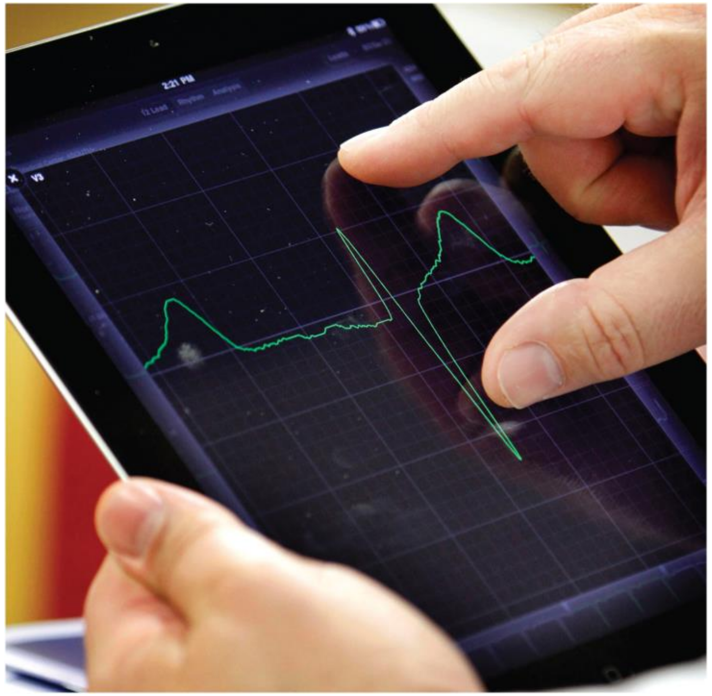
ELECTRIC POTENTIAL

Chapter 23



Alessandro Bombelli

Operations & Environment

Faculty of Aerospace Engineering

Structure of the lecture

1. Electric Potential Energy and Potential Difference
2. Relation between Electric Potential and Field
3. Electric Potential due to Point Charges
4. Electric Potential due to Charge Distributions
5. Equipotential Lines and Surfaces
6. Electric Field from Electric Potential

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Learning objectives for today's lecture

After this lecture you should be able to:

- Understand the **relationship** between electric field and electric potential

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- Apply such a relationship to **compute the electric potential** due to a point charge or a continuous charge distribution

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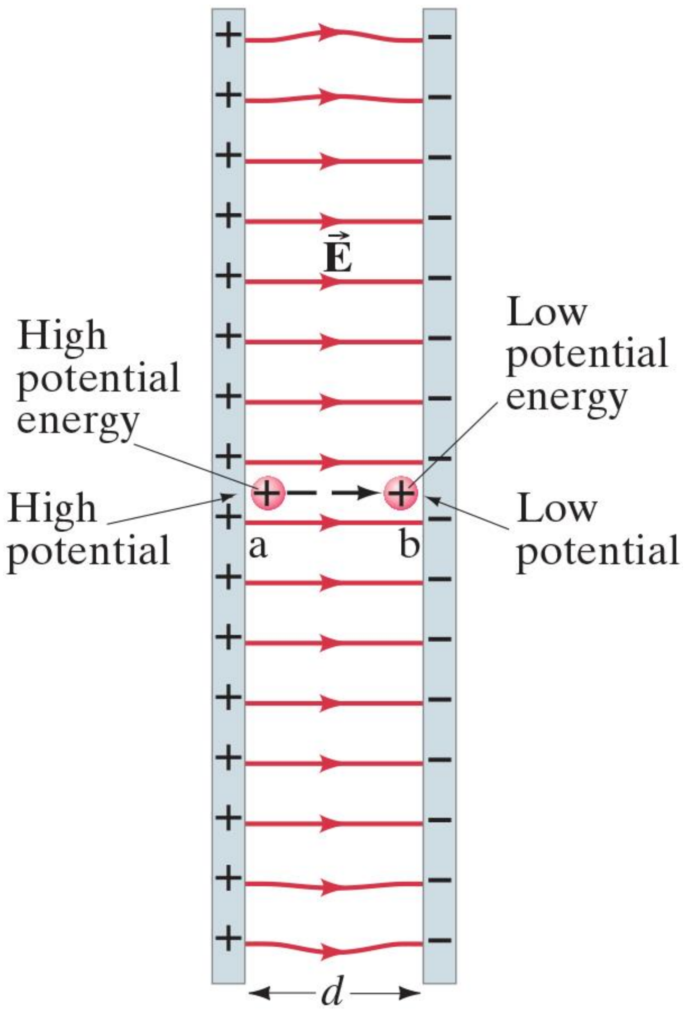
After this lecture you should be able to:

- Understand the **relationship** between electric field and electric potential
- Apply such a relationship to **compute the electric potential** due to a point charge or a continuous charge distribution
- Apply the **inverse relationship** to determine the electric field given a known electric potential

23.1 – Electric Potential Energy and Difference



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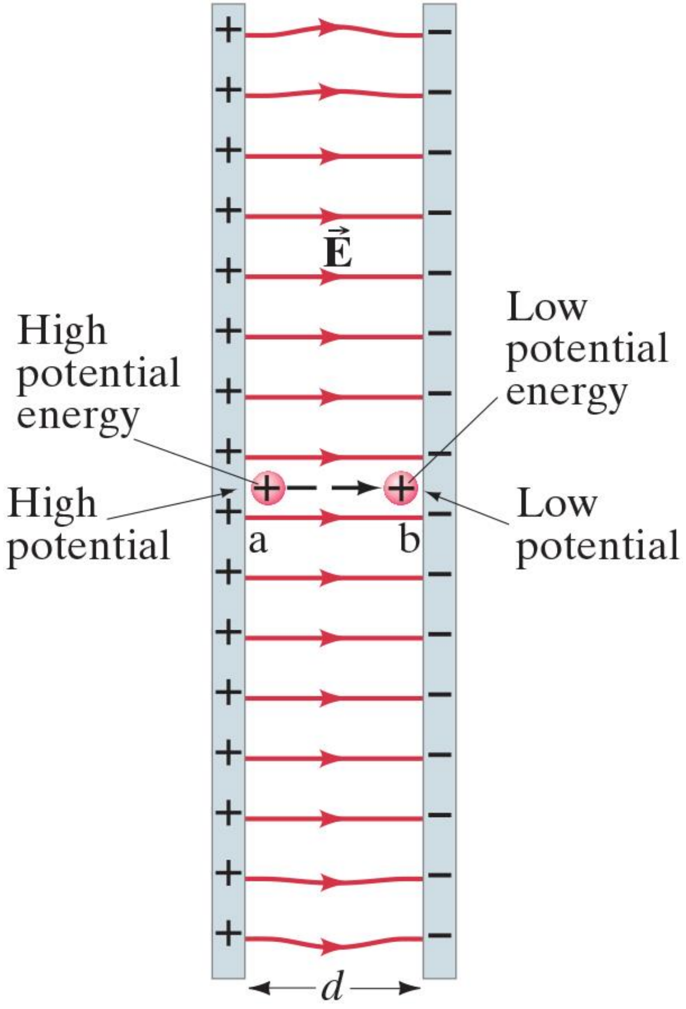

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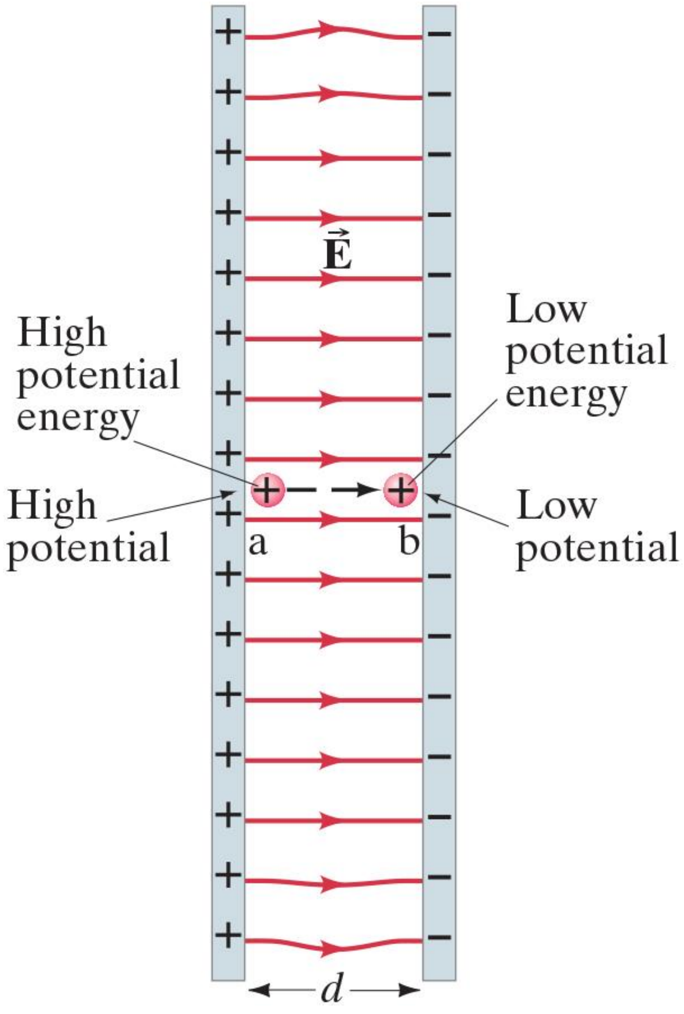

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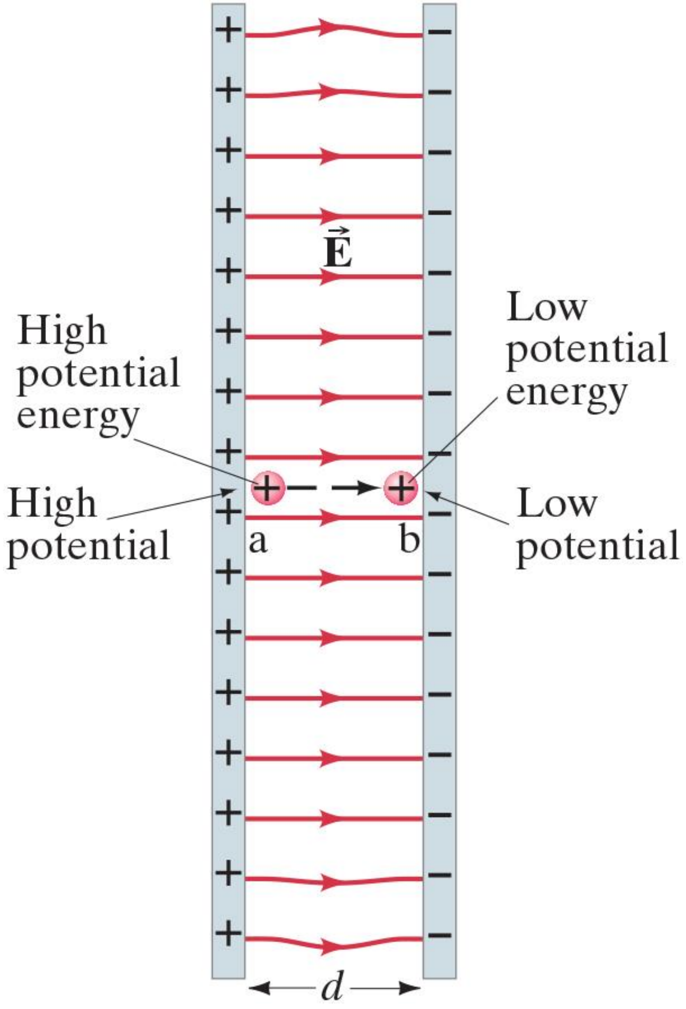

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Finally, the change in (any) potential energy is equal to the negative of the work done by the conservative force on the object being moved: $\boldsymbol{\Delta U = -W}$.

