تمرین سری 6 درس ~~[illegible]~~ سیگنال - رادین خیام - 99101579

سوال 1)

a)

$$Z(t) = x(t)\,y(t)$$

$$\rightarrow F\{Z(t)\} = \int_{-\infty}^{+\infty} x(t)\,y(t)\,e^{-j\omega t}\,dt$$

$$\xrightarrow[\text{برعکس}]{\text{طبق تبدیل فوریه}} x(t) = \frac{1}{2\pi}\int_{-\infty} X(t)\,e^{-j\omega t}\,d\omega$$

$$\rightarrow F\{Z(t)\} = \int_{-\infty}^{+\infty} \left(\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(a)\,e^{+jat}\,da\right) y(t)\,e^{-j\omega t}\,dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(a)\int_{-\infty}^{+\infty} y(t)\,e^{-j(\omega-a)t}\,dt\,da = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(a)\,Y(\omega-a)\,da$$

$$= \boxed{\frac{1}{2\pi} X(\omega) * Y(\omega)} = \frac{1}{2\pi}(\hat{x} * \hat{y})(\omega)$$

b)

(i) $x_1(t) = t e^{-\alpha|t|}\cos(\beta t)$

$$F\{e^{-\alpha|t|}\} = \frac{2\alpha}{\alpha^2+\omega^2} \rightarrow F\{t e^{-\alpha|t|}\} = j\frac{d}{d\omega}\frac{2\alpha}{\alpha^2+\omega^2}$$

$$= j \times \frac{0 - 4\omega\alpha}{(\alpha^2+\omega^2)^2} = \frac{-4j\omega\alpha}{(\alpha^2+\omega^2)^2}$$

$$F\{\cos(\beta t)\} = \pi\left(\delta(\omega-\beta) + \delta(\omega+\beta)\right)$$

$$\rightarrow \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{-4ja\alpha}{(\alpha^2+a^2)^2} \times \pi\left(\delta(\omega-a-\beta) + \delta(\omega-a+\beta)\right) da$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} \frac{-j \cdot 4a\alpha}{(\alpha^2+a^2)^2} \times \left(\delta(\omega-a-\beta)\right) da + \frac{1}{2}\int_{-\infty}^{+\infty} \frac{-j4a\alpha}{(\alpha^2+a^2)^2} \times \delta(\omega-a+\beta)\,da$$

ادامه سؤال 1 بخش b)

$$= \frac{1}{2}\left(\frac{-j\,4\times(\omega-\beta)\times\alpha}{(\alpha^2+(\omega-\beta)^2)^2} + \frac{-j\,4\times(\omega+\beta)\times\alpha}{(\alpha^2+(\omega+\beta)^2)^2} \right)$$

$$= \boxed{-j2\alpha\left(\frac{\omega-\beta}{(\alpha^2+(\omega-\beta)^2)^2} + \frac{\omega+\beta}{(\alpha^2+(\omega+\beta)^2)^2} \right)} \rightarrow X_1(\omega)$$

(ii) $$x_2(t) = \underbrace{\frac{\sin(\pi t)}{\pi t}}_{\mathrm{sinc}(t)} \times \underbrace{\frac{\sin(2\pi(t-1))}{\pi(t-1)}}_{2\,\mathrm{sinc}(2(t-1))}$$

$$F\{\mathrm{sinc}(t)\} = \Pi\left(\frac{\omega}{2\pi}\right)$$

$$F\{\mathrm{sinc}(t-1)\} = \Pi\left(\frac{\omega}{2\pi}\right) \times e^{-j\omega}$$

$$\rightarrow F\{\mathrm{sinc}(2(t-1))\} = \frac{1}{2}\Pi\left(\frac{\omega}{4\pi}\right)e^{-j\frac{\omega}{2}}$$

$$\rightarrow F\{2\,\mathrm{sinc}(2(t-1))\} = \Pi\left(\frac{\omega}{4\pi}\right)e^{-j\frac{\omega}{2}}$$

$$\int_{-\infty}^{\infty} \Pi\left(\frac{\omega-a}{2\pi}\right)\Pi\left(\frac{a}{4\pi}\right)e^{-j\frac{a}{2}}\,da = \int_{-2\pi}^{+2\pi} \Pi\left(\frac{\omega-a}{2\pi}\right)e^{-j\frac{a}{2}}\,da$$

