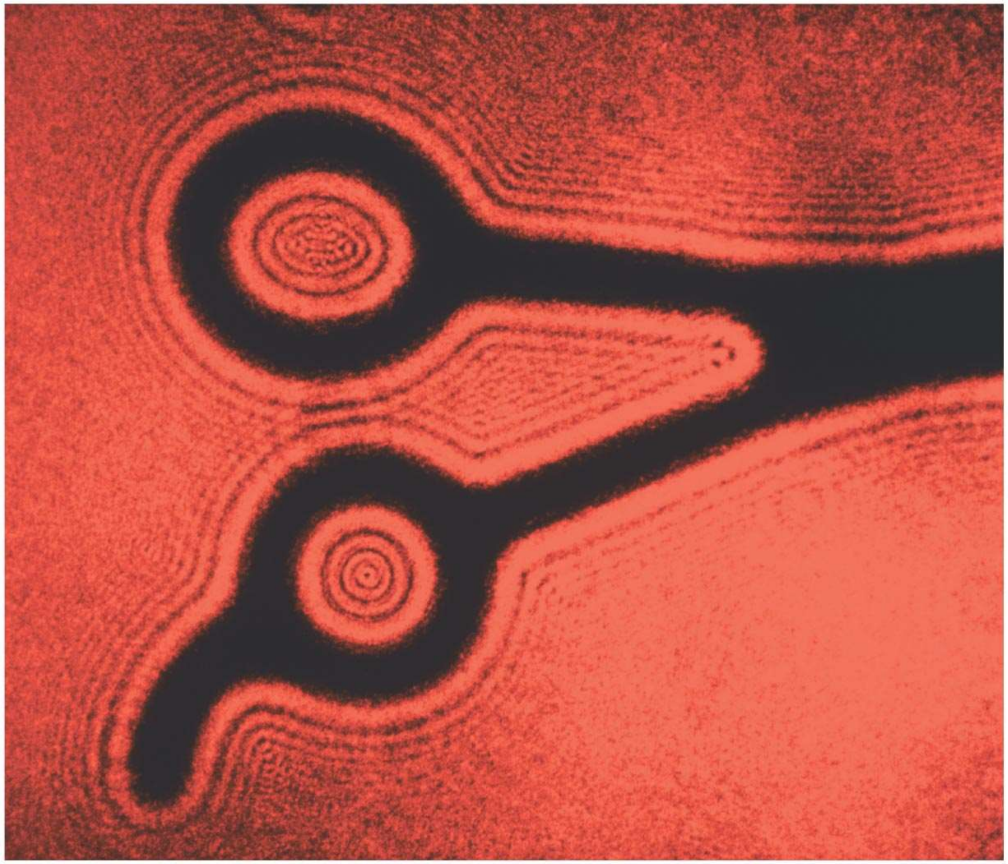
Chapter 35: Diffraction



Chapter 35: Diffraction

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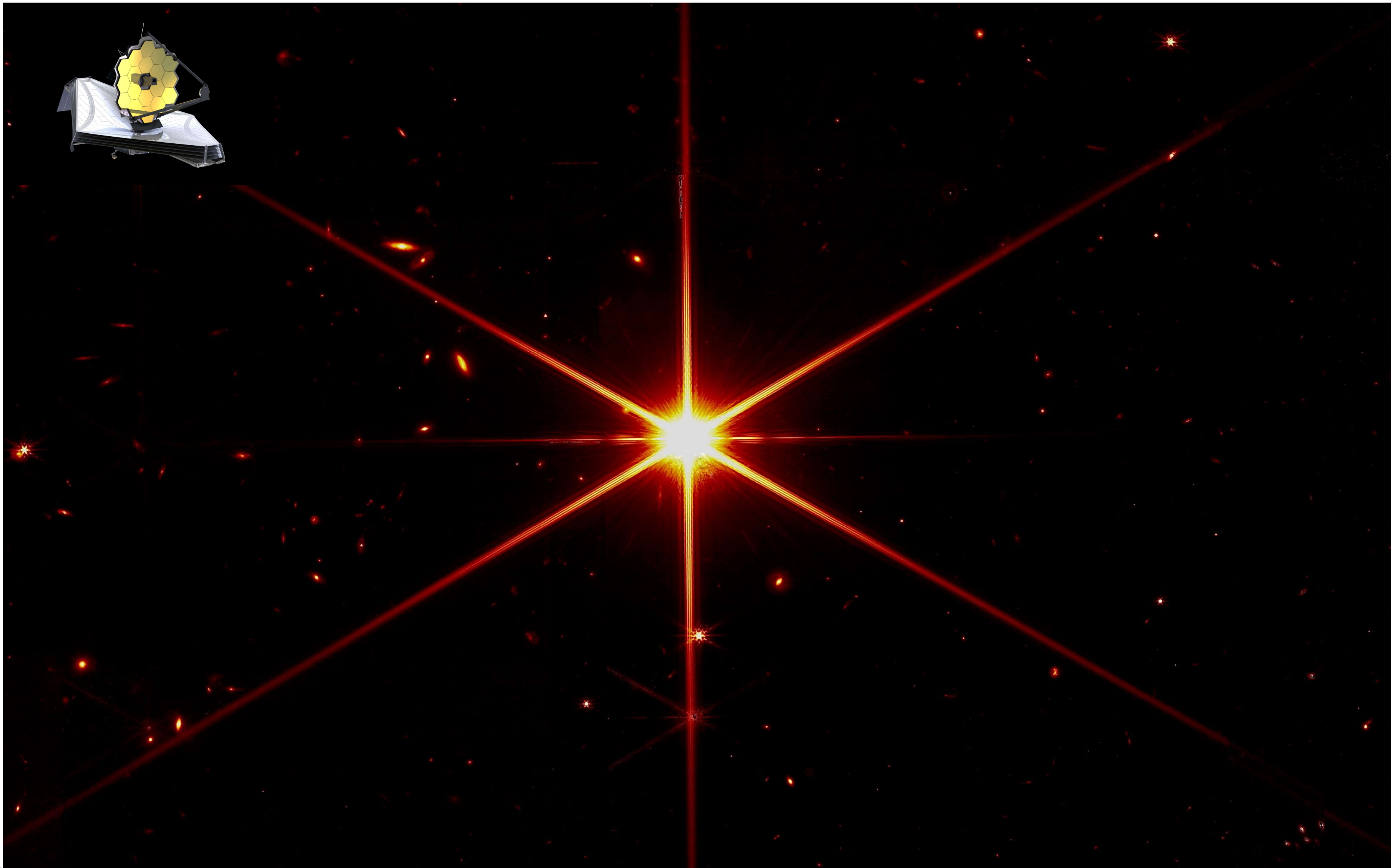

