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تمرین سری ۴ درس سیگنال و سیستم - رادین خیام - 99102579

سوال 1)

(1

$$x_1(t) = \sum_{k\in Z} a_k e^{j\frac{2\pi}{10}kt}$$

$$a_k = \frac{1}{T}\int_0^T x(s)e^{-j\frac{2\pi}{T}ks}ds = \frac{1}{10}\int_0^{10} x_1(s)e^{-j\frac{\pi}{5}ks}ds$$

$$= \frac{1}{10}\int_0^1 1\times e^{-j\frac{k\pi}{5}s}ds = \frac{1}{10}\times\frac{e^{-j\frac{k\pi}{5}s}}{-j\frac{k\pi}{5}}\Big|_0^1 = \frac{1}{10}\times\frac{e^{-j\frac{k\pi}{5}}-1}{-j\frac{k\pi}{5}} = \frac{e^{-j\frac{k\pi}{5}}-1}{-j2k\pi}$$

$$= \frac{\cos(\frac{k\pi}{5}) - j\sin(\frac{k\pi}{5}) - 1}{-j2k\pi} = \frac{\sin(\frac{k\pi}{5}) + j\cos(\frac{k\pi}{5}) - j}{2k\pi}$$

$$\rightarrow a_k = \frac{\sin(\frac{k\pi}{5}) + j(\cos(\frac{k\pi}{5}) - 1)}{2k\pi}$$

$$\rightarrow x_1(t) = \sum_{k\in Z}\frac{\sin(\frac{k\pi}{5}) + j(\cos(\frac{k\pi}{5}) - 1)}{2k\pi}\times\left(\cos(\frac{k\pi}{5}t) + j\sin(\frac{k\pi}{5}t)\right)$$

$$= \frac{1}{2\pi}\sum_{k\in Z}\frac{1}{k}\Big(\sin(\frac{k\pi}{5})\cos(\frac{k\pi}{5}t) - \cos(\frac{k\pi}{5})\sin(\frac{k\pi}{5}t) + \sin(\frac{k\pi}{5}t) + j\big(\cos(\frac{k\pi}{5})\cos(\frac{k\pi}{5}t) - \cos(\frac{k\pi}{5}t) + \sin(\frac{k\pi}{5})\sin(\frac{k\pi}{5}t)\big)\Big)$$

(2

$$x_2(t) = \sum_{k\in Z} a_k e^{j\frac{2\pi}{10}kt}$$

$$a_k = \frac{1}{10}\int_0^2 e^{-j\frac{\pi}{5}ks}ds = \frac{1}{10}\times\frac{e^{-j\frac{k\pi}{5}s}}{-j\frac{k\pi}{5}}\Big|_0^2 = \frac{e^{-j\frac{2k\pi}{5}}-1}{-j2k\pi}$$

$$= \frac{\cos(\frac{2k\pi}{5}) - j\sin(\frac{2k\pi}{5}) - 1}{-j2k\pi} = \frac{\sin(\frac{2k\pi}{5}) + j(\cos(\frac{2k\pi}{5}) - 1)}{2k\pi}$$

$$\rightarrow x_2(t) = \sum_{k\in Z}\frac{\sin(\frac{2k\pi}{5}) + j(\cos(\frac{2k\pi}{5}) - 1)}{2k\pi}\times\left(\cos(\frac{k\pi}{5}t) + j\sin(\frac{k\pi}{5}t)\right)$$

$$= \sum_{k\in Z}\sin(\frac{2k\pi}{5})\cos(\frac{k\pi}{5}t) - \cos(\frac{2k\pi}{5})\sin(\frac{k\pi}{5}t) + j\big(\cos(\frac{2k\pi}{5})\cos(\frac{k\pi}{5}t) + \sin(\frac{2k\pi}{5})\sin(\frac{k\pi}{5}t) - \cos(\frac{k\pi}{5}t)\big)$$

$$= \sum_{k\in Z}\sin(\frac{2k\pi}{5} - \frac{k\pi t}{5}) + \sin(\frac{k\pi}{5}t) + j\left(\cos(\frac{2k\pi}{5} - \frac{k\pi}{5}t) - \cos(\frac{k\pi}{5}t)\right)$$

$$\boxed{= \frac{1}{2\pi}\sum_{k\in Z}\frac{1}{k}\left(\sin(\frac{k\pi}{5}(2-t)) + \sin(\frac{k\pi}{5}t) + j\left(\cos(\frac{k\pi}{5}(2-t)) - \cos(\frac{k\pi}{5}t)\right)\right)}$$

