A Research of Timing-error Estimation Method for BPSK Signals

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***Abstract*—this paper mainly aims at BPSK signals to establish and validate a timing estimation method named as WP.Z algorithm that uses two samples/symbol sampling rate and its modified algorithm. Both of them have been carried on modeling and simulating in detail and their performance is evaluated under various parameters. It is not only discussed the accuracy, astringency and applied bound between two algorithms, still simulated and verified each parameter's influence to algorithm, the result is that in a low SNR and short interval length environment, the modified algorithm can be improved significantly compared to the basic algorithm.**

Keywords-Symbol Synchronization, Timing Error Estimation, WP.Z algorithm

I. INTRODUCTION

Symbol synchronization [1] is a key technique in the digital receiver of wireless communication, and it mostly fulfills picking up timing signal from receiving signal whose frequency is code velocity, aims at optimal time, and makes the receiver sample the optimal place of code to minimize BER. For single carrier, we need to find the best sampled points correctly, because a little offset will cause the dispersing of the constellation picture. There are existing two kinds of timing estimation algorithm: data-aided (DA) and non-data-aided (NDA). As this method mentioned in this paper needs a preamble inserted in each data frame, it belongs to the category of data/preamble aided feedforward symbol synchronization.

The Oerder and Meyr algorithm [2] is perhaps one of the earliest and most efficient feed forward recovery algorithms. It extracts the timing information from the squared signal and yields an unbiased estimate of the timing phase. But the O&M algorithm requires at least four samples/symbol sampling rate. However, WP.Z timing recovery algorithm [3] that uses only two samples/symbol sampling rate for timing estimation, which is one of its advantages. Certainly, in the experiment, we find that using four or eight samples/symbol sampling rate can reach more accurate simulation results.

The synchronization task, such as the symbol timing recovery in a communication receiver, can be eventually implemented by hardware such as ASIC and DSP chips. The computational and implementational complexity is directly related to the cost of the receiver. This method requires only two samples per symbol period and is very suitable for DSP/ASIC implementation. In view of the cost of receiver, this method is better than the traditional ways.

This paper is organized as follows. Primarily, it has been introduced the principles of WP.Z algorithm and modified algorithm, compared their structure and computation complexity; Secondly, it has been used BPSK signal to simulate and compare estimating performance in different parameters and estimating variance in different SNR values; Finally according to these simulation results has been produced the corresponding beneficial conclusions.

II. FUNDAMENTAL

A. The Structure of Feedforward Timing Estimator

WP.Z Algorithm [3] is a typical feedforward symbol synchronization scheme for digital receiver. Dashed frame in Fig. 1 shows the structure of the timing estimator for a two-dimensional (2-D) modem such as quadrature phase-shift keying (QPSK) and quadrature amplitude modulation (QAM), where the input signal $I(t)$ and $Q(t)$ represent, respectively, the in-phase or quadrature signals which are sampled at two samples/symbol sampling rate. In the basic algorithm we use either I or Q to estimate the timing phase. As shown in the whole structure, an improved algorithm can be obtained if both I and Q are employed. Compared to the basic algorithm, it can be improved. It is of interest to note that the timing variance can be improved significantly at low SNR values such less than 10 dB. However, in a high SNR environment, the modified algorithm gives a similar estimation result compared to the basic algorithm [3].

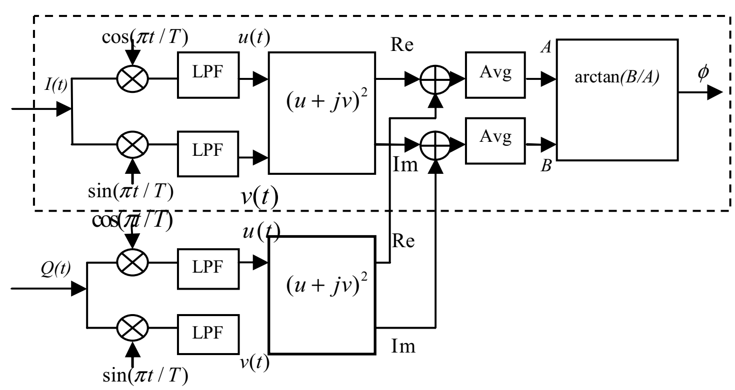

Fig. 1. Block diagram for timing estimator and it's modified algorithm

B. Timing-error Estimation of BPSK Signals

The algorithm is deduced as follows:

