1)}{-\frac{1}{2}} \longrightarrow \frac{E_{alpha}}{E_{delta}} \simeq \boxed{2^{-30}}$$

$$\longrightarrow \boxed{\left.\frac{E_{alpha}}{E_{delta}}\right|_{N=1024} \simeq \frac{1}{2} \left.\frac{E_{alpha}}{E_{delta}}\right|_{N=512}}$$

2-

$$f_k = \frac{k}{N} f_s$$

we want $f_{k+1} - f_k \leq 100\,\text{Hz}$

$$\longrightarrow \left(\frac{k+1}{N} - \frac{k}{N}\right) f_s \leq 100 \longrightarrow \boxed{\frac{f_s}{N} \leq 100}$$

because of nyquist rate : $f_s \geq 2\,\text{kHz}$

$$\xrightarrow{\text{at least}} \boxed{f_s = 2\,\text{kHz}} \longrightarrow \frac{2k}{N} \leq 100 \longrightarrow N \geq 20$$

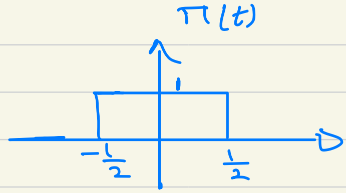
$$\xrightarrow{\text{at least}} \boxed{N = 20}$$

$$L \geq 20 \times \frac{1}{2k} \xrightarrow{\text{at least}} \boxed{L = 0.01\,s}$$

length of signal

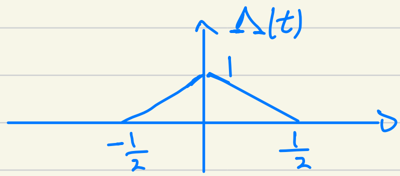
3 -

$\Pi(t) \xleftrightarrow{\mathcal{F}} \text{sinc}(f)$

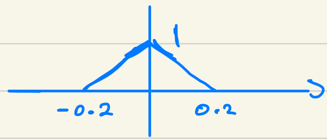


$\Lambda(\frac{t}{2}) = \Pi(t) * \Pi(t)$

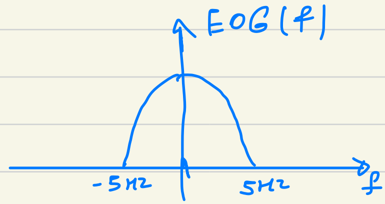
$\rightarrow \quad \Lambda(\frac{t}{2}) \xleftrightarrow{\mathcal{F}} \text{sinc}^2(f)$



$EOG(f) = \frac{1}{5}\,\text{sinc}^2\left(\frac{f}{5}\right)$



$\rightarrow \quad EOG(f) = 0$ for $|f| > 5HZ$



So this noise can have effect on delta and theta bands.

Made with Goodnotes

4 -

a)

peek at 0 HZ : it's DC and it doesn't have any useful information. it could be happen cause to our messuring method or poor electrode connection.

peek at 50 HZ : it's because of power line noise.

peek at 10 HZ : it can show activity of alphe band in brain. it can be happen by SSVEP too.

peek at 22 HZ : it shows activity of β Band.

this band is active when person is awake and do something with open eyes.

I think this data is collected through an SSVEP task. the person was looking at monitor and because of that, we can observe Beta band. also we have a peek at 10 Hz caused by turning off and on of monitor.

b)

as we can see, we don't have so much information below 50 HZ, so we can use a lowpass filter with $f_c = 45\,HZ$ and then Sample the signal with $f_s = 90\,HZ$.

with doing this we can remove 50 HZ noise and prevent aliasing too. most of the time default sampling frequency of EEG cap is around 500 HZ, but we can down sample the signal.

c)

we know if a signal is real then its DFT is symmetric conjugate.