We can define the change in electric potential energy as $\boldsymbol{U_b - U_a}$, where a point charge q is being moved from a to b.

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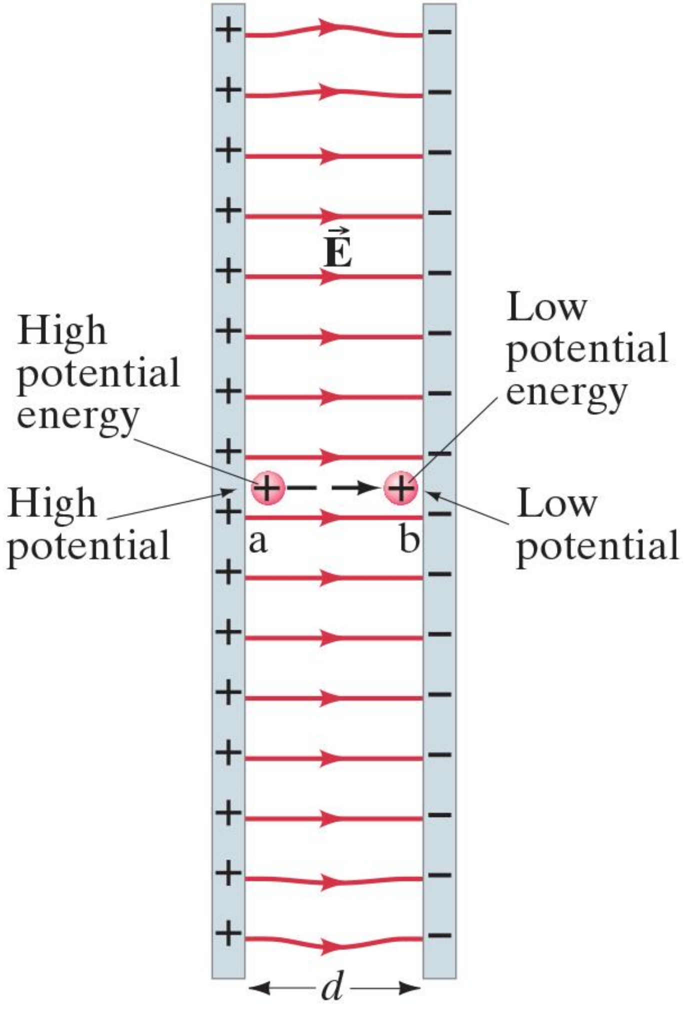

In the example to the left, if we are far enough from the edges of the capacitor, the electric field is constant. If we "release" a charge q from point a, it will be accelerated by the electric field towards the negative plate and towards point b.

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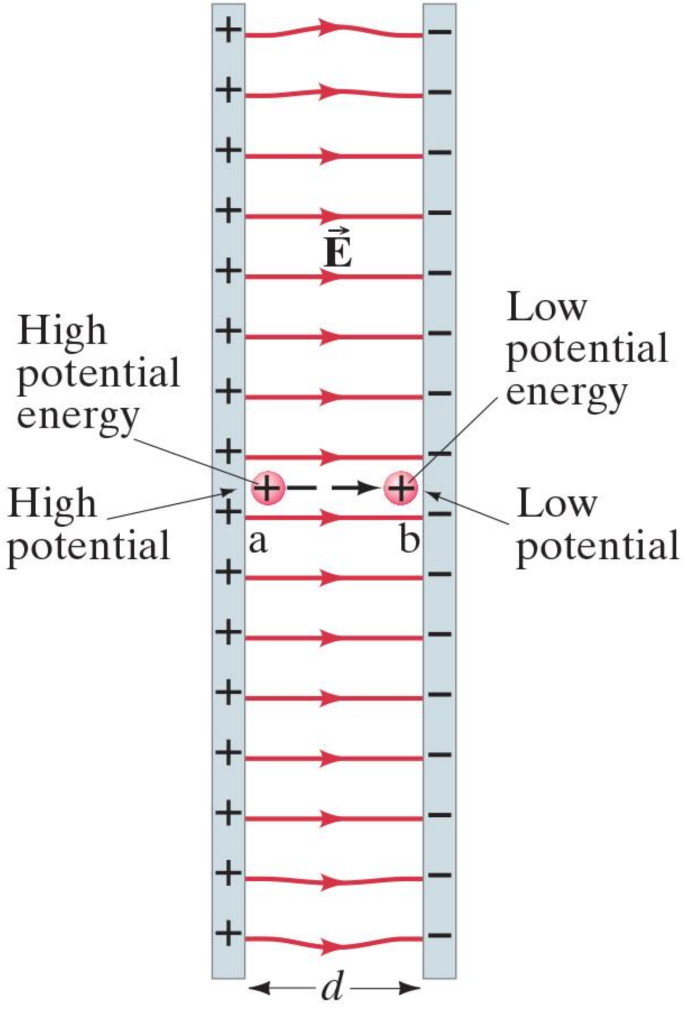

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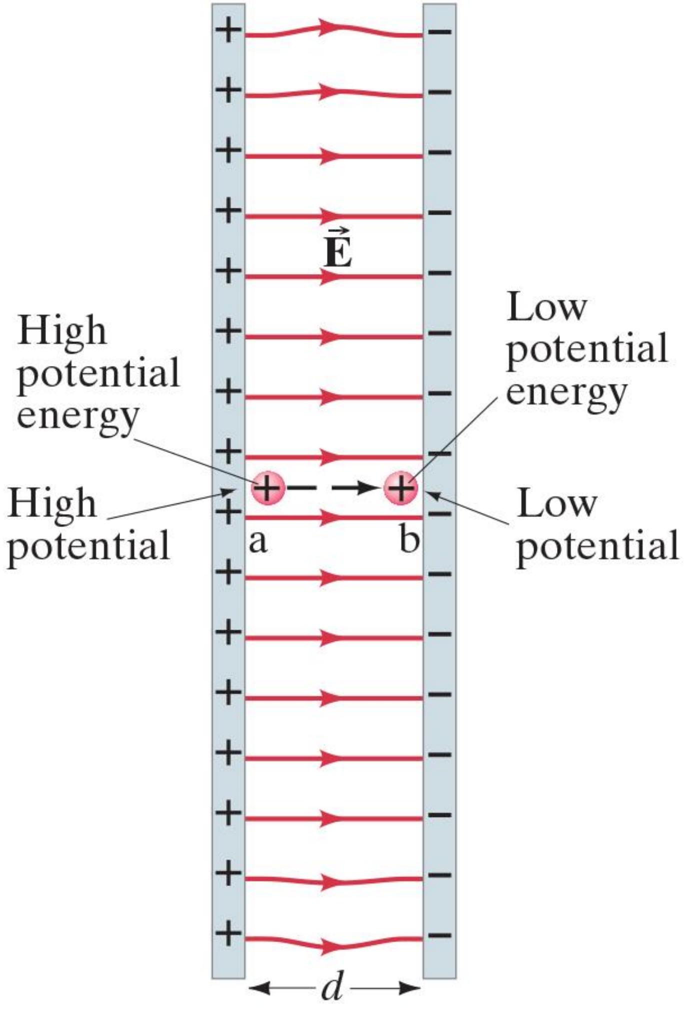

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The work done by the electric field is

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and the change in potential energy can be expressed as

$$\boldsymbol{U_b - U_a = -W = -qEd}$$

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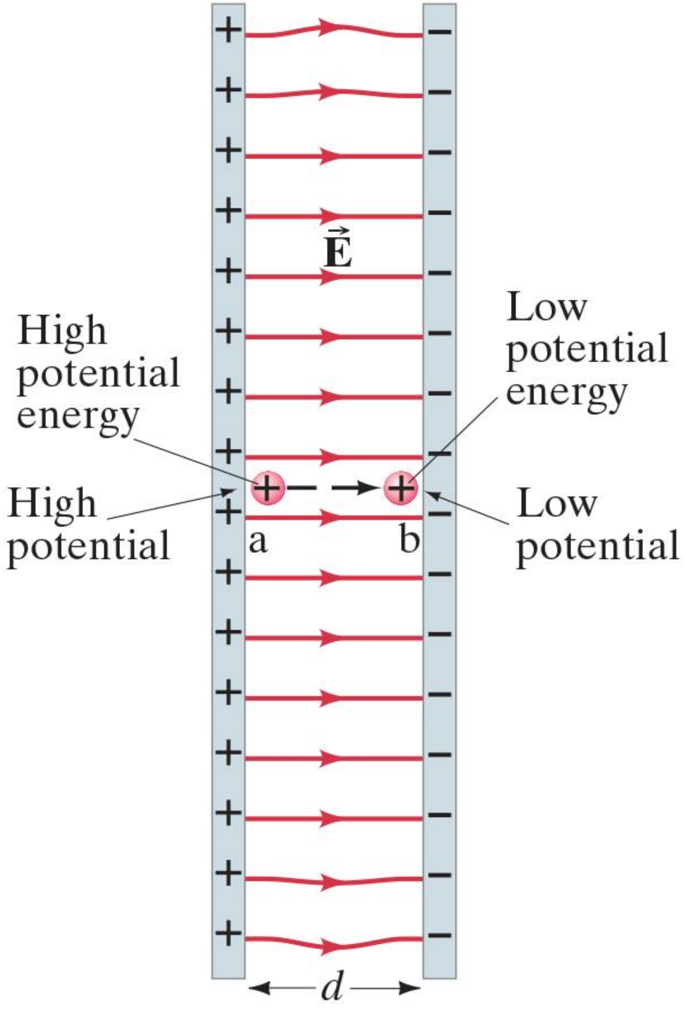

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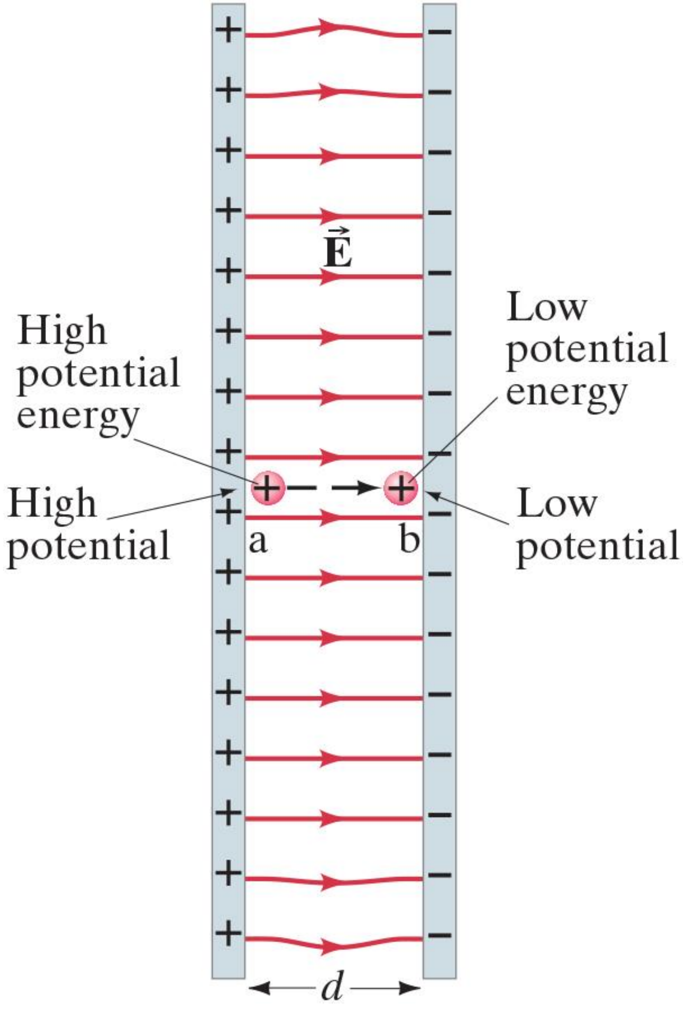