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The input baseband signal, I(t) or Q(t), in Fig. 1 can be expressed as.

$$x(t)=\sum_k a_k h\left(t-\tau-kT\right)+n(t),(t=nT_s) \quad (1)$$

where a_k is represent the transmitted symbols with a symbol duration T, τ indicates the timing offset, h(t) represents the overall baseband impulse response and n(t) is additive Gaussian noise. We assume a raised cosine filter is used for pulse shaping, giving an impulse response for the overall baseband transmission,

$$h(t)=\frac{T\sin(\pi t/T)\cos(\alpha\pi t/T)}{\pi t\left[1-\left(2\alpha t/T\right)^2\right]} \quad (2)$$

where α is the roll-off factor. In the following discussion, the noise term in (1) is omitted for simplicity in justification of the timing estimation algorithm.

We assume that the low-pass filter in Fig.1 is an ideal filter with a cutoff frequency $\alpha/2T$. It can easily be shown that

$$u(t)+jv(t)=\frac{T}{2}e^{j\pi\tau/T}\sum_k a_k g\left(t-\tau-kT\right)e^{jk\pi} \quad (3)$$

where g(t) is given by

$$g(t)=g_1(t)+jg_2(t) \quad (4)$$

$$g_1(t)=\frac{1}{2\pi}\int_{-\alpha\pi/T}^{\alpha\pi/T}e^{jwt}dw=\frac{\sin(\alpha\pi t/T)}{\pi t} \quad (5)$$

$$g_2(t)=\frac{1}{2}[g_1(t-\frac{T}{2\alpha})-g_1(t+\frac{T}{2\alpha})] \quad (6)$$

The squared complex signal can then be expressed as

$$\left[u(t)+jv(t)\right]^2=\frac{T^2}{4}e^{j2\pi\tau/T}\sum_k\sum_l a_k a_l g\left(t-\tau-kT\right)g\left(t-\tau-lT\right)e^{j(k+l)\pi} \quad (7)$$

By using $E\left(a_k a_l\right)=\begin{cases}E\left(a_k^{\,2}\right)=E_s & k=l\\ 0 & k\neq l\end{cases}$

where E_s represents the symbol energy, one can find the mean of the squared complex signal,

$$E\left\{\left[u(t)+jv(t)\right]^2\right\}=\frac{T^2}{4}e^{j2\pi\tau/T}E_s\sum_k g^2\left(t-\tau-kT\right) \quad (8)$$

Note that

$$\sum_k g^2(t-\tau-kT)=\sum_k[g_1^{\,2}(t-\tau-kT)-g_2^{\,2}(t-\tau-kT)]-j2\sum_k[g_1(t-\tau-kT)g_2(t-\tau-kT)]$$

Using (6) and noting that the statistical mean can be computed by the time average, one can obtain

$$E[g_1\left(t-\tau-kT\right)g_2\left(t-\tau-kT\right)]=0$$

as long as the interval for averaging is large enough. Therefore, (8) is simplified as

$$E\left\{\left[u(t)+jv(t)\right]^2\right\}=E[u^2(t)-v^2(t)]+j2E[u(t)v(t)]$$

$$=\frac{T^2}{4}e^{j2\pi\tau/T}E_s\sum_k[g_1^{\,2}\left(t-\tau-kT\right)-g_2^{\,2}\left(t-\tau-kT\right)] \quad (9)$$

which gives a timing estimate,

$$\phi=2\pi\tau/T=\arctan\left\{\frac{2E\left[u(t)v(t)\right]}{E[u^2(t)-v^2(t)]}\right\} \quad (10)$$

III. SIMULATION AND VALIDATION

A. Veracity of Algorithm Estimation

To simulate the veracity of WP.Z algorithm with matlab, in an ideal case, without noise, parameters selected as follow, BPSK signal, interval length $L=128$ 1/s, $T=1/2000$ s, $T_s=T/2$,roll-off factor $\alpha=0.5$,an ideal filter with a cutoff frequency $\alpha/2T=500$ Hz.

