تمرین سری 3 درس ML - رادین خیام - 99101579

سوال 1 -

$$E[\hat{p}(x)] = \int \frac{1}{h}\phi\left(\frac{x-\lambda}{h}\right)p(\lambda)\,d\lambda$$

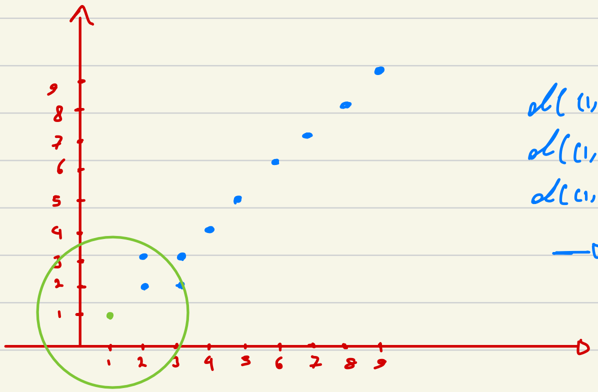
$$var(\hat{p}(x)) = var\left(\frac{1}{N}\sum_{k=1}^{N}\frac{1}{h}\phi\left(\frac{x-x_k}{h}\right)\right)$$

$$= \sum_{k=1}^{N} var\left[\frac{1}{N}\frac{1}{h}\phi\left(\frac{x-x_k}{h}\right)\right] = N\times\left(E\left[\left(\frac{1}{Nh}\phi\left(\frac{x-x_k}{h}\right)\right)^2\right] - \left(E[\hat{p}(x)]\right)^2\right)$$

$$= \frac{1}{Nh}E\left[\frac{1}{h}\phi^2\left(\frac{x-x_k}{h}\right)\right] - N\left(E[\hat{p}(x)]\right)^2$$

$$= \frac{1}{Nh}\int\frac{1}{h}\phi^2\left(\frac{x-\lambda}{h}\right)p(\lambda)\,d\lambda - N\left(\int\frac{1}{h}\phi\left(\frac{x-\lambda}{h}\right)p(\lambda)\,d\lambda\right)^2$$

سؤال 2 -



$d((1,1),(2,3)) = \sqrt{5}$

$d((1,1),(3,2)) = \sqrt{5}$

$d((1,1),(2,2)) = \sqrt{2}$

$\rightarrow V = (\sqrt{5})^2 \pi = 5\pi$

$\rightarrow \hat{P}((1,1)) = \frac{K}{NV} = \frac{3}{10 \times 5\pi} = \boxed{\frac{3}{50\pi}}$

سؤال 3 -

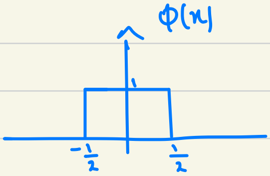
الف)

$$\mu_1 = \frac{1+1.3+1.5+1.6+1.8}{5} = 1.44$$

$$\mu_2 = \frac{1.7+1.9+2+2.2+2.5}{5} = 2.06$$

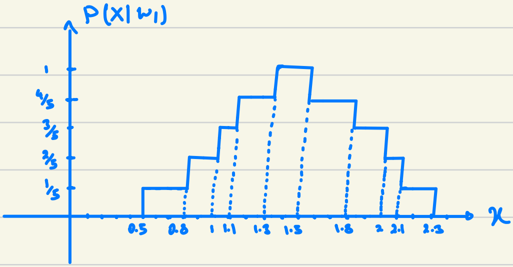
	$\lVert x_K - \mu_1 \rVert$	$\lVert x_K - \mu_2 \rVert$	$\hat{w}$
1.4	0.04	0.66	w_1
1.65	0.21	0.41	w_1
1.75	0.31	0.31	N
1.85	0.41	0.21	w_2
2.1	0.66	0.04	w_2

ب)

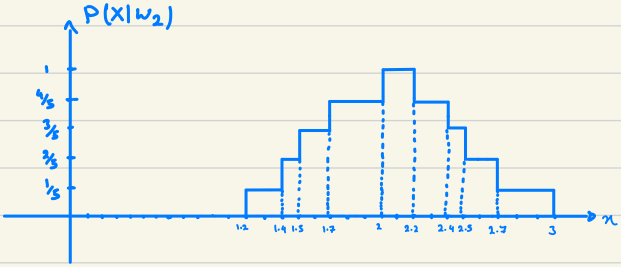


$$\hat{P}(x|w_1) = \frac{1}{5}\sum_{k=1}^{5}\phi(x-x_k)$$

$$\rightarrow \hat{P}(x|w_1) = \frac{1}{5}\big(\phi(x-1)+\phi(x-1.3)+\phi(x-1.5)+\phi(x-1.6)+\phi(x-1.8)\big)$$



$$\hat{P}(x|w_2) = \frac{1}{5}\big(\phi(x-1.7)+\phi(x-1.9)+\phi(x-2)+\phi(x-2.2)+\phi(x-2.5)\big)$$



