Faceted Search in Mathematics

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Overview

Why math search?

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- Textual search engines cannot index math.

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Figure : Typical Formula

Why faceted search?

- It’s important to allow the user to refine the query.

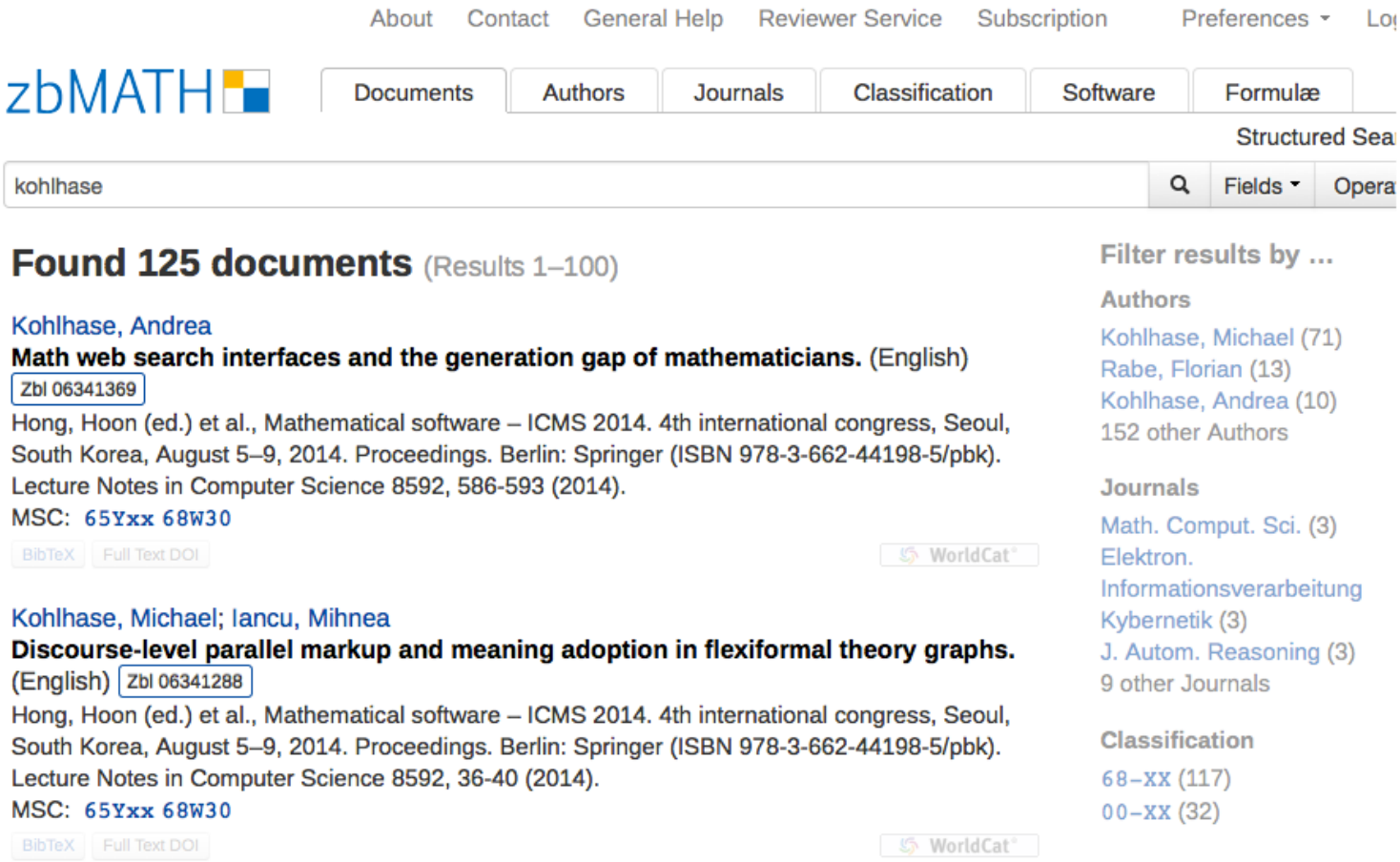

Figure : Faceted Search Example

Formula Facets

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$$\int_M \Phi(d_p f) dvol$$

$$\lambda X.h(H^1 X) \cdots H^n X$$

$$\frac{\Gamma \vdash A \gg \alpha}{D}$$

Figure : Formula Schemata as Formula Facets

Preliminaries

- MathWebSearch (MWS)
- Elasticsearch (ES)

