ELECTRIC CHARGE AND FIELD

Chapter 21

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TUDelft

Structure of the lecture

1. Static electricity and electric charge
2. Coulomb's law
3. Electric field
4. Continuous distribution of charge
5. Electric field lines
6. Electric fields and conductors
7. Electric dipole

Learning objectives for today's lecture

After this lecture you should be able to:

- Apply Coulomb's law to determine the net electric force acting in a point due to a set of charges

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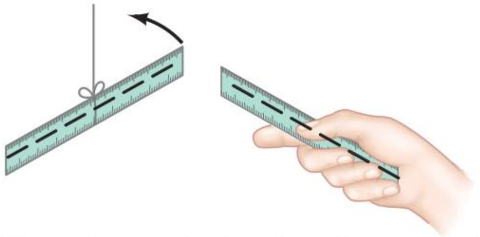
- Apply Coulomb's law to determine the net electric force acting in a point due to a set of charges

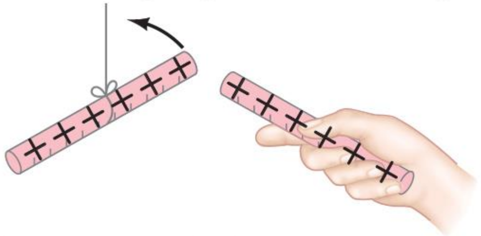
- Determine the electric field in a point in space due to the effect of a point charge, set of point charges, or charge distribution

- Understand the concept of electric dipole

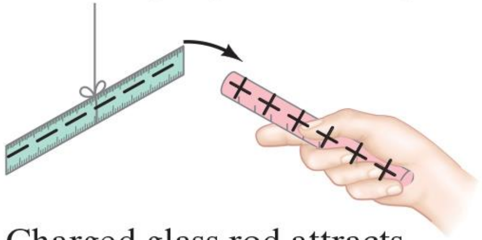
21.1 - Static electricity



(a) Two charged plastic rulers repel



(b) Two charged glass rods repel



(c) Charged glass rod attracts
charged plastic ruler

Charge comes in two types, positive and negative; like charges repel and opposite charges attract.

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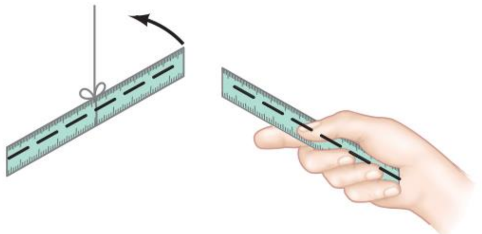

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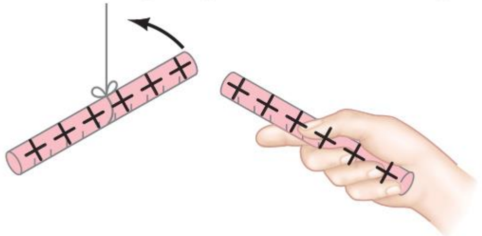

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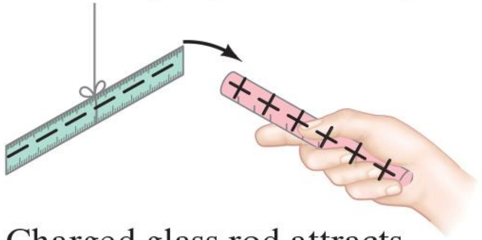
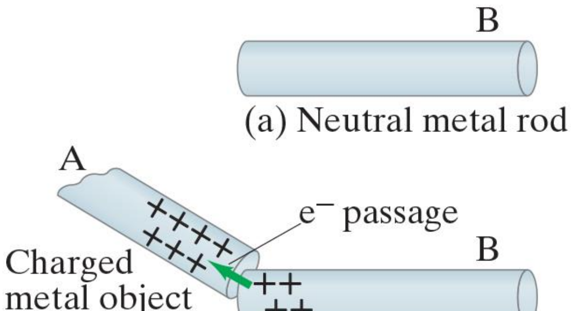


(c) Charged glass rod attracts charged plastic ruler

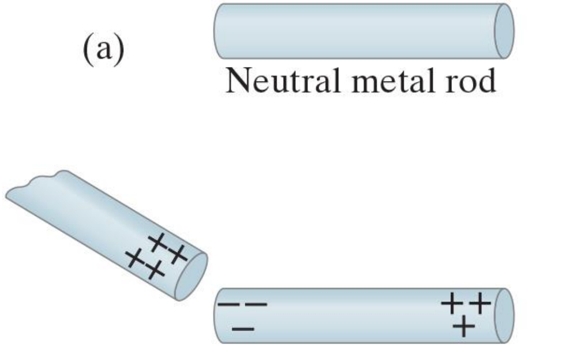
Charge comes in two types, positive and negative; like charges repel and opposite charges attract.

It is possible to separate positive and negative charges in a conducting material by using a charged object



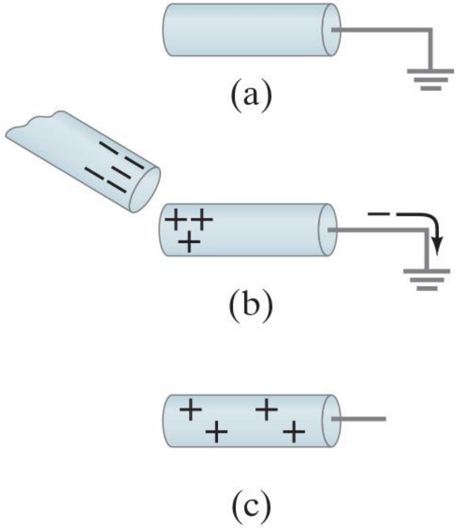
(a) Neutral metal rod

(b) Metal rod acquires charge by contact



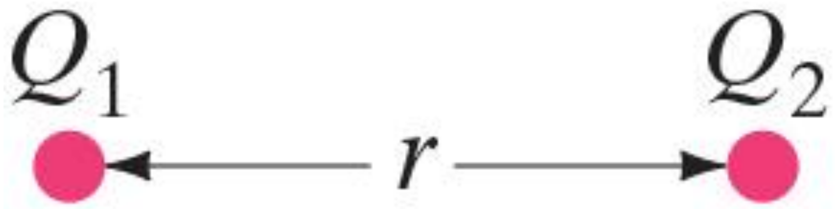
Neutral metal rod

Metal rod still neutral, but with a separation of charge



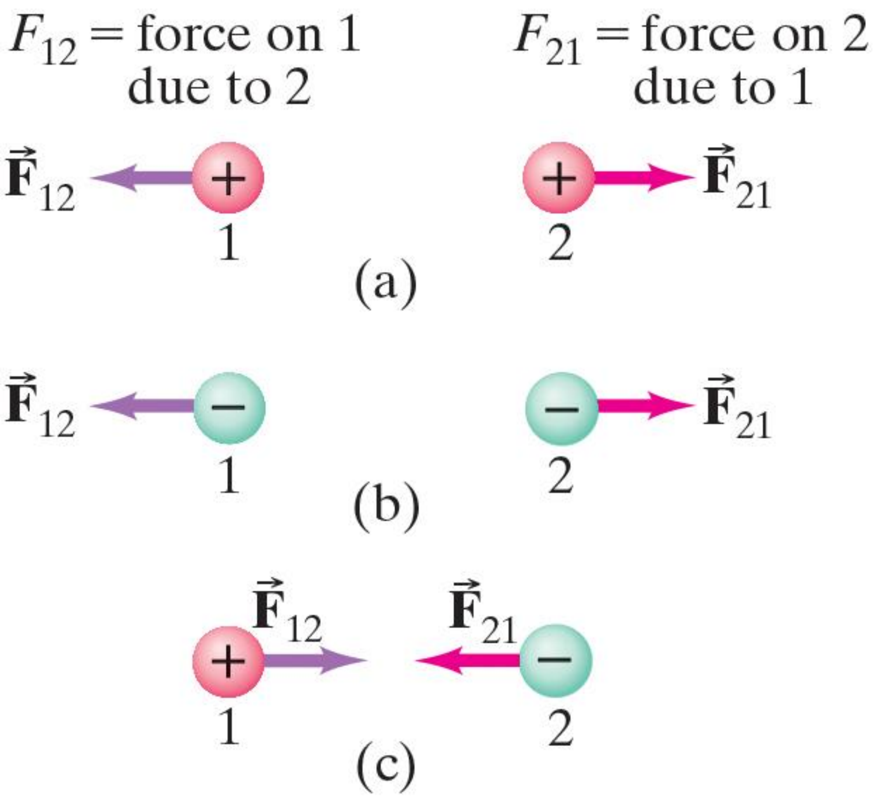
21.5 – Coulomb's law

Experiment shows that the electric force between two charges is proportional to the product of the charges and inversely proportional to the distance between them.



21.5 – Coulomb's law

The force is along the line connecting the charges, and is attractive if the charges are opposite, and repulsive if they are the same.



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- Magnitude of the force represented in the formula. Direction is given by the line connecting the 2 point sources

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$$\boldsymbol{F} = \frac{\boldsymbol{k\, Q_1 Q_2}}{\boldsymbol{r^2}}$$

- Magnitude of the force represented in the formula. Direction is given by the line connecting the 2 point sources

- Formula precise (and defined) for point charges. For other types of charges, the accuracy might vary to the point of not being applicable at all

21.5 – Coulomb's law

The unit of charge is the Coulomb (C), while the proportionality constant can be defined as

$$k = 8.99 \times 10^9 \frac{N\,m^2}{C^2}$$

Hence, two charges of $1\ C$ each will exert on each other a force equal to $8.99 \times 10^9\ N$ when placed exactly $1\ m$ apart.

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To get some insights, charges produced by rubbing are generally in the order of the $\mu C\ (1 \times 10^{-6}\ C)$

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The charge of a single electron is

$$\boldsymbol{e = 1.602 \times 10^{-19}\ C}$$

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which means that the charge due to rubbing entails an amount of electrons of roughly

$$\boldsymbol{1\ \mu C \rightarrow 10^{13}}$$

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Oftentimes, Coulomb's law is rewritten as

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where ϵ_0 is the permittivity of free space (will come back when talking about capacitors), and is equal to $8.85 \times 10^{-12}\ C^2/Nm^2$

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The principle of superimposition states that the overall electrostatic force acting on a charged particle due to the effect of n distinct particles can be computed by singularly computing the n forces and then vectorially summing them

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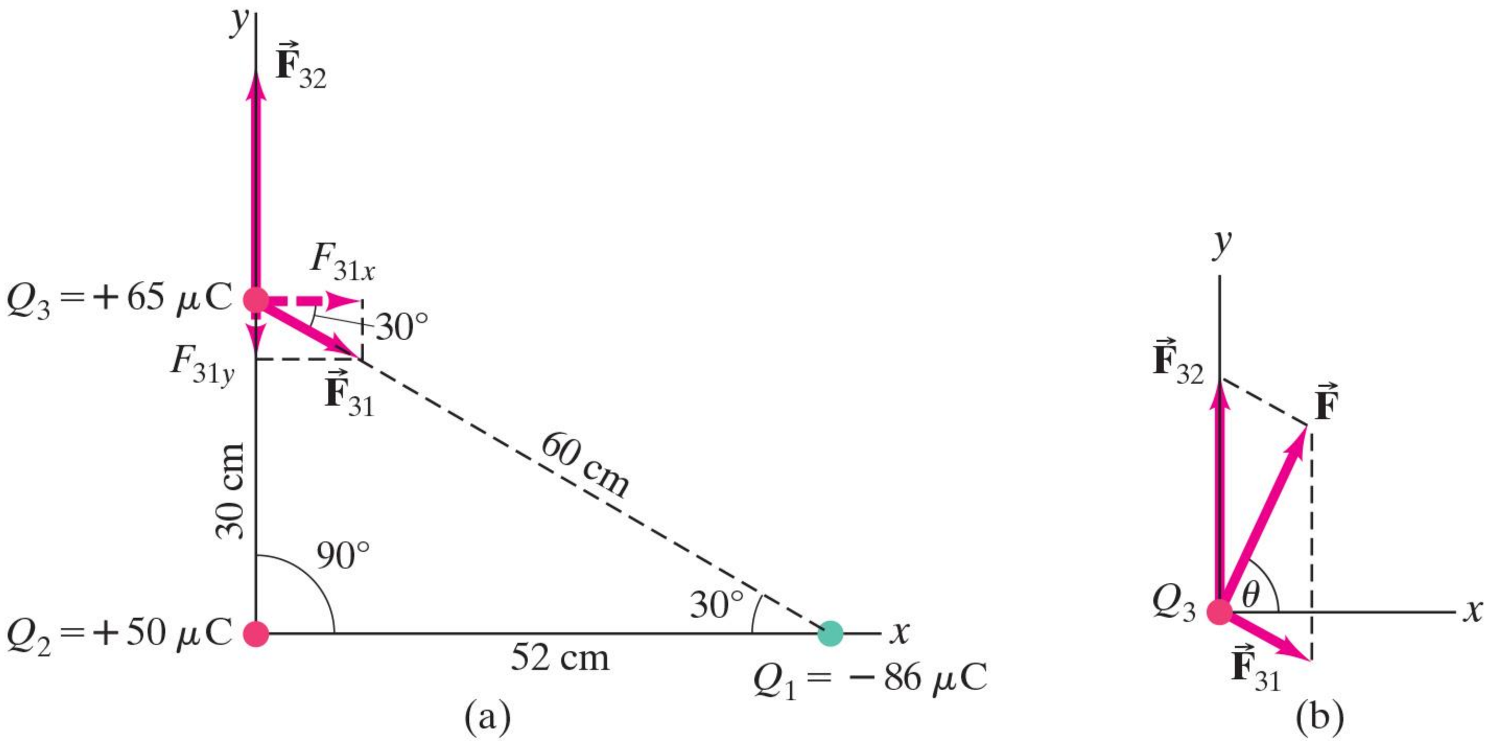
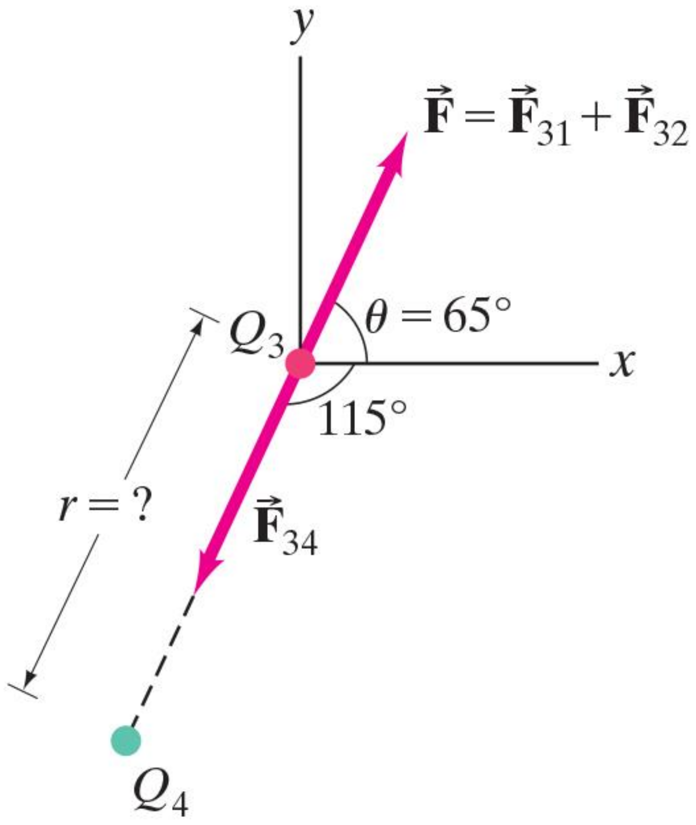
The electrostatic force on Q_3 can be computed by first determining separately the two contributions due to Q_1 and Q_2



21.5 – Coulomb's law

At which distance shall we place Q_4 $(-50\mu C)$ to have a zero net effect on Q_3?



