

## Implement Feed Forward

```
In [3]: import random
import numpy as np
import matplotlib.pyplot as plt
```

```
In [4]: class Perceptron:

    def __init__(self, num_inputs):
        self.weights = []
        self.num_inputs = num_inputs
        for _ in range(0, num_inputs):
            self.weights.append(random.random() * 2 - 1)
        print(self.weights)

    def feed_forward(self, inputs):
        self.inputs = inputs

        sum = 0
        for i in range(0, self.num_inputs):
            sum += self.weights[i] * inputs[i]

        self.output = self.activate(sum)
        return self.output

    def activate(self, x):
        if x > 0:
            return 1
        return 0
```

```
In [ ]: a = Perceptron(5)
```

```
In [ ]: a.feed_forward([1, 2, 3, 4, 5])
```

## Define the Line

```
In [ ]: def line(x):
        return 0.5 * x
```

```
In [ ]: def graph(formula):
    x = np.array(range(0, 1000))
    y = formula(x)
    plt.plot(x, y)
    plt.xlim(0, 1000), plt.ylim(0, 1000)
    plt.show()
```

```
In [ ]: graph(line)
```

## Test for Point with Line

```
In [ ]: p = Perceptron(2)
```

```
In [ ]: x_coord = random.random() * 1000
        y_coord = random.random() * 1000
        line_y = line(x_coord)

        print(x_coord, y_coord)
        print(x_coord, line_y)

        if y_coord > line_y:
            answer = 1
        else:
            answer = 0

        print(answer)
```

```
In [ ]: correct = 0

        for _ in range(0,1000):
            x_coord = random.random() * 1000
            y_coord = random.random() * 1000
            line_y = line(x_coord)

            is_above = y_coord > line_y
            guess_above = p.feed_forward([x_coord, y_coord])

            if (is_above == True and guess_above >= 0.5):
                correct += 1
            if (is_above == False and guess_above < 0.5):
                correct += 1

        print(correct)
```

## Implement Back Propagation

```
In [13]: #Modified from above
class Perceptron:

    def __init__(self, num_inputs):
        self.weights = []
        self.num_inputs = num_inputs
        for _ in range(0, num_inputs):
            self.weights.append(random.random() * 2 - 1)
        print(self.weights)

    def feed_forward(self, inputs):
        self.inputs = inputs

        sum = 0
        for i in range(0, self.num_inputs):
            sum += self.weights[i] * inputs[i]

        self.output = self.activate(sum)
        return self.output

    def activate(self, x):
        if x > 0:
            return 1
        return 0

    def backward_pass(self, error):
        learning_rate = 0.01 # hyperparameter
        for i in range(0, self.num_inputs):
            self.weights[i] -= error * self.inputs[i] * learning_rate

    def get_weights(self):
        return self.weights
```

```
In [ ]: p = Perceptron(2)
```

```
In [ ]: print(p.get_weights())

for _ in range(0, 10000):
    x_coord = random.random() * 1000
    y_coord = random.random() * 1000
    line_y = line(x_coord)

    if y_coord > line_y:
        answer = 1
    else:
        answer = 0

    guess = p.feed_forward([x_coord, y_coord])
    p.backward_pass(guess - answer)

print(p.get_weights())
```

