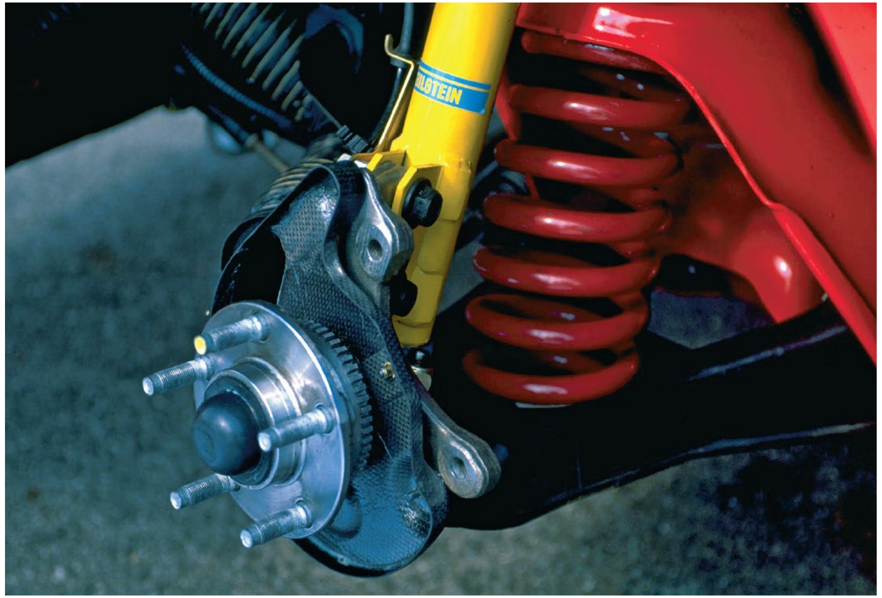
OSCILLATIONS

Chapter 14



Dr. Roberto Merino-Martinez

Operations & Environment section

Faculty of Aerospace Engineering

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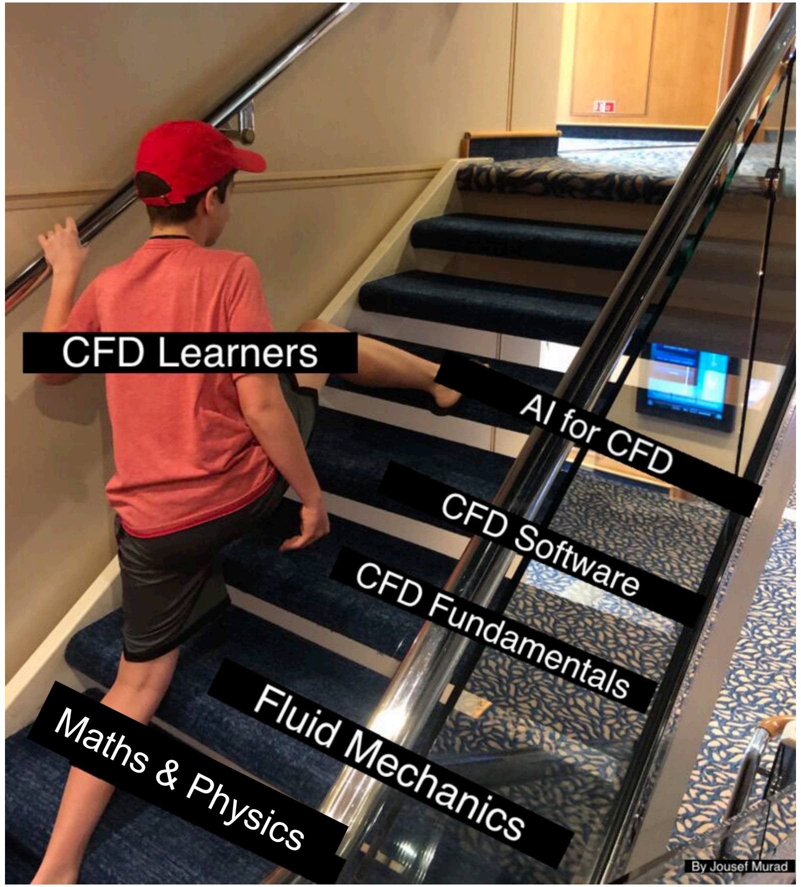
Before we start…

… why do we need this course? We have AI now!



Position in the syllabus

14. **Oscillations**
15. Waves
16. Sound
17. Temperature and the ideal gas law
18. Thermodynamics
19. Electricity and circuits
20. Electromagnetism
21. Optics

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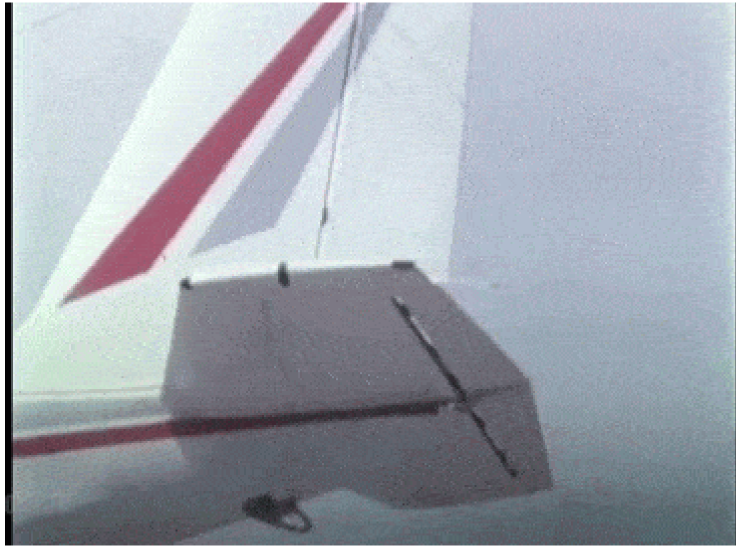
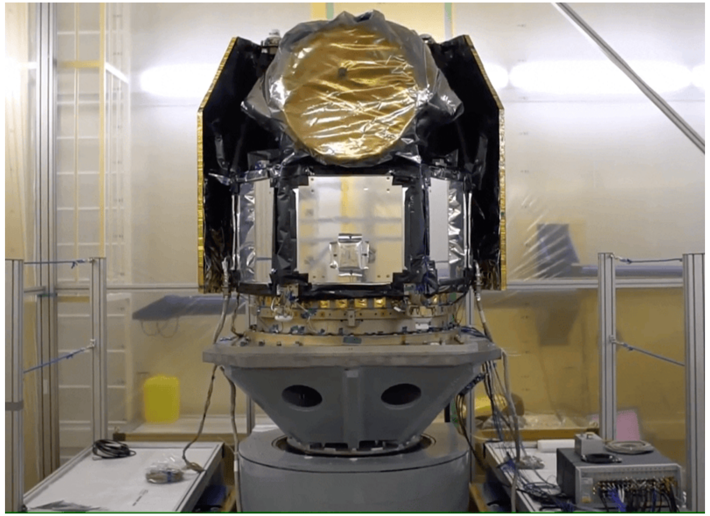
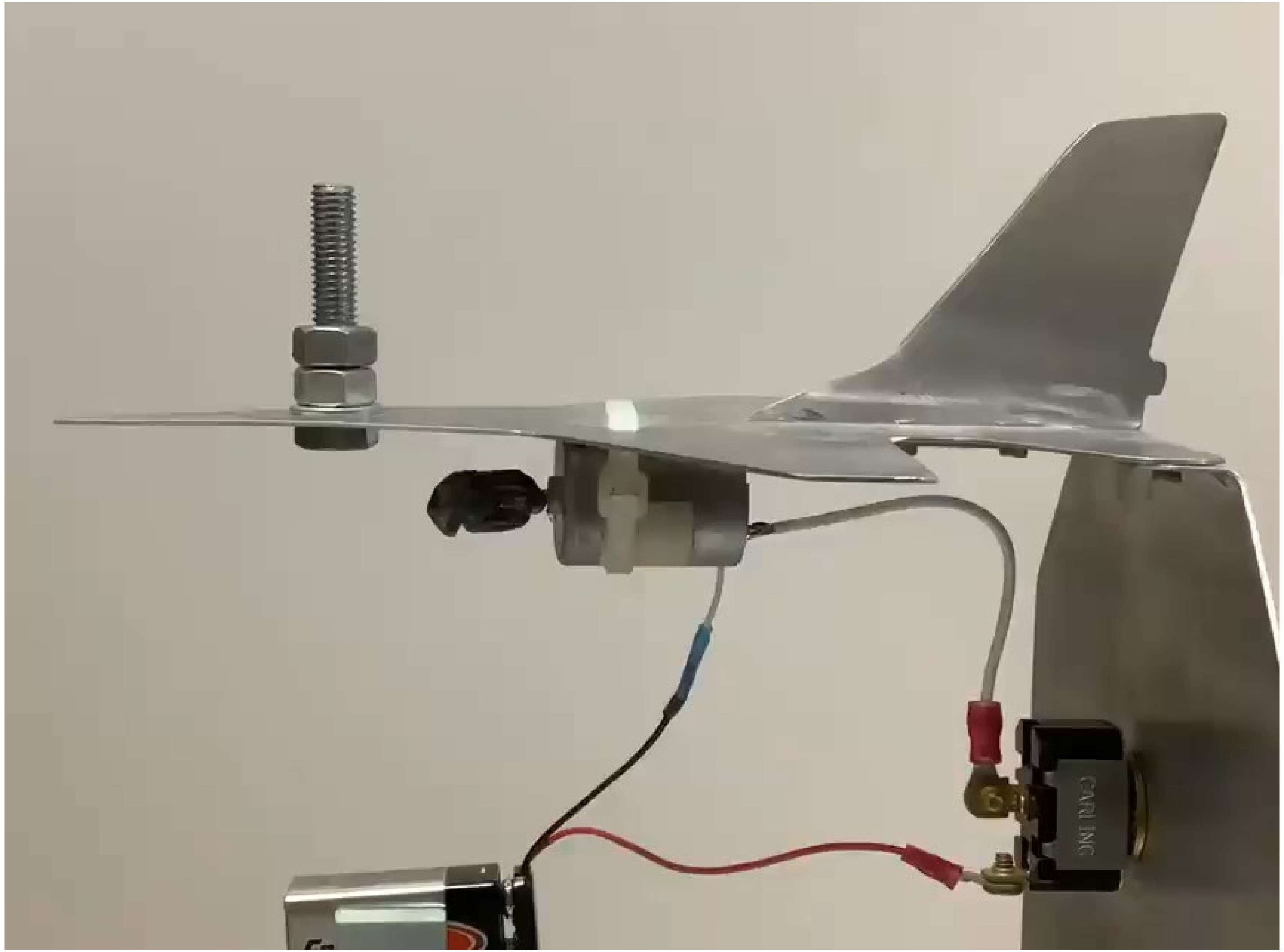
Relevant topic in aerospace engineering





The importance of oscillations



If not properly fastened, nuts can get unscrewed by vibrations!

