Study of functions using Hill Climbing and Simulated Annealing algorithms

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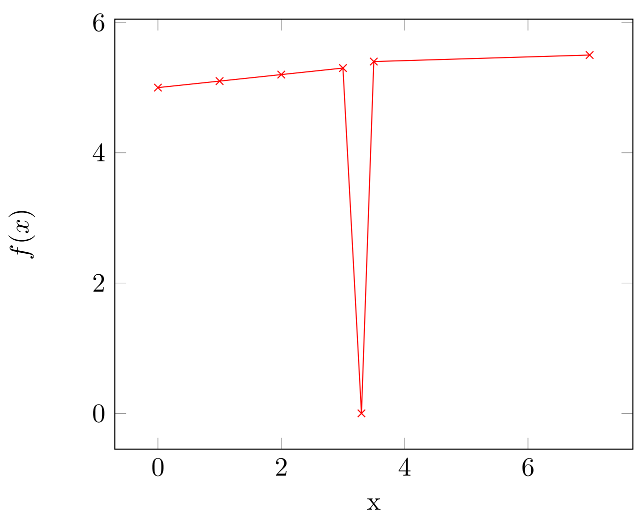
1 Introduction

1.1 Motivation

The problem of exploring the values of functions and finding the global minimum of said function for a specified domain has useful aplications, yet it is dificult to solve with a deterministic algorithm. That is because some functions have a very steep path to the minimum, like in the example below.



2 Method

Nondeterministic algorithms can be used to overcome this problem, as they have a better chance to explore the function and find the minimum .
The representation of the input variables will be a string of n bits such that they can accuately represent the function domain.

$$x = a + decimal_{represenation}(bit_{str}) \cdot (b-a)/(2^n - 1), x \in [a, b]$$

Using this represenattion, a random input called candidate solution can be generated, and its vecinity can be explored by negating one bit, such that the hamming distance between the candidate solution and the vecinity is one. This leads to the following aproaches:

Hill Climbing:

Select a candidate solution for each iteration and try to improove it using either the first better vecinity or the best vecinity. This algorithm finds the minimumm by exploring the basin of the candidate solution.

Simulated annealing:

Select a candidate solution at the start and explore its vecinity. This algorithm better explores the domain of the function by choosing worse vecinities base on the probability given by this expression:

$$random.uniform(0, 1) < math.exp(-abs((evaln - evalc)/temperature))$$

This algorithm makes use of the hot iron concept. At the begining the temperature is high and the chance to choose a worse solution is high but it decreeses over each iteration based on this formula:

$$temperature = temperature * 0.9$$

3 Experiment

For this experiment, a python program will analyse theese functions on 5, 10 and 30 dimensions with 10^{-5} precissionn. Each test is run 30 times to ensure consistancy

De Jong's Function

$$f(x) = \sum_{i=1}^{n} \left[x_i^2\right], x_i \in [-5.12, 5.15]$$

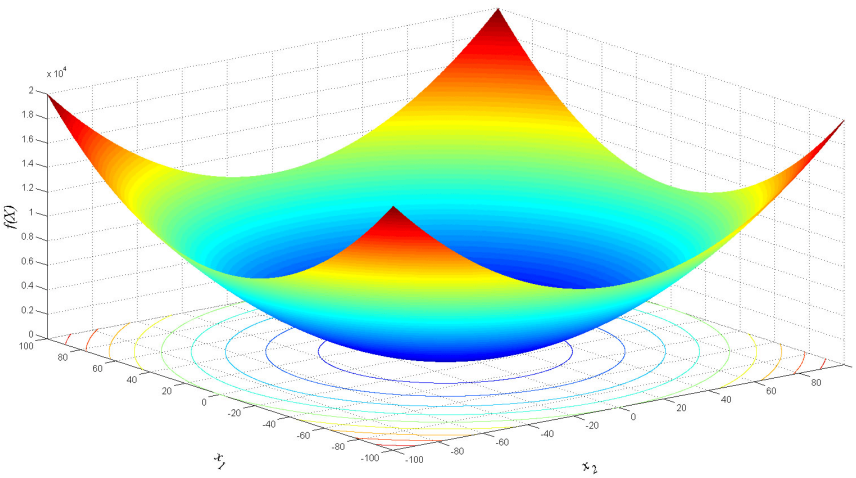


Figure 1: De Jong's Function.