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The difference in potential energy is equal to the negative of the work done by the electric field to move a charge from its initial to its final location (and potential)

$$V_{ba} = \Delta V = V_b - V_a = \frac{U_b - U_a}{q} = -\frac{W_{ba}}{q}$$

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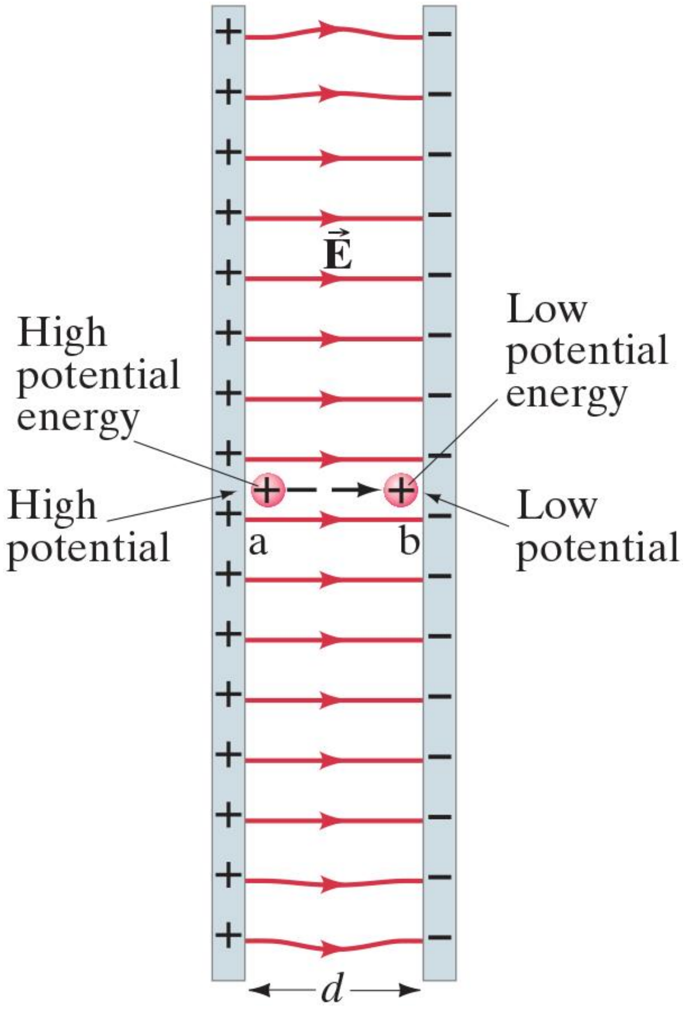

The electric potential, like the electric field, does not depend on the test charge q, but on the other charges that created the field.

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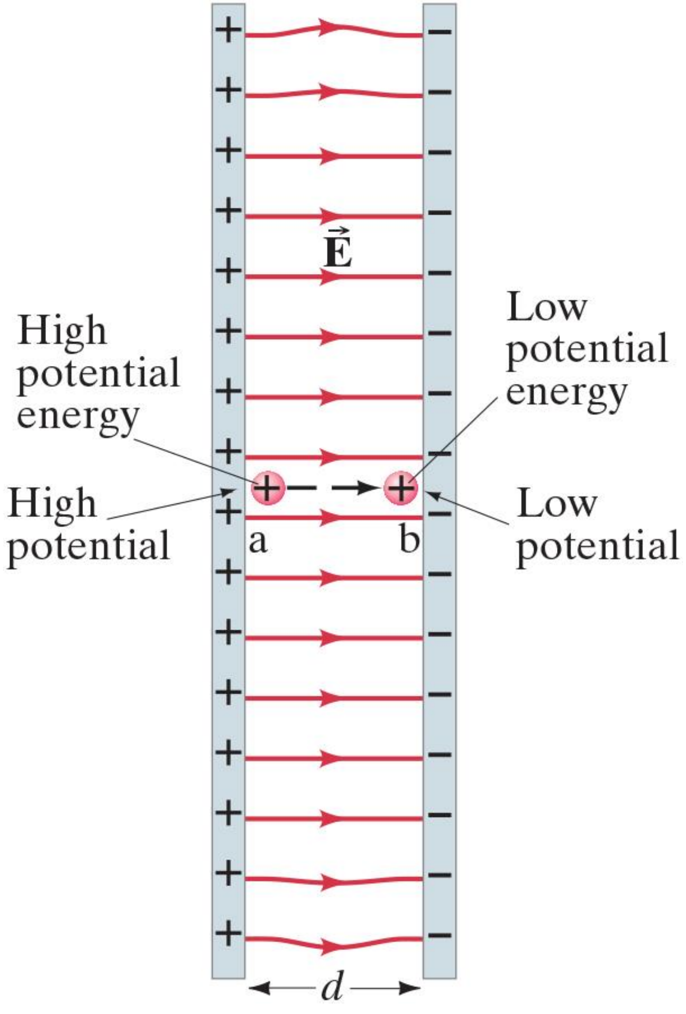

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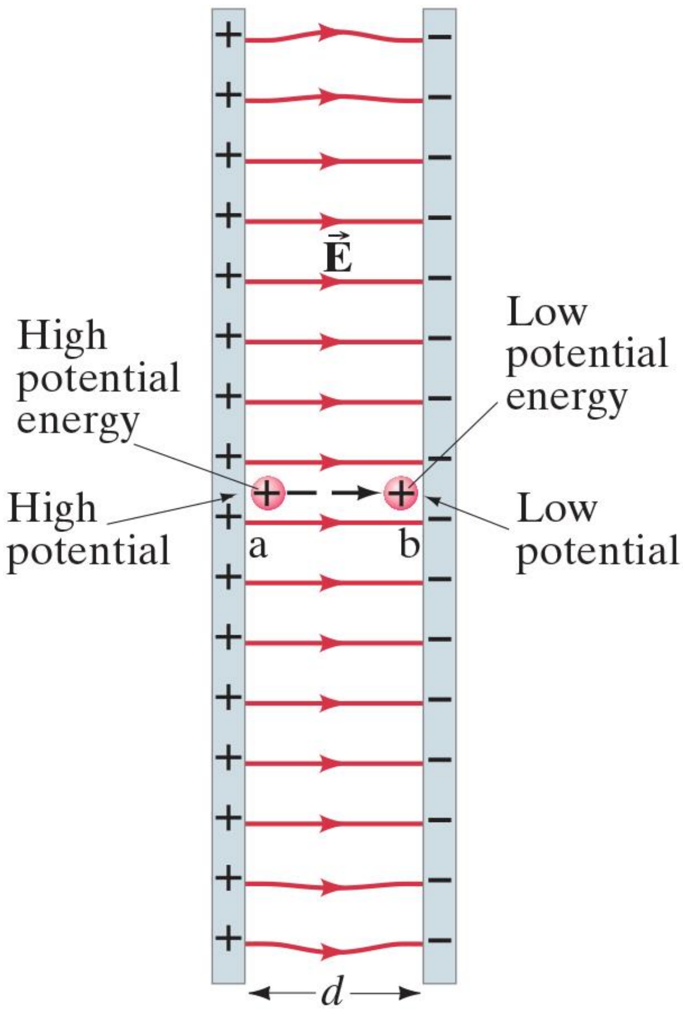

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Similarly to the gravitational potential, electric potential is always relative to a reference point, as only differences in potential can be computed. In general, either the ground or a point at ∞ distance are set to be at zero potential.

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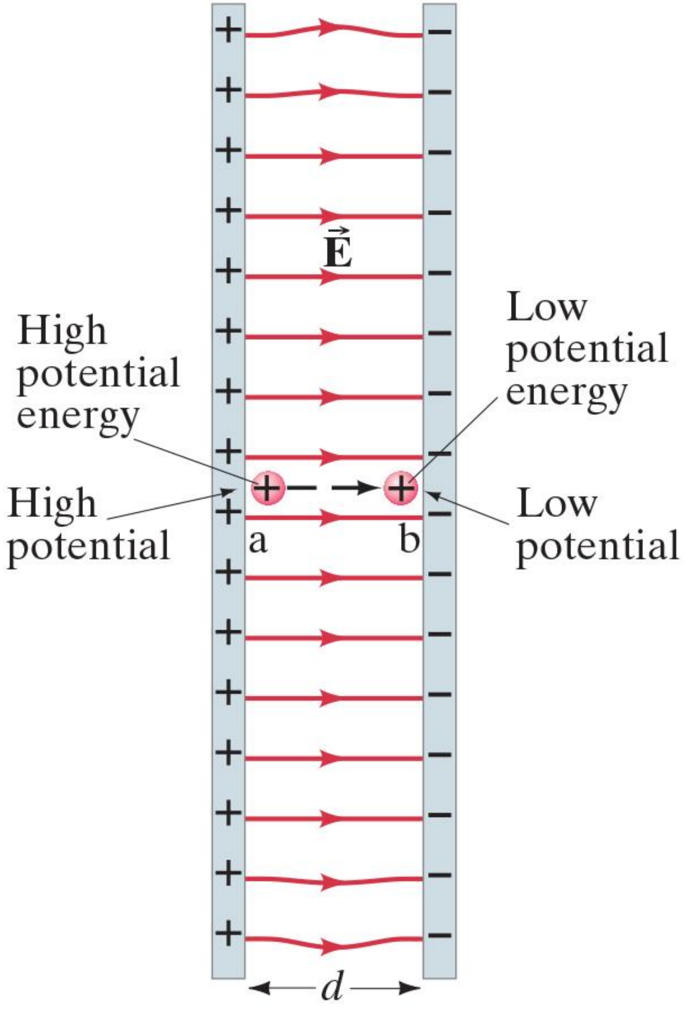

Because the potential energy is defined as the potential energy per unit charge, then the change in potential energy of a charge q when moving from point a to point b is

$$\boldsymbol{\Delta U = U_b - U_a = q(V_b - V_a) = qV_{ba}}$$

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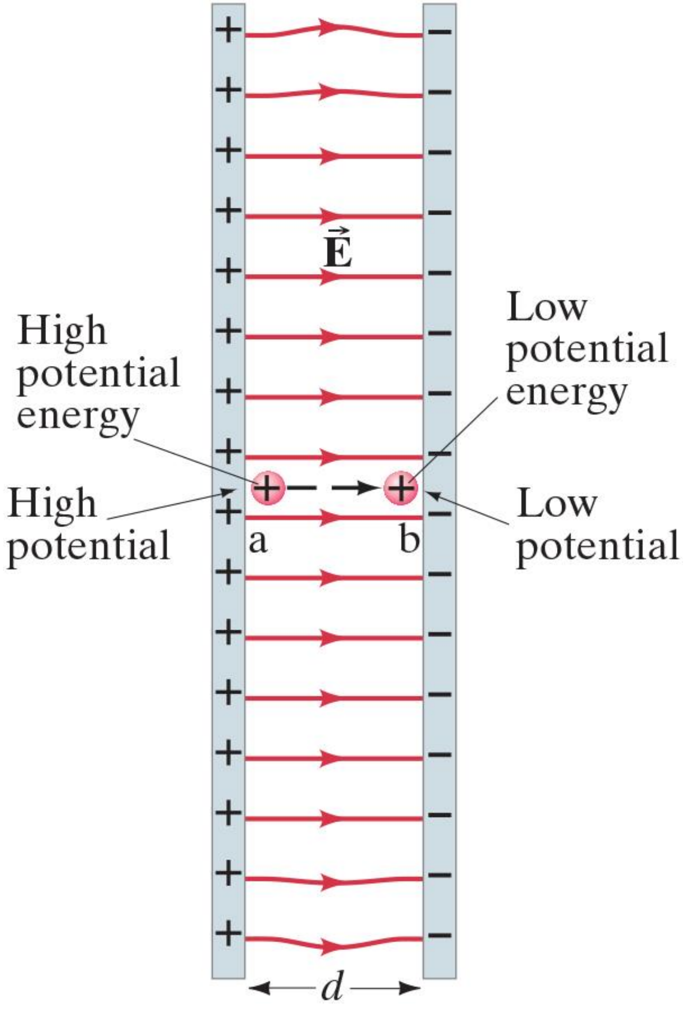

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If the potential difference between the two plates to the left is $6\,V$, then a $+1\,C$ charge moved from b to a acquires $6\,J$ of electric potential energy. Note that we need to provide an equivalent amount of work to move the charge.