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ادامه سوال 1)

3)

$$x_3(t)=\sum_{k\in Z} a_k e^{+j\frac{k\pi}{5}t}$$

$$a_k=\frac{1}{10}\left(\int_0^1 e^{-j\frac{k\pi}{5}s}ds-\int_2^3 e^{-j\frac{k\pi}{5}s}ds\right)=\frac{1}{10}\left(\frac{e^{-j\frac{k\pi}{5}s}}{-j\frac{k\pi}{5}}\Big|_0^1-\frac{e^{-j\frac{k\pi}{5}s}}{-j\frac{k\pi}{5}}\Big|_2^3\right)$$

$$=\frac{e^{-j\frac{k\pi}{5}}-1}{-j2k\pi}+\frac{e^{-j2\frac{k\pi}{5}}-e^{-j3\frac{k\pi}{5}}}{-j2k\pi}=\frac{e^{-j\frac{k\pi}{5}}+e^{-j2\frac{k\pi}{5}}-e^{-j2\frac{k\pi}{5}}-1}{-j2k\pi}$$

$$=\frac{\left(\cos(\frac{k\pi}{5})+\cos(\frac{2k\pi}{5})-\cos(\frac{3k\pi}{5})-1\right)+j\left(-\sin(\frac{k\pi}{5})-\sin(\frac{2k\pi}{5})+\sin(\frac{3k\pi}{5})\right)}{-j2k\pi}$$

$$\Rightarrow a_k=\frac{\sin(\frac{k\pi}{5})+\sin(\frac{2k\pi}{5})-\sin(\frac{3k\pi}{5})+j\left(\cos(\frac{k\pi}{5})+\cos(\frac{2k\pi}{5})-\cos(\frac{3k\pi}{5})-1\right)}{2k\pi}$$

$$x_3(t)=\sum_{k\in Z} a_k e^{j\frac{k\pi}{5}t}=\sum_{k\in Z} a_k\left(\cos(\tfrac{k\pi}{5}t)+j\sin(\tfrac{k\pi}{5}t)\right)$$

$$=\frac{1}{2\pi}\sum_{k\in Z}\frac{1}{k}\left(\sin\left(\tfrac{k\pi}{5}(1-t)\right)+\sin\left(\tfrac{k\pi}{5}(2-t)\right)+\sin\left(\tfrac{k\pi}{5}(t-3)\right)-\sin(\tfrac{k\pi}{5}t)+j\left(\cos\left(\tfrac{k\pi}{5}(1-t)\right)+\cos\left(\tfrac{k\pi}{5}(2-t)\right)+\cos\left(\tfrac{k\pi}{5}(t-3)\right)-\cos(\tfrac{k\pi}{5}t)\right)\right)$$

4)

$$x_4(t)=\sum_{k\in Z} a_k e^{+j\frac{k\pi}{5}t}$$

$$a_k=\frac{1}{10}\left(\int_0^1 s e^{-j\frac{k\pi}{5}s}ds+\int_1^2 e^{-j\frac{k\pi}{5}s}ds+\int_2^3 (3-s)e^{-j\frac{k\pi}{5}s}ds\right)$$

$$*\int s e^{as}ds=\frac{s}{a}e^{as}-\int\frac{e^{as}}{a}ds=\frac{e^{as}}{a}\left(s-\frac{1}{a}\right)$$