21.6 – Electric Field

The electric field (which is a vectorial field) is defined as the force on a small test charge q, divided by the magnitude of that charge

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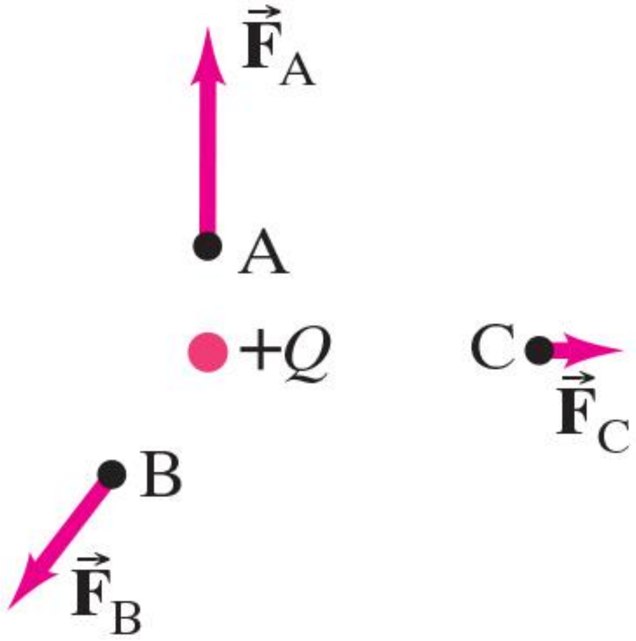
Charge Q defines an electric field in points A, B, and C whose direction is the same as the direction of the forces $\vec{F}_A$ etc., and whose magnitude is scaled w.r.t. the force by q



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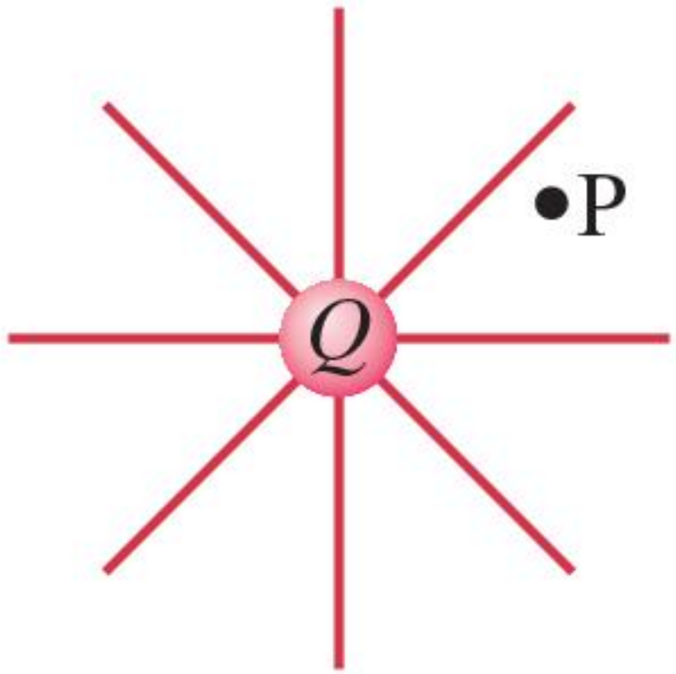
21.6 – Electric Field

An electric field surrounds every charge



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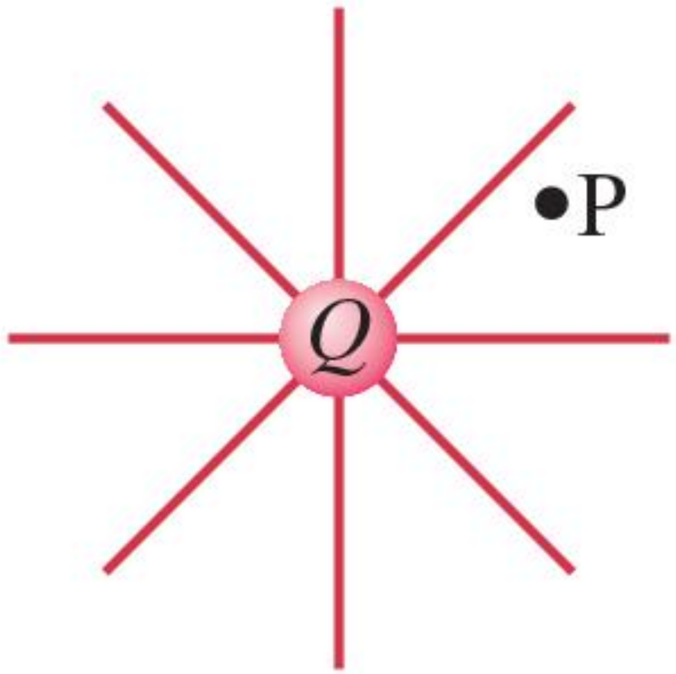

For a point charge, we have that

$$|\boldsymbol{E}| = \frac{|\boldsymbol{F}|}{\boldsymbol{q}} = \boldsymbol{k}\frac{\boldsymbol{qQ}}{\boldsymbol{r^2}}\frac{\boldsymbol{1}}{\boldsymbol{q}} = \frac{\boldsymbol{kQ}}{\boldsymbol{r^2}}$$

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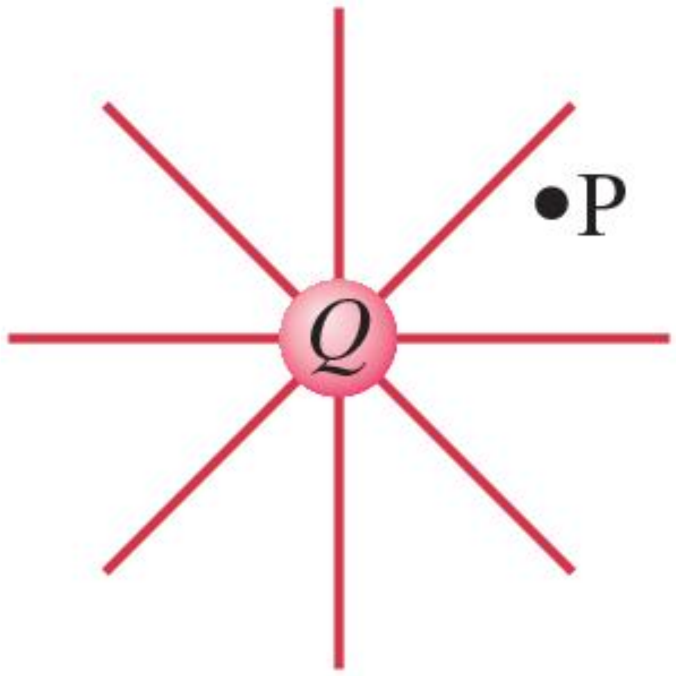

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while, given that $\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}}{\boldsymbol{q}}$, the direction of $\vec{\boldsymbol{E}}$ is the same as $\vec{\boldsymbol{F}}$ if $q > 0$ and is opposite otherwise

21.7 – Electric Field: continuous distribution of charge

A continuous distribution of charge may be treated as a succession of infinitesimal (point) charges. The total field is then the integral of the infinitesimal fields due to each bit of charge

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} \rightarrow E = \int dE$$

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Remember that the electric field is a vector, and usually the integral can be simplified if the problem at hand is characterized by symmetry (e.g., one component of the field is identically zero)

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- Define and (try to) solve the integral without pausing to think about symmetries, how the integral itself can be simplified, etc.

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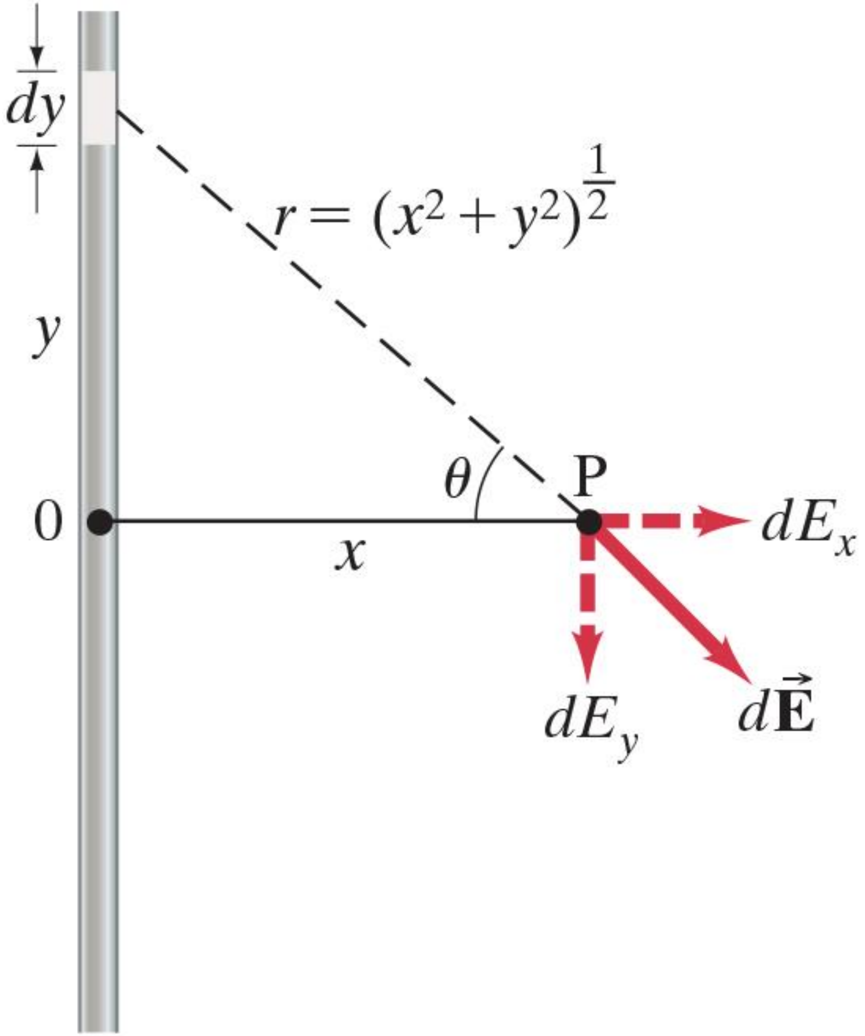
- Define and (try to) solve the integral without pausing to think about symmetries, how the integral itself can be simplified, etc.

- Take some time to do that

21.7 – Infinite (or reasonably long) wire



Compute the electric field generated by an infinite wire in a point distant x from the wire. Some preliminary considerations:

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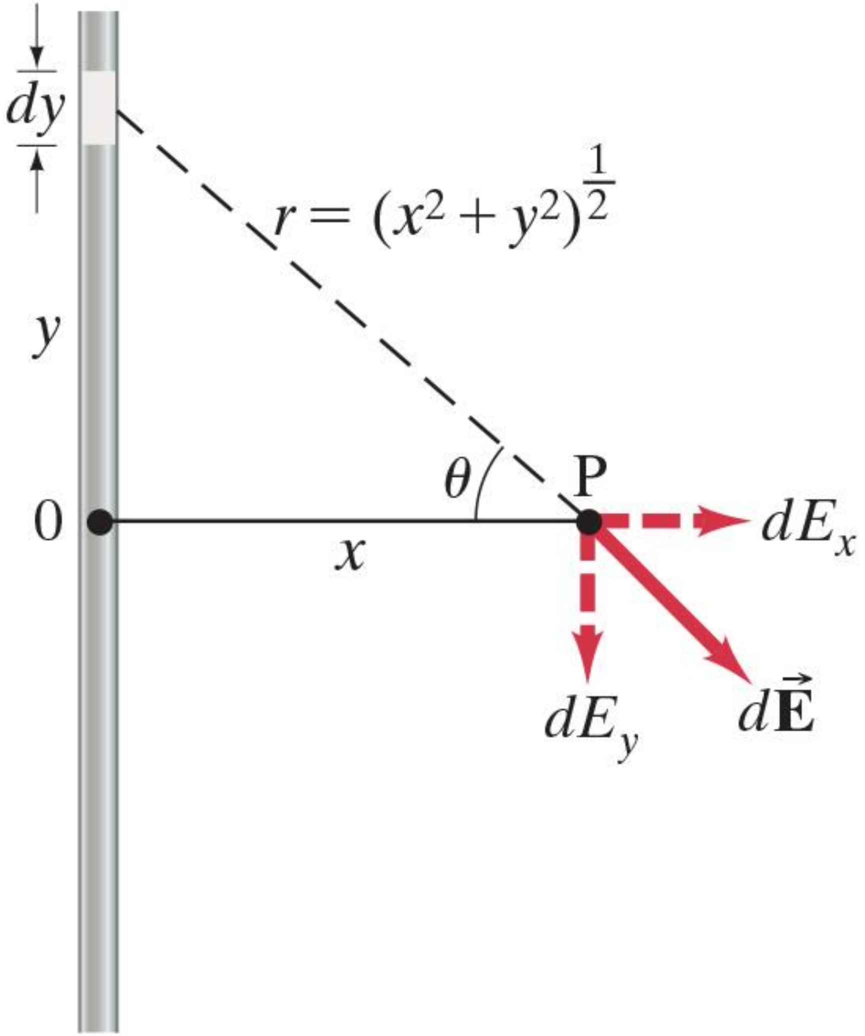

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- Because the wire is very long, it does not really matter where we place the 0 y-coordinate
- λ is the charge per unit length

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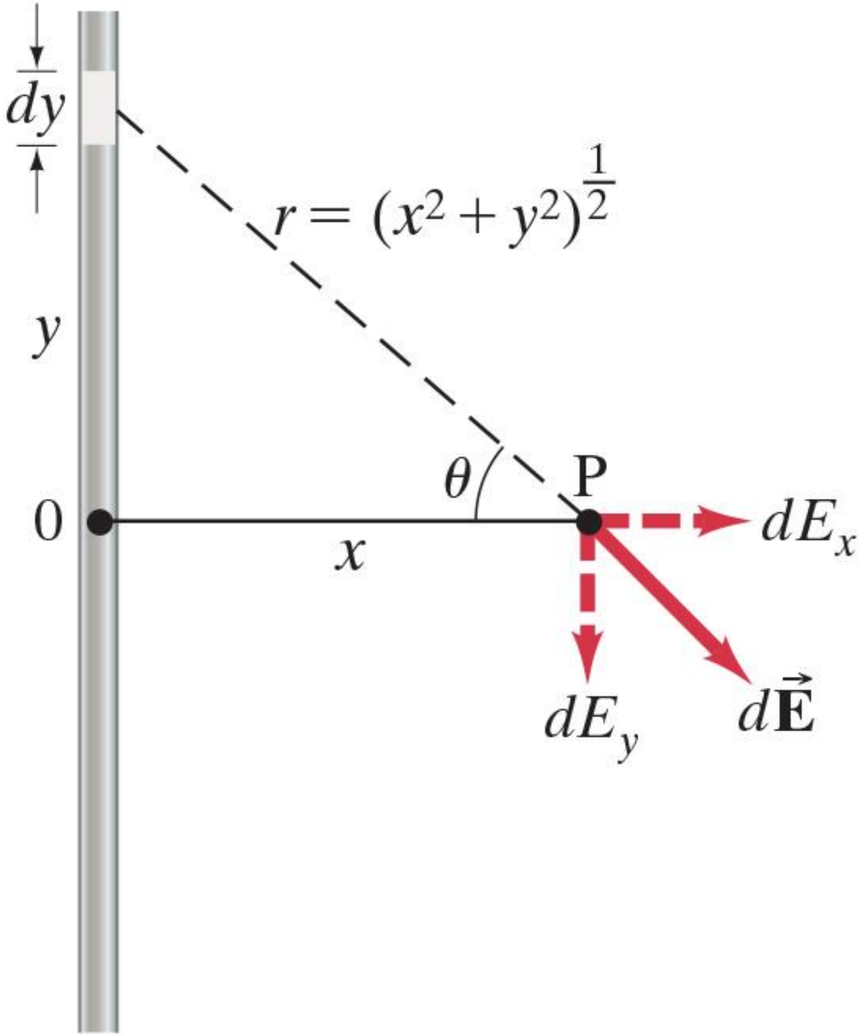

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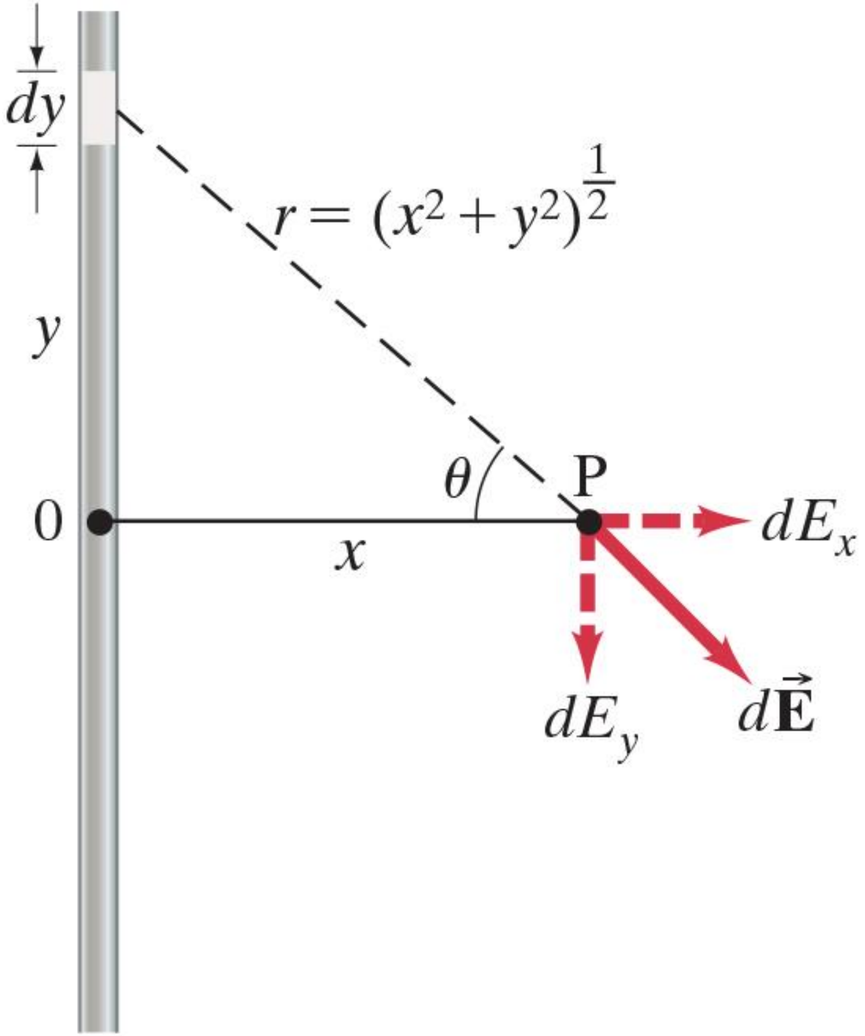

