Numerical Methods (Introduction)

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General-1

Motivation for study

- To introduce numerical algorithms

Instructor

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- Dean's Complex

Grading

- CT(2): 20%
- Mid Sem 20% End-Sem: 40%
- Assignments: 20%

General-2

Books

- Books have been listed in a separate Document
- Fairly Comprehensive Power Points will be uploaded as notes.

General-3

Emphasis

- Engineering rather than mathematical
- Fundamentals to be stressed
- Theorems and Lemmas not be stressed but just mentioned
- General concepts to be stressed than very specific ones
- Extensive use of algorithms will be the focus

Material to be covered

- Single non-linear equation
- System of linear and non-linear equations
- Interpolation, extrapolation and regression
- Differentiation and integration
- Ordinary Differential Equations (ODEs) including Initial Value Problem (IVP) and (BVP)
- Partial Differential Equations (PDEs) involving parabolic, elliptic and hyperbolic systems
- Detailed topics have been outlined on a separate Document

Problem Solving in Engineering

- Analytical
  - Needs complex mathematics and applicable only for ideal cases.
- Experimental
  - Limited validity
  - Expensive and Time consuming
- Numerical
  - Simple to apply
  - Quick and economical

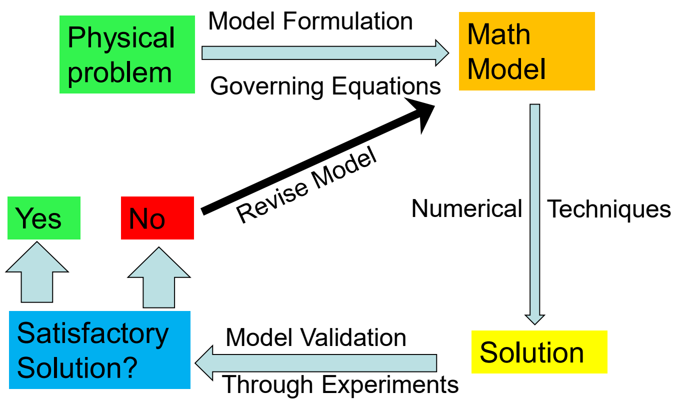
Advantages of Numerical methods

- Complex problems can be solved with modest mathematical background
- Large parametric solutions can be obtained economically to reinforce the physical understanding
- Graphical visualization possible to locate hot spots, high velocity zones, etc.

Goals and Objectives

- To lay foundation on Numerical Methods so that more advance courses can be built on them
- To give exposure to a wide spectrum of methods
- To instill confidence in problem solving skills

Problem Solving Methodology



Elements of Numerical Solution

Algorithm Design

- Aim of the course

Program Implementation

- Students to learn independently through weekly homework

Debugging and Testing

- Benchmark problems to be utilised

Documentation, Storage and Retrieval

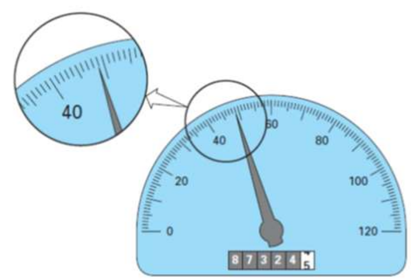
- Should have a clear write-up, stored in pendrives and hard discs.

General Tips

- Programs should be structured
- Should have several comment statements
- Should be modular and made of several functions and subroutines
- Should use structured blocks such as,
  IF-THEN-ELSE-ENDIF
  DO-ENDDO
- Should not assume to converge

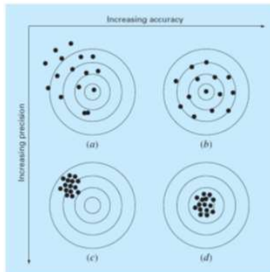
Significant Figures

- The best estimate for speed is say 48.9.
- The same for odometer is 87324.45.
- The former has an uncertainty in the 3rd digit, while the latter has uncertainty in the 7th digit.



Accuracy and Precision

- Accuracy implies that the measured value agrees with true value.
- Precision refers to measurements that are close to each other.
- When we make a bunch of measurements, accuracy implies that average value is close to true value.
- Precision implies that the standard deviation is small.



