Faceted Search in Mathematics

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Overview

1. Introduction
2. Preliminaries
   - MathWebSearch
   - Elasticsearch
3. The Formula Schematizer
   - Purpose
   - Working Principle
4. Evaluation of Results
   - SchemaSearch
   - TeMa v2
5. Future Work
6. Discussion

Why math search?

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- Textual search engines cannot index math.

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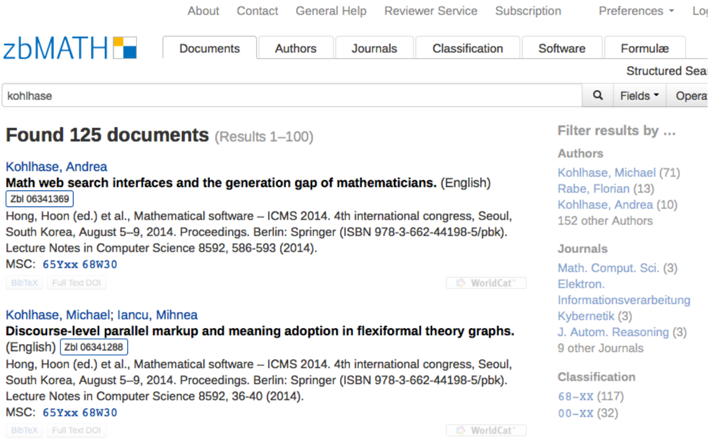
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Figure : Typical Formula

Why faceted search?

- It’s important to allow the user to refine the query.



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MSC: 65Yxx 68W30

Kohlhase, Michael; Iancu, Mihnea
Discourse-level parallel markup and meaning adoption in flexiformal theory graphs. (English) Zbl 06341288
Hong, Hoon (ed.) et al., Mathematical software – ICMS 2014. 4th international congress, Seoul, South Korea, August 5–9, 2014. Proceedings. Berlin: Springer (ISBN 978-3-662-44198-5/pbk). Lecture Notes in Computer Science 8592, 36-40 (2014).
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J. Autom. Reasoning (3)
9 other Journals

Classification
68-XX (117)
00-XX (32)

Figure : Faceted Search Example

Formula Facets

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$$\int_M \Phi(d_p f)dvol$$

$$\lambda X.h(H^1 X)\cdots H^n X$$

$$\frac{\Gamma \vdash A \gg \alpha}{D}$$

Figure : Formula Schemata as Formula Facets

Preliminaries

- MathWebSearch (MWS)
- Elasticsearch (ES)

MathWebSearch

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- formulae are inserted in the index according to their DFS traversal
- the index nodes are unique integers corresponding to MathML elements.
- a FormulaID is assigned to each formula.

Example

- Formula: $\frac{2}{x+3}$

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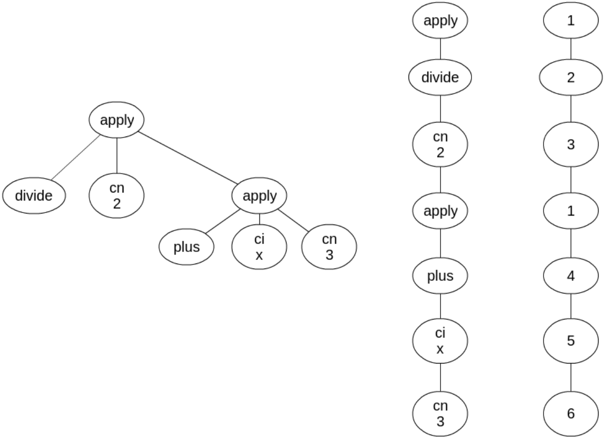

Figure : MWS Index

Elasticsearch

- powerful & efficient text search and analytics engine

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- massively scalable & fault tolerant
- provides faceted search features (aggregations)
- We can use it to run aggregations on formulae.