سؤال 1)

c)

$$\int_{-\infty}^{+\infty} x(t)\, y^*(t)\, dt = I$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{x}(\omega)\, e^{j\omega t}\, d\omega \quad , \quad y(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{y}(\omega)\, e^{j\omega t}\, d\omega$$

$$\rightarrow I = \int_{-\infty}^{+\infty} \left(\frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{x}(\omega)\, e^{j\omega t}\, d\omega\right)\left(\frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{y}(\omega')^*\, e^{-j\omega' t}\, d\omega'\right) dt$$

$$\rightarrow I = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \hat{x}(\omega)\, \hat{y}(\omega')^* \left(\int_{-\infty}^{+\infty} e^{j(\omega-\omega')\tau}\, d\tau\right) d\omega'\, d\omega$$

میدانیم که:

$$\delta(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{\pm j\tau\omega}\, d\tau$$ (نمایش انتگرالی تابع ضربه)

$$\rightarrow I = \left(\frac{1}{2\pi}\right)^2 \times 2\pi \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \hat{x}(\omega)\, \hat{y}(\omega')^*\, \delta(\omega-\omega')\, d\omega'\, d\omega$$

$$\rightarrow I = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{x}(\omega)\left(\int_{-\infty}^{+\infty} \hat{y}(\omega')^*\, \delta(\omega-\omega')\, d\omega'\right) d\omega$$

$$\rightarrow \boxed{I = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{x}(\omega)\, \hat{y}(\omega)^*\, d\omega}$$

d)

(i) $$I_1 = \int_0^{+\infty} \frac{1}{(a^2+x^2)^2}\, dx = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{1}{(a^2+x^2)^2}\, dx$$

سؤال 8)

4.25)

a) $y(t) = x(t+1)$ یک سیگنال زوج است پس می شود گفت که $Y(j\omega)$ حقیقی،

هم یک سیگنال حقیقی و زوج است $\longrightarrow$ $\measuredangle Y(j\omega) = 0$ همچنین طبق خواص فوریه میدانیم که:

$$Y(j\omega) = e^{j\omega} X(j\omega) \longrightarrow \boxed{\measuredangle X(j\omega) = -\omega}$$

b)

$$X(j0) = \int_{-\infty}^{+\infty} x(t)\, dt = 2 + \frac{3}{2} + \frac{3}{2} + 2 = \boxed{7}$$

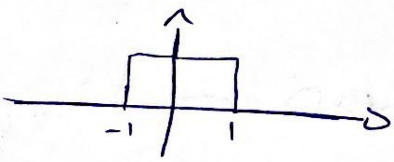
c)

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega) e^{-j\omega t}\, dt = x(t) \xrightarrow{t=0} \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, dt = x(0)$$

$$\longrightarrow \int_{-\infty}^{+\infty} X(j\omega)\, dt = 2\pi x(0) = \boxed{4\pi}$$

d)

میدانیم که:

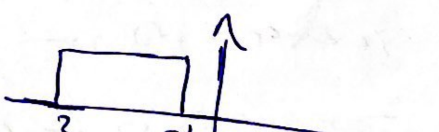


$$\xrightarrow{F} \frac{2\sin(\omega)}{\omega}$$

حال در اینجا:

$$\int_{-\infty}^{+\infty} X(j\omega) \underbrace{\frac{2\sin(\omega)}{\omega} e^{j2\omega}}_{Y(j\omega)} d\omega$$

اگر فرض شود $Y(j\omega)$ $\longrightarrow y(t) =$



* ضرب شدن $e^{j2\omega}$ در حوزه فرکانس معادل شیفت 2 واحدی به سمت چپ در حوزه زمان است

$$\longrightarrow \int_{-\infty}^{+\infty} X(j\omega) Y(j\omega)\, d\omega = 2\pi \left(x(t) * y(t)\right)\Big|_{t=0} = 2\pi \times \left(\frac{3}{2} + 2\right) = \boxed{7\pi}$$

