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Numerical Methods in Engineering (Solution of Linear Equations-1)

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Review

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- ❑ Understood that
 - ❑ Fixed point iteration method has linear convergence
 - ❑ Newton's method has quadratic convergence
 - ❑ Secant Method has the rate of convergence as 1.62
 - ❑ For difficult equations we can use continuation method to find the root

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General

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- ❑ Solution of a set of Linear Equations
 - ❑ Network Analysis
 - ❑ Curve Fitting
 - ❑ Solution of ODE's and PDE's
- ❑ Its solution forms a basic part of most numerical algorithms

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Matrix Notation

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Consider a set of linear equation

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

The above set may be represented by

$$\begin{matrix} \swarrow \\ \text{Coefficient} \\ \text{Matrix} \end{matrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix} \begin{matrix} \searrow \\ \text{Source} \\ \text{vector} \end{matrix}$$

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Types of Coefficient Matrices-I

Diagonal Matrix Identity Matrix Tri-diagonal Matrix

$$\begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix}$$

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Types of Coefficient Matrices-II

Lower Diagonal Matrix [L] Upper Diagonal Matrix [U]

$$\begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$$

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Matrix Operations-I

Addition and Subtraction

$$A + B = C \Rightarrow a_{ij} + b_{ij} = c_{ij}$$

$$\begin{bmatrix} 3 & 6 & 2 \\ 4 & 1 & 7 \\ 8 & 5 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 6 & 4 \\ 3 & 7 & 1 \\ 6 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 12 & 6 \\ 7 & 8 & 8 \\ 14 & 6 & 6 \end{bmatrix}$$

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Matrix Operations-II

Multiplication

$$A_{P \times Q} \times B_{Q \times R} = C_{P \times R} \Rightarrow \sum_{k=1}^Q a_{ik} b_{kj} = c_{ij}$$

$$\begin{matrix} 3 \times 2 & & 2 \times 4 & & 3 \times 4 \\ \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} & \times & \begin{bmatrix} 9 & 8 & 7 & 6 \\ 5 & 4 & 3 & 2 \end{bmatrix} & = & \begin{bmatrix} 19 & 16 & 13 & 10 \\ 47 & 40 & 33 & 26 \\ 75 & 64 & 53 & 42 \end{bmatrix} \end{matrix}$$

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Determinant

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

$$a_{11}(-1)^2 \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{12}(-1)^3 \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}(-1)^4 \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

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Towards Solution of Linear Equations-I

$$\text{If } \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

$$\text{Then } x_1 = \frac{b_1}{d_{11}} \quad x_2 = \frac{b_2}{d_{22}} \quad x_3 = \frac{b_3}{d_{33}}$$

$$\text{Logic} \Rightarrow \text{For } i = 1, N \quad x_i = \frac{b_i}{d_{ii}}$$

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Towards Solution of Linear Equations-II

$$\text{If } \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

$$\Rightarrow \begin{aligned} x_1 &= \frac{b_1}{l_{11}} \\ x_2 &= \frac{b_2 - l_{21}x_1}{l_{22}} \\ x_3 &= \frac{b_3 - l_{31}x_1 - l_{32}x_2}{l_{33}} \end{aligned}$$

Logic \Rightarrow

$$x_1 = \frac{b_1}{l_{11}}$$

For $i = 2, N$

$$x_i = \frac{b_i - \sum_{j=1}^{i-1} l_{ij}x_j}{l_{ii}}$$

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Towards Solution of Linear Equations-III

$$\text{If } \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

$$\Rightarrow \begin{aligned} x_3 &= \frac{b_3}{u_{33}} \\ x_2 &= \frac{b_2 - u_{23}x_3}{u_{22}} \\ x_1 &= \frac{b_1 - u_{12}x_2 - u_{13}x_3}{u_{11}} \end{aligned}$$

Logic \Rightarrow

$$x_N = \frac{b_N}{u_{NN}}$$

For $i = N-1, 1$

$$x_i = \frac{b_i - \sum_{j=i+1}^N u_{ij}x_j}{u_{ii}}$$

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