	$\hat{P}(x\|w_1)$	$\hat{P}(x\|w_2)$	$\hat{\omega}$
1.4	1	0.2	w_1
1.65	0.8	0.6	w_1
1.75	0.8	0.8	N
1.85	0.6	0.8	w_2
2.1	0.2	1	w_2

پ)

$$\hat{P}(x) = \frac{K}{N\,V(x,K)} = \frac{3}{5\,V(x,3)}$$

$$1.4 \left\{ \begin{array}{l} \nearrow 1.3, 1.5, 1.6 \longrightarrow V = 0.04\pi \longrightarrow \hat{P}(x|\omega_1) = \frac{3}{0.2\pi} \\ \searrow 1.7, 1.9, 2 \longrightarrow V = 0.36\pi \longrightarrow \hat{P}(x|\omega_2) = \frac{3}{1.8\pi} \end{array} \right\} \longrightarrow \omega_1$$

$$1.65 \left\{ \begin{array}{l} \nearrow 1.6, 1.8, 1.5 \longrightarrow V = 0.0225\pi \longrightarrow \hat{P}(x|\omega_1) = \frac{5}{0.1125\pi} \\ \searrow 1.7, 1.9, 2 \longrightarrow V = 0.1225\pi \longrightarrow \hat{P}(x|\omega_2) = \frac{5}{0.6125\pi} \end{array} \right\} \longrightarrow \omega_1$$

$$1.75 \left\{ \begin{array}{l} \nearrow 1.5, 1.6, 1.8 \longrightarrow V = 0.0625\pi \longrightarrow \hat{P}(x|\omega_1) = \frac{5}{0.3125\pi} \\ \searrow 1.7, 1.9, 2 \longrightarrow V = 0.0625\pi \longrightarrow \hat{P}(x|\omega_2) = \frac{5}{0.3125\pi} \end{array} \right\} \longrightarrow N$$

$$1.85 \left\{ \begin{array}{l} \nearrow 1.5, 1.6, 1.8 \longrightarrow V = 0.1225\pi \longrightarrow \hat{P}(x|\omega_1) = \frac{5}{0.6125\pi} \\ \searrow 1.7, 1.9, 2 \longrightarrow V = 0.0225\pi \longrightarrow \hat{P}(x|\omega_2) = \frac{5}{0.1125\pi} \end{array} \right\} \longrightarrow \omega_2$$

$$2.1 \left\{ \begin{array}{l} \nearrow 1.5, 1.6, 1.8 \longrightarrow V = 0.36\pi \longrightarrow \hat{P}(x|\omega_1) = \frac{5}{1.8\pi} \\ \searrow 1.9, 2, 2.2 \longrightarrow V = 0.04\pi \longrightarrow \hat{P}(x|\omega_2) = \frac{5}{0.2\pi} \end{array} \right\} \longrightarrow \omega_2$$

ت)

$K=3$:

$$1.4 \longrightarrow \overset{\omega_1}{1.3}, \overset{\omega_1}{1.5}, \overset{\omega_2}{1.7} \longrightarrow \omega_1$$

$$1.65 \longrightarrow \overset{\omega_1}{1.6}, \overset{\omega_2}{1.7}, \overset{\omega_1}{1.5} \longrightarrow \omega_1$$

$$1.75 \longrightarrow \overset{\omega_2}{1.7}, \overset{\omega_1}{1.8}, \overset{\omega_1}{1.6} \text{ or } \overset{\omega_2}{1.9} \longrightarrow N$$

$$1.85 \longrightarrow \overset{\omega_1}{1.8}, \overset{\omega_2}{1.9}, \overset{\omega_2}{1.7} \longrightarrow \omega_2$$

$$2.1 \longrightarrow \overset{\omega_2}{2}, \overset{\omega_2}{2.2}, \overset{\omega_1}{1.8} \longrightarrow \omega_2$$