ادامه سؤال 8)

4.25)

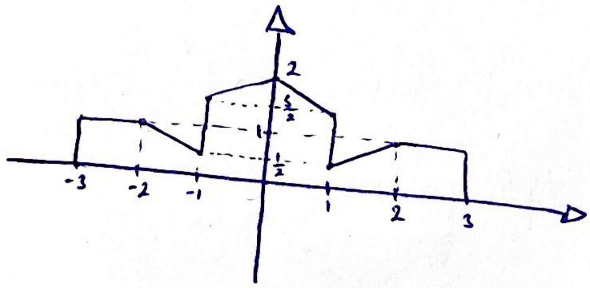
e)

$$\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega = 2\pi \int_{-\infty}^{+\infty} |x(t)|^2 dt = \boxed{26\pi}$$

f) چون سیگنال حقیقی است تبدیل فوریه برعکس جزء حقیقی $X(j\omega)$ برابر می شود با

$Ev\{x(t)\}$ که بدین صورت است :

$$\frac{x(t) + x(-t)}{2}$$



4.38)

a)

$$X(j(\omega-\omega_0)) = \int_{-\infty}^{+\infty} x(t) e^{-j(\omega-\omega_0)t} dt = \int_{-\infty}^{+\infty} x(t) e^{j\omega_0 t} \times e^{-j\omega t} dt$$

$$= \boxed{\mathcal{F}\{x(t) e^{j\omega_0 t}\}}$$

b)

$$y(t) = e^{j\omega_0 t} \longleftrightarrow Y(j\omega) = 2\pi \delta(\omega-\omega_0)$$

$$x(t) y(t) \longleftrightarrow \frac{1}{2\pi}[X(j\omega) * Y(j\omega)] = X(j\omega) * \delta(\omega-\omega_0)$$

$$= \boxed{X(j(\omega-\omega_0))}$$ ✓

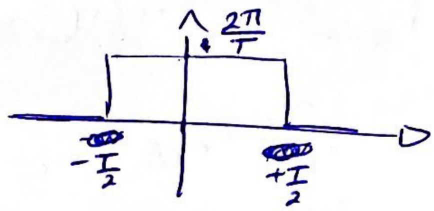
(سئوال 7

$$h(t) = \frac{\sin(3\pi(t-2))}{\pi(t-2)}$$

$$\text{if } h(t) = h_1(t-2) \longrightarrow h_1(t) = \frac{\sin 3\pi t}{\pi t} = 3\,\text{sinc}(3t)$$

از طرفی میدانیم که :

$$\frac{\sin(\frac{tT}{2})}{\frac{tT}{2}} \xleftrightarrow{F}$$



$$\longrightarrow H_1(\omega) = \begin{cases} 1 & |\omega| < 3\pi \\ 0 & \text{o.w} \end{cases}$$

خواص تبدیل فوریه

$$\longrightarrow H(\omega) = \begin{cases} e^{-j2\omega} & |\omega| < 3\pi \\ 0 & \text{o.w} \end{cases}$$

$$x_1(t) = \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k \sin(2kt) = \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k \times \frac{e^{j2kt} - e^{-j2kt}}{2j}$$ (a

$$\longrightarrow X_1(j\omega) = \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k \times \frac{\cancel{2}\pi\delta(\omega-2k) - \cancel{2}\pi\delta(\omega+2k)}{\cancel{2}j}$$

$$= \left(\frac{\pi}{j}\right)\left(\sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k \times \left(\delta(\omega-2k) - \delta(\omega+2k)\right)\right)$$

$$\longrightarrow Y_1(j\omega) = X_1(j\omega) H(j\omega) = \frac{\pi}{j}\left(\frac{1}{3} \times \left(\delta(\omega-2) - \delta(\omega+2)\right)\right) \times e^{-j2\omega}$$

