Homework Assignments #2

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Assessment policy:

- Give full points when correct, 1/n for solving each n subproblems. 0 for totally wrong or none, -1 for each errors.
- There may be partial points for proofs if the direction is correct.

1. Apply master theorem (12 pts)

Give tight asymptotic bounds for the following recurrences. *(2 pts each)*

- a. $T(n) = 2T(n/4) + 1$
- b. $T(n) = 2T(n/4) + \sqrt{(n)}$
- c. $T(n) = 2T(n/4) + n$
- d. $T(n) = 2T(n/4) + n^2$
- e. $T(n) = T(n/2) + \Theta(1)$
- f. $T(n) = 4T(n/2) + n^2 \lg n$

a. $\Theta(\sqrt{n})$
b. $\Theta(\sqrt{n}\log n)$
c. $\Theta(n)$
d. $\Theta(n^2)$
e. $\Theta(\log n)$
f. $\Theta(n^2 \log^2 n)$

$n^2 \log \frac{1}{2}$
$n^2 \log \frac{n}{2^2}$

$\sum_{i=0}^{\log n - 1} n^2 \log \frac{n}{2^i} + \Theta(4^{\log n})$

$n^2 \sum \log n - i \quad \left[(\log n - 1)\log n - \frac{(\log n - 2)(\log n - 3)}{2} \right] \times n^2 + n^2 \log^2 n$

2. Apply CUT-ROD (5 pts)

For the rod cutting problem, you are given a price table as below.

length i	1	2	3	4	5	6	7	8	9	10
price p_i	1	5	8	9	10	15	15	17	18	21

Apply the EXTENDED-BOTTOM-UP-CUT-ROD procedure, and compute for both tables r and s.

- (a) What is the best revenue you could make with rod of legnth 6? How should you cut the rod? 16 (3,3)
- (b) What about for length 10? 26 (3,3,2,2)

3. EXTENDED-MEMOIZED-CUT-ROD (5 pts)

Modify MEMOIZED-CUT-ROD and MEMOIZED-CUT-ROD-AUX to return not only the value but also the actual solutions, as we did for EXTENDED-BOTTOM-UP-CUT-ROD. Write pseudocode for both functions.

```
MEMOIZED-CUT-ROD(p,n)
let r[0...n] be a new array
let s[0...n] be a new array
for i = 0 to n
    r[i] = -∞
    s[i] = -∞
return MEMOIZED-CUT-ROD-AUX(p,n,r)

MEMOIZED-CUT-ROD-AUX(p,n,r)
if r[n] ≥ 0
    return r[n]
if n == 0
    q = 0
else q = -∞
    for i = 1 to n
        temp = q
        q = max(q, p[i] + MEMOIZED-CUT-ROD-AUX(p, n-i, r))
        if temp != q
            s[n] = i
r[n] = q
return q
```

4. Apply matrix-chain multiplication (5 pts)

Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $\langle 5, 10, 3, 12, 5, 50, 6 \rangle$.

P_0 P_1 P_2 P_3 P_4 P_5 P_6

5x10 10x3 3x12 12x5 5x50 50x6

m	1	2	3	4	5	6
1	0	150	330	405	1655	2010
2		0	360	330	2430	1950
3			0	180	930	1770
4				0	3000	1860
5					0	1500
6						0

s	1	2	3	4	5	6
1	0	1	2	2	4	2
2		0	2	2	2	2
3			0	3	4	4
4				0	4	4
5					0	5
6						0

$\Rightarrow (m_1 m_2)((m_3 m_4)(m_5 m_6))$ -1