

# SOUND

## *Chapter 16*



**Dr. Roberto Merino-Martinez**

Operations & Environment section

Faculty of Aerospace Engineering

# Position in the syllabus

14. Oscillations

15. Waves



16. Sound

17. Temperature and the ideal gas law

18. Thermodynamics

19. Electricity and circuits

20. Electromagnetism

21. Optics

# Structure of the lecture

1. Characteristics of sound
2. Mathematical representation of longitudinal waves
3. Intensity of sound: Decibels
4. Sources of sound: Vibrating strings and air columns
5. Quality of sound and noise: Superposition
6. Interference of sound waves, Beats
7. Doppler effect
8. Shock waves and sonic boom
9. Applications: Sonar, ultrasound, and medical imaging

# Learning objectives for today's lecture

After this lecture you should be able to:



- Describe the **main characteristics** of sound waves



- Calculate the **mathematical representation** of longitudinal waves



- Quantify the **intensity of sound** using the decibel scale

# Assumed prior knowledge

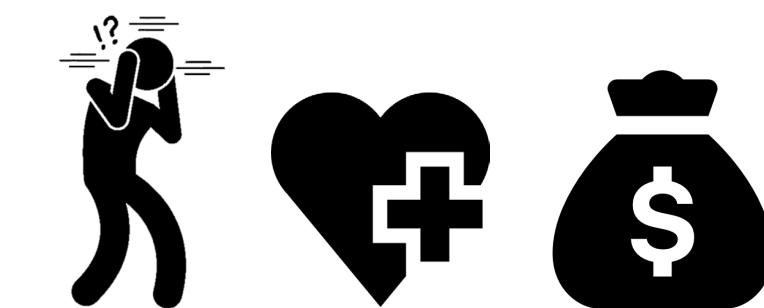


- Basic trigonometry (cosine, etc.)
- Basic math (**logarithm**)
- Basic mechanics and kinematics (Newton's laws, etc.)
- Differential equations
- Concepts learned in **Chapters 14 (Oscillations) and 15 (Wave motion)**

# Very relevant issue in aerospace engineering!



# And a very hot topic nowadays



Nieuws

## Vliegtuiglawaaï Schiphol kwelt omgeving: ‘Continu overlast, daar worden mensen gek van’

Omwonenden van luchthavens zijn gevoeliger voor vliegtuighet en twintig jaar geleden. Dat blijkt uit onderzoek van het RIVM. Het geluid van opstijgende en landende vliegtuigen zorgt vaker voor ergernis, frustratie en een onderbroken nachtrust.

Edwin Timmer 21 februari 2023, 09:29

**Het Parool**

2 minute read · June 24, 2022 6:14 PM GMT+2 · Last Updated a year ago

REUTERS®

## Schiphol flights to be limited to 11% below 2019 levels to cut noise, emissions

FRIDAY, 21 APRIL 2023 - 07:00

Thousands living near Schiphol will be eligible for noise disturbance compensation

NL#TIMES

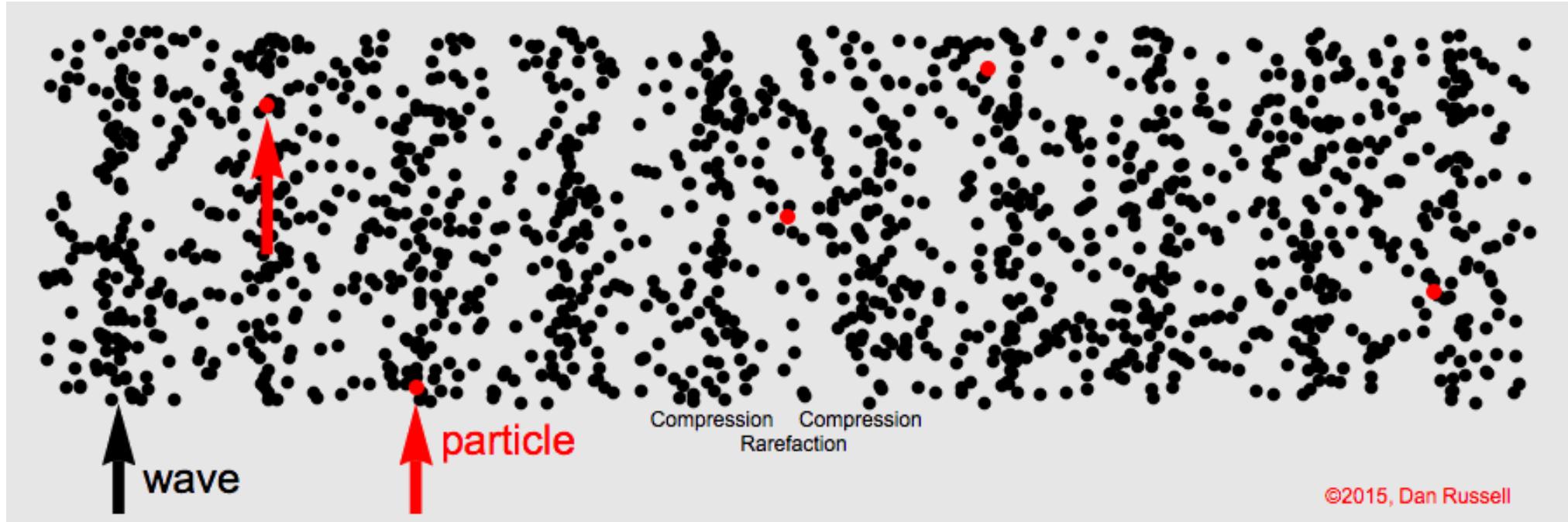
1 minute read · April 3, 2023 10:37 PM GMT+2 · Last Updated a month ago

REUTERS®

## Amsterdam's Schiphol Airport to cut late-night flights -Het Parool

# 16.1 – Characteristics of sound

Sound is a **longitudinal wave** that can travel through any kind of **matter**.



The (oscillation) velocity of a **particle** is not equal to the **wave velocity**!

See: <https://musiclab.chromeexperiments.com/Sound-Waves/>

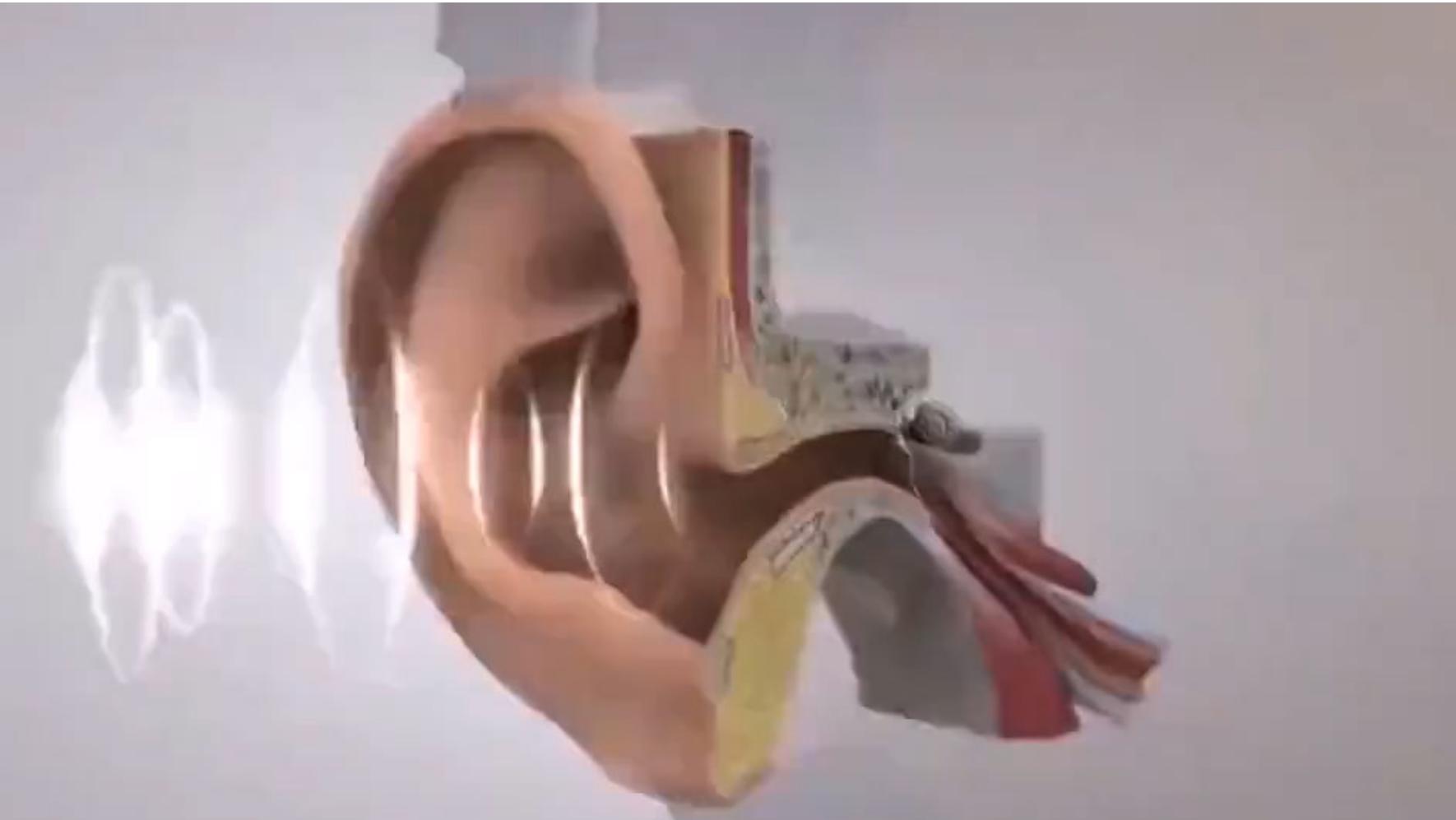
# 16.1 – Sound is a pressure wave



Source: Twitter @Sci\_Phile

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# 16.1 – How sound is perceived by humans



# 16.1 – Sound needs matter to propagate!

Sound is a **longitudinal wave** that can travel through any kind of **matter**, i.e. sound does **not** propagate through vacuum.



© <https://tenor.com>

# 16.1 – Speed of sound

**TABLE 16–1 Speed of Sound in Various Materials (20°C and 1 atm)**

Material	Speed (m/s)
Air	343
Air (0°C)	331
Helium	1005
Hydrogen	1300
Water	1440
Sea water	1560
Iron and steel	≈5000
Glass	≈4500
Aluminum	≈5100
Hardwood	≈4000
Concrete	≈3000

The speed of sound is different in different materials; in general, it is slowest in gases, faster in liquids, and fastest in solids.

$$v = \sqrt{\frac{\text{elastic force}}{\text{inertia}}}$$

Solids:  $v = \sqrt{\frac{E}{\rho}}$

Fluids:  $v = \sqrt{\frac{B}{\rho}}$

The propagation velocity depends somewhat on the **temperature**, especially for gases.

The speed of sound in air is

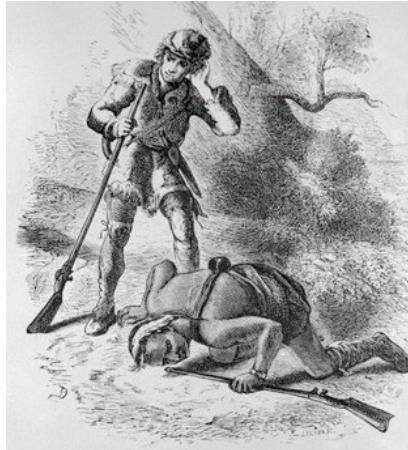
$$v = (331 + 0.60 T) \text{ m/s}$$

with  $T$  the temperature in °C

# 16.1 – Speed of sound

**TABLE 16–1 Speed of Sound in Various Materials (20°C and 1 atm)**

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Aluminum	≈5100
Hardwood	≈4000
Concrete	≈3000



$$v_{\text{ground}} \gg v_{\text{air}}$$

© <https://www.gettyimages.nl/>



© <https://www.businessinsider.com>

$$v_{\text{Helium}} > v_{\text{air}}$$

$$f = \frac{v}{\lambda}$$

$$f_{\text{Helium}} > f_{\text{air}}$$

$$\lambda = \text{constant}$$

# 16.1 – Characteristics of sound - Definitions

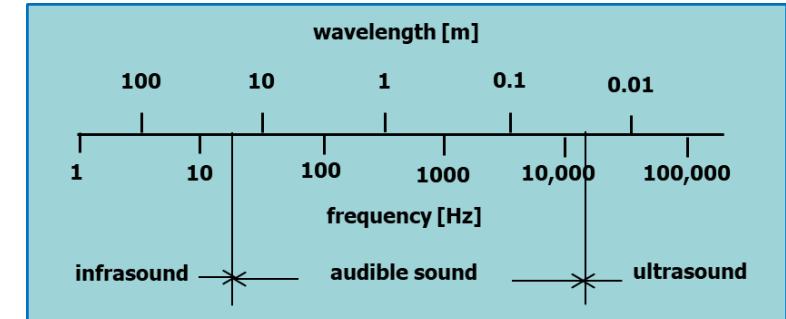
- **Loudness** is related to the intensity of the sound wave.
- **Pitch** is related to the frequency of sound.
- The audible human range goes from about **20 Hz** to **20 kHz**. The upper limit decreases with age.

- Ultrasound is above 20 kHz

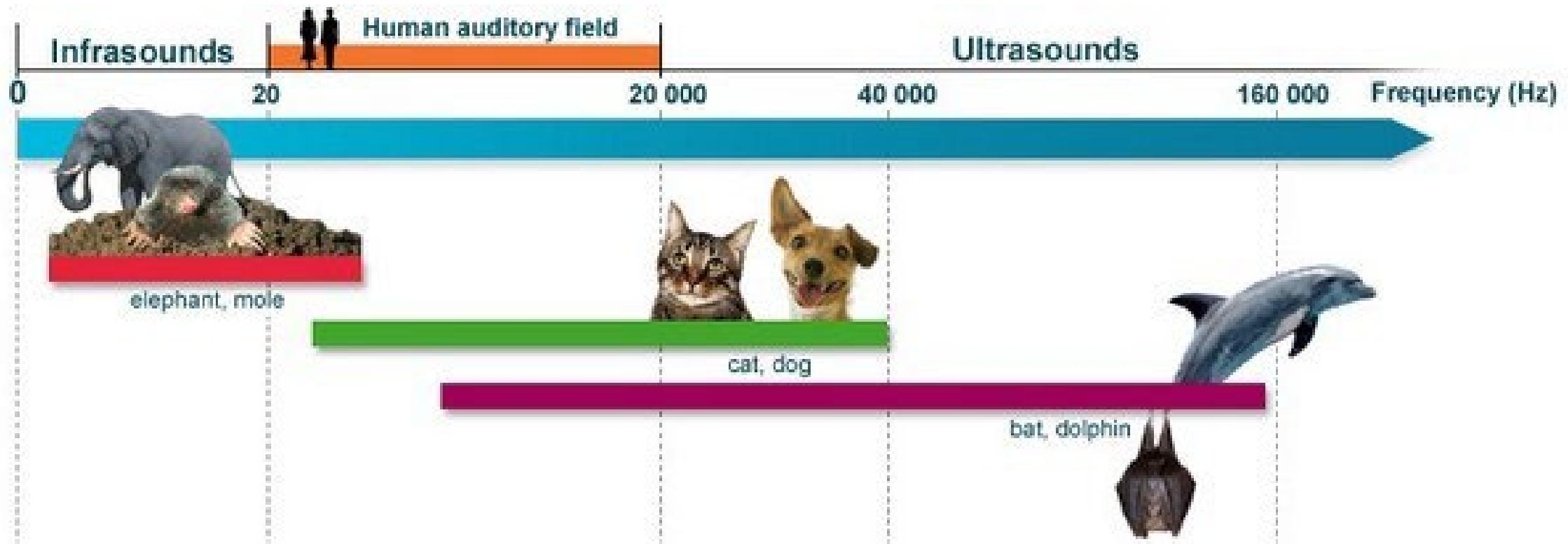
$$\lambda = \frac{343}{20,000} = 17 \text{ mm}$$
$$v = \frac{\lambda}{T} = \lambda f$$

- Infrasound is below 20 Hz

$$\lambda = \frac{343}{20} = 17 \text{ m}$$

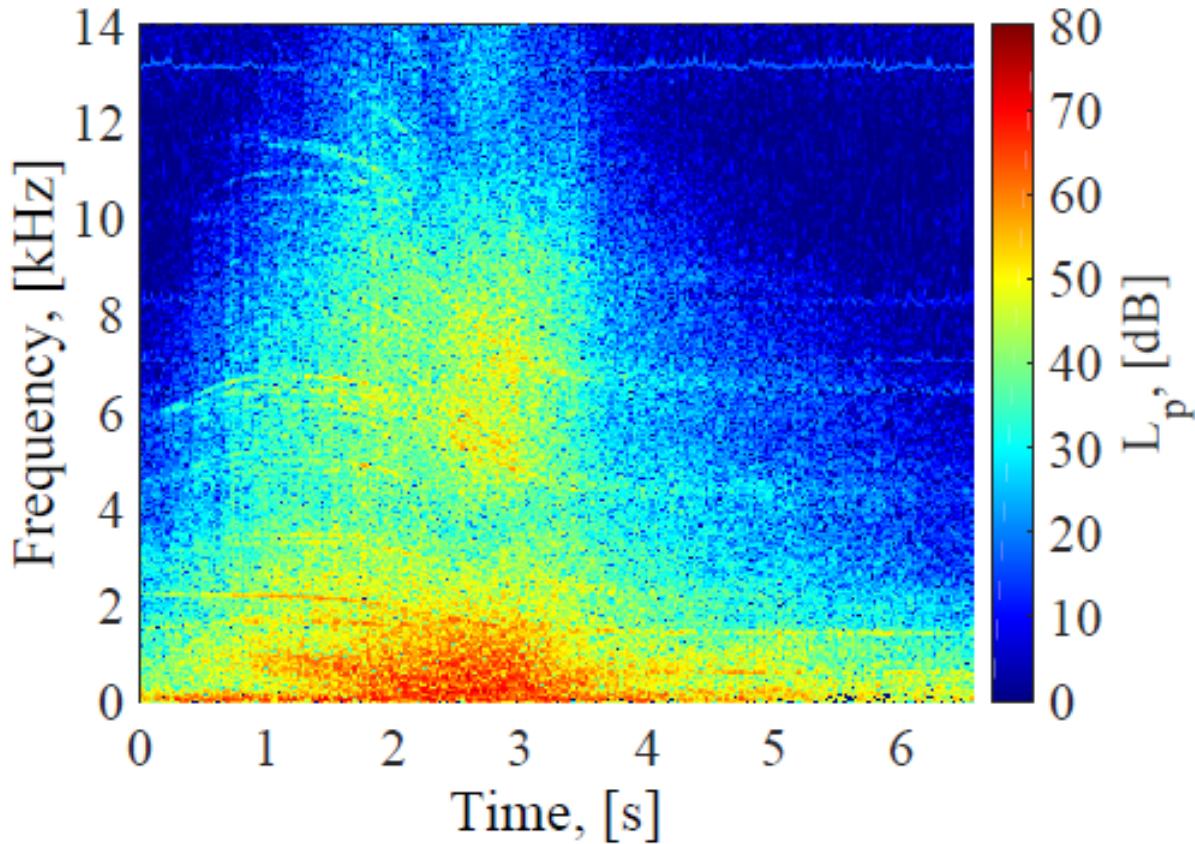


# 16.1 – Hearing frequency range



© <https://www.cochlea.org>

# 16.1 – Sound visualization (time and frequency)



An spectrogram enables us to simultaneously visualize sound in terms of **time** and **frequency** (remember Fourier transform from chapter 15).