Link to video

The importance of oscillations

Chinook CH-47 helicopter ground resonance test

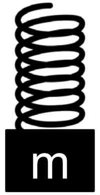
Link to video

Structure of the lecture

1. Oscillations of a spring
2. Simple harmonic motion (SHM)
3. Energy in the simple harmonic oscillator (SHO)
4. Simple harmonic motion related to uniform circular motion
5. The simple pendulum
6. The physical pendulum and the torsion pendulum
7. Damped harmonic motion
8. Forced oscillations. Resonance

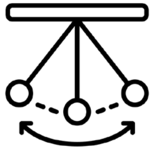
Learning objectives for today’s lecture

After this lecture you should be able to:



- Explain the fundamentals of **simple harmonic motion**, using the basic example of a mass attached to a spring.

- Analyze the **energy** contained in a simple harmonic oscillator.



- Explain the **pendulum motion** (starting with the simple pendulum and then moving to the physical and torsion pendulums).

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Assumed prior knowledge

- Basic trigonometry (cosine, etc.) (from *high school*)
- Basic mechanics and kinematics (Newton’s laws, moment of inertia, etc.) (from *Statics* and *Dynamics*)
- Differential equations (from *Calculus*)
- Integrals (from *Calculus*)

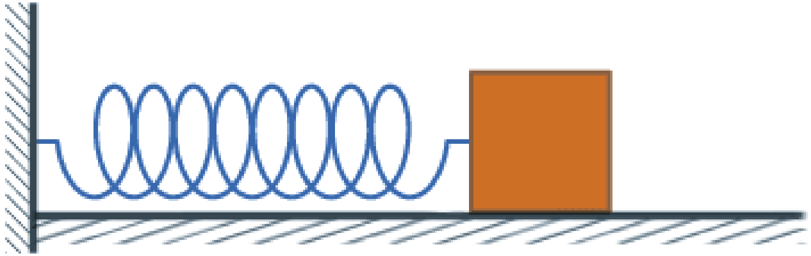
14.1 – Oscillations of a spring

If an object vibrates or oscillates back and forth over the same path, each cycle taking the same amount of time, the motion is called **periodic**.

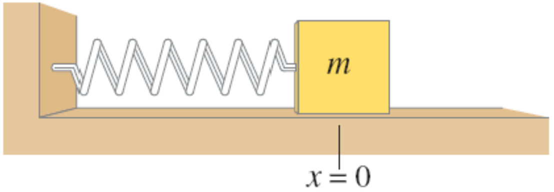
The mass and spring system is a useful model for a periodic system.

Assumptions:

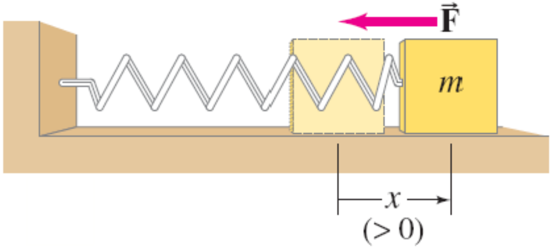
- Surface is frictionless.
- Mass of the spring is negligible.



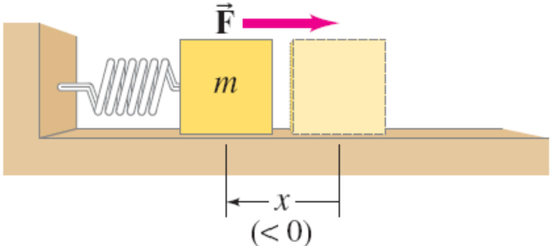
Equilibrium



Extension

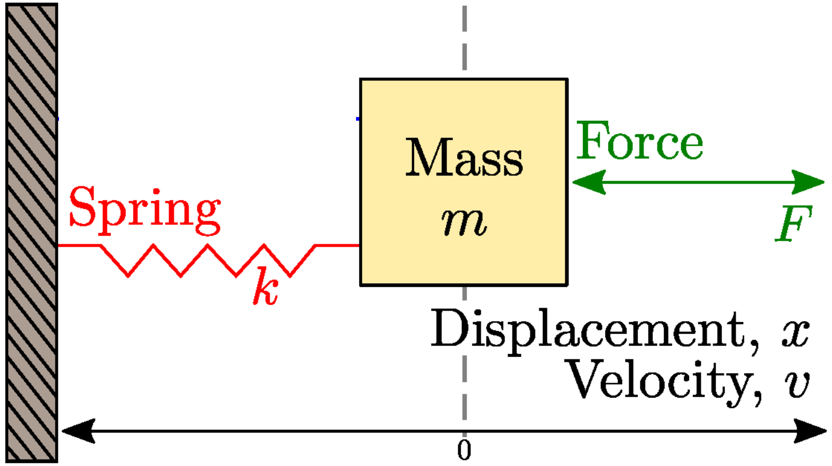


Compression

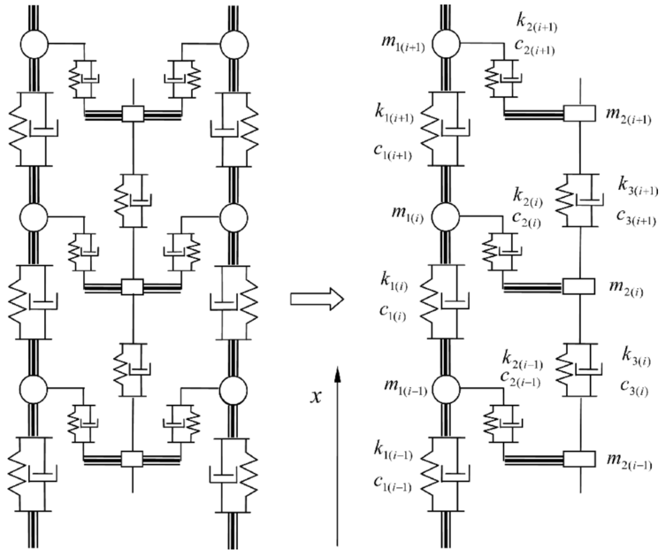


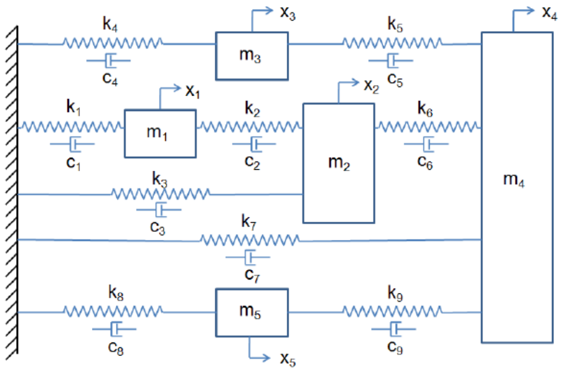
14.1 – A useful tool to model complex systems

We can also add damping (later in 14.7)