$K=1$

$$1.4 \longrightarrow \overset{\omega_1}{1.3} \longrightarrow \omega_1$$

$$1.65 \longrightarrow \overset{\omega_1}{1.6} \text{ or } \overset{\omega_2}{1.7} \longrightarrow N$$

$$1.75 \longrightarrow \overset{\omega_1}{1.8} \text{ or } \overset{\omega_2}{1.7} \longrightarrow N$$

$$1.85 \longrightarrow \overset{\omega_1}{1.8} \text{ or } \overset{\omega_2}{1.9} \longrightarrow N$$

$$2.1 \longrightarrow \overset{\omega_2}{2} \longrightarrow \omega_2$$

سئوال 4 -

الف)

$$P(x|\theta) = \sum_{j=1}^{2} P(x|z=j;\theta)P(z=j) = \sum_{j=1}^{2} N(x;\mu_j,\sigma_j^2)P(z=j)$$

$$= \alpha N(x;\mu_1,\sigma_1^2) + (1-\alpha) N(x;\mu_2,\sigma_2^2)$$

ب)

$$P(z_n=j|x_n,\theta) = \frac{P(x_n|z_n,\theta)P_{z_n}}{P(x_n,\theta)}$$

$$\to P(z_n=1|x_n,\theta) = \frac{\alpha N(x_n;\mu_1,\sigma_1^2)}{\alpha N(x_n;\mu_1,\sigma_1^2)+(1-\alpha)N(x_n;\mu_2,\sigma_2^2)}$$

$$P(z_n=2|x_n,\theta) = \frac{(1-\alpha)N(x_n;\mu_2,\sigma_2^2)}{\alpha N(x_n;\mu_1,\sigma_1^2)+(1-\alpha)N(x_n;\mu_2,\sigma_2^2)}$$

پ)

$$Q(\theta) = \sum_{Z} P(Z|X,\theta) Ln P(X,Z|\theta)$$

$$= \sum_{n=1}^{N}\sum_{z_n=1}^{2} P(z_n|x_n,\theta) Ln(P(x_n|z_n,\theta)P_{z_n})$$

$$= \sum_{n=1}^{N} P(z_n=1|x_n,\theta)\ Ln(\alpha N(x_n;\mu_1,\sigma_1^2)) + P(z_n=2|x_n,\theta)$$

$$Ln((1-\alpha)N(x_n;\mu_2,\sigma_2^2))$$

$$= \sum_{n=1}^{N} P(z_n=1|x_n,\theta)\left(-\frac{1}{2}\ln\sigma_1^2 - \frac{1}{2\sigma_1^2}(x_n-\mu_1)^2 + \ln\alpha\right)$$

$$+ P(z_n=2|x_n,\theta)\left(-\frac{1}{2}\ln\sigma_2^2 - \frac{1}{2\sigma_2^2}(x_n-\mu_2)^2 + \ln(1-\alpha)\right)$$

ش)

$$\frac{\partial(Q(\theta;\theta(t)))}{\partial\mu_j} = \sum_{n=1}^{N} P(z_n=j|x_n;\theta(t))\left(-\frac{\mu_j-x_K}{2\sigma_j^2}\right)=0$$

$$\rightarrow \mu_j(t+1) = \frac{\sum_{n=1}^{N} P(z_n=j|x_K;\theta(t))\,x_n}{\sum_{n=1}^{N} P(z_n=j|x_K;\theta(t))}$$

$$\frac{\partial(Q(\theta;\theta(t)))}{\partial\sigma_j^2} = \sum_{n=1}^{N} P(z_n=j|x_n;\theta(t))\left(-\frac{1}{2}\frac{1}{\sigma_j^2} + \frac{1}{2\sigma_j^4}(x_n-\mu_j)^2\right)=0$$

$$\rightarrow \sigma_j^2(t+1) = \frac{\sum_{n=1}^{N} P(z_n=j|x_n;\theta(t))(x_n-\mu_j)^2}{\sum_{n=1}^{N} P(z_n=j|x_n;\theta(t))}$$

سؤال 5 -

الف)

$$\text{sensitivity} = \frac{TP}{TP+FN} = 1 \rightarrow FN = 0$$

$$\text{Accuracy} = \frac{TP+TN}{TP+TN+FP+\overset{0}{\cancel{FN}}}$$

حالت احتمال برابر برای دو کلاس:

$$TP + \overset{0}{\cancel{FN}} = TN + FP \rightarrow \text{Accuracy} = \frac{2TN+FP}{2TN+2FP} = 1 - \frac{FP}{2(TN+FP)}$$

$$= 1 - \frac{1}{2}(1 - \text{specificity}) = \frac{1}{2} + \frac{1}{2}\,\text{specificity}$$

$$\rightarrow \boxed{\%50 < Acc < \%100}$$

حالت دو کلاس با احتمال نابرابر:

$$Acc = \frac{TP+TN}{TP+TN+FP} = 1 - \frac{FP}{TP+TN+FP}$$

$$\rightarrow \boxed{0 < ACC < 1}$$

ب)

$$spec = \frac{TN}{TN+FP} = 1 \rightarrow FP = 0$$

$$ACC = \frac{TP+TN}{TP+TN+FN+\cancel{FP}^{0}} = 1 - \frac{FN}{TP+TN+FN}$$

$$\rightarrow \boxed{0 < ACC < 1}$$

پ)

$$PP = \frac{TP}{TP+FP} = 1 \rightarrow FP = 0$$

$$\rightarrow ACC = \frac{TP+TN}{TP+TN+FN+\cancel{FP}0} \rightarrow ACC = 1 - \frac{FN}{TP+TN+FN} \rightarrow \boxed{0 \; ACC < 1}$$