$$\xrightarrow{*} a_k=\frac{1}{10}\times\left(\frac{e^{-j\frac{k\pi}{5}s}}{-j\frac{k\pi}{5}}\left(s+\frac{1}{j\frac{k\pi}{5}}\right)\Big|_0^1+\left(\frac{e^{-j\frac{k\pi}{5}s}}{-j\frac{k\pi}{5}}\right)\Big|_1^2+\left(3\times\frac{e^{-j\frac{k\pi}{5}s}}{-j\frac{k\pi}{5}}\right)\Big|_2^3-\left(\frac{e^{-j\frac{k\pi}{5}s}}{-j\frac{k\pi}{5}}\left(s+\frac{1}{j\frac{k\pi}{5}}\right)\right)\Big|_2^3\right)$$

$$=\frac{1}{10}\times\left(\frac{e^{-j\frac{k\pi}{5}}}{-(\frac{k\pi}{5})^2}+\frac{e^{-j\frac{k\pi}{5}}}{-j\frac{k\pi}{5}}-\frac{1}{-(\frac{k\pi}{5})^2}+\frac{e^{-j2\frac{k\pi}{5}}-e^{-j\frac{k\pi}{5}}}{-j\frac{k\pi}{5}}+\frac{3e^{-j3\frac{k\pi}{5}}-3e^{-j2\frac{k\pi}{5}}}{-j\frac{k\pi}{5}}-\frac{3e^{-j3\frac{k\pi}{5}}}{-j\frac{k\pi}{5}}-\frac{e^{-j3\frac{k\pi}{5}}}{-(\frac{k\pi}{5})^2}+\frac{2e^{-j2\frac{k\pi}{5}}}{-j\frac{k\pi}{5}}+\frac{e^{-j2\frac{k\pi}{5}}}{-(\frac{k\pi}{5})^2}\right)$$

$$\Rightarrow a_k=\frac{5-5e^{-j\frac{k\pi}{5}}}{2k^2\pi^2}+\frac{e^{-j\frac{k\pi}{5}}}{-j2k\pi}+\frac{e^{-j2\frac{k\pi}{5}}-e^{-j\frac{k\pi}{5}}}{-j2k\pi}+\frac{3e^{-j3\frac{k\pi}{5}}-3e^{-j2\frac{k\pi}{5}}}{-j2k\pi}-\frac{3e^{-j3\frac{k\pi}{5}}}{-j2k\pi}+\frac{5e^{-j3\frac{k\pi}{5}}}{2k^2\pi^2}+\frac{2e^{-j2\frac{k\pi}{5}}}{-j2k\pi}-\frac{5e^{-j2\frac{k\pi}{5}}}{2k^2\pi^2}$$

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ادامه سؤال 1 مورد 4)

$$\rightarrow a_k = \cancel{\frac{e^{-j\frac{k\pi}{5}} + 3e^{-j\frac{2k\pi}{5}} - 3e^{-j\frac{2k\pi}{5}} + e^{-j2k\pi} - e^{-j\frac{k\pi}{5}} - 3e^{-j\frac{2k\pi}{5}} + 2e^{-j\frac{2k\pi}{5}}}{-j2k\pi}}$$

$$+ \frac{5 - 5e^{-j\frac{k\pi}{5}} + 5e^{-j\frac{3k\pi}{5}} - 5e^{-j\frac{2k\pi}{5}}}{2k^2\pi^2}$$

$$\rightarrow a_k = \frac{5}{2} \times \frac{\left(e^{-j\frac{3k\pi}{5}} - e^{-j\frac{2k\pi}{5}} - e^{-j\frac{k\pi}{5}} + 1\right)}{k^2\pi^2}$$

$$\rightarrow x_4(t) = \sum_{k\in Z} \frac{5}{2\pi^2} \times \left( \frac{\cos(\frac{3k\pi}{5}) - \cos(\frac{2k\pi}{5}) - \cos(\frac{k\pi}{5}) + 1 + j\left(-\sin(\frac{3k\pi}{5}) + \sin(\frac{2k\pi}{5}) + \sin(\frac{k\pi}{5})\right)}{k^2} \right)$$