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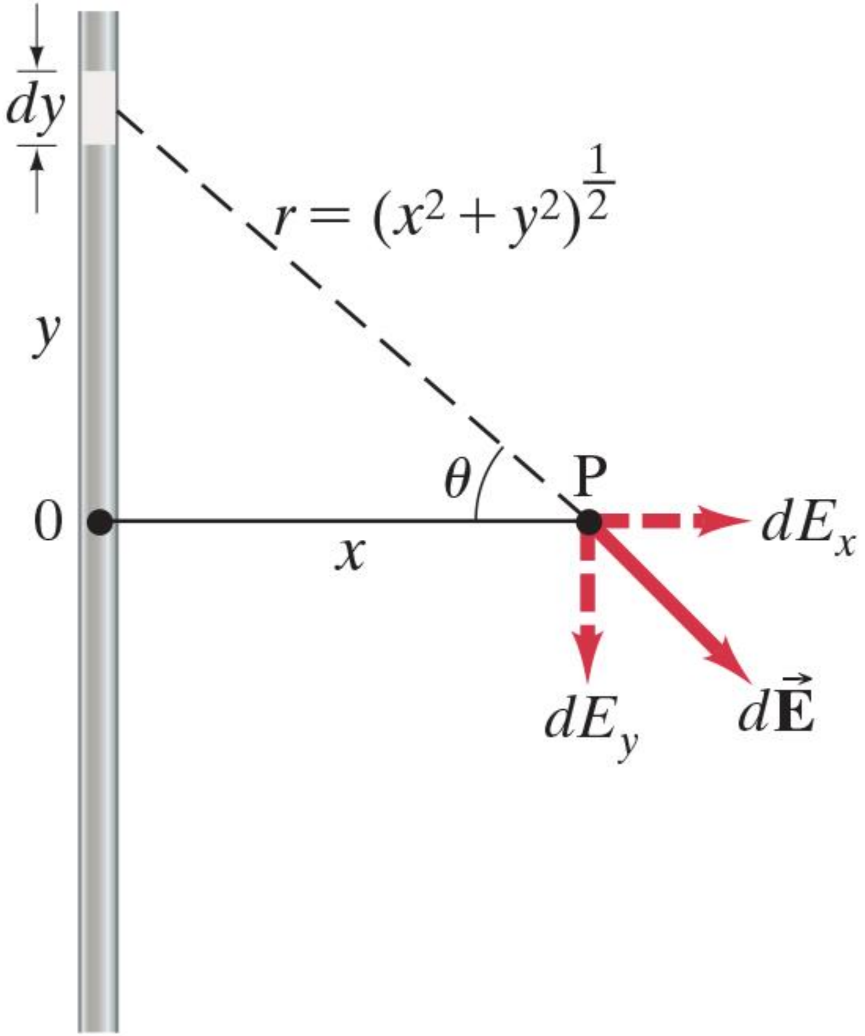

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{\lambda dy}{x^2 + y^2}\cos\theta$$

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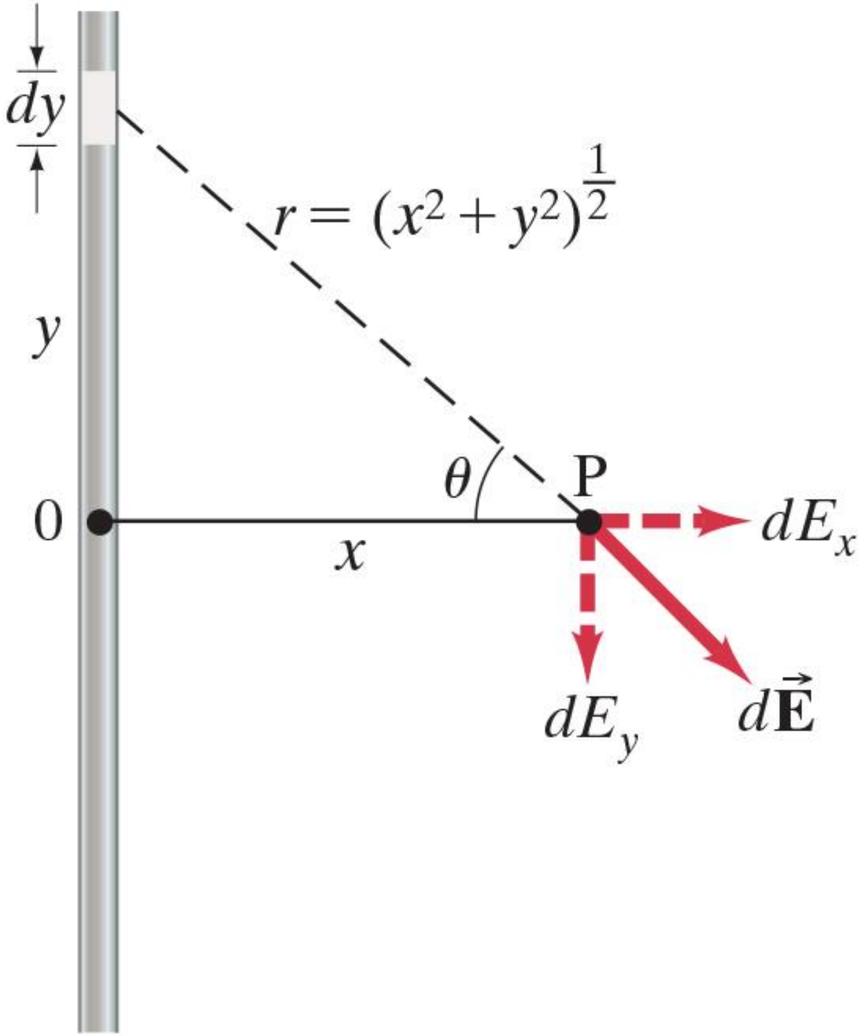

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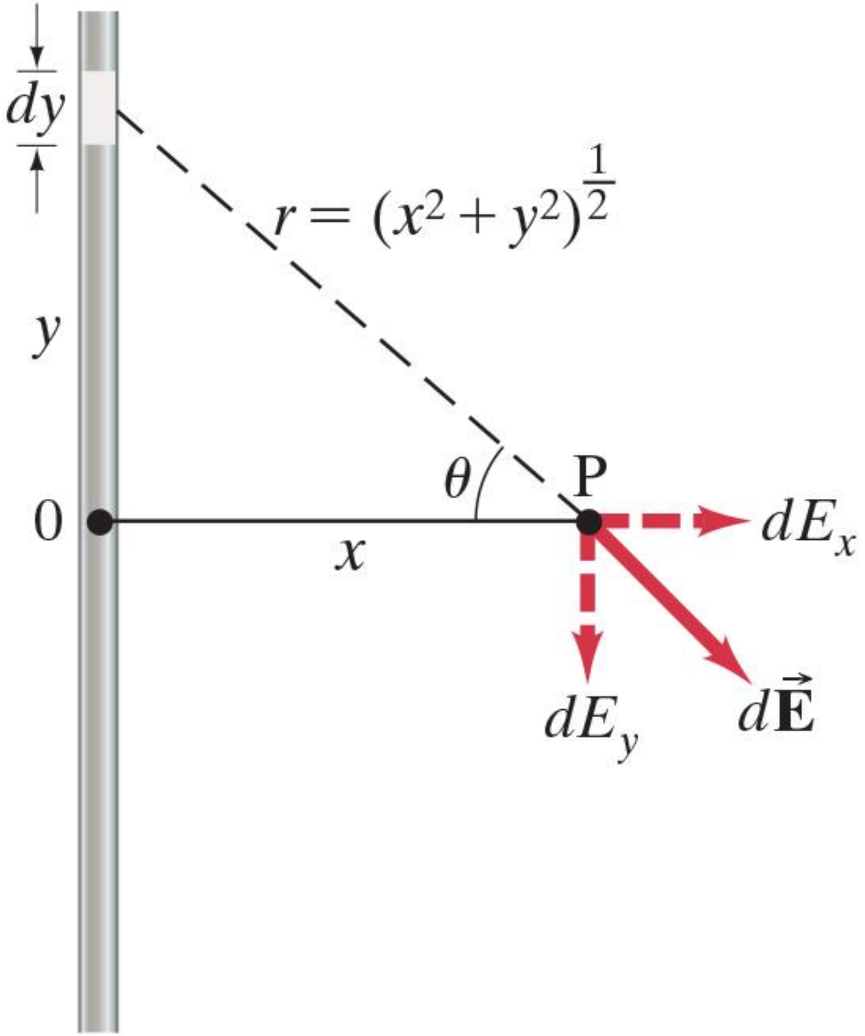

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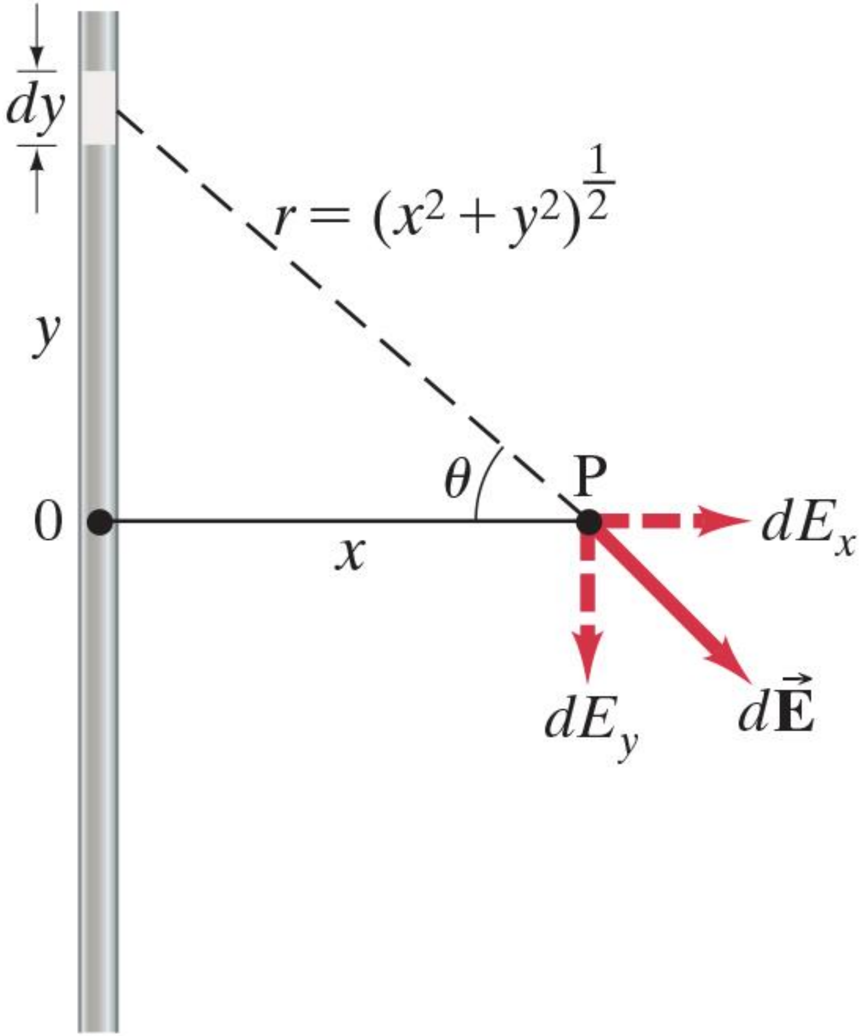

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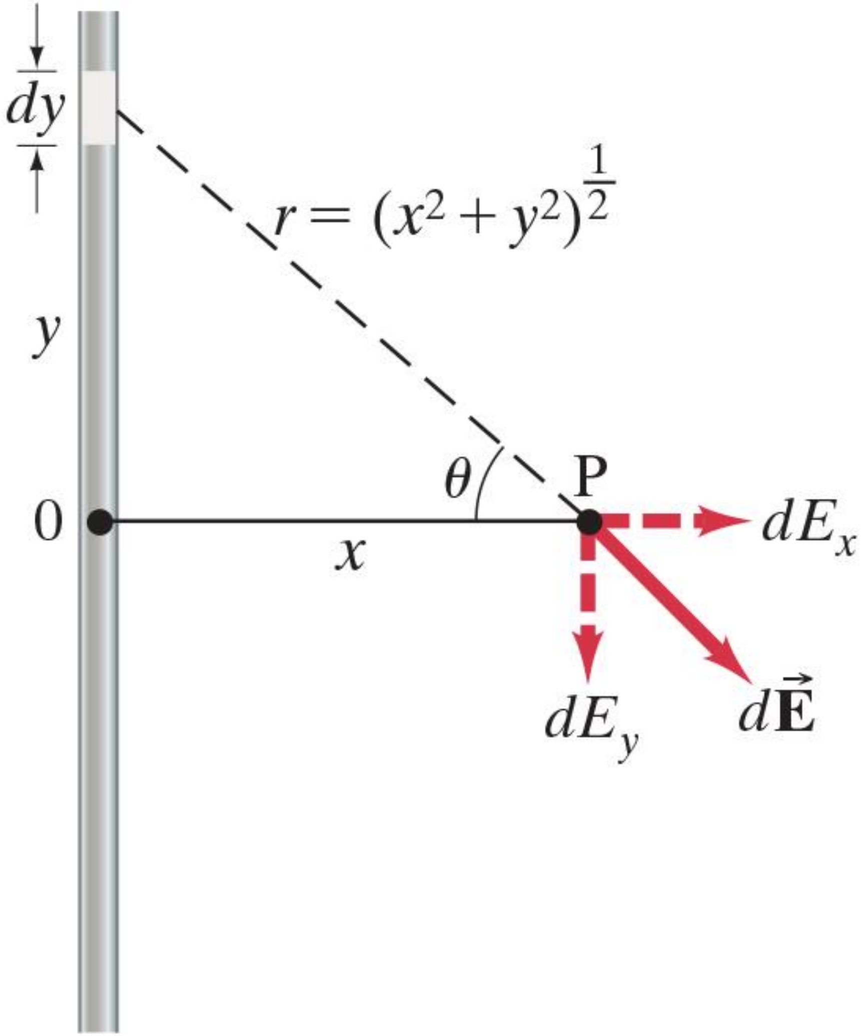

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As our integral in y would range from $-\infty$ to $+\infty$, our integral in θ ranges from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$

In addition, we have that $\frac{1}{x^2+y^2} = \frac{\cos^2\theta}{x^2}$

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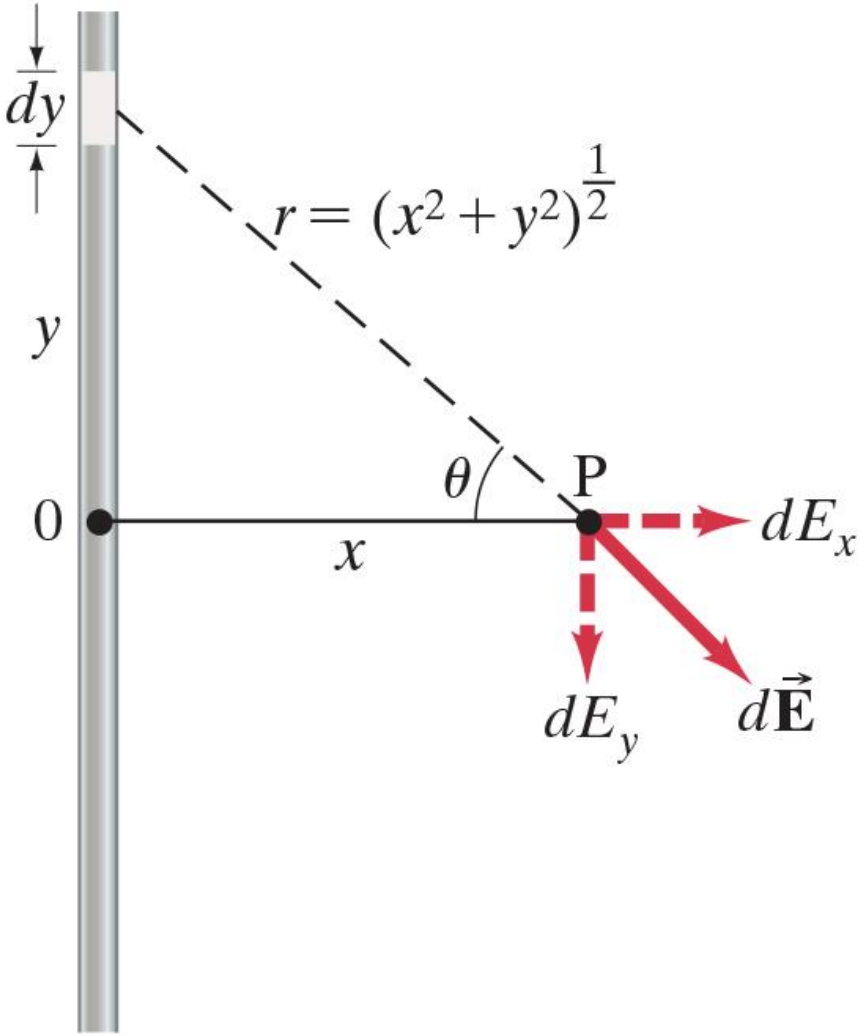

$$|E_x| = \frac{\lambda}{4\pi\epsilon_0 x}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta d\theta = \frac{2\lambda}{4\pi\epsilon_0 x}\left(\sin\frac{\pi}{2}\right)$$