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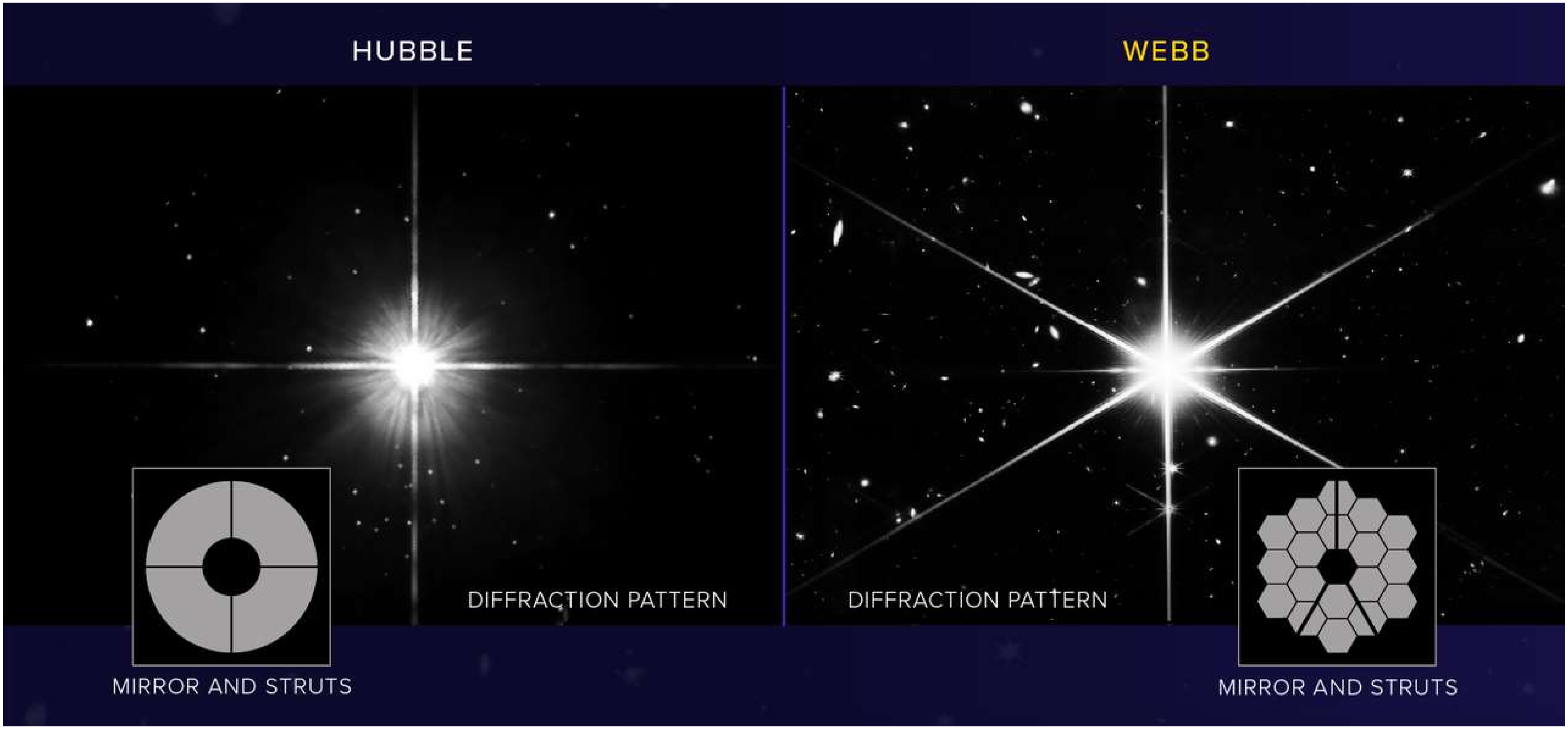
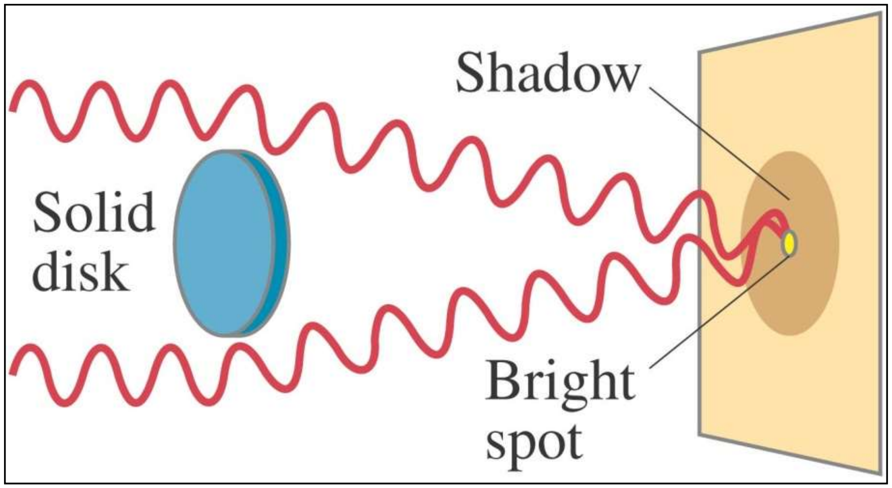
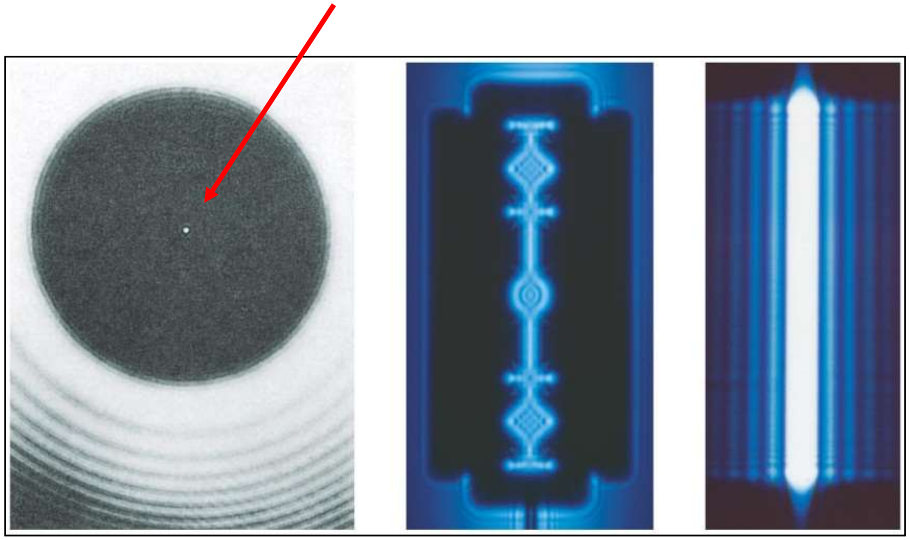

HUBBLE
WEBB
DIFFRACTION PATTERN
DIFFRACTION PATTERN
MIRROR AND STRUTS
MIRROR AND STRUTS

35 -0 Introduction: Diffraction

- If light is a wave, a bright spot will appear at the centre of the shadow.
- The experimental observation is strong evidence for the wave theory.

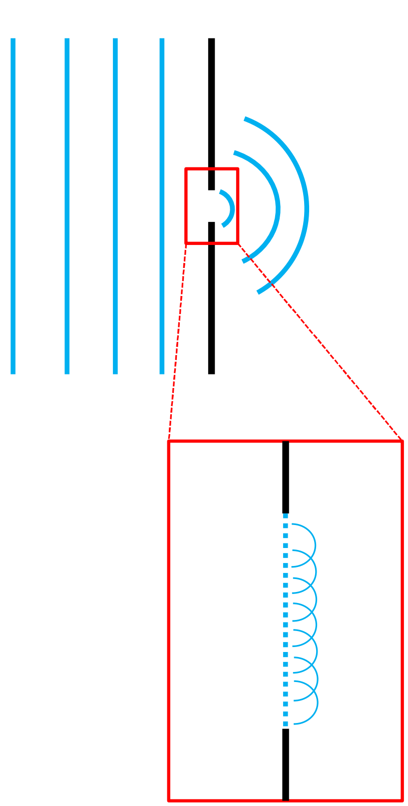


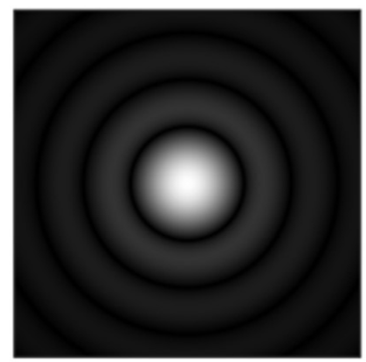


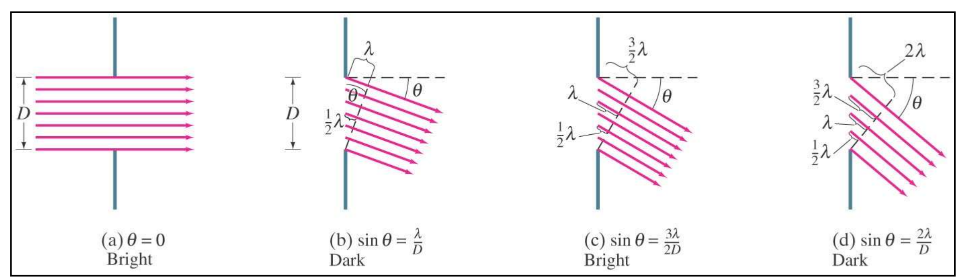
- Diffraction pattern of a circular disk, razor blade and a single slit, each illuminated by a coherent point source of monochromatic light.

35-1 Diffraction by a Single Slit or Disk

- This pattern arises because different points along a slit create wavelets that **interfere with each other** just as a double slit would.