Purpose

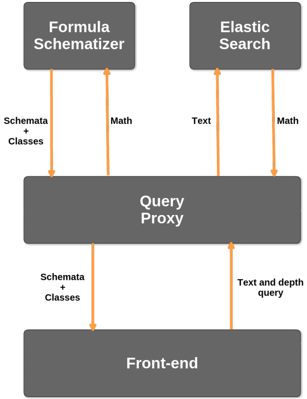

Figure : FS Engine Architecture

Working Principle

- Idea: use the index to generate similar schemata

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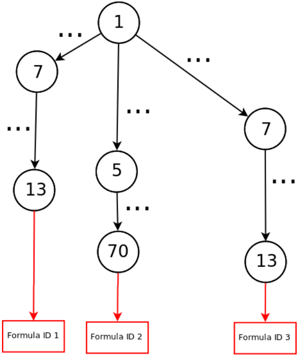

Figure : Simplified Index at depth 1

Working Principle

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1. Obtain MathML representation of formulae set.

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2. Create the table of signatures using a cutoff heuristic.

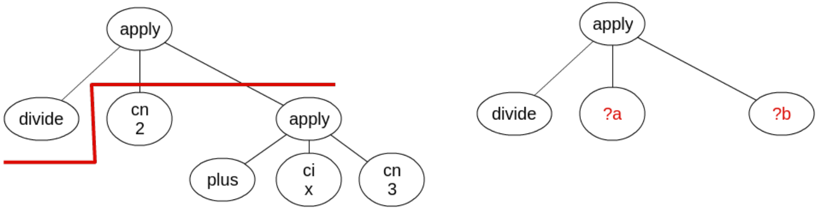

Figure : Dynamic Cutoff

3. Process the table.

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4. Generate Content MathML schemata.

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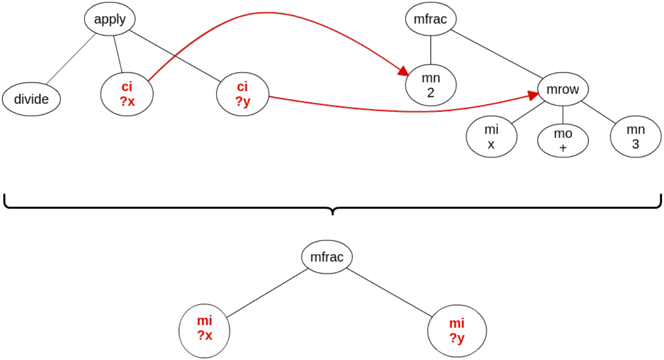
4. Generate Content MathML schemata.
5. Create Presentation MathML schemata (**presentation by replacement**).



SchemaSearch

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- Text-only search

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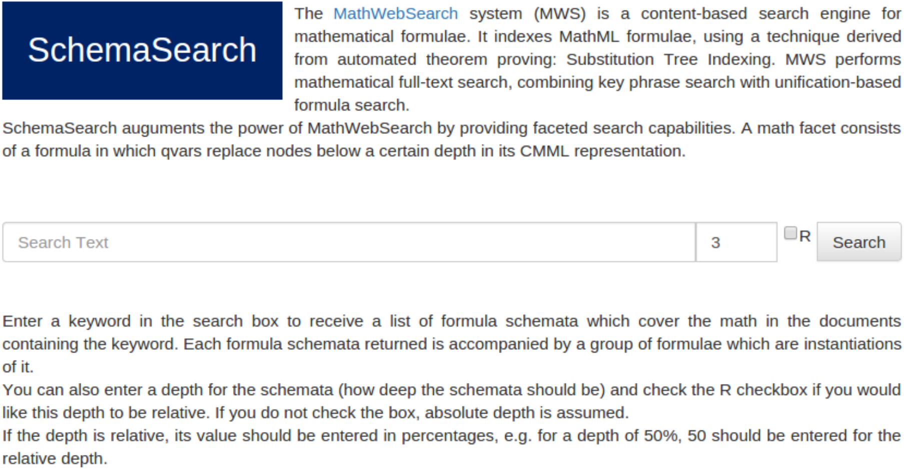
SchemaSearch

- Text-only search
- Ideal for exploring a corpus
- It returns schemata & formula classes
- Used mainly to showcase to Schematizer