MathWebSearch

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- formulae are inserted in the index according to their DFS traversal
- the index nodes are unique integers corresponding to MathML elements.
- a FormulaID is assigned to each formula.

- Idea: use the index to generate similar schemata

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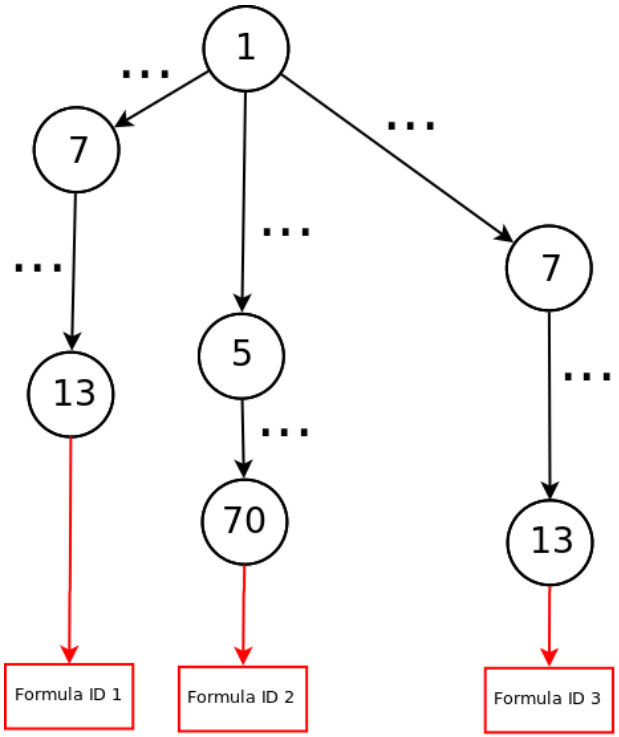

Figure : Simplified Index at depth 1

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- massively scalable & fault tolerant
- provides faceted search features (aggregations)
- We can use it to run aggregations on formulae.

Purpose

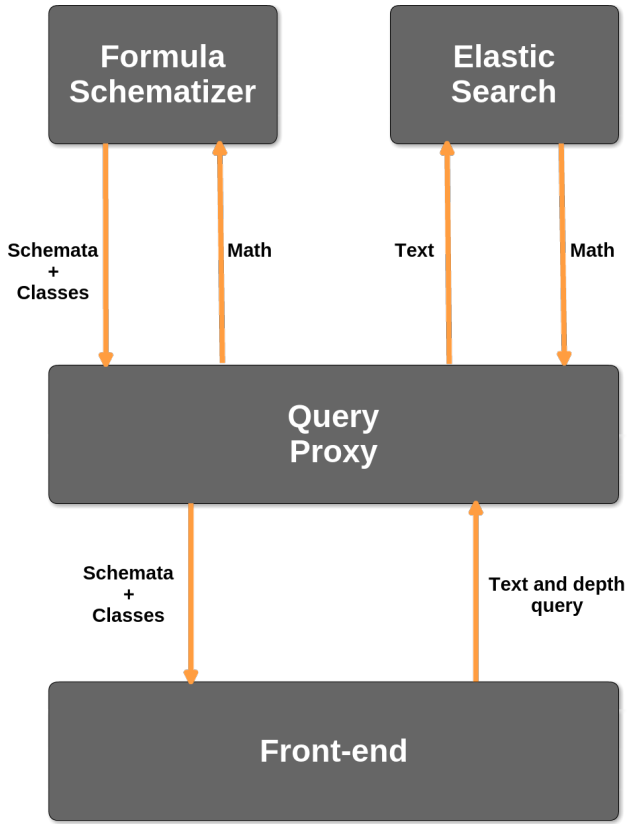

Figure : FS Engine Architecture

Working Principle

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2. Create the table of signatures using a cutoff heuristic.