$$\rightarrow X_1[500] = X_1[1500]^* = 2-3j$$

$$f_k = \frac{k}{N} f_s \rightarrow f_{500} = \frac{500}{2000} \times 700 = \boxed{175\,HZ}$$

$$f_{1500} = \frac{1500}{2000} \times 700 = \boxed{525\,HZ}$$

$$\rightarrow x_1(e^{j2\pi \times 175}) = 2-3j \quad , \quad x_1(e^{j2\pi \times 525}) = 2+3j$$

d)

$$f_K = \frac{K}{N} f_s \longrightarrow K = \frac{N f_K}{f_s}$$

$$\xrightarrow{12\,Hz} K = \frac{12\cancel{0}\cancel{0} \times 12}{5\cancel{0}\cancel{0}} = 28.8$$

$$\xrightarrow{22\,Hz} K = \frac{12\cancel{0}\cancel{0} \times 22}{5\cancel{0}\cancel{0}} \doteq 52.8$$

$$\longrightarrow [12\,Hz, 22\,Hz] \sim K = [28, 53]$$

5-

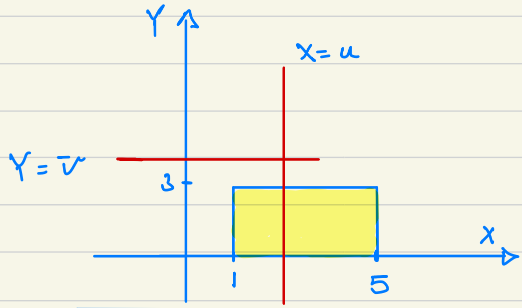
a)

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_{XY}(x,y)\,dx\,dy = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \alpha x y\,dx\,dy = 1$$

$$\to \int_0^3 y\int_1^5 x\,dx\,dy = \frac{1}{\alpha} \to 12\,\frac{y^2}{2}\Big|_0^3 = \frac{1}{\alpha}$$

$$\to \boxed{\alpha = \frac{1}{54}} \qquad \to f_{XY}(x,y) = \begin{cases} \frac{xy}{54} & 1<x<5,\ 0<y<3 \\ 0 & \text{o.w} \end{cases}$$

$$F_{XY}(u,v) = P(X \leq u, Y \leq v) = \int_{-\infty}^{y}\int_{-\infty}^{x} f_{XY}(u,v)\,du\,dv$$



if $x<1$ or $y<0$ $\to F_{XY}(x,y) = \boxed{0}$

if $x>5$ and $y>3$ $\to F_{XY}(x,y) = \boxed{1}$

if $1<x<5$ and $0<y<3$:

$$F_{XY}(x,y) = \int_0^y\int_1^x \frac{uv}{54}\,du\,dv = \frac{1}{54}\times\frac{x^2-1}{2}\times\frac{y^2}{2} = \boxed{\frac{y^2(x^2-1)}{216}}$$

if $x>5$ and $0<y<3$:

$$F_{XY}(x,y) = F_{XY}(5,y) = \frac{24y^2}{216} = \boxed{\frac{y^2}{9}}$$

if $1<x<5$ and $y>3$:

$$F_{XY}(x,y) = F_{XY}(x,3) = \boxed{\frac{x^2-1}{24}}$$

$$F_{XY}(x,y) \begin{cases} 0, & x<1 \text{ or } y<0 \\ 1, & x>5 \text{ and } y>3 \\ \frac{y^2(x^2-1)}{216}, & 1<x<5 \text{ and } 0<y<3 \\ \frac{y^2}{9}, & x>5 \text{ and } 0<y<3 \\ \frac{x^2-1}{24} & 1<x<5 \text{ and } y>3 \end{cases}$$

b)

$$Z = X + 2Y$$

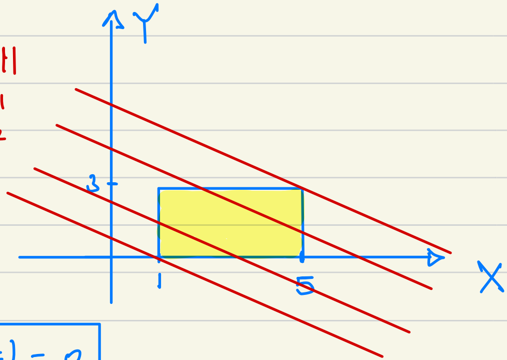
$$F_Z(z) = P(Z \leq z) = P(X + 2Y \leq z) = P\left(Y \leq -\frac{X}{2} + \frac{z}{2}\right)$$

