

Cramer's Rule-I								
$\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$	-1 2 -2	2 3 -1	$\begin{cases} x_I \\ x_2 \\ x_3 \end{cases}$		Sol	$l = \begin{cases} 1 & \text{if } l = l \end{cases}$	$\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$	
Solution	12	-1 2	2		3	12	2	
	11	2	3		1	11	3	
$x_1 = \underline{\hspace{1cm}}$	2	-2	-1	$x_2 = $	2	2	-1	
	3	-1	2		3	-1	2	
	1	2	3		1	2	3	
	2	-2	-1		2	-2	-1	
CH-2-16/	MO\ Ni	narical M	thade I	acture 5 Set	of Lines	- Faustia	no.ll	

Cramer's Rule-II

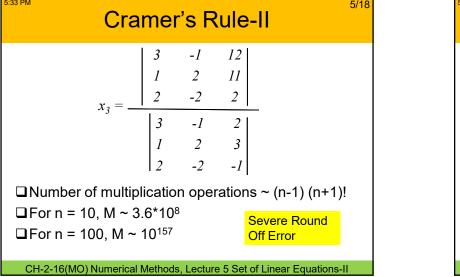
$$x_3 = \frac{\begin{vmatrix} 3 & -l & 12 \\ l & 2 & 1l \\ 2 & -2 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & -l & 2 \\ l & 2 & 3 \\ 2 & -2 & -l \end{vmatrix}}$$

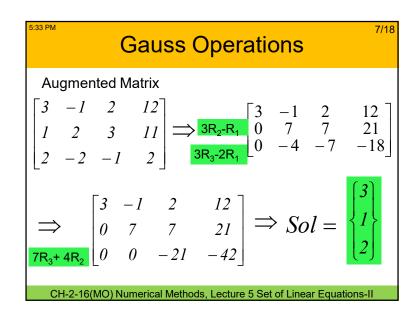
$$\square \text{ Number of multiplication operations } \sim \text{(n-1) (n+1)!}$$

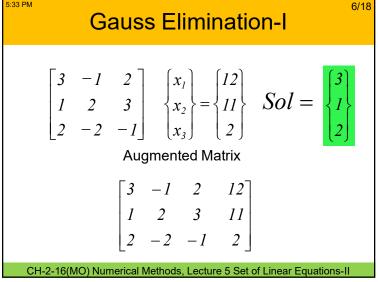
$$\square \text{ For n = 10, M} \sim 3.6*10^8$$

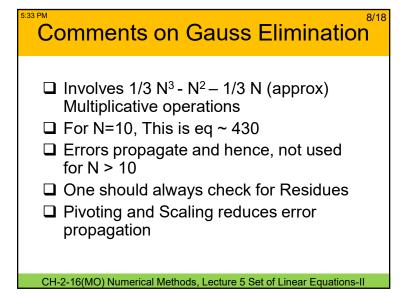
$$\square \text{ For n = 100, M} \sim 10^{157}$$

$$\square \text{ CH-2-16(MO) Numerical Methods, Lecture 5 Set of Linear Equations-II}$$

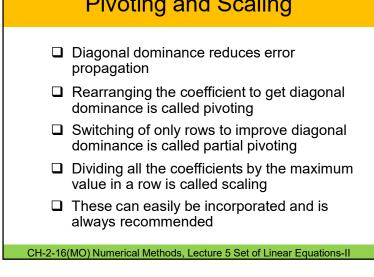


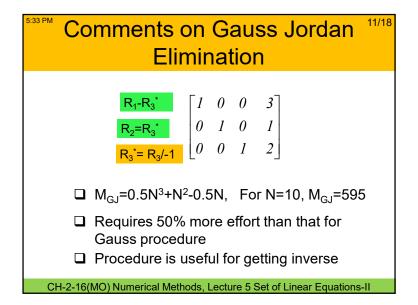


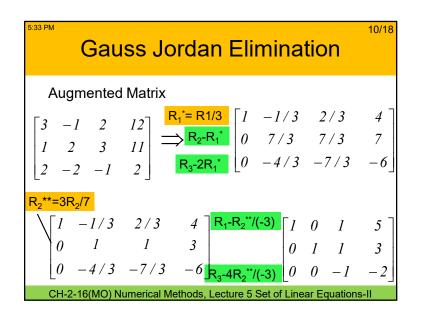




5:33 PM	Pivoting and Scaling	9/18
	Diagonal dominance reduces error propagation	
	Rearranging the coefficient to get diagonal dominance is called pivoting	
	Switching of only rows to improve diagonal dominance is called partial pivoting	
	Dividing all the coefficients by the maximum value in a row is called scaling	1
	These can easily be incorporated and is always recommended	
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Towards Solution of Linear Equations-III

$$\begin{bmatrix} 3 & -1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 2 & -2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & -0.5714 & 0.7143 & 1 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0.8571 & -0.5714 & -1 \end{bmatrix}$$
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L-U Decomposition-I

☐ If the problem has to be repeated with several source vectors for the same coefficient vector. L-U decomposition is recommended

$$[A] = [L][U]$$

- ☐ Such a decomposition speeds up calculation
- ☐ In general two methods are available
 - □ Crout's Decomposition
 - Dolittle's Decomposition

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Crout's Decomposition

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

- \Box $a_{12} = I_{11} \ u_{12}$, $a_{13} = I_{11} \ u_{13}$
- $\Box a_{22} = I_{21} u_{12} + I_{22}$, $a_{32} = I_{31} u_{12} + I_{32}$
- $\Box a_{23} = I_{21} u_{13} + I_{22} u_{23}$
- \Box $a_{33} = I_{31} U_{13} + I_{32} U_{23} + I_{33}$

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Logic for Crout's Method

$$l_{i1} = a_{i1}, \text{ for } i = 1, n$$

$$u_{1j} = a_{1j} / l_{11}, \text{ for } j = 2, n$$

$$\text{for } j = 2, n-1$$

$$l_{ij} = a_{ij} - \sum_{k=1}^{j-1} l_{ik} u_{kj} \quad (\text{ for } i = j, n)$$

$$u_{ji} = \left(a_{ji} - \sum_{k=1}^{j-1} l_{jk} u_{ki}\right) / l_{jj} \quad (\text{ for } i = j+1, n)$$

$$l_{nn} = a_{nn} - \sum_{k=1}^{n} l_{nk} u_{kn}$$

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Solution for Crout's Method

$$[A]{x} = {b} \Rightarrow [L][U]{x} = {b}$$

- \square Introducing $[U]\{x\} = \{d\}$ 1
- This implies $[L]{d} = {b}$
- ☐ Since [L] and {b} are known, {d} can be found from Eq.(2)by forward sweep
- □Once {d} is found out, {x} can be found from Eq. (1) by backward sweep

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Comments on Crout's Method-I

M_{crout} = M_{Gauss}

But, back substitution M = n² - n

Therefore for a large set one may save substantial effort (n³ vs n²)

It is possible to store the coefficients of [L] and [U] in [A] itself as [A] is no longer required. This saves memory

Other decompositions are similar

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Comments on Crout's Method-II

☐ It is possible to store L and U in A itself and conserve memory and logic written accordingly

$$\begin{bmatrix} l_{11} & u_{12} & u_{13} \\ l_{21} & l_{22} & u_{23} \\ l_{31} & l_{32} & u_{33} \end{bmatrix}$$

- ☐ Indices have to be carefully addressed
- ☐ Since memory is cheap, this no longer may be required

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