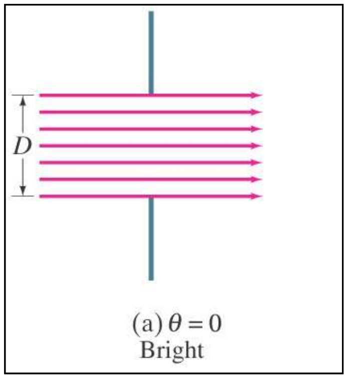




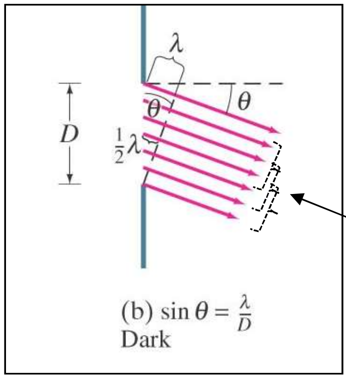


35-1 Diffraction by a Single Slit or Disk

- First we consider rays that pass straight through the hole.
  - They are all in phase. ⇒ **constructive interference**
  - A central bright spot on the screen



- We consider rays reaching the screen that move at an angle θ
  - Such that the ray from the top of the slit travels exactly one wavelength farther than the ray from the bottom
  - The ray from the center will travel one-half wavelength $(\frac{\lambda}{2})$ farther than the ray at the bottom, these two rays will be out of phase ⇒ they will cancel out **(destructive interference)**.
  - This process occurs such as $\sin(\theta) = \frac{\lambda}{D}$
  - The light intensity is maximum at $\theta = 0°$ and decreases to 0 at $\theta = \text{asin}\left(\frac{\lambda}{D}\right)$

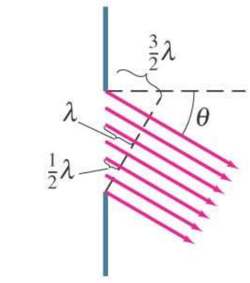


Rays cancel out 2-by-2

35-1 Diffraction by a Single Slit or Disk

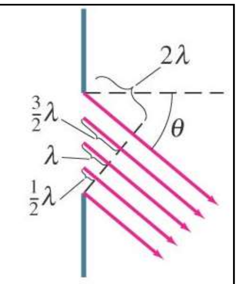
- Now, we consider a larger θ such as the top ray travels $\frac{3}{2}\lambda$ farther than the bottom ray.
  - Rays from the bottom third will cancel out in pairs with those in the middle third as they are in phase opposition
  - The top third is not cancelled out then light reaches the screen.

  ⇒ **« constructive » interference** at $sin(\theta) = \frac{3\lambda}{2D}$



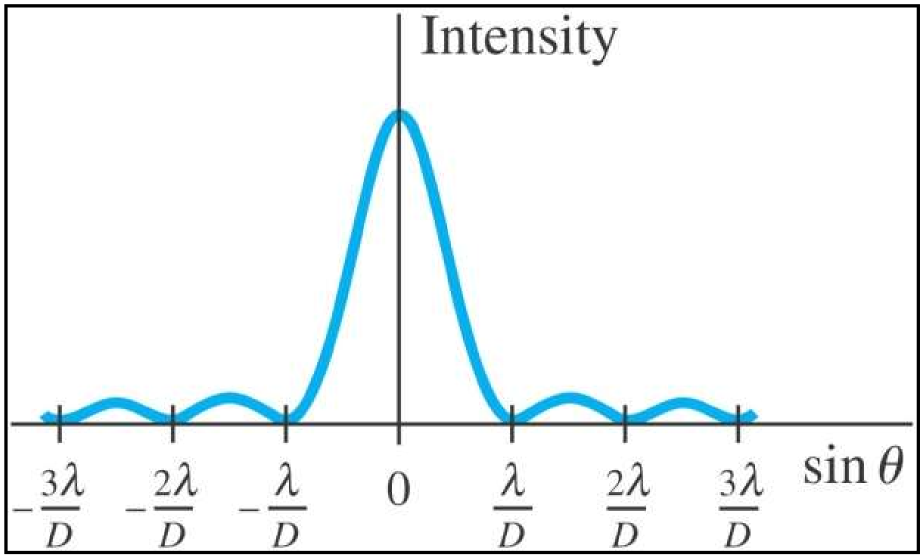
(c) $\sin\theta = \frac{3\lambda}{2D}$
Bright

- we consider a larger θ such as the top ray travels 2λ farther than the bottom ray.
  - Rays cancel out by quarter.
  - ⇒ **destructive interference**



(d) $\sin\theta = \frac{2\lambda}{D}$
Dark

35-1 Diffraction by a Single Slit or Disk



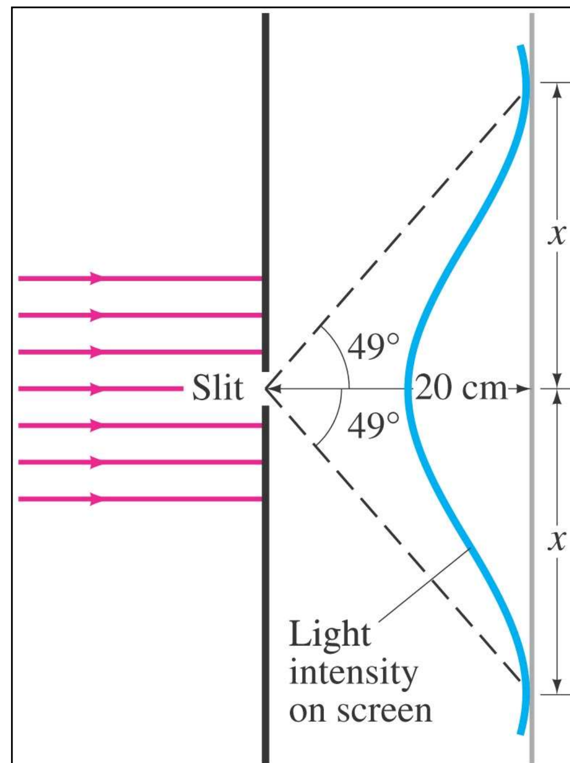
Maxima: $\theta = 0$ (strong), $\sin(\theta) = \frac{m\lambda}{D}$ with $m = \pm\frac{3}{2}, \pm\frac{5}{2}, \ldots$

Minima: $\sin(\theta) = \frac{m\lambda}{D}$ with $m = \pm 1, \pm 2, \pm 3, \ldots$

35-1 Diffraction by a Single Slit or Disk

- Example 35-1: Single-slit diffraction maximum.
  - Light of wavelength 750 nm passes through a slit $1{,}0\ 10^{-3} mm$ wide.
  - How wide is the central maximum ?
    - (a) in degrees
    - (b) in cm, if a screen is placed a 20 cm away

    - $\sin(\theta) = \frac{m\lambda}{D}$



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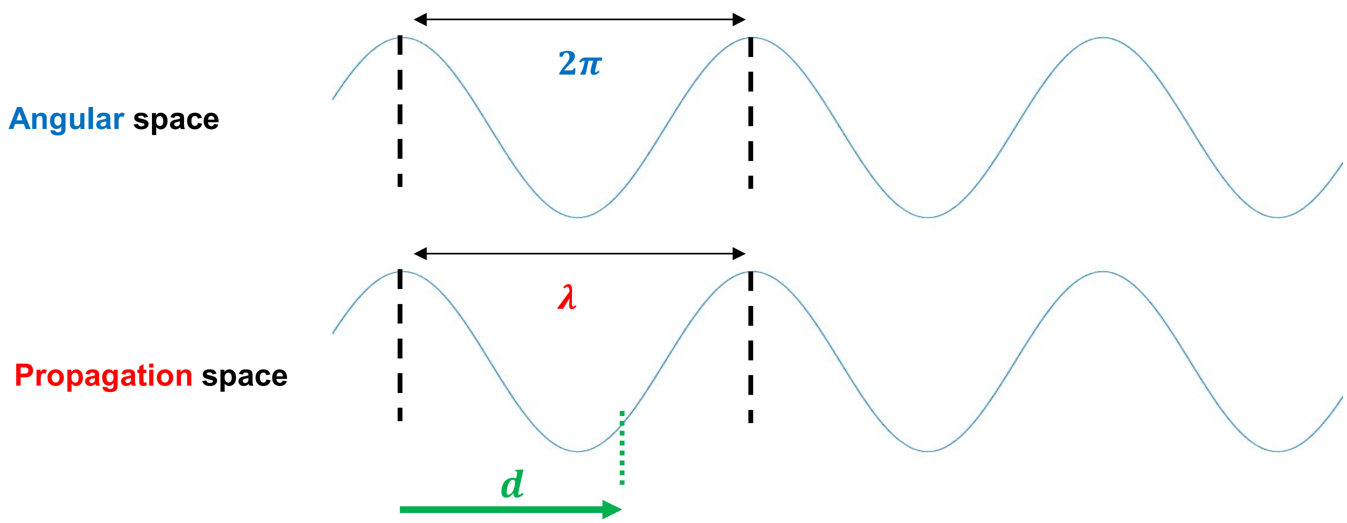
35-1 Diffraction by a Single Slit or Disk