$$\times \left(\cos(\tfrac{k\pi}{5}t) + j\sin(\tfrac{k\pi}{5}t)\right)$$

$$\boxed{= \frac{5}{2\pi^2} \sum \frac{1}{k^2} \times \left( \cos(\tfrac{k\pi}{5}(3-t)) - \cos(\tfrac{k\pi}{5}(2-t)) - \cos(\tfrac{k\pi}{5}(1-t)) + \cos(\tfrac{k\pi}{5}t) + j\left(\sin(\tfrac{k\pi}{5}(t-3)) + \sin(\tfrac{k\pi}{5}(2-t)) + \sin(\tfrac{k\pi}{5}(1-t))\right)\right)}$$

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سؤال 2)

$$x_1(t) = \sum_{-\infty}^{+\infty} (-1)^n \delta(t-nT) \longrightarrow \text{Period} = 2T$$

(1

$$a_k = \frac{1}{2T} \int_{-\frac{3}{2}T}^{+\frac{T}{2}} x_1(s) \times e^{-jk\frac{s\pi}{T}} ds = \frac{1}{2T} \int_{-\frac{3T}{2}}^{+\frac{T}{2}} (\delta(t) - \delta(t+T)) \times e^{-jk\frac{s\pi}{T}} ds$$

$$= \frac{1}{2T} \times (1 - e^{jk\pi}) = \frac{1}{2T} \times (1 - \cos(k\pi) - \cancel{j\sin(k\pi)}) = \frac{1}{2T}(1 - \cos(k\pi))$$

$$\rightarrow \begin{array}{l} \text{if } k = 2P \xrightarrow{P\in Z} \\ \text{if } k = 2P+1 \longrightarrow \end{array} \boxed{\begin{array}{l} a_k = 0 \\ a_k = \frac{1}{T} \end{array}} \rightarrow \boxed{x_1(t) = \sum_{\substack{k\in\{2P+1\} \\ P\in Z}} \frac{1}{T} \times \left(\cos(\tfrac{k\pi}{T}t) + j\sin(\tfrac{k\pi}{T}t)\right)}$$

(2

$$x_2(t) = \sum_{-\infty}^{+\infty} \delta'(t-nT) \longrightarrow \text{Period} = T$$

$$\rightarrow a_k = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \delta'(t) \times e^{-jk\frac{2\pi}{T}t} dt = \frac{1}{T} \times \left( \int_{-\frac{T}{2}}^{\frac{T}{2}} (\delta'(t) \times 1) dt - \int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t) \times (-jk\tfrac{2\pi}{T}) \right)$$

$$= \frac{1}{T} \times \left(0 + jk\frac{2\pi}{T}\right) = \frac{jk2\pi}{T^2}$$

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ادامه سؤال 2 مورد 2)

$$\rightarrow x_2(t) = \sum_{k\in Z} \frac{jk2\pi}{T^2} \times e^{j\frac{2k\pi}{T}t}$$

$$= \frac{2\pi}{T^2} \sum_{k\in Z} jk \times \left(\cos\left(\frac{2k\pi}{T}t\right) + j\sin\left(\frac{2k\pi}{T}t\right)\right)$$

$$= \boxed{\frac{2\pi}{T^2} \sum_{k\in Z} -k\sin\left(\frac{2k\pi}{T}t\right) + jk\cos\left(\frac{2k\pi}{T}t\right)}$$