## Graph Results

```
In [ ]: # Modified from above
def graph(formula, weights=[]):
    x = np.array(range(0, 1000))
    y = formula(x)
    plt.plot(x, y)

    if len(weights) == 2:
        y2 = (-weights[0] / weights[1]) * x
        plt.plot(x, y2)

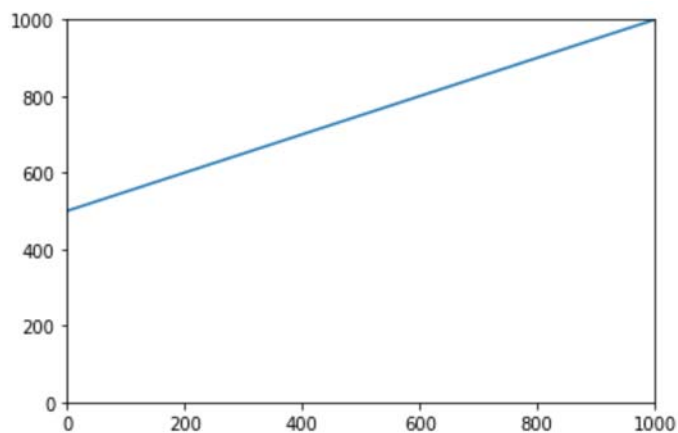
    plt.xlim(0, 1000), plt.ylim(0, 1000)
    plt.show()
```

```
In [ ]: graph(line, [-19.56805979028088, 5.751733481531401])
```

### Line with Bias

```
In [14]: def line(x):
         return 0.5 * x + 500
```

```
In [15]: graph(line)
```



Try optimizing this with the current 2-input Perceptron

### Implement Bias

```
In [164]: p = Perceptron(3)
          [0.10627913462087979, 0.03666328157163057, -0.8696577734425568]
```

```
In [165]: # Modified from above
from IPython import display
import pylab as pl

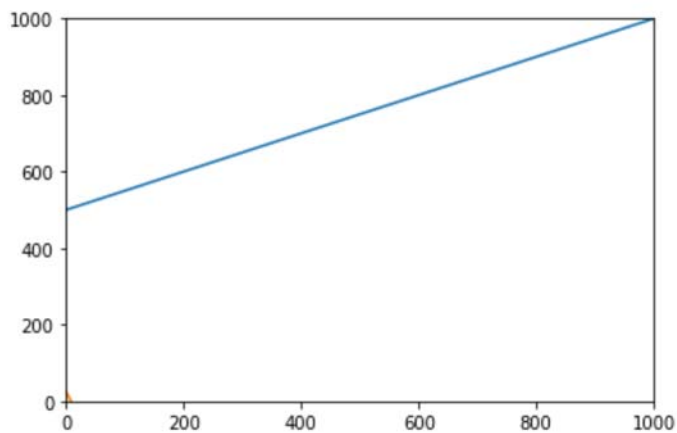
def graph(formula, weights=[]):
    plt.clf()
    x = np.array(range(0, 1000))
    y = formula(x)
    plt.plot(x, y)

    if len(weights) == 2:
        y2 = (-weights[0] / weights[1]) * x
        plt.plot(x, y2)

    if len(weights) == 3:
        y2 = (-weights[0] / weights[1]) * x - (weights[2] / weights[1])
        plt.plot(x, y2)

    plt.xlim(0, 1000), plt.ylim(0, 1000)
    display.display(pl.gcf())
    display.clear_output(wait=True)
```

```
In [166]: graph(line, p.get_weights())
```



```
In [170]: #Modified from above
correct = 0

for _ in range(0,1000):
    x_coord = random.random() * 1000
    y_coord = random.random() * 1000
    line_y = line(x_coord)

    is_above = y_coord > line_y
    guess_above = p.feed_forward([x_coord, y_coord, 1])

    if (is_above == True and guess_above >= 0.5):
        correct += 1
    if (is_above == False and guess_above < 0.5):
        correct += 1

print(correct)
```

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```
In [169]: #Modified from above
print(p.get_weights())

for i in range(0, 100000000):
    x_coord = random.random() * 1000
    y_coord = random.random() * 1000
    line_y = line(x_coord)