- How phase difference is defined in a wave?

  relationship between phase and wavelength



2π in the angular space is equivalent to λ in the "propagation" space

A fraction of wavelength gives a phase shift of $\delta = \frac{2\pi}{\lambda} * d$

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35-2 Intensity in Single-Slit Diffraction Pattern

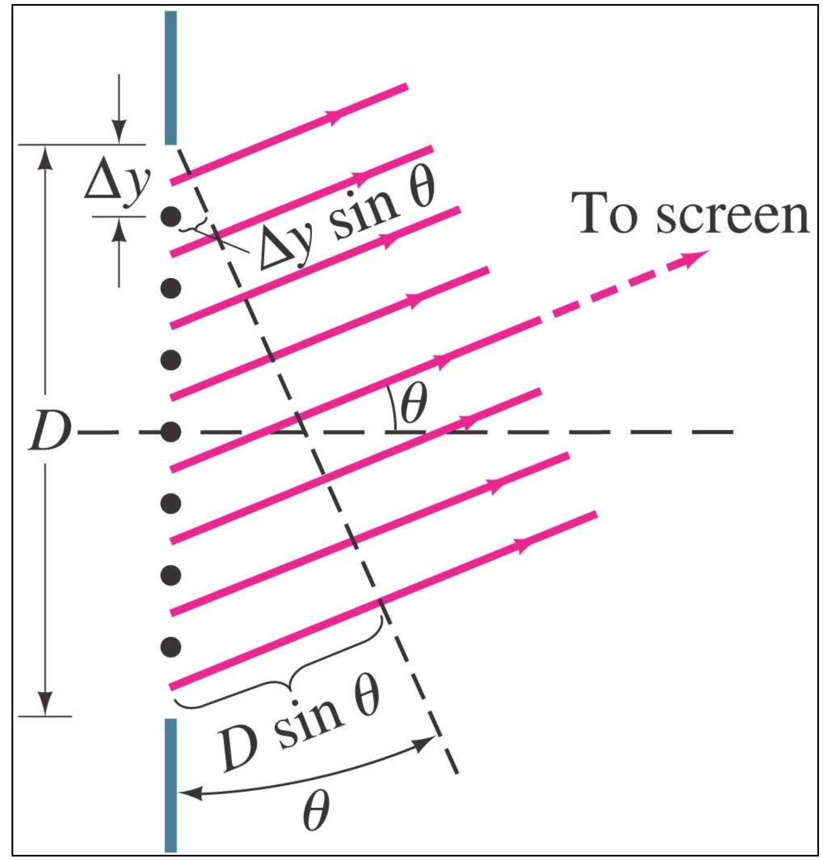
- Light passing through a single slit can be divided into a series of narrower strips;
- each contributes the same amplitude to the total intensity on the screen,
- but the phases differ due to the differing path lengths:

$$\boxed{\delta = \Delta y \sin(\theta)}$$

(optical path difference between one strip to each other)

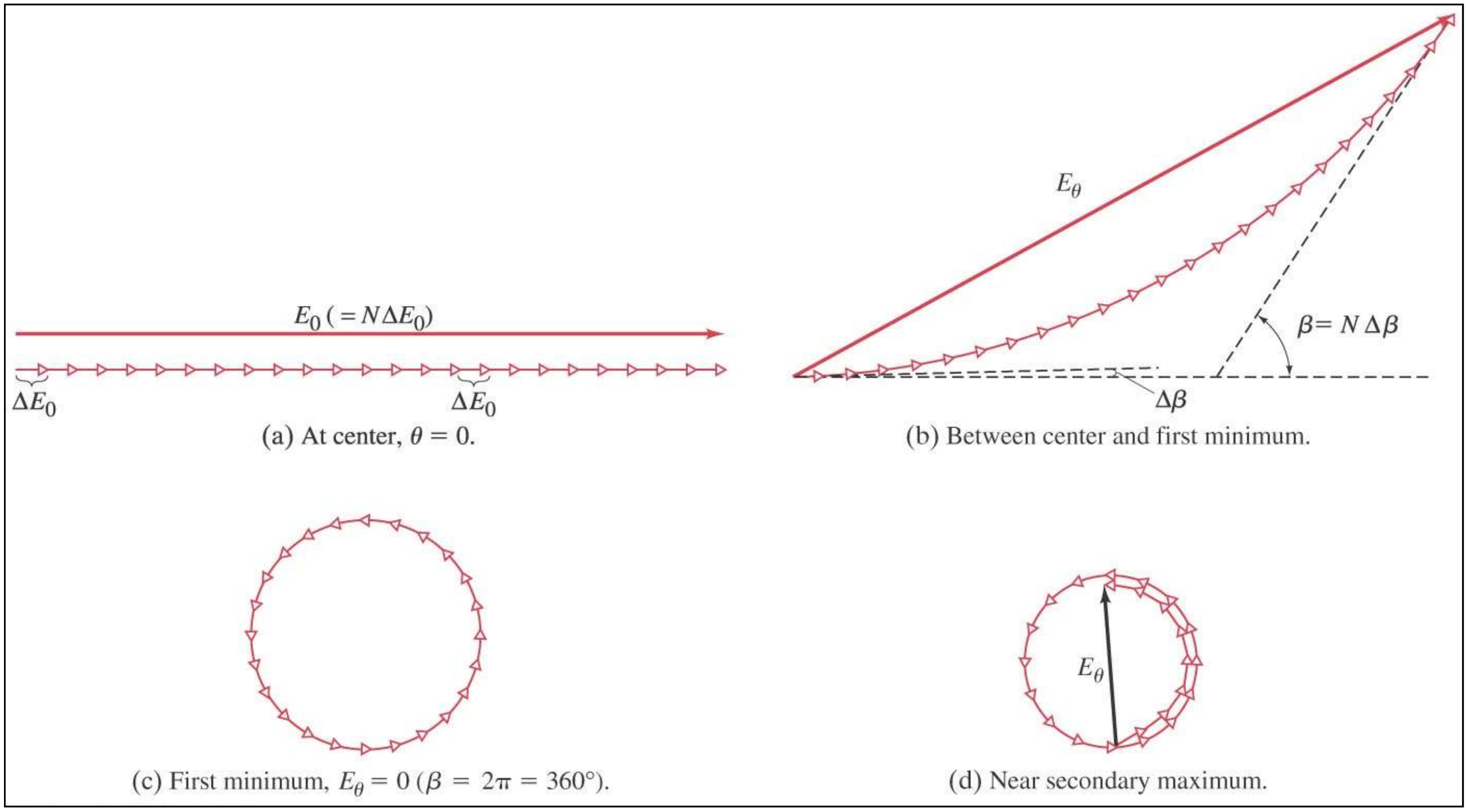
$$\boxed{\Delta\beta = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda}\Delta y \sin(\theta)}$$

(phase difference)



35-2 Intensity in Single-Slit Diffraction Pattern

- Phasor diagrams give us the intensity as a function of angle.



(a) At center, $\theta = 0$.

(b) Between center and first minimum.

(c) First minimum, $E_\theta = 0$ ($\beta = 2\pi = 360°$).