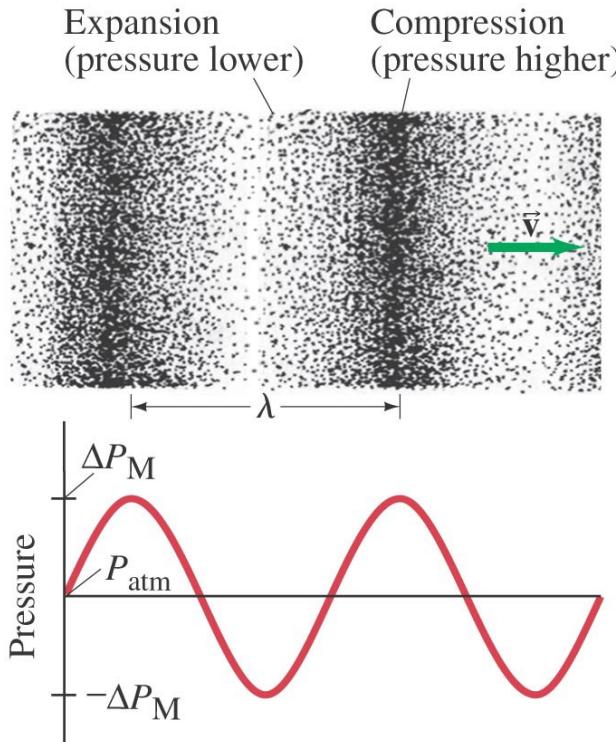
MATLAB, Python, etc. have built-in spectrogram functions.

A nice online tool (no installation) is:

<https://musiclab.chromeexperiments.com/spectrogram>

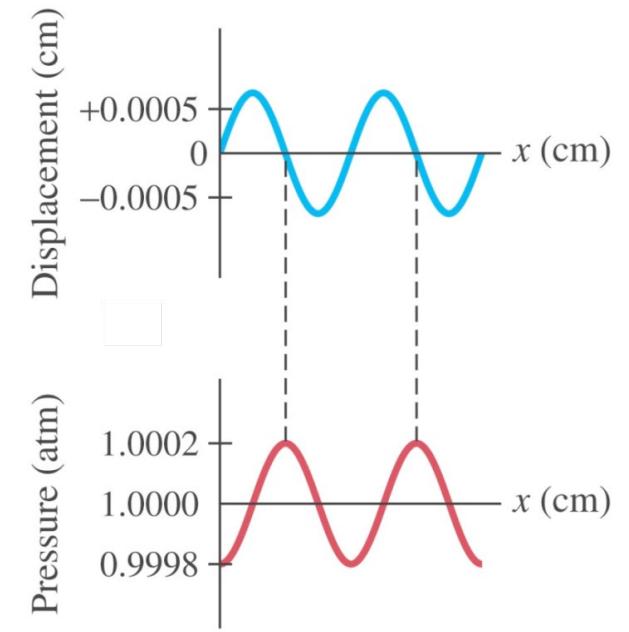
## 16.2 – Math. representation of longitudinal waves

Longitudinal waves are often called pressure waves. For sound in air, we consider the pressure fluctuations  $\Delta P$  with respect to the atmospheric pressure:



The particle displacement  $D(x, t)$  is  $90^\circ$  out of phase with respect to the pressure:

$$D(x, t) = A \sin(kx - \omega t)$$

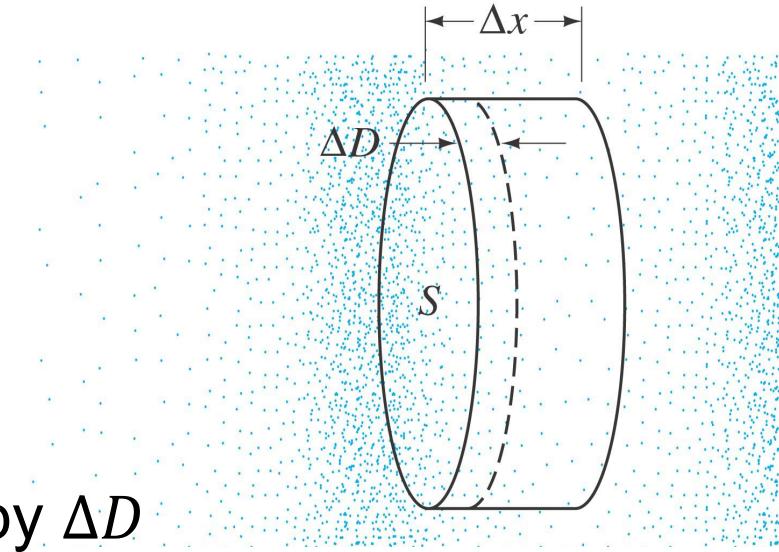


## 16.2 – Math. representation of longitudinal waves

Similarly as in Chapter 15 for the wave equation, we now consider a small cylindrical region within the fluid of base surface  $S$  and thickness  $\Delta x$ . The volume of the cylinder is, therefore,  $V = S\Delta x$ .

Using the definition of the bulk modulus  $B$  of the fluid:

$$B = -V \frac{\Delta P}{\Delta V} \quad \Delta P = -B \frac{\Delta V}{V} = -B \frac{S\Delta D}{S\Delta x}$$



The sound wave modifies the thickness of our cylinder by  $\Delta D$

Taking  $\Delta x \rightarrow 0$ :

$$\Delta P = -B \frac{\partial D}{\partial x}$$

## 16.2 – Math. representation of longitudinal waves

$$\Delta P = -B \frac{\partial D}{\partial x}$$

$$D(x, t) = A \sin(kx - \omega t)$$

$$\Delta P = -BAk \cos(kx - \omega t)$$

$$\Delta P_{max} = BAk$$

And hence the  
90° phase shift:

$$v = \sqrt{\frac{B}{\rho}} = \frac{\omega}{k}$$

$$\Delta P_{max} = \rho v^2 A \frac{\omega}{v} = 2\pi f \rho v A$$

$$v = \sqrt{\frac{B}{\rho}} = \frac{\omega}{k}$$

Acoustic impedance

# 16.3 – Intensity of sound: Decibels

From chapter 15 (Wave motion) we remember that:

$$I = \frac{P}{S} = 2\pi^2 \rho v f^2 A^2, \quad [\text{W/m}^2]$$

$$\Delta P_{max} = 2\pi f \rho v A \quad A = \frac{\Delta P_{max}}{2\pi f \rho v}$$

$$I = 2\pi^2 \rho v f^2 \left( \frac{\Delta P_{max}}{2\pi f \rho v} \right)^2 = \frac{\Delta P_{max}^2}{2v\rho}$$

# 16.3 – Intensity of sound: Decibels

$$I = \frac{P}{S} = 2\pi^2 \rho v f^2 A^2, \quad [\text{W/m}^2]$$

The human ear can detect sounds with an intensity as low as  $10^{-12} \text{ W/m}^2$  (**threshold of hearing**) and as high as  $1 \text{ W/m}^2$  (**threshold of pain**).

The perceived loudness of sound, however, is **not** proportional to the intensity in  $\text{W/m}^2$ .

**TABLE 16–2**  
**Intensity of Various Sounds**

Source of the Sound	Sound Level (dB)	Intensity ( $\text{W/m}^2$ )
Jet plane at 30 m	140	$100$
Threshold of pain	120	$1$
Loud rock concert	120	$1$
Siren at 30 m	100	$1 \times 10^{-2}$
Truck traffic	90	$1 \times 10^{-3}$
Busy street traffic	80	$1 \times 10^{-4}$
Noisy restaurant	70	$1 \times 10^{-5}$
Talk, at 50 cm	65	$3 \times 10^{-6}$
Quiet radio	40	$1 \times 10^{-8}$
Whisper	30	$1 \times 10^{-9}$
Rustle of leaves	10	$1 \times 10^{-11}$
Threshold of hearing	0	$1 \times 10^{-12}$

## 16.3 – Sound intensity level

Instead, the loudness is much more closely related to the **logarithm (in base 10)** of the intensity.

The sound intensity level (SIL – in the book they call it  $\beta$ ) is measured in **decibels** (dB) and is defined as:

$$SIL = 10 \log \frac{I}{I_0}$$

where  $I$  is the sound intensity and  $I_0$  is the sound intensity corresponding to the threshold of hearing ( $I_0 = 10^{-12} \text{ W/m}^2$ ).

**TABLE 16–2**  
**Intensity of Various Sounds**

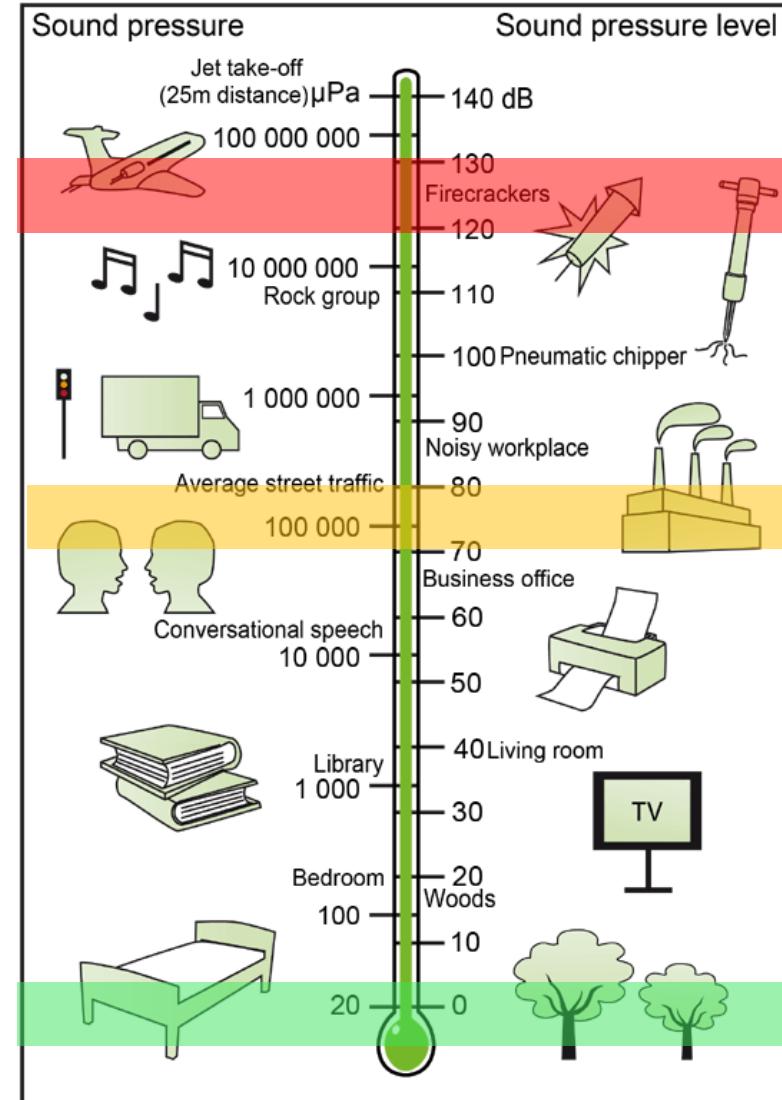
Source of the Sound	Sound Level (dB)	Intensity (W/m <sup>2</sup> )
Jet plane at 30 m	140	100
Threshold of pain	120	1
Loud rock concert	120	1
Siren at 30 m	100	$1 \times 10^{-2}$
Truck traffic	90	$1 \times 10^{-3}$
Busy street traffic	80	$1 \times 10^{-4}$
Noisy restaurant	70	$1 \times 10^{-5}$
Talk, at 50 cm	65	$3 \times 10^{-6}$
Quiet radio	40	$1 \times 10^{-8}$
Whisper	30	$1 \times 10^{-9}$
Rustle of leaves	10	$1 \times 10^{-11}$
Threshold of hearing	0	$1 \times 10^{-12}$

## 16.3 – Sound pressure level

In practice, sound intensity is quite challenging to measure. Instead we normally measure sound pressures (with a microphone for example) and use the analogous **sound pressure level** (SPL) metric, also measured in dB:

$$SPL = 10 \log \frac{p^2}{p_0^2} = 20 \log \frac{p}{p_0}$$

where  $p$  is the sound pressure (with respect to the atmospheric pressure) and  $p_0$  is the threshold of human hearing pressure ( $p_0 = 2 \times 10^{-5} \text{ Pa} = 20 \mu\text{Pa}$ ).



# 16.3 – 0 dB does not mean no sound!



[Snail eating lettuce – link to video](#)

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## 16.3 – Intensity of sound: Decibels

An important consideration when working with logarithms is **that linear operations do not hold**. For example if we have two speakers emitting sound at 80 dB each:

$$80 \text{ dB} + 80 \text{ dB} \neq 160 \text{ dB}$$

Instead, we need to convert back to acoustic powers ( $p^2$ ) or intensities ( $I$ ) and do the summation there, and convert back to SPL in dB. Therefore:

$$\text{SPL} = 10 \log (10^{80/10} + 10^{80/10}) = 83 \text{ dB}$$

## 16.3 – Intensity of sound: Decibels

Useful to remember:

$$\text{SPL} = 10 \log (N_{sources} 10^{80/10}) = 80 + 10 \log (N_{sources}) = 83 \text{ dB}$$

Useful to remember:

$$10 \log 2 \sim 3 \text{ dB}$$

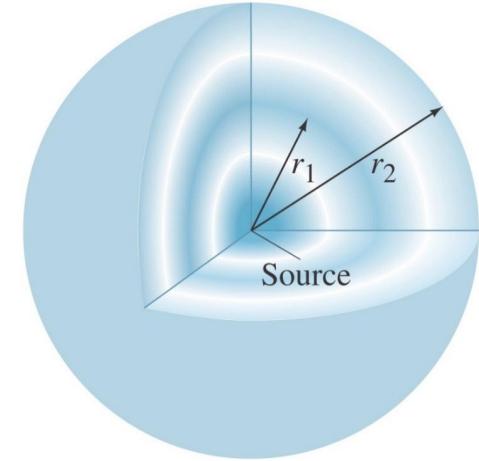
$$10 \log 4 = 10 \log 2^2 = 20 \log 2 \sim 6 \text{ dB}$$

# 16.3 – Sound propagation

In **open areas** (free-field), assuming **spherical spreading**, the sound intensity diminishes with distance as:

$$I \propto \frac{1}{r^2}$$

$$SPL_{r_2} = SPL_{r_1} - 20 \log \left( \frac{r_2}{r_1} \right)$$



The air in the **atmosphere** also **attenuates** the intensity of sound waves, especially the higher frequencies.

However, in **enclosures** this does not apply because of reflections in walls, ceiling, floor, etc.

## 16.3 – Loudness

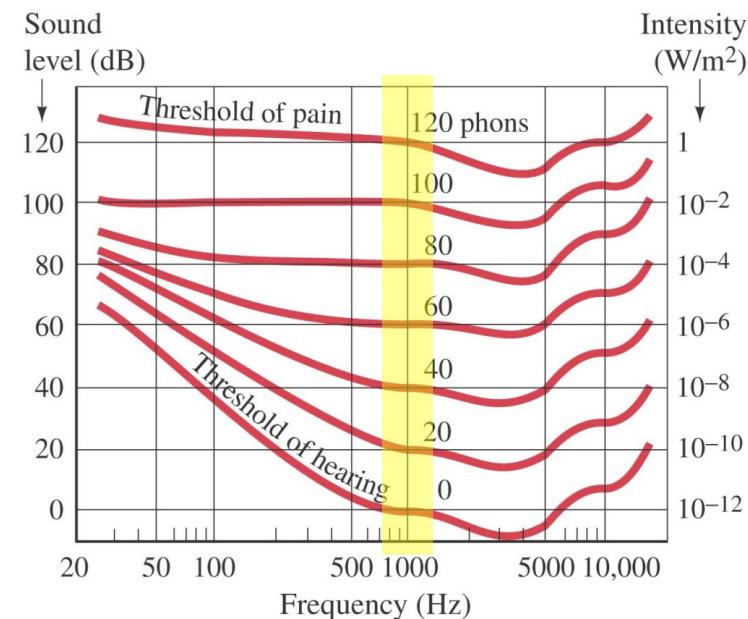
The **sensitivity of our ears is quite complex** and it varies with the frequency and sound level.

E.g., **very low or very high frequencies** are not very sensitive for our ears.

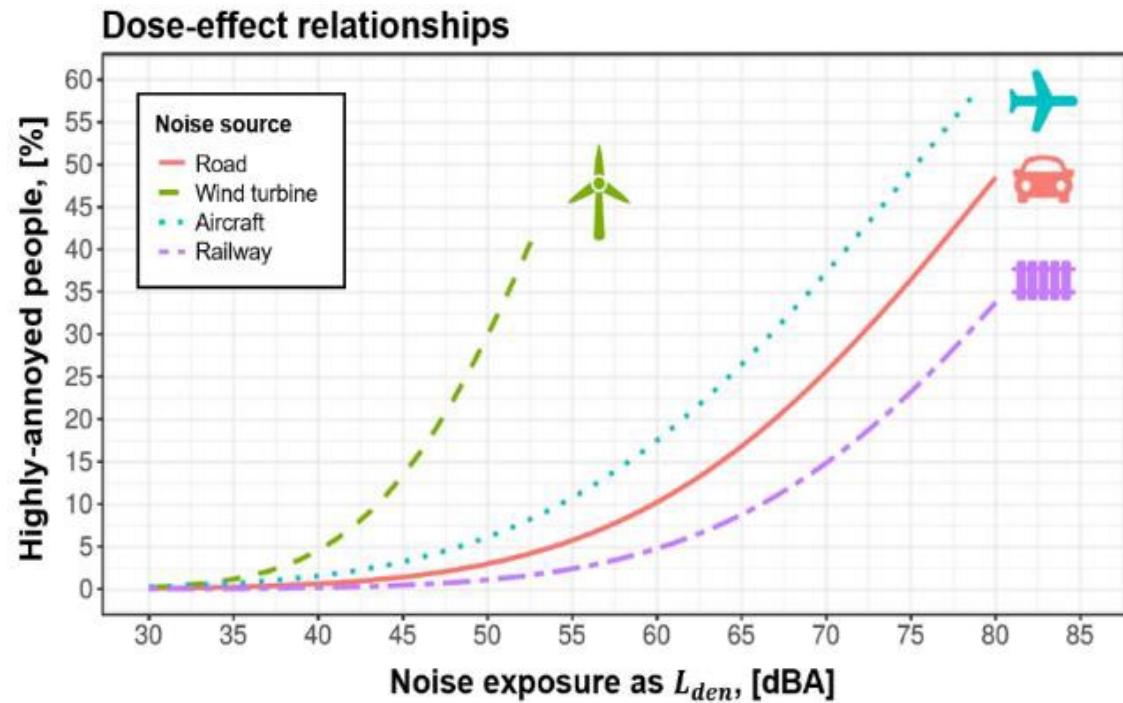
For discussing human perception, we should consider psychoacoustics and perception-based metrics like **loudness**.

For the particular case of 1000 Hz, there is a similarity between SPL [dB] = loudness [phon]

In general +10 phon  $\approx$  double perceived loudness.



# 16.3 – Sound perception - Competition



Fredianelli, L. et al, Science of the Total Environment, 2019.

The deadline for submission is  
Monday 24<sup>th</sup> at 17:00

Lower SPL does not always mean lower (perceived) **noise annoyance**.

To test this claim yourselves, there is a small “competition” in BrightSpace inside the *Noise\_annoynce\_competition.zip* file with some instructions.