Absolute and Relative Error

- True Error = True Value – Approximate Value

$$E_t = TV - AV$$

- Most of the time we do not know the true value!
- Fractional Relative Error = True error/True value.

$$e_t = \frac{E_t}{TV}$$

- % Relative Error = e_t x 100
- Often when estimating a true value, the procedure may need several computations to increase accuracy

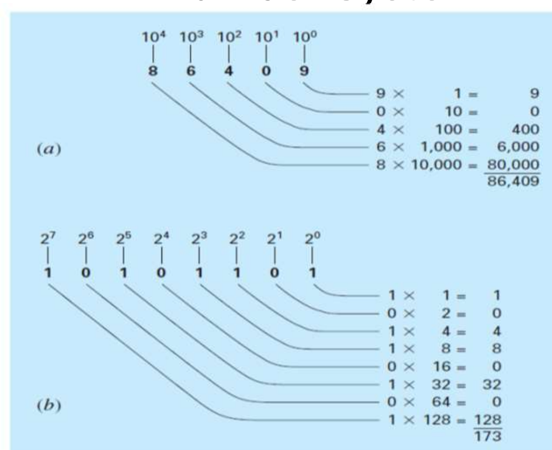
Absolute and Relative Error

- For e.g. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ \ldots.$
- Approximate relative error

$$e_a = \frac{More\ accurate value - Less\ accurate\ value}{More\ accurate\ value}$$

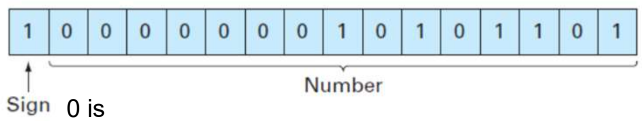
- In such cases we take as many terms that is required for a prescribed e_a.
- Usually the computation is taken into a recursive loop and the program comes out of the loop once the set e_a is satisfied.

Number System



Integer Representation

- Integer Representation



0 is positive, 1 is negative

- Largest Integer in a 16 bit computer

$(1 \times 2^{14}) + (1 \times 2^{13}) + \cdots + (1 \times 2^{1}) + (1 \times 2^{0})$ = 32,767

- Range

 - 32,768 to 32767   - 0 is taken as -32,768

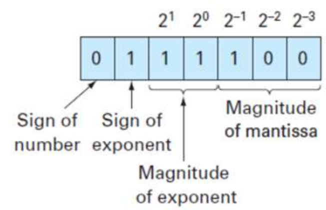
Real Number Representation

- Real Number Representation

$m \times b^{e}$

e.g., -0.1234×10^{23}

- Storage in a computer (7-bit)



Represents

$+\left(0.\left(\frac{1}{2}\right)\right)^{-(2+1)}$

$= +0.5 \times 2^{-3}$

Round-off Error

- Round off error is the error introduced when we store an irrational number in a digital form.
- For e.g., 10/11=0.90909090909….and if we have the limitation of storing only five digits, it would be stored as 0.90909.
- Mathematical operations introduce round off error. Let us assume that we have four digit machine

e.g. 0.2546E1+0.4346E-3 = 2.546+0.004346

 = 2.550346 = 0.2550 E3

- Note the drop in number of significant digits
- Subtraction of near equal numbers leads to even more serious issue.

Round-off Error

- Consider the following:.

 0.577237-0.577128 = 0.000109

- In a four digit accuracy machine, it will be

 0.5772E0+0.5771E0 = 0.0001E0 = 0.1000E-3

- The answer has only one significant digit. Note the drop in number of significant digits
- The relative error = (0.000109-0.0001)/0.000109 = (0.000009/0.000109)=9/109 =~ 9%
- This is is too much of an error in a four digit machine!

Truncation Error

- There is another type of error introduced when working with differential equations.
- When solving differential equations such as

  $\frac{dy}{dx} = f(x), where\ f(x)$ is a complicated function
- The derivative dy/dx is often dersibed as

$$\frac{dy}{dx}(x=0) = f(0) = \frac{y(h) - y0}{h}$$

- This is would imply that
- $y(h) = y0 + f(0) \times h$

Truncation Error-2

- The basis for such solution comes from Taylor Series

$$y(x_0 + h)$$
$$= y(x_0) + hy'(x_0) + \frac{h^2}{2!}y''(x_0) + \cdots \frac{h^n}{2!}y''(x_0)$$

- Often it is truncated and expressed as:

$$= y(x_0 + h) = y(x_0) + hy'(x_0) + O(h^2)$$

- For $x_0 = 0$, we get

$$y(h) = y0 + f(0) \times h$$

- The error introduced by truncation of the Taylor series is called truncation error.
- We shall discuss these later in the course.