Assignment 10

1. Note that above scheme is a subset of the theta-method given in the mid sem. Assume that you are solving for the diffusion equation in the domain t>0, 0<x<1 with the following initial and boundary conditions

   f(x,0) = sin (π x), f(0,t) = 0 = f(1,t). Show that the analytical solution is

   f(x,t) = exp(- $\alpha\pi^2$ t) sin(π x) (This you can show by mere substitution).
2. Solve it by explicit method with Take the value of alpha = 0.01. and delx = 0.1. Choose delt such that Diffusion number is 0.5. Plot the results for the numerical and analytical at t = 10 and t = 20.
3. Repeat the same at D = 1/6 and plot the results at t = 10 and t = 20.
4. Re-run this case for D = 2.0 and plot the results. In this case carry the calculations till t=40 plot the results at t = 10, 20 and 40.
5. Write a computer code to solve the diffusion equation using the Theta method. Do as directed. Carry all calculations upto t = 40 or just close to it.

(i) Solve for theta = 1.0, with D = 2.0 and Comment on the results by plotting the data at t = 10, and 40.

(ii) Solve for theta = 0.5, theta = 1. with D= 2.0. Comment on the results by plotting the data at t = 10, and 40.

(iii) Solve for theta = 0.5-(1/12D). with D= 2.0. Comment on the results by plotting the data at t = 10, and 40.

(iv) Solve for theta = 0.5,-1/(12D). with D= 1/sqrt(20). Comment on the results by plotting the data at t = 10, and 40.

Typical Coding

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      Tin = 0.
      Tmax = 42.
      Dee =5
      Theta = 0.5
      Tright=0.
      Tleft=0.
      Alpha=0.01
      el=1.
      Nodes=11
      delx=El/Float(Nodes-1)
      delt=Dee*delx*delx/alpha
C       Grid Generation
      x(1)=0.
      DO  I=2,Nodes
      x(I)=x(I-1)+ delx
      End Do

C  ****** TEMPERATURE INITIALIZATION ***
      pi=4.*Atan(1.)
      DO I=1,Nodes+2
        T0(I)=sin(pi*x(I))
      End Do

      tt=Tin+delt
C  ****** COMPUTATION of TRIDIAGONAL COEFFICIENTS FOR INT. NODES ****
10     DO 50 I=2,Nodes-1
      AA(I,1)=-Theta*dee
      AA(I,3)=AA(I,1)
      AA(I,2)=1.-AA(I,1)-AA(I,3)
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50      C(I)=(1.-Theta)*dee*T0(i+1)+(1.-2*(1-Theta)*Dee)*T0(i)+
     1  (1.-Theta)*dee*T0(i-1)

C  ******TREATMENT OF BOUNDARY NODES *****
       AA(1,2)=1.
       AA(1,3)=0.
       C(1)=0.
       AA(Nodes,1)=0.
       AA(Nodes,2)=1.
       C(Nodes)=0.
C  ***
C  ******COMPUTATION OF TEMPERATURES ****
       CALL THOMAS(Nodes,AA,C,T)

C**Computation of Analytical Solution
       write(13,*)tt
       Do 100 I = 1,Nodes
       T_ana(i)=exp(-alpha*pi*pi*tt)*sin(pi*x(i))
       write(13,*)x(i),T(I),T_ana(I)
100      EndDo

C  ****** Update, Continue/Terminate
       tt=tt+delt
       If(tt.gt.Tmax) Then
         Stop
       Else
         Do I = 1,Nodes
           T0(i)=T(i)
         End Do
       Goto 10
       Endif
C  ******
       END
```