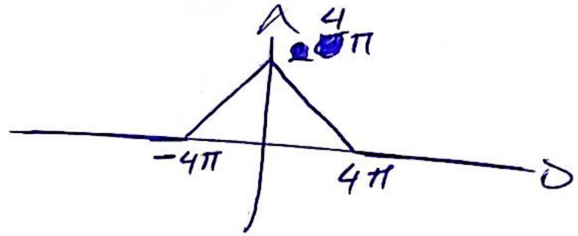
$$\longrightarrow \boxed{y(t) = \frac{1}{3}\sin(2t-2)}$$

ادامه سؤال 7)

b)

$$x_2(t) = \left(\frac{\sin(2\pi t)}{\pi t}\right)^2 = 4\,\mathrm{sinc}^2(2t)$$

$$\longrightarrow X_2(j\omega) =$$



$$Y_2(j\omega) = X_2(j\omega)H(j\omega) = X_2(j\omega)e^{-j2\omega}$$

$$\rightarrow y_2(t) = x_4(t-2) = \left(\frac{\sin\left(2\pi(t-1)\right)}{\pi(t-1)}\right)^2$$

سوال 5)

اگر فرض کنیم که $y(t)=x(t+1)$ آنگاه $y(t)$ یک سیگنال حقیقی و فرد است

در نتیجه می توانیم بگوییم که تبدیل فوریه آن یعنی $Y(j\omega)$ موهومی خالص است پس:

$$\measuredangle Y(j\omega) = \frac{\pi}{2}$$

از طرفی:

$$Y(j\omega) = X(j\omega)e^{j\omega} \longrightarrow \boxed{\measuredangle X(j\omega) = \frac{\pi}{2} - \omega}$$

$$\hat{x}(0) = \int_{-\infty}^{+\infty} x(t)\,dt = \boxed{0}$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{x}(\omega)\, e^{j\omega t}\, d\omega = x(t) \xrightarrow{t=0} \int_{-\infty}^{+\infty} \hat{x}(\omega)\, d\omega = 2\pi x(0) = \boxed{2\pi}$$

$$\int_{-\infty}^{+\infty} \hat{x}(\omega)\, \underbrace{\frac{2\sin\omega}{\omega} e^{j2\omega}}_{\hat{y}(\omega)}\, d\omega = 2\pi\left(x(t) * y(t)\right)\Big|_{t=0} = 2\pi \times -1 = \boxed{-2\pi}$$

$$\hookrightarrow y(t) = \begin{cases} 1 & -3<t<-1 \\ 0 & o.w \end{cases}$$

(5 ادامه سؤال

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}|\hat{x}(\omega)|^2 d\omega = \int_{-\infty}^{+\infty}|x(t)|^2 dt$$

$$\rightarrow \int_{-\infty}^{+\infty}|\hat{x}(\omega)|^2 d\omega = 2\pi\int_{-\infty}^{+\infty}|x(t)|^2 dt = 2\pi\times\left(\int_{-1}^{0}(t+1)^2dt+\int_{0}^{1}(-t+1)^2dt+\int_{1}^{2}(t-1)^2dt+\int_{2}^{3}(-t+3)^2dt\right)$$

$$= 2\pi\times\left(\left(\frac{t^3}{3}+t^2+t\right)\Big|_{-1}^{0}+\left(+\frac{t^3}{3}-t^2+t\right)\Big|_{0}^{1}+\left(\frac{t^3}{3}-t^2+t\right)\Big|_{1}^{2}+\left(\frac{t^3}{3}-3t^2+9t\right)\Big|_{2}^{3}\right)$$

$$= 2\pi\left(\cancel{+\frac{1}{3}}\cancel{-1}\cancel{+1}+\frac{1}{3}\cancel{-1}\cancel{+1}\cancel{+\frac{8}{3}}-4+2\cancel{-\frac{1}{3}}\cancel{+1}\cancel{-1}+9\cancel{-27}\cancel{+27}\cancel{-\frac{8}{3}}+12-18\right)$$

$$= 2\pi\times\left(\frac{4}{3}\right) = \boxed{\frac{8\pi}{3}}$$

$$\int_{-\infty}^{+\infty}\omega\,\hat{x}(\omega)e^{j\omega}$$

$$Y(j\omega) = \omega\hat{x}(\omega) \leftarrow y = \frac{d}{dt}x(t)\times\frac{1}{j}$$ اگر