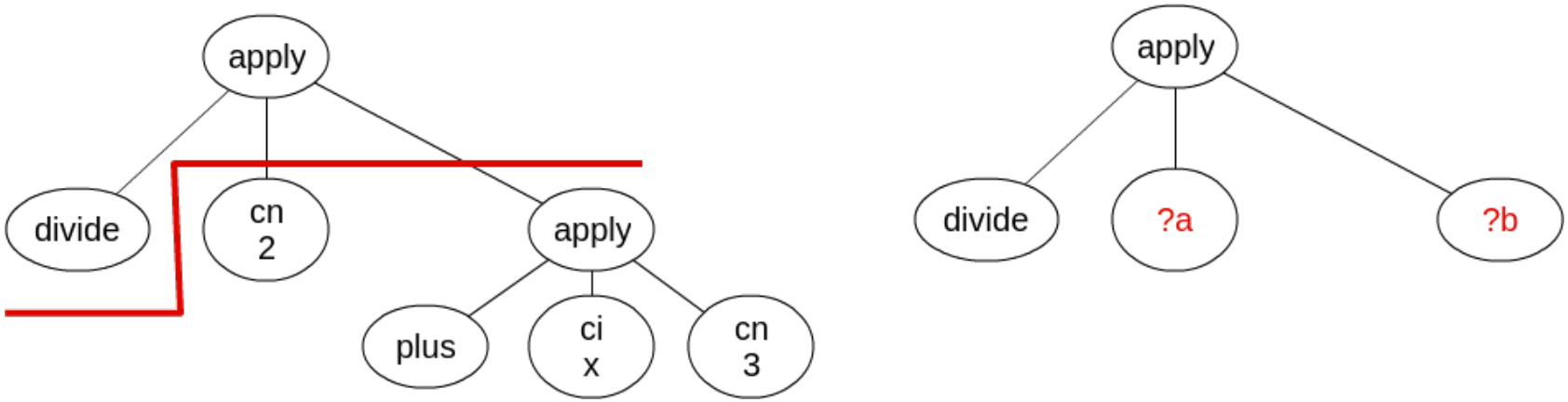

Figure : Dynamic Cutoff

3. Process the table.

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4. Generate Content MathML schemata.