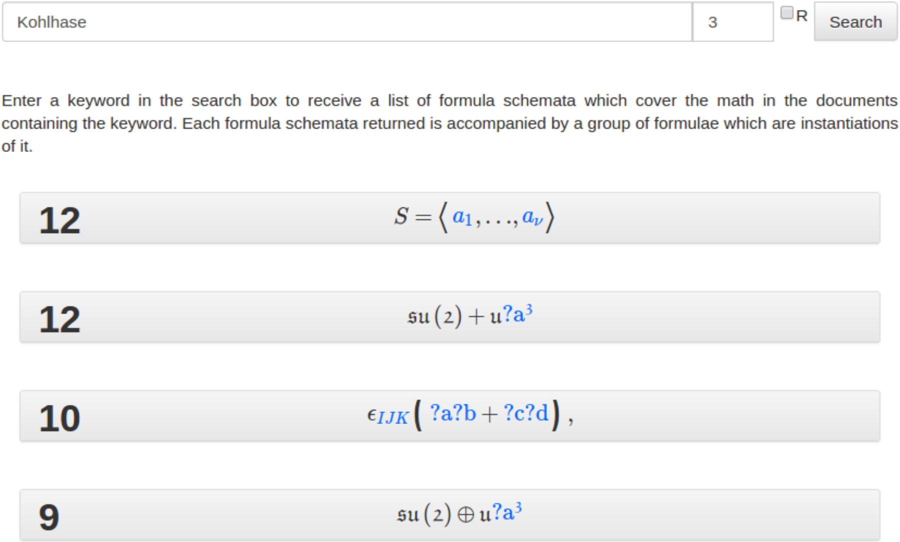
The MathWebSearch system (MWS) is a content-based search engine for mathematical formulae. It indexes MathML formulae, using a technique derived from automated theorem proving: Substitution Tree Indexing. MWS performs mathematical full-text search, combining key phrase search with unification-based formula search.

SchemaSearch auguments the power of MathWebSearch by providing faceted search capabilities. A math facet consists of a formula in which qvars replace nodes below a certain depth in its CMML representation.

Enter a keyword in the search box to receive a list of formula schemata which cover the math in the documents containing the keyword. Each formula schemata returned is accompanied by a group of formulae which are instantiations of it.

You can also enter a depth for the schemata (how deep the schemata should be) and check the R checkbox if you would like this depth to be relative. If you do not check the box, absolute depth is assumed.

If the depth is relative, its value should be entered in percentages, e.g. for a depth of 50%, 50 should be entered for the relative depth.



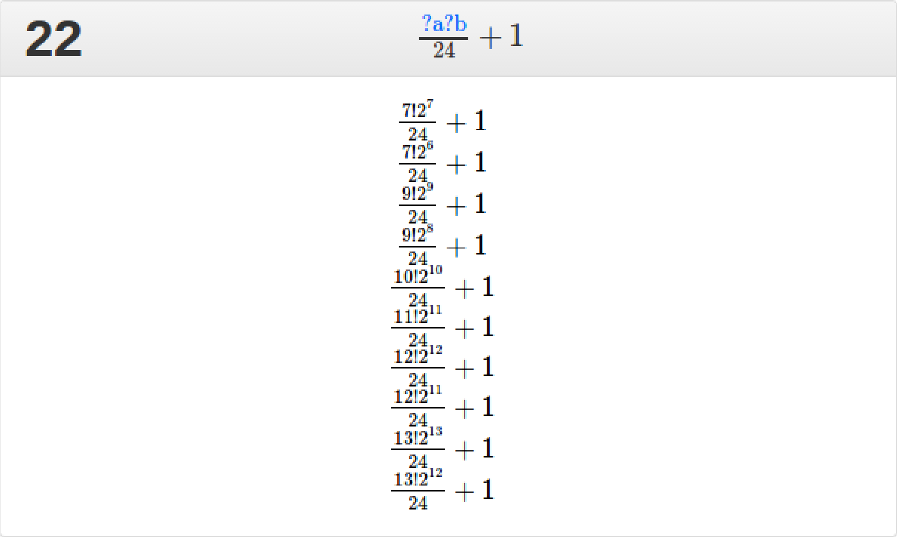
Kohlhase

3

□R

Enter a keyword in the search box to receive a list of formula schemata which cover the math in the documents containing the keyword. Each formula schemata returned is accompanied by a group of formulae which are instantiations of it.