I $z \geq 11$

II $7 \leq z \leq 11$

III $1 \leq z \leq 7$

IV $z \leq 1$



$$\text{I}: \quad F_Z(z) = 1 \xrightarrow{\frac{d}{dz}} \boxed{f_Z(z) = 0}$$

$$\text{IV}: \quad F_Z(z) = 0 \xrightarrow{\frac{d}{dz}} \boxed{f_Z(z) = 0}$$

$$\text{II}: \quad F_Z(z) = 1 - \left(\int_{z-6}^{5} \int_{-\frac{x}{2}+\frac{z}{2}}^{3} \frac{xy}{54}\, dy\, dx\right) = 1 - \frac{(z-11)^2(z^2+22z-3)}{5184}$$

$$\rightarrow \boxed{f_Z(z) = \frac{d}{dz} F_Z(z) = \frac{-z^3 + 183z - 682}{1296}}$$

$$\text{III}: \quad F_Z(z) = \int_{1}^{z} \int_{0}^{-\frac{x}{2}+\frac{z}{2}} \frac{xy}{54}\, dy\, dx = \frac{(z-1)^3(z+3)}{5184}$$

$$\rightarrow \boxed{f_Z(z) = \frac{d}{dz} F_Z(z) = \frac{z^3 - 3z + 2}{1296}}$$

$$\rightarrow f_Z(z) = \begin{cases} 0, & z \geq 11 \\ \frac{-z^3+183z-682}{1296}, & 7 \leq z \leq 11 \\ \frac{z^3-3z+2}{1296}, & 1 \leq z \leq 7 \\ 0, & z \leq 1 \end{cases}$$

6-

a)

$$X(t) = A\cos(\omega t + \theta)$$

* A and θ are two independent random variable

$$\rightarrow E[X(t)] = E[A]E[\cos(\omega t+\theta)]$$

$$E[A] = \int_0^{B} x \times \frac{1}{B} dx = \frac{B}{2}$$

$$E[\cos(\omega t+\theta)] = \int_0^{2\pi} \frac{1}{2\pi}\cos(\omega t+\theta)\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} \cos(\omega t)\cos(\theta) - \sin(\omega t)\sin(\theta)\, d\theta$$

$$= \frac{1}{2\pi}\left(\cos(\omega t)\underbrace{\cancel{\int_0^{2\pi}\cos(\theta)\, d\theta}}_{0} - \sin(\omega t)\underbrace{\cancel{\int_0^{2\pi}\sin\theta\, d\theta}}_{0}\right) = \boxed{0}$$

$$\rightarrow \boxed{E[X(t)] = 0} \quad \mathrm{I}$$

$$R_X(t_1,t_2) = E[X(t_1)X(t_2)] = E[A^2\cos(\omega t_1+\theta)\cos(\omega t_2+\theta)]$$

$$= E\left[A^2 \times \frac{1}{2} \times \left(\cos(\omega(t_1+t_2)+2\theta) + \cos(\omega(t_1-t_2))\right)\right]$$

$$= \frac{1}{2}E[A^2]\Big(\underbrace{\cancel{E[\cos(\omega(t_1+t_2)+2\theta)]}}_{0} + E[\underbrace{\cos(\omega(t_1-t_2))}_{\text{it's a number} \rightarrow = \cos(\omega(t_1-t_2))}]\Big)$$

$$E[A^2] = \int_0^{B} x^2 \times \frac{1}{B} dx = \frac{x^3}{3B}\Big|_0^{B} = \frac{B^2}{3} \quad \rightarrow \boxed{R_X(t_1,t_2) = \frac{B^2}{6}\cos(\omega(t_1-t_2))} \quad \mathrm{II}$$

$\mathrm{I}, \mathrm{II} \rightarrow$ E is constant and R_X only depends on $t_1 - t_2$

So this random process is WSS.

Made with Goodnotes

b)

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{+T} x(t)\,dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{+T} A\cos(\omega t+\theta)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} A \times \left(\cos(\theta)\int_{-T}^{+T}\cos(\omega t)\,dt - \sin\theta \underbrace{\cancel{\int_{-T}^{+T}\sin(\omega t)\,dt}}_{0} \right)$$

$$= \lim_{T\to\infty} \frac{A}{2T} \times \Big(\cos\theta \big(\sin(\omega T) - \sin(-\omega T)\big)\Big) = \lim_{T\to\infty} \frac{A\cos(\theta)\sin(\omega T)}{T}$$

$$= \boxed{0}$$

from part a we know: $E[x(t)] = 0$

$\longrightarrow \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{+T} x(t)\,dt = E[x(t)] \longrightarrow$ it's mean ergodic.

