

Relationship Between the Acceleration and Mass

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Abstract

The acceleration and the mass are two fundamental physics quantities that I found interesting. Every single object has mass as its intrinsic property, while every moving object has acceleration. When I was in middle school, our physics teacher told us that the mass will affect how far the car can move with the given constant acceleration. It's not hard for me to imagine but really hard for me to explain why. Then in high school I learned about kinematics. I began to realize maybe it's because there are some sorts of relationship between the acceleration of the object and the object's mass since this explains everything.

The Newton's second law enabled me to make a link from the theory we learned in class and my question. Therefore I attempted to find out the relationship between the acceleration and the mass.

1 Theoretical Information

1.1 Basis Theory

The Newton's second law of motion is been used. The law:

When a body is acted upon by a net force, the body's acceleration multiplied by its mass is equal to the net force.

can be represented in equation as:

$$F = ma.$$

In the experiment I designed, the force is kept unchanged to make life easier, and thus we can deduct:

$$a = \frac{F}{m}.$$

This leads to our consequence that the acceleration and the mass is inversely proportional.



Figure 1: The equipment used in the experiment

1.2 Theoretical Deduction

The equipments are placed in figure 1.

Since the two parts of the equipment is connected by a string and their acceleration is the same, we can see them as one object, and there is two forces acting on it:

1. The weight of part b of the object W_b ;
2. The friction exerted on part a of the object f_d .

Therefore, we can take these variables and apply them using Newton's second law:

$$a = \frac{F_n}{m}.$$

To simplify, we also assume that the surface is perfectly smooth and has no friction force $f_d = 0\text{N}$. So the net force of this object is:

$$F_n = W + f_d = W.$$

So now the relationship we found between the acceleration and the mass is:

$$a = \frac{W}{m} = \frac{m_b \cdot g}{m_a + m_b}.$$

The acceleration is equivalent to the quotient of the product of the object b's mass and gravitational acceleration and the total mass of the object.

2 Variables

2.1 Independent Variables

We say a variable is independent means that the variable that changes and affects the value of the variable which we are going to evaluate. Here, it is the

mass of the entire object, or more specifically, the mass of the object a which is represented as m_a . In this experiment, this variable will be changed by adding heavy objects which mass is a constant on the object a . Its mass is measured with a electronic scale.

2.2 Dependent Variables

The dependent variable changes according to the independent variable, and in our case is the acceleration a . We use a sensor to plot its speed-time graph, and calculate the acceleration by applying $a = \frac{v}{t}$ to it.

2.3 Controlled Variables

- The mass of the object b is controlled to ensure the initial force is the same every time.
- The mass of the heavy objects to be added on the car is a constant.
- The coefficient friction of the surface μ is 0 for simplicity.

3 Methodology

3.1 Experimental Apparatus

1. A track that has very little friction and is long enough for the car to move.
2. A car which is tied to a heavy object.
3. A sensor which collects the time-displacement data.
4. A scale which can measure the mass of the car and the heavy object.
5. A heavy object which is heavy enough to ensure the robustness of the data measured.
6. Some heavy object to add extra weight to the car.

3.2 Experimental Procedure

1. Set up all the apparatus mentioned as shown in figure 2.
2. At first do not place any heavy objects on the car.
3. Connect the sensor to the computer with Vernier Logger Pro software installed and opened.
4. Place the car at a suitable position on the track so that there is a relatively long distance for the car to move before it stops.
5. Release the car and click the collect button on the PC.

6. Measure for three times and add two heavy object to the car.
7. Iterate from step 4.

3.3 Experimental Set-up

The set-up diagram of the experiment is shown in figure 2

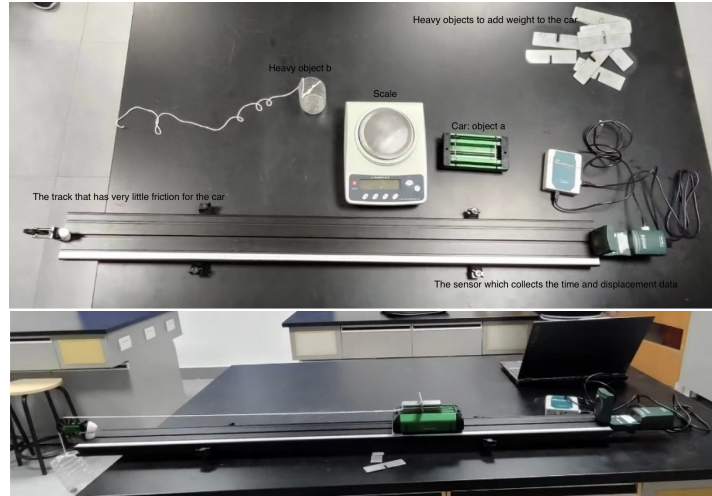


Figure 2: Set-up diagram for this experiment

3.4 Safety, Environmental or Ethical Concerns

- The car tied to a heavy object is very dangerous since it moves at a fast speed. To ensure safety, we surround the equipment with fences.
- The heavy objects added to the car is sharp and may cut people very easily. Thus we wear gloves when operating on these objects to prevent harm.