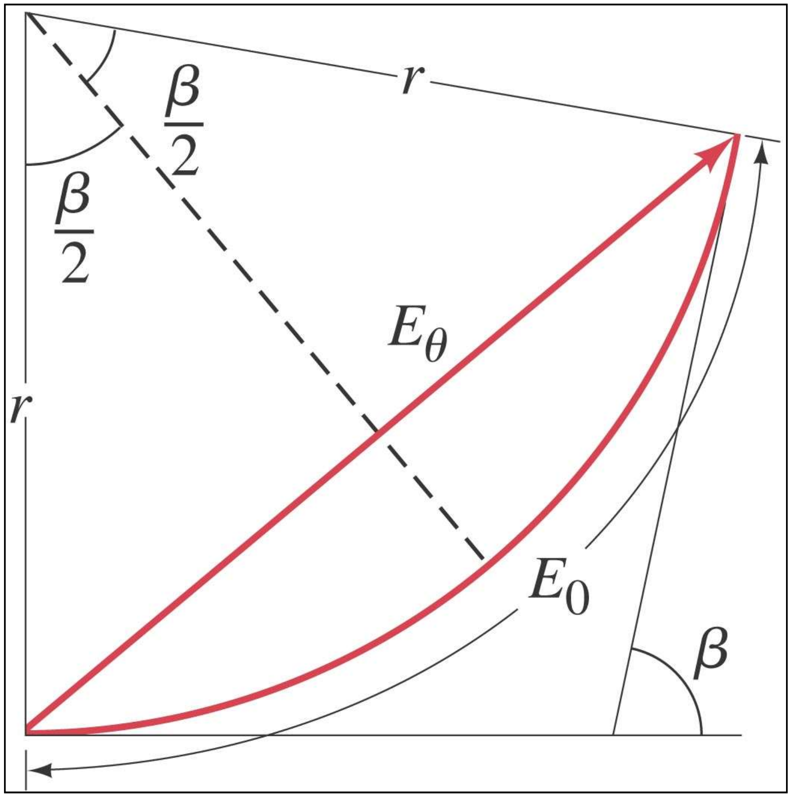
(d) Near secondary maximum.

35-2 Intensity in Single-Slit Diffraction Pattern

- Taking the limit as the width becomes infinitesimally small ($\Delta y \rightarrow 0$) gives the field as a function of angle:

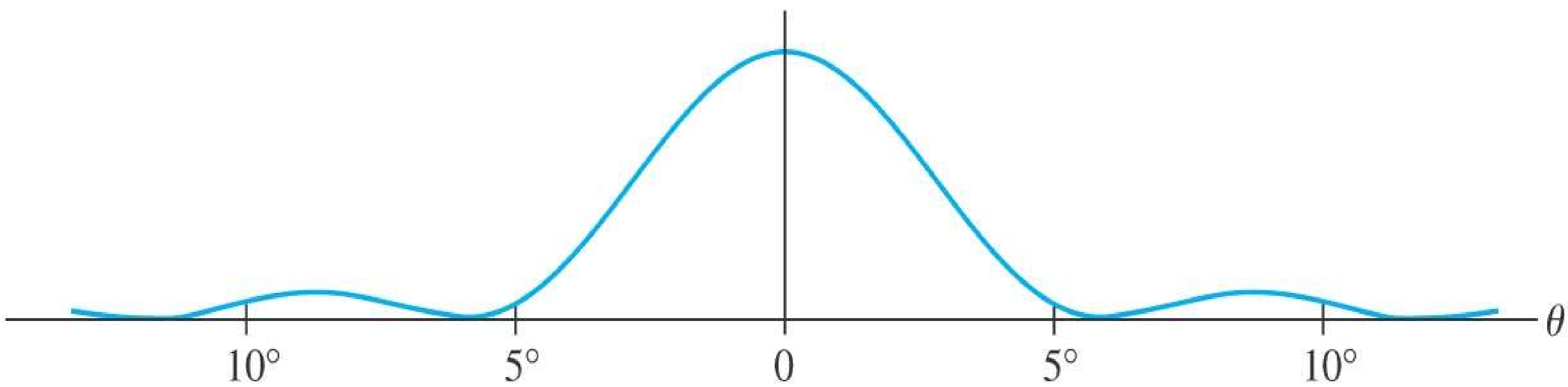
$$\boxed{E_\theta = E_0 \frac{\sin\left(\frac{\beta}{2}\right)}{\frac{\beta}{2}}}$$

With $\beta = \frac{2\pi}{\lambda} D \sin(\theta)$ and $I_\theta = \boldsymbol{I}_0 \left(\frac{\sin\left(\frac{\beta}{2}\right)}{\frac{\beta}{2}}\right)^2$



35-2 Intensity in Single-Slit Diffraction Pattern

Sinc function : $\frac{\sin(x)}{x}$



(a) Diffraction factor, $(\sin^2\beta/2)/(\beta/2)^2$ vs. θ

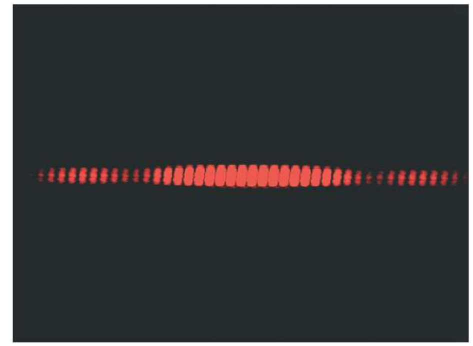
35-3 Diffraction in the Double-Slit Experiment

- The double-slit experiment also exhibits diffraction effects, as the slits have a finite width. This means the amplitude at an angle θ will be modified by the same factor as in the single-slit experiment:

$$E_{\theta 0} = 2E_0 \underbrace{\left(\frac{\sin\left(\frac{\beta}{2}\right)}{\frac{\beta}{2}}\right)}_{\text{Diffraction}} \overbrace{\cos\left(\frac{\delta}{2}\right)}^{\text{Young Interference}}$$

with $\beta = \frac{2\pi}{\lambda} D \sin(\theta)$ and $\delta = \frac{2\pi}{\lambda} d \sin(\theta)$

- Remark: The intensity is, as usual, proportional to the square of the field.

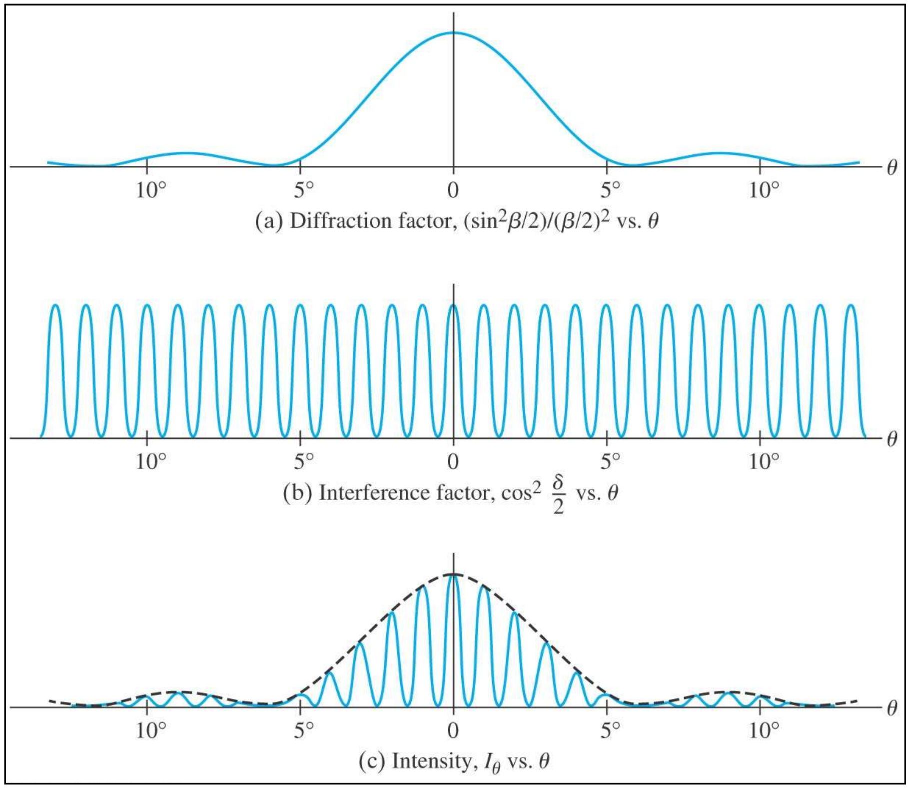


(a)

(b)