3)

$$x_3(t) = \underbrace{\cos(t) + \cos(2.5t)}_{\text{Period}=4\pi} = \frac{e^{jt} + e^{-jt}}{2} + \frac{e^{j2.5t} + e^{-j2.5t}}{2}$$

$$\rightarrow x_3(t) = \sum_{k\in Z} a_k \times e^{j\frac{kt}{2}}$$

$$x_3(t) = \frac{1}{2}e^{j-2.5t} + \frac{1}{2}e^{-jt} + \frac{1}{2}e^{jt} + \frac{1}{2}e^{+j2.5t}$$

$$\rightarrow \boxed{a_2 = a_{-2} = a_5 = a_{-5} = \frac{1}{2}}$$

4)

$$x_4(t) = e^{t-n} \quad \text{for } n \leq t < n+1 \rightarrow \text{Period} = 1$$

$$a_k = \frac{1}{T}\int_{\langle T\rangle} x_4(t) e^{-j\frac{2k\pi}{T}t} \rightarrow a_k = \frac{1}{1} \times \int_0^1 e^t \times e^{-j2k\pi t}\, dt$$

$$\rightarrow a_k = \frac{e^{t(1-j2k\pi)}}{1-j2k\pi}\Big|_0^1 = \frac{e^{1-j2k\pi} - 1}{1-j2k\pi}$$

$$\rightarrow x_4(t) = \sum_{k\in Z} \frac{e^{1-j2k\pi} - 1}{1-j2k\pi} \times e^{j2k\pi t} = \boxed{\sum_{k\in Z} \frac{e - e^{j2k\pi t}}{1-j2k\pi}}$$

5)

$$x_5(t) = |\cos(2\pi f_0 t)| \rightarrow \text{period} = \frac{2\pi}{2\pi f_0} \times \frac{1}{2} = \boxed{\frac{1}{2f_0}}$$

$$a_k = 2f_0 \int_{-\frac{1}{8f_0}}^{\frac{3}{8f_0}} |\cos(2\pi f_0 t)| \times e^{-j4kf_0\pi t}\, dt$$

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ادامه سؤال 2 قسمت 5)

$$a_K = 2f_0 \underbrace{\int_{-\frac{1}{8f_0}}^{+\frac{1}{8f_0}} \cos(2\pi f_0 t) \times e^{-j4kf_0\pi t} dt}_{I} + \underbrace{\int_{\frac{1}{8f_0}}^{\frac{3}{8f_0}} -\cos(2\pi f_0 t) \times e^{-j4kf_0\pi t}}_{II}$$

$$I = \underbrace{\int_{-\frac{1}{8f_0}}^{\frac{1}{8f_0}} \cos(2\pi f_0 t) \times \cos(4k\pi f_0 t)}_{*} - j \underbrace{\int_{-\frac{1}{8f_0}}^{+\frac{1}{8f_0}} \cos(2\pi f_0 t) \times \sin(4k\pi f_0 t)}_{\text{چون تابع فرد است} = 0}$$

$$* = \int_{-\frac{1}{8f_0}}^{\frac{1}{8f_0}} \frac{\cos(t(2\pi f_0 + 4k\pi f_0)) + \cos(t(2\pi f_0 - 4k\pi f_0))}{2}$$

$$= \frac{1}{2}\left(\frac{\sin(t(2\pi f_0 + 4k\pi f_0))}{2\pi f_0 + 4k\pi f_0} + \frac{\sin(t(2\pi f_0 - 4k\pi f_0))}{2\pi f_0 - 4k\pi f_0}\right)\Bigg|_{-\frac{1}{8f_0}}^{\frac{1}{8f_0}}$$

$$= \frac{1}{\cancel{2}} \times \cancel{2} \times \frac{\sin\left(\frac{2\pi + 4k\pi}{8}\right)}{2\pi f_0 + 4k\pi f_0} + \frac{\sin\left(\frac{2\pi - 4k\pi}{8}\right)}{2\pi f_0 - 4k\pi f_0}$$

$$= \frac{\sin\left(\frac{\pi}{4} + \frac{k\pi}{2}\right)}{2\pi f_0 + 4k\pi f_0} + \frac{\sin\left(\frac{\pi}{4} - \frac{k\pi}{2}\right)}{2\pi f_0 - 4k\pi f_0}$$

$$II = -\int_{\frac{1}{8f_0}}^{\frac{3}{8f_0}} \cos(2\pi f_0 t) \times \cos(4k\pi f_0 t) -$$