The idea is that you can submit different sounds that you find **very annoying** or **very pleasing** and see which one scores higher.

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# WRAP-UP

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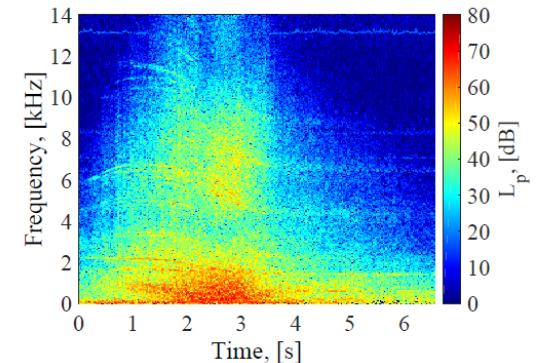
# Wrap-up: revisit learning objectives

After this lecture you should be able to:



- Describe the **main characteristics** of sound waves

$$v = \frac{\lambda}{T} = \lambda f$$



- Calculate the **mathematical representation** of longitudinal waves

$$\Delta P = -BAk \cos(kx - \omega t)$$

$$\Delta P_{max} = \rho v^2 A \frac{\omega}{v} = 2\pi f \rho v A$$

$$I = \frac{\Delta P_{max}^2}{2v\rho}$$



- Quantify the **intensity of sound** using the decibel scale

$$SPL = 10 \log \frac{p^2}{p_0^2} = 20 \log \frac{p}{p_0}$$

$$SPL_{r_2} = SPL_{r_1} - 20 \log \left( \frac{r_2}{r_1} \right)$$

# For next lecture – Finish chapter 16

4. Sources of sound: Vibrating strings and air columns
5. Quality of sound and noise: Superposition
6. Interference of sound waves, Beats
7. Doppler effect
8. Shock waves and sonic boom
9. Applications: Sonar, ultrasound, and medical imaging

# SOUND

## *Chapter 16*



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# Position in the syllabus

14. Oscillations

15. Waves



16. Sound

17. Temperature and the ideal gas law

18. Thermodynamics

19. Electricity and circuits

20. Electromagnetism

21. Optics

# Structure of the lecture

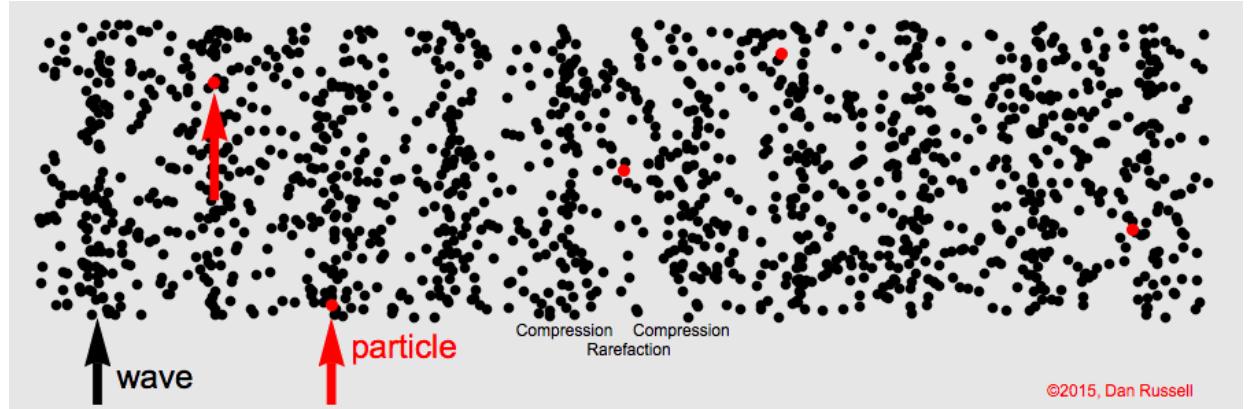
1. Characteristics of sound
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# Quick reminder or last lecture

Sound is a longitudinal wave

$$v = \frac{\lambda}{T} = \lambda f$$

---



Mathematical representation of longitudinal waves:

$$\Delta P = -BAk \cos(kx - \omega t)$$

---

$$\Delta P_{max} = \rho v^2 A \frac{\omega}{v} = 2\pi f \rho v A$$

$$I = \frac{\Delta P_{max}^2}{2v\rho}$$

Sound intensity:

$$SPL = 10 \log \frac{p^2}{p_0^2} = 20 \log \frac{p}{p_0}$$

$$SPL_{r_2} = SPL_{r_1} - 20 \log \left( \frac{r_2}{r_1} \right)$$

# Adding SPL in the logarithmic dB scale

$$SPL = 10 \log \frac{p^2}{p_0^2} = 20 \log \frac{p}{p_0}$$

Ratio of acoustic powers ( $\text{Pa}^2$ )

Ratio of acoustic pressures (Pa)

Threshold of human hearing:

$$p_0 = 20 \mu\text{Pa}$$

$$10 \log(N 10^{\frac{SPL_1}{10}}) = 10 \log N + 10 \log(10^{\frac{SPL_1}{10}}) = SPL_1 + 10 \log N$$

$$SPL_1 + SPL_2 + \dots + SPL_N =$$

To add logarithms, we need to first convert to acoustic powers and then apply the logarithm again:

$$10 \log(10^{\frac{SPL_1}{10}} + 10^{\frac{SPL_2}{10}} + \dots + 10^{\frac{SPL_N}{10}})$$

For the particular case with:

$$SPL_1 = SPL_2 = \dots = SPL_N$$

We assume that the sound sources are **incoherent** (e.g. white noise)

# Useful ratios to remember

$$10 \log(N 10^{\frac{SPL_1}{10}}) = SPL_1 + 10 \log N$$

**NOTE:** For coherent sound sources (e.g. 2 sinusoidal waves of the same frequency), the total SPL depends on the relative phase of the waves (recall the constructive/destructive interferences).

$$N = 2 \quad 10 \log 2 \sim 3 \text{ dB}$$

Twice the acoustic power

$$N = 4 \quad 10 \log 4 \sim 6 \text{ dB}$$

Four times the acoustic power  
Twice the acoustic pressure

$$N = 1/2 \quad 10 \log 1/2 \sim -3 \text{ dB}$$

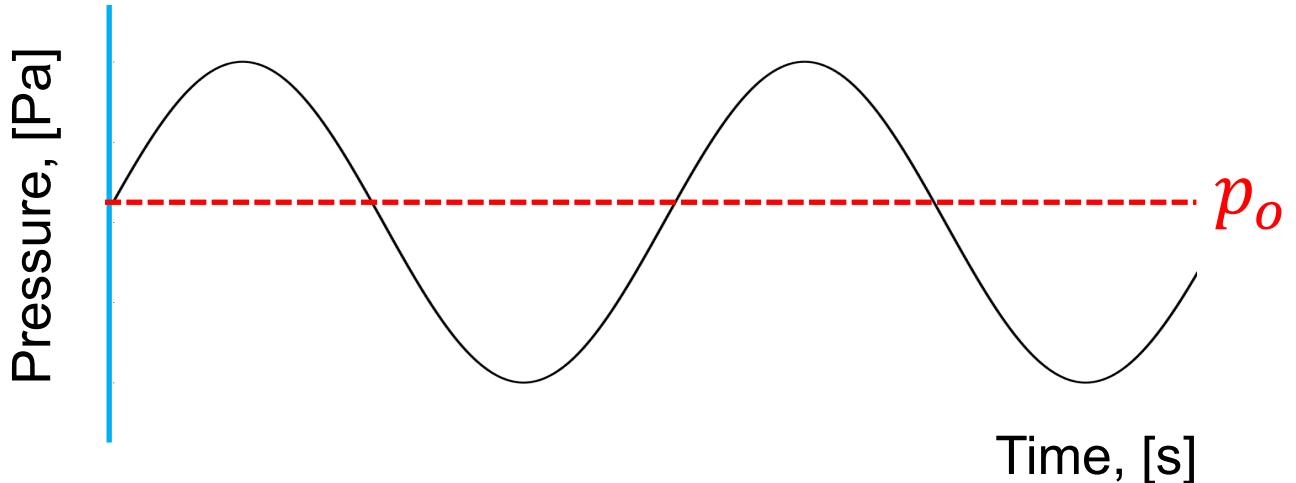
Half the acoustic power

$$N = 1/4 \quad 10 \log 1/4 \sim -6 \text{ dB}$$

One fourth of the acoustic power  
Half the acoustic pressure

We assume that the sound sources are **incoherent** (e.g. white noise)

# Maximum SPL in air (in theory)



**Acoustic pressures** are oscillations with respect to the static pressure of the medium (e.g. the atmospheric pressure  $p_{atm} \approx 101,325 \text{ Pa}$ ):

$$p' = p_{\text{total}} - p_{atm} \ll p_{atm}$$

$$SPL = 20 \log \frac{p'}{p_0}$$

$$p_0 = 20 \mu\text{Pa}$$

The maximum SPL in air (in theory) would be due to a pressure wave of the same magnitude as the **atmospheric pressure** since we cannot have negative absolute pressures:

Practically impossible!

$$SPL_{\max} = 20 \log \frac{101325 \text{ Pa}}{20 \mu\text{Pa}} = 194 \text{ dB}$$

# Learning objectives for today's lecture

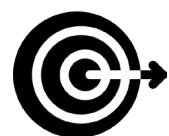
After this lecture you should be able to:



- Explain the **sound generation** from different sound sources (e.g. vibrating strings and air columns).



- Explain the phenomena **sound interference** (beats) and superposition.



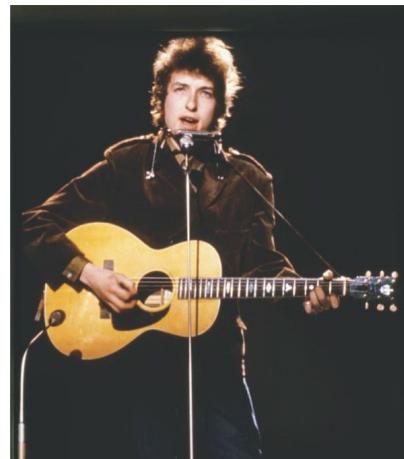
- Calculate the effects of moving sources: **Doppler effect** and shock waves (**sonic boom**)

## 16.4 – Sources of sound

In general, any vibrating object is a source of sound.

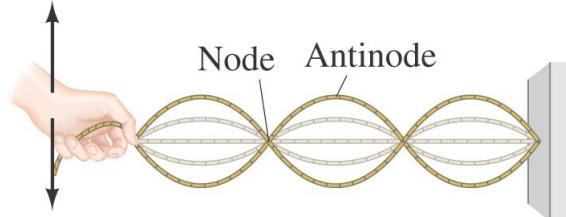
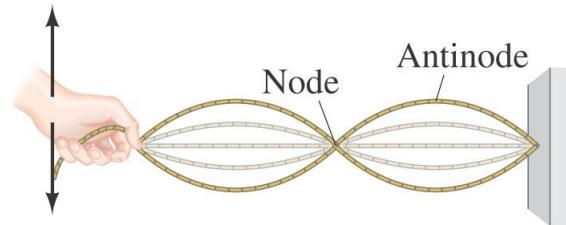
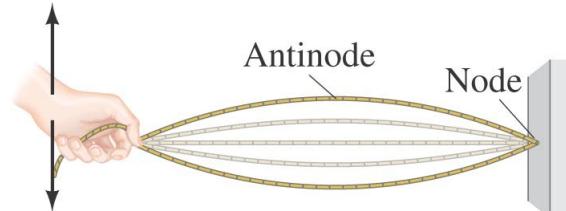
**Musical instruments** produce sounds in various ways: vibrating strings, vibrating membranes, vibrating metal or wood shapes, vibrating air columns.

The **vibration** may be started by plucking, striking, bowing, or blowing. The vibrations are transmitted to the air and then to our ears.



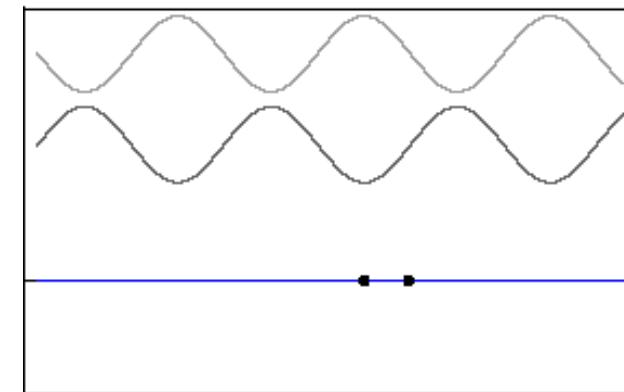
## 16.4 – Sources of sound: Vibrating strings

Recall the concept of **standing waves** from Chapter 15.9

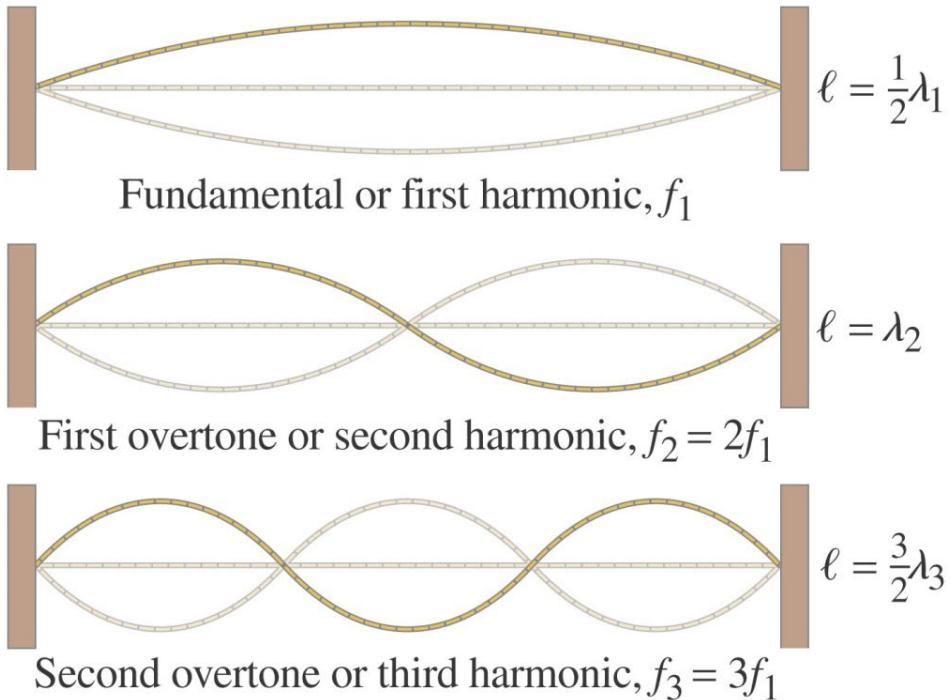


**Standing waves** occur when both ends of a string are fixed. In that case, only waves which are **motionless at the ends of the string can persist**.

There are **nodes**, where the amplitude is always zero, and **antinodes**, where the amplitude varies from zero to the maximum value.



# 16.4 – Sources of sound: Vibrating strings



**Frequencies on the string and in the air are the same** (string and air are in contact).

$$f_n = n f_1 = \frac{n\nu}{2l}, \quad n = 1, 2, 3 \dots$$

**Wavelengths are however different**, because the wave speed on the string is different than that in air.

In the string:  $\nu = \sqrt{\frac{F_T}{\mu}}$

$$f = \frac{\nu}{\lambda}$$

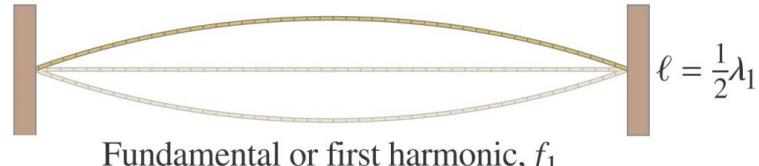
In air:  $\nu \approx 343 \text{ m/s}$

## 16.4 – Sources of sound: Vibrating strings example

Imagine a violin string of 0.32 m length that is tuned for a frequency of 440 Hz.

The wavelength of the fundamental string vibration is:

$$\lambda_{string} = 2l = 0.64 \text{ m}$$



On the other hand, the wavelength of the sound wave produced (in air) is:

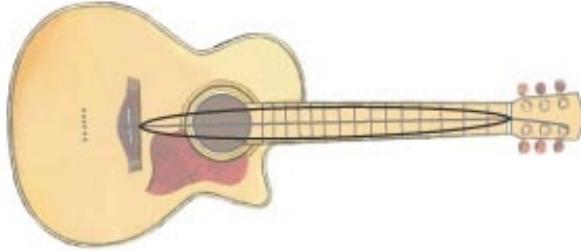
$$\lambda_{air} = \frac{v_{air}}{f} = \frac{343}{440} = 0.78 \text{ m}$$

On the string we have:

$$v_{string} = f\lambda_{string} = 440 \times 0.64 = 282 \text{ m/s}$$

## 16.4 – Sources of sound: Vibrating strings

For example, on a guitar the strings can be **effectively shortened** to raise the fundamental pitch.



$$f_n = \frac{n\nu}{2l}, \quad n = 1, 2, 3 \dots \quad \lambda = \frac{n}{2} l$$

A visualization of this can be found in:

<https://musiclab.chromeexperiments.com/Strings/>

The pitch of a string of a given length can also be altered by using a string material of a **different density** (change  $\mu$ ).

$$f = \frac{\nu}{\lambda} = \frac{1}{\lambda} \sqrt{\frac{F_T}{\mu}}$$

## 16.4 – Sources of sound: Vibrating strings



The sound waves from vibrating strings are usually quite low in sound amplitude, so they need to be amplified in order to obtain a relevant loudness.

This is done in practice in acoustical instruments by using a sounding board or box, in order to create a **resonant chamber**.



The sound can also be amplified **electronically**.

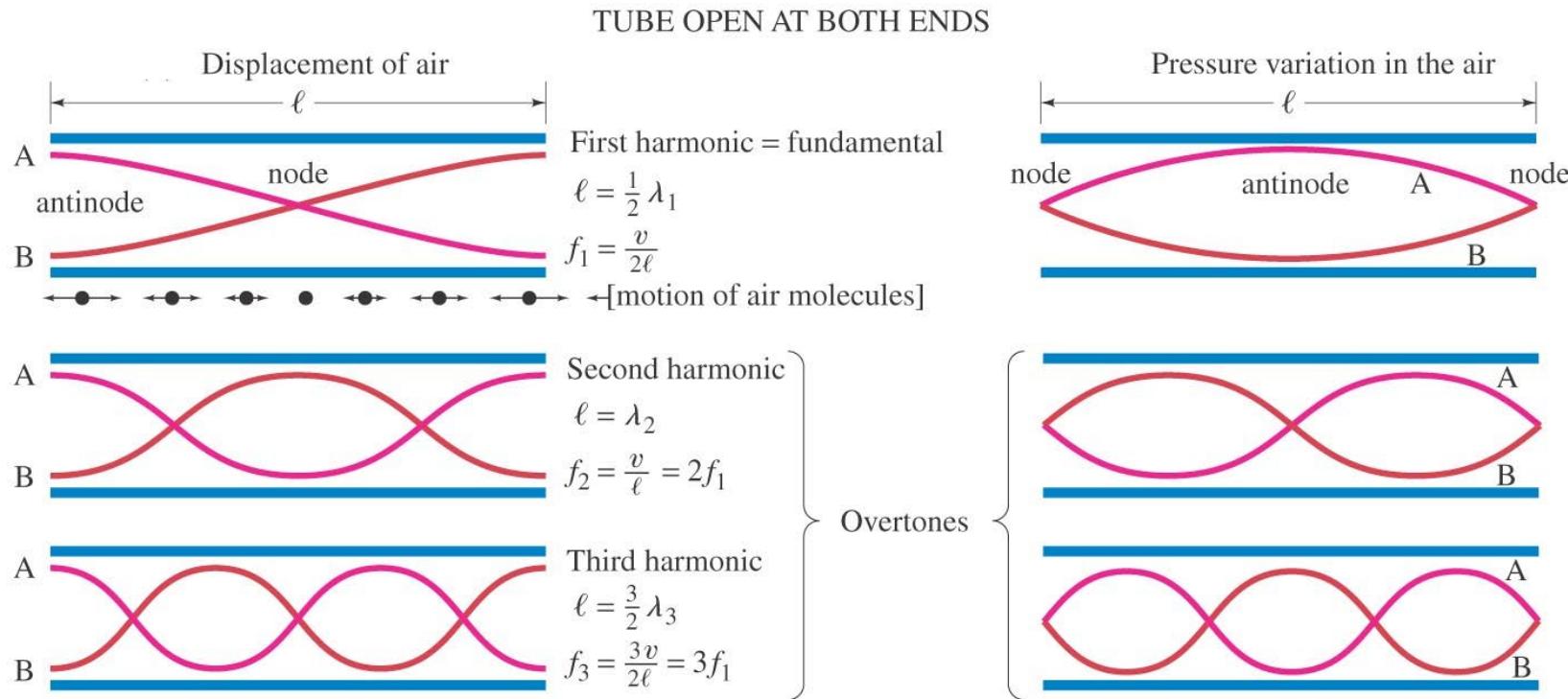
## 16.4 – Sources of sound: Air columns

**Wind instruments** create sound through **standing waves in a column of air**. The air vibrates in a variety of frequencies but only those corresponding to standing waves prevail. In most instruments: **length >> diameter**.