12	$S = \langle a_1, \ldots, a_\nu \rangle$
12	$\mathfrak{su}(2) + \mathfrak{u}?\mathrm{a}^3$
10	$\epsilon_{IJK}\left(?\mathrm{a}?\mathrm{b} + ?\mathrm{c}?\mathrm{d} \right),$
9	$\mathfrak{su}(2) \oplus \mathfrak{u}?\mathrm{a}^3$



22

$$\frac{?a?b}{24}+1$$

$$\frac{7!2^{7}}{24}+1$$

$$\frac{7!2^{6}}{24}+1$$

$$\frac{9!2^{9}}{24}+1$$

$$\frac{9!2^{8}}{24}+1$$

$$\frac{10!2^{10}}{24}+1$$

$$\frac{11!2^{11}}{24}+1$$

$$\frac{12!2^{12}}{24}+1$$

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$$\frac{13!2^{13}}{24}+1$$

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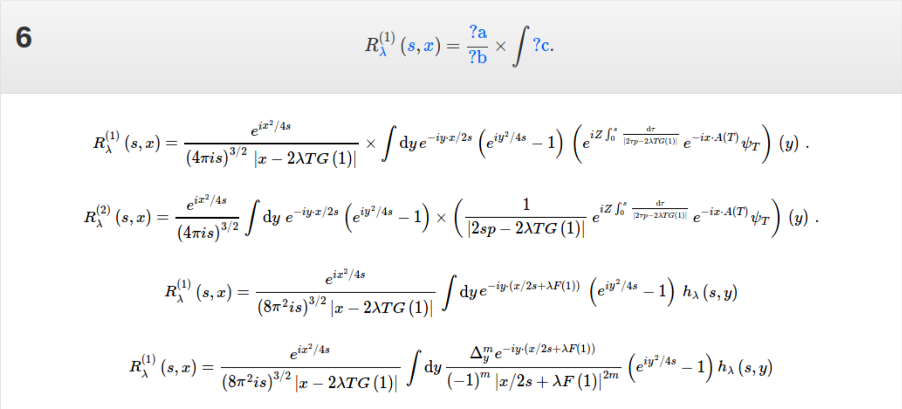

6

$$R_{\lambda}^{(1)}\left(s,x\right)=\frac{?\mathrm{a}}{?\mathrm{b}}\times\int ?\mathrm{c}.$$

$$R_{\lambda}^{(1)}\left(s,x\right)=\frac{e^{ix^{2}/4s}}{\left(4\pi is\right)^{3/2}\left|x-2\lambda TG\left(1\right)\right|}\times\int\mathrm{d}y\,e^{-iy\cdot x/2s}\left(e^{iy^{2}/4s}-1\right)\left(e^{iZ\int_{0}^{s}\frac{\mathrm{d}\tau}{\left|2\tau p-2\lambda TG(1)\right|}}e^{-ix\cdot A(T)}\psi_{T}\right)(y)\;.$$

$$R_{\lambda}^{(2)}\left(s,x\right)=\frac{e^{ix^{2}/4s}}{\left(4\pi is\right)^{3/2}}\int\mathrm{d}y\;e^{-iy\cdot x/2s}\left(e^{iy^{2}/4s}-1\right)\times\left(\frac{1}{\left|2sp-2\lambda TG\left(1\right)\right|}e^{iZ\int_{0}^{s}\frac{\mathrm{d}\tau}{\left|2\tau p-2\lambda TG(1)\right|}}e^{-ix\cdot A(T)}\psi_{T}\right)(y)\;.$$

