# Study of functions using Hill Climbing and Simulated Annealing algorithms

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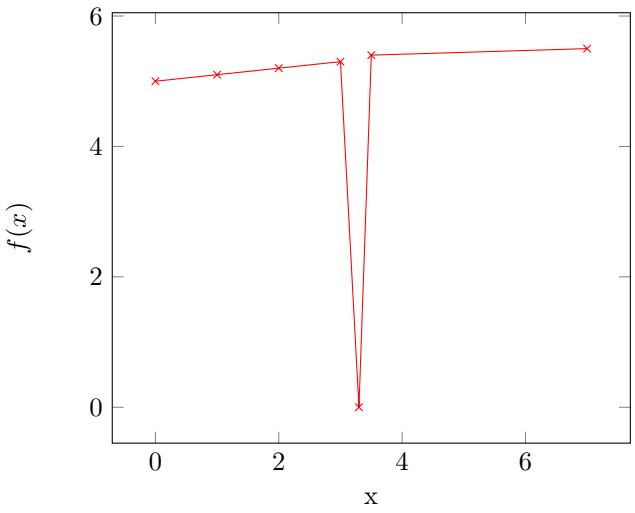
## 0.1 Introduction

### 0.1.1 Motivation

The problem of exploring the values of functions and finding the global minimum of said function for a specified domain has useful aplications, yet it is dificult to solve with a deterministic algorithm. That is because some functions have a very steep path to the minimum, like in the example below.




## 0.2 Method

Nondeterministic algorithms can be used to overcome this problem, as they have a better chance to explore the function and find the minimum .
The representation of the input variables will be a string of n bits such that they can accuately represent the function domain.

$$x = a + decimal_{represenation}(bit_{str}) \cdot (b-a)/(2^n - 1), x \in [a, b]$$

Using this represenattion, a random input called candidate solution can be generated, and its vecinity can be explored by negating one bit, such that the hamming distance between the candidate solution and the vecinity is one. This leads to the following aproaches:

Hill Climbing:

Select a candidate solution for each iteration and try to improove it using either the first better vecinity or the best vecinity. This algorithm finds the minimumm by exploring the basin of the candidate solution.

Simulated annealing:

Select a candidate solution at the start and explore its vecinity. This algorithm better explores the domain of the function by choosing worse vecinities base on the probability given by this expression:

$$random.uniform(0,1) < math.exp(-abs((evaln - evalc)/temperature))$$

This algorithm makes use of the hot iron concept. At the begining the temperature is high and the chance to choose a worse solution is high but it decreeses over each iteration based on this formula:

$$temperature = temperature * 0.9$$

## 0.3 Experiment

For this experiment, a python program will analyse theese functions on 5, 10 and 30 dimensions with $10^{-5}$ precissionn. Each test is run 30 times to ensure consistancy

$$f(x) = \sum_{i=1}^{n} \left[x_i^2\right], x_i \in [-5.12, 5.15]$$

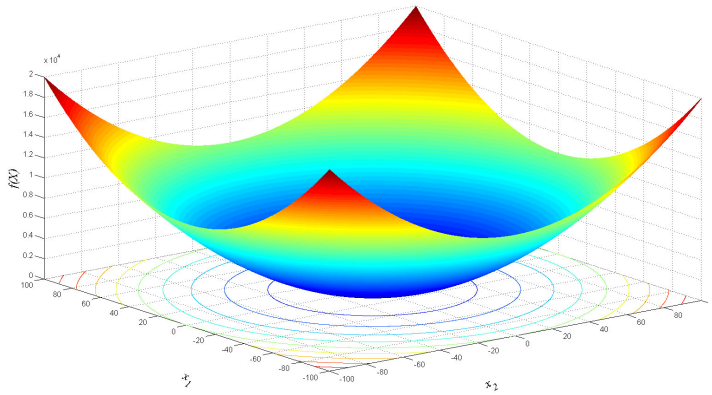



Figure 1: Image De Jong's Function.[1]

$$f(x) = \sum_{i=1}^{n} \left[ -x_i \cdot sin(sqrt(|x_i|)) \right], x_i \in [-500, 500]$$