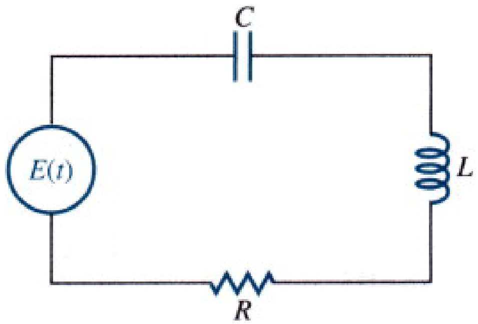


Using this concept, we can model very complex systems



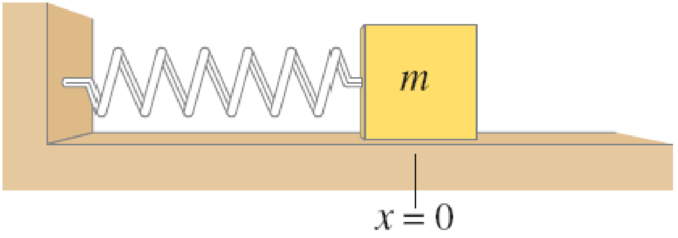


"Similar" to electrical circuits



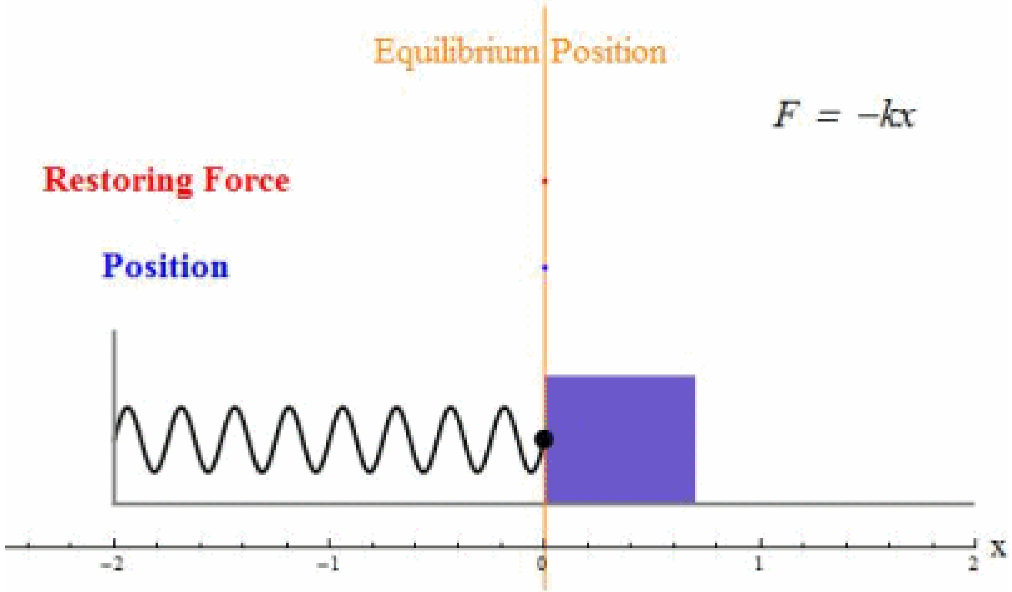
14.1 – Oscillations of a spring

Equilibrium $\sum \vec{F} = \vec{0}$



The **equilibrium position** is defined as the location where the spring is neither stretched nor compressed (i.e. the **spring's natural length**).

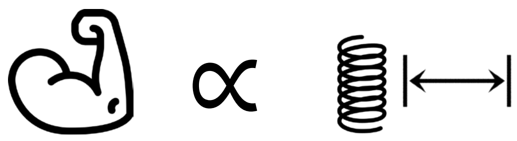
We measure the displacement x with respect to that position.



The **restoring force** F exerted by the spring is proportional to the **displacement** x:

Hooke's law $$\boxed{F = -kx}$$

14.1 – Oscillations of a spring – Hooke's law



$$\boxed{F = -kx}$$

- The **minus** sign indicates that it is a **restoring** force (i.e. it is directed to restore the mass to its equilibrium position).
- k is the **spring (stiffness) constant** measured in [N/m].
- The force is **<u>not constant</u>**! Thus, the acceleration is not constant either.
- This formula is only valid within the **elastic region** of the spring.

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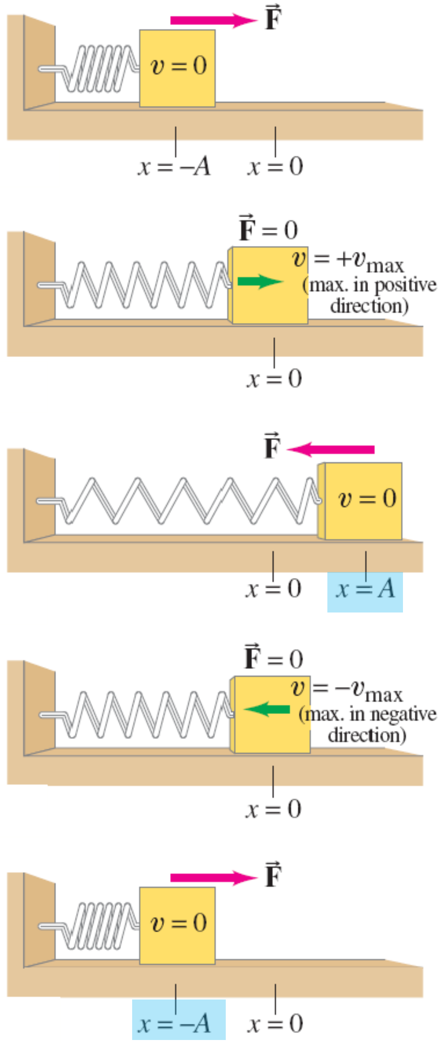
14.1 – Oscillations of a spring – Definitions



- **Displacement** (x) is measured from the equilibrium point, [m].
- **Amplitude** (A) is the maximum displacement, [m].
- A **cycle** is a full back-and-forth motion.
- **Period** (T) is the time required to complete one cycle, [s].
- **Frequency** (f) is the number of cycles completed per second, [Hz = 1/s].

$$\boxed{f = \frac{1}{T}}$$

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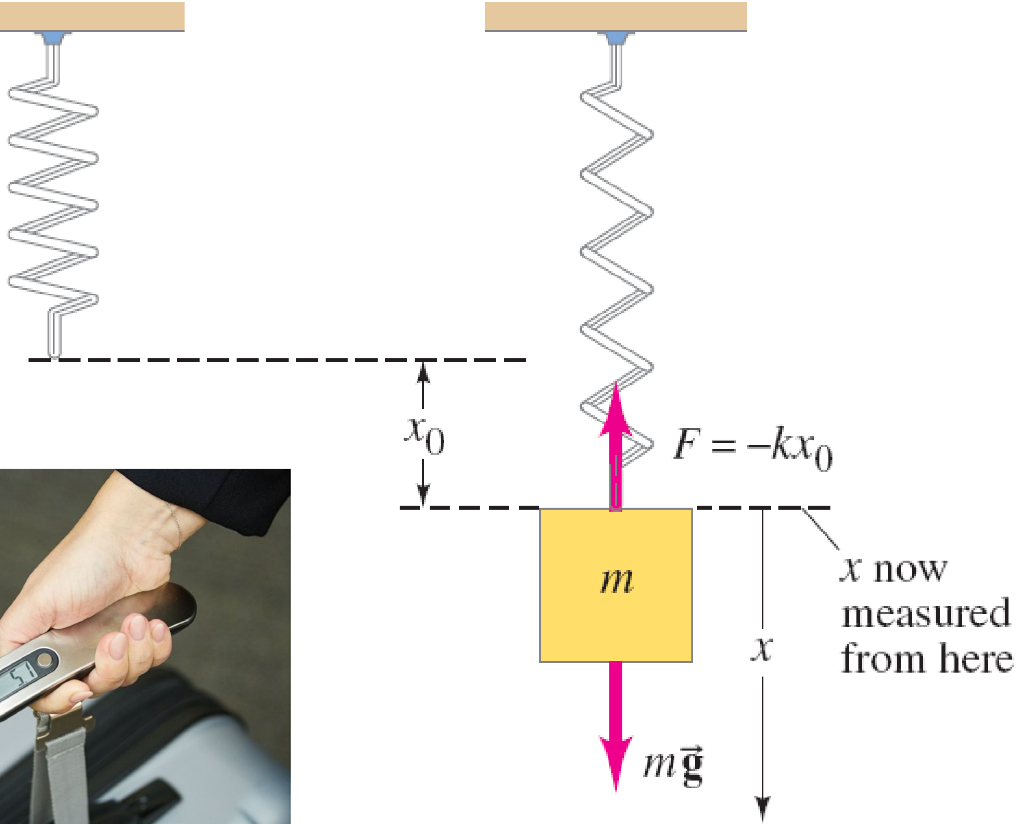
14.1 – Oscillations of a spring – Vertical springs

If the spring is hung **vertically**, the only change is in the equilibrium position.

Now it is at the point where the spring force equals the **gravitational force**.

$$\sum F = 0 = mg - kx_0$$

$$x_0 = \frac{mg}{k}$$



14.2 – Simple harmonic motion (SHM)

Any vibrating system where the **restoring force is proportional to the negative of the displacement** (e.g. Hooke's law) is in **simple harmonic motion** (SHM) and is normally called a **simple harmonic oscillator** (SHO).

Hooke's law $F = -kx$

Newton's second law $F = ma$

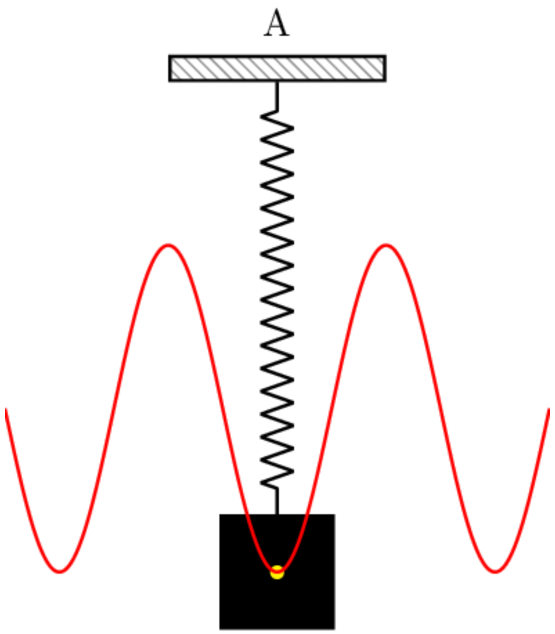
Acceleration $a = \frac{d^2x}{dt^2}$

$$-kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

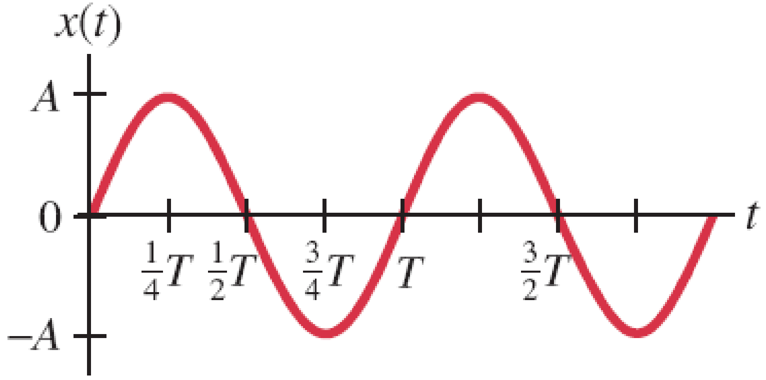
14.2 – Simple harmonic motion (SHM)