## 16.4 – Sources of sound: Tubes open at both ends

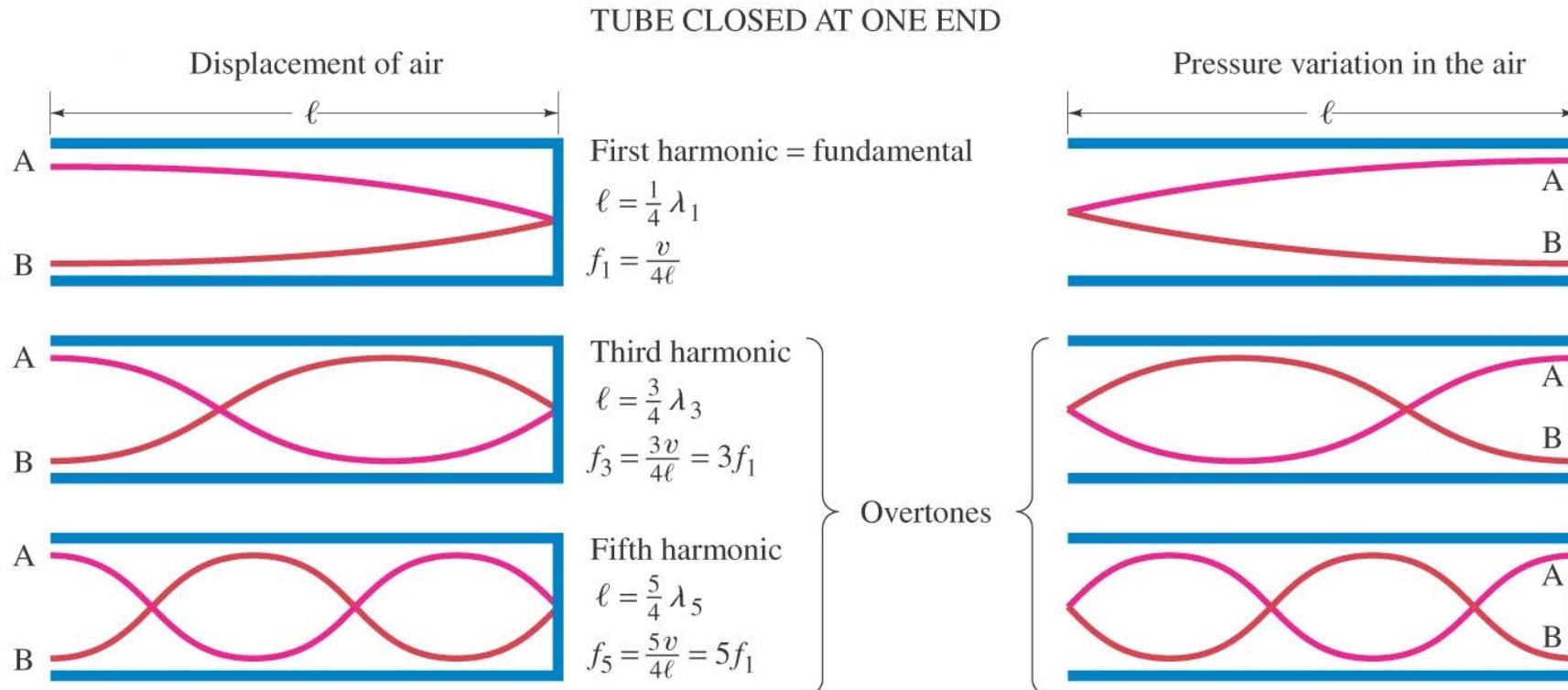
A tube **open at both ends** (most wind instruments) has **pressure nodes**, and therefore **displacement antinodes**, at the ends (remember the  $90^\circ$  phase offset).



Another visualization of this can be found in: <https://musiclab.chromeexperiments.com/Harmonics>

# 16.4 – Sources of sound: Tubes closed at one end

A tube **closed at one end** (some organ pipes) has a **displacement node** (and **pressure antinode**) at the **closed end**.



Only odd harmonics present!

# 16.4 – Sources of sound: Air columns summary

## Tube open at both ends

$$\lambda_n = \frac{2}{n} l$$

$$f_n = \frac{v}{\lambda} = \frac{n\nu}{2l} = nf_1$$

For n = 1, 2, 3 ...

## Tube closed at one end

$$\lambda_n = \frac{4}{n} l$$

$$f_n = \frac{v}{\lambda} = \frac{n\nu}{4l} = nf_1$$

For n = 1, 3, 5 ...

# 16.4 – Sources of sound: Boundary conditions

## Open end

Maximum displacement  
(antinode)

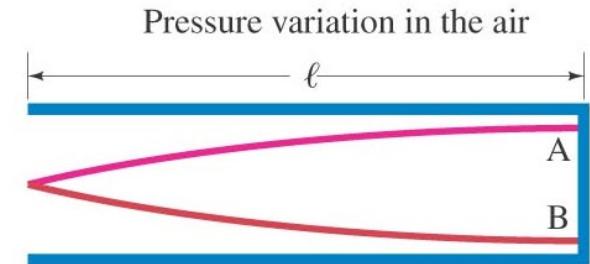
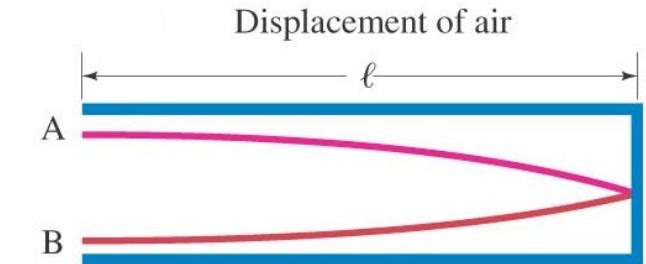
No pressure variation  
(node)

$$p = p_{atm}$$

## Closed end

No displacement (node)

Maximum pressure  
variation (antinode)



# 16.4 – World largest organ

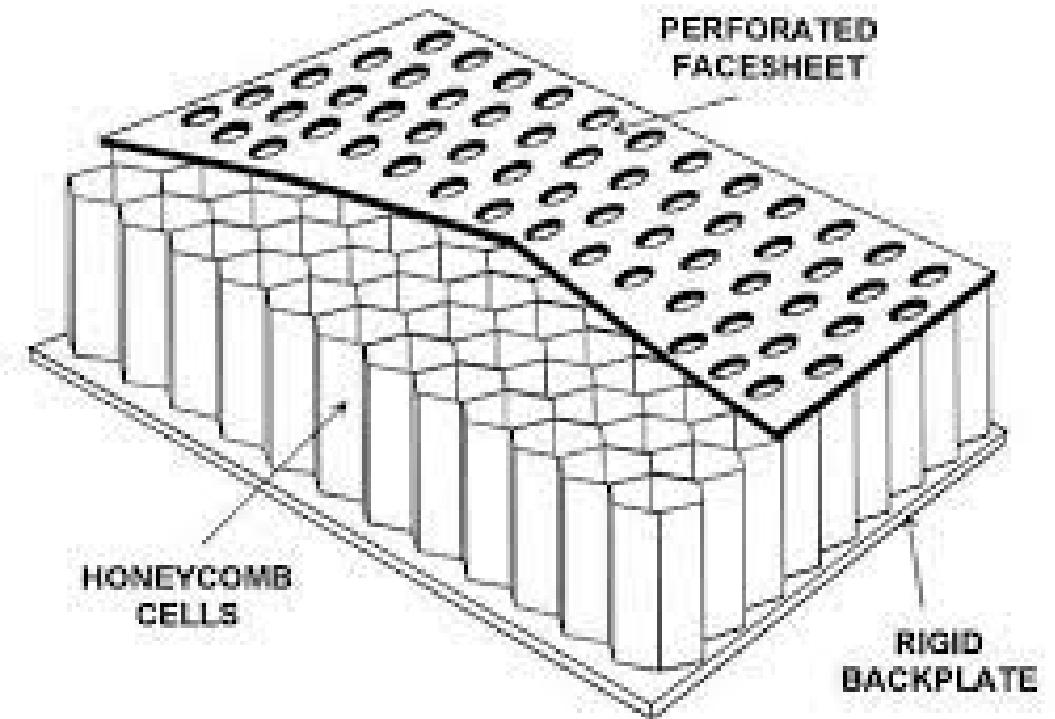


Interesting YouTube video about using a cave as a gigantic organ

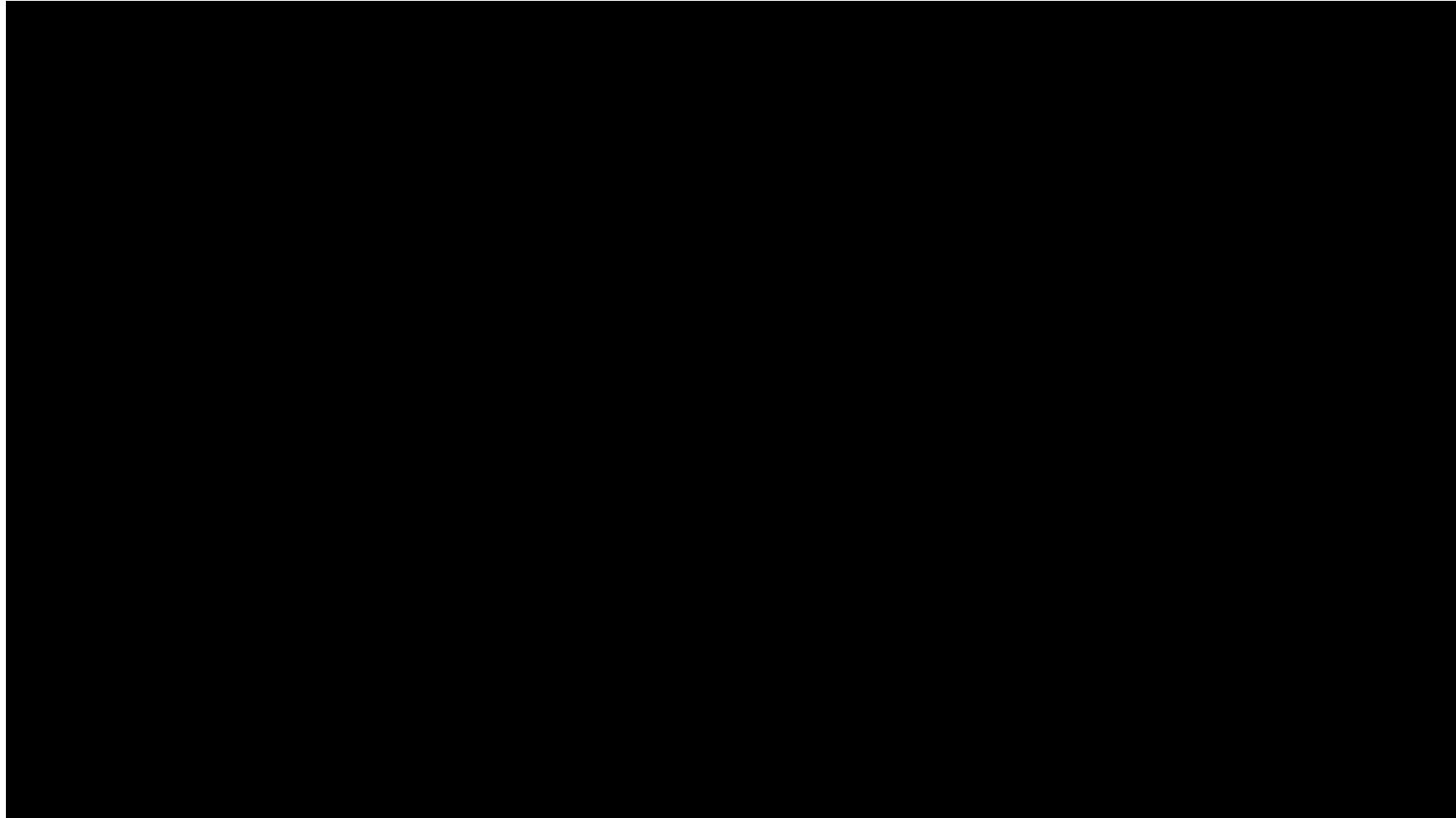
[Link to full video \(Veritasium\)](#)

# Application in turbofan engines (acoustic lining)

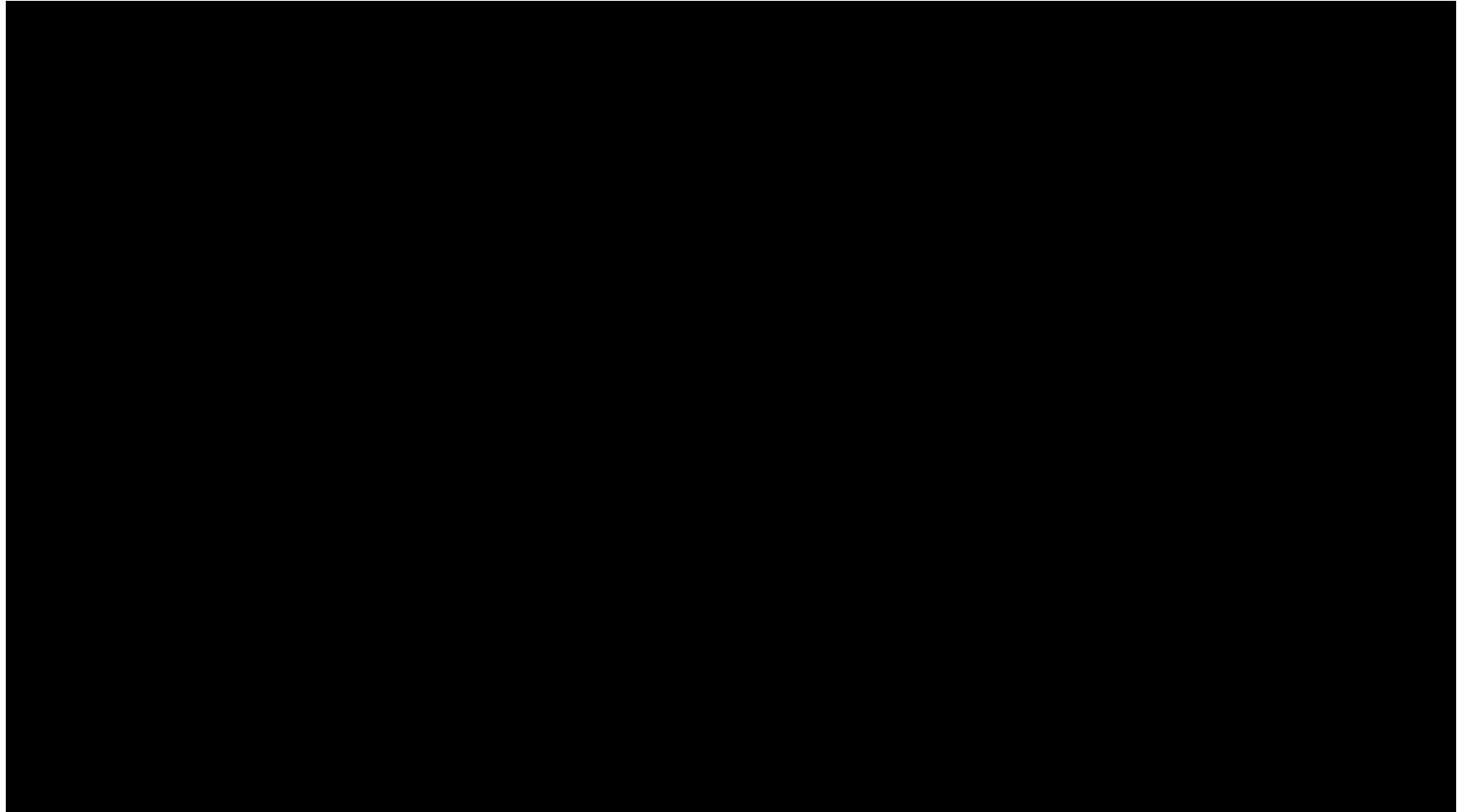
On a similar note (Helmholtz resonators), we can tailor the shape of cavities to excite specific frequencies to attenuate tonal sounds, such as fan noise.



