تمرین سری ۵ درس سیگنال - رادین خیام - 99101579

سئوال 1)

a)

$$\hat{x}(e^{j\omega})\Big|_{\omega=0} = \sum_{n=-\infty}^{\infty} x[n] = (1-3j)+(2+j)+(3-2j)+(3+2j)+(2-j)+(1+3j) = \boxed{12}$$

b)

$$\hat{x}(e^{j\omega})\Big|_{\omega=\pi} = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\pi n} = \sum_{n=-\infty}^{+\infty} (-1)^n x[n]$$

$$\rightarrow \hat{x}(e^{j\omega})\Big|_{\omega=\pi} = 6-6j-6-6j = \boxed{-12j}$$

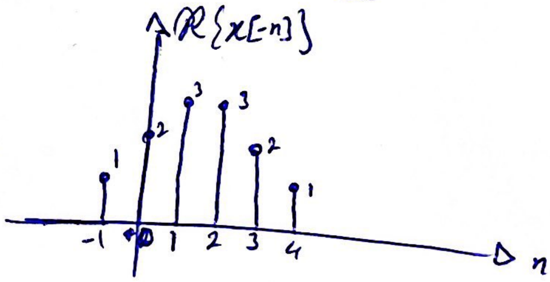
c)

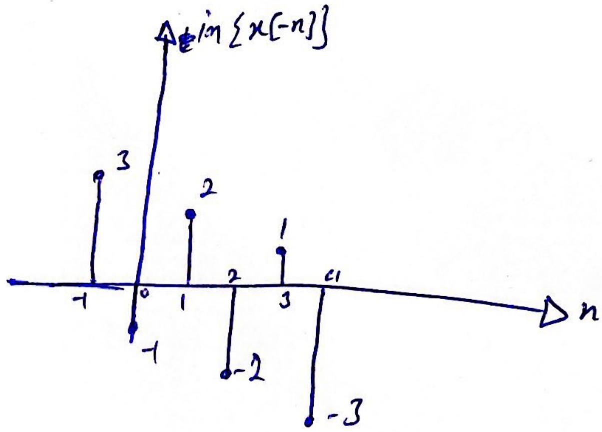
$$\int_{-\pi}^{+\pi} \hat{x}(e^{j\omega})\,d\omega = 2\pi x[0] = 2\pi(2-j) = \boxed{4\pi-2\pi j}$$

d) طبق خواص فوریه میدانیم که:

if $x[n] \xleftrightarrow{DTFT} \hat{x}(e^{j\omega})$

$\rightarrow x[-n] \xleftrightarrow{DTFT} X(e^{-j\omega}) \rightarrow$



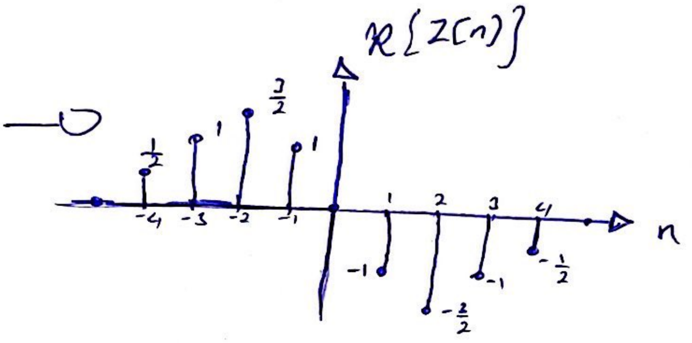
ادامه سوال ۱)

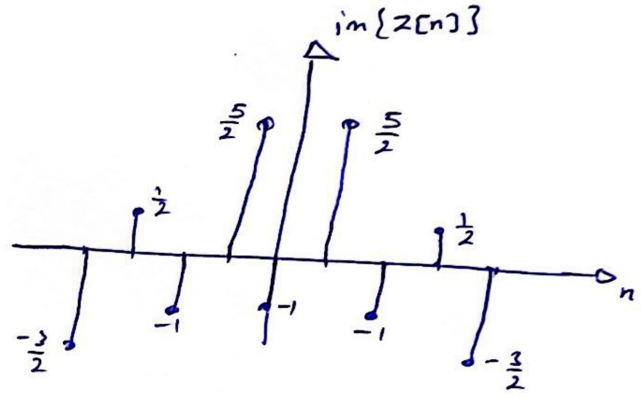
e) باز هم طبق خواص فوریه میدانیم که :

if $x[n] \longleftrightarrow \hat{X}(e^{j\omega})$

$$\longrightarrow \underbrace{\frac{1}{2}\left(x[n] - \bar{x}[-n]\right)}_{z[n]} \longleftrightarrow j\, im\{\hat{X}(e^{j\omega})\}$$