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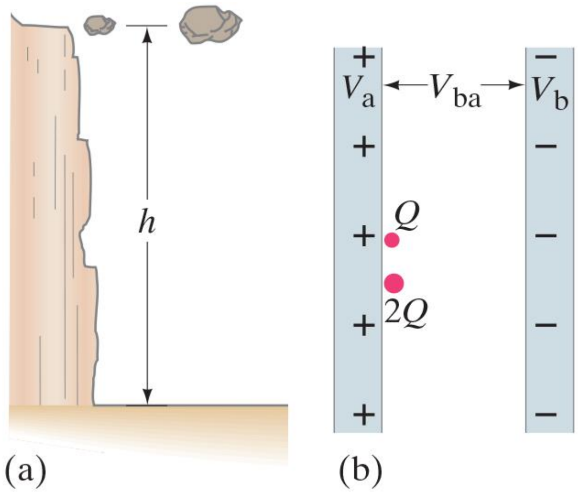

Similarly to the gravitational case, two different charges can have the same electric potential if in the same position (akin to two different masses at the same height), but the potential energy will be different as it is proportional to the actual charge (resp. mass).

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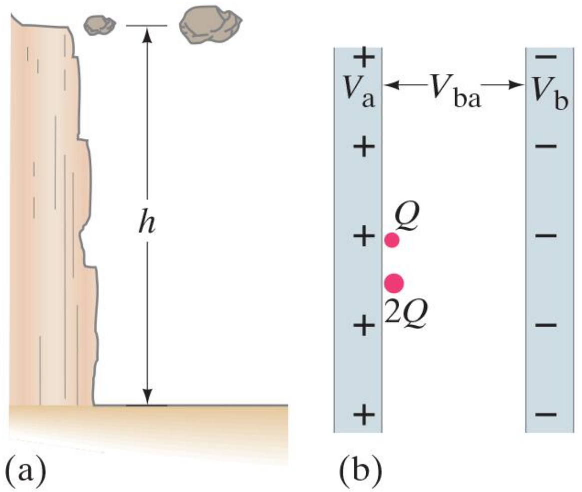

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A (not so) subtle differences between the gravitational and electric potentials is that in the former mass can only be positive, whereas in the second one charges come with a sign.

23.2 – Electric Potential Energy and Field

Given a conservative force $\vec{F}$ and the potential energy U associated with that force, we can write

$$\boldsymbol{U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{l}}$$

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For the electric case, we are generally interested in the potential difference, hence we can divide both sides by q

$$\boldsymbol{V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}}$$

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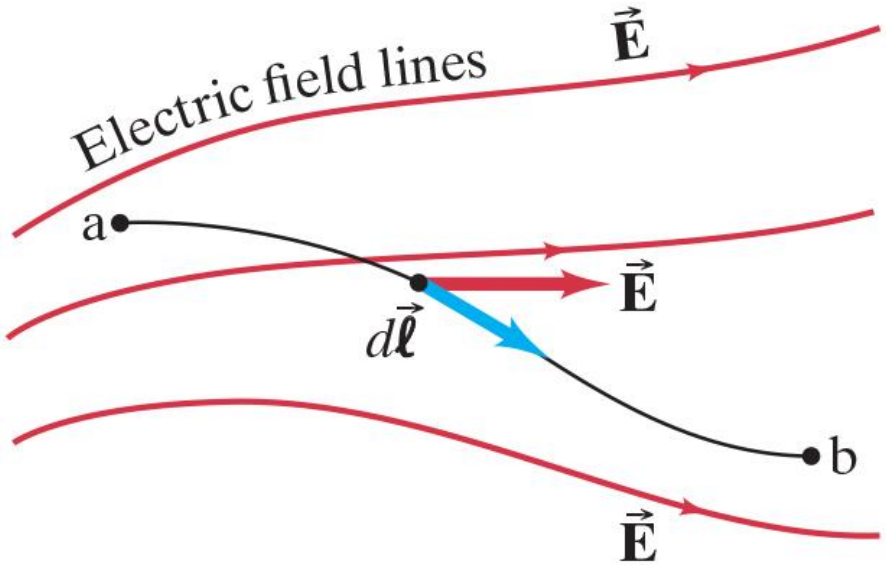

In a generic non-uniform field, we need to compute the aforementioned line integral either analytically (if an analytical form exists) or numerically.

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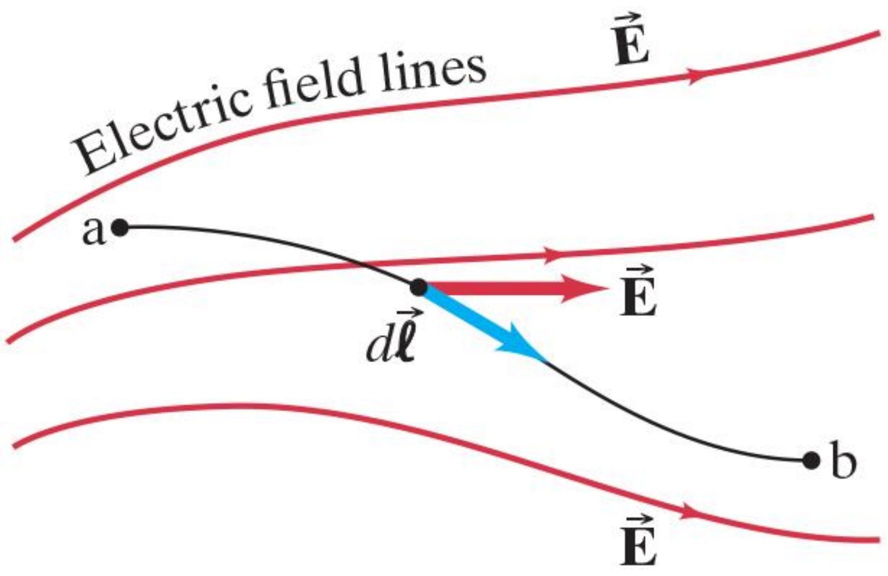

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In the case of a capacitor, for example, where $\vec{E}$ is constant, the potential difference between the plates can be expressed as

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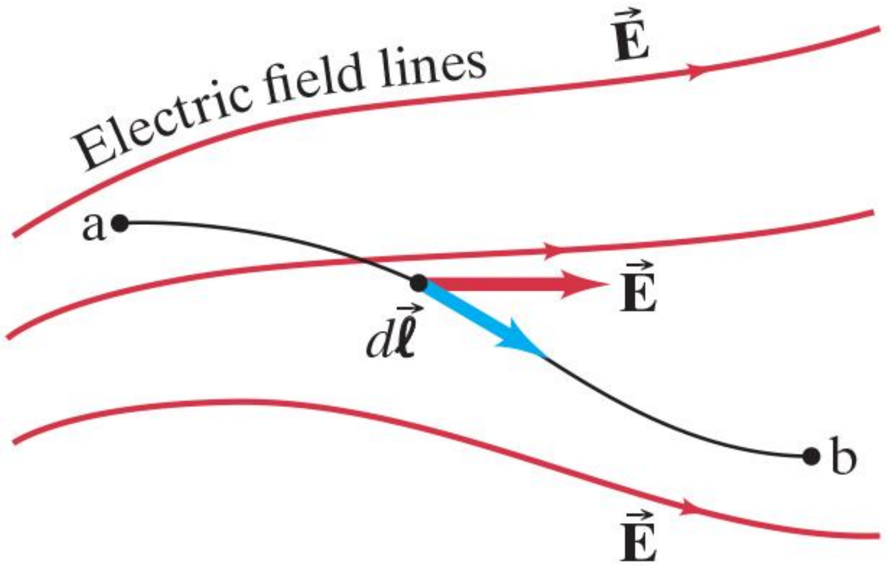

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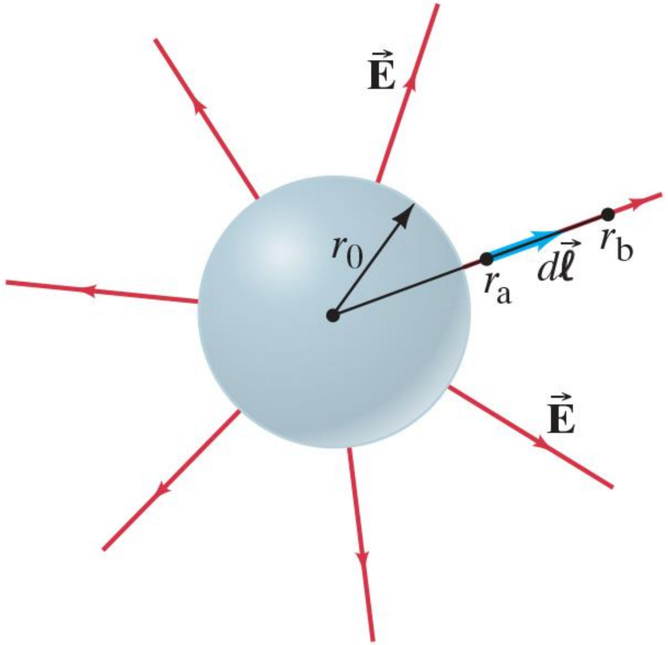
with d being the distance between the plates

23.2 – Electric Potential of charged conducting sphere



For such a sphere, we have that

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

for $r \geq r_0$ as all the charge lies on the surface of the sphere.

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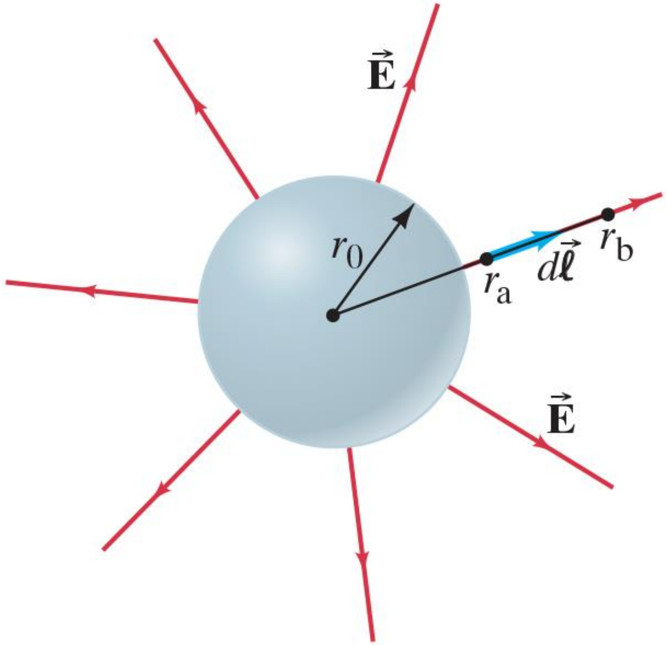