4 Analysis

4.1 Raw Data Collection

We use the set of data given in table 1 to give an example on how we processed each group of datum.

4.2 Data Analysis

Since we deduced before that acceleration is:

$$\frac{m_b \cdot g}{m_a + m_b}$$

Total Mass ($m_a + m_b$)/g $\Delta(m_a + m_b) \pm 0.01\text{g}$	Acceleration $a/m \cdot s^{-2}$ $\Delta c = \pm 0.001m \cdot s^{-2}$		
	Trail 1	Trail 2	Trail 3
537.20	0.432	0.433	0.430
587.42	0.365	0.362	0.370
637.40	0.321	0.320	0.316
737.39	0.255	0.255	0.256
787.68	0.233	0.232	0.227

Table 1: Raw data

Reciprocal of Total Mass $\frac{1}{m_a + m_b} / \text{kg}^{-1}$	Uncertainty of Reciprocal of Total Mass $\Delta\left(\frac{1}{m_a + m_b}\right) / \text{kg}^{-1}$	The Average of Acceleration $\bar{a} / \text{m} \cdot \text{s}^{-1}$	The Uncertainty of Acceleration $\Delta\bar{a} / \text{m} \cdot \text{s}^{-1}$
1.8615×10^{-3}	3×10^{-8}	0.432	0.002
1.6976×10^{-3}	3×10^{-8}	0.366	0.004
1.5686×10^{-3}	3×10^{-8}	0.319	0.003
1.4544×10^{-3}	3×10^{-8}	0.281	0.004
1.3553×10^{-3}	3×10^{-8}	0.255	0.001
1.2697×10^{-3}	3×10^{-8}	0.231	0.003

Table 2: Processed Data

therefore we should first find the reciprocal of $m_a + m_b$ which is:

$$\frac{1}{537.20\text{g}} = 1.86150 \times 10^{-3}\text{g}$$

while its uncertainty is:

$$\Delta\left(\frac{1}{m_a + m_b}\right) = \left(\frac{0.01\text{g}}{537.20\text{g}}\right)^2 \cdot 1 = 3 \times 10^{-8}\text{g}^{-1}.$$

Next we calculate the average of the velocity:

$$\bar{a} = \frac{a_1 + a_2 + a_3}{3} = \frac{0.432 + 0.433 + 0.430}{3} \text{m/s}^{-2} = 0.432\text{m/s}^{-2}$$

which uncertainty is:

$$\Delta\bar{a} = \frac{a_{\max} - a_{\min}}{2} = \frac{0.433 - 0.430}{2} \text{m/s}^{-2} = 0.002\text{m/s}^{-2}.$$

The table 2 shows the processed data.

4.3 Processed Data Presentation

The graph of the processeed data is shown in figure 3, and according to the deduction, we expect a proportional relationship while the graph shows that

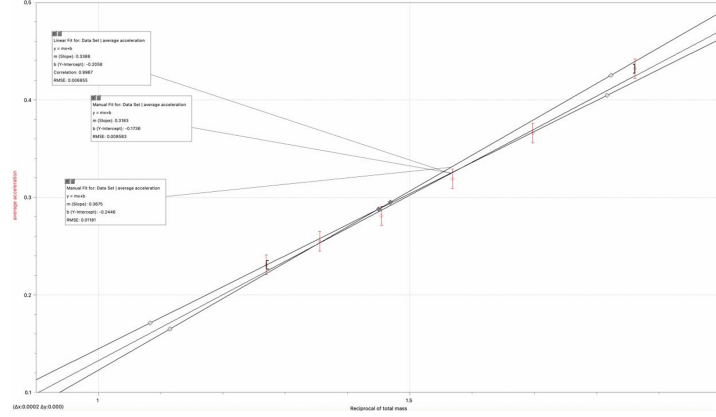


Figure 3: Relationship between the average acceleration and the reciprocal of total mass

relationship after we draw a best fit line that passes through all the points. Also, the $\Delta\left(\frac{1}{m_a+m_b}\right)$ is smaller than the single measurement therefore can be neglected.

From this figure we can know that the gradient of best fit is 0.339, while the maximum is 0.368 and the minimum is 0.318. Its uncertainty calculated is:

$$\Delta k = \frac{k_{\max} - k_{\min}}{2} = 0.025 \approx 0.03.$$

The relative uncertainty of k is therefore 7.37%.

We can also know the y -intercept of the best fit line is $i_{\text{best}} = -0.206$.

Finally we can calculate the acceleration which is:

$$a = \frac{k}{m_a + m_b} + i_{\text{best}} = (0.339 \pm 0.025) \times \frac{1}{m_a + m_b} - 0.206.$$