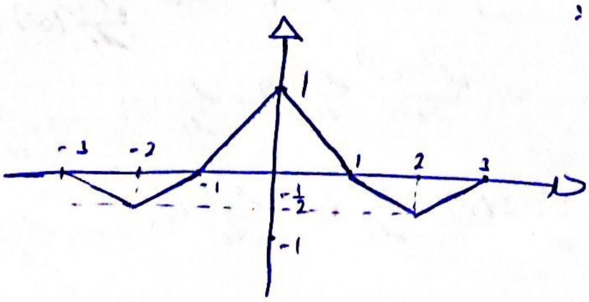
$$\rightarrow \frac{1}{2\pi}\int_{-\infty}^{+\infty}\omega\,\hat{x}(\omega)e^{j\omega} = \frac{1}{j}\times\left(\frac{d}{dt}x(t)\right)\Big|_{t=1}$$

$$\rightarrow \int_{-\infty}^{+\infty}\omega\,\hat{x}(\omega)e^{j\omega} = \frac{2\pi}{j}\times -1 = -\frac{2\pi}{j} = \boxed{2\pi j}$$

چون سیگنال حقیقی است قسمت حقیقی تبدیل فوریه متناظر است با قسمت زوج تابع $x(t)$

در نتیجه کافیست که:

$$\frac{x(t)+x(-t)}{2}$$



سؤال 6)

a)

$$y(t) = x(t)\cos(\underbrace{2\pi f_0}_{w_0} t) = x(t) \times \frac{e^{jw_0t} + e^{-jw_0t}}{2}$$

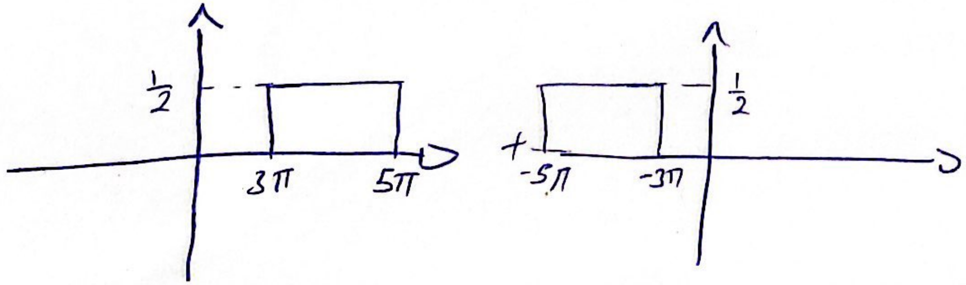
$$\rightarrow \boxed{\hat{y}(w) = \frac{1}{2}\left(\hat{x}(w-w_0) + \hat{x}(w+w_0)\right)}$$

b)

$$x(t) = \frac{\sin(\pi t)}{\pi t} = \operatorname{sinc}(t) \rightarrow \hat{x}(w) = \begin{cases} 1 & |w| < \pi \\ 0 & \text{o.w} \end{cases}$$

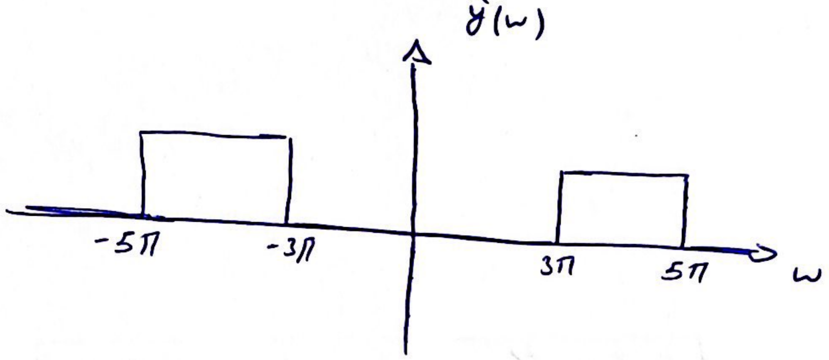
$$f_0 = 2 \rightarrow \boxed{w_0 = 4\pi}$$

$$\rightarrow \hat{y}(w) = \frac{1}{2}\left(\hat{x}(w-4\pi) + \hat{x}(w+4\pi)\right)$$