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$



This 2nd order differential equation has solutions of the form:

$$\boxed{x = A\cos(\omega t + \phi)}$$



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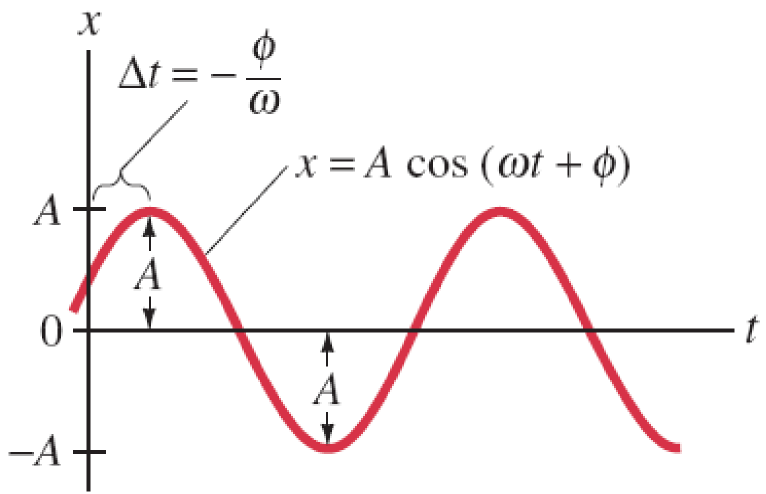
14.2 – Simple harmonic motion - definitions

$$\boxed{x = A\cos(\omega t + \phi)}$$



- x is the displacement with respect to the equilibrium position, [m].
- A is the amplitude, [m].
- ω is the **angular frequency**, [rad/s].
- t is the time, [s].
- ϕ is the **phase** of the motion at t = 0.

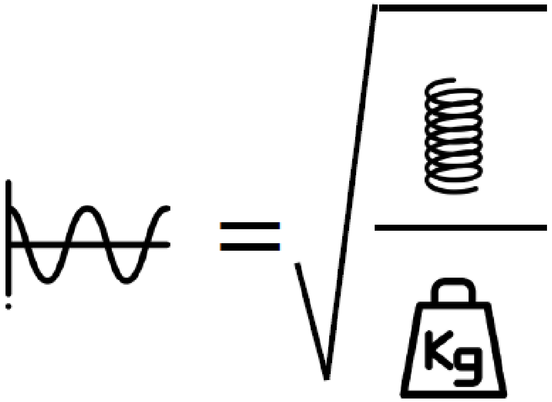
14.2 – Simple harmonic motion - definitions

$$x = A\cos(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$-\cancel{A}\,\omega^2 \cancel{\cos(\omega t + \phi)} + \frac{k}{m}\cancel{A}\cancel{\cos(\omega t + \phi)} = 0$$

$$-\omega^2 + \frac{k}{m} = 0$$



$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

14.2 – Simple harmonic motion - frequencies

Relation between **linear** frequency f and **angular** frequency ω:

$$\omega = 2\pi f \quad \begin{cases} f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} \\[2em] T = 2\pi\sqrt{\dfrac{m}{k}} \end{cases}$$

ω, f, and T do **<u>not</u>** depend on the amplitude!

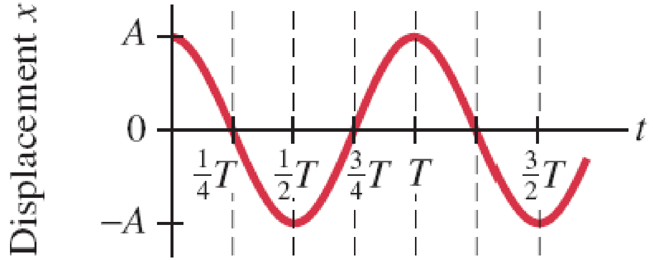
Pro tip: Always check the units in formulas!

$$[\omega] = [1/\mathrm{s}]$$

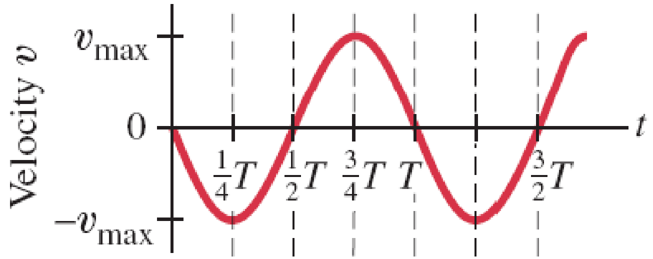
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\mathrm{N/m}}{\mathrm{kg}}} = \sqrt{\frac{(\mathrm{kg}\,\frac{\mathrm{m}}{\mathrm{s}^2})/\mathrm{m}}{\mathrm{kg}}} = \sqrt{\frac{1}{\mathrm{s}^2}} = 1/\mathrm{s}$$ ✓

14.2 – Simple harmonic motion - kinematics

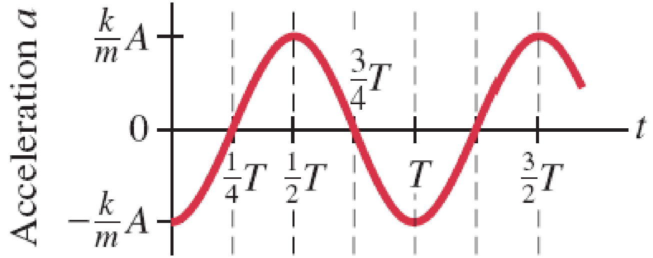
$$x = A\cos(\omega t + \phi)$$



$$v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$$



$$a = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$$



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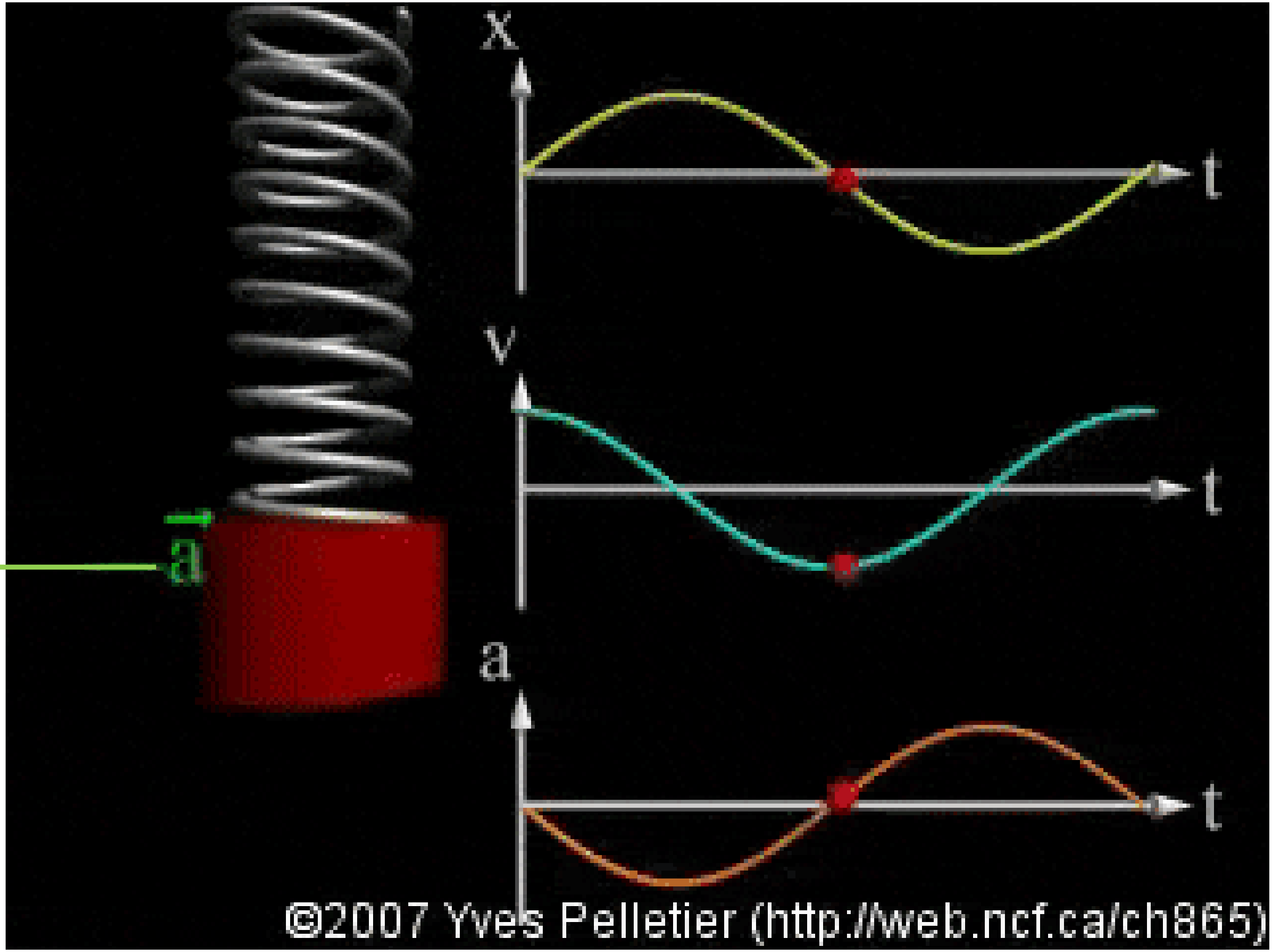
14.2 – Simple harmonic motion - kinematics