$\longrightarrow$



سؤال 2)

(a

$$y[n] = -2x[n] + 4x[n-1] - 2x[n-2]$$

$$\rightarrow h[n] = -2\delta[n] + 4\delta[n-1] - 2\delta[n-2]$$

(b

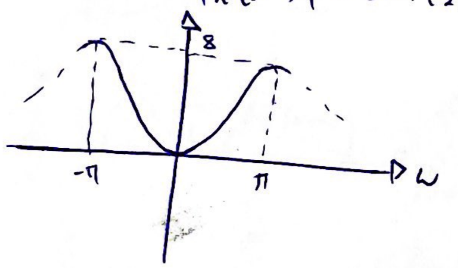
$$\hat{h}(e^{j\omega}) = -2 + 4e^{-j\omega} - 2e^{-j2\omega} = -2e^{-j\omega}(\underbrace{e^{j\omega} + e^{-j\omega}}_{2\cos(\omega)} - 2)$$

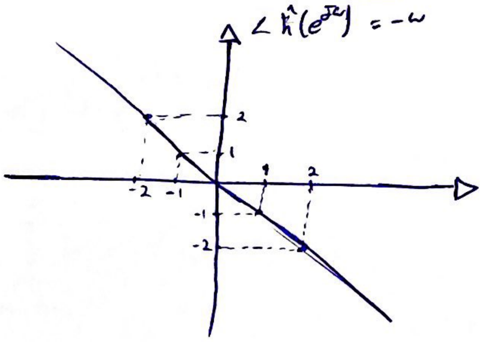
$$\rightarrow \hat{h}(e^{j\omega}) = 4e^{-j\omega}(\underbrace{1-\cos(\omega)}_{2\sin^2(\frac{\omega}{2})}) = \boxed{8\sin^2(\frac{\omega}{2})e^{-j\omega}}$$

$$\rightarrow \boxed{a(\omega) = 8\sin^2(\frac{\omega}{2})}$$

$$\boxed{n_d = 1}$$

(c




(d

می دانیم که:

$$x[n] = e^{jan} \rightarrow \boxed{\hat{h}(e^{j\omega})} \rightarrow y[n] = e^{jan} \times \hat{h}(e^{ja})$$

$$\rightarrow x_1[n] = 1 + e^{j\frac{\pi}{2}n} = e^{j0n} + e^{j\frac{\pi}{2}n}$$

$$\rightarrow y_1[n] = \hat{h}(e^{j0})e^{j0n} + \hat{h}(e^{j\frac{\pi}{2}})e^{j\frac{\pi}{2}n} = 0 + 8\sin^2(\frac{\pi}{4})e^{-j\frac{\pi}{2}} \times e^{j\frac{\pi}{2}n}$$

$$= 8 \times \frac{1}{2} \times e^{+j\frac{\pi}{2}(n-1)} = \boxed{4e^{j\frac{\pi}{2}(n-1)}}$$

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ادامه سئوال 2)

e)

$$x_2[n] = \left(1+e^{j\frac{\pi}{2}n}\right)u[n]$$

$$\longrightarrow y_2[n] = \sum_{m=-\infty}^{+\infty} h[m]\,x_2[n-m] = \sum_{m=-\infty}^{+\infty} h[m]\left(1+e^{j\frac{\pi}{2}(n-m)}\right)u[n-m]$$

$$= \sum_{m=-\infty}^{n} h[m]\left(1+e^{j\frac{\pi}{2}(n-m)}\right)$$

از آنجایی که $h[n]$ برای $n$ های کوچکتر از صفر برابر با صفر است می‌شود گفت که:

$$y_2[n] = 0 \quad , \text{ if } n<0$$

$$y_2[n] = \underbrace{\sum_{m=0}^{n} h[m]\left(1+e^{j\frac{\pi}{2}(n-m)}\right)}_{I} \quad , \text{ if } n>0$$

$$I = \sum_{m=0}^{\infty} h[m]\left(1+e^{j\frac{\pi}{2}(n-m)}\right) - \sum_{m=n+1}^{\infty} h[m]\left(1+e^{j\frac{\pi}{2}(n-m)}\right)$$

$$= \sum_{m=0}^{\infty} h[m] + \left(\sum_{m=0}^{\infty} h[m]\,e^{-j\frac{\pi}{2}m}\right)e^{j\frac{\pi}{2}n} - \sum_{m=n+1}^{\infty} h[m]\left(1+e^{j\frac{\pi}{2}(n-m)}\right)$$

$$= \hat{h}(e^{j0}) + \left(\hat{h}(e^{j\frac{\pi}{2}}) \times e^{j\frac{\pi}{2}n}\right) - \underbrace{\sum_{m=n+1}^{\infty} h[m]\left(1+e^{j\frac{\pi}{2}(n-m)}\right)}_{(*)}$$