ادامه سؤال 6)

b)



c)

$$Z(t) = \underbrace{x(t)\cos(2\pi f_0 t)}_{y(t)} - x_H(t)\sin(2\pi f_0 t)$$

$$F\{Z(t)\} = \hat{y}(\omega) - \underbrace{F\{x_H(t)\sin(2\pi f_0 t)\}}_{I}$$

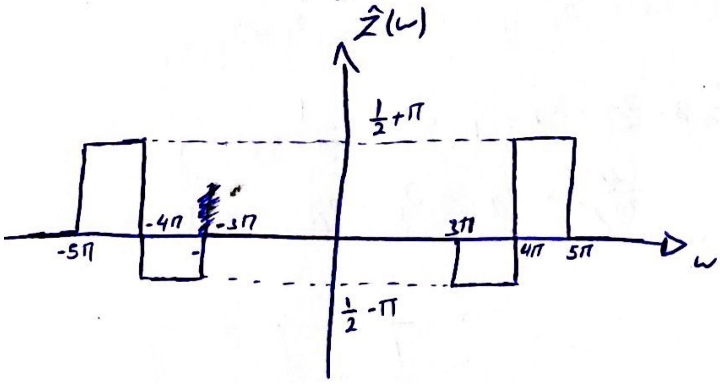
$$I = F\{x_H(t)\} * F\{\sin(2\pi f_0 t)\}$$

$$\rightarrow I = \left(\hat{x}(\omega) \times -j\,\text{sign}(\omega)\right) * \left(-j\pi\left(\delta(\omega - 2\pi f_0) - \delta(\omega + 2\pi f_0)\right)\right)$$

$f_0 = 2 \quad \rightarrow \hat{Z}(\omega) = \hat{y}(\omega) + \pi\left(\hat{x}(\omega - 4\pi)\,\text{sign}(\omega - 4\pi)\right) - \pi\left(\hat{x}(\omega + 4\pi)\,\text{sign}(\omega + 4\pi)\right)$

$$\rightarrow \hat{Z}(\omega) = \frac{1}{2}\hat{x}(\omega - 4\pi) + \frac{1}{2}\hat{x}(\omega + 4\pi) + \pi\,\text{sign}(\omega - 4\pi)\,\hat{x}(\omega - 4\pi) - \pi\,\text{sign}(\omega + 4\pi)\,\hat{x}(\omega + 4\pi)$$

$$= \hat{x}(\omega - 4\pi)\left(\frac{1}{2} + \pi\,\text{sign}(\omega - 4\pi)\right) + \hat{x}(\omega + 4\pi)\left(\frac{1}{2} - \pi\,\text{sign}(\omega + 4\pi)\right)$$



بله با نمودار قسمت قبل تفاوت دارد.

سئوال 4)

1a

$$\hat{x}(\omega) = \int_{-2}^{0} -t e^{-j\omega t} dt + \int_{0}^{2} t e^{-j\omega t} dt$$

$$= e^{-j\omega t}\left(\frac{-t}{j\omega} + \frac{1}{\omega^2}\right)\Big|_0^2 - e^{-j\omega t}\left(\frac{-t}{j\omega} + \frac{1}{\omega^2}\right)\Big|_{-2}^{0}$$

$$= \frac{1}{\omega^2}\left(e^{2j\omega} + e^{-2j\omega}\right) + \frac{2}{j\omega}\left(e^{2j\omega} - e^{-2j\omega}\right) - \frac{2}{\omega^2}$$

$$= \frac{2\cos(2\omega)}{\omega^2} + \frac{4\sin(2\omega)}{\omega} - \frac{2}{\omega^2} = \frac{4}{\omega^2}\left(\omega\sin(2\omega) + \frac{\cos(2\omega)}{2} - \frac{1}{2}\right)$$