$x > 0$



14.3 – Energy in the simple harmonic oscillator

The **potential energy** U of a spring is given by:

$$U = -\int F\,dx = \int kx\,dx = \frac{1}{2}kx^2$$

The **kinetic energy** K of the moving mass is given by:

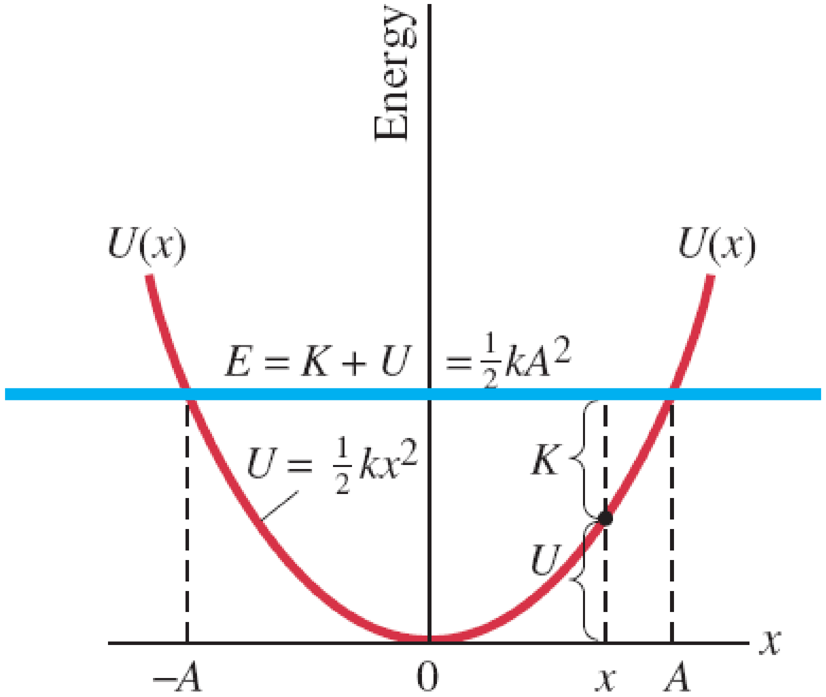
$$K = \frac{1}{2}mv^2$$

14.3 – Energy in the simple harmonic oscillator

The **total mechanical energy will be conserved**.

Remember that one of our assumptions is that the system is **frictionless**.

$$E = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$



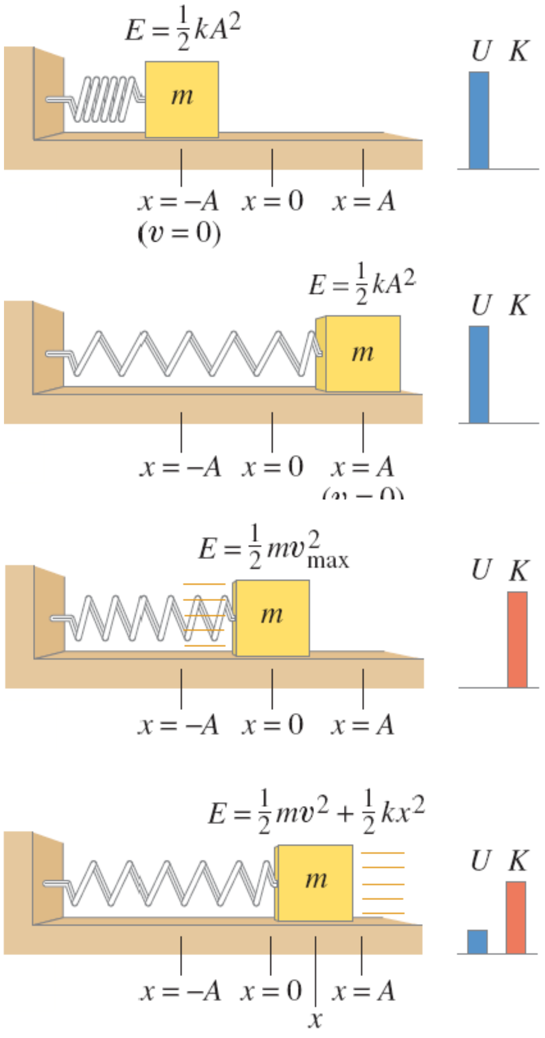
14.3 – Energy in the simple harmonic oscillator

$$E = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

If the mass is at the limits of its motion, the energy is all potential (U), since $v = 0$ there.

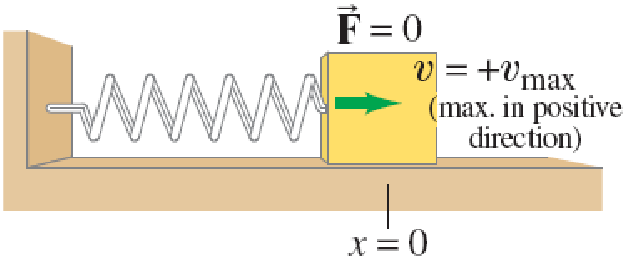
If the mass is at the equilibrium point, the energy is all kinetic (K), since $x = 0$ there.

In any other point, the total energy is a mix of both.

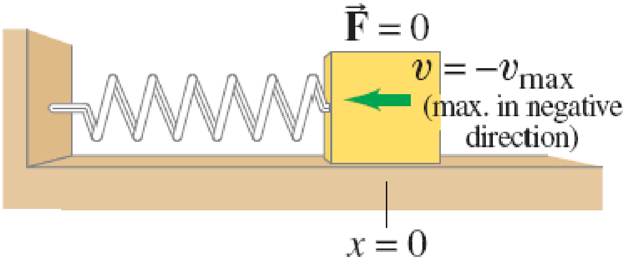


14.3 – Energy in the simple harmonic oscillator

$$E = U_{max} = K_{max} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$



$$E = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$$



Maximum velocity:

$$\boxed{v_{max} = \pm A\sqrt{\frac{k}{m}} = \pm A\omega}$$

14.3 – Energy in the simple harmonic oscillator

For a generic displacement x we can solve for v:

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$$

$$\boxed{v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = v_{max}\sqrt{1 - \frac{x^2}{A^2}} = \pm\omega\sqrt{A^2 - x^2}}$$

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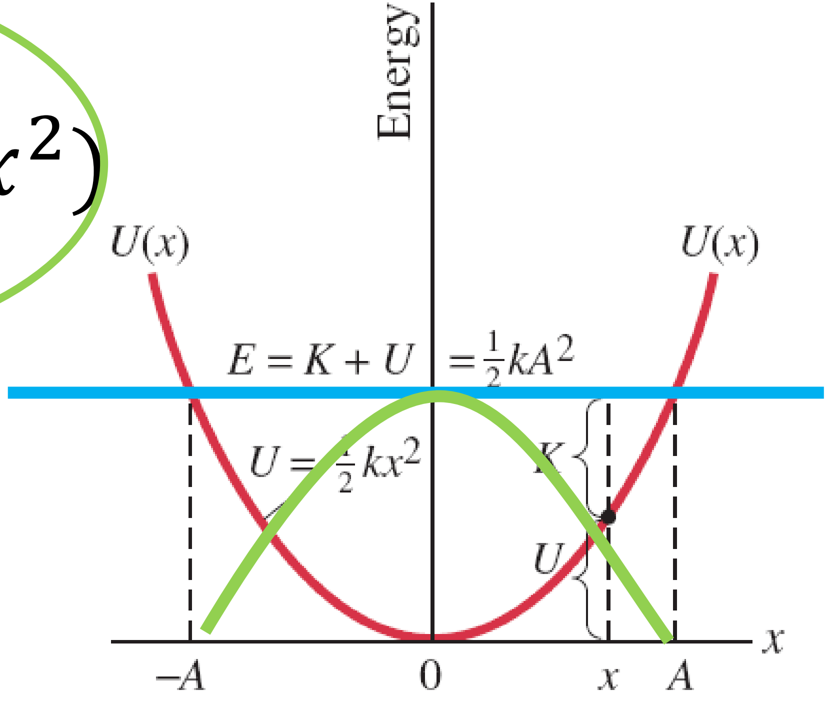
14.3 – Energy in the simple harmonic oscillator

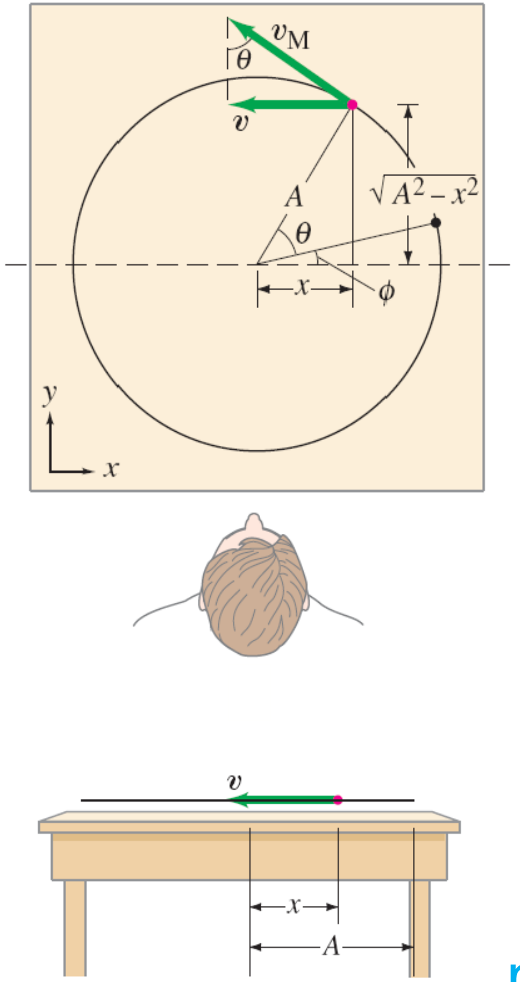
$$E = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$E = U + K = \frac{1}{2}kx^2 + \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$E = f(k, x, A, \omega, m)$$