Schwefel's Function

$$f(x) = \sum_{i=1}^{n} \left[-x_i \cdot sin(sqrt(|x_i|))\right], x_i \in [-500, 500]$$

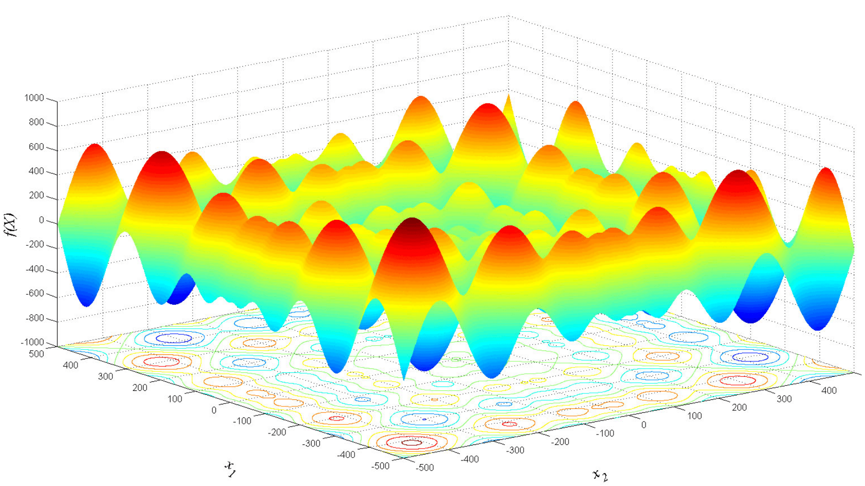


Figure 2: Schwefel's Function.

Rastrigin's Function

$$f(x) = A \cdot n + \sum_{i=1}^{n} \left[x_i^2 - A \cdot cos(2\pi x_i) \right], A = 10, x_i \in [-5.12, 5.15]$$

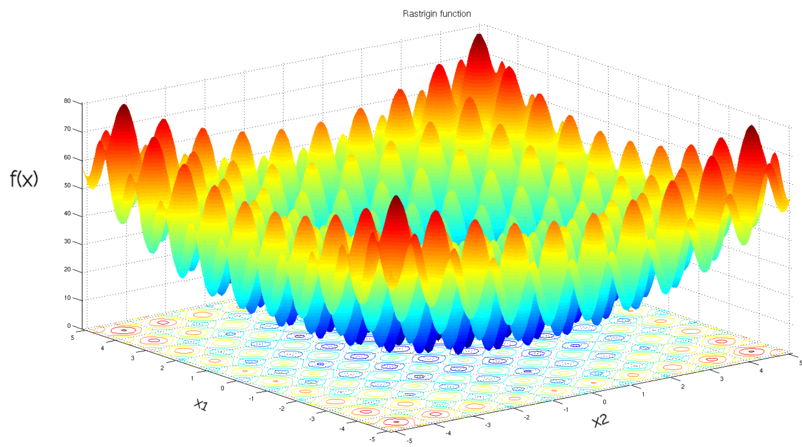


Figure 3: Rastrigin's Function.

Michalewicz's Function

$$f(x) = -\sum_{i=1}^{n} \left[sin(x_i) \cdot \left(sin\left(\frac{i \cdot x_i^2}{\pi} \right) \right)^{2 \cdot m} \right], x_i \in [0, \pi], m = 10$$

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4 Results

4.1 Interpretation

Hill Climbing with the first and best neighbour choice give different resaults and times for different functions. Yet in all of them the time to solve large inputs is high in the python implementation that was used. However, the simulated annealing approach gives marginally worse resaults and significanly better time.