$$= \frac{4}{\omega^2}\left(\omega\sin(2\omega) - \sin^2(\omega)\right)$$

(b

$$\hat{x}(\omega) = \int_{-2}^{-1} 2e^{-j\omega t} dt + \int_{-1}^{0} (-t+1) e^{-j\omega t} dt + \int_{0}^{1} (t+1) e^{-j\omega t} dt + \int_{1}^{2} 2e^{-j\omega t} dt$$

$$= \frac{-2}{j\omega} e^{-j\omega t}\Big|_{-2}^{-1} - e^{-j\omega t}\left(\frac{-t}{j\omega} + \frac{1}{\omega^2}\right)\Big|_{-1}^{0} - \frac{1}{j\omega} e^{-j\omega t}\Big|_{-1}^{0} + e^{-j\omega t}\left(\frac{-t}{j\omega} + \frac{1}{\omega^2}\right)\Big|_{0}^{1}$$

$$- \frac{1}{j\omega} e^{-j\omega t}\Big|_{0}^{1} - \frac{2}{j\omega} e^{-j\omega t}\Big|_{1}^{2} = \frac{-2}{j\omega}\left(e^{j\omega} - e^{2j\omega}\right) - \frac{1}{\omega^2} + e^{j\omega}\left(\frac{1}{j\omega} + \frac{1}{\omega^2}\right)$$

$$- \frac{1}{j\omega} + \frac{1}{j\omega} e^{j\omega} + e^{-j\omega}\left(\frac{-1}{j\omega} + \frac{1}{\omega^2}\right) - \frac{1}{\omega^2} - \frac{1}{j\omega} e^{-j\omega} + \frac{1}{j\omega} - \frac{2}{j\omega} e^{-2j\omega} + \frac{2}{j\omega} e^{-2j\omega}$$

ساده‌شده $$= \frac{2}{j\omega}\left(e^{2j\omega} - e^{-2j\omega}\right) + \frac{1}{\omega^2}\left(e^{j\omega} + e^{-j\omega}\right) - \frac{2}{\omega^2} = \boxed{\frac{4\sin(2\omega)}{\omega} + \frac{2\cos(\omega)}{\omega^2} - \frac{2}{\omega^2}}$$

ادامه سؤال 4)

$$\hat{x}(\omega) = \int_{-1}^{0}(t+1)e^{-j\omega t}\,dt + \int_{0}^{1}(t-1)e^{-j\omega t}\,dt \qquad (c$$

$$= e^{-j\omega t}\left(\frac{-t}{j\omega}+\frac{1}{\omega^2}\right)\Big|_{-1}^{0} - \frac{1}{j\omega}e^{-j\omega t}\Big|_{-1}^{0} + e^{-j\omega t}\left(\frac{-t}{j\omega}+\frac{1}{\omega^2}\right)\Big|_{0}^{1} + \frac{1}{j\omega}e^{-j\omega t}\Big|_{0}^{1}$$

$$= e^{j\omega}\left(-\frac{1}{j\omega}-\frac{1}{\omega^2}+\frac{1}{j\omega}\right) + e^{-j\omega}\left(\frac{-1}{j\omega}+\frac{1}{\omega^2}+\frac{1}{j\omega}\right) - \frac{2}{j\omega}$$

$$= \frac{1}{\omega^2}\left(e^{+j\omega}+e^{-j\omega}\right) - \frac{2}{j\omega} = \boxed{\frac{2j\sin(\omega)+2j\omega}{\omega^2}}$$

$$\hat{x}(\omega) = \int_{-2}^{-1}(t+2)e^{-j\omega t}\,dt + \int_{-1}^{1}(-t)e^{-j\omega t}\,dt + \int_{1}^{2}(t-2)e^{-j\omega t}\,dt \qquad (d$$

$$= e^{-j\omega t}\left(\frac{-t}{j\omega}+\frac{1}{\omega^2}\right)\Big|_{-2}^{-1} - \frac{2}{j\omega}e^{-j\omega t}\Big|_{-2}^{-1} - e^{-j\omega t}\left(\frac{-t}{j\omega}+\frac{1}{\omega^2}\right)\Big|_{-1}^{1}$$