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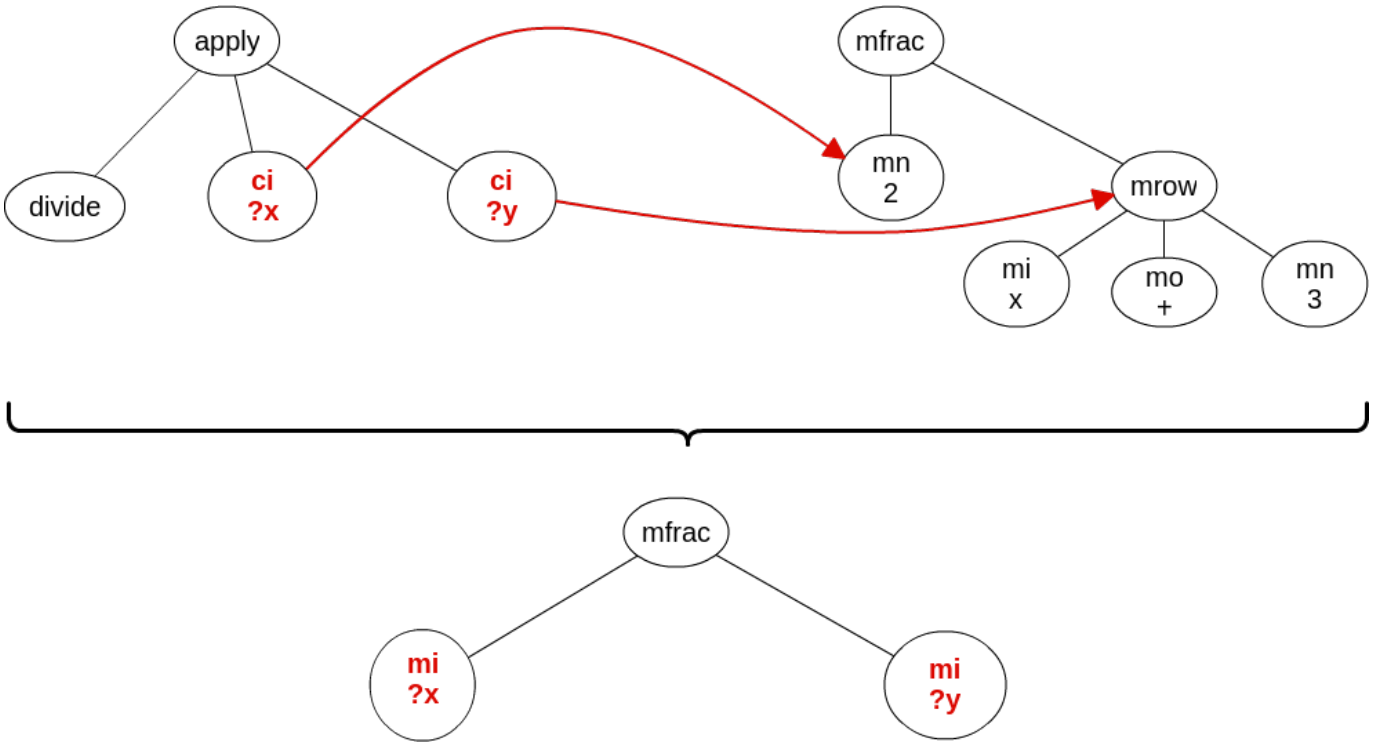
4. Generate Content MathML schemata.
5. Create Presentation MathML schemata (**presentation by replacement**).



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- Text-only search
- Ideal for exploring a corpus
- It returns schemata & formula classes
- Used mainly to showcase to Schematizer

SchemaSearch

The MathWebSearch system (MWS) is a content-based search engine for mathematical formulae. It indexes MathML formulae, using a technique derived from automated theorem proving: Substitution Tree Indexing. MWS performs mathematical full-text search, combining key phrase search with unification-based formula search.

SchemaSearch auguments the power of MathWebSearch by providing faceted search capabilities. A math facet consists of a formula in which qvars replace nodes below a certain depth in its CMML representation.

Search Text | 3 | R | Search

Enter a keyword in the search box to receive a list of formula schemata which cover the math in the documents containing the keyword. Each formula schemata returned is accompanied by a group of formulae which are instantiations of it.

You can also enter a depth for the schemata (how deep the schemata should be) and check the R checkbox if you would like this depth to be relative. If you do not check the box, absolute depth is assumed.

If the depth is relative, its value should be entered in percentages, e.g. for a depth of 50%, 50 should be entered for the relative depth.

Kohlhase | 3 | R | Search

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12 $S = \langle a_1, \ldots, a_\nu \rangle$

12 $\mathfrak{su}(2) + \mathfrak{u}?a^3$

10 $\epsilon_{IJK}\left(?a?b + ?c?d\right),$

9 $\mathfrak{su}(2) \oplus \mathfrak{u}?a^3$

22 $\frac{?a?b}{24} + 1$

$$\frac{712^{7}}{24} + 1$$
$$\frac{712^{6}}{24} + 1$$
$$\frac{912^{9}}{24} + 1$$
$$\frac{912^{8}}{24} + 1$$
$$\frac{1012^{10}}{24} + 1$$
$$\frac{1112^{11}}{24} + 1$$
$$\frac{1212^{12}}{24} + 1$$
$$\frac{1212^{11}}{24} + 1$$
$$\frac{1312^{13}}{24} + 1$$
$$\frac{1312^{12}}{24} + 1$$