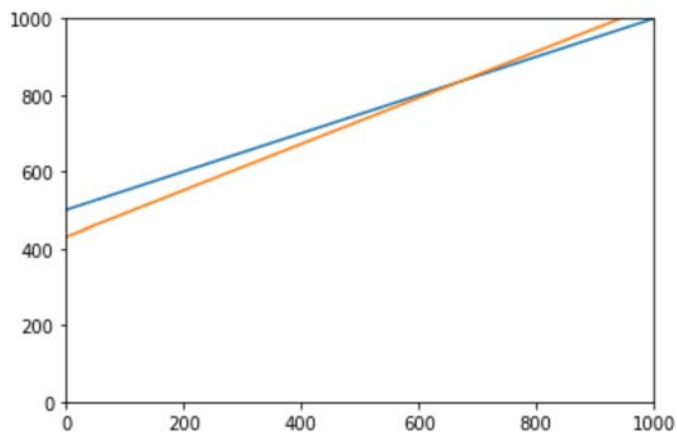
    if y_coord > line_y:
        answer = 1
    else:
        answer = 0

    guess = p.feed_forward([x_coord, y_coord, 1])
    p.backward_pass(guess - answer)

    if i % 100000 == 0:
        graph(line, p.get_weights())

print(p.get_weights())
```

```
[-19.815261771600923, 36.175675609834016, -17177.00965795771]
```

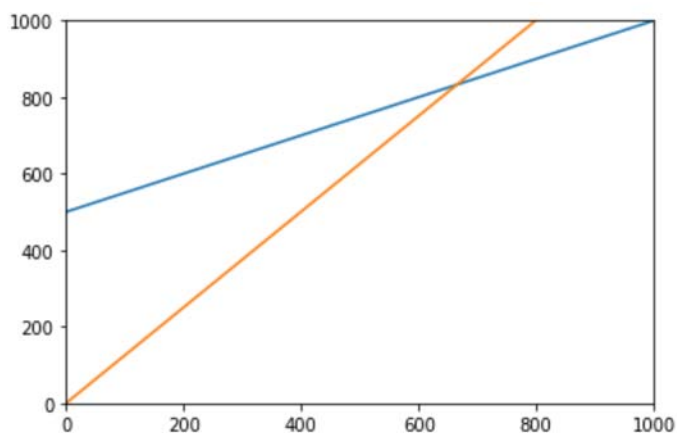


### Input Normalization

```
In [233]: p = Perceptron(3)
```

```
[0.9030310356758318, -0.7215011889549581, 0.04429049265388407]
```

```
In [234]: graph(line, p.get_weights())
```



```
In [237]: #Modified from above
correct = 0

for _ in range(0,1000):
    x_coord = random.random() * 1000
    y_coord = random.random() * 1000
    line_y = line(x_coord)

    x_coord_norm = x_coord / 1000
    y_coord_norm = y_coord / 1000

    is_above = y_coord > line_y
    guess_above = p.feed_forward([x_coord_norm, y_coord_norm, 1])

    if (is_above == True and guess_above >= 0.5):
        correct += 1
    if (is_above == False and guess_above < 0.5):
        correct += 1

print(correct)
```

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```
In [236]: #Modified from above
print(p.get_weights())

for i in range(0, 10000):
    x_coord = random.random() * 1000
    y_coord = random.random() * 1000
    line_y = line(x_coord)

    x_coord_norm = x_coord / 1000
    y_coord_norm = y_coord / 1000

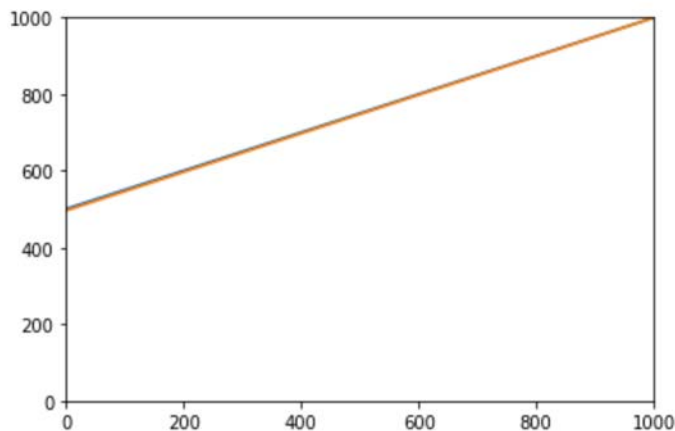
    if y_coord > line_y:
        answer = 1
    else:
        answer = 0

    guess = p.feed_forward([x_coord_norm, y_coord_norm, 1])
    p.backward_pass(guess - answer)

    if i % 100 == 0:
        w = p.get_weights().copy()
        w[-1] = w[-1] * 1000
        graph(line, w)

print(p.get_weights())

[-0.07705697890516727, 0.15291340468688674, -0.07570950734611594]
```