$$R_{\lambda}^{(1)}\left(s,x\right)=\frac{e^{ix^{2}/4s}}{\left(8\pi^{2}is\right)^{3/2}\left|x-2\lambda TG\left(1\right)\right|}\int\mathrm{d}y\,e^{-iy\cdot\left(x/2s+\lambda F(1)\right)}\left(e^{iy^{2}/4s}-1\right)h_{\lambda}\left(s,y\right)$$

$$R_{\lambda}^{(1)}\left(s,x\right)=\frac{e^{ix^{2}/4s}}{\left(8\pi^{2}is\right)^{3/2}\left|x-2\lambda TG\left(1\right)\right|}\int\mathrm{d}y\,\frac{\Delta_{y}^{m}e^{-iy\cdot\left(x/2s+\lambda F(1)\right)}}{\left(-1\right)^{m}\left|x/2s+\lambda F\left(1\right)\right|^{2m}}\left(e^{iy^{2}/4s}-1\right)h_{\lambda}\left(s,y\right)$$

TeMa v2

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- Text and formula search

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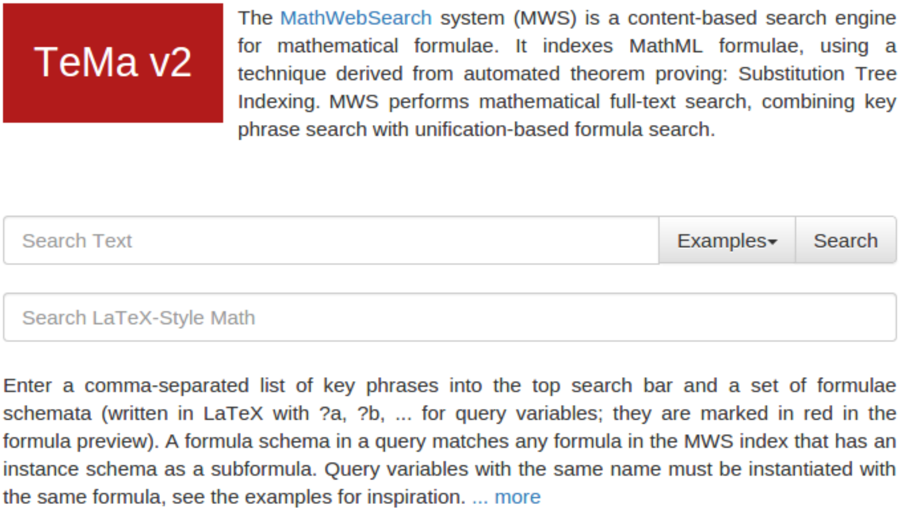
- Schemata filter query results

TeMa v2

- Text and formula search
- Schemata filter query results
- Main demo for faceted search



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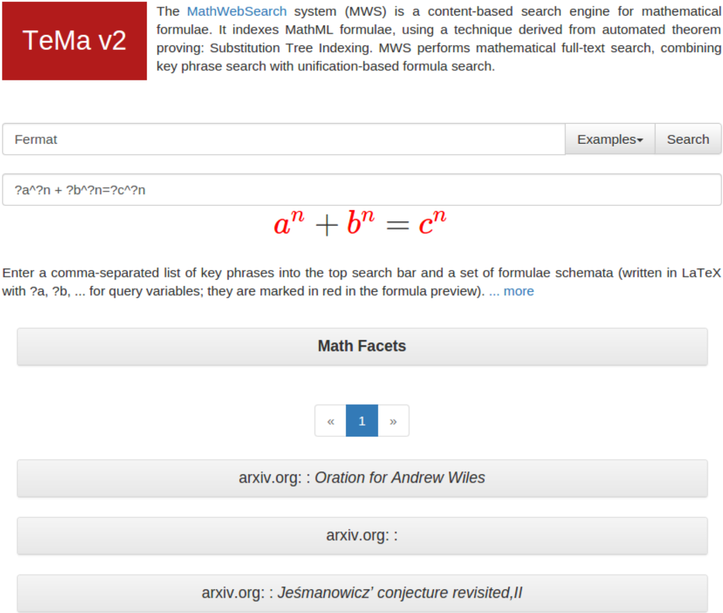
Enter a comma-separated list of key phrases into the top search bar and a set of formulae schemata (written in LaTeX with ?a, ?b, ... for query variables; they are marked in red in the formula preview). A formula schema in a query matches any formula in the MWS index that has an instance schema as a subformula. Query variables with the same name must be instantiated with the same formula, see the examples for inspiration. ... more