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سوال 3)

$x(t) = 1 - \frac{t^2}{2} \quad -2<t<2$

$\xrightarrow{\text{سری فوریه}} x(t) = \sum_{k\in Z} a_k \times e^{j\frac{k\pi}{2}t} = \sum a_k \left(\cos(\frac{k\pi}{2}t) + j\sin(\frac{k\pi}{2}t)\right)$

1)

$\sum_{k\in Z} a_k \times (-1)^k$

$\xrightarrow{\text{طبق سری فوریه}} x(2) = \sum_{k\in Z} a_k \times \left(\cos(k\pi) + j\cancel{\sin(k\pi)}^{0}\right) = \sum a_k \times \cos(k\pi)$

$= \sum a_k \times (-1)^k$

$\rightarrow \sum a_k (-1)^k = x(2) = 1 - \frac{2^2}{2} = \boxed{-1}$

2)

$\sum_{k\in Z} a_k \times j^k$

$\xrightarrow{\text{طبق سری فوریه}} x(1) = \sum_{k\in Z} a_k \times \overbrace{\left(\cos(\frac{k\pi}{2}) + j\sin(\frac{k\pi}{2})\right)}^{A}$

$\rightarrow$ if $k=4s \rightarrow A=1$
if $k=4s+1 \rightarrow A=j$
if $k=4s+2 \rightarrow A=-1$
if $k=4s+3 \rightarrow A=-j$

$\rightarrow \sum_{k\in Z} a_k \times j^k = x(1) = 1 - \frac{1}{2} = \boxed{\frac{1}{2}}$

3)

$\sum_{k\in Z} |a_k|^2 = \frac{1}{T}\int_0^T |x(s)|^2 ds$ قضیه پارسوال

$\rightarrow \sum_{k\in Z} |a_k|^2 = \frac{1}{4}\int_{-2}^{2} (1-\frac{t^2}{2})^2 dt = \frac{1}{4}\int_{-2}^{2} (1 + \frac{t^4}{4} - t^2) dt$

$= \frac{1}{4}\left( t\Big|_{-2}^{2} + \frac{t^5}{20}\Big|_{-2}^{2} - \frac{t^3}{3}\Big|_{-2}^{2}\right) = \frac{1}{4}\left(4 + \frac{64}{20} - \frac{16}{3}\right)$

$= \frac{1}{4}\left(\frac{240+192-320}{60}\right) = \frac{1}{4} \times \frac{432-320}{60} = \frac{1}{\cancel{4}} \times \frac{\cancel{112}^{28}}{60} = \boxed{\frac{7}{15}}$

ادامه سوال 3)

4)

$$\sum_{k\in Z} a_{2k}$$

طبق قسمت ب میدانیم که :

$$\sum_{k\in Z}(-1)^k a_k = \sum_{k\in Z} a_{2k} - a_{2k+1} = -1$$

از طرفی :

$$\sum_{k\in Z} a_k = x(0) = 1 \rightarrow \sum_{k\in Z} a_{2k} + a_{2k+1} = +1$$

حال دو معادله دو مجهول داریم :

$$\sum_{k\in Z} a_{2k} - \sum_{k\in Z} a_{2k+1} = -1$$

$$\sum_{k\in Z} a_{2k} + \sum_{k\in Z} a_{2k+1} = 1$$

$$\rightarrow \boxed{\sum_{k\in Z} a_{2k} = 0}$$

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سؤال 4)

a)