درنتیجه برای $\boxed{n\geq 2}$

برای $n\geq 2$ چون $h[n]$ صفر می‌شود پس $(*)$ هم برابر صفر می‌شود و حذف می‌شود بنابراین:

$$\text{if } n\geq 2 : \; y_2[n] = \cancelto{0}{\hat{h}(e^{j0})} + \hat{h}(e^{j\frac{\pi}{2}}) \times e^{j\frac{\pi}{2}n} = 8\sin^2\left(\frac{\pi}{4}\right)e^{-j\frac{\pi}{2}} \times e^{j\frac{\pi}{2}n} = \boxed{4e^{j\frac{\pi}{2}(n-1)}}$$

از طرفی می‌دانیم که:

$$y_1[n] = 4e^{j\frac{\pi}{2}(n-1)}$$

$$\longrightarrow \boxed{\text{for } n\geq 2 : \quad y_2[n] = y_1[n]}$$

سئوال 3)

a)

$$\hat{h}(e^{j\omega}) = \frac{1}{1-0.8e^{-j\omega}} + \frac{e^{-j2\omega}}{1-0.8e^{-j\omega}}$$

$$\longrightarrow h[n] = (0.8)^n u[n] + (0.8)^{n-2} u[n-2]$$

b)

$$\hat{h}(e^{j\omega}) = \frac{\hat{y}(e^{j\omega})}{\hat{x}(e^{j\omega})} \longrightarrow \frac{\hat{y}(e^{j\omega})}{\hat{x}(e^{j\omega})} = \frac{1+e^{-j2\omega}}{1-0.8e^{-j\omega}}$$

$$\longrightarrow \hat{y}(e^{j\omega}) - 0.8\hat{y}(e^{j\omega})e^{-j\omega} = \hat{x}(e^{j\omega}) + \hat{x}(e^{j\omega})e^{-j2\omega}$$

$$\xrightarrow{\text{تبدیل فوریه معکوس}} y[n] - 0.8y[n-1] = x[n] + x[n-2]$$

$$\longrightarrow \boxed{y[n] = 0.8y[n-1] + x[n] + x[n-2]}$$

c)

$$x[n] = 4 + 2\cos(\omega_0 n) = 4e^{j0n} + e^{j\omega_0 n} + e^{-j\omega_0 n}$$

$$\longrightarrow y[n] = 4e^{j0n} \times \overset{10}{\cancel{\hat{h}(e^{j0})}} + e^{j\omega_0 n} \times \hat{h}(e^{j\omega_0}) + e^{-j\omega_0 n} \times \hat{h}(e^{-j\omega_0})$$

$$= 40 + \left(e^{j\omega_0 n} \times \underbrace{\frac{1+e^{-j2\omega_0}}{1-0.8e^{-j\omega_0}}}_{B}\right) + \left(e^{-j\omega_0 n} \times \underbrace{\frac{1+e^{j2\omega_0}}{1-0.8e^{j\omega_0}}}_{C}\right)$$

$$\boxed{\omega_0 = \frac{\pi}{2}}$$ ⟵ برای اینکه $y[n]$ عدد ثابت شود باید B و C صفر شوند

$$\longrightarrow y[n] = 4 \times 10 = \boxed{40 = A}$$

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سؤال 4)

a)

$$y[n] = x[n] + 2x[n-1] + x[n-2]$$

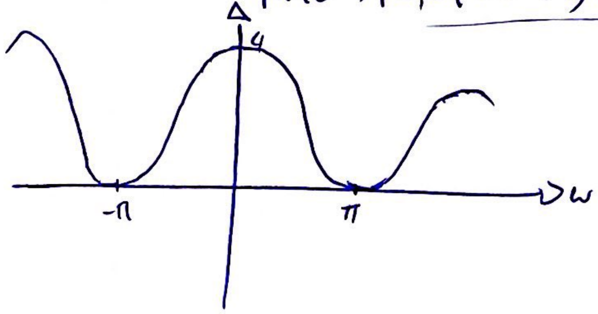
$$\rightarrow h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$$