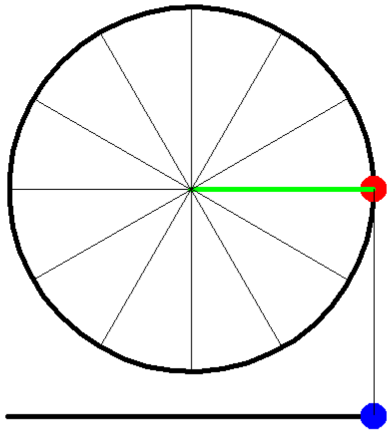
14.4 – SHM related to uniform circular motion



Consider an object moving in a **circle** of radius A at a **constant speed** v_{max}.

If we project its velocity onto the x axis it follows:

$$v = v_{max}\sqrt{\left(1 - \frac{x^2}{A^2}\right)}$$



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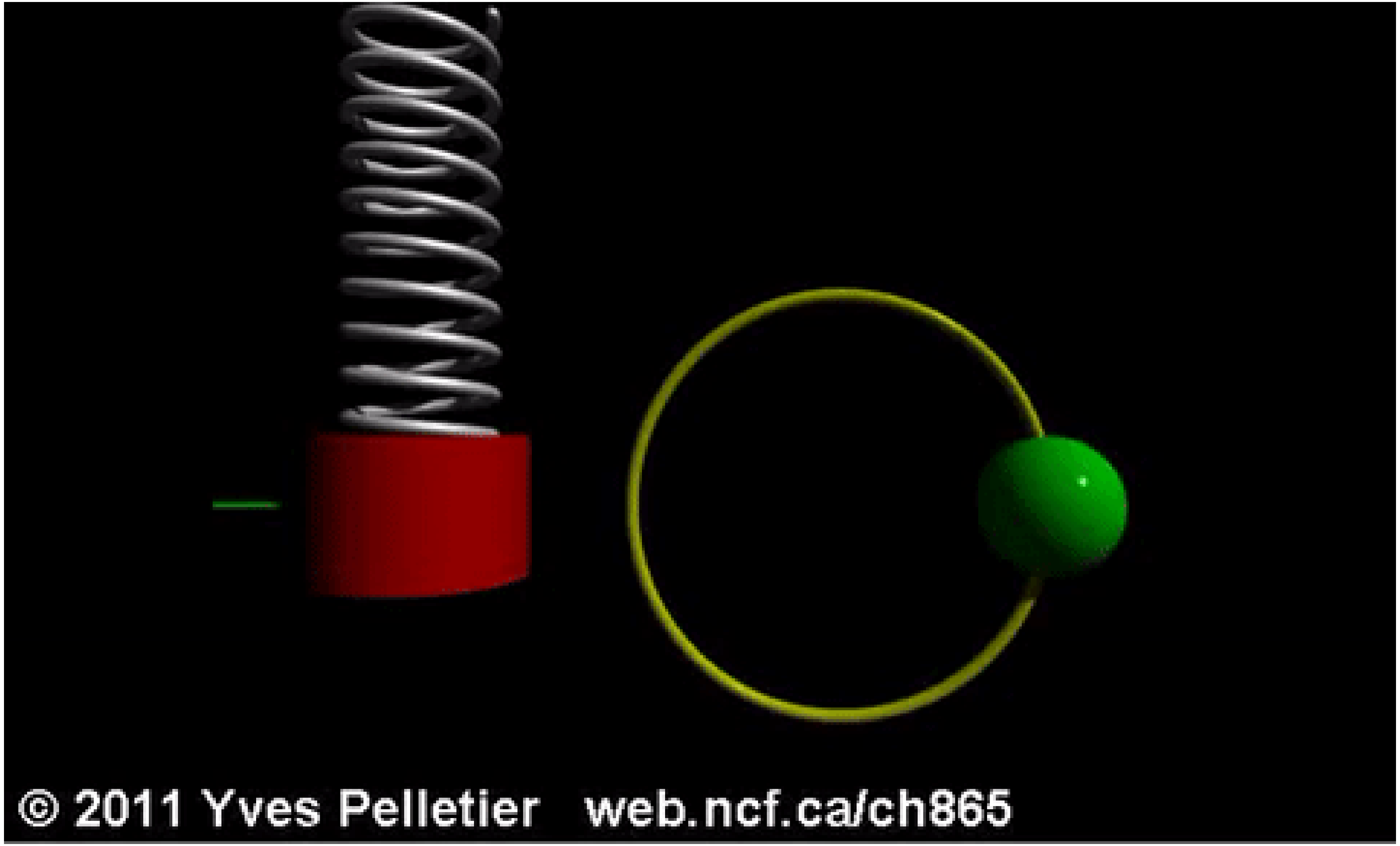
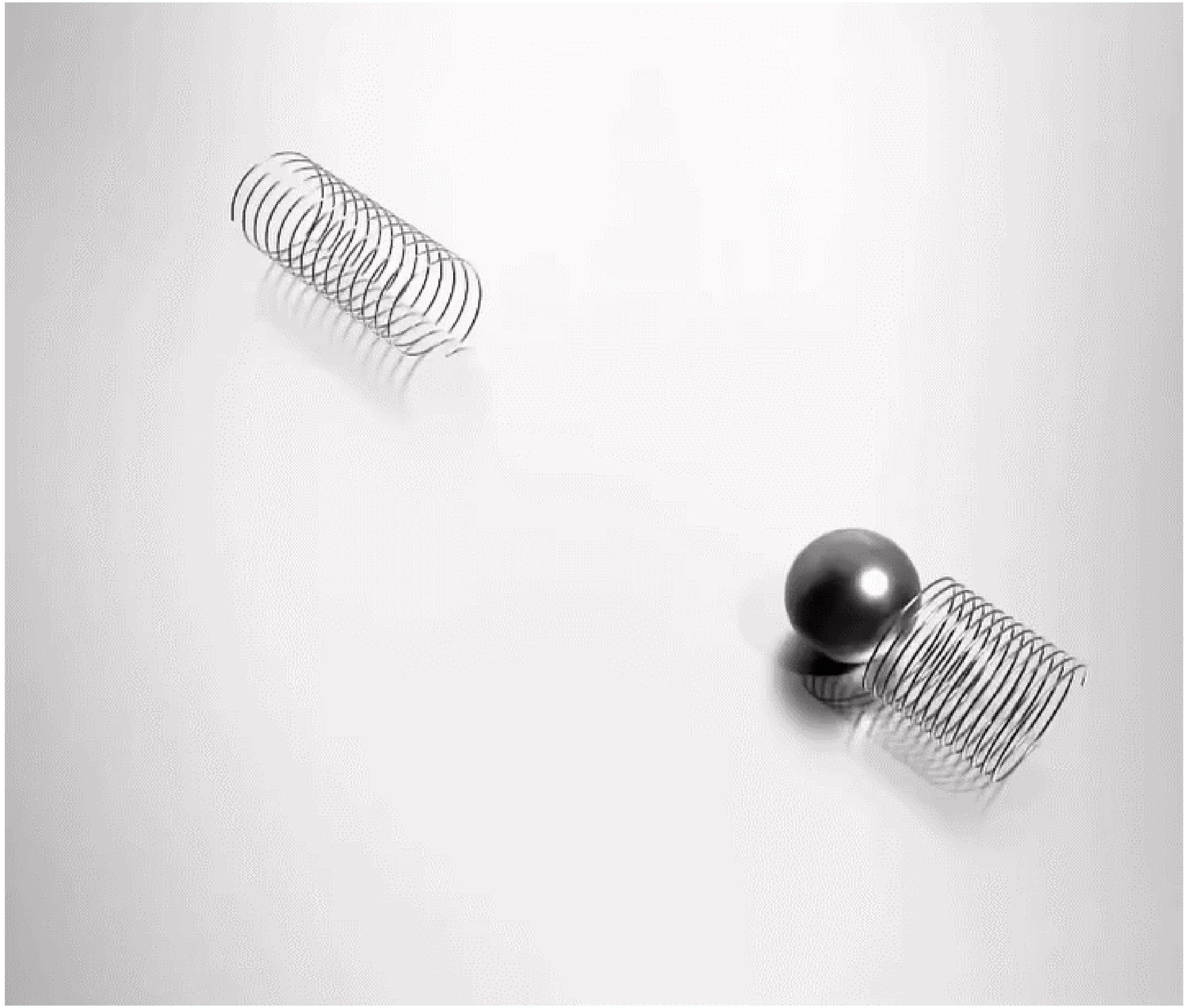
Therefore, the projection of a circular motion onto a straight line, follows a SHM