Simulation results:

The fixed value in advance is $delay=6.25\times10^{-5}$ s ;

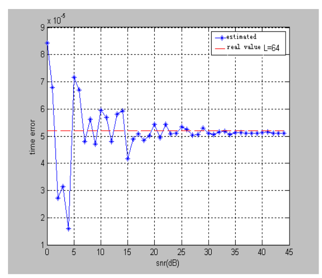
WP.Z algorithm: $terror1=6.2494\times10^{-5}$ $\text{offset}1=6.0\times10^{-8}$;

Modified WP.Z algorithm:

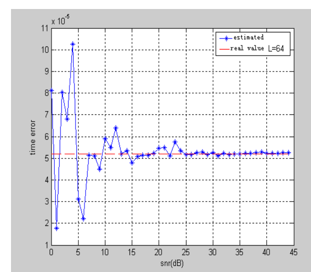
$terror2=6.2356\times10^{-5}$; $\text{offset}2=1.44\times10^{-7}$;

It is obvious that the veracity of WP.Z algorithm is good in an ideal case, but in the same environment, $L=128$ 1/s, the veracity of modified WP.Z algorithm is not better than the basic algorithm, even stability is worse. In the extended experiment, it has been shown that the variance of timing estimate can be reduced significantly in low interval length by using the modified algorithm.

B. Astringency and Mean Squared Error of Algorithm

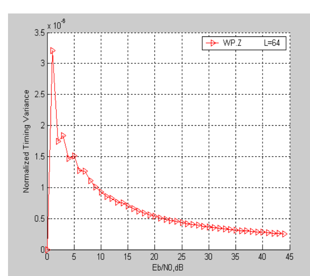


(a)WP.Z algorithm

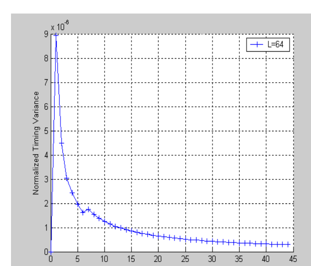


(b) modified WP.Z algorithm

Fig.2.Variance in timing estimate in different SNR values



(a)WP.Z algorithm



(b) modified WP.Z algorithm

Fig.3.Variance in mean squared error in different SNR values

To analyze the results of experiment: Fig.2 shows that with higher and higher SNR value, timing estimate value can be more and more close to the real value. As shown in Fig.3, with higher and higher SNR value, the timing variance in mean squared error can be less and less, and its estimation performance is better and better. As seen in comparing with Fig.2 (a), (b): in short estimation length (here L=64), the modified algorithm gives a better estimation result compared to the basic algorithm, namely, the basic algorithm can be not fluctuant after SNR is higher than 35dB,however,the modified can be close to real value after 25dB. As shown in Fig.3, the curve of mean squared error is steeper than the basic one, clearly, its astringency of estimation is better.

C. The Influence of Interval Length on Algorithm

To more discuss the applied bound of two algorithms, in this section, we will show the timing estimation performance for different choices of estimation interval length. Fig.4 shows the comparisons of two algorithms in a short interval length, and Fig.5 shows the variances of timing estimate with different estimation interval lengths.