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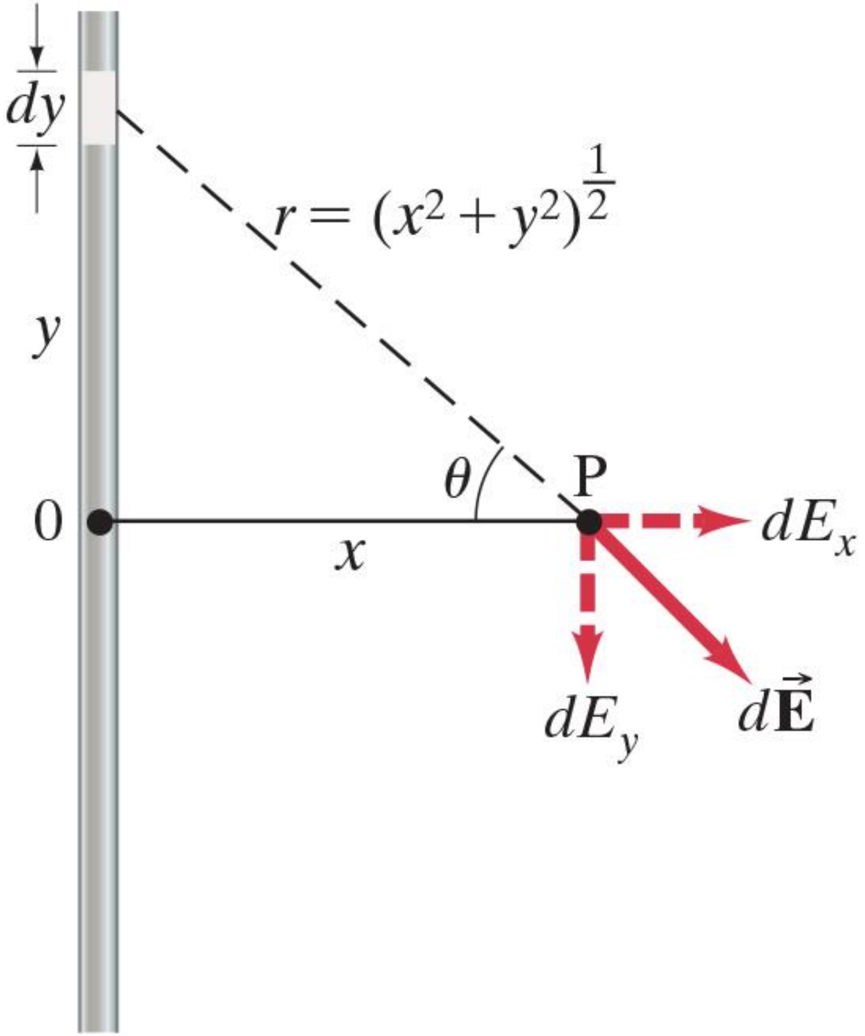

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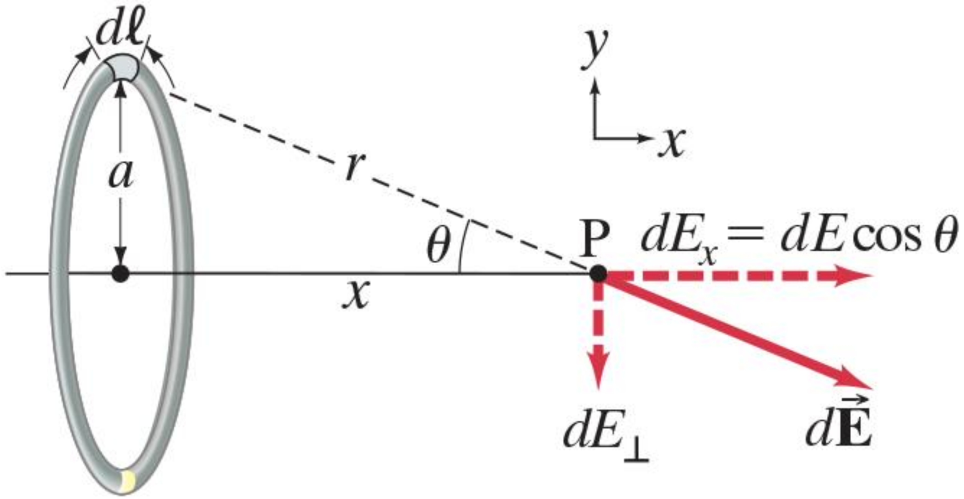
Decrease of electric field is proportional to inverse of the distance and not to the inverse of the square distance as in the point charge case

21.7 – Ring of charge



Compute the electric field generated by a ring of charge at a point distant x from the wire. Some preliminary considerations:

- λ is the charge per unit length
- Q is the full charge of the ring, i.e., $2\pi a\lambda$

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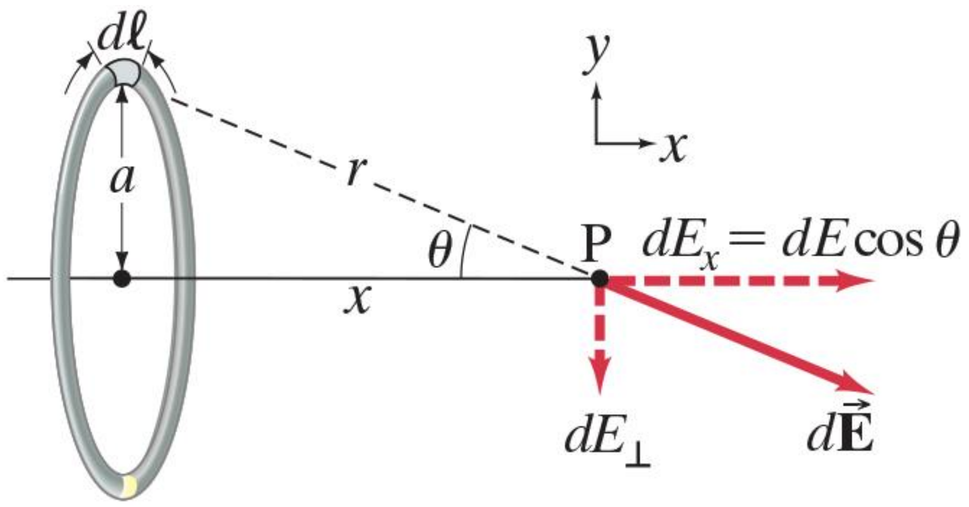

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Skipping a few steps (e.g., $\mathrm{r}\cos\theta = x$) we get to the following (not that here we decided to integrate over dl)

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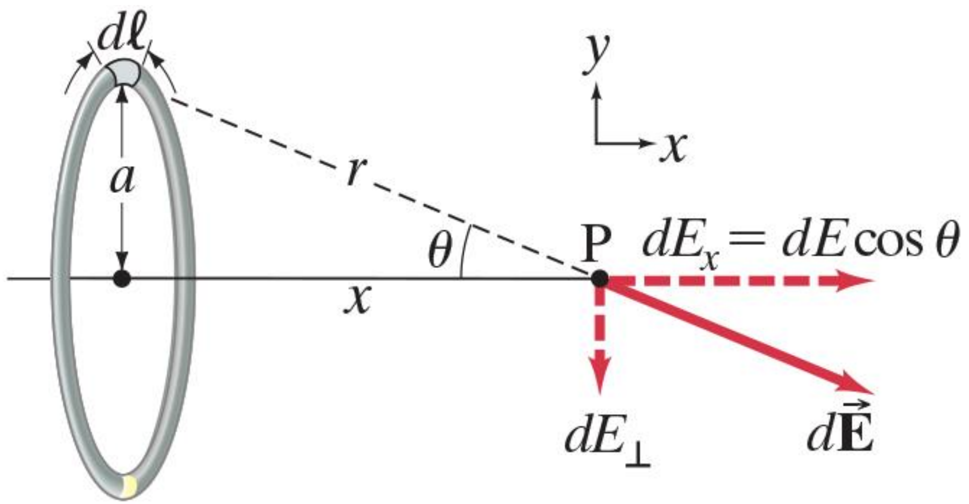

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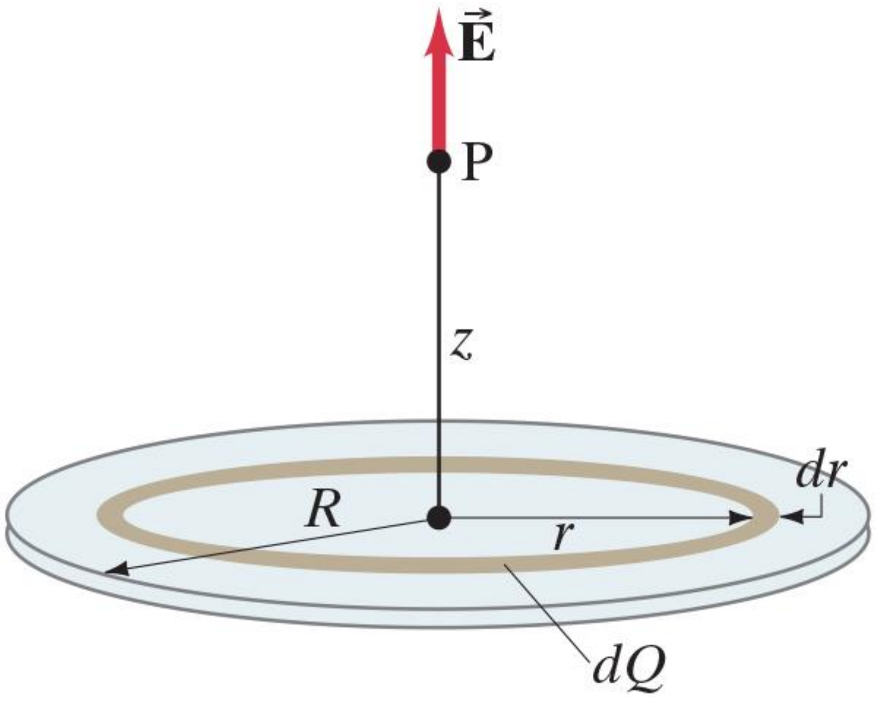
$$|E| = |E_x| = \frac{\lambda}{4\pi\epsilon_0 x}\int_0^{2\pi a}\frac{dl}{r^2}\cos\theta$$

$$= \frac{\lambda}{4\pi\epsilon_0}\frac{x}{(x^2+a^2)^{\frac{3}{2}}}\int_0^{2\pi a} dl = \frac{1}{4\pi\epsilon_0}\frac{\lambda x(2\pi a)}{(x^2+a^2)^{\frac{3}{2}}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{\frac{3}{2}}}$$

21.7 – Uniformly charged thin flat disk



Determine electric field at a point P on the axis of the disk, a distance z above it.
Some preliminary considerations:

- σ is charge per unit area
- We could consider the disk as a sequence of very thin rings (check previous slides)

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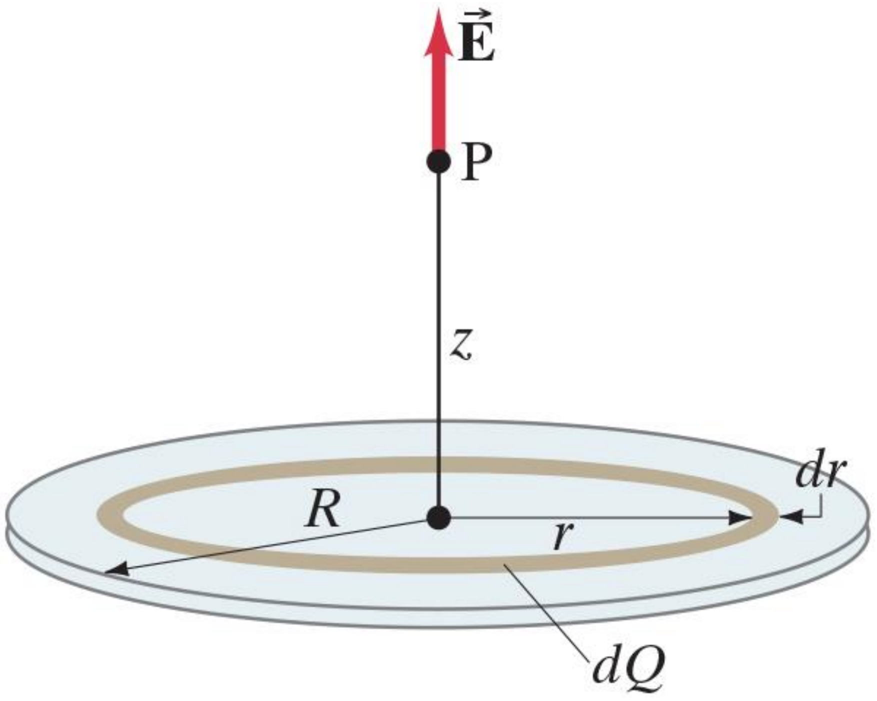

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The contribution to the electric field in P of a small ring with radius r is

$$dE = \frac{1}{4\pi\epsilon_0}\frac{z\,dQ}{(z^2+r^2)^{\frac{3}{2}}}$$

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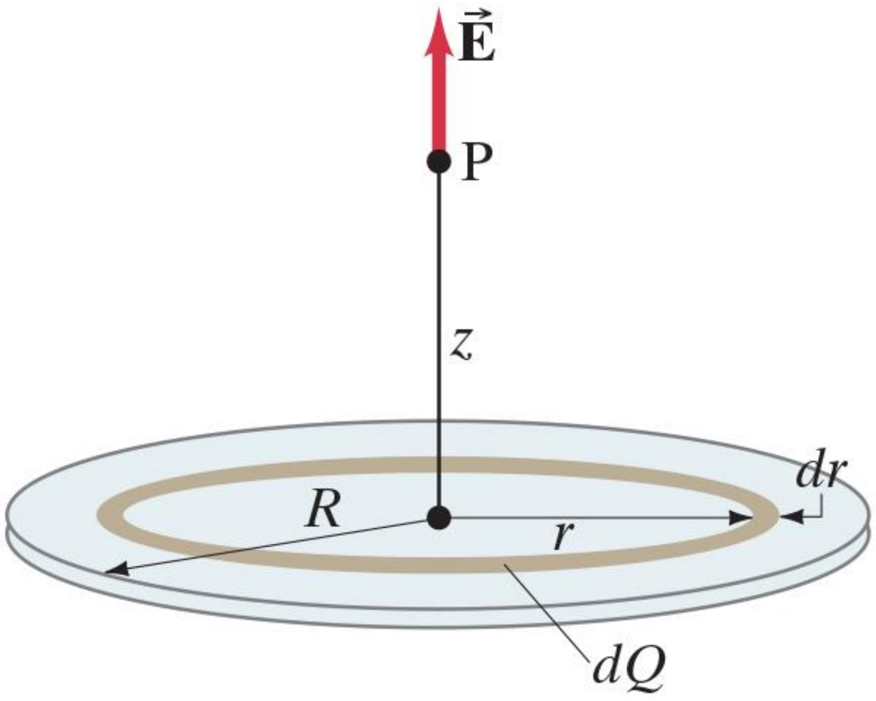

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where

$$dQ = \sigma 2\pi r dr$$

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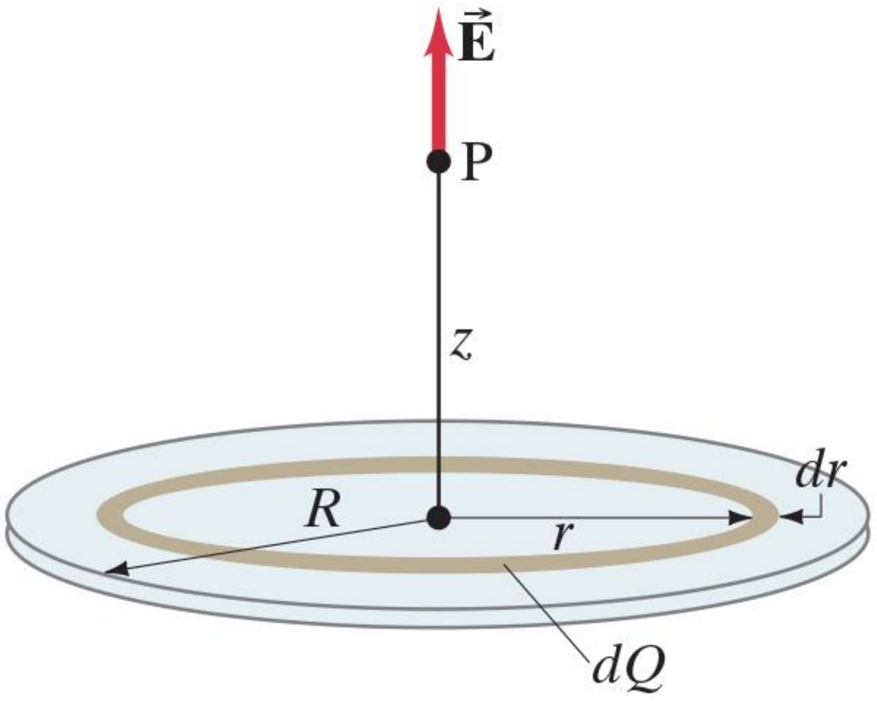

Hence

$$dE = \frac{z\sigma r dr}{2\epsilon_0 (z^2 + r^2)^{\frac{3}{2}}}$$

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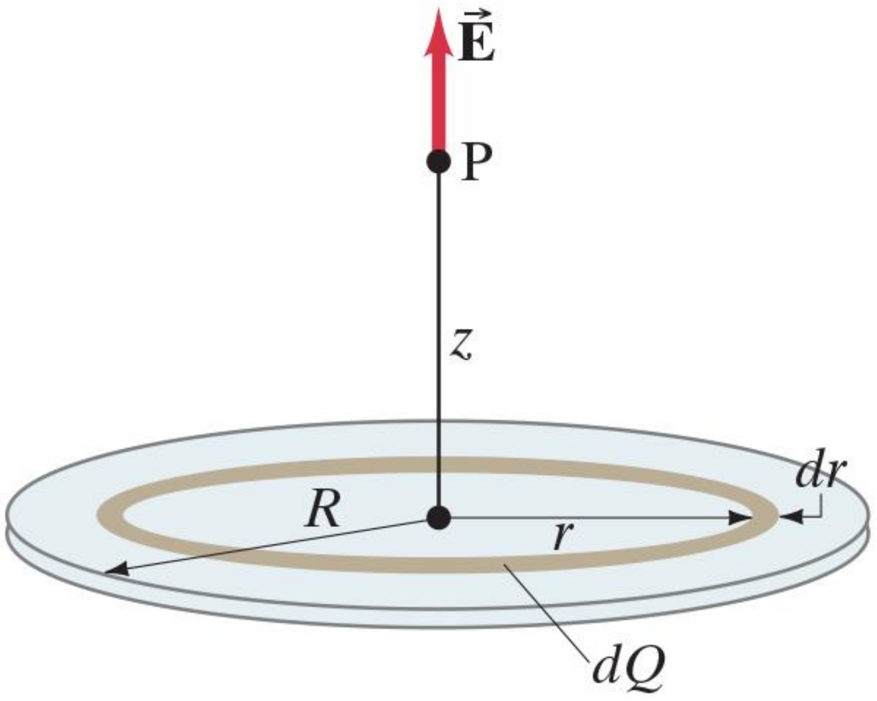


