


نہین سر 6 درس ML - رابین خیم - 99101579

سؤال 1-

$$P(w_i | x) = \frac{1}{c} \prod_{j=1}^L (P_j(w_i | x))^{1/L}$$

$$D_{av} = \frac{1}{L} \sum_{j=1}^L D_j, \quad D_j = \sum_{i=1}^m P(w_i | x) \ln \frac{P(w_i | x)}{P_j(w_i | x)}$$

$$\min \left(\frac{1}{L} \sum_{j=1}^L \sum_{i=1}^m P(w_i) \log \left(\frac{P(w_i)}{P_j(w_i)} \right) \right)$$

$$\sum_{i=1}^m P(w_i) = 1$$

$$L(P(w_i)) = \frac{1}{L} \sum_{j=1}^L \sum_{i=1}^m P(w_i) \log \frac{P(w_i)}{P_j(w_i)} - \lambda \left(\sum_{i=1}^m P(w_i) - 1 \right)$$

$$\rightarrow \frac{1}{L} \sum_{j=1}^L \log(P(w_i)) + 1 - \frac{1}{L} \sum_{j=1}^L \log P_j(w_i) - \lambda = 0$$

$$P(w_i) = \exp(\lambda - 1) \exp\left(\frac{1}{L} \sum_{j=1}^L \log P_j(w_i)\right)$$

$$\sum_{i=1}^m P(w_i) = 1 \rightarrow \exp(\lambda - 1) = \frac{1}{\sum_{i=1}^m \exp\left(\frac{1}{L} \sum_{j=1}^L \log P_j(w_i)\right)}$$

$$P(w_i) = \frac{\exp\left(\frac{1}{L} \log \prod_{j=1}^L P_j(w_i)\right)}{\sum_{i=1}^m \exp\left(\frac{1}{L} \log \prod_{j=1}^L P_j(w_i)\right)} \rightarrow P(w_i) = \frac{\exp\left(\frac{1}{L} \log \prod_{j=1}^L P_j(w_i)\right)}{\sum_{i=1}^m \exp\left(\frac{1}{L} \log \prod_{j=1}^L P_j(w_i)\right)}$$

$$\rightarrow P(w_i) = \frac{\prod_{j=1}^L (P_j(w_i))^{1/L}}{\sum_{i=1}^m \prod_{j=1}^L (P_j(w_i))^{1/L}}$$

سؤال 2-

$$L(P(w_i)) = \frac{1}{L} \sum_{j=1}^L \sum_{i=1}^m P_j(w_i) \log P_j(w_i) - \frac{1}{L} \sum_{j=1}^L \sum_{i=1}^m P_j(w_i) \log P(w_i) \\ - \lambda \left(\sum_{i=1}^m P(w_i) - 1 \right)$$

$$\left. \begin{aligned} P(w_i) &= -\frac{1}{\lambda L} \sum_{j=1}^L P_j(w_i) \\ \lambda &= -\frac{1}{L} \sum_{i=1}^m \sum_{j=1}^m P_j(w_i) \end{aligned} \right\} \rightarrow P(w_i) = \frac{1}{L} \sum_{j=1}^L P_j(w_i)$$

سؤال 3 -

$$\sum_{i=1}^n \exp(-y_i F(x_i)) = \sum_{y_i \Phi(x_i, \theta) > 0} \exp(-y_i F(x_i)) + \sum_{y_i \Phi(x_i, \theta) < 0} \exp(-y_i F(x_i))$$

$$\sum_{i=1}^n \exp(-y_i F(x_i)) \geq \sum_{y_i \Phi(x_i, \theta) < 0} \exp(-y_i F(x_i)) \geq \sum_{y_i \Phi(x_i, \theta) < 0} 1$$

سؤال 4 -

$$\alpha_m = \arg \min (e^{-\alpha} (1 - p_m) + e^{\alpha} p_m)$$

$$\frac{d}{d\alpha} (e^{-\alpha} (1 - p_m) + e^{\alpha} p_m) = 0$$

$$\rightarrow -e^{-\alpha} (1 - p_m) + e^{\alpha} p_m = 0$$

$$\ln(e^{\alpha} p_m) = \ln(e^{-\alpha} (1 - p_m)) \rightarrow \alpha + \ln(p_m) = -\alpha + \ln(1 - p_m)$$

$$\rightarrow 2\alpha = \ln(1 - p_m) - \ln(p_m) \rightarrow \alpha = \frac{1}{2} \ln\left(\frac{1 - p_m}{p_m}\right)$$

سؤال 5 -

$$Z_m = \sum_{i=1}^N w_i^{(m)} \exp(-y_i \alpha_m \phi(x_i, \theta_m))$$

$$w_i^{(m)} = \frac{\exp(-y_i F_{m-1}(x_i))}{Z_{m-1}}$$

$$\rightarrow Z_m Z_{m-1} = \sum_{i=1}^N \exp(-y_i F_{m-1}(x_i)) \exp(-y_i \alpha_m \phi(x_i, \theta_m))$$

$$w_i^{(1)} = \frac{1}{N} \rightarrow \prod_{m=1}^K Z_m = \frac{1}{N} \sum_{i=1}^N \exp(-y_i F(x_i))$$

سؤال 6 -

$$P_1 = \{0.1, 0.2, 0.3, 0.4\}$$

$$P_2 = \{0.4, 0.3, 0.2, 0.1\}$$

$$D = - \sum_i P_1(x) \ln\left(\frac{P_2(x)}{P_1(x)}\right) = -(0.1 \ln(4) + 0.2 \ln(\frac{3}{2})$$

$$+ 0.3 \ln(\frac{2}{3}) + 0.4 \ln(\frac{1}{4})) = \boxed{0.46}$$