## Logical AND and OR

```
In [277]: a = Perceptron(3)

[0.29543212526446827, 0.8275084905441301, -0.4436670042916313]
```

```
In [280]: print("Logical AND")
print(a.feed_forward([1, 1, 1]))
print(a.feed_forward([1, 0, 1]))
print(a.feed_forward([0, 1, 1]))
print(a.feed_forward([0, 0, 1]))
```

```
Logical AND
1
0
0
0
```



```
In [279]: for _ in range(0,1000):
            first = random.choice([0, 1])
            second = random.choice([0, 1])
            a_out = a.feed_forward([first, second, 1])

            if (first and second):
                answer = 1
            else:
                answer = 0
            a.backward_pass(a_out - answer)

        print(a.get_weights())

[0.29543212526446827, 0.6275084905441299, -0.6436670042916315]
```

```
In [269]: # Modified from above
            from IPython import display
            import pylab as pl

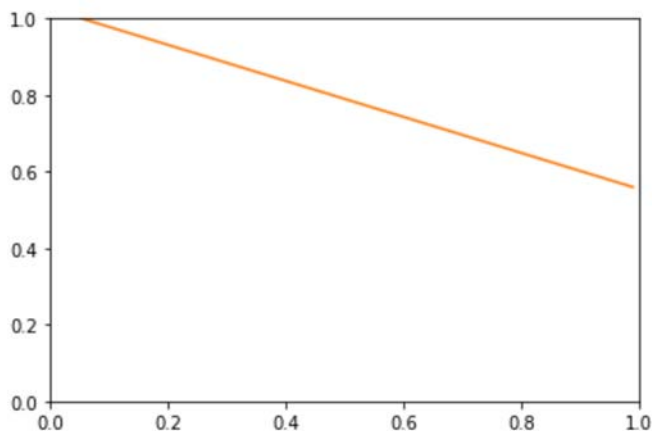
            def graph(formula, weights=[], size=1000):
                plt.clf()
                x = np.array(range(0, size))
                if size == 1:
                    x = np.arange(0, size, .01)
                y = formula(x)
                plt.plot(x, y)

                if len(weights) == 2:
                    y2 = (-weights[0] / weights[1]) * x
                    plt.plot(x, y2)

                if len(weights) == 3:
                    y2 = (-weights[0] / weights[1]) * x - (weights[2] / weights[1])
                    plt.plot(x, y2)

                plt.xlim(0, size), plt.ylim(0, size)
                display.display(pl.gcf())
                display.clear_output(wait=True)
```

```
In [281]: graph(line, a.get_weights(), size=1)
```



```
In [295]: b = Perceptron(3)

[-0.18923118637178127, 0.9056137514888873, 0.22100557748683358]
```

```
In [296]: print("Logical OR")
print(b.feed_forward([1, 1, 1]))
print(b.feed_forward([1, 0, 1]))
print(b.feed_forward([0, 1, 1]))
print(b.feed_forward([0, 0, 1]))
```