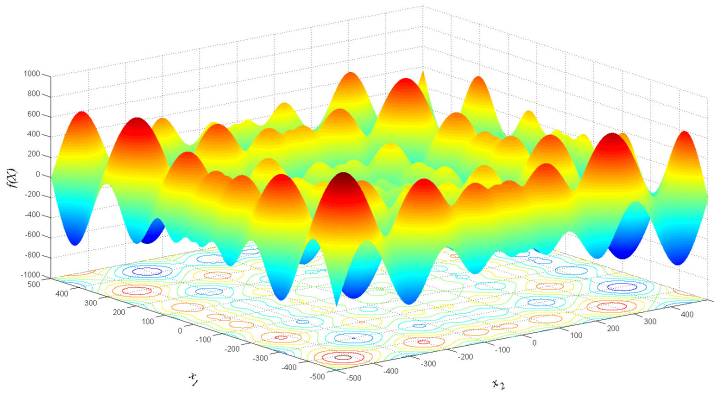



Figure 2: Image Schwefel's Function. [2]

$$f(x) = A \cdot n + \sum_{i=1}^{n} \left[ x_i^2 - A \cdot cos(2\pi x_i) \right], A = 10, x_i \in [-5.12, 5.15]$$

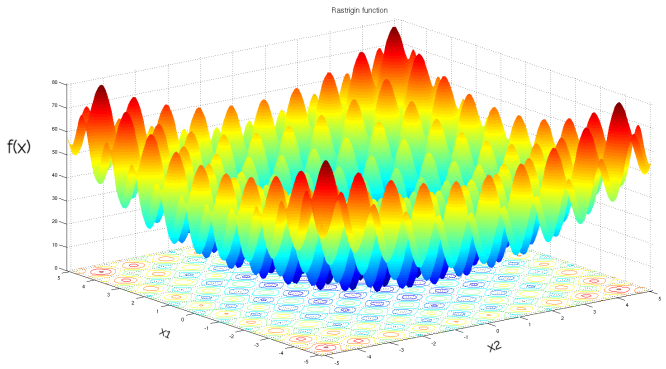



Figure 3: Image Rastrigin's Function. [3]

$$f(x) = -\sum_{i=1}^{n} \left[ sin(x_i) \cdot \left( sin\left( \frac{i \cdot x_i^2}{\pi} \right) \right)^{2 \cdot m} \right], x_i \in [0, \pi], m = 10$$

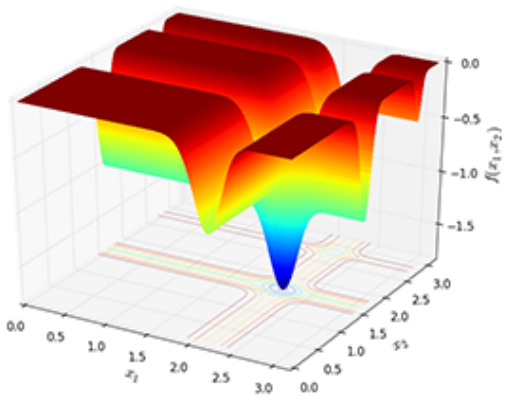



Figure 4: Michalewicz's Function. [4]

## 0.4 Results

### 0.4.1 Interpretation

Hill Climbing with the first and best neighbour choice give different resaults and times for different functions. Yet in all of them the time to solve large inputs is high in the python implementation that was used. However, the simulated annealing approach gives marginally worse resaults and significanly better time.