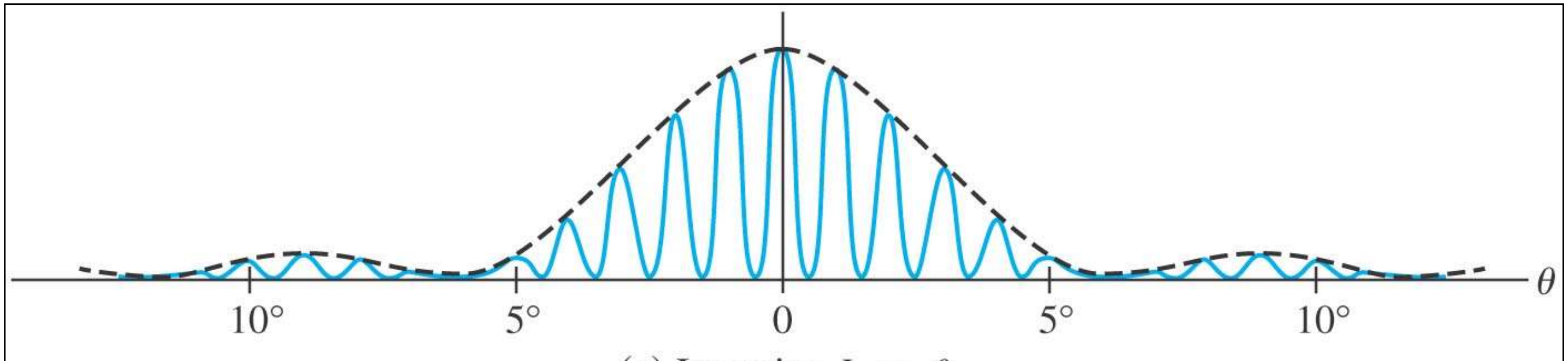
35-3 Diffraction in the Double-Slit Experiment

- The diffraction factor (depends on β) appears as an "envelope" modifying the more rapidly varying interference factor (depends on δ)



35-3 Diffraction in the Double-Slit Experiment

- Example 35-4: Diffraction plus interference.
- Show why the central diffraction peak shown, plotted for the case where $d = 6D = 60\lambda$, contains 11 interference fringes.



(c) Intensity, I_θ vs. θ

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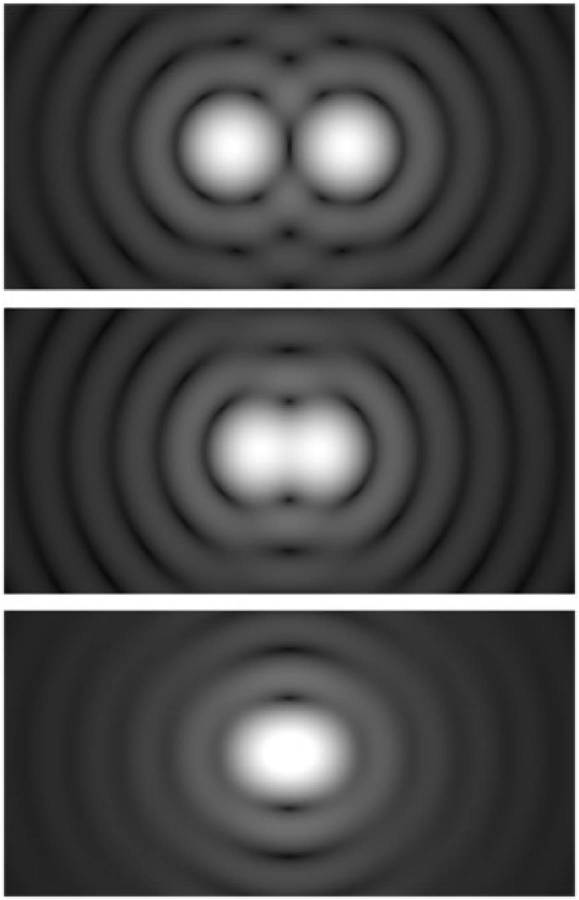
35-4 Limits of Resolution; Circular Apertures

- Resolution is the
  - minimal distance between two separate objects at which an optical system can barely distinguish them.
  - Can be expressed in distance or in angle (far objects)

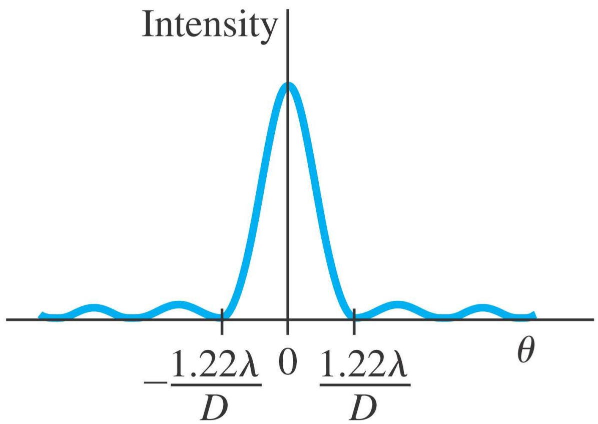
- Resolution is limited by aberrations and by diffraction.
  - Aberrations can be minimized,
  - but diffraction is unavoidable;
  - it is due to the size of the lens compared to the wavelength of the light.

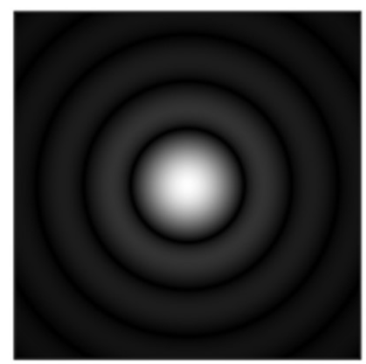


35-4 Limits of Resolution; Circular Apertures

- For a circular aperture of diameter D, the central maximum has an angular width:

$$\theta = \frac{1.22\lambda}{D}. \qquad [\theta \text{ in radians}]$$

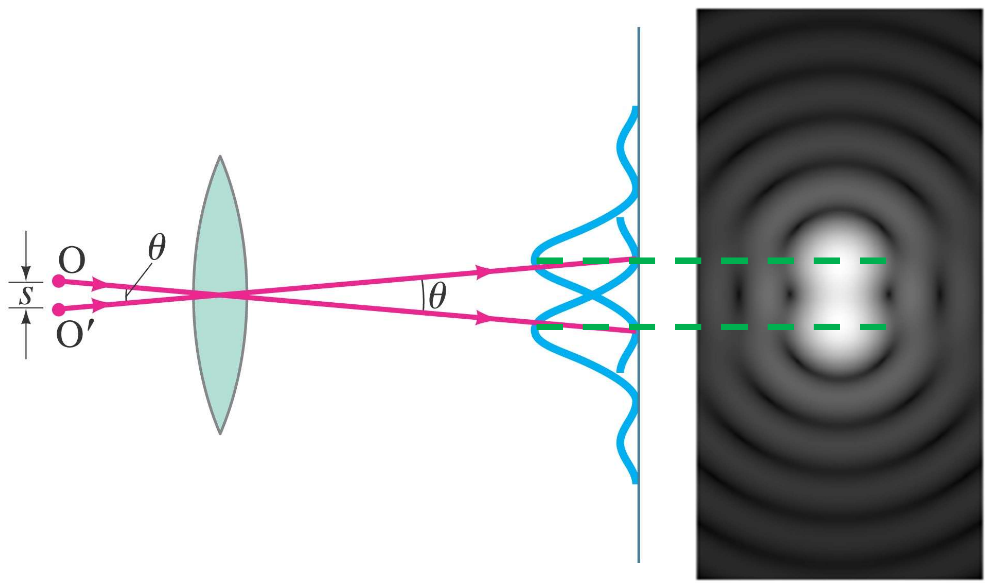




Remark: 1.22 comes from the 1st Bessel function → solution for a circular aperture

35-4 Limits of Resolution; Circular Apertures

- The Rayleigh criterion states that two images are just resolvable when the center of one peak is over the first minimum of the other.