Logical OR

1  
1  
1  
1

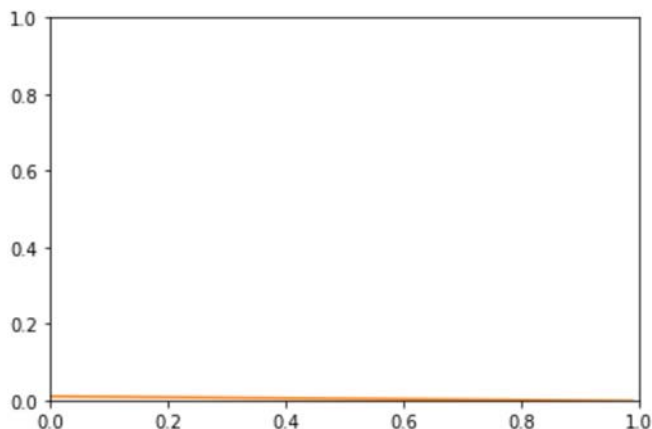
```
In [297]: for _ in range(0,1000):
    first = random.choice([0, 1])
    second = random.choice([0, 1])
    b_out = b.feed_forward([first, second, 1])

    if (first or second):
        answer = 1
    else:
        answer = 0
    b.backward_pass(b_out - answer)

print(b.get_weights())
```

[0.010768813628218766, 0.9056137514888873, -0.008994422513166478]

```
In [298]: graph(line, b.get_weights(), size=1)
```



## XOR with 2 Perceptrons

```
In [404]: a = Perceptron(3)
b = Perceptron(4)
```

[0.3396846050617881, 0.24252979675626563, -0.005778131976126311]  
[-0.05957581809648693, -0.18670489951449754, -0.5669880081788032, -0.020868857746876524]

```
In [405]: def network(first, second):
    a_out = a.feed_forward([first, second, 1])
    b_out = b.feed_forward([first, a_out, second, 1])
    return b_out
```

```
In [408]: print(network(1, 1))
          print(network(1, 0))
          print(network(0, 1))
          print(network(0, 0))
```

```
0
1
1
0
```

```
In [343]: #Modified from above
          class Perceptron:
```

```
    def __init__(self, num_inputs):
        self.weights = []
        self.num_inputs = num_inputs
        for _ in range(0, num_inputs):
            self.weights.append(random.random() * 2 - 1)
        print(self.weights)

    def feed_forward(self, inputs):
        self.inputs = inputs

        sum = 0
        for i in range(0, self.num_inputs):
            sum += self.weights[i] * inputs[i]

        self.output = self.activate(sum)
        return self.output

    def activate(self, x):
        if x > 0:
            return 1
        return 0

    def backward_pass(self, error):
        learning_rate = 0.01 # hyperparameter
        back_error = []
        for i in range(0, self.num_inputs):
            back_error.append(error * self.weights[i])
            self.weights[i] -= error * self.inputs[i] * learning_rate
        return back_error

    def get_weights(self):
        return self.weights
```

```
In [407]: for _ in range(0,1000):
            first = random.choice([0, 1])
            second = random.choice([0, 1])

            a_out = a.feed_forward([first, second, 1])
            b_out = b.feed_forward([first, a_out, second, 1])

            if (first != second):
                answer = 1
            else:
                answer = 0

            back_error = b.backward_pass(b_out - answer)
            a.backward_pass(back_error[1])

        print(a.get_weights())
        print(b.get_weights())

[0.3263164090812082, 0.25628308192619154, -0.019912229966416256]
[-0.019575818096486924, 0.23329510048550256, -0.21698800817880293, -0.0008688577
468765231]
```