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

$$y(t) = \sum_{k=-\infty}^{+\infty} b_k e^{jk\omega_0 t}$$

$$x(t).y(t) = \sum_{n=-\infty}^{+\infty} C_k e^{jk\omega_0 t}$$

$$C_k = \frac{1}{T}\int_{t=0}^{T} x(t)y(t) e^{-jk\omega_0 t} = \frac{1}{T}\int_0^T \left(\sum_{s=-\infty}^{+\infty} a_s e^{js\omega_0 t} \times \sum_{d=-\infty}^{+\infty} b_d e^{jd\omega_0 t}\right) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{T}\int_0^T \left(\sum_{s=-\infty}^{+\infty}\sum_{d=-\infty}^{+\infty} a_s b_d e^{j(s+d)\omega_0 t}\right) e^{-jk\omega_0 t} d\tau$$

$$= \sum_{s=-\infty}^{+\infty}\sum_{d=-\infty}^{\infty} a_s b_d \underbrace{\left[\frac{1}{T}\int_0^T e^{-j(k-s-d)\omega_0 t} dt\right]}_{\delta[k-s-d]}$$

$$= \sum_{s=-\infty}^{+\infty}\sum_{d=-\infty}^{+\infty} a_s b_d \delta[k-s-d]$$

$$= \sum_{s=-\infty}^{+\infty} a_s \sum_{d=-\infty} b_d \delta[k-s-d] = \sum_{s=-\infty}^{+\infty} a_s b_{k-s} \longrightarrow \boxed{C_k = \sum_{s=-\infty}^{+\infty} a_s b_{k-s}}$$

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سوال 4 قسمت ب)

طبق صورت سوال میدانیم که:

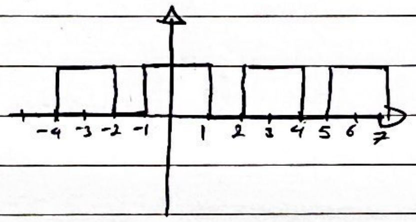
$$\Pi(t)=\begin{cases}1 & |t|\le \frac{1}{2}\\ 0 & |t|>\frac{1}{2}\end{cases}$$

$$\rightarrow \text{if } \frac{1}{2}|(t-3k)|\le\frac{1}{2} \rightarrow |t-3k|\le 1 \rightarrow -1\le t-3k\le 1$$

$$\rightarrow \boxed{3k-1\le t\le 3k+1 \ : \ \pi\left(\frac{t-3k}{2}\right)=1}$$

$$\text{if } \frac{1}{2}|t-3k|>\frac{1}{2} \rightarrow |t-3k|>1 \rightarrow \boxed{\begin{matrix} t>3k+1 \\ t<3k-1\end{matrix} \ : \pi\left(\frac{t-3k}{2}\right)=0}$$

بنابراین حال می‌توان شکل سیگنال $\sum_{-\infty}^{+\infty}\pi\left(\frac{1}{2}(t-3k)\right)$ را رسم کنیم:




میبینیم که یک سیگنال متناوب با دوره تناوب 3 هست بنابراین می‌توان ضرایب سری فوریه آن را در آورد:

$$a_k=\frac{1}{3}\int_{-1}^{2}\sum_{-\infty}^{+\infty}\pi\left(\frac{1}{2}(t-3k)\right)\times e^{-j\frac{2\pi}{3}kt}\,dt=\frac{1}{3}\int_{-1}^{+1}e^{-j\frac{2\pi}{3}kt}$$

$$=\frac{1}{3}\times\frac{e^{-j\frac{2\pi}{3}k}-e^{j\frac{2\pi}{3}k}}{-j\frac{2\pi}{3}k}=\frac{e^{j\frac{2\pi}{3}k}-e^{-j\frac{2\pi}{3}k}}{j2k\pi}$$

از طرفی میدانیم که:

$$\sin(20\pi t)=\frac{e^{j20\pi t}-e^{-j20\pi t}}{2j}=\frac{1}{2j}\times e^{j20\pi t}-\frac{1}{2j}e^{-j20\pi t}$$

ضرایب سری فوریه $\rightarrow b_{30}=\frac{1}{2j}\ ,\ b_{-30}=-\frac{1}{2j}$

طبق قسمت قبل $\rightarrow c_k=\sum_{n=-\infty}^{+\infty}a_n b_{k-n} \rightarrow$

$$\boxed{c_k=-\frac{1}{2j}\times\frac{e^{j\frac{2\pi}{3}(k+30)}-e^{-j\frac{2\pi}{3}(k+30)}}{j2\pi(k+30)}+\frac{1}{2j}\times\frac{e^{j\frac{2\pi}{3}(k-30)}-e^{-j\frac{2\pi}{3}(k-30)}}{j2\pi(k-30)}}$$