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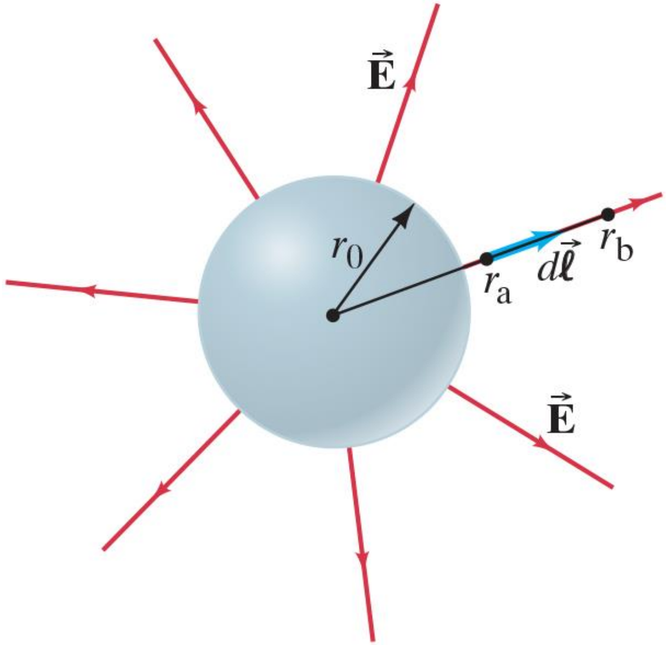

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If we let $V_b = 0$ for $r_b \to \infty$, then

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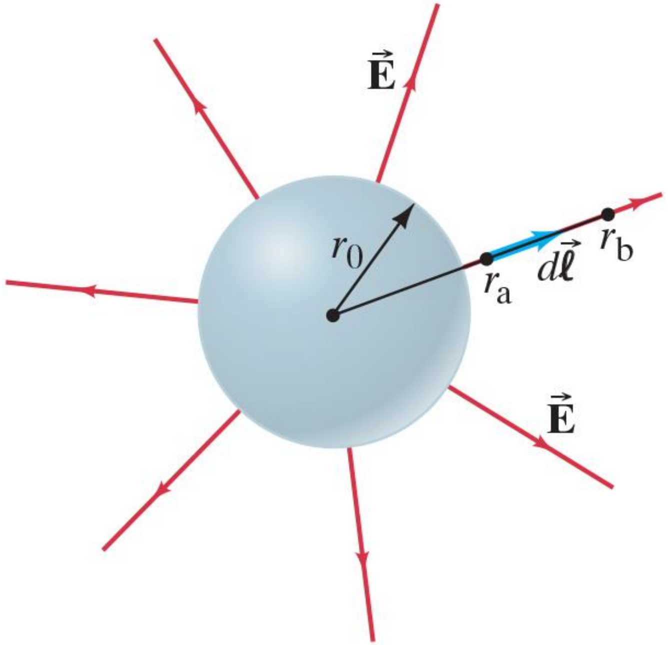

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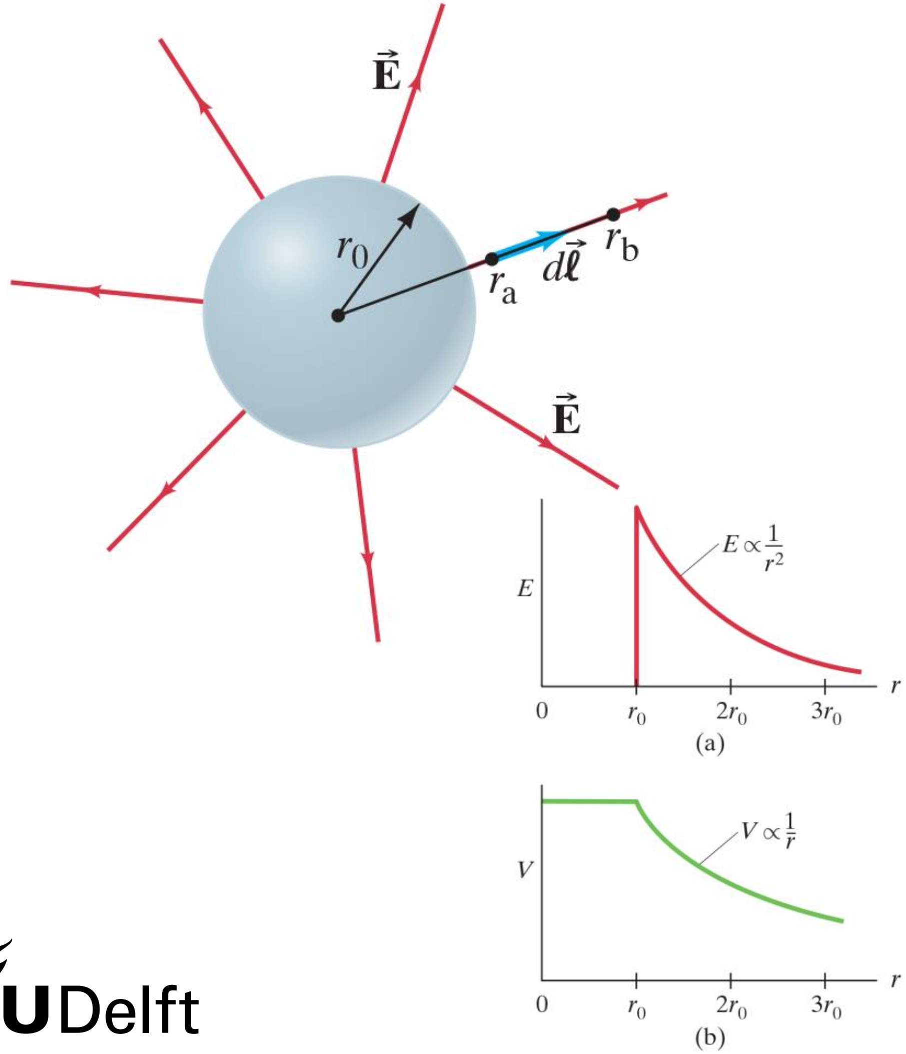

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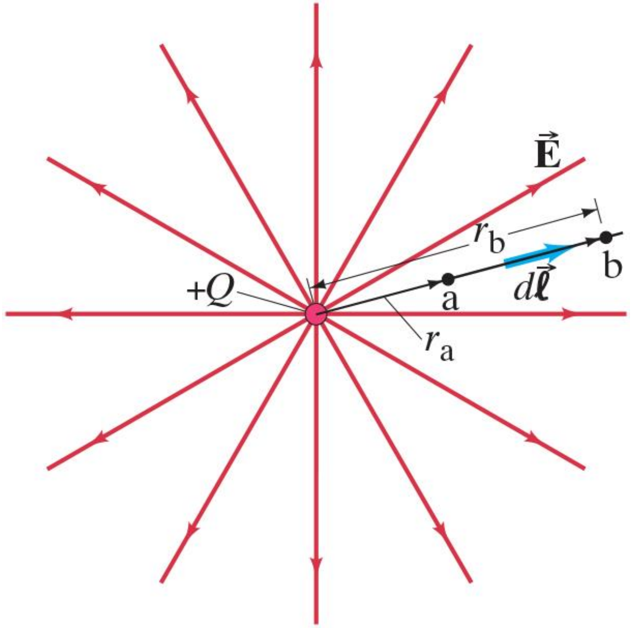
Inside the sphere there is no electric field, hence the integral $\int \vec{E} \cdot d\vec{l}$ is zero and hence there is no potential difference as we move from the center of the sphere to its surface. The potential is constant inside the sphere and equal to the potential of the surface.

23.3 – Electric Potential due to point charges



The electric potential of a point charge is very similar to the previous case (as a matter of fact, it is the previous case where $r_o \to 0$)

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$$

$$= -\frac{Q}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

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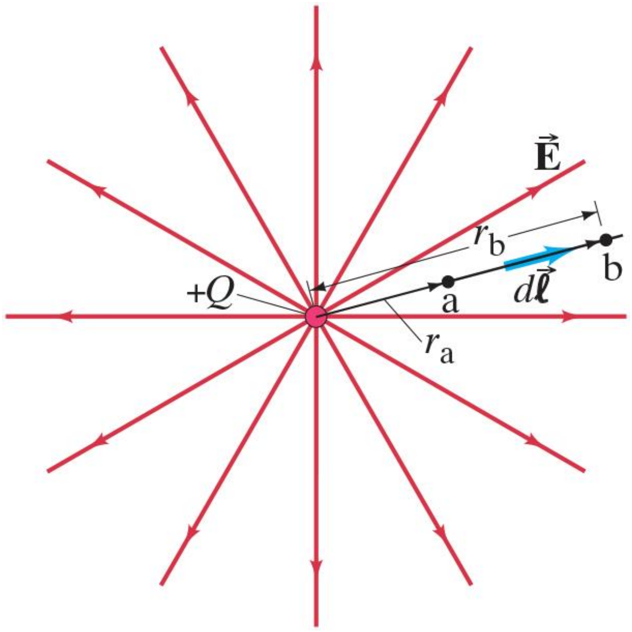

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and letting again $V_b = 0$ for $r_b \to \infty$, then

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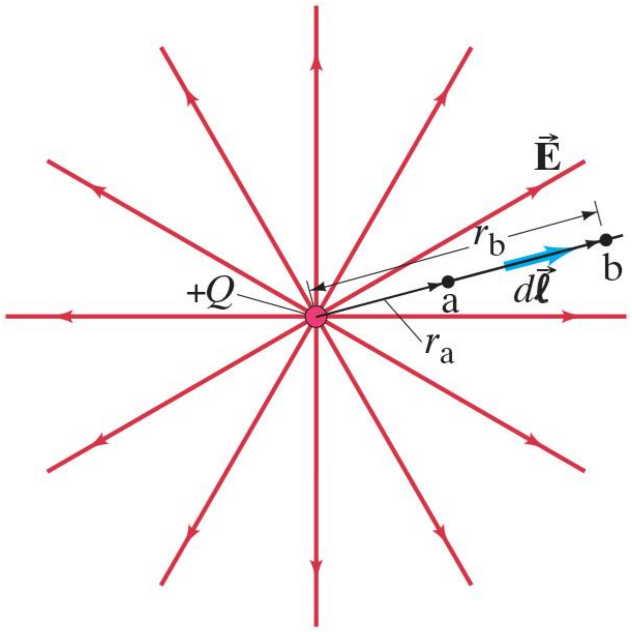

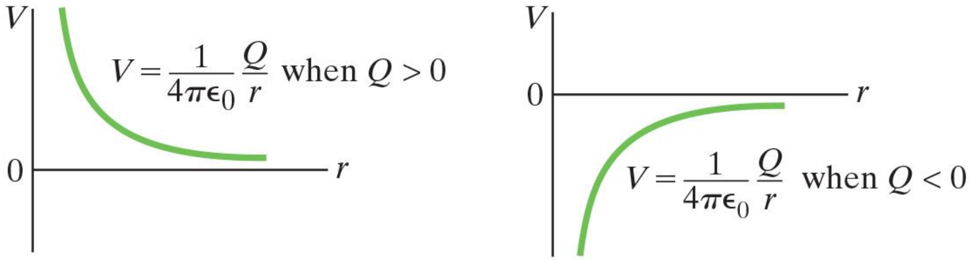

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$$V(r) = \frac{Q}{4\pi\epsilon_0}\frac{1}{r}$$

The potential decreases towards zero as we move away from a positive charge and increases towards zero as we move away from a negative charge

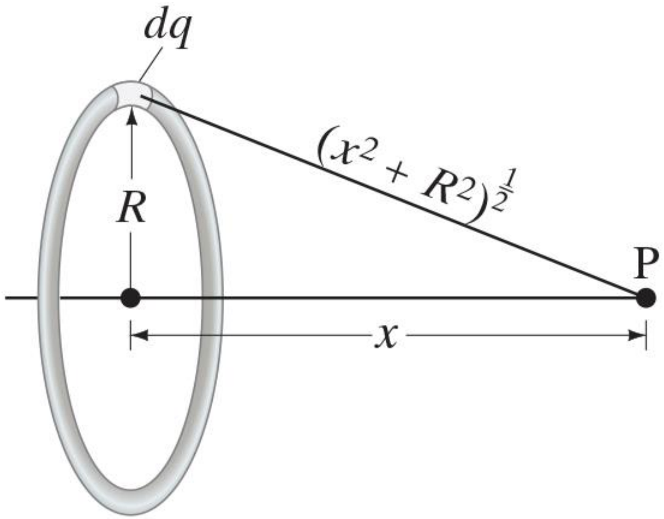
23.4 – Electric Potential due to any charge distribution