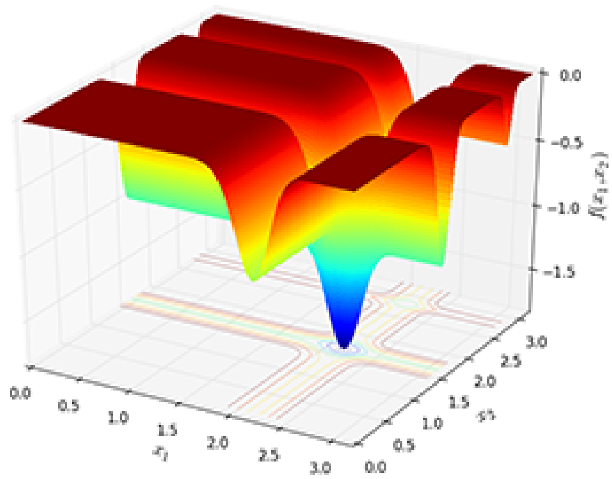

Figure 4: Michalewicz's Function.

5 Conclusions

As demonstrated in the experiment, nondeterministic algorithms can be used to find the global minimum of functions.

References

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[3] Sfu
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Function formulas. http://www.geatbx.com/docu/fcnindex-01.html

[5] Course Page. https://profs.info.uaic.ro/~eugennc/teaching/ga/1

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function	avg	min	max	time
DeJong 1 5D - Hill Climbing first	0	0	0	7
DeJong 1 5D -Hill Climbing best	0	0	0	14
DeJong 1 5D - Simulated Annealing	0	0	0	2
DeJong 1 10D - Hill Climbing first	0	0	0	56
DeJong 1 10D - Hill Climbing best	0	0	0	108
DeJong 1 10D - Simulated Annealing	0	0	0	4
DeJong 1 30D - Hill Climbing first	0	0	0	973
DeJong 1 30D - Hill Climbing best	0	0	0	1485
DeJong 1 30D - Simulated Annealing	0	0	0	10
Schwefel 5D - Hill Climbing first	-1939	-1992	-1853	11
Schwefel 5D -Hill Climbing best	-1967	-2094	-1893	10
Schwefel 5D - Simulated Annealing	-1825	-1963	-1776	3
Schwefel 10D - Hill Climbing first	-3652	-3872	-3492	131
Schwefel 10D -Hill Climbing best	-3623	-3772	-3506	83
Schwefel 10D - Simulated Annealing	-3642	-3905	-3246	7
Schwefel 30D - Hill Climbing first	-9453	-9923	-9369	1525
Schwefel 30D -Hill Climbing best	-10153	-10453	-10063	1511
Schwefel 30D - Simulated Annealing	-9746	-11279	-9365	20
Rastrigin 5D - Hill Climbing first	2.59497	1.88367	3.69702	13
Rastrigin 5D -Hill Climbing best	2.23592	1.579144	2.98702	14
Rastrigin 5D - Simulated Annealing	2.23086	1.71005	3.102193	3
Rastrigin 10D - Hill Climbing first	7.18790	3.93217	15.19234	161
Rastrigin 10D -Hill Climbing best	8.18790	3.67834	14.0925	121
Rastrigin 10D - Simulated Annealing	9.18790	3.19593	15.2394	7
Rastrigin 30D - Hill Climbing first	23.72184	17.12235	38.63187	1617
Rastrigin 30D -Hill Climbing best	22.43780	16.97201	37.29053	1392
Rastrigin 30D - Simulated Annealing	25.18790	17.67834	38.44140	21
Michalewicz 5D - Hill Climbing first	-4.44109	-4.62793	-3.97123	11
Michalewicz 5D -Hill Climbing best	-4.37306	-4.65685	-4.02012	10
Michalewicz 5D - Simulated Annealing	-4.28916	-4.6328	-3.26917	2
Michalewicz 10D - Hill Climbing first	-8.20880	-9.01029	-6.89132	107
Michalewicz 10D -Hill Climbing best	-9.09037	-9.24031	-8.25910	79
Michalewicz 10D - Simulated Annealing	-8.70903	-9.22690	-6.15737	6
Michalewicz 30D - Hill Climbing first	-20.79776	-22.61219	-17.31592	560
Michalewicz 30D -Hill Climbing best	-24.09166	-25.14376	-22.21472	423
Michalewicz 30D - Simulated Annealing	-23.66259	-24.29173	-21.53991	15