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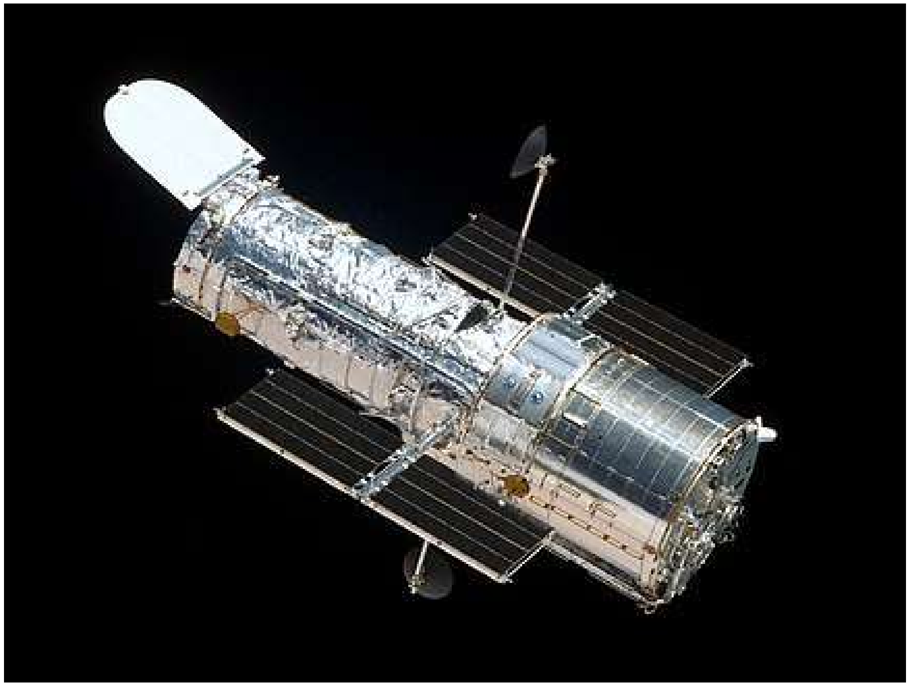
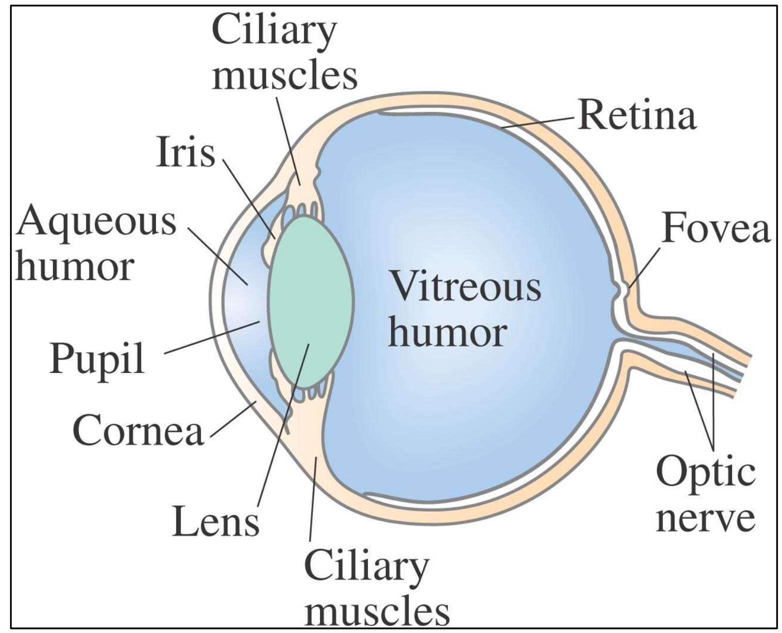
35-4 Limits of Resolution; Circular Apertures

- **Example 35-5: Hubble Space Telescope.**
  - The Hubble Space Telescope (HST) is a reflecting telescope that was placed in orbit above the Earth's atmosphere
  - its resolution would not be limited by turbulence in the atmosphere.
  - Its objective diameter is 2.4 m, focal length is 57.6 m
  - For visible light, say $\lambda = 550\ nm$, estimate the improvement in resolution the Hubble offers over Earth-bound telescopes ($\approx half\ arcsec$).

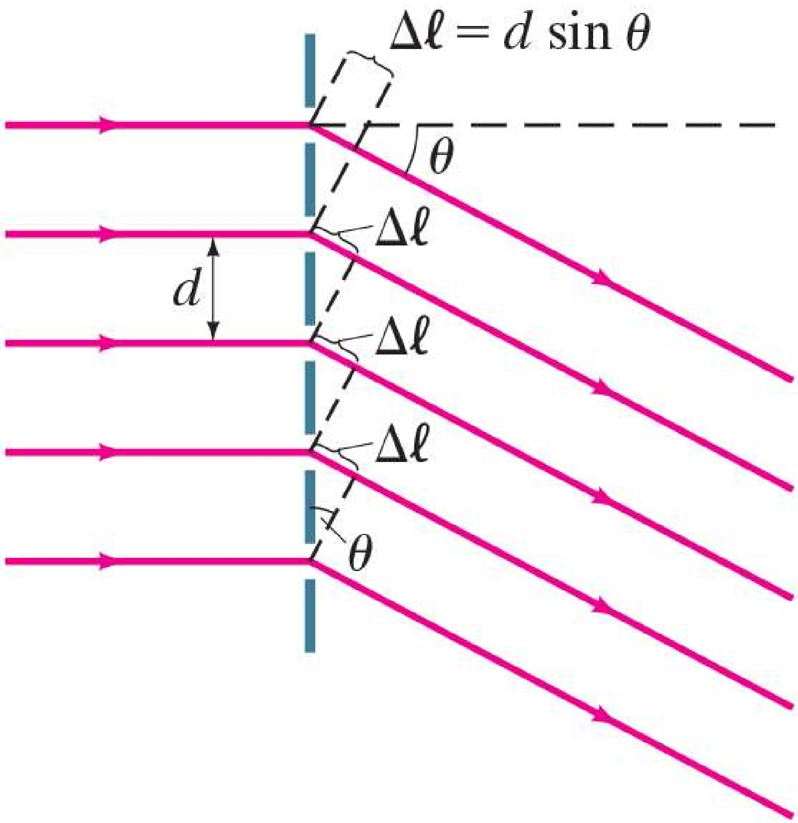


35-4 Limits of Resolution; Circular Apertures

- **Example 35-6: Eye resolution.**
  - You are in an airplane at an altitude of 10,000 m. If you look down at the ground,
  - estimate the minimum separation s between objects that you could distinguish.
  - Could you count cars in a parking lot? Consider only diffraction, and assume your pupil is about 3.0 mm in diameter and $\lambda = 550\ nm$.



35-7 Diffraction Grating



A diffraction grating consists of a large number of equally spaced narrow slits or lines.

A transmission grating has slits, while a reflection grating has lines that reflect light.