c)

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{+T} x(t)\,x(t-\tau)\,dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{+T} A^2\cos(\omega t+\theta)\cos(\omega(t-\tau)+\theta)\,dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2T}\left(\int_{-T}^{+T}\cos(2\omega t-\omega\tau+2\theta)\,dt + \underbrace{\int_{-T}^{+T}\cos(\omega\tau)\,dt}_{2T\cos(\omega\tau)} \right)$$

$$\longrightarrow \cos(2\theta-\omega\tau)\int_{-T}^{+T}\cos(2\omega t)\,dt + \sin(2\theta-\omega\tau)\underbrace{\cancel{\int_{-T}^{+T}\sin(2\omega t)\,dt}}_{0}$$

$$= \cos(2\theta-\omega\tau) \times \frac{\cancel{2}}{\cancel{2}\omega} \times \sin(2\omega T) = \frac{\cos(2\theta-\omega\tau)\sin(2\omega T)}{\omega}$$

$$= \lim_{T\to\infty} \frac{A^2}{2T} \times \underbrace{\cancel{\frac{\cos(2\theta-\omega\tau)\sin(2\omega T)}{\omega}}}_{0} + \lim_{T\to\infty} \frac{A^2}{\cancel{2T}} \times \cancel{2T}\cos(\omega\tau)$$

$= A^2\cos(\omega\tau)$, from part a we know: $R_x(\tau) = \frac{B^2}{6}\cos(\omega\tau)$

$\longrightarrow$ it's not correlation ergodic.

7-

a)

$$\mathrm{cum}(Y) = (-j)^k \frac{d^k \ln \Phi_Y(w)}{dw^k}\Big|_{w=0}$$

$$\Phi_Y(w) = E[e^{jwY}] = E[e^{jw(X+C)}] = e^{jwc} E[e^{jwX}]$$

$$= e^{jwc}\Phi_X(w)$$

$$\rightarrow \mathrm{cum}(Y) = (-j)^k \frac{d^k \ln(e^{jwc}\Phi_X(w))}{dw^k}\Big|_{w=0}$$

$$= \mathrm{cum}(X) + (-j)^k \frac{d^k \ln e^{jwc}}{dw^k}\Big|_{w=0} = \mathrm{cum}(X) + (-j)^k \frac{d^k jwc}{dw^k}\Big|_{w=0}$$

$$\rightarrow \mathrm{cum}(Y) = \begin{cases} \mathrm{cum}(X) + C, & k=1 \\ \mathrm{cum}(X), & k>1 \end{cases}$$

b)

the purpose of this part is to find $C_n(Y)$ for $n \geq 2$. as I mentioned in part a $C_n(Y)$ for $n \geq 2$ is equal to $C_n(X)$

Made with Goodnotes

c)

$$\text{cum}(X,Y) = -j \frac{\partial}{\partial \omega_X \partial \omega_Y} \ln \Phi_{XY}(\omega_X, \omega_Y) \Big|_{\omega_X=\omega_Y=0}$$

$$\Phi_{X_1+Y_1, X_2+Y_2, \dots, X_m+Y_m}(\omega_1, \omega_2, \dots, \omega_m) = E\left[e^{(X_1+Y_1)\omega_1 + \dots + (X_m+Y_m)\omega_m}\right]$$

cause to independency between $\{X_1, X_2, \dots, X_m\}$ and $\{Y_1, Y_2, \dots, Y_m\}$

we can write:

$$= E\left[e^{X_1\omega_1 + \dots + X_m\omega_m}\right] \cdot E\left[e^{Y_1\omega_1 + \dots + Y_m\omega_m}\right]$$

$$= \Phi_{X_1, X_2, \dots, X_m} \cdot \Phi_{Y_1, Y_2, \dots, Y_m}$$

$$\Rightarrow \text{cum}(X_1+Y_1, \dots, X_m+Y_m) = -j \frac{\partial}{\partial \omega_1 \dots \partial \omega_m} \ln \Phi_{X_1+Y_1, \dots, X_m+Y_m}(\omega_1, \dots, \omega_m) \Big|_{\omega_1=\dots=\omega_m=0}$$

$$= -j \frac{\partial}{\partial \omega_1 \dots \partial \omega_m} \left(\ln \Phi_{X_1, \dots, X_m} + \ln \Phi_{Y_1, \dots, Y_m}\right) \Big|_{\omega_1=\dots=\omega_m=0}$$

$$= \boxed{\text{cum}(X_1, X_2, \dots, X_m) + \text{cum}(Y_1, Y_2, \dots, Y_m)}$$