$$+ e^{-j\omega t}\left(\frac{-t}{j\omega}+\frac{1}{\omega^2}\right)\Big|_{1}^{2} + \frac{2}{j\omega}e^{-j\omega t}\Big|_{1}^{2}$$

$$= e^{j\omega}\left(\cancel{\frac{1}{j\omega}}+\frac{1}{\omega^2}-\cancel{\frac{2}{j\omega}}+\cancel{\frac{1}{j\omega}}+\frac{1}{\omega^2}\right) + e^{2j\omega}\left(\cancel{\frac{-2}{j\omega}}-\frac{1}{\omega^2}+\cancel{\frac{2}{j\omega}}\right)$$

$$+ e^{-j\omega}\left(\cancel{\frac{1}{j\omega}}-\frac{1}{\omega^2}+\cancel{\frac{1}{j\omega}}-\frac{1}{\omega^2}-\cancel{\frac{2}{j\omega}}\right) + e^{-2j\omega}\left(\cancel{\frac{-2}{j\omega}}+\frac{1}{\omega^2}+\cancel{\frac{2}{j\omega}}\right)$$

$$= \frac{1}{\omega^2}\left(2e^{j\omega}-2e^{-j\omega}\right) + \frac{1}{\omega^2}\left(e^{-2j\omega}-e^{2j\omega}\right) = \frac{4j\sin(\omega)-2j\sin(2\omega)}{\omega^2}$$

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ادامه سؤال 14

$$\hat{x}(\omega)=\int_{-2}^{-1}(t+2)e^{-j\omega t}dt+\int_{-1}^{1}e^{-j\omega t}dt+\int_{1}^{2}(-t+2)e^{-j\omega t}dt$$ (e

$$=e^{-j\omega t}\left(\frac{-t}{j\omega}+\frac{1}{\omega^2}\right)\Big|_{-2}^{-1}-\frac{2}{j\omega}e^{-j\omega t}\Big|_{-2}^{-1}-\frac{1}{j\omega}e^{-j\omega t}\Big|_{-1}^{1}$$

$$-e^{-j\omega t}\left(\frac{-t}{j\omega}+\frac{1}{\omega^2}\right)\Big|_{1}^{2}+\frac{-2}{j\omega}e^{-j\omega t}\Big|_{1}^{2}$$

$$=e^{j\omega}\left(\cancel{\frac{1}{j\omega}}+\frac{1}{\omega^2}-\cancel{\frac{2}{j\omega}}+\cancel{\frac{1}{j\omega}}\right)+e^{2j\omega}\left(\cancel{\frac{-2}{j\omega}}-\frac{1}{\omega^2}+\cancel{\frac{2}{j\omega}}\right)+e^{-j\omega}\left(\cancel{\frac{-1}{j\omega}}-\cancel{\frac{1}{j\omega}}+\frac{1}{\omega^2}+\cancel{\frac{2}{j\omega}}\right)$$

$$+e^{-2j\omega}\left(\cancel{\frac{2}{j\omega}}-\frac{1}{\omega^2}-\cancel{\frac{2}{j\omega}}\right)=\frac{1}{\omega^2}\left(e^{j\omega}+e^{-j\omega}\right)-\frac{1}{\omega^2}\left(e^{2j\omega}+e^{-2j\omega}\right)$$

$$=\frac{2}{\omega^2}\left(\cos(\omega)-\cos(2\omega)\right)$$

$$\hat{x}(\omega)=-\int_{-\frac{1}{2f_0}}^{0}\sin(2\pi f_0 t)e^{-j\omega t}dt+\int_{0}^{\frac{1}{2f_0}}\sin(2\pi f_0 t)e^{-j\omega t}dt$$ (f

$$=\underbrace{\int_{-\frac{1}{2f_0}}^{0}\frac{e^{2\pi f_0 t j}-e^{-2\pi f_0 t j}}{2j}e^{-j\omega t}dt}_{I}+\int_{0}^{\frac{1}{2f_0}}I\,dt$$

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