Given a generic charge distribution, we can consider the contribution to the electric potential of each infinitesimal portion of charge dq and sum over all contributions. If the distribution is continuous, then the summation becomes an integral

$$\boldsymbol{V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}}$$

where we again assumed that the potential goes to zero as we move towards infinity

23.4 – Electric Potential of ring of charge



We want to determine the potential at a point P at a distance x from the center of a ring of radius R that carries a uniformly distributed charge (overall charge is Q).

$$\boldsymbol{V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}}$$

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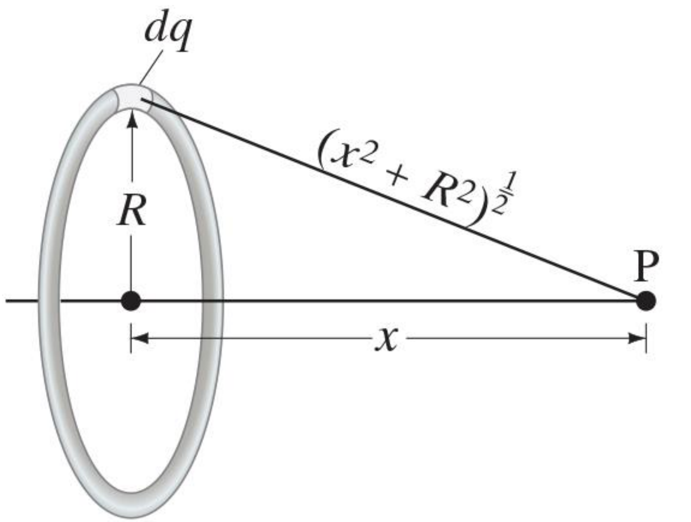

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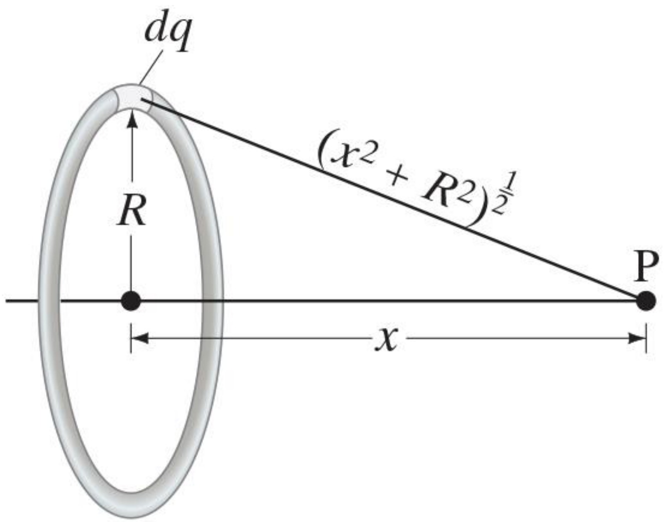

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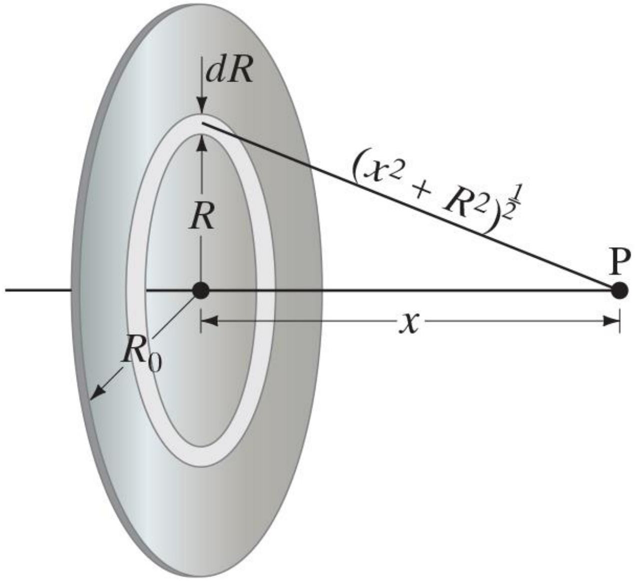
If $x \gg R$, then the potential reduces to the potential due to a point charge (we used r instead of x before), which is intuitively correct

23.4 – Electric Potential of charged disk



We want to determine the potential at a point P at a distance x from the center of a disk of radius R_0 that carries a uniformly distributed charge (overall charge is Q).

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

where $r = \sqrt{x^2 + R^2}$ (note that now $0 \le R \le R_0$). In addition, we need to properly define dq

$$dq = \frac{2\pi R dR}{\pi R_0^2} Q = \frac{2 R dR}{R_0^2} Q$$

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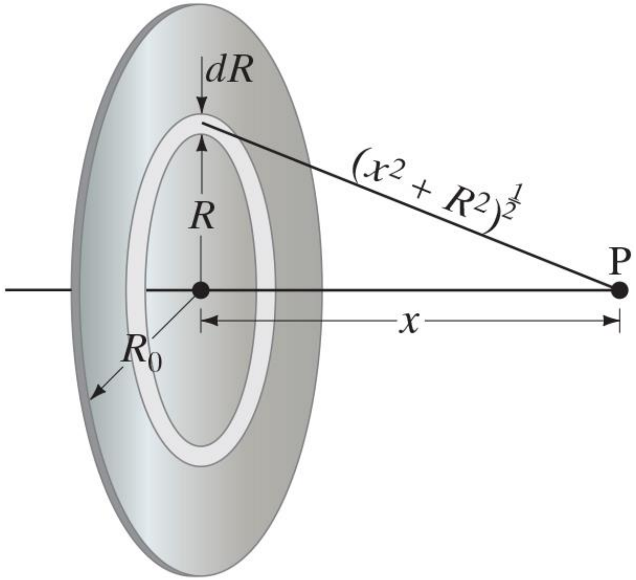

$$V = \frac{Q}{2\pi\epsilon_0 R_0^2}\int_0^{R_0}\frac{R\,dR}{\sqrt{x^2+R^2}} =$$

$$\frac{Qx}{2\pi\epsilon_0 R_0^2}\left[\left(1+\frac{R_0^2}{x^2}\right)^{\frac{1}{2}} - 1\right]$$

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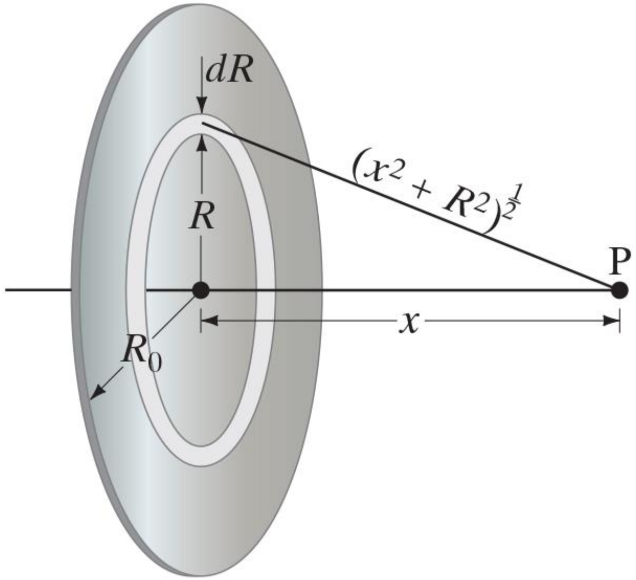

$$V=\frac{Q}{2\pi\epsilon_0 R_0^2}\int_0^{R_0}\frac{RdR}{\sqrt{x^2+R^2}}=$$

$$\frac{Qx}{2\pi\epsilon_0 R_0^2}\left[\left(1+\frac{R_0^2}{x^2}\right)^{\frac{1}{2}}-1\right]$$

If $x \gg R$, then the potential reduces to the potential due to a point charge (we used r instead of x before), which is intuitively correct. Here the conversion is slightly less straightforward as we need binomial expansion

23.5 – Equipotential lines and surfaces

The electric potential can be characterized (and visualized) using equipotential lines (or surfaces in 3D cases), i.e., lines defining sets of points in space characterized by the same electric potential.

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The electric potential can be characterized (and visualized) using equipotential lines (or surfaces in 3D cases), i.e., lines defining sets of points in space characterized by the same electric potential.

These lines should be perpendicular to the electric field, as along an equipotential line it holds that

$$\boldsymbol{V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = 0 \rightarrow \vec{E} \cdot d\vec{l} = 0 \rightarrow \vec{E} \perp d\vec{l}}$$

23.6 – Electric field from electric potential

We can also reverse $\boldsymbol{V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}}$ to express the electric field as a function of the electric potential. We can take an infinitesimal contribution of the aforementioned expression

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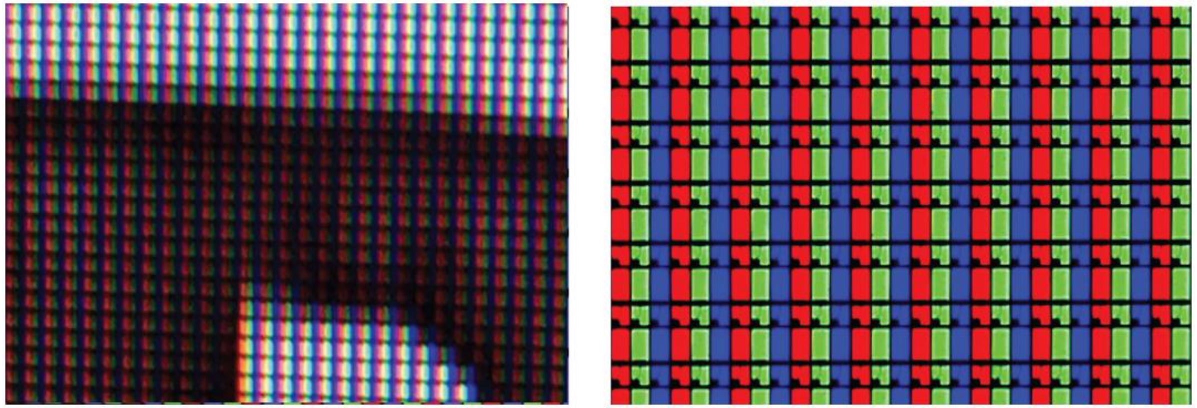
The component of the electric field in any direction is equal to the negative of the rate of change of the electric potential along that direction, i.e., the gradient. In a 3D frame we have

$$\boldsymbol{E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}}$$

23.10 – LCD televisions



Modern TVs are characterized by roughly 1,080 rows of pixels, each with 1,920 pixels, roughly 2M pixels overall (2K TVs). Each pixel has 3 subpixels, Red, Green, Blue (RGB) so that any combination can generate any color.

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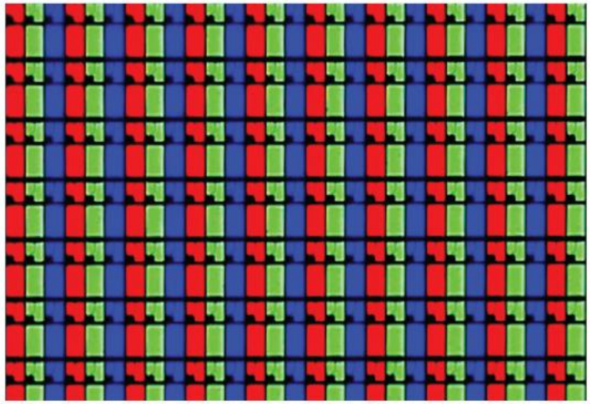

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There exist also 4K and 8K TVs, with a higher pixel density. The brightness of each subpixel depends on the dV between the voltage on the front and back of the screen at that point.