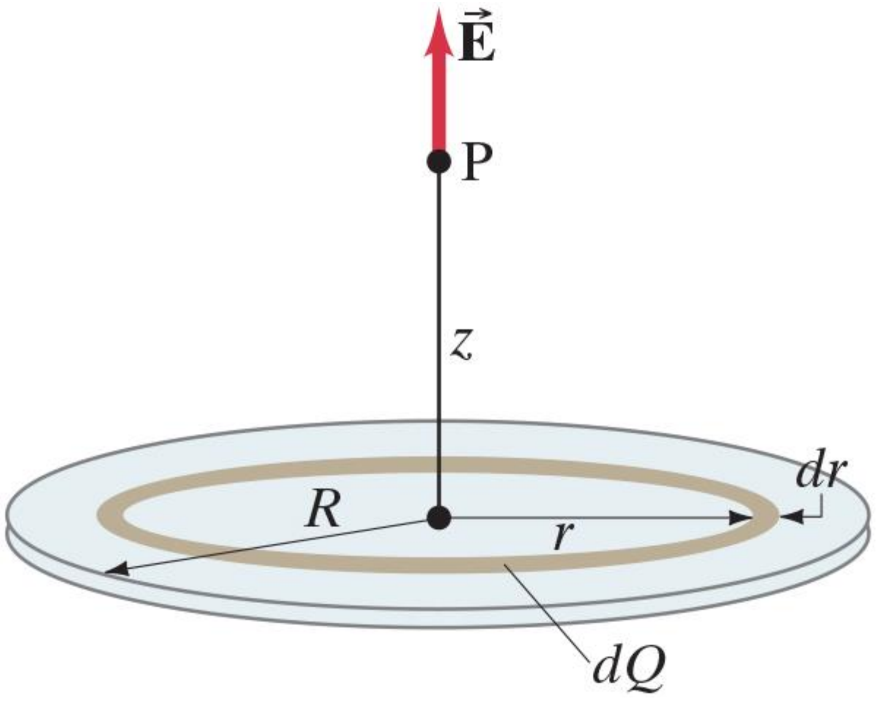
Hence

$$dE = \frac{z\sigma r dr}{2\epsilon_0 (z^2 + r^2)^{\frac{3}{2}}}$$

and then

$$|E| = \frac{z\sigma}{2\epsilon_0}\int_0^R \frac{r dr}{(z^2 + r^2)^{\frac{3}{2}}} = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{(z^2+R^2)^{\frac{1}{2}}}\right]$$

21.7 – Uniformly charged thin flat disk



Hence

$$dE = \frac{z\sigma r dr}{2\epsilon_0 (z^2 + r^2)^{\frac{3}{2}}}$$

and then

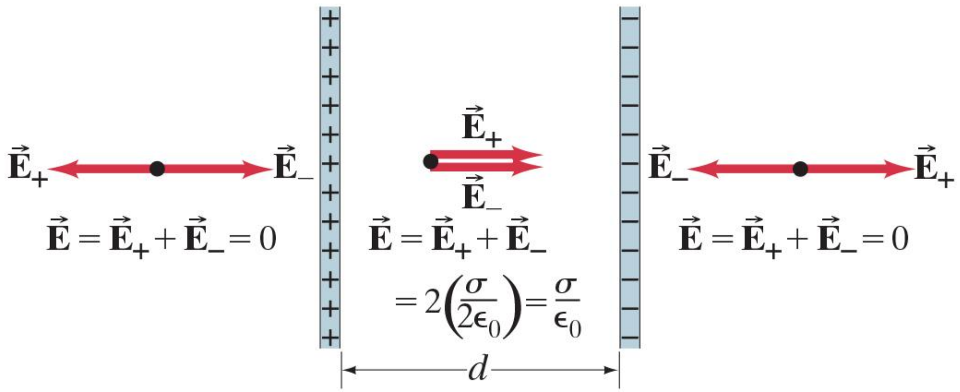
$$|E| = \frac{z\sigma}{2\epsilon_0} \int_0^R \frac{r dr}{(z^2 + r^2)^{\frac{3}{2}}} = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{(z^2+R^2)^{\frac{1}{2}}}\right]$$

What if $\mathrm{R} \gg z$, i.e., the disk radius is much larger than the distance where we compute the field?

$$|E| = \frac{\sigma}{2\epsilon_0}$$

Electric field close to an infinite plane (of any shape)

21.7 – Uniformly (oppositely) charged plates



Electric field in between (due to the opposite charge):

$$|E| = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

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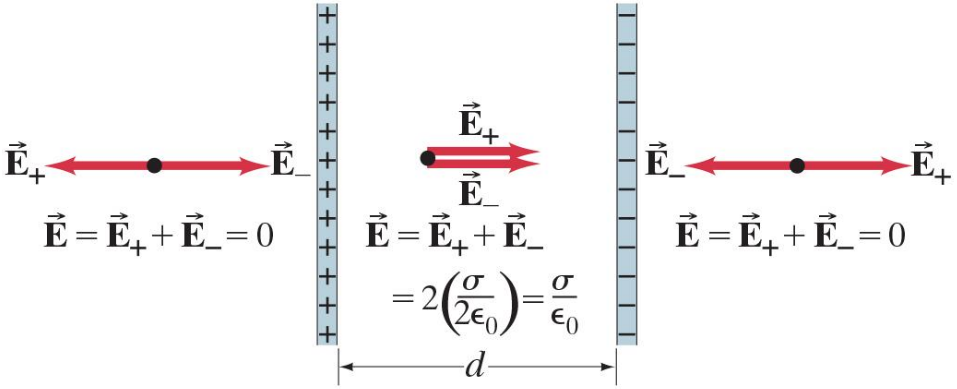

Electric field in between (due to the opposite charge):

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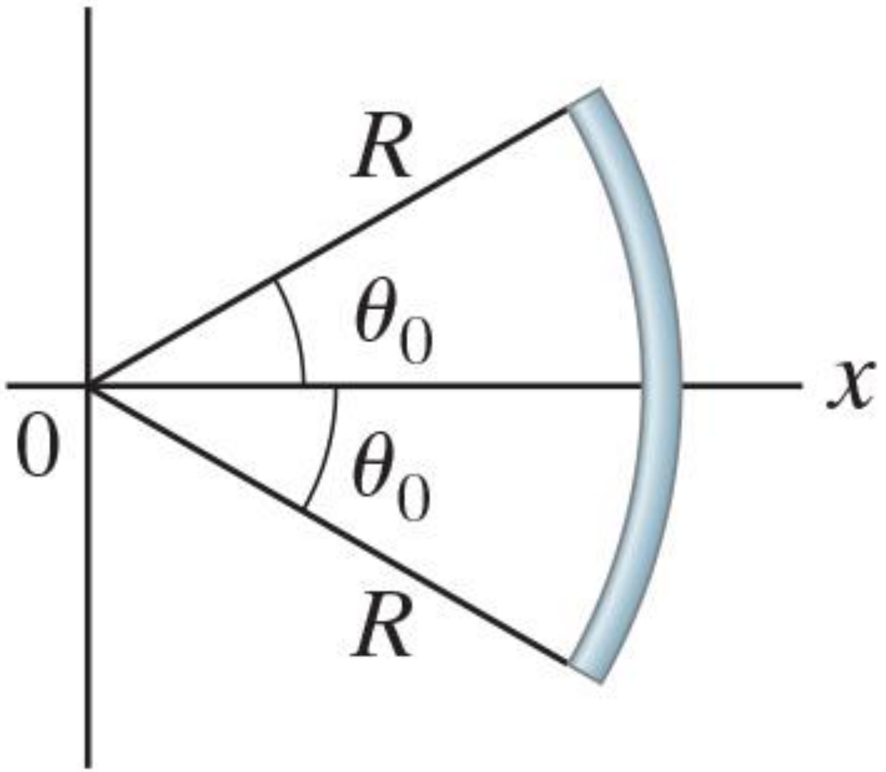
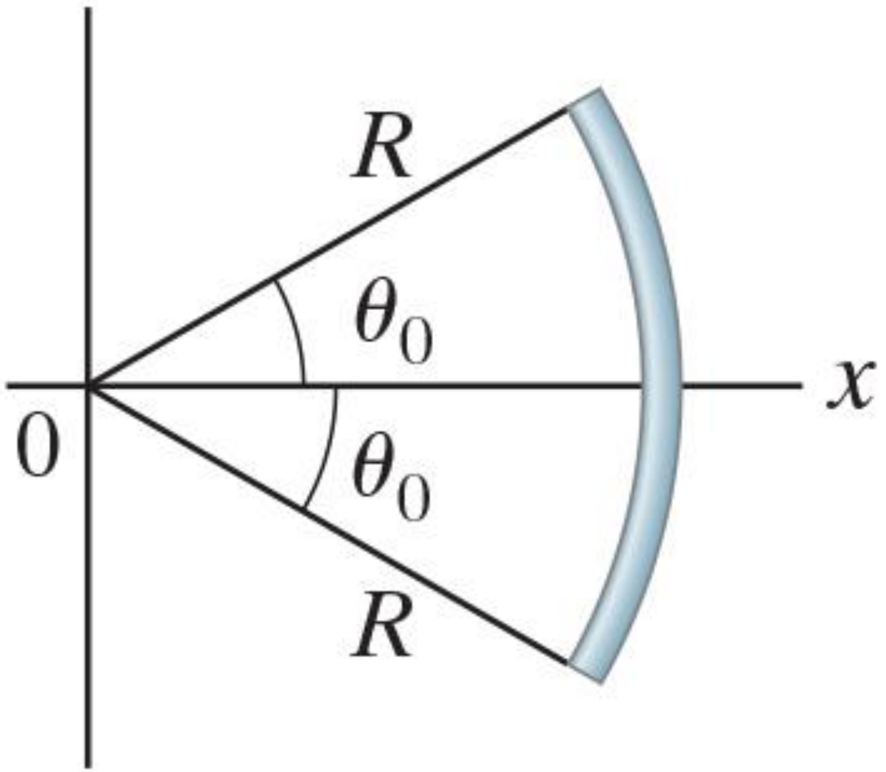
Underlying principle of a capacitor (more coming in the coming lectures)

21.7 – Portion of circular ring



Compute the electric field generated by a portion of circular wire in a point distant R from the wire (center of the circle). Some preliminary considerations:

21.7 – Portion of circular ring



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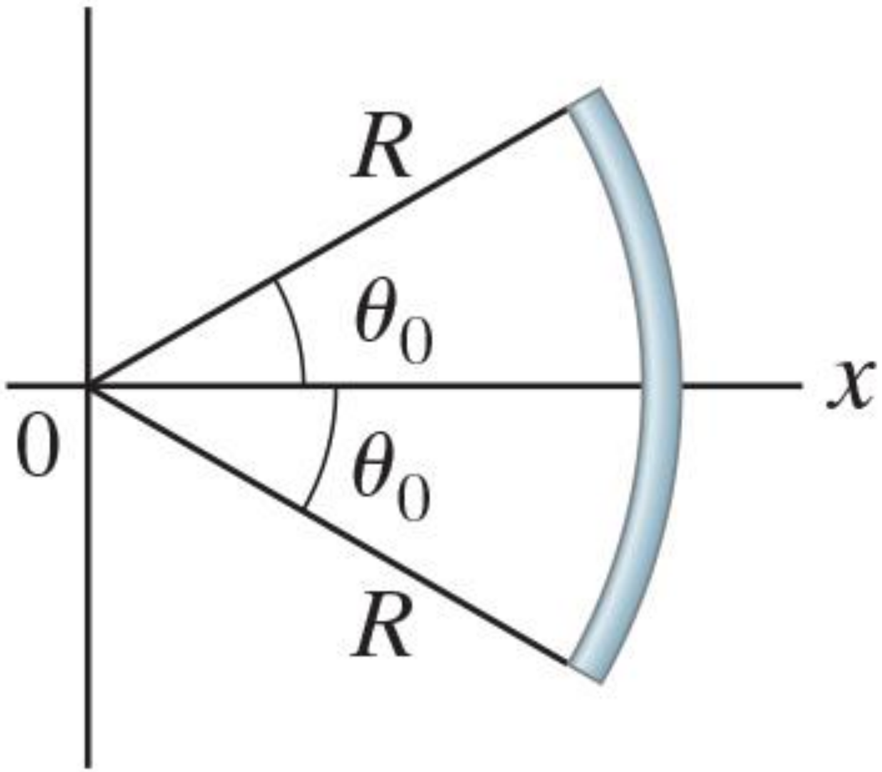
- λ is the charge per unit length

21.7 – Portion of circular ring



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Because for any contribution stemming from the half-plane above we have an equal contribution from the half-plane below, the only component of the electric field is along the x-axis (pointing left)

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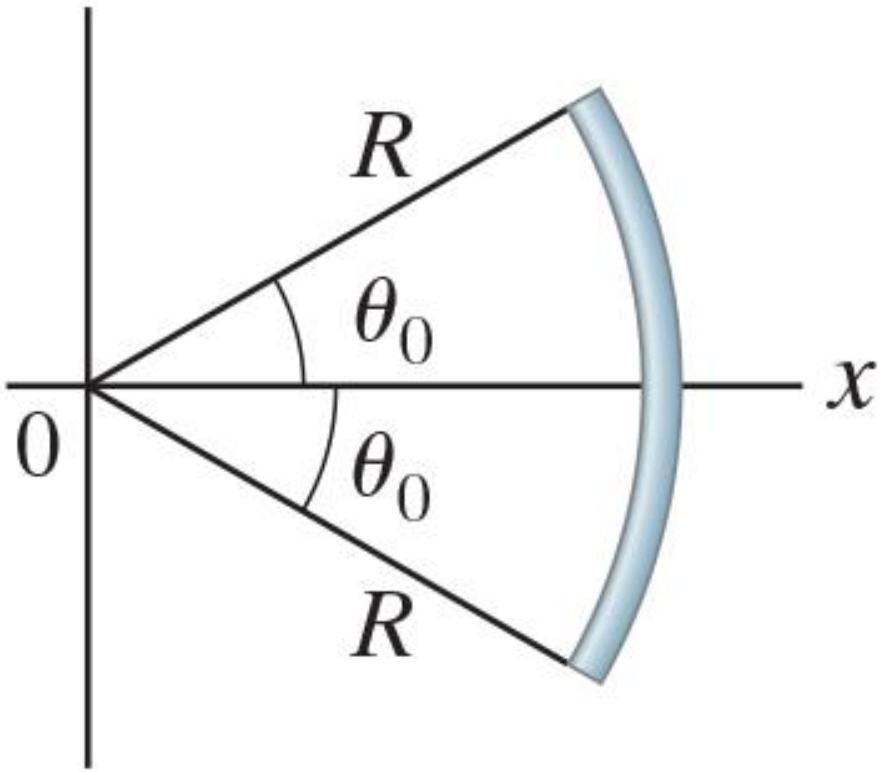