### XOR with 3 Perceptrons

```
In [360]: a = Perceptron(3)
            b = Perceptron(3)
            c = Perceptron(3)

[-0.9060909691995083, -0.8396727866613862, 0.8670974138325274]
[-0.7903703175883345, -0.25264914886399636, 0.2055522551052451]
[-0.9547053142489783, 0.33788439995699093, -0.03749374516832882]
```

```
In [361]: def network(first, second):
            a_out = a.feed_forward([first, second, 1])
            b_out = b.feed_forward([first, second, 1])
            c_out = c.feed_forward([a_out, b_out, 1])
            return c_out
```

```
In [364]: print(network(1, 1))
            print(network(1, 0))
            print(network(0, 1))
            print(network(0, 0))
```

```
0
1
1
0
```

```
In [363]: for _ in range(0,1000000):
            first = random.choice([0, 1])
            second = random.choice([0, 1])
            c_out = network(first, second)

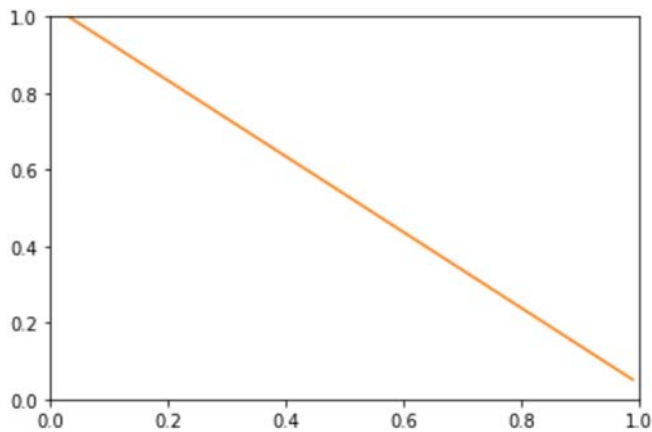
            if (first != second):
                answer = 1
            else:
                answer = 0

            back_error = c.backward_pass(c_out - answer)
            a.backward_pass(back_error[0])
            b.backward_pass(back_error[1])

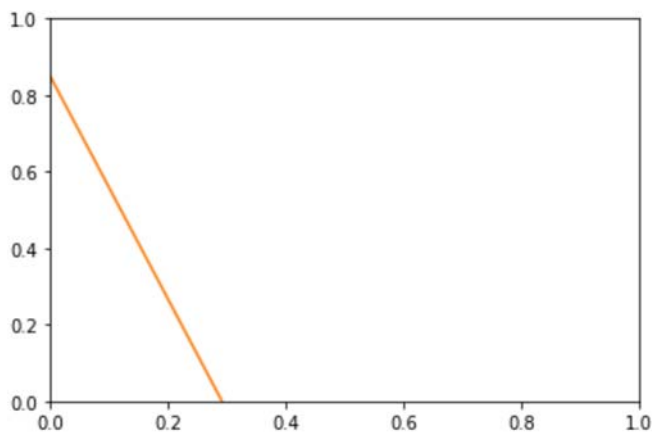
            print(a.get_weights())
            print(b.get_weights())
            print(c.get_weights())

[-0.7339970560698764, -0.7409082455291834, 0.7642562544050576]
[-0.865208701572851, -0.29713562485152334, 0.25268878710395487]
[0.015294685751022392, -0.022115600043009234, -0.007493745168328807]
```

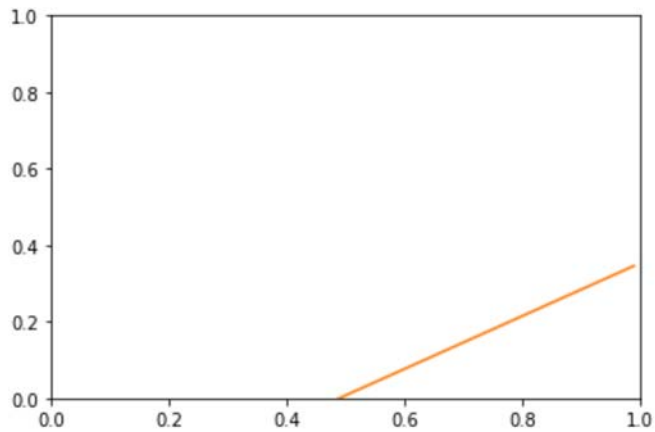
```
In [365]: graph(line, a.get_weights(), size=1)
```



```
In [366]: graph(line, b.get_weights(), size=1)
```