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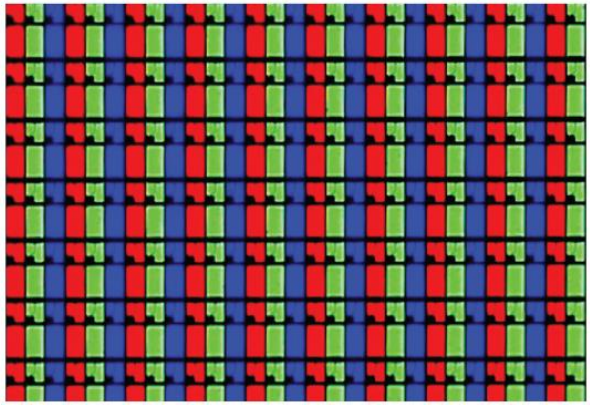

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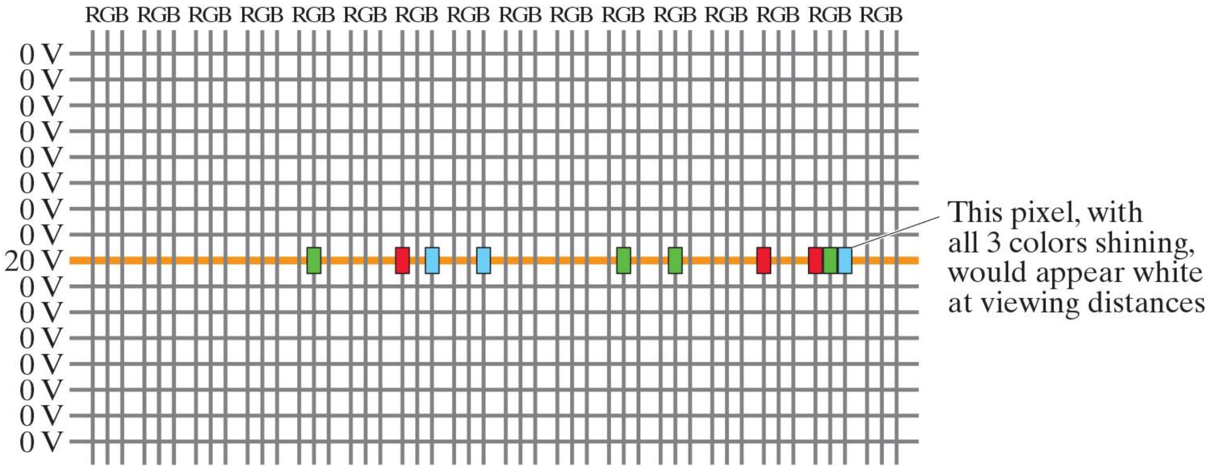
The front of each subpixel is generally kept at a positive voltage, whereas the back is changed for each intersection of a horizontal row and vertical row. In a 2K TV, there are roughly 6,000 vertical wires (1,920 pixels × 3 colors).

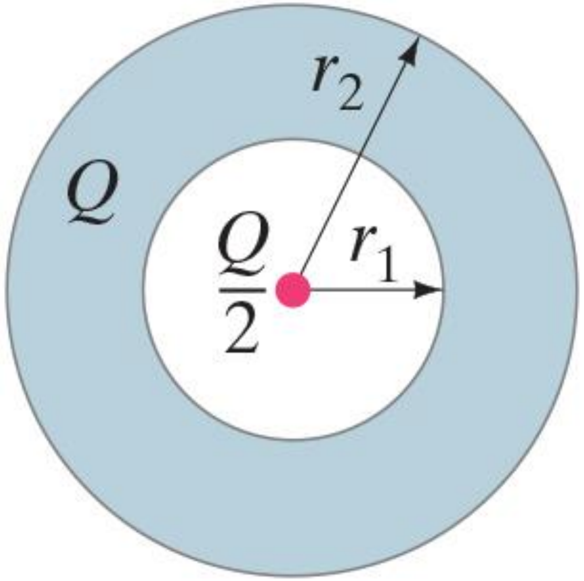
23.10 – LCD televisions



Horizontal lines are activated one by one, with each vertical line (subpixel) applying the correct voltage to achieve the desired color.

A full activation of all horizontal lines occurs, in general, in $\frac{1}{60}\,s$ as most TVs operate at $60\,Hz$, i.e., in 1 second the image is refreshed 60 times.

Example: hollow spherical conductor



A hollow spherical conductor, carrying a net charge equal to Q, has inner radius r_1 and outer radius $r_2 = 2r_1$. At the center there is a point charge whose charge is $\frac{Q}{2}$. Our goal is to determine the electric field and potential at any radial point $0 \leq r \leq \infty$
We start with the electric field.

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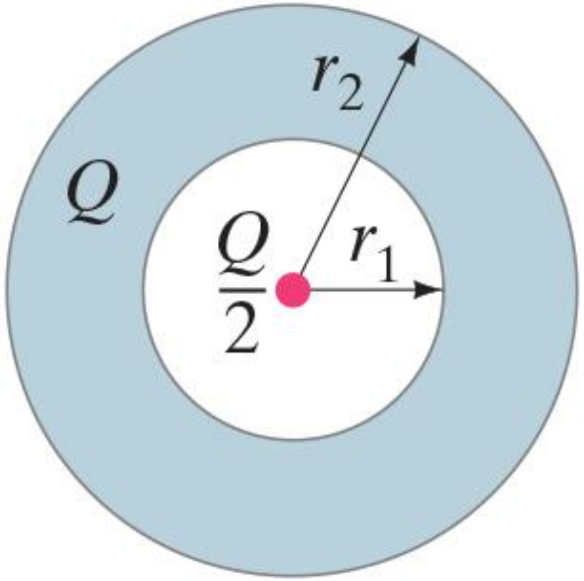

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For $0 \leq r \leq r_1 \rightarrow E(r) = \frac{1}{4\pi\epsilon_0}\frac{\frac{1}{2}Q}{r^2}$

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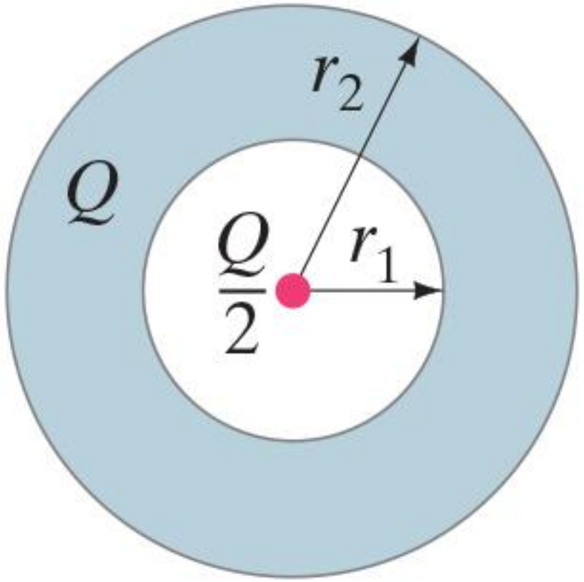

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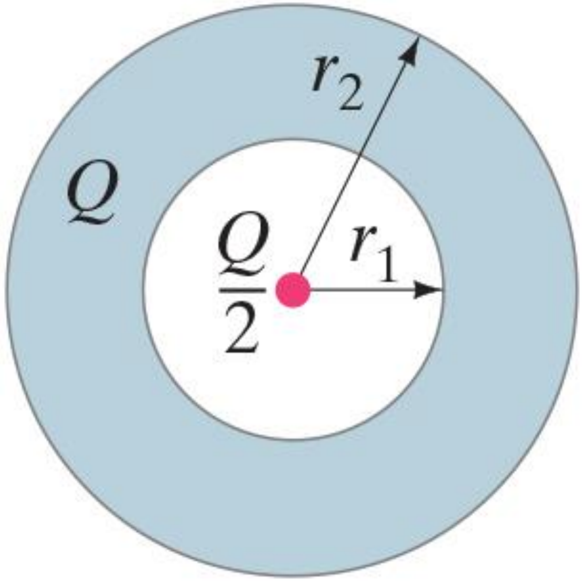

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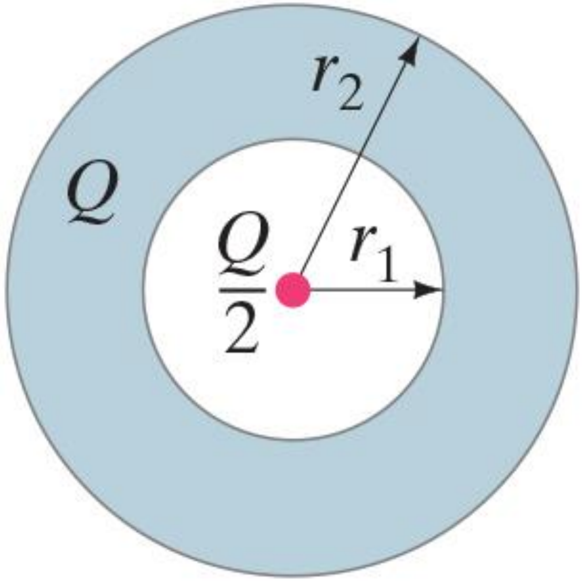
For $r_2 \leq r \leq \infty \rightarrow E(r) = \frac{1}{4\pi\epsilon_0}\frac{\frac{3}{2}Q}{r^2}$

Example: hollow spherical conductor



We now move on to the electric potential. Outside the sphere, the effect is the same as point charge of value $\frac{3}{2}Q$. Assuming $V_\infty = 0$ we have for $r_2 \leq r \leq \infty$:

$$\boldsymbol{V(r) = \frac{3}{8\pi\epsilon_0}\frac{Q}{r}}$$

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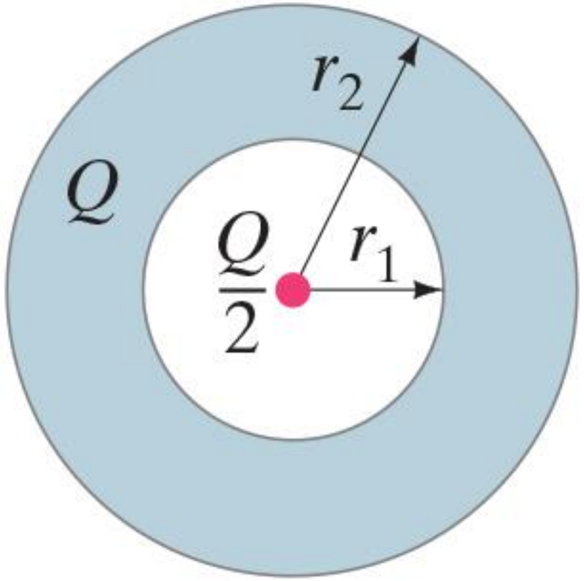
Example: hollow spherical conductor