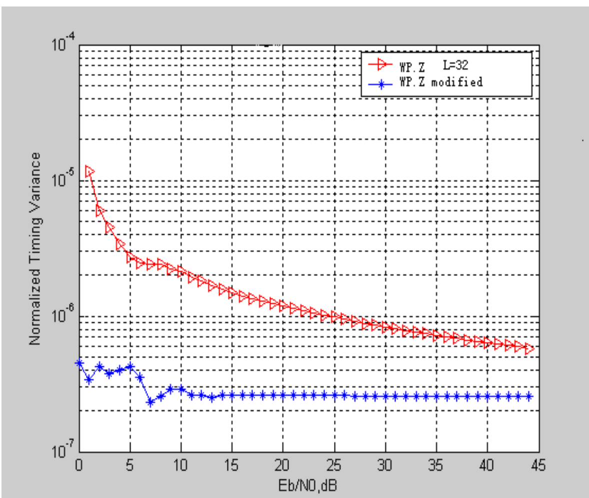


Fig.4. $L = 32$ Timing variance of two algorithms

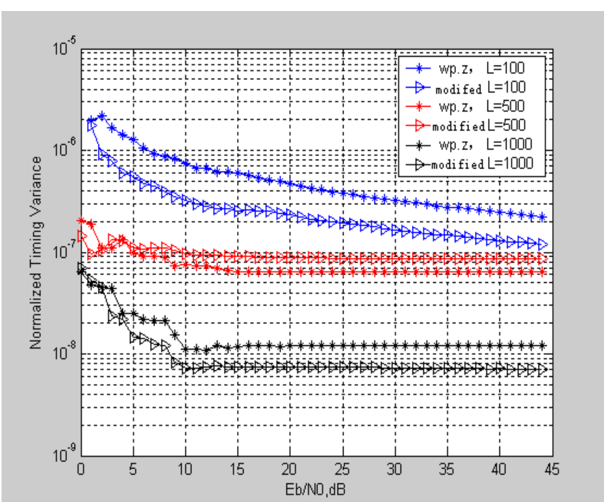


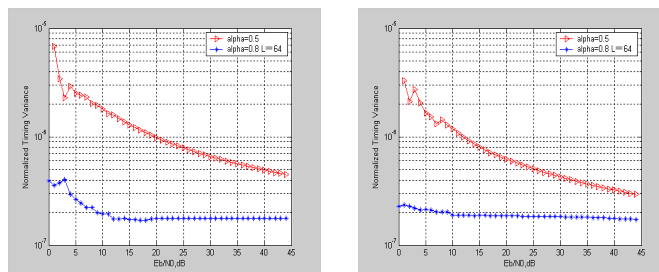
Fig.5. Timing variance with different lengths

As shown in Fig.4, the modified algorithm can be improved compared to the basic algorithm in the quality of estimate. Especially, at low SNR values, it has been shown that the variance of timing estimate can be reduced significantly by using the modified algorithm. However, in a high SNR environment (for instance), the modified algorithm gives a similar estimation result compared to the basic algorithm.

As shown in Fig.5, Clearly, the variances of two algorithms are both reduced as the estimation length is increased. But as the length is more and more increased, the advantage of the modified algorithm is less and less evident compared to the basic algorithm. Especially, in and high SNR values, the basic algorithm works even well than the modified one. In view of implementation complexity, in the same estimation performance, the basic WP.Z algorithm is given first priority.

D. The Influence of Roll-off Factor on Algorithm

In the sinc shaping filter, roll-off factor interval length, other parameter is the same. In the comparison of two algorithms, the simulation result is showed as Fig.6 (a) and (b). We can know that the variances of two algorithms are both reduced as roll-off factor is increased as shown in Fig.6. Furthermore, in the short interval length, the modified algorithm can be improved significantly at low SNR values.



(a)WP.Z algorithm (b) modified WP.Z algorithm

Fig.6.The influence of estimate variance on roll-off factor

TABEL I. COMPARISON ON WP.Z ALGORITHM AND MODIFIED ALGORITHM

CHARACTERISTIC	WP.Z ALGORITHM	WP.Z Modified algorithm
Veracity	better	good
Astringency	fine	better
MRS	small	less
Interval length	good in large length	good in short length
Roll-off factor	Influence	Evident influence
Complexity	low	high
Applied bound	High SNR and low length	Low SNR and large length

IV. CONCLUSION

The estimation performance of WP.Z and WP.Z Modified algorithm using two samples per symbol has been compared mostly. The influence of each factor on estimation performance and actual applied bound between two algorithms are discussed separately. Research indicates that with the increasing of the length of code, roll-off factor and sampling ratio, the veracity of WP.Z algorithm can be well enhanced. WP.Z modified algorithm can be improved significantly at low SNR values and short interval length. In view of implementation complexity, in the high SNR values (larger than 10 dB) and short interval length (longer than 128) environment, the basic WP.Z algorithm is given first priority.

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