b) بله سیستم پایدار است چون $h[n] \in L_1$

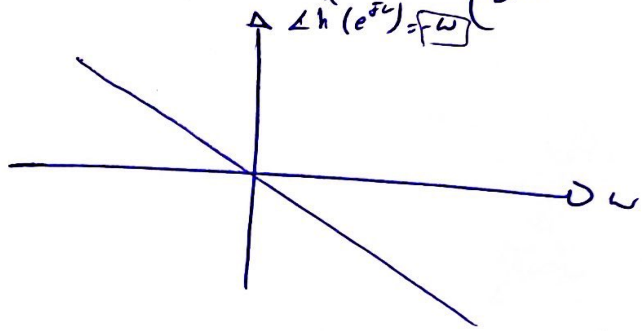
c)

$$\hat{h}(e^{j\omega}) = 1 + 2e^{-j\omega} + e^{-j2\omega} = \boxed{2e^{-j\omega}(1+\cos(\omega))}$$

$$|\hat{h}(e^{j\omega})| = \boxed{2(1+\cos(\omega))}$$




d) $\angle \hat{h}(e^{j\omega}) = \boxed{-\omega}$




e) طبق خواص فوریه میدانیم که:

$$e^{j\omega_0 n}x[n] \longleftrightarrow \hat{x}(e^{j(\omega-\omega_0)})$$

$$\rightarrow h_1[n] = e^{-j\pi n}h[n] = (\overbrace{\cos(\pi n)}^{(-1)^n} - \overbrace{\cancel{j\sin(\pi n)}}^{0})(\delta[n] + 2\delta[n-1] + \delta[n-2])$$

$$= \boxed{\delta[n] - 2\delta[n-1] + \delta[n-2]}$$

سؤال 6)

طبق نمودار فاز و اندازه‌ای که داده شده است می‌شود فهمید که :

$$\hat{x}(e^{j\omega}) = \sin\left(\frac{\omega}{2}\right) \times -j e^{-j\frac{\omega}{2}} = -\cancel{j} \times e^{-j\frac{\omega}{2}} \times \left(\frac{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}}{2\cancel{j}}\right)$$

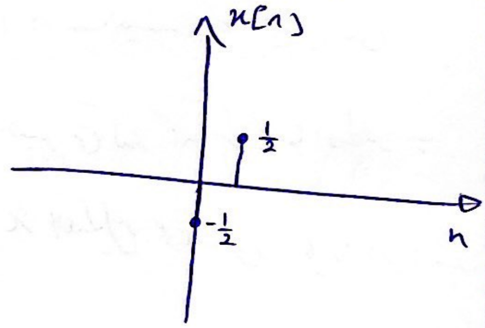
$$= \boxed{\frac{1}{2}\left(e^{-j\omega} - 1\right)}$$

$$\longrightarrow x[n] = \frac{1}{4\pi}\int_{2\pi}\left(e^{-j\omega} - 1\right) e^{j\omega n}\, d\omega = \frac{1}{4\pi}\int_{2\pi}\left(e^{j\omega(n-1)} - e^{j\omega n}\right) d\omega$$

$\longrightarrow$ if $n=0$ : $x[n] = -\frac{1}{2}$

if $n=1$ : $x[n] = \frac{1}{2}$ $\longrightarrow$

o.w : $x[n] = 0$



سئوال 5)

a)

$$\hat{h}(e^{j\omega}) = \frac{\hat{y}(e^{j\omega})}{\hat{x}(e^{j\omega})} = \frac{1-1.25e^{-j\omega}}{1-0.8e^{-j\omega}}$$

$$\longrightarrow \hat{y}(e^{j\omega}) - 0.8\,\hat{y}(e^{j\omega})e^{-j\omega} = \hat{x}(e^{j\omega}) - 1.25\,\hat{x}(e^{j\omega})e^{-j\omega}$$

$$\xrightarrow{\text{تبدیل فوریه معکوس}} y[n] - 0.8\,y[n-1] = x[n] - 1.25\,x[n-1]$$

$$\longrightarrow \boxed{y[n] = 0.8\,y[n-1] + x[n] - 1.25\,x[n-1]}$$

ادامه سؤال 5)

(b

$$\hat{h}(e^{j\omega}) = \frac{1-1.25e^{-j\omega}}{1-0.8e^{-j\omega}}$$

$$\longrightarrow |\hat{h}(e^{j\omega})| = \frac{|1-1.25e^{-j\omega}|}{|1-0.8e^{-j\omega}|}$$

$$|1-1.25e^{-j\omega}| = |1-1.25\cos(\omega) + j1.25\sin(\omega)|$$

$$= \sqrt{(1-1.25\cos(\omega))^2 + (1.25\sin(\omega))^2} = \sqrt{1+(1.25)^2 - 2.5\cos(\omega)}$$

$$|1-0.8e^{-j\omega}| = \sqrt{(1-0.8\cos(\omega))^2 + (0.8\sin(\omega))^2} = \sqrt{1+(0.8)^2 - 1.6\cos(\omega)}$$