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Inside the conductor, the potential is constant (no electric field), and can be computed from the expression above if $r = r_2$. For $r_1 \leq r \leq r_2$:

$$\boldsymbol{V(r) = \frac{3}{8\pi\epsilon_0}\frac{Q}{r_2}}$$

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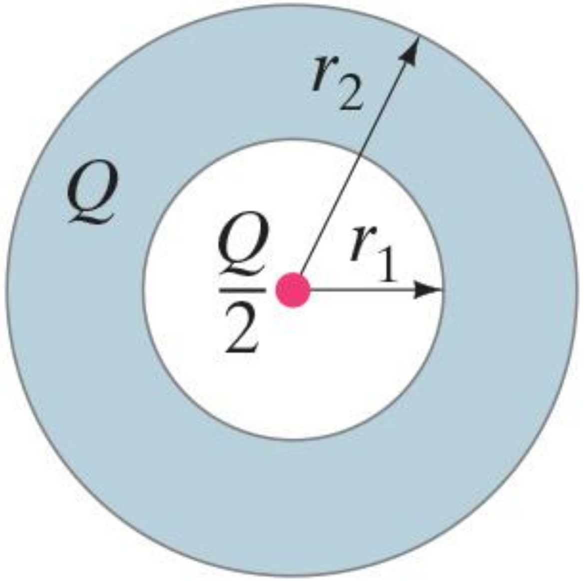
Example: hollow spherical conductor

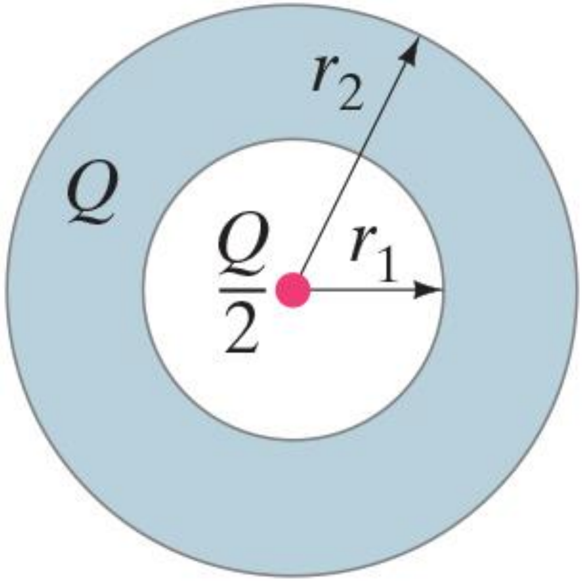


Inside the cavity ($0 \leq r \leq r_1$), we have the potential due to a point charge of value $\frac{1}{2}Q$:

$$\boldsymbol{V(r) = \frac{Q}{8\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{r_1}\right) + C}$$

where C is a constant that we need to determine to ensure continuity of the potential. We can do it as we know the value of the potential for $r = r_1$

Example: hollow spherical conductor



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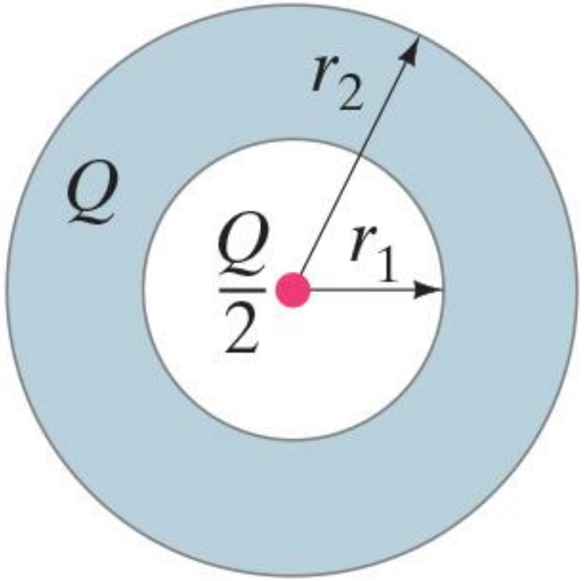

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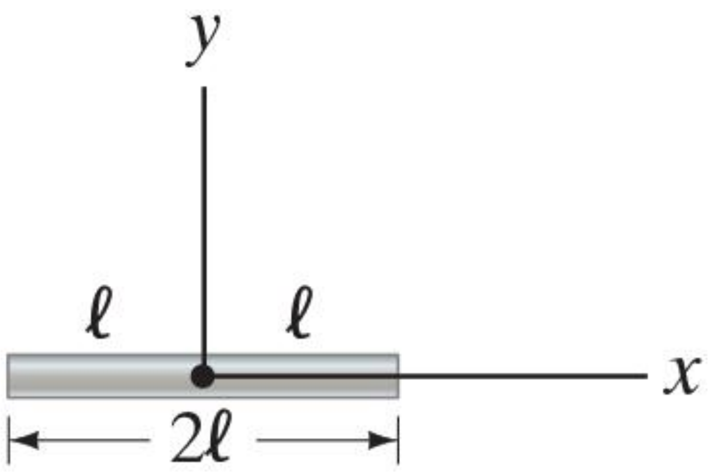
$$\boldsymbol{V(r_1) = \frac{Q}{8\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_1}\right) + C = \frac{3}{8\pi\epsilon_0}\frac{Q}{r_2} \rightarrow C = \frac{3}{8\pi\epsilon_0}\frac{Q}{r_2}}$$

We can now plug such a value back in the expression above to get

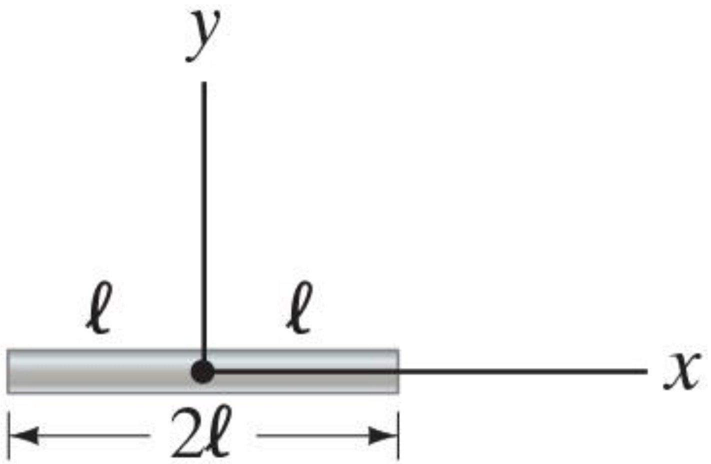
$$\boldsymbol{V(r) = \frac{Q}{8\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{r_1}\right) + \frac{3}{8\pi\epsilon_0}\frac{Q}{r_2} = \frac{Q}{8\pi\epsilon_0}\left(\frac{1}{r} + \frac{1}{2r_1}\right)}$$

Example: uniformly charged rod



Determine the potential $V(y)$ along the vertical axis of symmetry of a thin rod of length $2l$ whose charge Q is uniformly distributed.

Example: uniformly charged rod



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We should use $V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$

Hence, we need to properly define dq and r.

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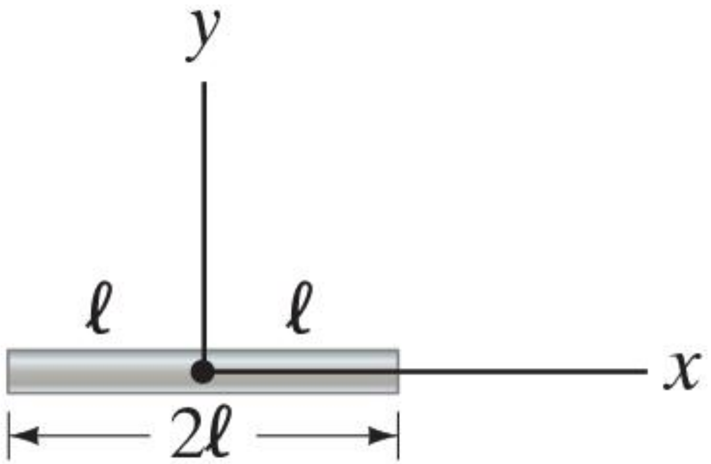

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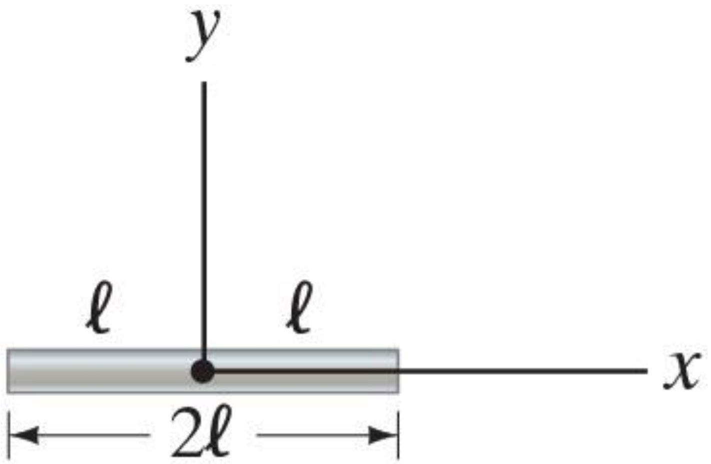
$$\boldsymbol{dq = \frac{Q}{2l} dx}$$
$$\boldsymbol{r = \sqrt{x^2 + y^2}}$$

Example: uniformly charged rod



$$V(y) = \frac{1}{4\pi\epsilon_0}\int_{-l}^{l}\frac{Q}{2l\sqrt{x^2+y^2}}dx$$

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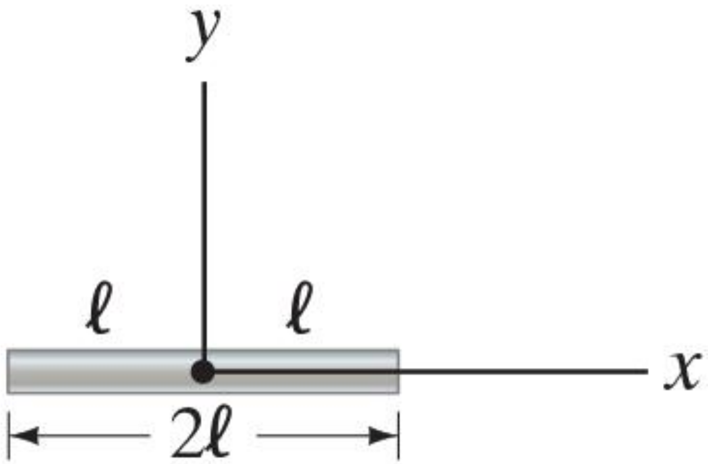
Example: uniformly charged rod



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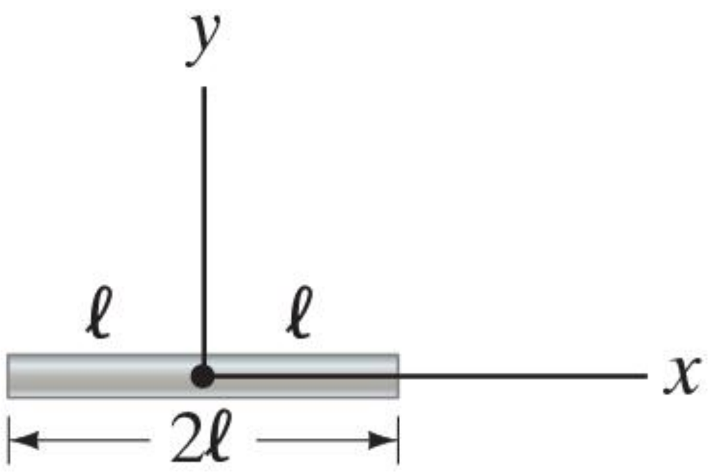

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$$V(y) = \frac{Q}{8\pi l\epsilon_0}\left[\ln\left(\frac{\sqrt{l^2+y^2}+l}{\sqrt{l^2+y^2}-l}\right)\right]$$

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Wrap-up: revisiting Learning objectives

After this lecture you should be able to:

- Understand the **relationship** between electric field and electric potential
- Apply such a relationship to **compute the electric potential** due to a point charge or a continuous charge distribution
- Apply the **inverse relationship** to determine the electric field given a known electric potential

Wrap-up: revisiting Learning objectives

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