## 0.5 Conclusions

As demonstrated in the experiment, nondeterministic algorithms can be used to find the global minimum of functions.

| function | avg | min | max | time |
|---|---|---|---|---|
| DeJong 1 5D - Hill Climbing first | 0 | 0 | 0 | 7 |
| DeJong 1 5D -Hill Climbing best | 0 | 0 | 0 | 14 |
| DeJong 1 5D - Simulated Annealing | 0 | 0 | 0 | 2 |
| DeJong 1 10D - Hill Climbing first | 0 | 0 | 0 | 56 |
| DeJong 1 10D - Hill Climbing best | 0 | 0 | 0 | 108 |
| DeJong 1 10D - Simulated Annealing | 0 | 0 | 0 | 4 |
| DeJong 1 30D - Hill Climbing first | 0 | 0 | 0 | 973 |
| DeJong 1 30D - Hill Climbing best | 0 | 0 | 0 | 1485 |
| DeJong 1 30D - Simulated Annealing | 0 | 0 | 0 | 10 |
| Schwefel 5D - Hill Climbing first | -1939 | -1992 | -1853 | 11 |
| Schwefel 5D -Hill Climbing best | -1967 | -2094 | -1893 | 10 |
| Schwefel 5D - Simulated Annealing | -1825 | -1963 | -1776 | 3 |
| Schwefel 10D - Hill Climbing first | -3652 | -3872 | -3492 | 131 |
| Schwefel 10D -Hill Climbing best | -3623 | -3772 | -3506 | 83 |
| Schwefel 10D - Simulated Annealing | -3642 | -3905 | -3246 | 7 |
| Schwefel 30D - Hill Climbing first | -9453 | -9923 | -9369 | 1525 |
| Schwefel 30D -Hill Climbing best | -10153 | -10453 | -10063 | 1511 |
| Schwefel 30D - Simulated Annealing | -9746 | -11279 | -9365 | 20 |
| Rastrigin 5D - Hill Climbing first | 2.59497 | 1.88367 | 3.69702 | 13 |
| Rastrigin 5D -Hill Climbing best | 2.23592 | 1.579144 | 2.98702 | 14 |
| Rastrigin 5D - Simulated Annealing | 2.23086 | 1.71005 | 3.102193 | 3 |
| Rastrigin 10D - Hill Climbing first | 7.18790 | 3.93217 | 15.19234 | 161 |
| Rastrigin 10D -Hill Climbing best | 8.18790 | 3.67834 | 14.0925 | 121 |
| Rastrigin 10D - Simulated Annealing | 9.18790 | 3.19593 | 15.2394 | 7 |
| Rastrigin 30D - Hill Climbing first | 23.72184 | 17.12235 | 38.63187 | 1617 |
| Rastrigin 30D -Hill Climbing best | 22.43780 | 16.97201 | 37.29053 | 1392 |
| Rastrigin 30D - Simulated Annealing | 25.18790 | 17.67834 | 38.44140 | 21 |
| Michalewicz 5D - Hill Climbing first | -4.44109 | -4.62793 | -3.97123 | 11 |
| Michalewicz 5D -Hill Climbing best | -4.37306 | -4.65685 | -4.02012 | 10 |
| Michalewicz 5D - Simulated Annealing | -4.28916 | -4.6328 | -3.26917 | 2 |
| Michalewicz 10D - Hill Climbing first | -8.20880 | -9.01029 | -6.89132 | 107 |
| Michalewicz 10D -Hill Climbing best | -9.09037 | -9.24031 | -8.25910 | 79 |
| Michalewicz 10D - Simulated Annealing | -8.70903 | -9.22690 | -6.15737 | 6 |
| Michalewicz 30D - Hill Climbing first | -20.79776 | -22.61219 | -17.31592 | 560 |
| Michalewicz 30D -Hill Climbing best | -24.09166 | -25.14376 | -22.21472 | 423 |
| Michalewicz 30D - Simulated Annealing | -23.66259 | -24.29173 | -21.53991 | 15 |

# Bibliography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[2] Al-roomi
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Function formulas. http://www.geatbx.com/docu/fcnindex-01.html

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