

Lab-2 Session Set of Linear Equations Gauss Elimination

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Step-1

- Step-1
- Read N, A(N,N), B(N)

$$\begin{bmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \\ 2 & -2 & -1 \end{bmatrix} \quad \begin{Bmatrix} 12 \\ 11 \\ 2 \end{Bmatrix}$$

A(N,N)

B(N)

Write A(3,3), B(3)

Upper Trangular Matrix-Sol-I

$$\text{If } \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

$$\Rightarrow \begin{aligned} x_3 &= \frac{b_3}{u_{33}} \\ x_2 &= \frac{b_2 - u_{23}x_3}{u_{22}} \\ x_1 &= \frac{b_1 - u_{12}x_2 - u_{13}x_3}{u_{11}} \end{aligned}$$

Logic \Rightarrow

$$x_N = \frac{b_N}{u_{NN}}$$

For $i = N-1, 1$

$$x_i = \frac{b_i - \sum_{j=i+1}^N u_{ij}x_j}{u_{ii}}$$

Logic

```

Read N, a(3,3), b(3)
Write N, a(3,3), b(3)
C Solution of UTM
x(n)=b(n)/a(n,n)
Do 40 I= n-1,1,-1
    sum=0.
    Do J = i+1,n
        sum=sum+a(i,j)*x(j)
    Enddo
    x(i)=(b(i)-sum)/a(i,i)
Enddo
Write x(I)
  
```

Upper Triangular Matrix-Sol-III

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 7 \\ 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} -8 \\ -2 \\ -20 \end{Bmatrix}$$

Sol

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} -11 \\ 5 \\ 4 \end{Bmatrix}$$

Gauss Elimination-I

$$\begin{bmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \\ 2 & -2 & -1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 12 \\ 11 \\ 2 \end{Bmatrix} \quad \text{Sol} = \begin{Bmatrix} 3 \\ 1 \\ 2 \end{Bmatrix}$$

Augmented Matrix

$$\begin{bmatrix} 3 & -1 & 2 & 12 \\ 1 & 2 & 3 & 11 \\ 2 & -2 & -1 & 2 \end{bmatrix}$$

Gauss Operations-Pedestrian

Augmented Matrix

$$\begin{bmatrix} 3 & -1 & 2 & 12 \\ 1 & 2 & 3 & 11 \\ 2 & -2 & -1 & 2 \end{bmatrix} \xRightarrow{\substack{3R_2-1R_1 \\ 3R_3-2R_1}} \begin{bmatrix} 3 & -1 & 2 & 12 \\ 0 & 7 & 7 & 21 \\ 0 & -4 & -7 & -18 \end{bmatrix}$$

$$\xRightarrow{7R_3+4R_2} \begin{bmatrix} 3 & -1 & 2 & 12 \\ 0 & 7 & 7 & 21 \\ 0 & 0 & -21 & -42 \end{bmatrix} \Rightarrow \text{Sol} = \begin{Bmatrix} 3 \\ 1 \\ 2 \end{Bmatrix}$$

Gauss Operations-Systematic

Augmented Matrix

$$\begin{bmatrix} 3 & -1 & 2 & 12 \\ 1 & 2 & 3 & 11 \\ 2 & -2 & -1 & 2 \end{bmatrix} \xRightarrow{\substack{R_2-1/3R_1 \\ R_3-2/3R_1}} \begin{bmatrix} 3 & -1 & 2 & 12 \\ 0 & 7/3 & 7/3 & 7 \\ 0 & -4/3 & -7/3 & -6 \end{bmatrix}$$

$$\xRightarrow{R_3 - (-4/7)R_2} \begin{bmatrix} 3 & -1 & 2 & 12 \\ 0 & 7/3 & 7/3 & 7 \\ 0 & 0 & -1 & -2 \end{bmatrix} \Rightarrow \text{Sol} = \begin{Bmatrix} 3 \\ 1 \\ 2 \end{Bmatrix}$$

Gauss Operation

```
Read N, a(N,N), b(N)
do 10 k = 1,n-1
c ***** Row operation loop begins
  do 20 l= k+1,N
    factor=a(i,k)/a(k,k)
c ***** column operations
    do 30 j = k+1,n
      a(i,j)=a(i,j)-factor*a(k,j)
    enddo
    b(i)=b(i)-factor*b(k)
  enddo
enddo
c ***** This completes Gauss
Append Upper Triangular Matrix Solver
```