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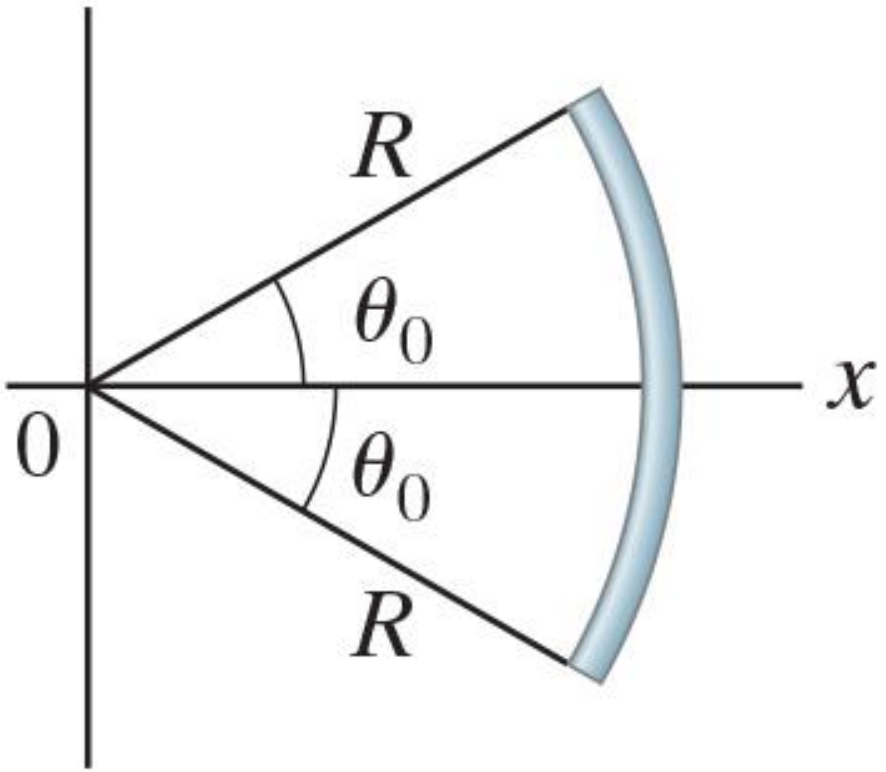
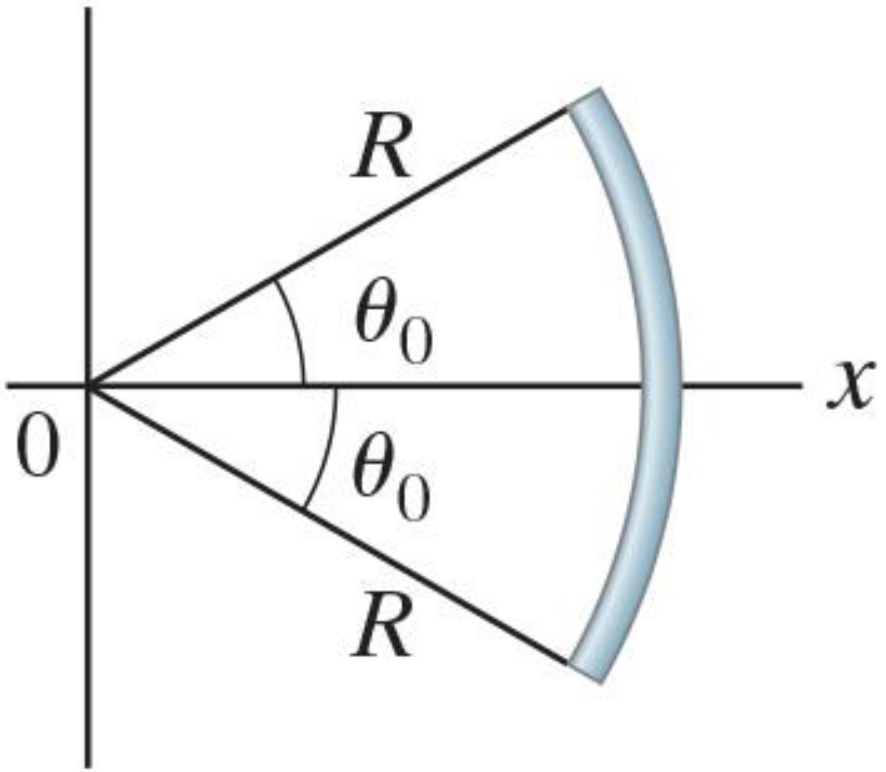
$$|\boldsymbol{E}| = |\boldsymbol{E}_x| = \int d\boldsymbol{E}_x$$

21.7 – Portion of circular ring



$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{\lambda R d\theta_0}{R^2}\cos\theta_0$$

21.7 – Portion of circular ring



$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{\lambda R d\theta_0}{R^2}\cos\theta_0$$

$$|E_x| = \frac{\lambda}{4\pi R\epsilon_0}\int_{-\theta_0}^{\theta_0}\cos\theta\, d\theta$$

Note: I switched from θ_0 to θ in the integral as formally we cannot use the same symbol as the extremes of integration

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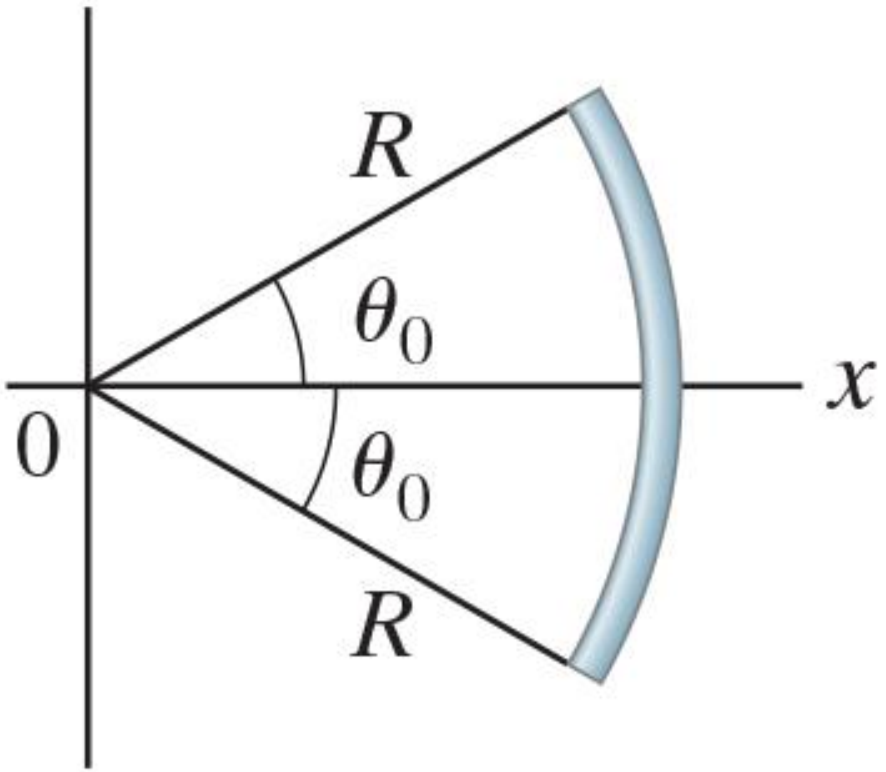

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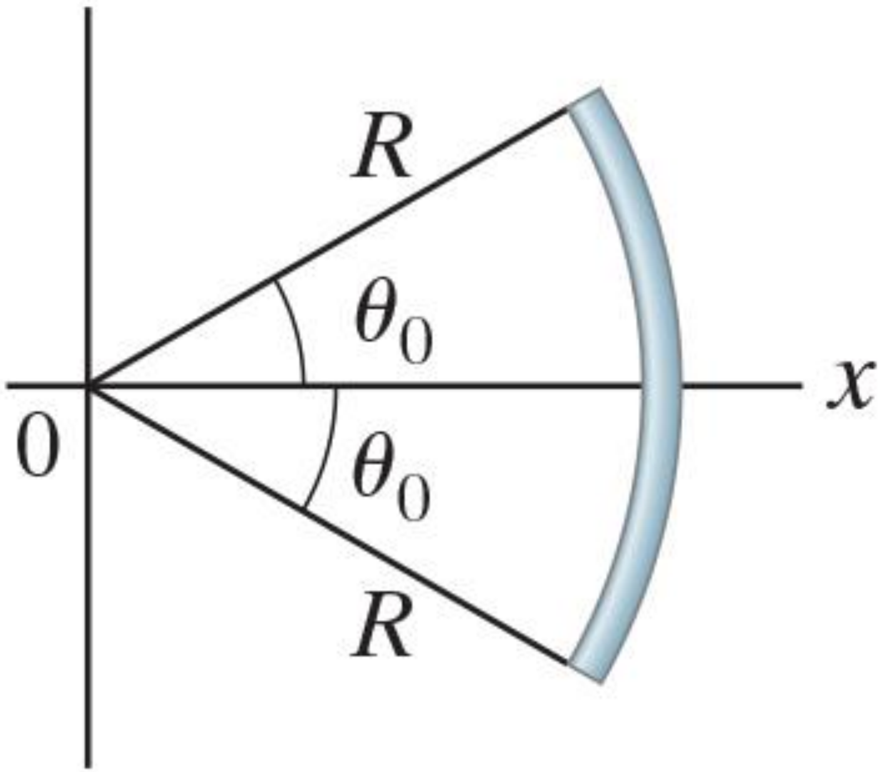

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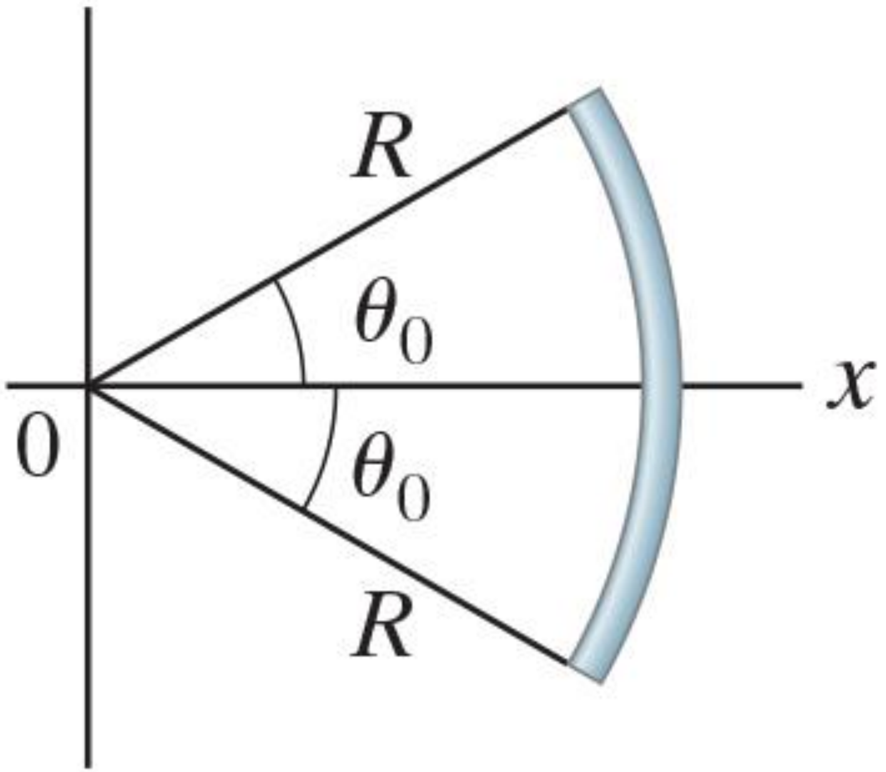
$$|E_x| = \frac{\lambda}{4\pi R\epsilon_0}2\int_{0}^{\theta_0}\cos\theta\, d\theta = \frac{\lambda\sin\theta_0}{2\pi R\epsilon_0}$$

The actual vectorial form is

$$E_x = -\frac{\lambda\sin\theta_0}{2\pi R\epsilon_0}\,\hat{\imath}$$

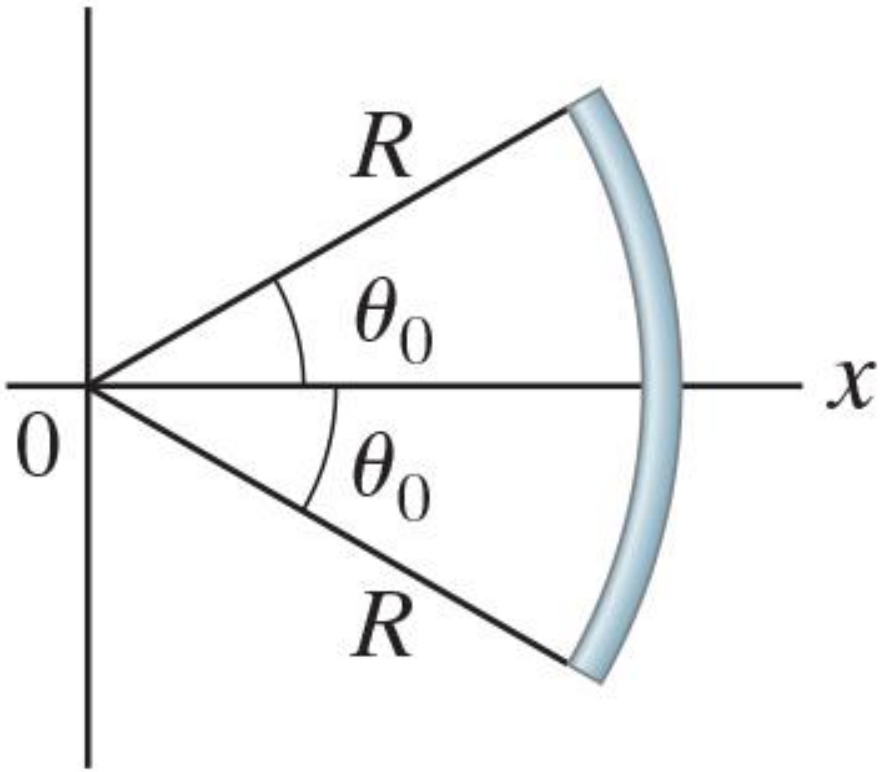
21.7 – Circular ring



$$\boldsymbol{E_x} = -\frac{\boldsymbol{\lambda} \sin \boldsymbol{\theta_0}}{\boldsymbol{2\pi R\epsilon_0}} \hat{\boldsymbol{\imath}}$$

For a circular ring we have that $\theta_0 = \pi$, hence the expression above yields 0: no electric field in the center of a uniformly distributed ring.

21.7 – Circular ring



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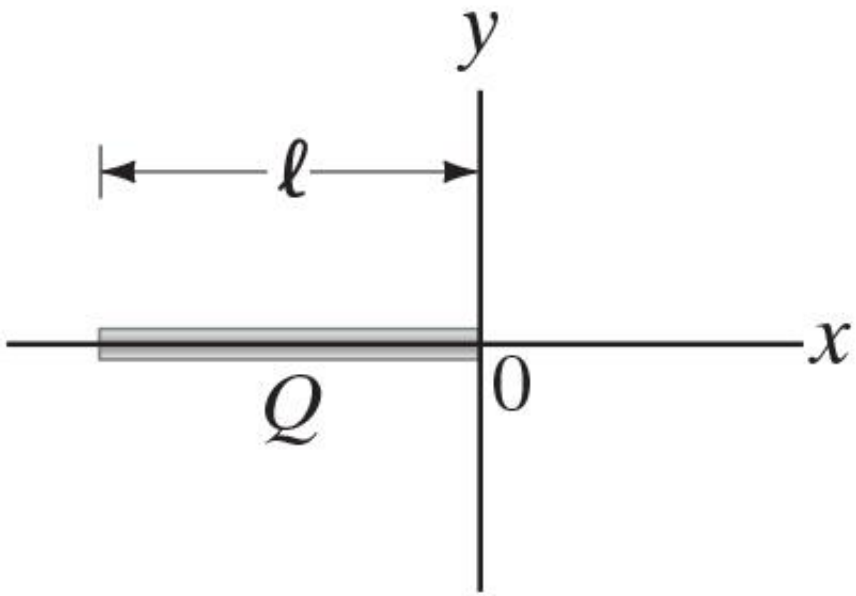
For a circular ring we have that $\theta_0 = \pi$, hence the expression above yields 0: no electric field in the center of a uniformly distributed ring.

This is also confirmed by the axial-symmetry of the problem. For every contribution $Rd\theta$ there is an opposite contribution diametrically opposite, so that the net contribution is 0.

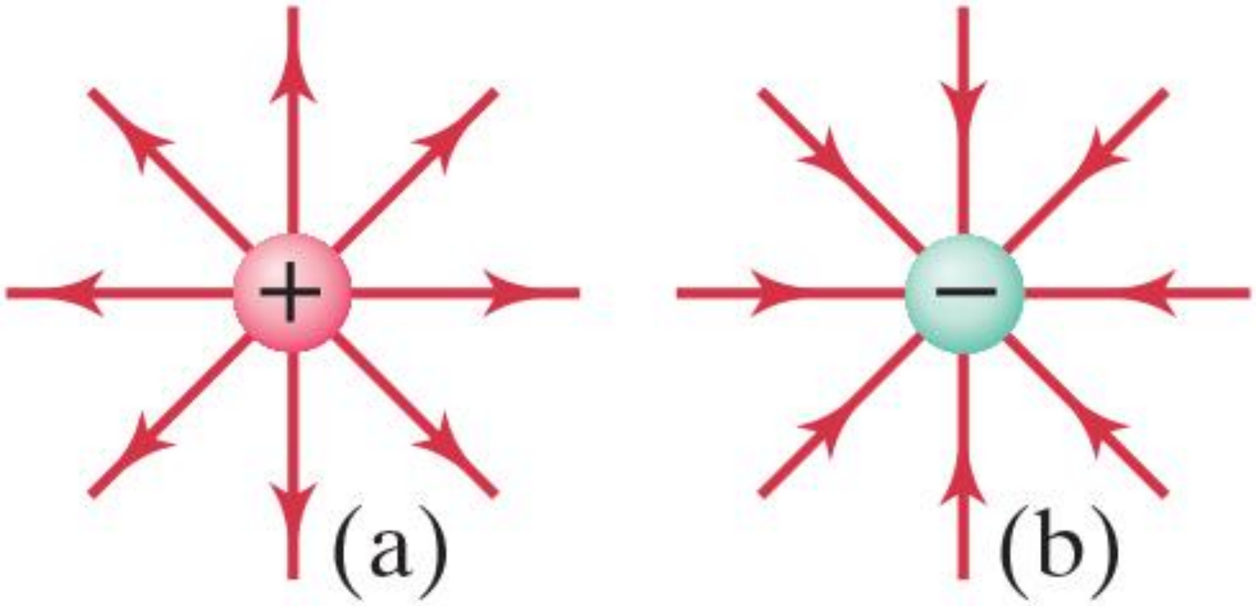
21.7 – Previous exam exercise



Determine electric field magnitude and direction due to a uniformly charged bar spanning from point $(-l, 0)$ to point $(-0,0)$ along the x-axis

21.8 – Field lines

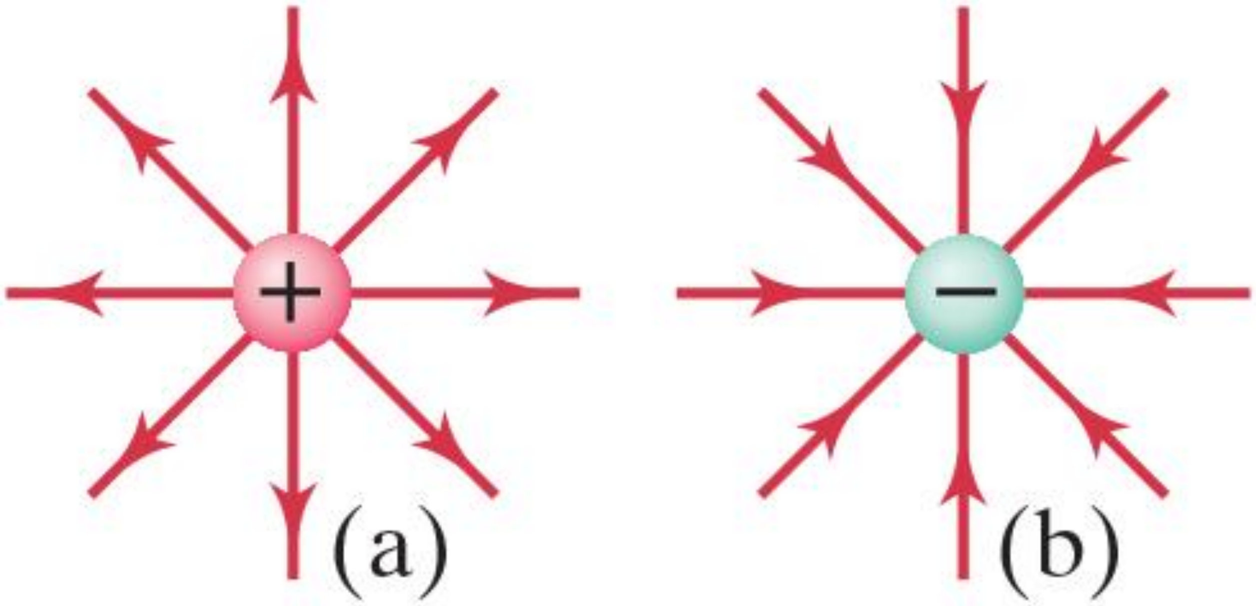
The electric field can be represented by field lines. These lines start on a positive charge and end on a negative charge.



