Assignment 12

1. Consider convection equation $\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = 0$, where u = 1 m/s, delx=0.2 m, for the domain 0<x<4., t > 0. The initial condition, T(0,x) = F(x), where F(x) varies as follows:

$F(x) = 1.25x \qquad for\ \ 0.0 \le x \le 0.8$

$F(x) = 1 - 1.25\ (x - 0.8) \qquad for\ \ 0.8 \le x \le 1.6$

$F(x) = 0 \qquad for \qquad x \ge 1.6$

   (a) The boundary condition T(t,0) = 0. The analytical solution for the above problem is T(t,x)=F(x-ut), for (x-ut) ≥0. Note that the boundary condition will propagate and make F(x-ut) < 0 = 0. The above equation represents a translating triangle as discussed in the class.
   (b) Verify that for Lax scheme the result is exact when $C = \frac{u\Delta t}{\Delta x} = 1$. The value of C can be adjusted choosing appropriate Δt. Note that Δx = 0.2 and u = 1 ⇒ $\Delta t = 0.2$.
   (c) Now repeat the solution for c= 0.25, 0.5 and 0.925 and witness the severe numerical diffusion resulting in dissipation of the maximum value.
   (d) Repeat (b) and (c) with Upwind Scheme and convince yourself that while this scheme also is exact for C = 1, good amount of numerical diffusion crreps in for C < 1
   (e) Repeat (b) and (c) same with Lax Wendroff Scheme and convince yourself the advantage of Lax Wendroff Scheme.
   (f) Finally check that FTCS scheme is an Unstable scheme for any Δt.