Assignment-7

1. Consider a first order differential equation, (dy/dx) = x-y with y = 0 at x = 0.
The exact solution of the above equation is y = x + exp(-x) – 1 (Solution given at the bottom).
Integrate the above equation numerically for x = 0 to x = 12 using a step size of 0.8 by (a) Euler's method, (b) modified Euler's method, (c) R-K fourth order method (classical), (d) Adams-Moulton (AM-4) predictor corrector method. Compare the results with the analytical solution. Now repeat the above problem with decreasing step sizes to 0.4 and 0.2. Now compare the normalised errors at x = 12 as a function of step size and correlate it with the order of the method.

$$\frac{dy}{dx} = (x - y) \quad Let\ z = (x - y) \quad \Rightarrow \frac{\mathrm{d(x - z)}}{\mathrm{dx}} = z \quad \Rightarrow 1 - \frac{\mathrm{dz}}{\mathrm{dx}} = z \quad \Rightarrow \frac{\mathrm{dz}}{1 - \mathrm{z}} = dx$$

Integrating, we get,

$$\ln(1 - z) = -x + c$$

Taking exponential of both sides, we get,

$$1 - z = ke^{-x}$$

Substituting for z, we get,

$$1 - (x - y) = ke^{-x} \quad \Rightarrow \mathrm{y} = ke^{-x} + x - 1$$

Putting the boundary conditions, y = 0 at x = 0, we get,

$$0 = k - 1 \ \ or \ \ k = 1$$

Thus,

$$\boldsymbol{y = e^{-x} + x - 1}$$