# 16.4 – Example of standing waves: 1D Ruben's tube



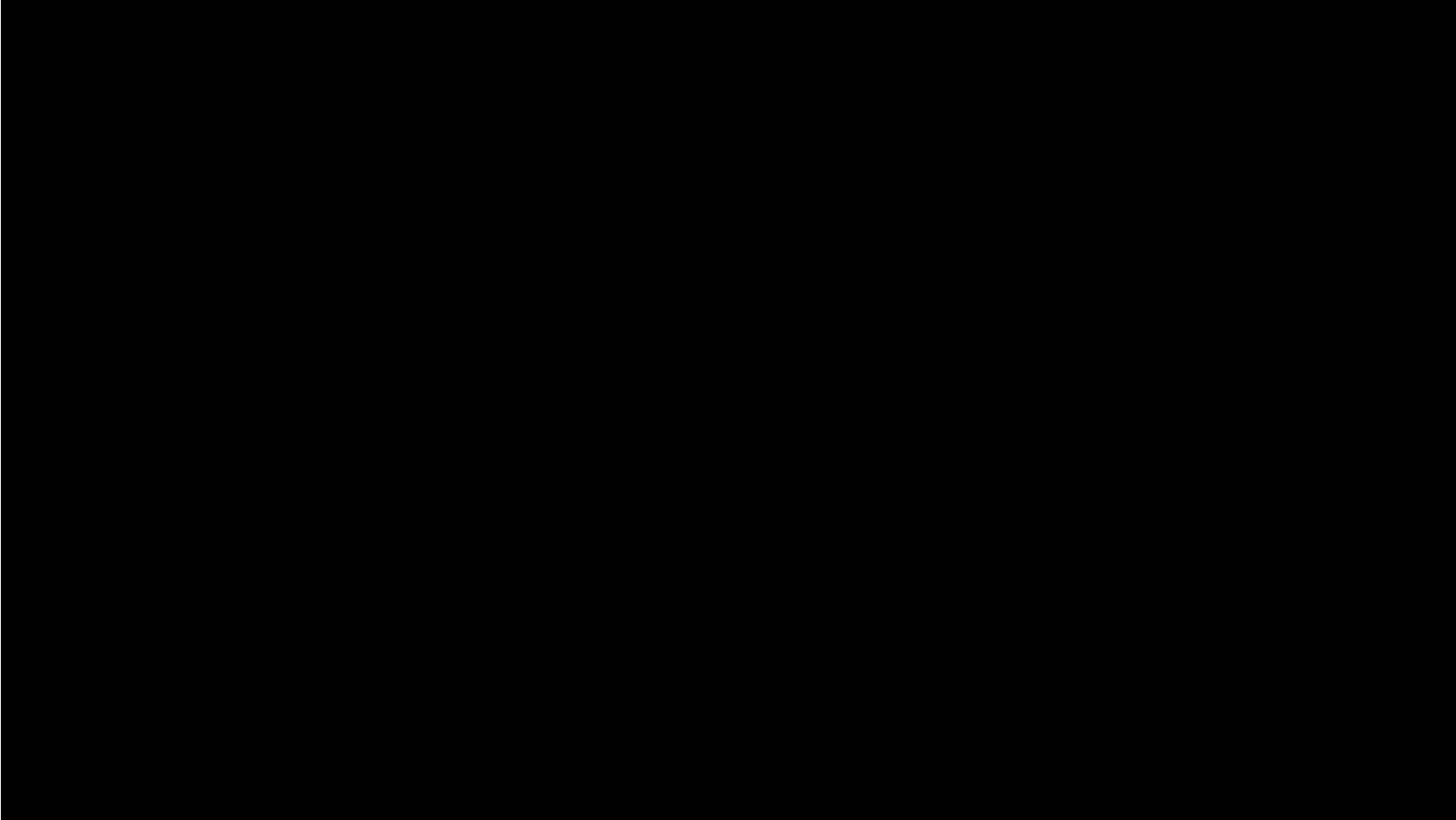
## 16.4 – Example of standing waves: 2D Ruben's tube



[Link to full video \(Veritasium\)](#)

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## 16.4 – Example of standing waves: 2D resonance



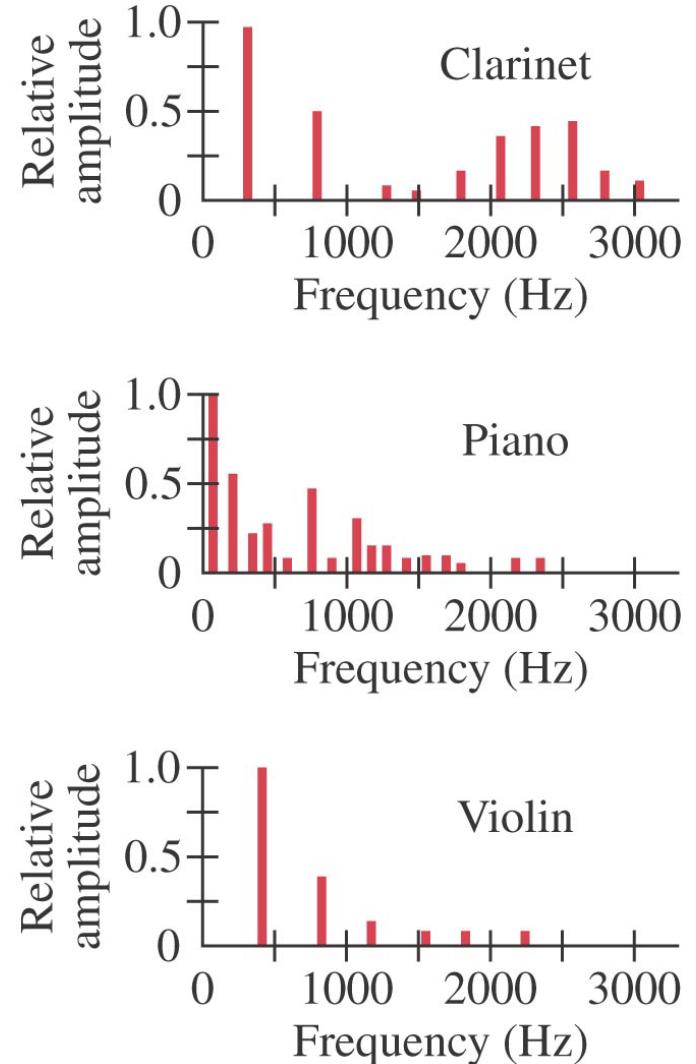
[Link to full video \(brussup\)](#)

# 16.5 – Quality of sound and noise - superposition

So why does a trumpet sound different from a flute?

The answer lies in **overtones**: **which ones are present, and how strong they are**, makes a big difference. The sound wave is the superposition of the fundamental and all the overtones. (Remember the superposition principle from Chapter 15.6)

Hence, **timbre**, “**quality**” or **tone color** is different (at the same loudness and pitch)!



# 16.5 – Quality of sound and noise - superposition

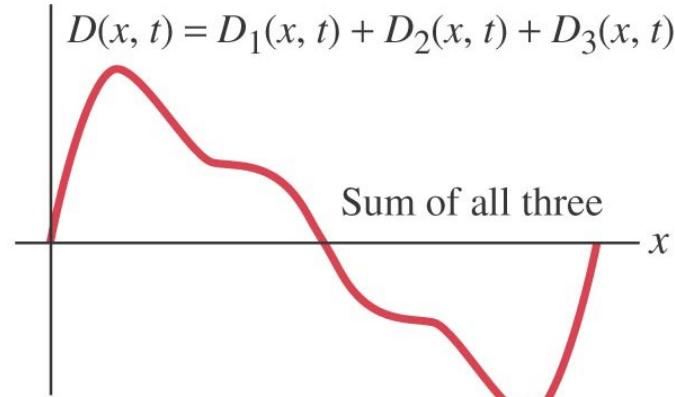
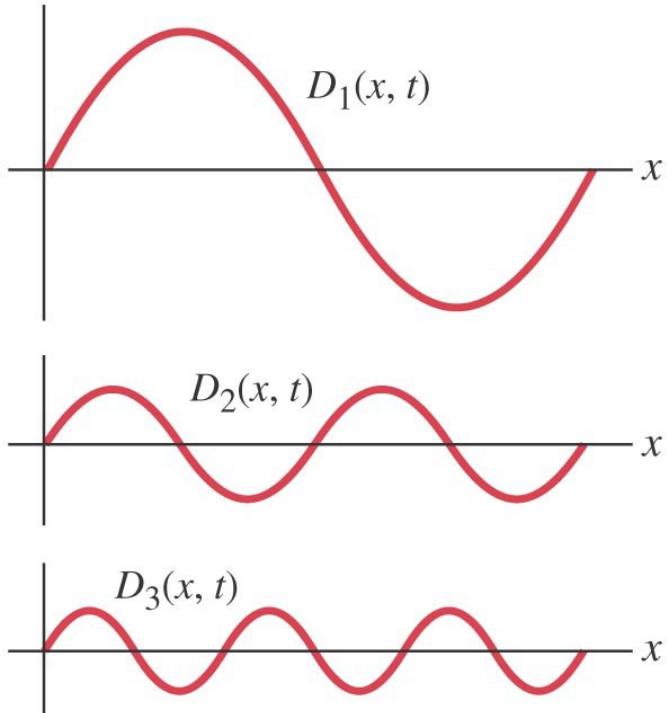


[Link to full video \(Veritasium\)](#)

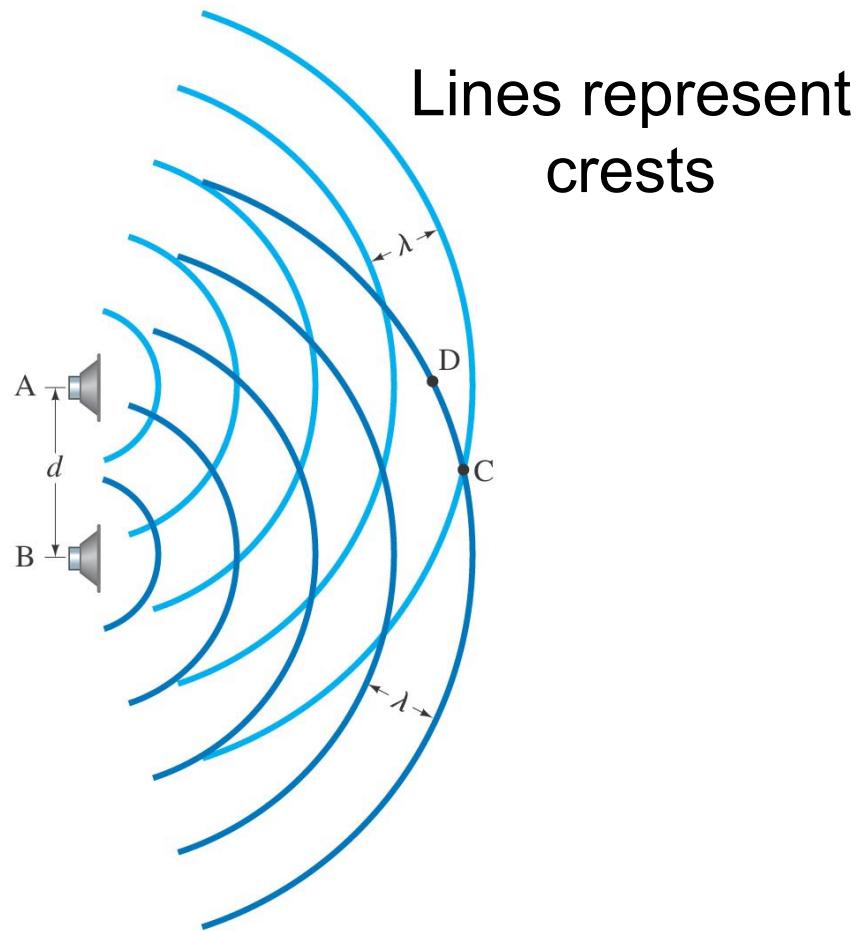
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## 16.5 – Superposition

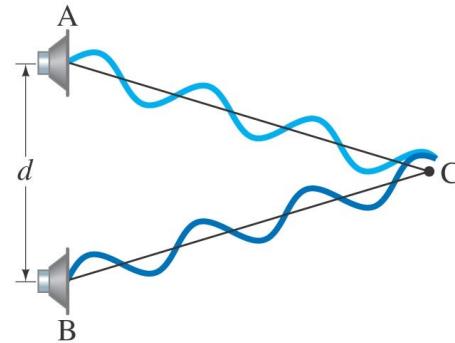
Recall Chapter 15.6: If  $D_1(x, t)$  and  $D_2(x, t)$  are solutions to the wave equation, then  $aD_1(x, t) + bD_2(x, t)$  is also a solution. This is called the superposition principle.



# 16.6 – Interference of sound waves

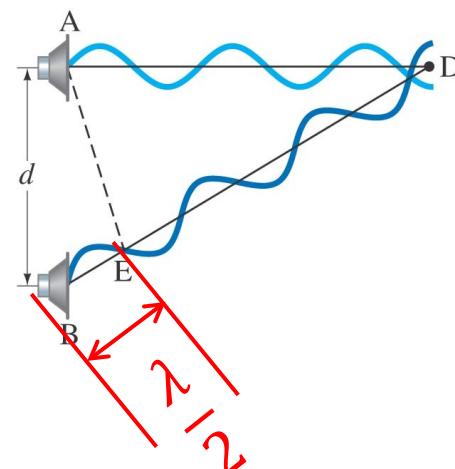


Sound waves **interfere** in the same way that other waves do in space.



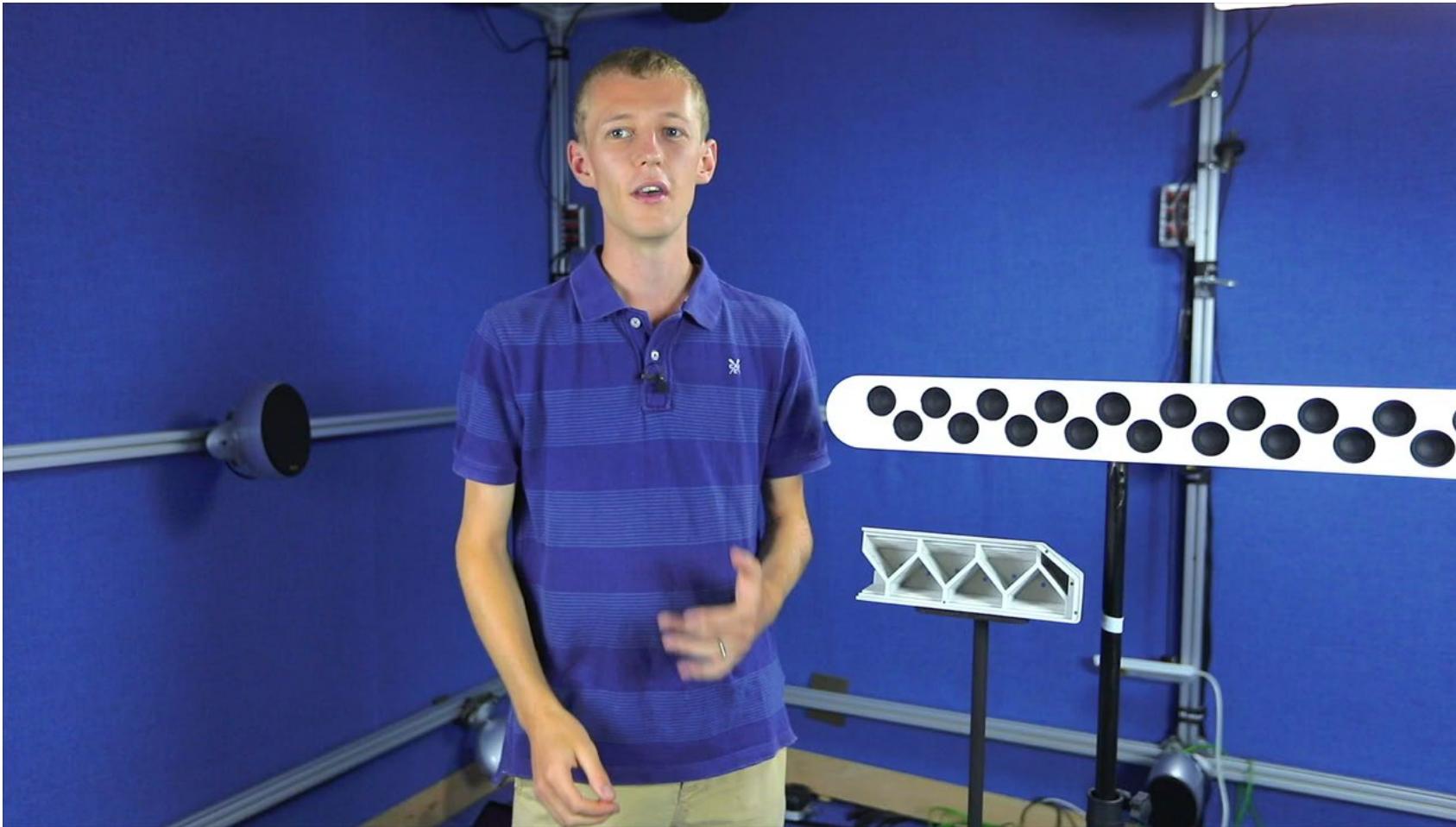
In phase

**Constructive**  
interference



Out of phase  
**Destructive**  
interference

## 16.6 – Example: Loudspeaker array

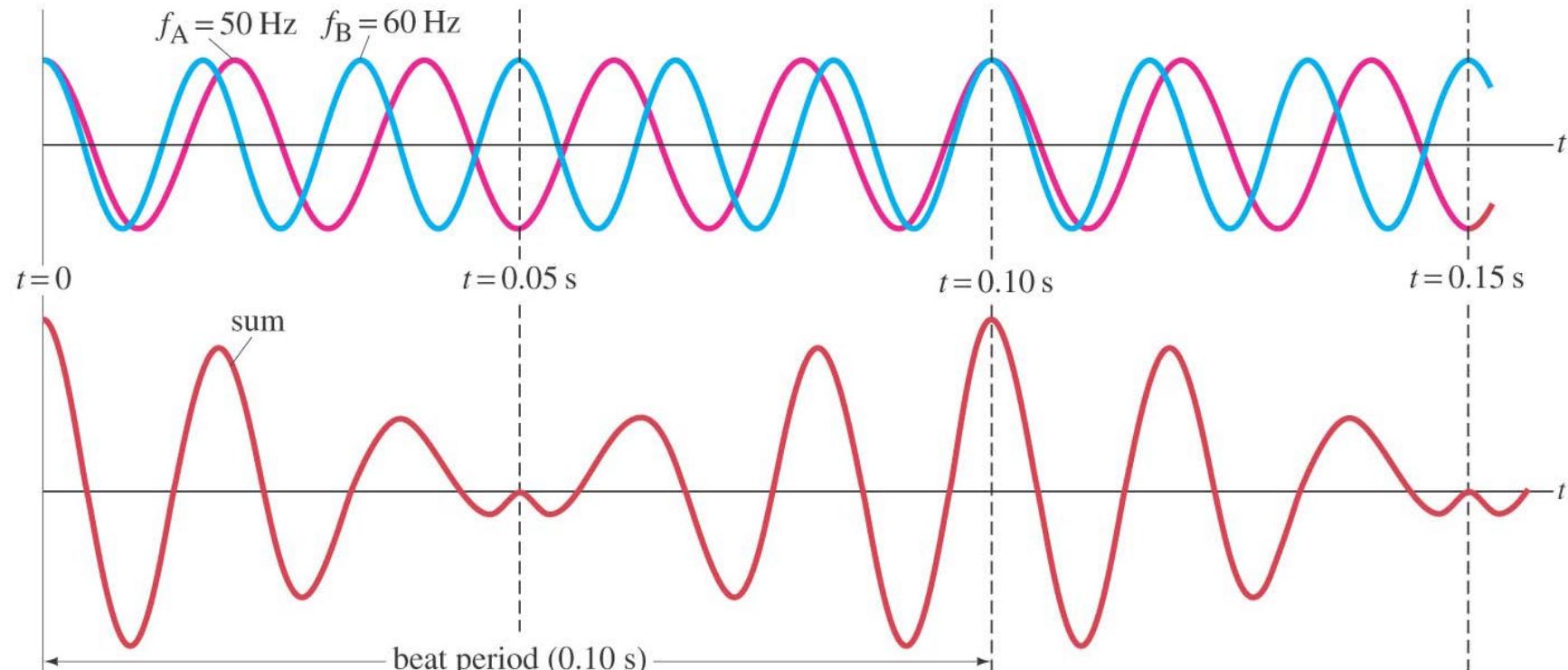


[Link to full video \(Charlie House Media\)](#)

## 16.6 – Beats

Waves can also **interfere in time**, causing a phenomenon called **beats**.

Beats are the **slow “envelope”** around two waves that are relatively close in frequency.