14.4 – SHM related to uniform circular motion



Another way to see it ☺

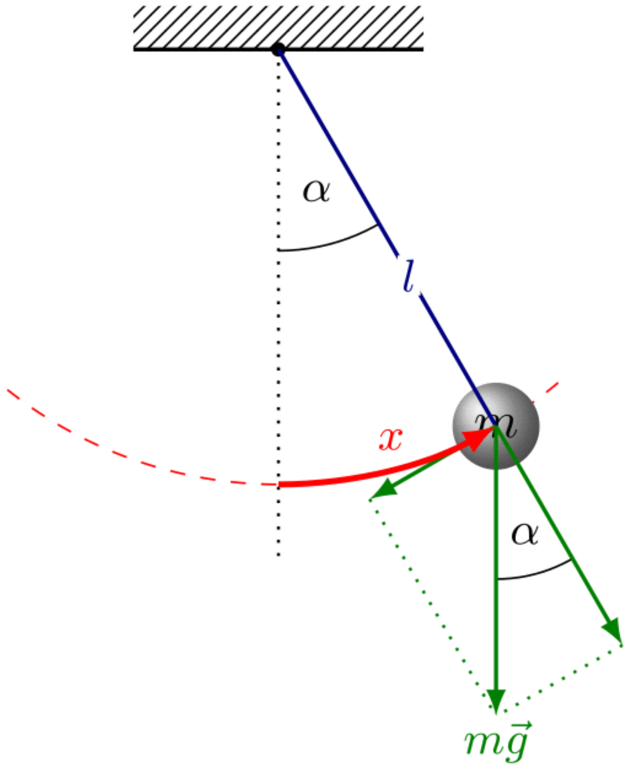


[Simple Harmonic Motion]

↕

[Uniform Circular Motion]

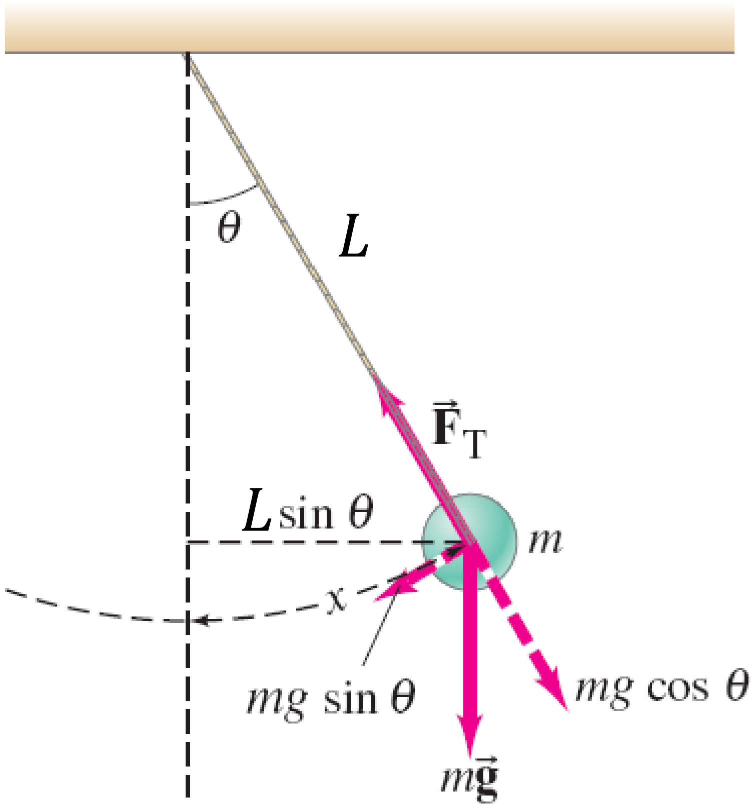
14.5 – The simple pendulum



A simple pendulum consists of a mass at the end of a lightweight cord. We assume that:

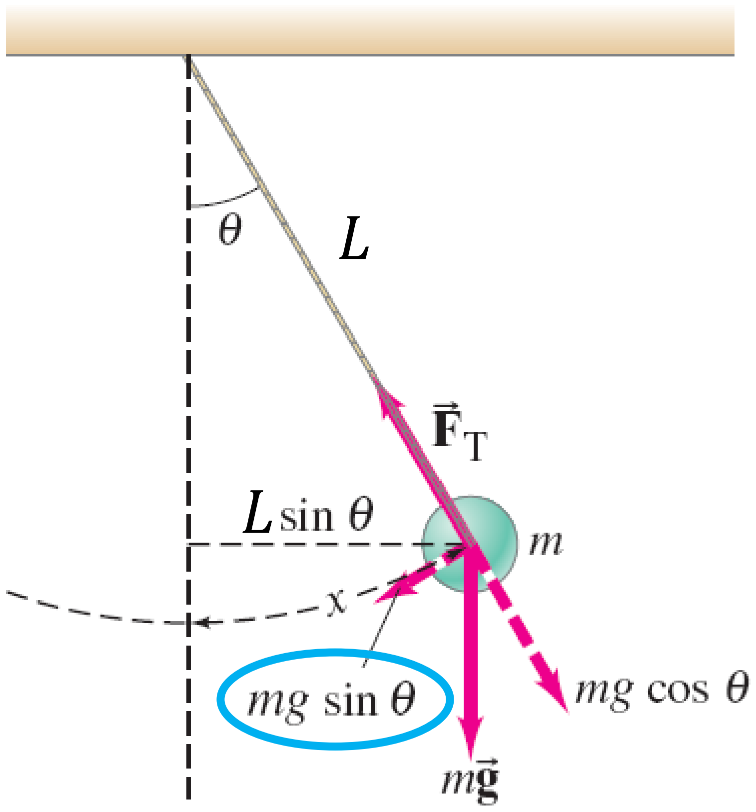
- The cord **does not stretch.**
- The **mass of the cord is negligible.**

14.5 – The simple pendulum - definitions



- m is the mass of the pendulum, [kg].
- L is the length of the pendulum cord, [m].
- θ is the **angle** the cord makes with respect to the vertical (equilibrium position), [rad].
- g is the gravitational acceleration, [m/s^2]
- F_T is the **tension force** of the cord, [N].

14.5 – The simple pendulum – Is it a SHM?



Does the simple pendulum motion follow a SHM?

The restoring force must be proportional to the negative of the displacement.

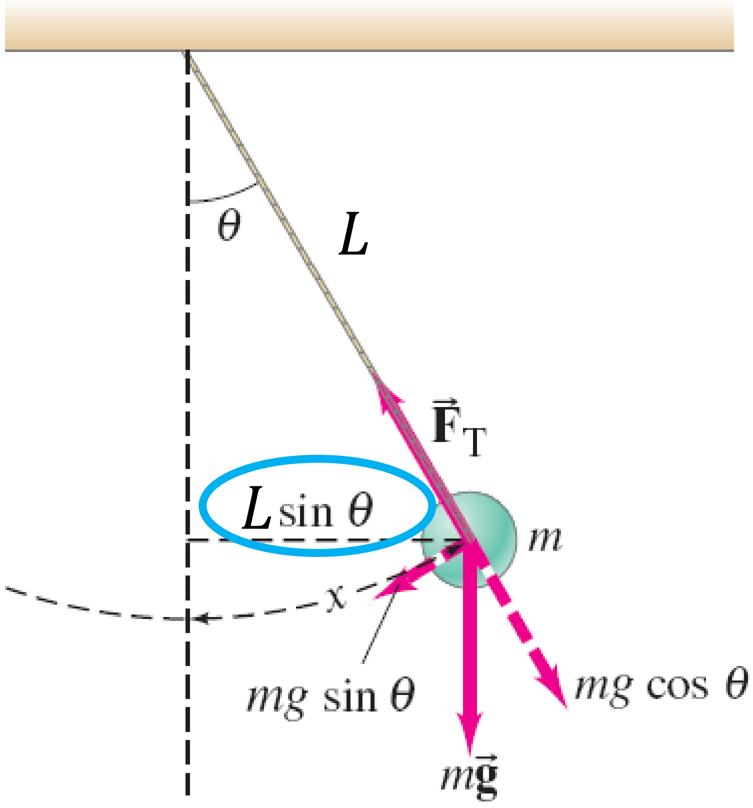
$$F = -mg\sin\theta$$

Which is **NOT** proportional to the displacement θ!

<u>HOWEVER</u>! For small angles ($\theta < 15° \approx 0.26\,\mathrm{rad}$):

$\sin\theta \approx \theta$ with θ in **<u>radians</u>!**

14.5 – The simple pendulum – Small angles



Therefore, for small angles, we have:

$$F = -mg\sin\theta \approx -mg\theta$$

Which **DOES** follow a SHM

$$\sin\theta = \frac{x}{L} \approx \theta \longrightarrow x = L\theta$$

$$F = -\frac{mg}{L}x$$

Effective force constant, k

14.5 – The simple pendulum – comparison to spring

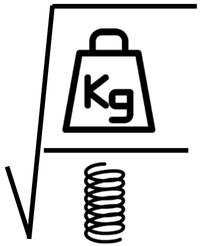
SPRING

$$F = -kx$$

$$x = A\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$



PENDULUM

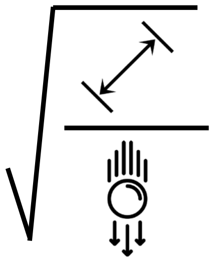
$$F = -\frac{mg}{L}x \qquad F = -mg\theta$$

$$\theta = \theta_{max}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{g}{L}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Huygens' equation