21.8 – Field lines

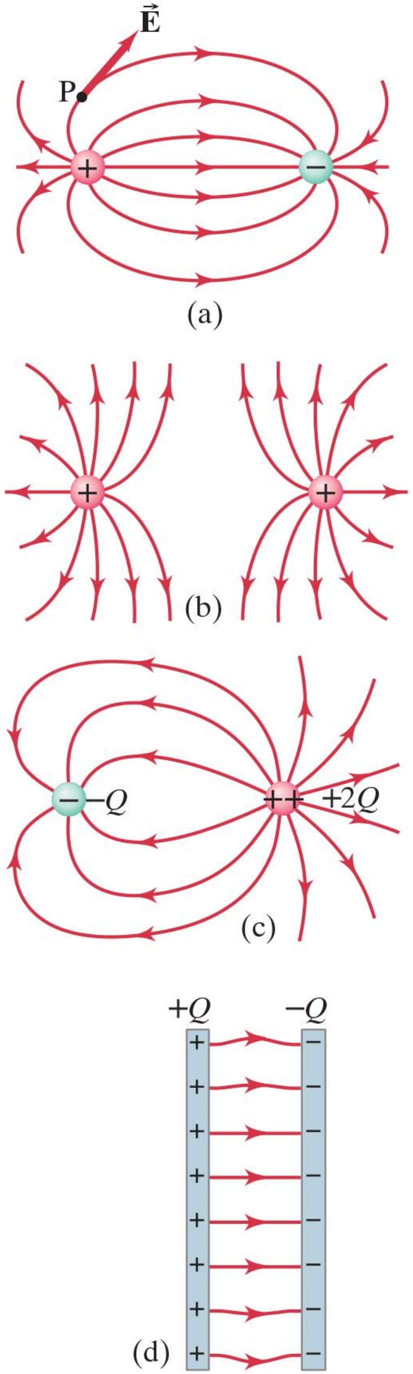
The electric field can be represented by field lines. These lines start on a positive charge and end on a negative charge.

- The number of field lines, starting (ending) on a positive (negative) charge, is proportional to the magnitude of the charge
- They indicate the direction of the electric field
- The electric field is stronger where the field lines are closer together



21.9 – Electric fields and Conductors

- Electric field equal to 0 inside a conductor



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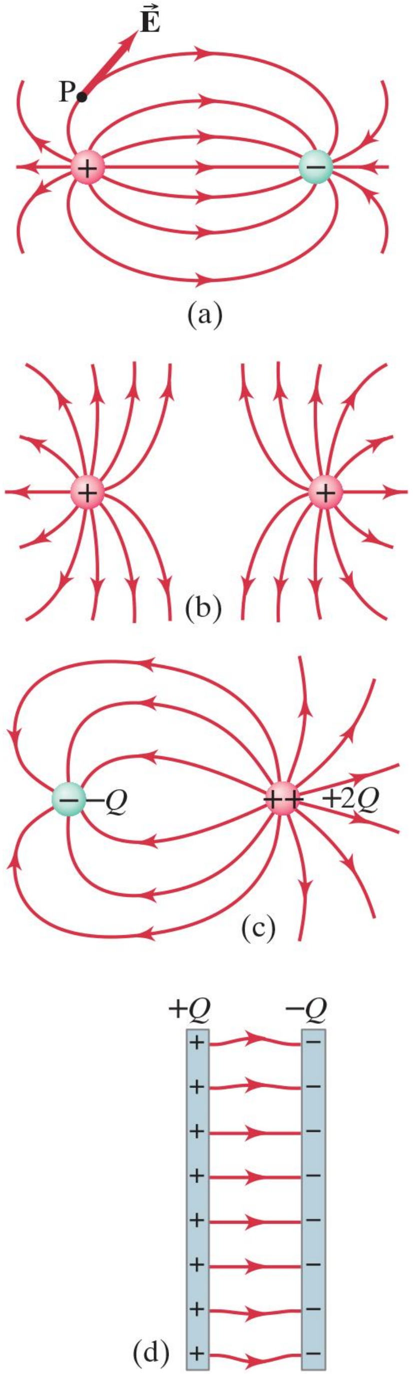
21.9 – Electric fields and Conductors

- Electric field equal to 0 inside a conductor
- Any net charge on a conductor distributes itself on the surface



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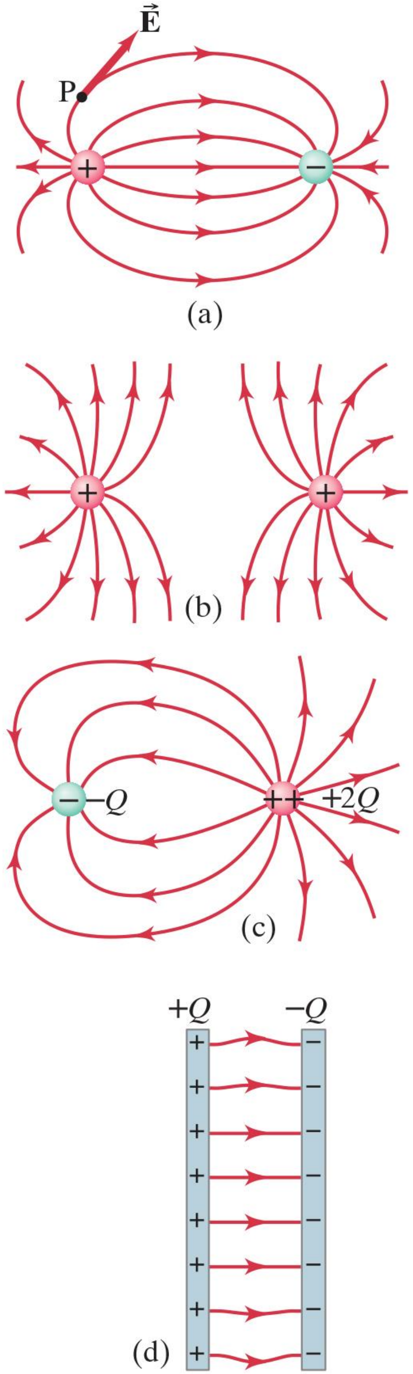
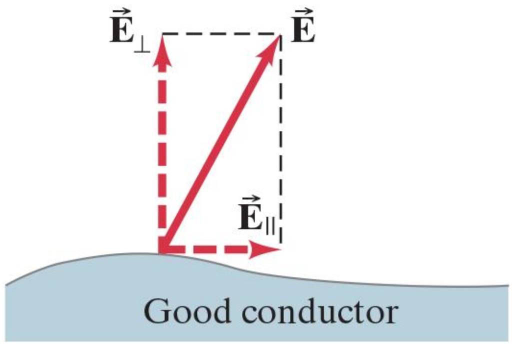


- Electric field equal to 0 inside a conductor
- Any net charge on a conductor distributes itself on the surface
- Electric field is always perpendicular to the surface outside of a conductor. If it was not, it would exert a force on the charges causing them to move to the equilibrium point (remember that $\vec{F} = q\,\vec{E}$)



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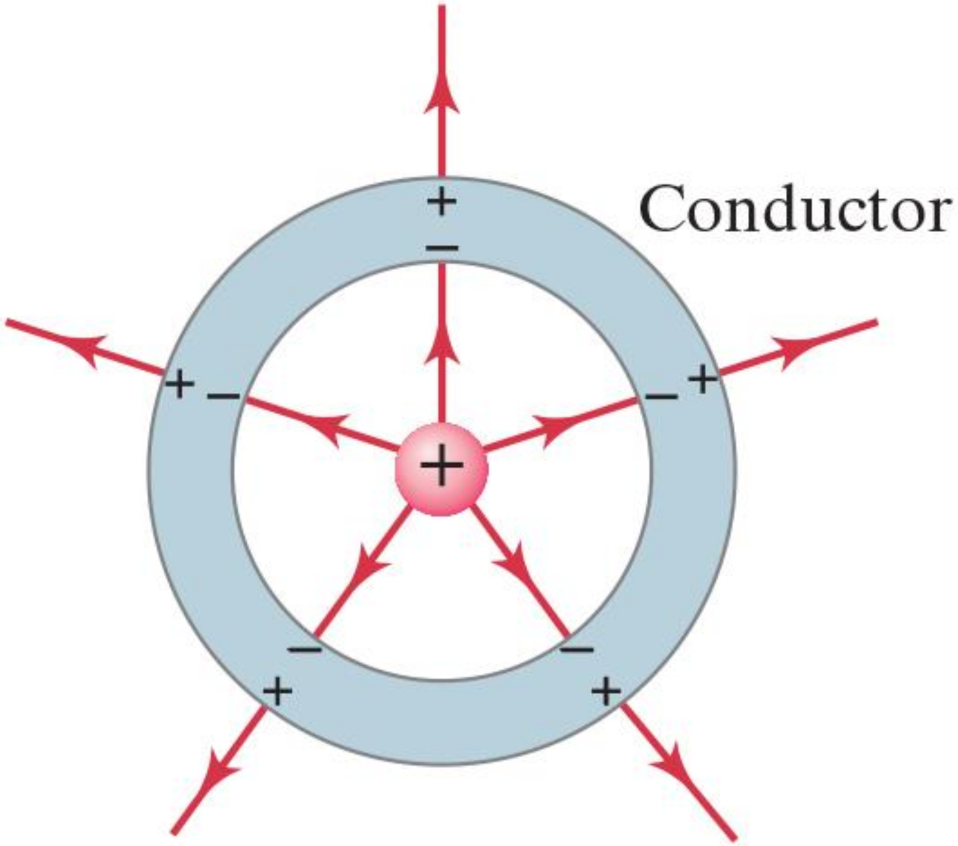

A charged particle ($+Q$) surrounded by an uncharged hollow metal conductor (spherical shell) induces an overall charge equal to $-Q$ and $+Q$ on the inner (resp. outer) side of the shell.

Hence, electric field lines stop on the inner side and start again on the outer side.

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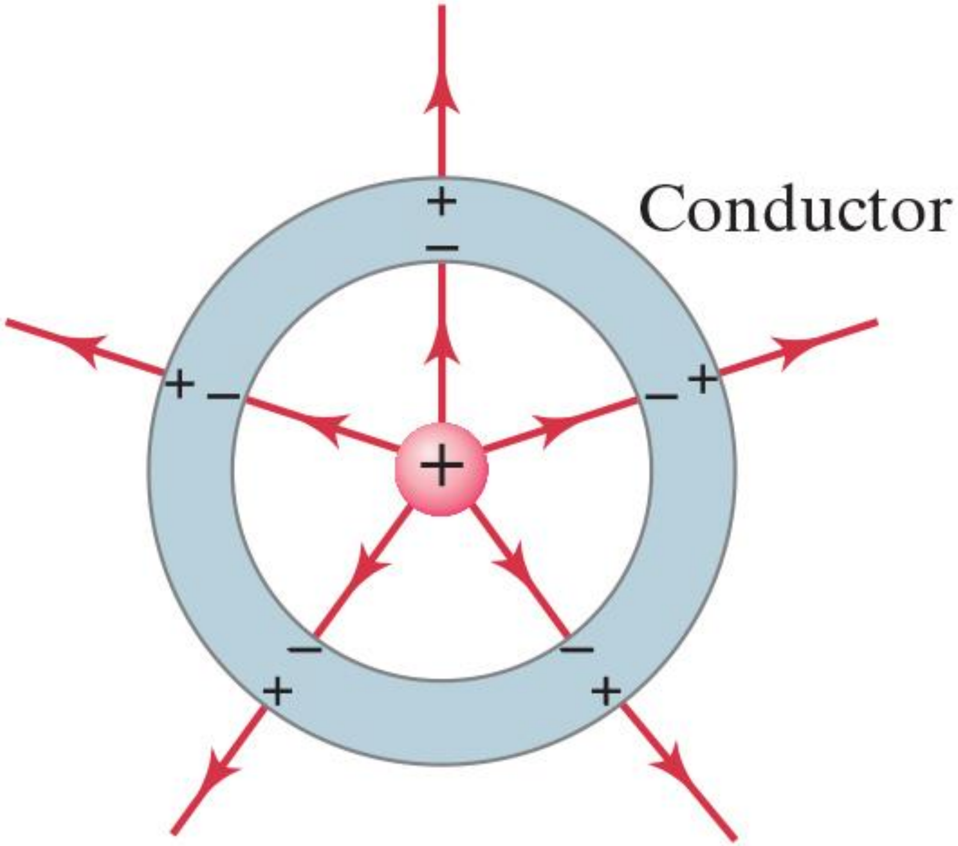

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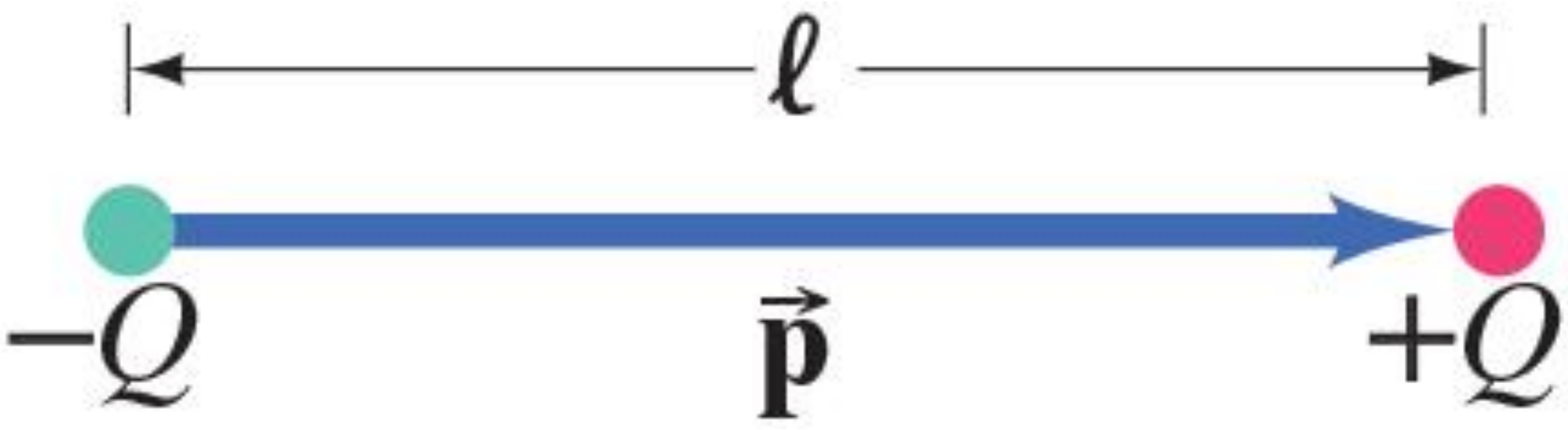
Hence, electric field lines stop on the inner side and start again on the outer side.