ت)

① $spec = 1 , sens = 1 \rightarrow FP = 0 , FN = 0$

$$\rightarrow AC = \frac{TP+TN}{TP+TN+\underset{0}{\cancel{FP}}+\underset{0}{\cancel{FN}}} = \boxed{1}$$

② $spec = 1 , PP = 1 \rightarrow FP = 0 \rightarrow \boxed{0 < ACC < 1}$

③ $sens = 1 , PP = 1 \rightarrow FP = 0 , FN = 0 \rightarrow \boxed{ACC = 1}$

پس جواب خیر است.

سؤال 6)

در نقطه $(0.25, 0.8)$ منحنی روابط رو می نویسیم :

$$sens = \frac{TP}{TP+FN} = \frac{4}{5} \rightarrow 5TP = 4TP + 4FN$$

$$\rightarrow \boxed{TP = 4FN}$$

$$1-spec = 1 - \frac{TN}{TN+FP} = \frac{FP}{TN+FP} = \frac{1}{4} \rightarrow 4FP = TN + FP$$

$$\rightarrow \boxed{TN = 3FP}$$

$$TP + FN = 100 \rightarrow 5FN = 100 \rightarrow FN = 20$$

$$TN + FP = 120 \rightarrow 4FP = 120 \rightarrow FP = 30$$

80	30
20	90

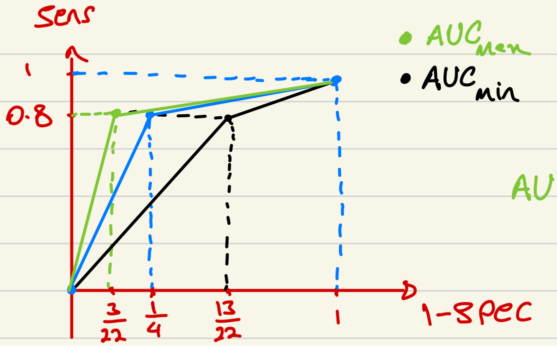
100 نفر اضافه شده بین TN و FP باید تقسیم بشوند در هر صورت مقدار sens عوض نمی‌شود اما مقدار spec تغییر می کند.

حالت حدی اول این هست که همه به TN اضافه شوند و حالت حدی دوم زمانی است که همه به FP اضافه شوند.

① $spec = \frac{190}{220} \rightarrow 1-spec = \frac{3}{22}$

② $spec = \frac{90}{220} \rightarrow 1-spec = \frac{13}{22}$

ادامه سؤال 6)



$$AUC_{max} = \frac{0.8 \times \frac{3}{22}}{2} + \frac{1+0.8}{2} \times \frac{19}{22}$$

$$= 0.832$$

$$AUC_{min} = \frac{0.8 \times \frac{13}{22}}{2} + \frac{1+0.8}{2} \times \frac{9}{22}$$

$$= 0.605$$

$$\rightarrow AUC_{max} - AUC_{min} = 0.832 - 0.605 = 0.227$$

سؤال 7)

الف)

TP 37	FP 8
FN 3	TN 52

$$ACC = \frac{TP+TN}{TP+TN+FP+FN} = \frac{89}{100}$$

$$sens = \frac{TP}{TP+FN} = \frac{37}{40}$$

$$Spec = \frac{TN}{TN+FP} = \frac{52}{60}$$

$$MR = \frac{FN}{TP+FN} = \frac{3}{40}$$

$$PPV = \frac{TP}{TP+FP} = \frac{37}{45}$$

$$FPR = \frac{FP}{TN+FP} = \frac{8}{60}$$

$$\text{False Alarm Rate} = \frac{FP}{TP+FP} = \frac{8}{45}$$

ب)

$$N_{pos} = TP+FN = 40$$

$$N_{neg} = TN+FP = 60$$

$$\rightarrow P(pos) = \frac{4}{10}$$

$$P(neg) = \frac{6}{10}$$

سؤال ۸)

در هر صورت :

$TP + FN = 100$

$TN + FP = 4900$

$$ACC = \frac{TP+TN}{TP+FN+TN+FP} = \frac{TP+TN}{5000}$$

اینکه TP و TN چقدری شوند را نمی‌شود هیچ جوره پیش بینی کرد.