$$\longrightarrow \frac{|1-1.25e^{-j\omega}|}{|1-0.8e^{-j\omega}|} = \frac{\sqrt{\frac{41}{16} - 2.5\cos(\omega)}}{\sqrt{\frac{41}{25} - 1.6\cos(\omega)}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

$$\longrightarrow \boxed{|\hat{h}(e^{j\omega})| = \frac{5}{4}}$$

(c

$$x[n] = \cos(0.2\pi n) \longrightarrow \hat{x}(e^{j\omega}) = \pi\sum_{m\in\mathbb{Z}} \delta(\omega - 0.2\pi - 2\pi m) + \pi\sum_{m\in\mathbb{Z}} \delta(\omega + 0.2\pi - 2\pi m)$$

$$= \pi\sum_{m\in\mathbb{Z}} \left(\delta(\omega - 0.2\pi - 2\pi m) + \delta(\omega + 0.2\pi - 2\pi m)\right)$$

$$\hat{h}(e^{j\omega}) = \frac{5}{4}\left(\frac{\frac{4}{5} - e^{-j\omega}}{1 - \frac{4}{5}e^{-j\omega}}\right)$$

ادامه سوال 5 قسمت C)

$$\rightarrow \hat{y}(e^{j\omega}) = \hat{h}(e^{j\omega})\,\hat{x}(e^{j\omega})$$

$$= \frac{5}{4}\times\pi\sum_{m\in Z}\left(\left(\frac{\frac{4}{5}-e^{-\frac{j\pi}{5}}}{1-\frac{4}{5}e^{-\frac{j\pi}{5}}}\right)\left(\delta(\omega-\frac{\pi}{5}-2\pi m\right) + \left(\frac{\frac{4}{5}-e^{+\frac{j\pi}{5}}}{1-\frac{4}{5}e^{\frac{j\pi}{5}}}\right)\left(\delta(\omega+\frac{\pi}{5}-2\pi m)\right)\right)$$

حال مخرج کسرها در مزدوج آن ها ضرب می کنیم یعنی در: $1-\frac{4}{5}e^{-\frac{j\pi}{5}}$ , $1-\frac{4}{5}e^{+\frac{j\pi}{5}}$

$$= \frac{5\pi}{4}\sum_{m\in Z}\left(\left(\frac{\frac{8}{5}-e^{-\frac{j\pi}{5}}-\frac{16}{25}e^{\frac{j\pi}{5}}}{\frac{41}{25}-\frac{4}{5}(\cos(\frac{\pi}{5}))}\right)\left(\delta(\omega-\frac{\pi}{5}-2\pi m)\right) + \left(\frac{\frac{8}{5}-\frac{16}{25}e^{-\frac{j\pi}{5}}-e^{\frac{j\pi}{5}}}{\frac{41}{25}-\frac{4}{5}(\cos\frac{\pi}{5})}\right)\left(\delta(\omega+\frac{\pi}{5}-2\pi m)\right)\right)$$

$\simeq 1$

$$= \frac{5\pi}{4}\sum_{m\in Z}\left(\frac{8}{5}-e^{-\frac{j\pi}{5}}-\frac{16}{25}e^{\frac{j\pi}{5}}\right)\left(\delta(\omega-\frac{\pi}{5}-2\pi m)\right) + \left(\frac{8}{5}-\frac{16}{25}e^{-\frac{j\pi}{5}}-e^{\frac{j\pi}{5}}\right)\left(\delta(\omega+\frac{\pi}{5}-2\pi m)\right)$$

حال می خواهیم تبدیل فوریه $A\cos(0.2\pi n+\theta)$ را بدست آوریم

$$A\cos(0.2\pi n+\theta) = A\,\frac{e^{j\theta}\times e^{\frac{j\pi n}{5}} + e^{-j\theta}\times e^{-\frac{j\pi n}{5}}}{2}$$

$$\rightarrow F\{A\cos(0.2\pi n+\theta)\} = A\times\left(e^{j\theta}\pi\sum_{m\in Z}\delta(\omega-\frac{\pi}{5}-2\pi m) + e^{-j\theta}\pi\sum_{m\in Z}\delta(\omega+\frac{\pi}{5}-2\pi m)\right)$$

$$\rightarrow \boxed{A=\frac{5}{4}}$$

برای پیدا کردن $\theta$ هم کافیست فاز عبارت $\frac{8}{5}-e^{-\frac{j\pi}{5}}-\frac{16}{25}e^{\frac{j\pi}{5}}$ را محاسبه کنیم.