TeMa v2

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Fermat

?a^?n + ?b^?n=?c^?n

$$a^n + b^n = c^n$$

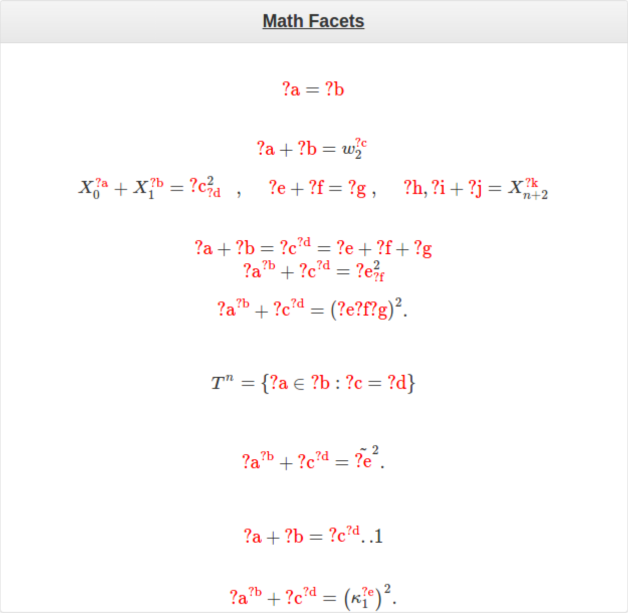
Enter a comma-separated list of key phrases into the top search bar and a set of formulae schemata (written in LaTeX with ?a, ?b, ... for query variables; they are marked in red in the formula preview). ... more

Math Facets

arxiv.org: : *Oration for Andrew Wiles*

arxiv.org: :

arxiv.org: : *Jeśmanowicz' conjecture revisited,II*



Math Facets

$$\mathrm{?a} = \mathrm{?b}$$

$$\mathrm{?a} + \mathrm{?b} = w_2^{\mathrm{?c}}$$

$$X_0^{\mathrm{?a}} + X_1^{\mathrm{?b}} = \mathrm{?c}_{\mathrm{?d}}^2 \quad , \quad \mathrm{?e} + \mathrm{?f} = \mathrm{?g}\ , \quad \mathrm{?h}, \mathrm{?i} + \mathrm{?j} = X_{n+2}^{\mathrm{?k}}$$

$$\begin{gathered} \mathrm{?a} + \mathrm{?b} = \mathrm{?c}^{\mathrm{?d}} = \mathrm{?e} + \mathrm{?f} + \mathrm{?g} \\ \mathrm{?a}^{\mathrm{?b}} + \mathrm{?c}^{\mathrm{?d}} = \mathrm{?e}_{\mathrm{?f}}^2 \end{gathered}$$

$$\mathrm{?a}^{\mathrm{?b}} + \mathrm{?c}^{\mathrm{?d}} = (\mathrm{?e?f?g})^2.$$

$$T^n = \{\mathrm{?a} \in \mathrm{?b} : \mathrm{?c} = \mathrm{?d}\}$$

$$\mathrm{?a}^{\mathrm{?b}} + \mathrm{?c}^{\mathrm{?d}} = \tilde{\mathrm{?e}}^2.$$

$$\mathrm{?a} + \mathrm{?b} = \mathrm{?c}^{\mathrm{?d}}..1$$

$$\mathrm{?a}^{\mathrm{?b}} + \mathrm{?c}^{\mathrm{?d}} = \left(\kappa_1^{\mathrm{?e}}\right)^2.$$

Future Work

- Similarity Search
- Improving NNexus

Demos

- **SchemaSearch**: http://jupiter.eecs.jacobs-university.de/schema
- **TeMa v2**: http://jupiter.eecs.jacobs-university.de/temaV2

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