## 16.6 – Beats

Using the **superposition principle**, we can consider two sound waves of the same amplitude and phase, but different frequencies:

$$D_1 = A \sin 2\pi f_1 t$$

$$D_2 = A \sin 2\pi f_2 t$$

$$D_1 + D_2 = A (\sin 2\pi f_1 t + \sin 2\pi f_2 t)$$

$$\sin A + \sin B = 2 \sin\left(\frac{1}{2}(A + B)\right) \cos\left(\frac{1}{2}(A - B)\right)$$

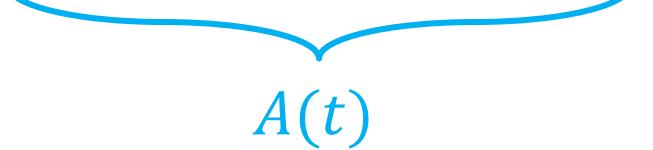
## 16.6 – Beats

$$\sin A + \sin B = 2 \sin\left(\frac{1}{2}(A+B)\right) \cos\left(\frac{1}{2}(A-B)\right)$$

$$D_1 = A \sin 2\pi f_1 t$$

$$D_2 = A \sin 2\pi f_2 t$$

$$D_1 + D_2 = 2A \cos\left[2\pi\left(\frac{f_1 - f_2}{2}\right)t\right] \sin\left[2\pi\left(\frac{f_1 + f_2}{2}\right)t\right]$$



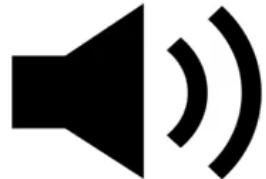
$$f_{beat} = f_1 - f_2$$

This represents a wave vibrating at the **average frequency**  $(f_1 + f_2)/2$  with an envelope  $A(t)$  at the **difference of the frequencies**  $(f_1 - f_2)$ , since the beat occurs every time the cosine term is 1 or -1 (twice per cycle).

## 16.6 – Beats example (part 1)

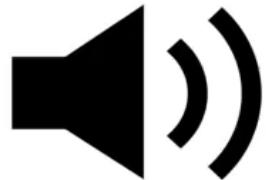
### Beats and Just Noticeable Difference

Note #1 500 Hz



Single  
Pitch

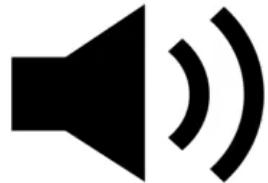
Note #2 500 Hz



## 16.6 – Beats example (part 2)

### Beats and Just Noticeable Difference

Note #1 500 Hz

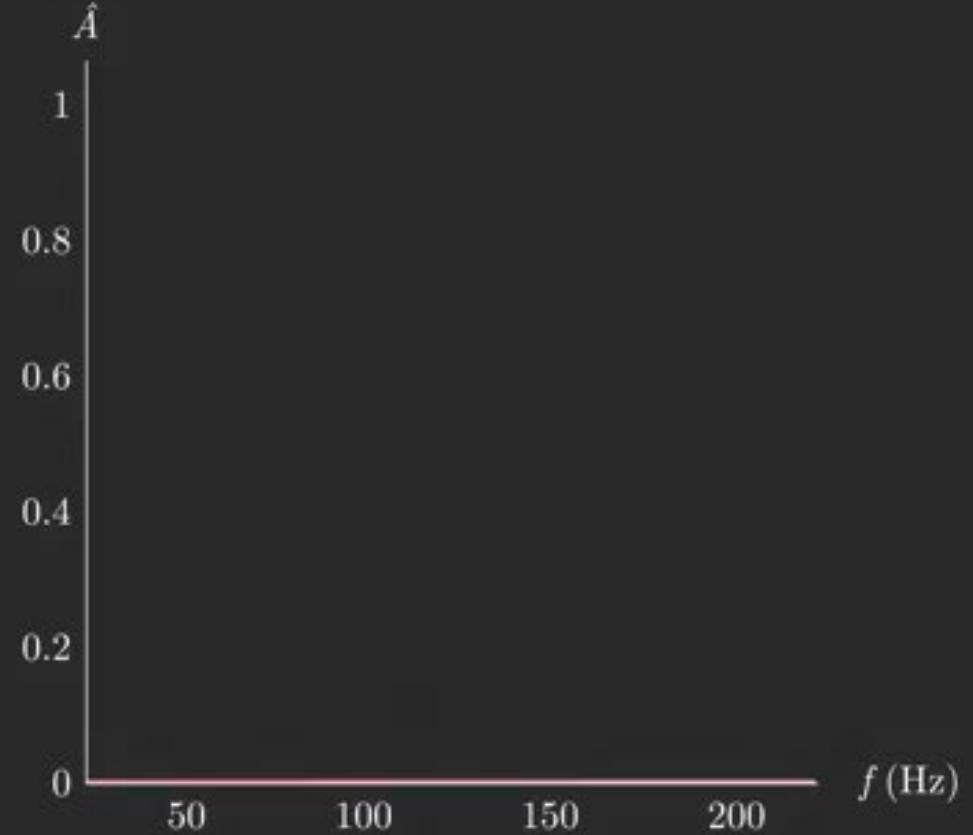


Single  
Pitch

Note #2 500 Hz



# 16.5 – Quality of sound and noise - superposition



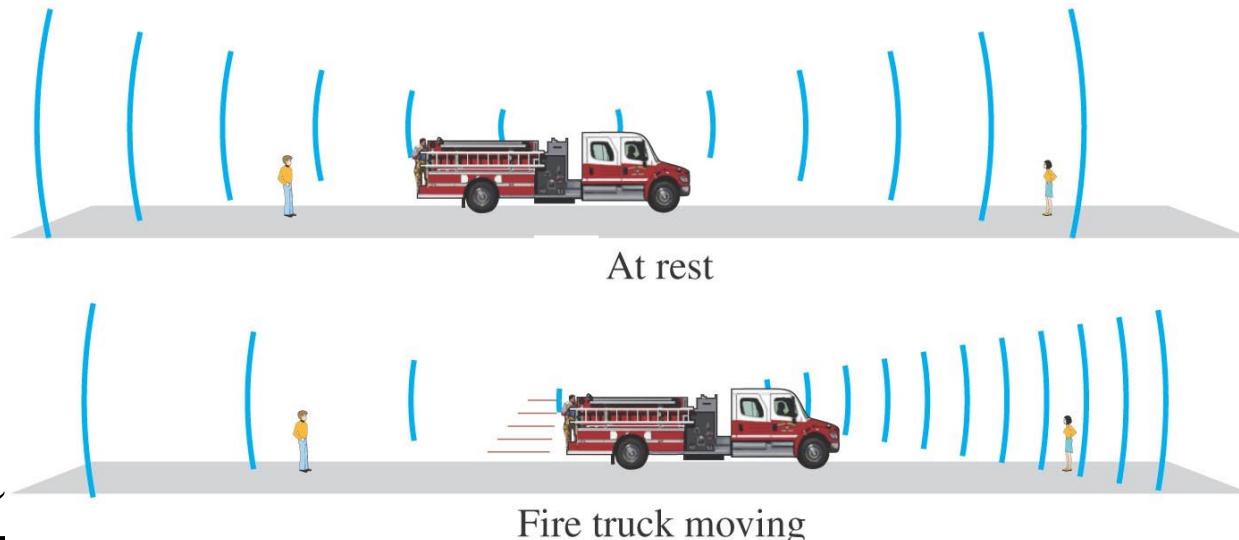
[Link to full video \(Veritasium\)](#)

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## 16.7 – Doppler effect

The Doppler effect occurs when there is a **relative motion** between the sound source and the observer. It also applies to other types of waves (e.g. light)

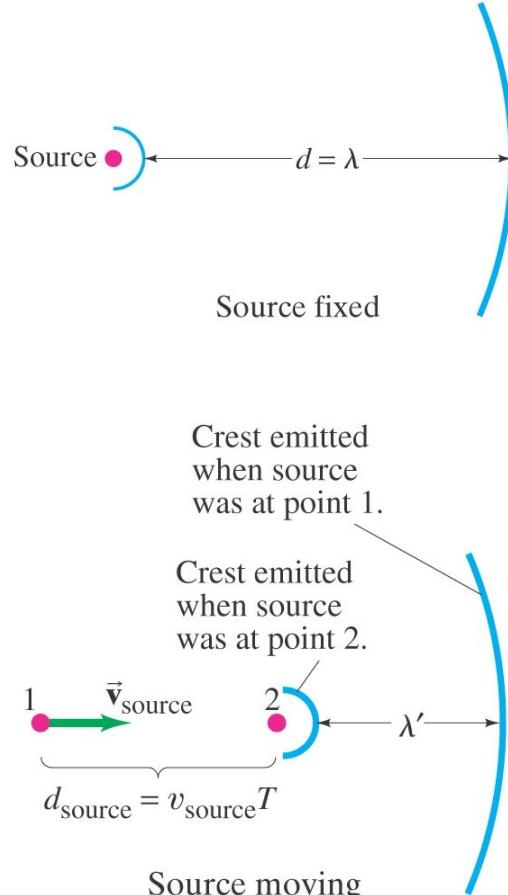
A source moving **toward** an observer appears to have a **higher frequency** and **shorter wavelength**; a source moving **away** from an observer appears to have a **lower frequency** and **longer wavelength**.



© <https://tenor.com>

# 16.7 – Doppler effect – Moving source

The **change in the wavelength** will determine the change in observed frequency.



For a stationary source, we consider the distance between successive wave crests as  $\lambda$ .

If the source moves towards the observer:

$$\lambda' = \lambda - v_{source}T$$

$$T = \frac{\lambda}{v_{sound}}$$

Mach number,  $M_s$

$$\lambda' = \lambda - v_{source} \frac{\lambda}{v_{sound}}$$

$$\lambda' = \lambda \left( 1 - \frac{v_{source}}{v_{sound}} \right)$$

## 16.7 – Doppler effect – Moving source

For a sound source **moving towards the observer**, the observed frequency is given by:

$$\lambda' = \lambda \left( 1 - \frac{v_{source}}{v_{sound}} \right)$$

$$f = \frac{v}{\lambda}$$

$$f' = \frac{f}{1 - \frac{v_{source}}{v_{sound}}} = \frac{f}{1 - M_s}$$

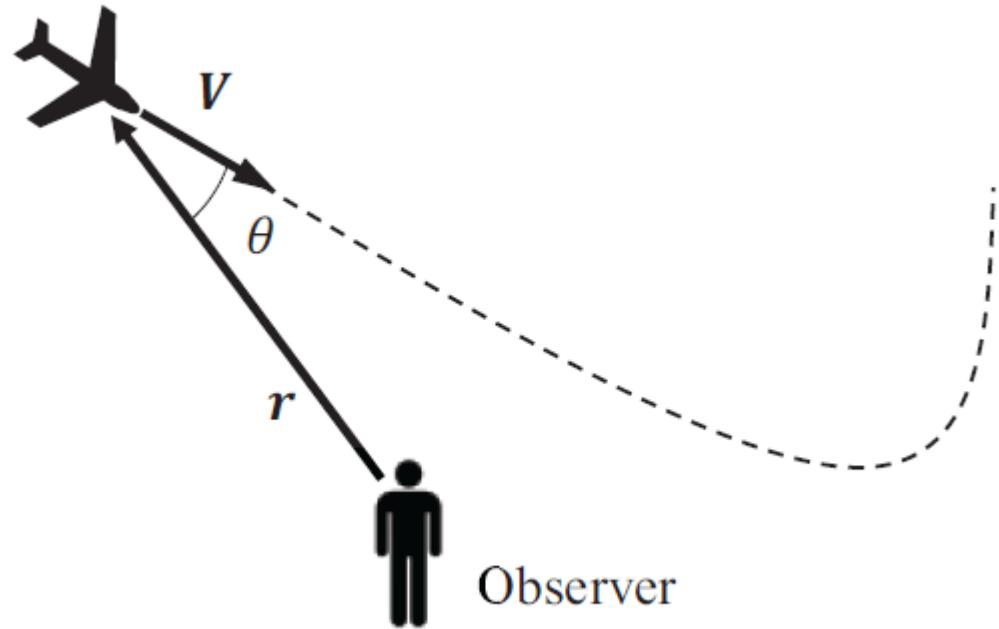
For a sound source **moving away from the observer**, the observed frequency is given by:

$$\lambda' = \lambda \left( 1 + \frac{v_{source}}{v_{sound}} \right)$$

$$f = \frac{v}{\lambda}$$

$$f' = \frac{f}{1 + \frac{v_{source}}{v_{sound}}} = \frac{f}{1 + M_s}$$

# 16.7 – Doppler effect – Moving source (general case)



$$f' = \frac{f}{1 - \|M_s\| \cos \theta}$$

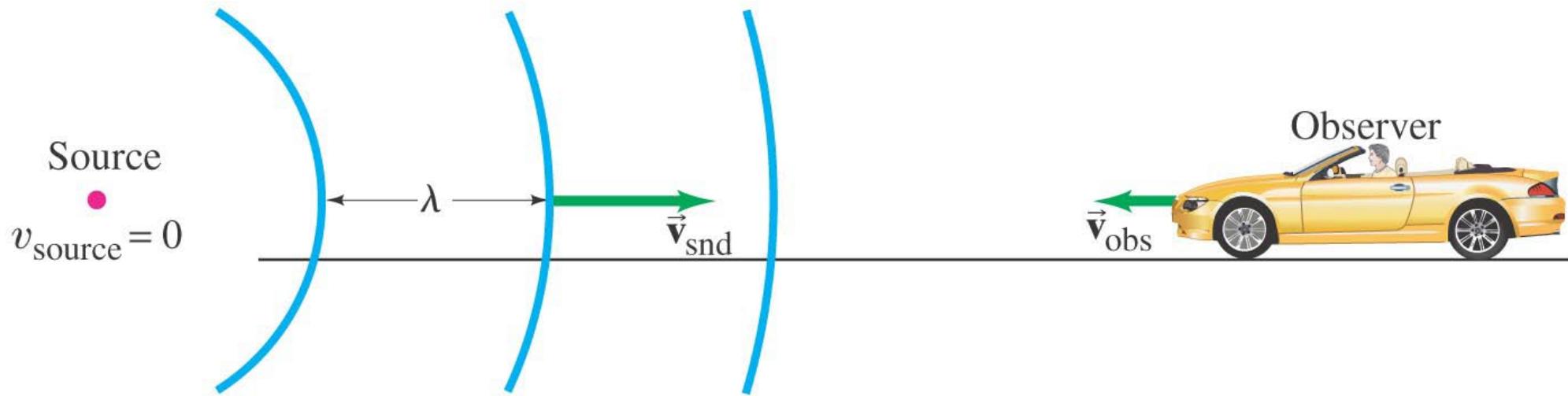
For extreme cases:

Incoming source ( $\theta \sim 0^\circ$ ):  $f' = \frac{f}{1 - M_s}$

Departing source ( $\theta \sim 180^\circ$ ):  $f' = \frac{f}{1 + M_s}$

## 16.7 – Doppler effect – Moving observer

In case it is the **observer that moves towards the source**, the wavelength remains the same but the propagation speed of the wave seems different for the observer:



$$f' = \frac{v_{\text{sound}} + v_{\text{observer}}}{\lambda} = f \frac{v_{\text{sound}} + v_{\text{observer}}}{v_{\text{sound}}} = f(1 + M_{\text{obs}})$$

## 16.7 – Doppler effect – Moving observer

For an observer **moving towards the sound source**, the observed frequency is given by:

$$f' = f(1 + M_{obs})$$

For an observer **moving away from the sound source**, the observed frequency is given by:

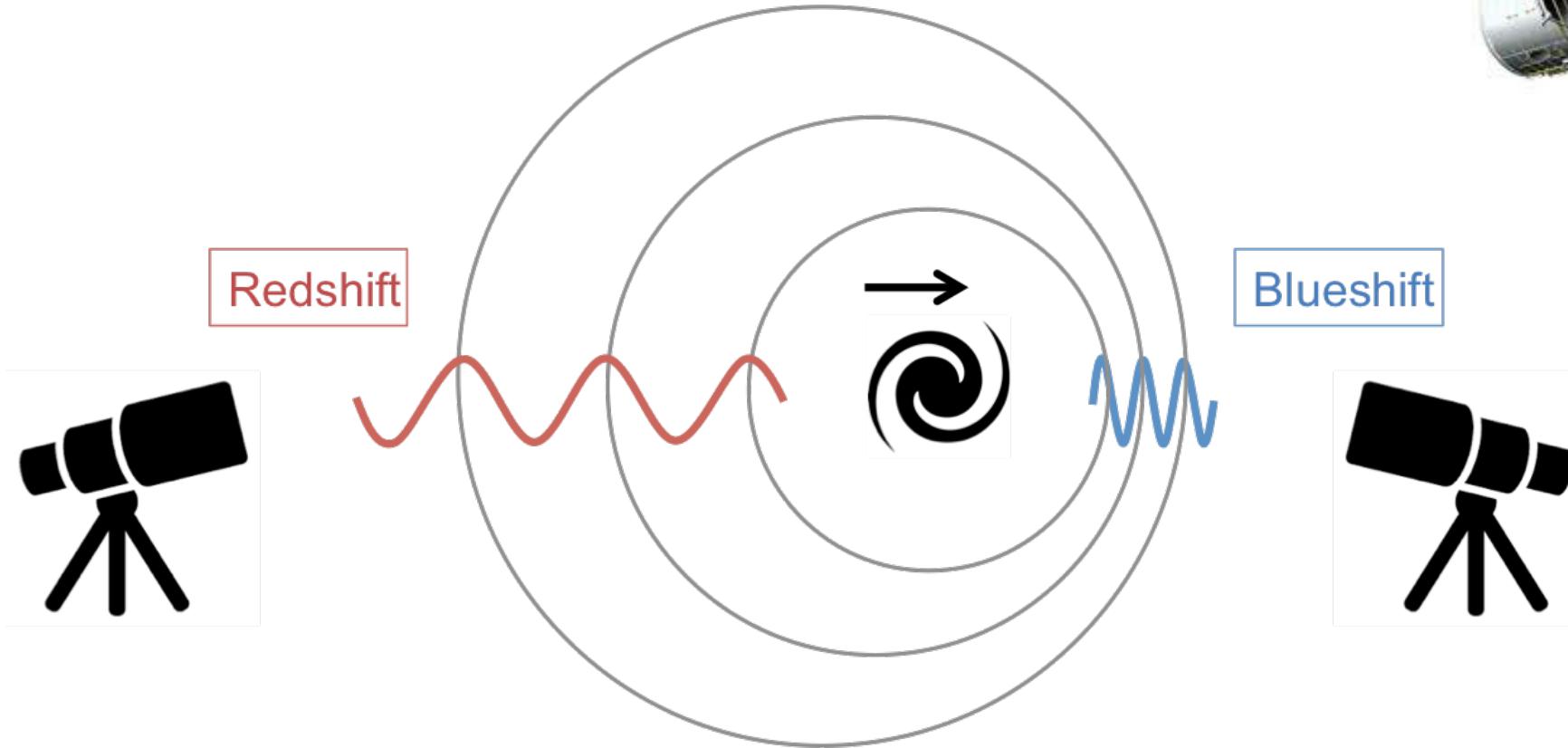
$$f' = f(1 - M_{obs})$$

## 16.7 – Doppler effect – Both effects combined

We can combine all four equations into one that takes into account all effects at once. **You need to keep track of the signs!**

$$f' = f \frac{v_{sound} \pm v_{observer}}{v_{sound} \mp v_{source}}$$

# 16.7 – Doppler effect – Also in space!

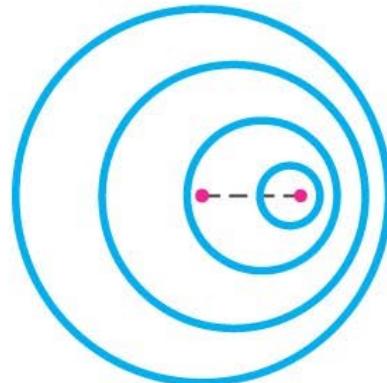


## 16.8 – Shock waves and the sonic boom

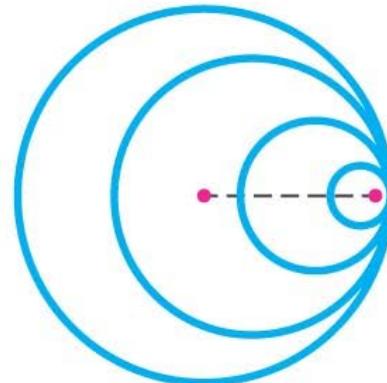
If a source is **moving faster than the wave speed in a medium**, waves cannot keep up and a **shock wave** is formed. It can be interpreted as the result of constructive interference of a large number of wave fronts.