Outside the shell, it is as if the shell did not exist at all in terms of electric field lines

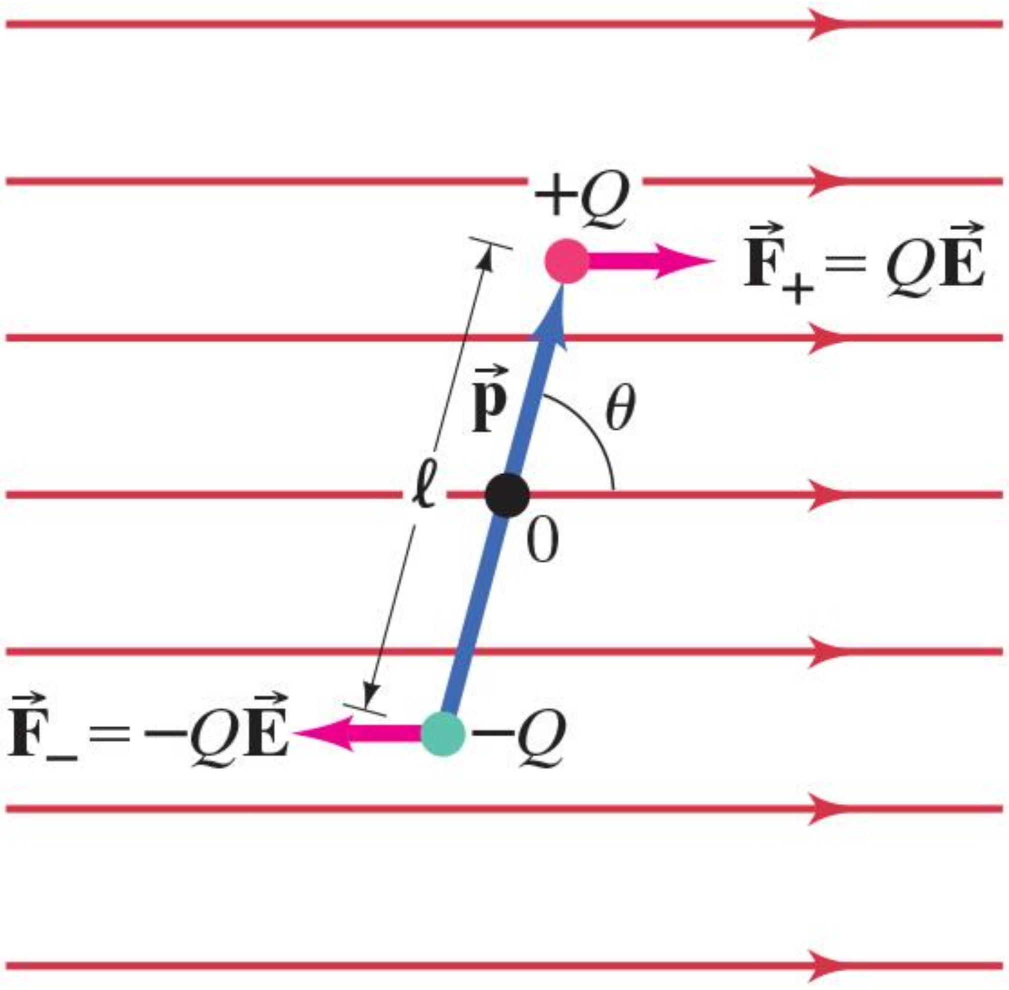
21.11 – Electric Dipole

An electric dipole consists of two charges Q, equal in magnitude and opposite in sign, separated by a distance l. The dipole moment, $\vec{p} = Q\vec{l}$, points from the negative to the positive charge.



21.11 – Electric Dipole

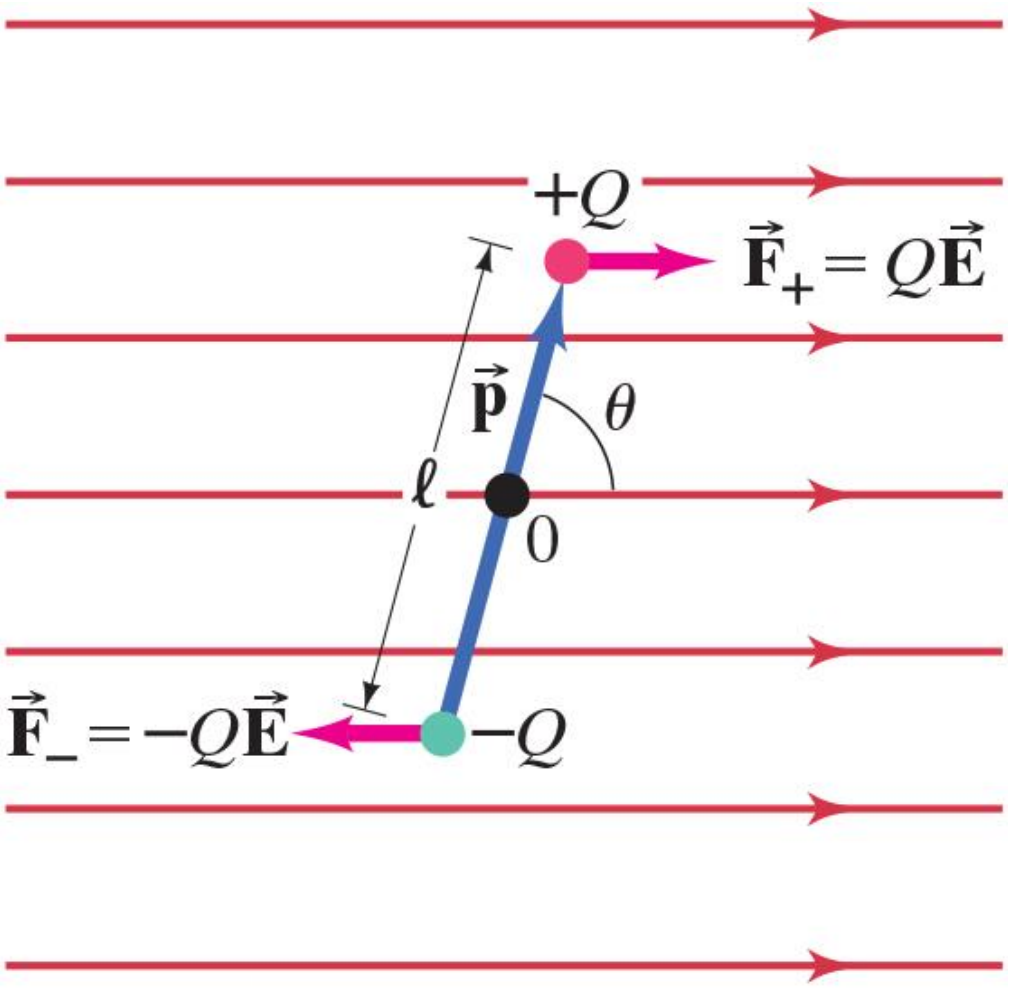
When immersed in a uniform electric field, a dipole experiences no net force, but a net torque



21.11 – Electric Dipole

When immersed in a uniform electric field, a dipole experiences no net force, but a net torque

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{p}} \times \vec{\boldsymbol{E}} \rightarrow |\vec{\tau}|$$
$$= \boldsymbol{QE}\frac{\boldsymbol{l}}{\boldsymbol{2}}\sin\boldsymbol{\theta} + \boldsymbol{QE}\frac{\boldsymbol{l}}{\boldsymbol{2}}\sin\boldsymbol{\theta}$$
$$= \boldsymbol{QEl}\sin\boldsymbol{\theta}$$

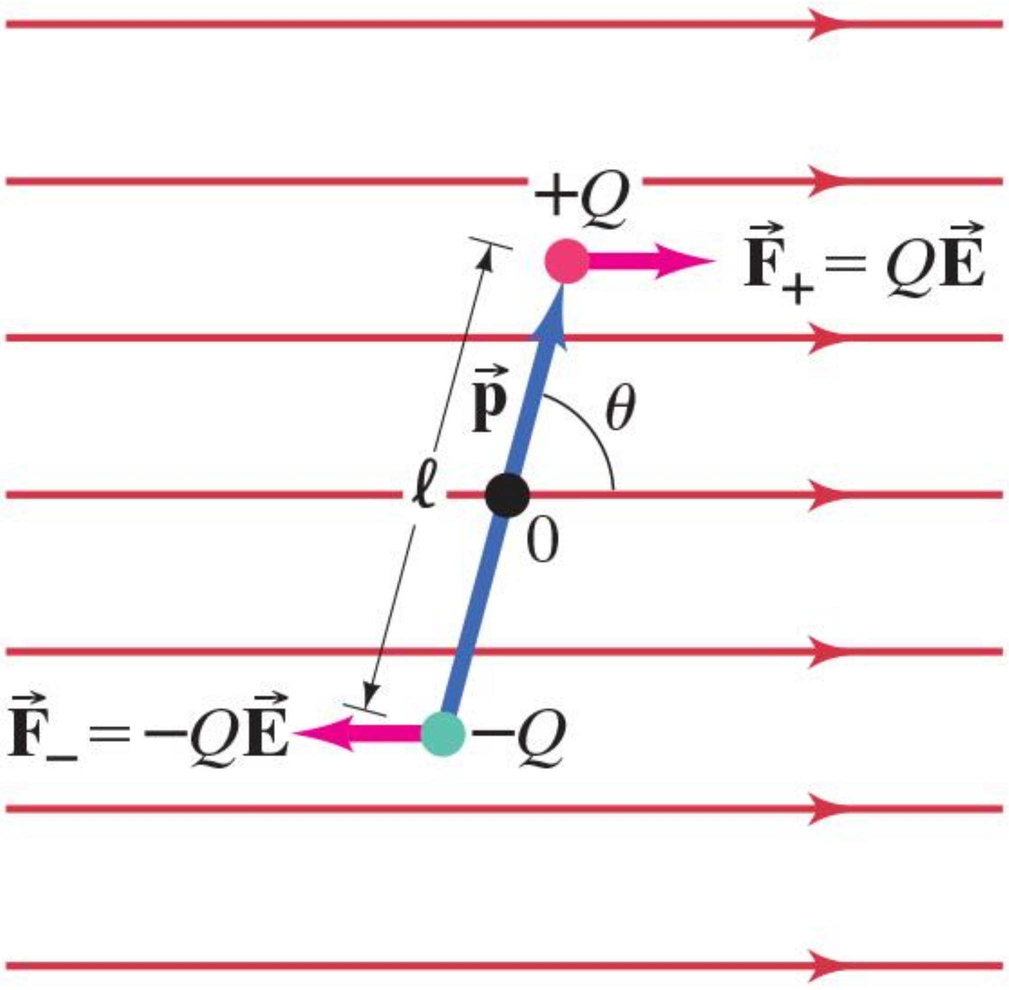


21.11 – Electric Dipole

The work done on the dipole by the electric field to move it from θ_1 to θ_2 is

$$\boldsymbol{W = \int_{\theta_1}^{\theta_2} \tau \, d\theta = -\int_{\theta_1}^{\theta_2} pE \sin\theta \, d\theta}$$

where $\tau = -pE\sin\theta$ as its direction (clockwise) is opposite to the direction of increase of θ.



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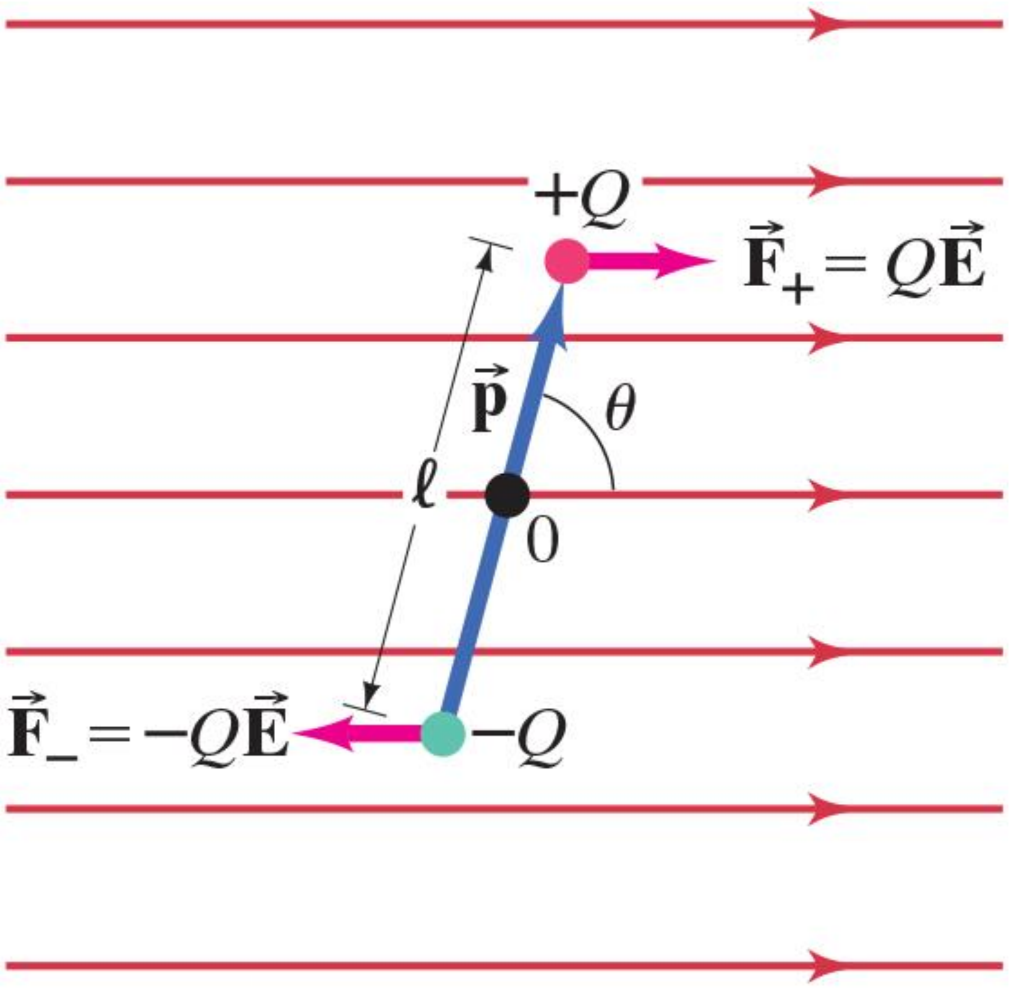
$$\boldsymbol{W = pE(\cos\theta_2 - \cos\theta_1)}$$

Positive work $(\theta_2 < \theta_1)$ reduces the potential energy U of the dipole



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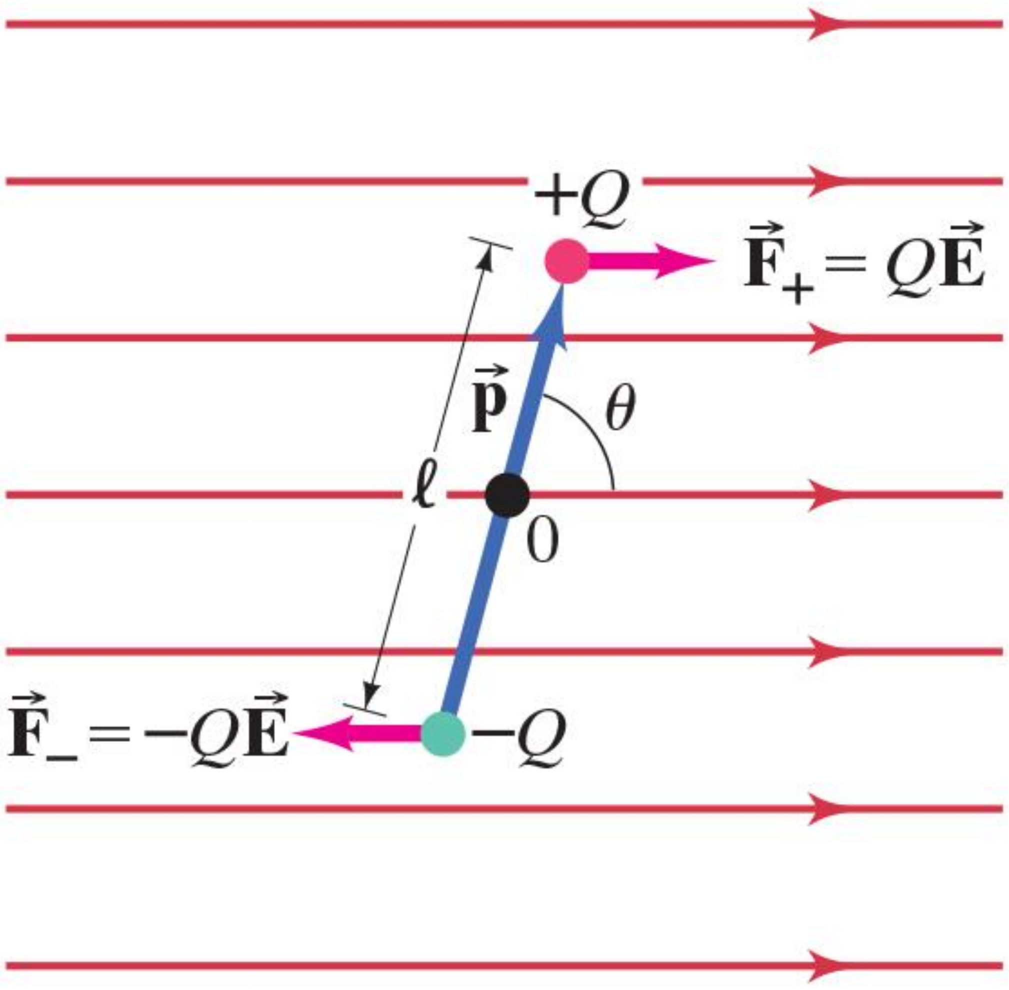

Example: Dipole Antenna
A **dipole antenna** consists of two conductive elements (such as metal rods or wires) separated by a small gap, where an alternating current is applied. This setup creates an oscillating electric dipole, which emits electromagnetic waves

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Wrap-up: revisiting learning objectives

After this lecture you should be able to:

- Apply **Coulomb's law** to determine the net electric force acting in a point due to a set of charges

- Determine the **electric field** in a point in space due to the effect of a point charge, set of point charges, or charge distribution

- Understand the concept of **electric dipole**

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Wrap-up: revisiting learning objectives

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- Apply **Coulomb's law** to determine the net electric force acting in a point due to a set of charges

$$\boldsymbol{F} = \frac{\boldsymbol{k\, Q_1 Q_2}}{\boldsymbol{r^2}}$$

Recall superimposition of effects

- Determine the **electric field** in a point in space due to the effect of a point charge, set of point charges, or charge distribution

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$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} \rightarrow E = \int dE$$

Recall to exploit symmetries

- Understand the concept of **electric dipole**

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- Understand the concept of **electric dipole**

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Opposite charges immersed in electric field. No net force, but net torque