سوال 5)

طبق صورت سوال گفته شده است که توان سیگنال مدنظر که با $x(t)$ آن را نشان می‌دهیم

محدود می‌باشد بنابراین می‌شود نوشت که ،

$$\frac{1}{T}\int_{\langle T\rangle} |x(t)|^2\, dt < \infty \xrightarrow{\text{طبق پارسوال}} \sum_{k\in Z} |a_k|^2 < \infty$$

همراه مثبتات

بنابراین طبق قضیه‌ای که در ریاضی 1 داشتیم با توجه به اینکه این سری همگرا شده است میتوان

گفت که به ازای $k\to\infty$ ، $a_k$ باید صفر بشود بنابراین حکم مسئله

ثابت شد .

* صورت قضیه این بود که اگر یک سری نامنفی همگرا باشد درنتیجه جمله عمومی آن وقتی $n\to\infty$

برود به سمت صفر میل می‌کند .

Subject ........ Date ........

سوال 6)

$$\frac{1}{6}\int_{-3}^{+3}|x(t)|^2 dt = \frac{1}{2}$$

$$\xrightarrow{\text{پارسوال}} \sum_{k\in Z}|a_k|^2 = \frac{1}{2}$$

$$a_0 = 0 \longrightarrow \frac{1}{6}\int_{-3}^{+3} x(t)\, dt$$

$$x(t) = -x(t-3) \longrightarrow -e^{-j\frac{2\pi}{6}\times k\times 3} a_k = a_k \longrightarrow -e^{-k\pi} a_k = a_k$$

$$\longrightarrow \left(\cos(k\pi) + j\sin(k\pi)\right) a_k = -a_k \longrightarrow a_k \cos(k\pi) = -a_k$$

به ازای $k$ های فرد که مشکلی پیش نیاید ولی به ازای $k$ های زوج این رابطه به شرطی برقرار است که $a_k = 0$ پس ضرایب $a_k$ های زوج صفر باشند.

$a_0 = 0$ و

(I) از طرفی طبق صورت سوال می‌دانیم به ازای $|k|$ های بزرگتر از 2، $a_k = 0$ در نتیجه:

$$\sum_{k\in Z}|a_k|^2 = (a_{-1})^2 + (a_1)^2 = \frac{1}{2} \quad , \quad \xrightarrow{\text{همچنین}} a_{+1} = \overline{a_{-1}}$$

$$\xrightarrow{\text{سری فوریه}} a_{-1}e^{-j\frac{\pi}{3}t} + a_1 e^{j\frac{\pi}{3}t}$$

از طرفی گفته شده که $a_1$ یک عدد حقیقی مثبت است پس می‌شود نتیجه گرفت که $a_1 = a_{-1}$

$$\longrightarrow (a_1)^2 + (a_1)^2 = \frac{1}{2} \longrightarrow \boxed{a_1 = a_{-1} = \frac{1}{2}}$$

$\boxed{A=1}$ $\boxed{B=\frac{\pi}{3}}$ $\boxed{C=0}$

$$\xrightarrow{\text{سری فوریه}} \frac{1}{2}\left(e^{j\frac{\pi}{3}t} + e^{-j\frac{\pi}{3}t}\right) = \frac{1}{2}\times 2\times \cos\left(\frac{\pi}{3}t\right) = \cos\left(\frac{\pi}{3}t\right)$$

* اثبات I: چون سیگنال حقیقی است: $a_k = \overline{a_{-k}}$

از طرفی طبق صورت سوال میدانیم به ازای $k>2$، $a_k = 0$ پس $a_{-k} = 0$ است.

بنابراین می‌شود گفت به ازای $|k|>2$، $a_k = 0$