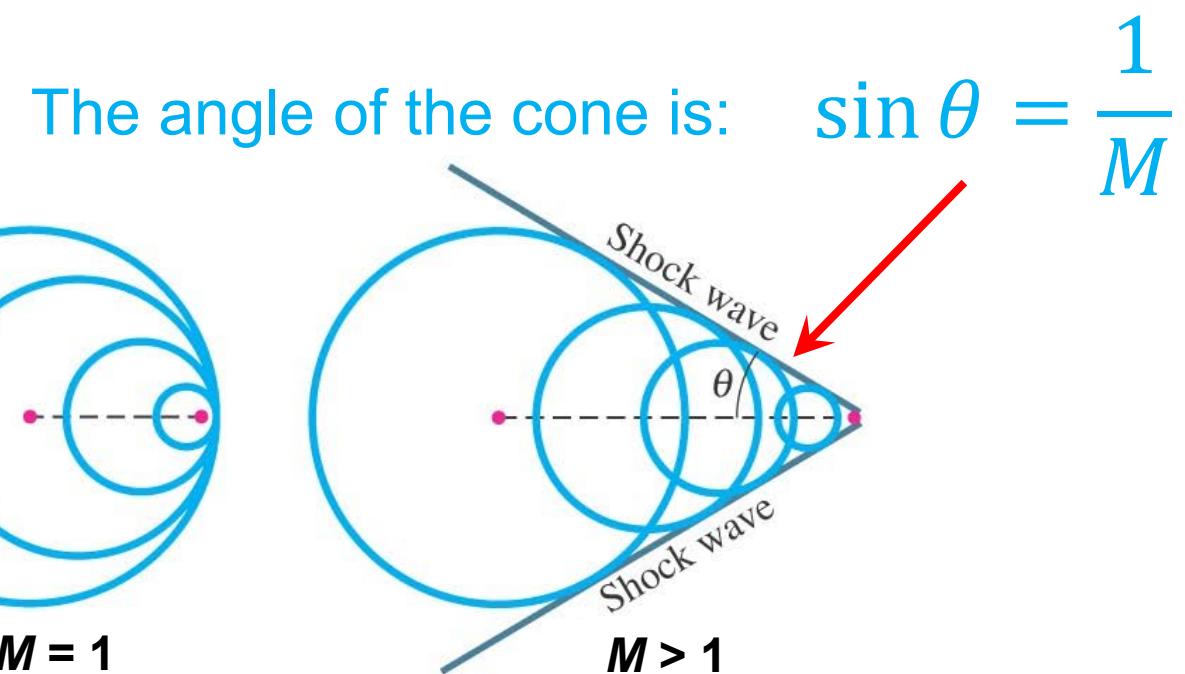
$$M = 0$$



$$M < 1$$



$$M = 1$$



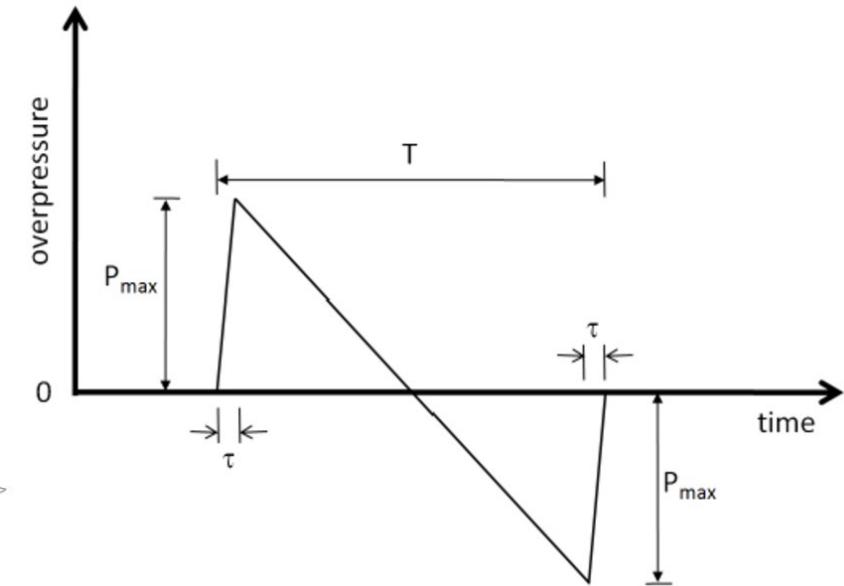
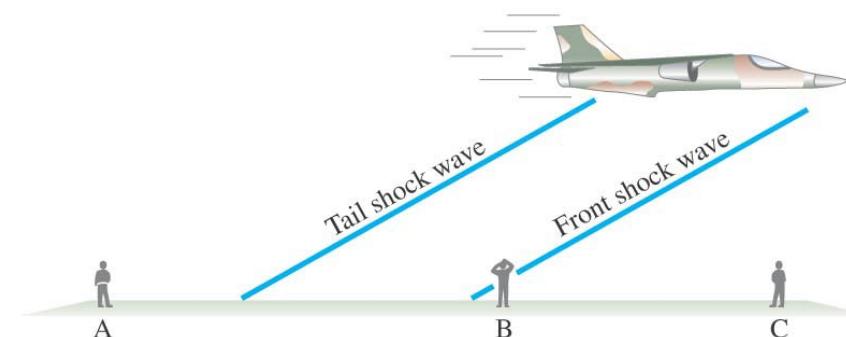
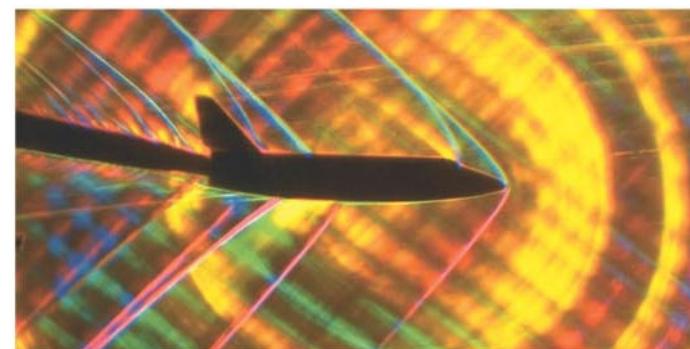
$$M > 1$$

# 16.8 – Shock waves and the sonic boom

Aircraft exceeding the speed of sound in air will produce **sonic booms**: usually one from the front and one from the tail. These shock waves contain a **tremendous amount of sound energy**. **The shock wave moves with the aircraft.**



<https://en.wikipedia.org/>



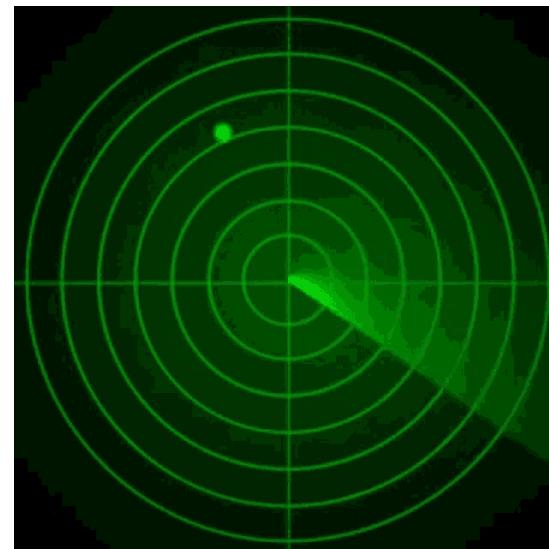
# 16.8 – Shock waves and the sonic boom - example



## 16.9 – Applications: Sonar

**Sonar** is used to locate objects underwater by measuring the time it takes a **sound pulse** to reflect back to the receiver. Similar techniques can be used to learn about the internal structure of the Earth.

Sonar usually uses **ultrasound waves**, as the shorter wavelengths are less likely to be diffracted by obstacles and smaller objects can be detected



**Radar** uses a similar technique but with **electromagnetic waves**

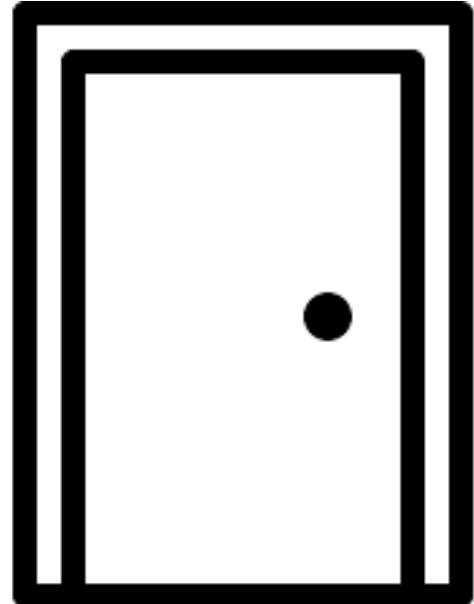
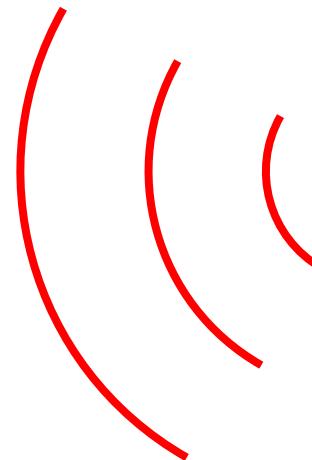
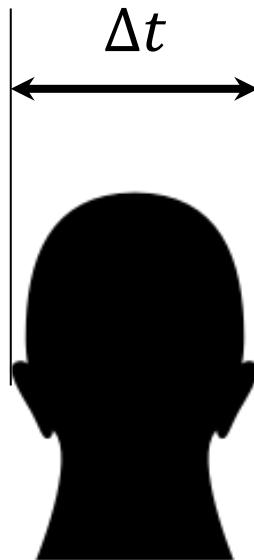
# 16.9 – Applications: Medical imaging

Ultrasound (~MHz) is also used for **medical imaging**. Repeated traces are made as the transducer is moved and a complete picture is build (even in 4D!)



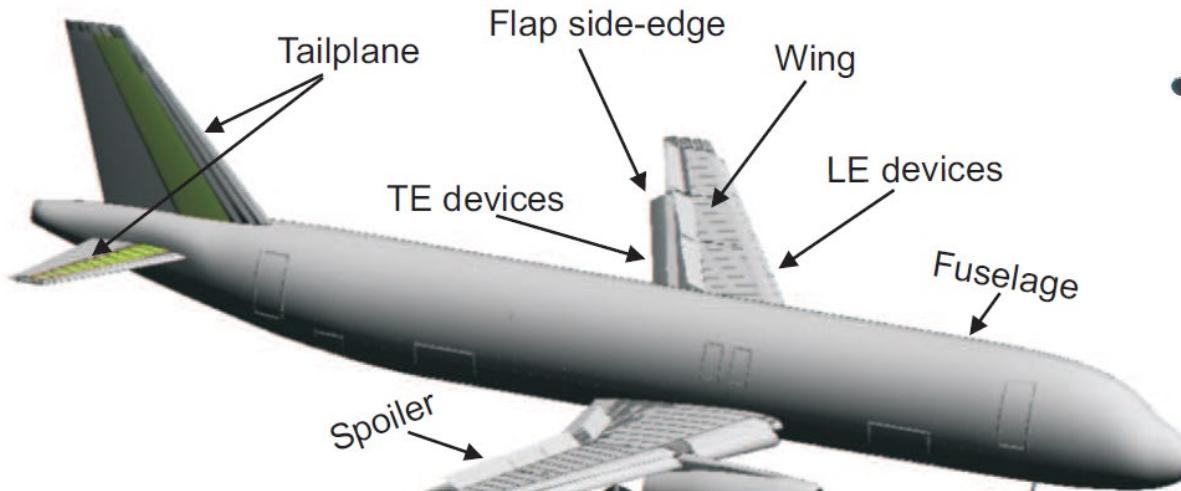
# 16.9 – Applications: Acoustic imaging

$$\Delta t = \frac{d}{v} \approx \frac{0.17 \text{ m}}{340 \frac{\text{m}}{\text{s}}} = 0.5 \text{ ms}$$

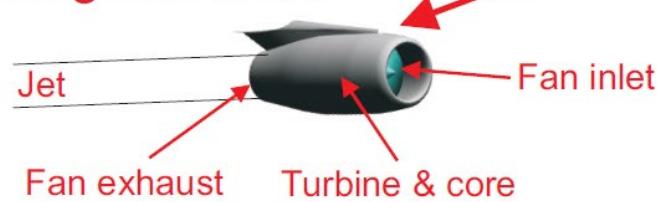


# 16.9 – Applications: Acoustic imaging of aircraft noise

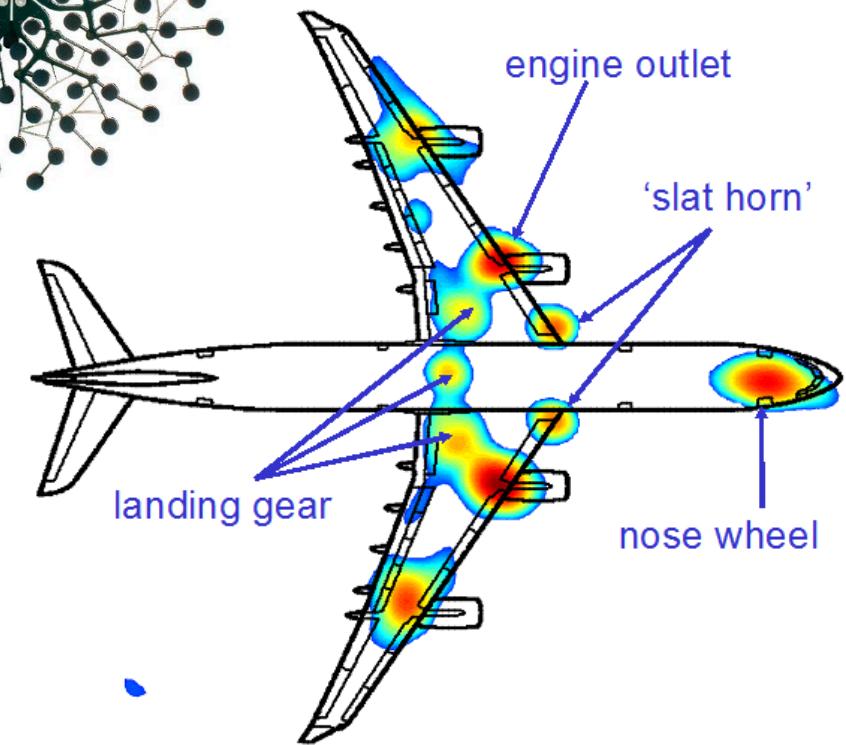
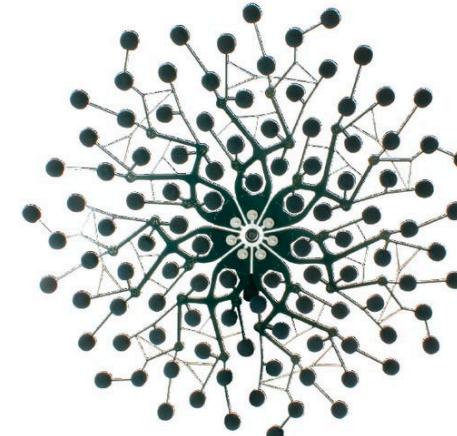
Airframe noise



Engine Noise

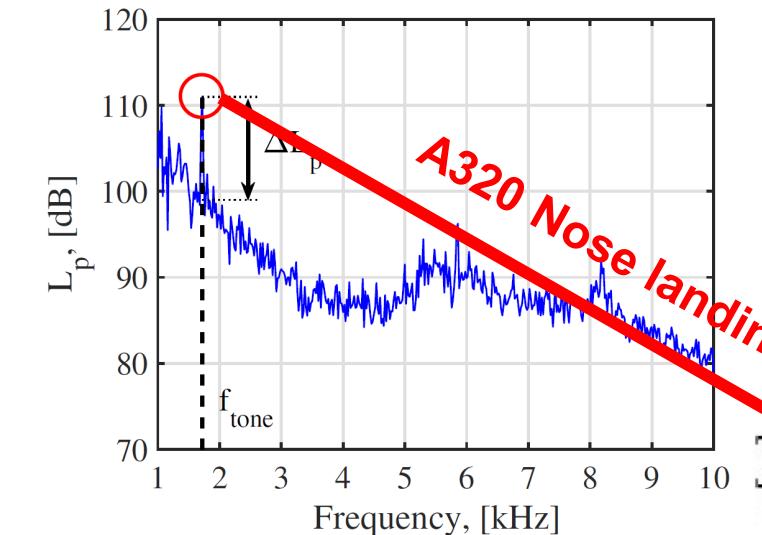
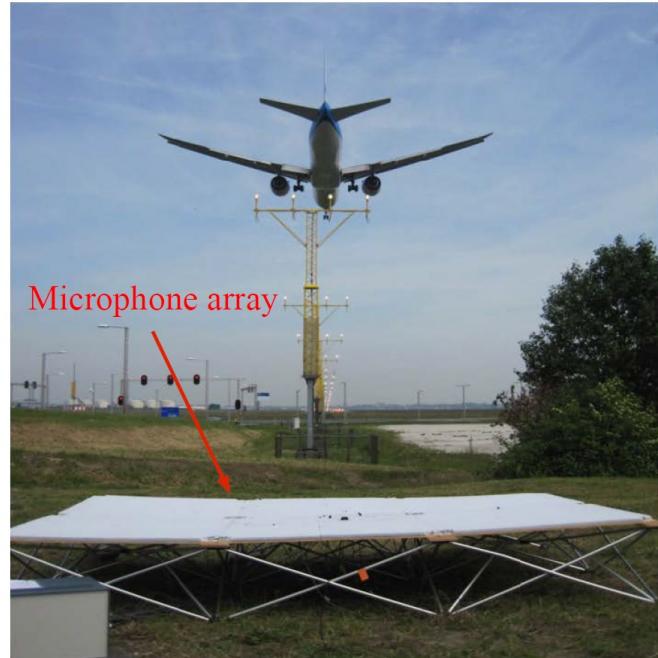