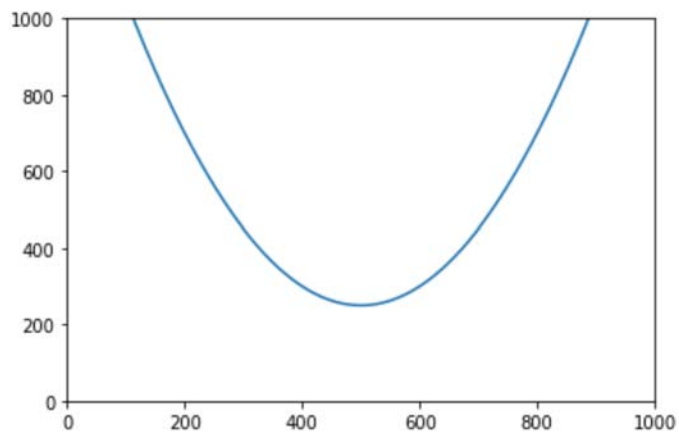
```
In [367]: graph(line, c.get_weights(), size=1)
```



### Parabola - Non-linear activation

```
In [368]: def parabola(x):
           return 0.005 * pow(x - 500, 2) + 250
```

```
In [369]: graph(parabola)
```



```
In [383]: a = Perceptron(3, act='sigmoid')
           b = Perceptron(3, act='sigmoid')
           c = Perceptron(3, act='sigmoid')

           def network(first, second):
               a_out = a.feed_forward([first, second, 1])
               b_out = b.feed_forward([first, second, 1])
               c_out = c.feed_forward([a_out, b_out, 1])
               return c_out

           [-0.9018061885208539, -0.6070839229460125, 0.5944619820191239]
           [0.18850649643752848, 0.025242870901806214, -0.9932353325299224]
           [0.5503966407290812, -0.2683264781866046, -0.2399859729541285]
```

```
In [386]: correct = 0

for _ in range(0,1000):
    x_coord = random.random() * 1000
    y_coord = random.random() * 1000
    curve_y = parabola(x_coord)
    x_norm = x_coord / 1000
    y_norm = y_coord / 1000

    is_above = y_coord > curve_y
    guess_above = network(x_norm, y_norm)

    if (is_above == True and guess_above >= 0.5):
        correct += 1
    if (is_above == False and guess_above < 0.5):
        correct += 1

print(correct)
```

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```
In [390]: for _ in range(0,1000000):
    x_coord = random.random() * 1000
    y_coord = random.random() * 1000
    curve_y = parabola(x_coord)
    x_norm = x_coord / 1000
    y_norm = y_coord / 1000

    c_out = network(x_norm, y_norm)

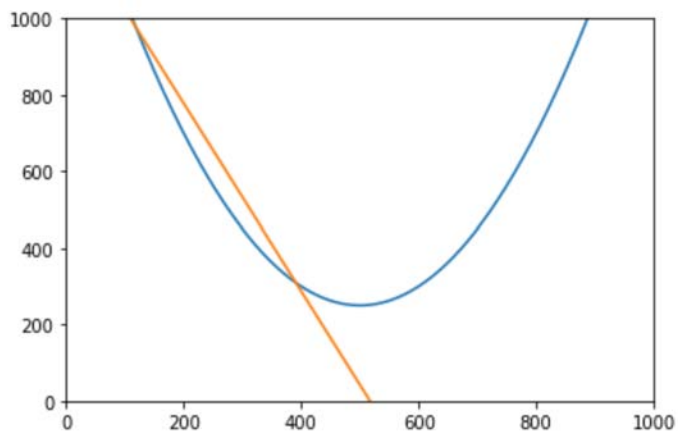
    if y_coord > curve_y:
        answer = 1
    else:
        answer = 0

    back_error = c.backward_pass(c_out - answer)
    a.backward_pass(back_error[0])
    b.backward_pass(back_error[1])