6

$$R_{\lambda}^{(1)}(s,x) = \frac{?a}{?b} \times \int ?c.$$

$$R_{\lambda}^{(1)}(s,x) = \frac{e^{ix^2/4s}}{(4\pi i s)^{3/2}\,|x - 2\lambda TG(1)|} \times \int \mathrm{d}y\, e^{-iy\cdot x/2s} \left(e^{iy^2/4s} - 1\right) \left(e^{iZ\int_0^s \frac{\mathrm{d}\tau}{|2\tau p - 2\lambda TG(1)|}}\, e^{-ix\cdot A(T)}\psi_T\right)(y)\,.$$

$$R_{\lambda}^{(2)}(s,x) = \frac{e^{ix^2/4s}}{(4\pi i s)^{3/2}} \int \mathrm{d}y\; e^{-iy\cdot x/2s} \left(e^{iy^2/4s} - 1\right) \times \left(\frac{1}{|2sp - 2\lambda TG(1)|}\, e^{iZ\int_0^s \frac{\mathrm{d}\tau}{|2\tau p - 2\lambda TG(1)|}}\, e^{-ix\cdot A(T)}\psi_T\right)(y)\,.$$

$$R_{\lambda}^{(1)}(s,x) = \frac{e^{ix^2/4s}}{(8\pi^2 i s)^{3/2}\,|x - 2\lambda TG(1)|} \int \mathrm{d}y\, e^{-iy\cdot(x/2s + \lambda F(1))} \left(e^{iy^2/4s} - 1\right) h_{\lambda}(s,y)$$

$$R_{\lambda}^{(1)}(s,x) = \frac{e^{ix^2/4s}}{(8\pi^2 i s)^{3/2}\,|x - 2\lambda TG(1)|} \int \mathrm{d}y\, \frac{\Delta_y^m e^{-iy\cdot(x/2s + \lambda F(1))}}{(-1)^m\,|x/2s + \lambda F(1)|^{2m}} \left(e^{iy^2/4s} - 1\right) h_{\lambda}(s,y)$$

TeMa v2

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- Text and formula search

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- Schemata filter query results

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- Text and formula search
- Schemata filter query results
- Main demo for faceted search

TeMa v2

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Search Text | Examples | Search

Search LaTeX-Style Math

Enter a comma-separated list of key phrases into the top search bar and a set of formulae schemata (written in LaTeX with ?a, ?b, ... for query variables; they are marked in red in the formula preview). A formula schema in a query matches any formula in the MWS index that has an instance schema as a subformula. Query variables with the same name must be instantiated with the same formula, see the examples for inspiration. ... more

TeMa v2

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Fermat | Examples | Search

?a^?n + ?b^?n=?c^?n

$$a^n + b^n = c^n$$

Enter a comma-separated list of key phrases into the top search bar and a set of formulae schemata (written in LaTeX with ?a, ?b, ... for query variables; they are marked in red in the formula preview). ... more

Math Facets

« 1 »

arxiv.org: : *Oration for Andrew Wiles*

arxiv.org: :

arxiv.org: : *Ješmanowicz' conjecture revisited,II*

Math Facets

$$?a = ?b$$

$$?a + ?b = w_2^{?c}$$

$$X_0^{?a} + X_1^{?b} = ?c_{?d}^2 \quad , \quad ?e + ?f = ?g \, , \quad ?h, ?i + ?j = X_{n+2}^{?k}$$

$$?a + ?b = ?c^{?d} = ?e + ?f + ?g$$
$$?a^{?b} + ?c^{?d} = ?e_{?f}^2$$
$$?a^{?b} + ?c^{?d} = (?e?f?g)^2.$$

$$T^n = \{?a \in ?b : ?c = ?d\}$$

$$?a^{?b} + ?c^{?d} = \tilde{?e}^{\,2}.$$

$$?a + ?b = ?c^{?d}..1$$

$$?a^{?b} + ?c^{?d} = \left(\kappa_1^{?e}\right)^2.$$

Future Work

- SimSearch
- NNexus