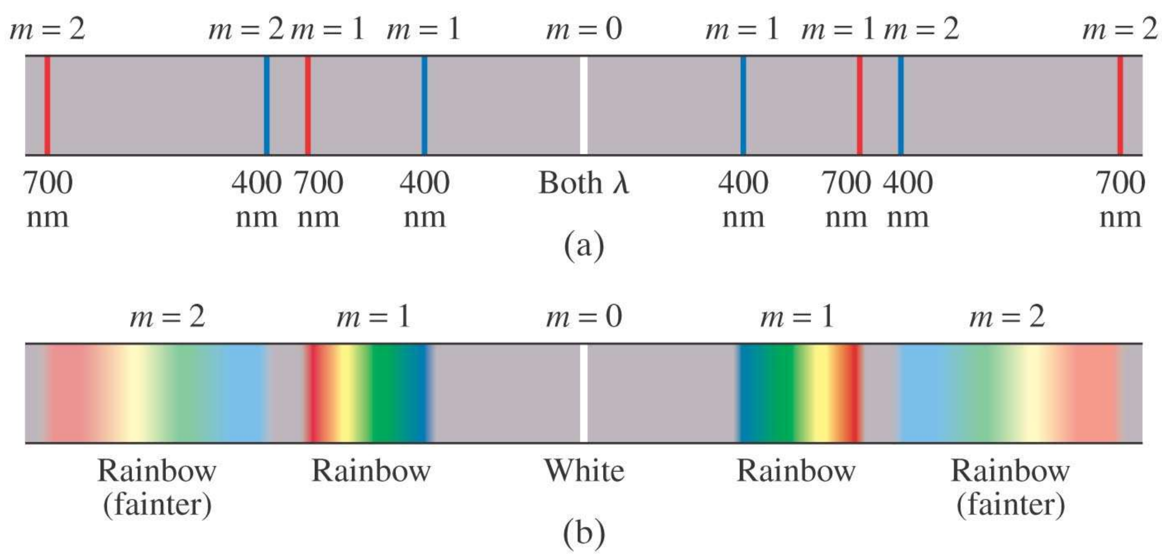
Grating equation

$$\sin(\theta) = \frac{m\lambda}{d}$$

35-7 Diffraction Grating

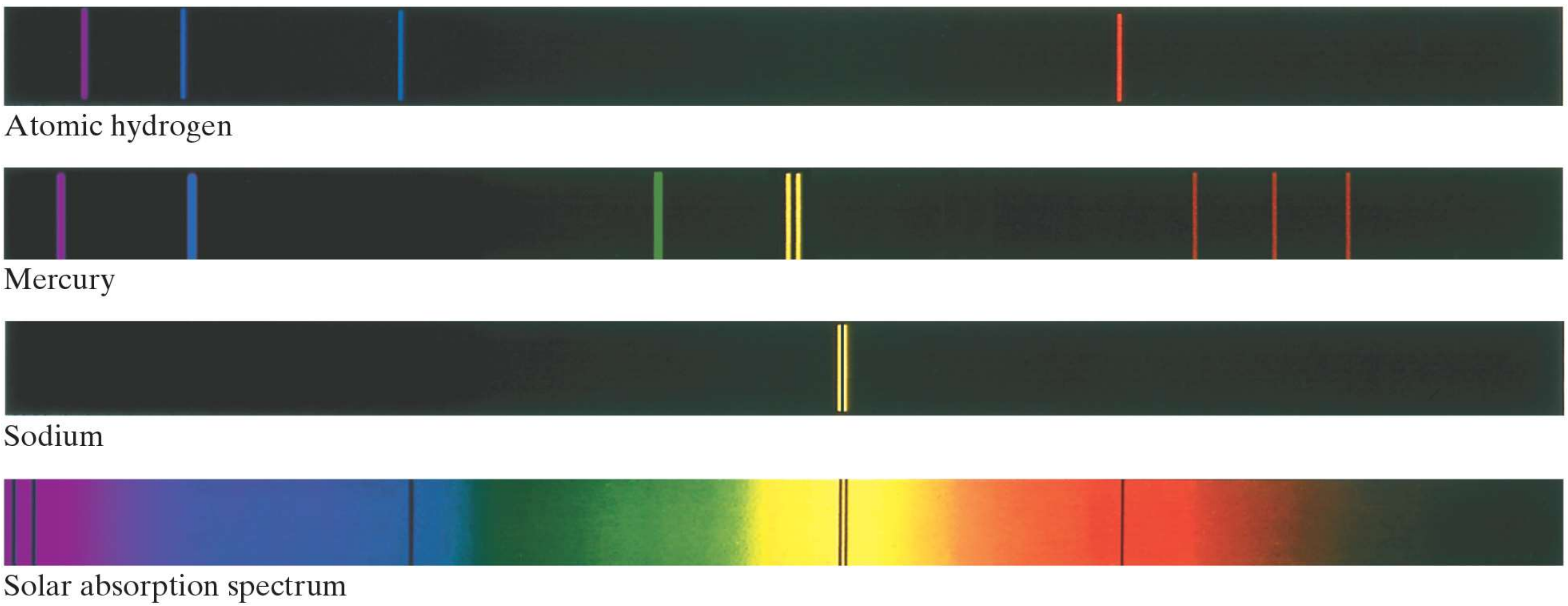
The maxima of the diffraction pattern are defined by

$$\sin\theta = \frac{m\lambda}{d}, \qquad m = 0, 1, 2, \cdots \qquad \left[\begin{array}{c}\text{diffraction grating,}\\ \text{principal maxima}\end{array}\right]$$

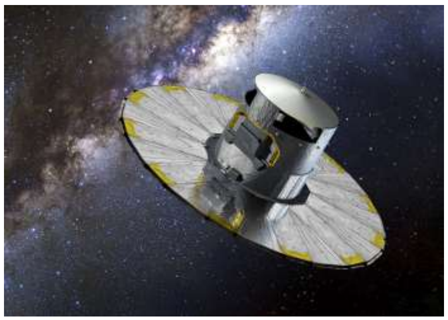


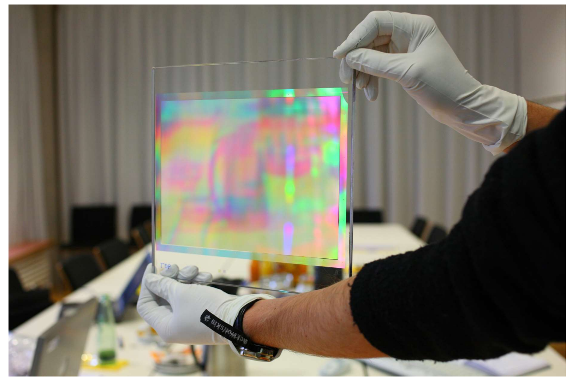
35-7 Diffraction Grating

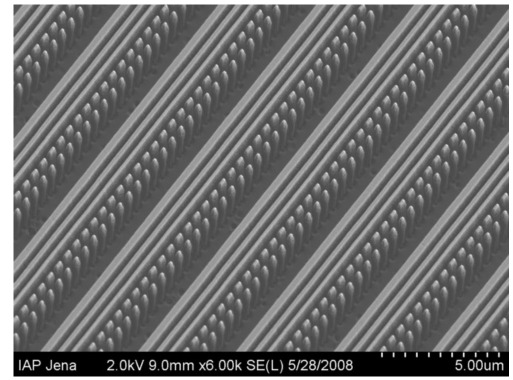
35-7 Diffraction Grating - Spectrum

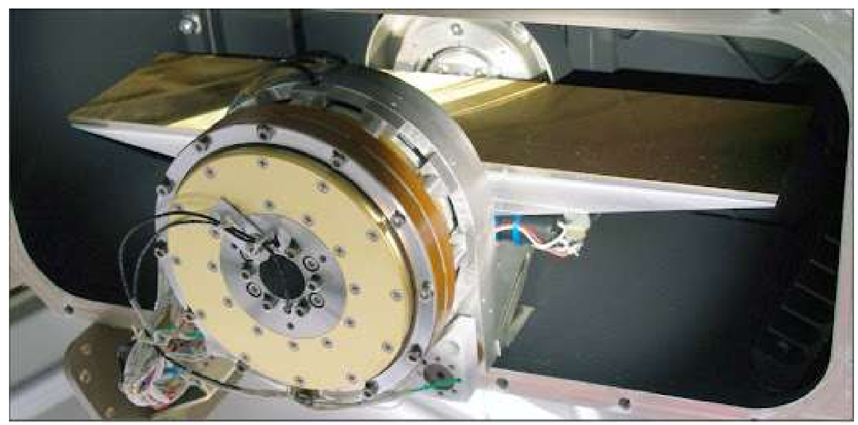


35-7 Diffraction Grating – Space



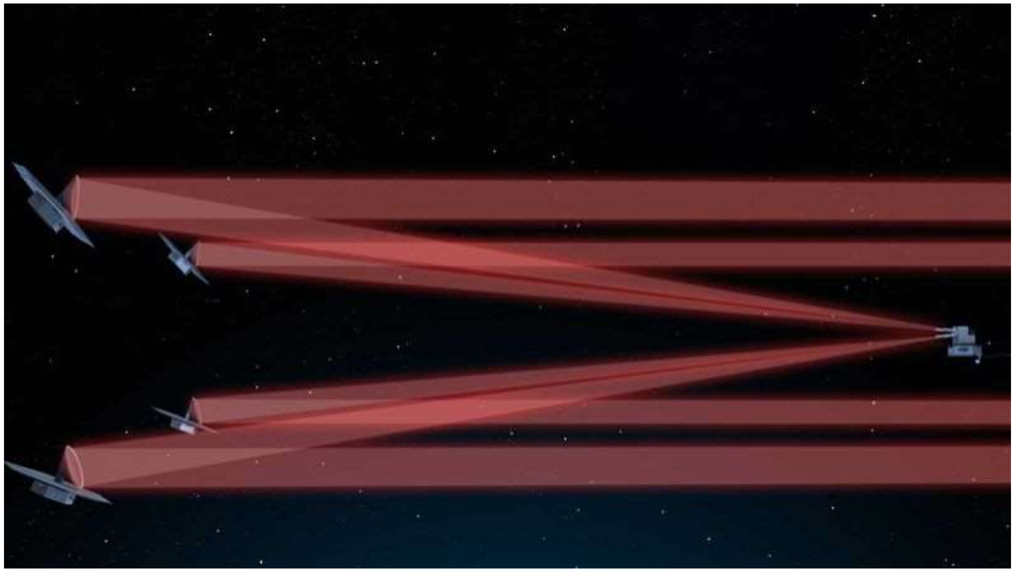






Interferometry for Astronomy

VLT – Very Large Telescope



LIFE – Large interferometer for exoplanets