Made with Goodnotes

9-

a) $A = X\Lambda X^{-1} = \begin{bmatrix}1&1\\2&1\end{bmatrix}\begin{bmatrix}1&0\\0&2\end{bmatrix}\begin{bmatrix}-1&1\\2&-1\end{bmatrix} = \begin{bmatrix}3&-1\\2&0\end{bmatrix}$

b) $A^3 = \begin{bmatrix}3&-1\\2&0\end{bmatrix}\begin{bmatrix}3&-1\\2&0\end{bmatrix}\begin{bmatrix}3&-1\\2&0\end{bmatrix} = \begin{bmatrix}7&-3\\6&2\end{bmatrix}\begin{bmatrix}3&-1\\2&0\end{bmatrix} = \begin{bmatrix}15&-7\\14&-6\end{bmatrix}$

c)

$X\Lambda^3 X^{-1} = \begin{bmatrix}1&1\\2&1\end{bmatrix}\begin{bmatrix}1&0\\0&8\end{bmatrix}\begin{bmatrix}-1&1\\2&-1\end{bmatrix} = \begin{bmatrix}1&8\\2&8\end{bmatrix}\begin{bmatrix}-1&1\\2&-1\end{bmatrix} = \begin{bmatrix}15&-7\\14&-6\end{bmatrix}$

d)

$A = X\Lambda X^{-1} \rightarrow A^n = \underbrace{(X\Lambda \cancel{X^{-1}})(\cancel{X}\Lambda \cancel{X^{-1}})(\cancel{X}\Lambda \cancel{X^{-1}}) \cdots\cdots (\cancel{X}\Lambda X^{-1})}_{n}$

$\rightarrow A^n = X\Lambda^n X^{-1}$

10-

real variables

$$R_X = E[XX^H] = \begin{bmatrix} \overline{X_1X_1^*} & \overline{X_1X_2^*} & \overline{X_3X_3^*} \\ \overline{X_2X_1^*} & \overline{X_2X_2^*} & \overline{X_2X_3^*} \\ \overline{X_3X_1^*} & \overline{X_3X_2^*} & \overline{X_3X_3^*} \end{bmatrix} = \begin{bmatrix} \overline{X_1^2} & \overline{X_1X_2} & \overline{X_1X_3} \\ \overline{X_2X_1} & \overline{X_2^2} & \overline{X_2X_3} \\ \overline{X_3X_1} & \overline{X_3X_2} & \overline{X_3^2} \end{bmatrix}$$

$$E[x_1^2] = Var(x_1) + E[x_1]^2 = 1 + 256 = 257$$

independency

$$E[x_1x_2] \overset{!}{=} E[x_1]\,E[x_2] = 256$$

Like above : $E[x_1x_3] = E[x_2x_3] = 256$

$E[x_2^2] = 258$, $E[x_3^2] = 259$

$$\rightarrow R_X = \begin{bmatrix} 257 & 256 & 256 \\ 256 & 258 & 256 \\ 256 & 256 & 259 \end{bmatrix}$$

$$C_X = E\left[(X - m_X)(X - m_X)^H\right] = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1x_2} & \sigma_{x_1x_3} \\ \sigma_{x_2x_1} & \sigma_{x_2}^2 & \sigma_{x_2x_3} \\ \sigma_{x_3x_1} & \sigma_{x_3x_2} & \sigma_{x_3}^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

11-

a) $X(t) = e^{W(t)}$, $W(t) \sim N(0,t)$

$E[X(t)] = E[e^{W(t)}]$

moment generating function

in each t it's equal to $M_W(1)$

if $X \sim N(\mu, \sigma^2) \to M_x(t) = E[e^{xt}] = \int e^{xt} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$

$z = \frac{x-\mu}{\sigma} \to M_x(t) = e^{\mu t} \int e^{z\sigma t} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}z^2} \times \sigma dz$

$= e^{\mu t} \int e^{z\sigma t} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz = e^{\mu t} e^{\frac{\sigma^2 t^2}{2}}$

