Lab Session 3

1. Write a code to decompose a matrix into L and U as outlined in the ppt and print the resulting L and U.

```
c  *****  initialise all el and u to 0 except for diagonal of u to 1 *******
      do i=1,n
      do j=1,n
        el(i,j)=0.
        if (i.eq.j) then
        u(i,j)=1
        else
        u(i,j)=0.
        endif
      enddo
      enddo

c  ***** Decomposition operation begins  *******
         Do i = 1,n
           el(i,1)=a(i,1)
         enddo
         do j = 2,n
            u(1,j)=a(1,j)/el(1,1)
         enddo

C*** El Operation
      do j = 2,n-1
         do i = j,n
           el(i,j)=a(i,j)
           do k=1,j-1
              el(i,j)=el(i,j)-el(i,k)*u(k,j)
           enddo
         enddo

C*** U Operation

         do i = j+1,n
         u(j,i)=a(j,i)
           do k=1, j-1
           u(j,i)=u(j,i)-el(j,k)*u(k,i)
           enddo
         u(j,i)=u(j,i)/el(j,j)
         enddo
      enddo

C  *** el(n,n) Operation

      el(n,n)= a(n,n)
         do k = 1,n-1
           el(n,n)=el(n,n)-el(n,k)*u(k,n)
         enddo
```

```
c  ***** This completes Decomposition Operations
C  ***** Write el and u

      write(16,*)'el- matrix'
      do i = 1,n
         write(16,*)(el(i,j),j=1,N)
      Enddo

      write(16,*)'u- matrix'
      do i = 1,n
         write(16,*)(u(i,j),j=1,N)
      Enddo
```

2. Perform necessary operations to obtain the solution as outlined in the ppt for the set;

$$\begin{bmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \\ 2 & -2 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 12 \\ 11 \\ 2 \end{Bmatrix}$$

The correct solution is $\{3,1,2\}^T$
The L and U Matrices are shown below:

$$\begin{bmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \\ 2 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 1 & \frac{7}{3} & 0 \\ 2 & -\frac{4}{3} & -1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Write the code for solving Tri-diagonal Matrix using Thomas algorithm as discussed in the ppt and solve the following equation

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix}$$

The correct solution is $\left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}^T$

Step-1  Read the matrix correctly

```
c *** n = number of unknowns
c     a(i,1) = sub diagonal
c     a(i,2) = diagonal
c     a(i,3) = super diagonal
c     r(i) = right hand side
```

```
c     x(i) = solution
      read(12,*)n
      do i=1,n
           read(12,*)(a(i,j),j=1,3),r(i)
      enddo
      call thomas(n,a,r,x)
      write(13,*)(x(i),i=1,n)
      stop
      end
```

Step-2 Perform Upper Triangulation

```
      AA(1,3)=AA(1,3)/AA(1,2)
      RHS(1)=RHS(1)/AA(1,2)
      Do I=2,N
         DEL=AA(I,2)-AA(I,1)*AA(I-1,3)
         IF(I.NE.N)   Then
              AA(I,3)=AA(I,3)/DEL
         Else
              RHS(I)=(RHS(I)-AA(I,1)*RHS(I-1))/DEL
          Endif
      Enddo
```

Step-3 Back substitution

```
      SOL(N)=RHS(N)
      DO 150 I=N-1,1,-1
150    SOL(I)=RHS(I)-AA(I,3)*SOL(I+1)
      RETURN
      END
```