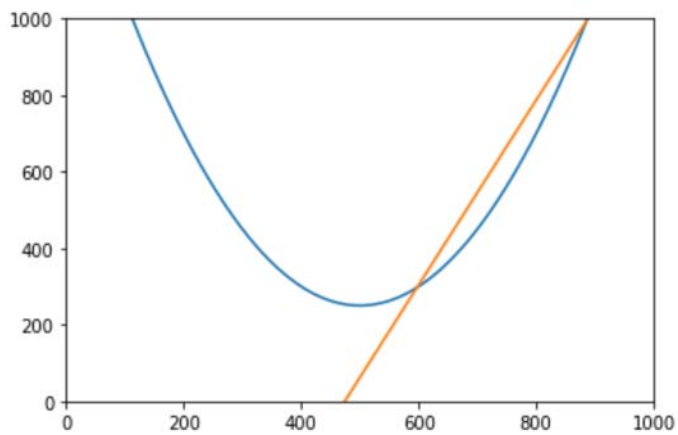
print(a.get_weights())
print(b.get_weights())
print(c.get_weights())

[-12.474348443313602, -5.084275184049197, 6.451473619935524]
[12.388689646694768, -5.126039559910979, -5.875989630522441]
[-14.387589128529227, -14.118700360720467, 8.175707532748401]
```

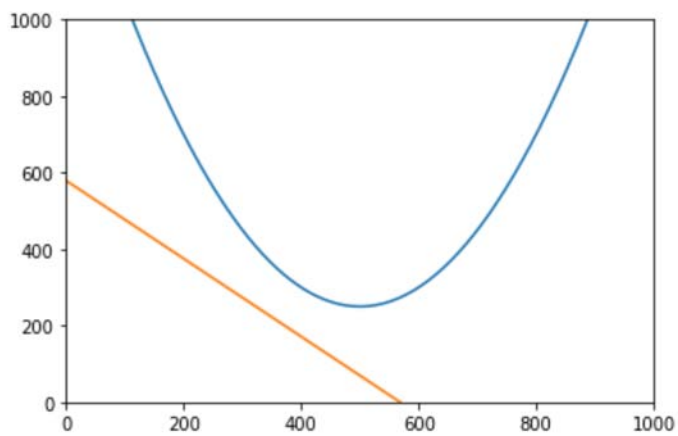
```
In [395]: w = a.get_weights().copy()  
w[-1] = w[-1] * 1000  
graph(parabola, w)
```



```
In [396]: w = b.get_weights().copy()  
w[-1] = w[-1] * 1000  
graph(parabola, w)
```



```
In [397]: w = c.get_weights().copy()  
w[-1] = w[-1] * 1000  
graph(parabola, w)
```





```
In [398]: print('%f' % c.feed_forward([1, 1, 1]))
print('%f' % c.feed_forward([1, 0, 1]))
print('%f' % c.feed_forward([0, 1, 1]))
print('%f' % c.feed_forward([0, 0, 1]))
```

```
0.000000
0.002001
0.002617
0.999719
```

```
In [382]: class Perceptron:
```

```
    def __init__(self, num_inputs, act='step'):
        self.weights = []
        self.num_inputs = num_inputs
        self.act = act # define activation function
        for _ in range(0, num_inputs):
            self.weights.append(random.random() * 2 - 1)
        print(self.weights)

    def get_weights(self):
        return self.weights

    def feed_forward(self, inputs):
        self.inputs = inputs
        sum = 0
        # multiply inputs by weights and sum them
        for i in range(0, self.num_inputs):
            sum += self.weights[i] * inputs[i]

        # 'activate' the sum and get the derivative
        self.output, self.output_prime = self.activate(sum)
        return self.output

    def activate(self, x):
        if (self.act == 'sigmoid'):
            activation = self.sigmoid(x)
            activation_prime = activation * (1 - activation)
        else:
            activation = self.step(x)
            activation_prime = 1 # use 1 since step activation is not differentiable
        return activation, activation_prime

    def sigmoid(self, x):
        return 1/(1 + np.exp(-x))

    def step(self, x):
        if x > 0:
            return 1
        return 0

    def backward_pass(self, error):
        learning_rate = 0.01 # hyperparameter
        back_error = [] # each element in list represent amount of error to send backward along that connection
        for i in range(0, self.num_inputs):
            back_error.append(error * self.output_prime * self.weights[i])
            self.weights[i] -= error * self.output_prime * self.inputs[i] * learning_rate
        return back_error
```