$\to M_W(1) = e^{\mu} e^{\frac{\sigma^2}{2}} = e^{\frac{t^2}{2}}$

$\to \boxed{E[e^{W(t)}] = e^{\frac{t^2}{2}}}$

b) $var(X(t)) = var(e^{W(t)}) = E[e^{2W(t)}] - E[e^{W(t)}]^2$

$E[e^{2W(t)}] = M_W(2) = e^{2t^2} \to var(X(t)) = e^{2t^2} - e^{\frac{t^2}{2}} = \boxed{e^{\frac{t^2}{2}}\left(e^{\frac{3t^2}{2}} - 1\right)}$

12-

if Y_1 and Y_2 are uncorrelated $\longrightarrow E[Y_1 Y_2] = E[Y_1]\,E[Y_2]$

$E[Y_1] = E[\cos(\alpha) X_1 + \sin(\alpha) X_2] = \cos\alpha\, \underbrace{E[X_1]}_{0} + \sin(\alpha)\, \underbrace{E[X_2]}_{0}$

$= 0 \longrightarrow E[Y_1 Y_2] = 0$

$E[Y_1 Y_2] = E\left[-\cos\alpha \sin\alpha X_1^2 + \sin\alpha\cos\alpha X_2^2 + (\cos^2\alpha - \sin^2\alpha) X_1 X_2\right]$

$= E[\sin\alpha\cos\alpha X_2^2] - E[\cos\alpha \sin\alpha X_1^2] + E[\cos 2\alpha X_1 X_2]$

$= \sin\alpha\cos\alpha\left(E[X_2^2] - E[X_1^2]\right) + \cos 2\alpha\, E[X_1 X_2]$

$= \sin\alpha\cos\alpha\,(\sigma_2^2 - \sigma_1^2) + \cos 2\alpha\, \rho\sigma_1\sigma_2$

$= \dfrac{\sin 2\alpha}{2}(\sigma_2^2 - \sigma_1^2) + \cos 2\alpha\, \rho\sigma_1\sigma_2 = 0$

$\div \cos 2\alpha \longrightarrow \quad \dfrac{1}{2}\tan(2\alpha)\,(\sigma_2^2 - \sigma_1^2) + \rho\sigma_1\sigma_2 = 0$

$\longrightarrow \tan(2\alpha) = \dfrac{-2\rho\sigma_1\sigma_2}{\sigma_2^2 - \sigma_1^2} = \dfrac{2\rho\sigma_1\sigma_2}{\sigma_1^2 - \sigma_2^2}$

$\longrightarrow \boxed{\alpha = \dfrac{1}{2}\tan^{-1}\left(\dfrac{2\rho\sigma_1\sigma_2}{\sigma_1^2 - \sigma_2^2}\right)}$

13 -

$$R_{yy}(t) = E[Y(t)Y(t-\tau)] = E[(X(at+b)+X(at-b))(X(at-a\tau+b)+X(at-a\tau-b))]$$

$$= E[X(at+b)X(at-a\tau+b)] + E[X(at+b)X(at-a\tau-b)]$$

$$+ E[X(at-b)X(at-a\tau+b)] + E[X(at-b)X(at-a\tau-b)]$$

$$= R_{XX}(a\tau) + R_{XX}(a\tau+2b) + R_{XX}(a\tau-2b) + R_{XX}(-a\tau)$$

$$= \boxed{2R_{XX}(a\tau) + R_{XX}(a\tau+2b) + R_{XX}(a\tau-2b)}$$

14 -

$$Z(t) = X(t) + Y(t)$$

$$E[Z(t)] = E[X(t)] + E[Y(t)] = m_X + m_Y \rightarrow \text{Constant}$$

$$E[Z(t_1)Z(t_2)] = E[(X(t_1)+Y(t_1))(X(t_2)+Y(t_2))]$$

$$= \underbrace{E[X(t_1)X(t_2)]}_{R_{XX}(\tau)} + \underbrace{E[X(t_1)Y(t_2)]}_{R_{XY}(\tau)} + \underbrace{E[Y(t_1)X(t_2)]}_{R_{YX}(\tau)} + \underbrace{E[Y(t_1)Y(t_2)]}_{R_{YY}(\tau)}$$

$\rightarrow$ $R_Z(t_1, t_2)$ only depends on $\tau = t_1 - t_2$

$\rightarrow$ $Z(t)$ is WSS