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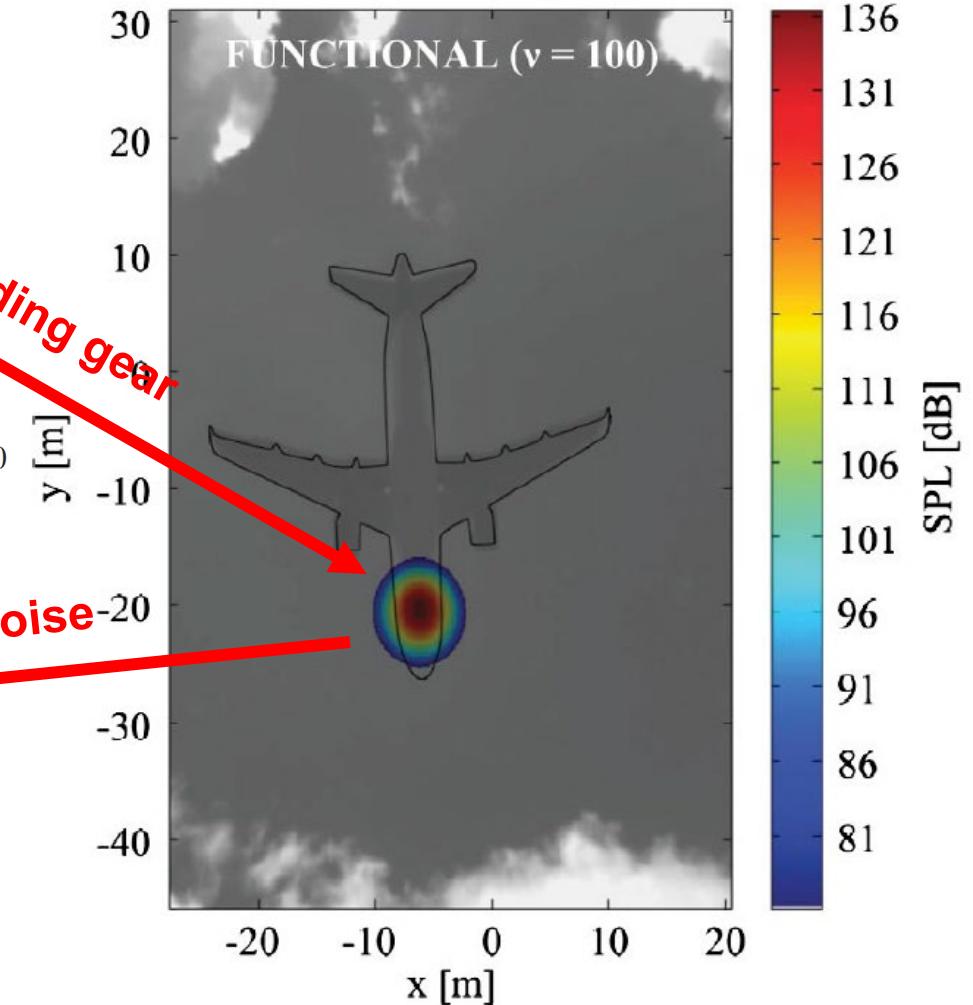


© Pieter Sijtsma

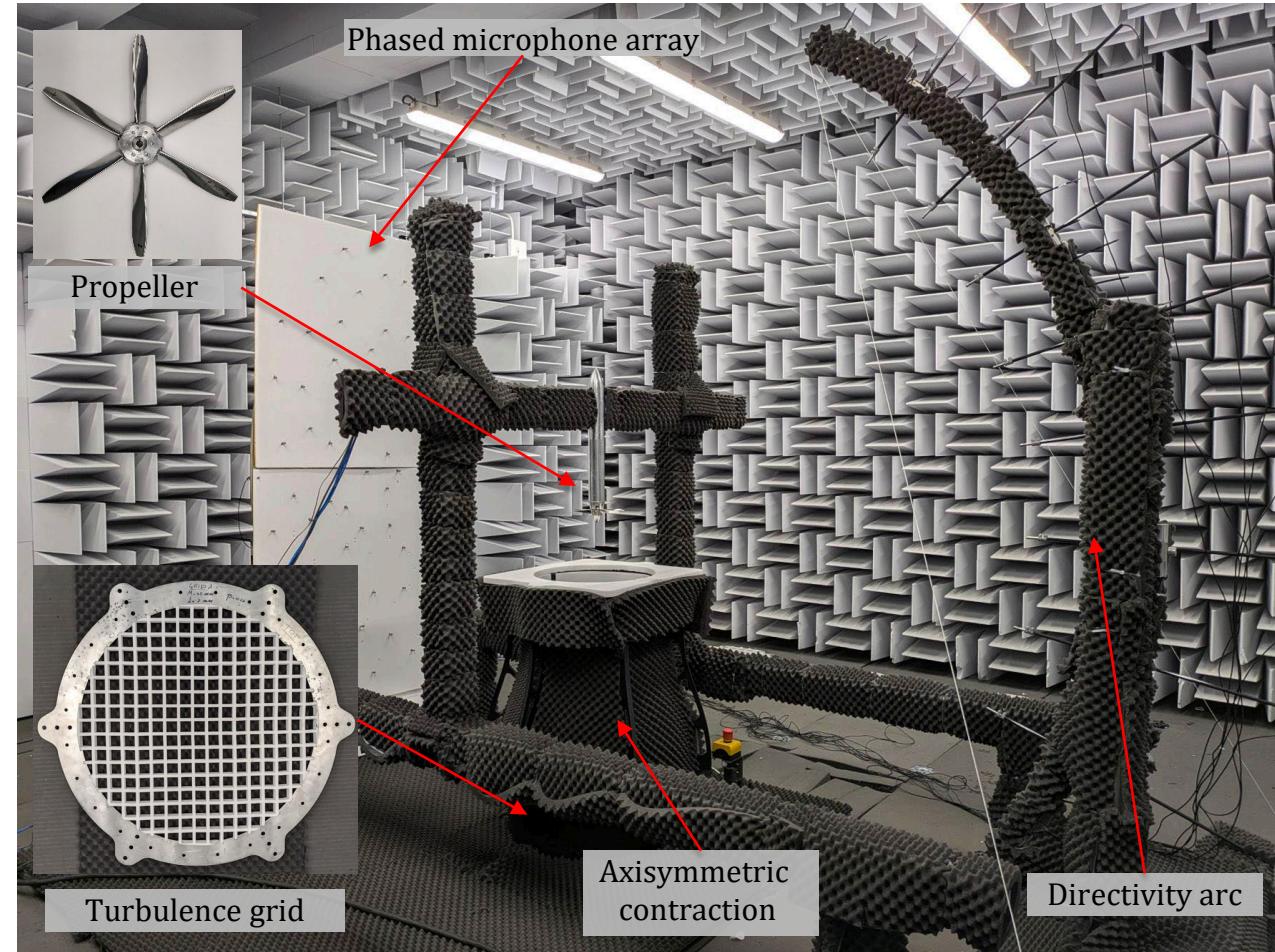
# 16.9 – Applications: Acoustic imaging of aircraft noise



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# 16.9 – Applications: Anechoic rooms



# 16.9 – Applications: Anechoic rooms



[Link to full video \(Veritasium\)](#)

**Another interesting YouTube video. This one about anechoic rooms:**

**“Can Silence Actually Drive You Crazy?”**

# 16.9 – Applications: Psychoacoustic listening tests

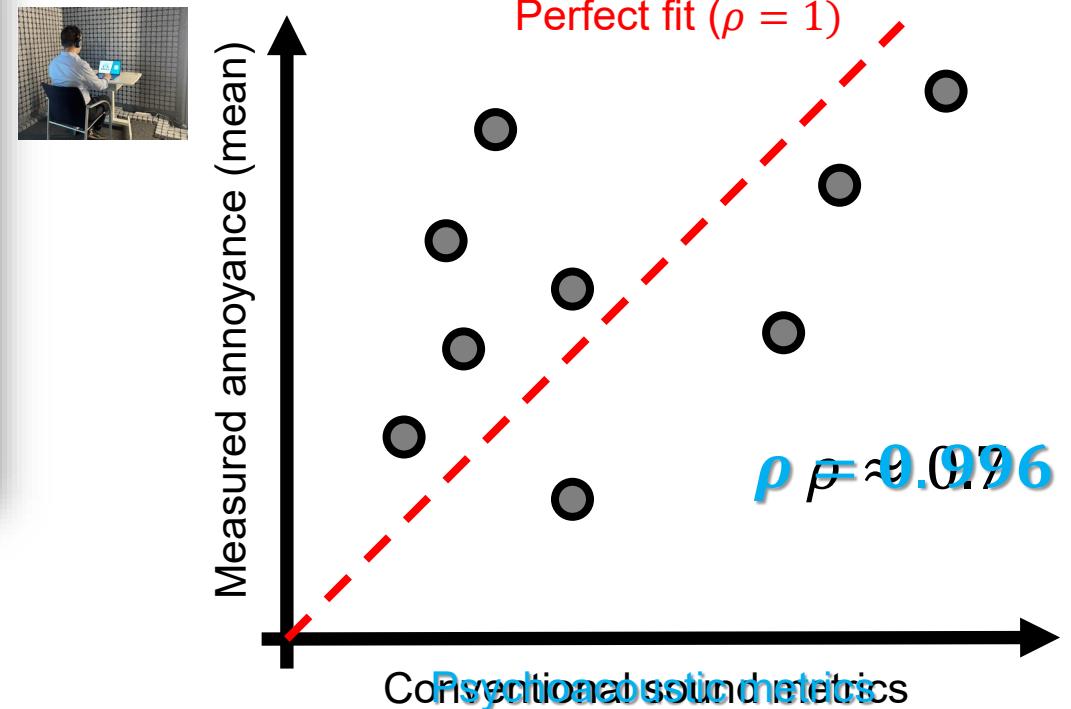
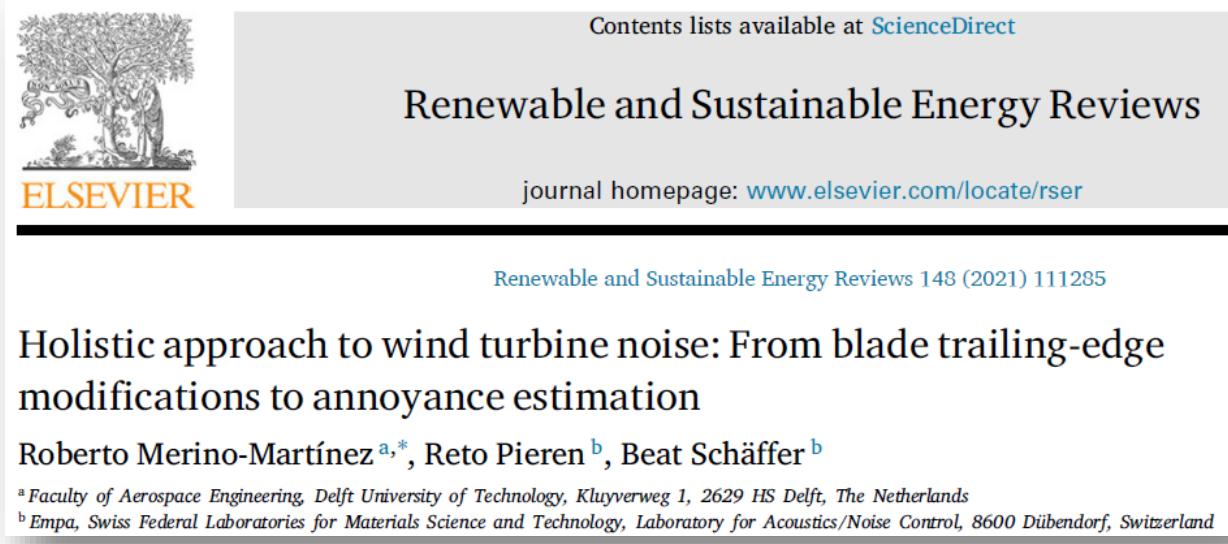


Subscribe as  
participant if you want



# 16.9 – Predicting noise annoyance

**Psychoacoustic perception-based metrics** are better indicators of noise annoyance than conventional metrics used in practice.



# An example: Optimized low-noise propeller

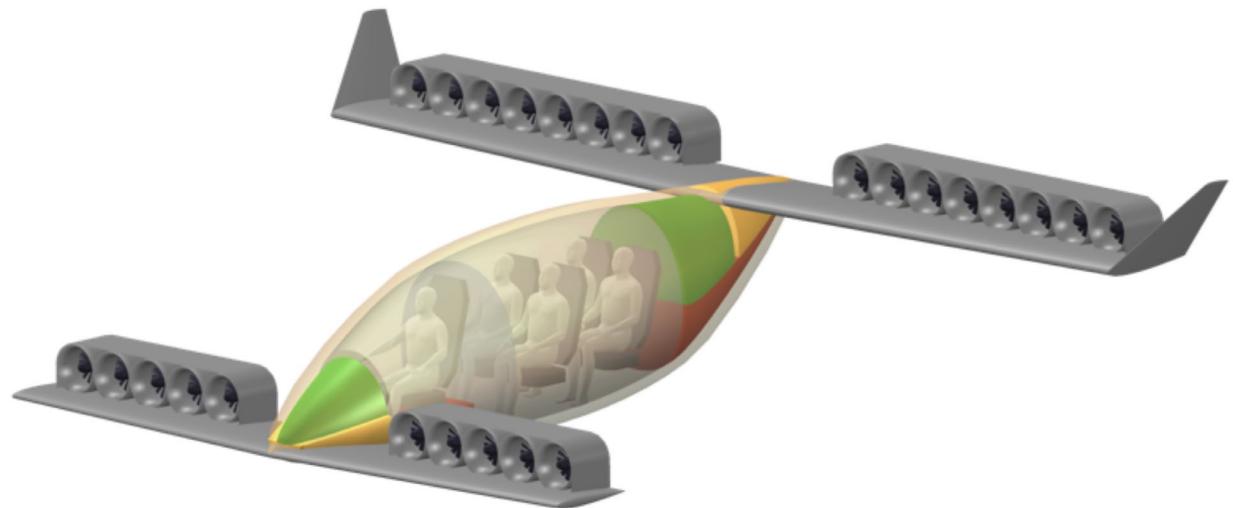
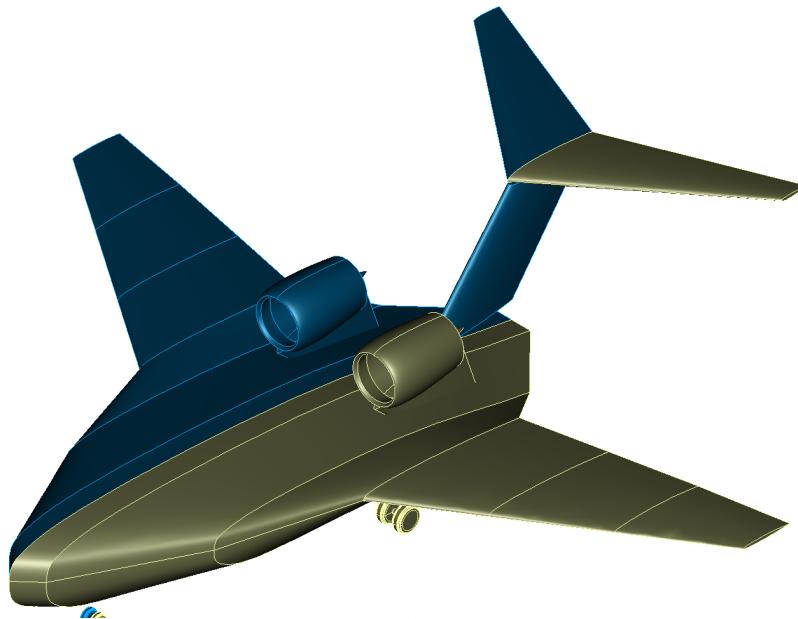


**Baseline  
propeller**



**Optimized  
propeller**

# An example: Making future aircraft audible



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# WRAP-UP

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# Wrap-up: revisit learning objectives

After this lecture you should be able to:

- Explain the **sound generation** from different sound sources (e.g. vibrating strings and air columns).


$$\text{Strings: } f_n = \frac{nv}{2l}, \quad n = 1, 2, 3 \dots \quad \text{Open tubes: } f_n = \frac{nv}{2l} \quad \text{Closed tubes: } f_n = \frac{nv}{4l}, \quad n = 1, 3, 5 \dots$$

- 
- Explain the phenomena **sound interference** (beats) and superposition.

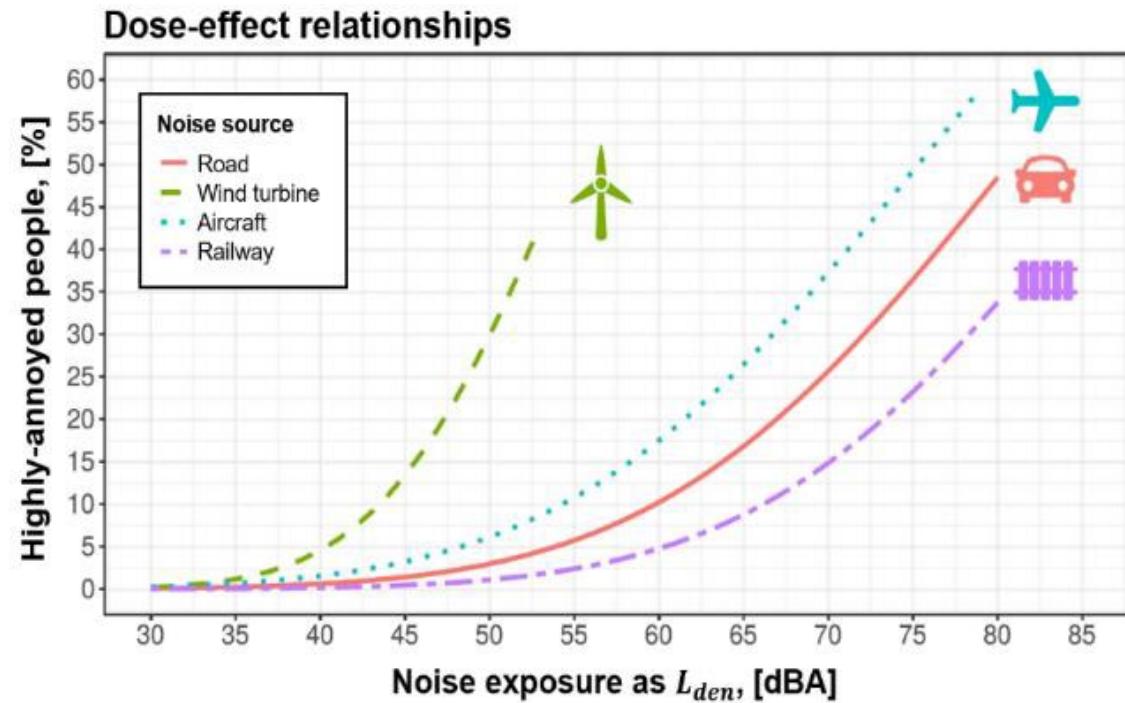
$$f_{beat} = f_1 - f_2$$

- 
- Calculate the effects of moving sources: **Doppler effect** and shock waves (**sonic boom**)

$$f' = f \frac{v_{sound} \pm v_{observer}}{v_{sound} \mp v_{source}} \quad f' = \frac{f}{1 - \|M_s\| \cos \theta}$$

$$\sin \theta = \frac{1}{M}$$

# Reminder: Sound perception - Competition



Fredianelli, L. et al, Science of the Total Environment, 2019.

The deadline for submission is  
**Monday 24<sup>th</sup> at 17:00**

Lower SPL does not always mean lower (perceived) **noise annoyance**.

To test this claim yourselves, there is a small “competition” in BrightSpace inside the *Noise\_annoynce\_competition.zip* file with some instructions.

The idea is that you can submit different sounds that you find **very annoying or very pleasing** and see which one scores higher.

# For next lecture – Chapter 17 (Temperature, thermal expansion, and the ideal gas law)

1. Atomic Theory of Matter
2. Temperature and Thermometers
3. Thermal Equilibrium and the Zeroth Law of Thermodynamics
4. Thermal Expansion
5. Thermal Stress
6. The Gas Laws and Absolute Temperature
7. The Ideal Gas Law
8. Problem Solving with the Ideal Gas Law
9. Ideal Gas Law in Terms of Molecules: Avogadro's Number
10. Ideal Gas Temperature Scale—a Standard

# SOUND

## *Chapter 16*



**Dr. Roberto Merino-Martinez**

Operations & Environment section

Faculty of Aerospace Engineering