$$T = 2\pi\sqrt{\frac{L}{g}}$$



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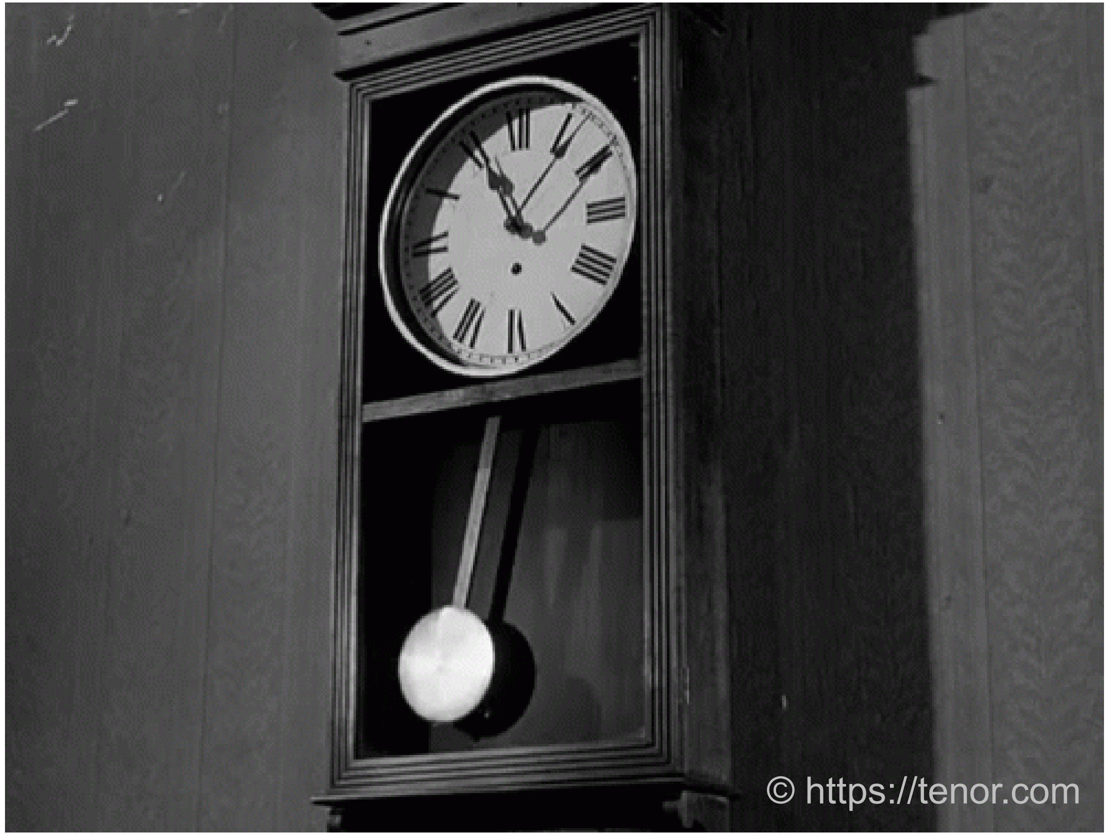
14.5 – The simple pendulum - Period

For small angles, the period of a pendulum does **NOT** depend on the **mass** nor **amplitude**:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T = f(L, g)$$



14.5 – The simple pendulum – Energy conservation

As with the simple harmonic oscillator, **energy is also conserved** ☺

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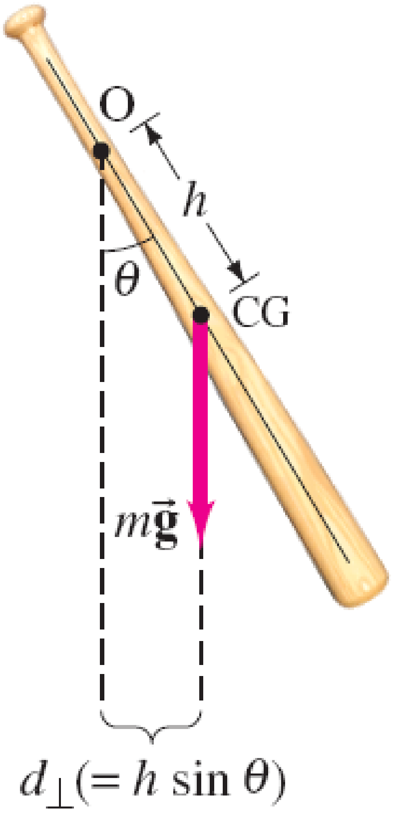
Link to video

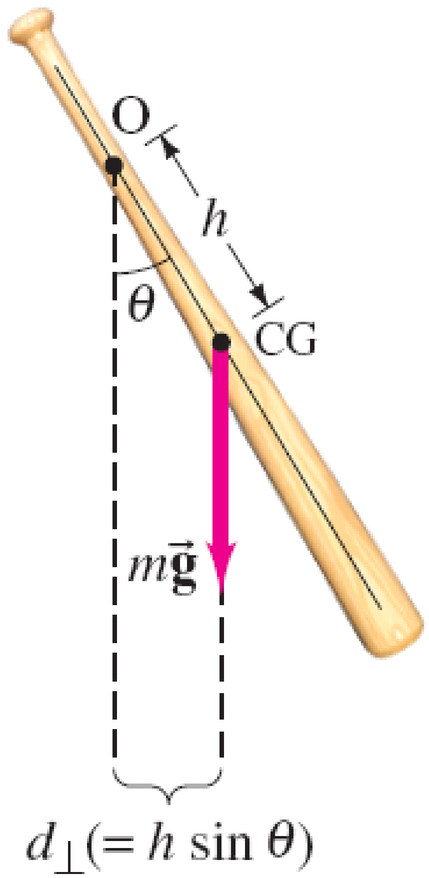
14.6 – The physical pendulum - definitions

A physical pendulum is any **real extended object** that oscillates back and forth.



- O is the rotation point
- CG is the center of gravity of the pendulum
- m is the mass of the pendulum, [kg].
- h is the **distance between the center of gravity CG and the rotation point** O, [m].
- θ is the angle the physical pendulum makes with respect to the vertical (equilibrium position), [rad].

14.6 – The physical pendulum - equations



The torque τ [N m] about point O is:

Small angles

$$\tau = -mgh\sin\theta \approx -mgh\theta$$

Using Newton's 2nd law for rotational motion, the torque about point O is:

$$I\frac{d^2\theta}{dt^2} = \sum \tau \approx -mgh\theta$$

I is the moment of inertia of the object about point O, [kg m^2]

$$\frac{d^2\theta}{dt^2} + \frac{mgh}{I}\theta = 0$$

14.6 – The physical pendulum – comparison to spring

SPRING

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$x = A\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

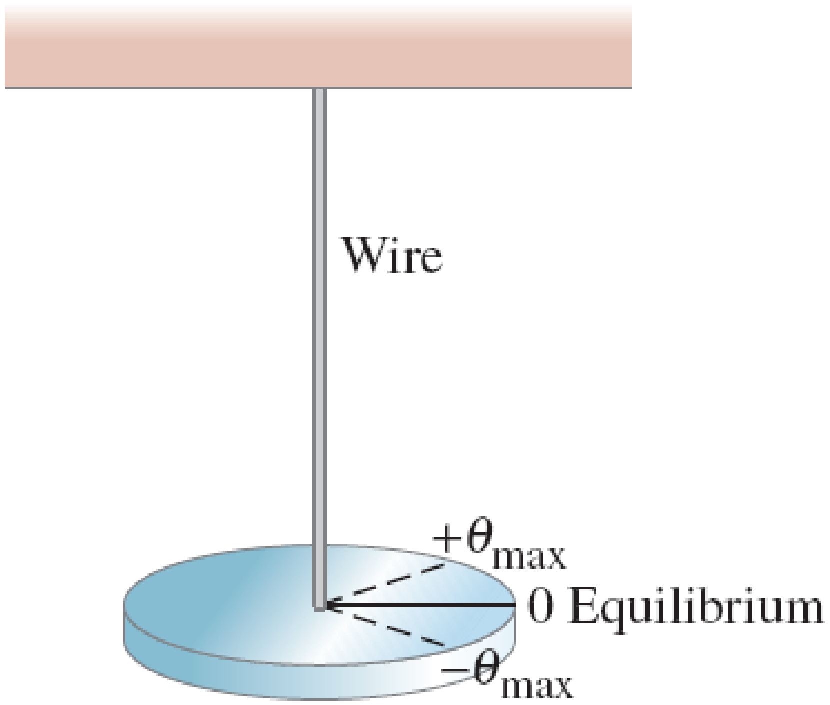
PHYSICAL PENDULUM

$$\frac{d^2\theta}{dt^2} + \frac{mgh}{I}\theta = 0$$

$$\theta = \theta_{max}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{mgh}{I}} \qquad T = 2\pi\sqrt{\frac{I}{mgh}}$$

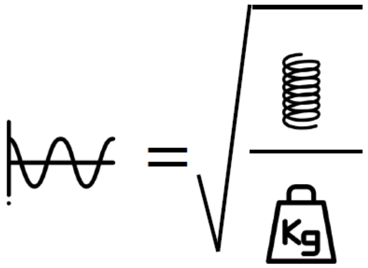
14.6 – Torsional pendulum



A torsional pendulum is one that **twists** rather than swings.

The motion is SHM as long as the wire obeys Hooke's law $\tau = -K\theta$, with

$$\omega = \sqrt{\frac{K}{I}}$$



- K is a constant that depends on the wire, [N m].
- I is the moment of inertia of the disk, [kg m^2].

WRAP-UP

Wrap-up: revisit learning objectives

- Explain the fundamentals of **simple harmonic motion**, using the oscillations of a spring as a basic example.

$$x = A\cos(\omega t + \phi)$$

- Analyze the **energy** contained in a simple harmonic oscillator.

$$E = U + K = \frac{1}{2}kx^2 + \frac{1}{2}m\omega^2(A^2 - x^2)$$

- Explain the **pendulum motion** (starting with the simple pendulum and then moving to the physical and torsion pendulums).

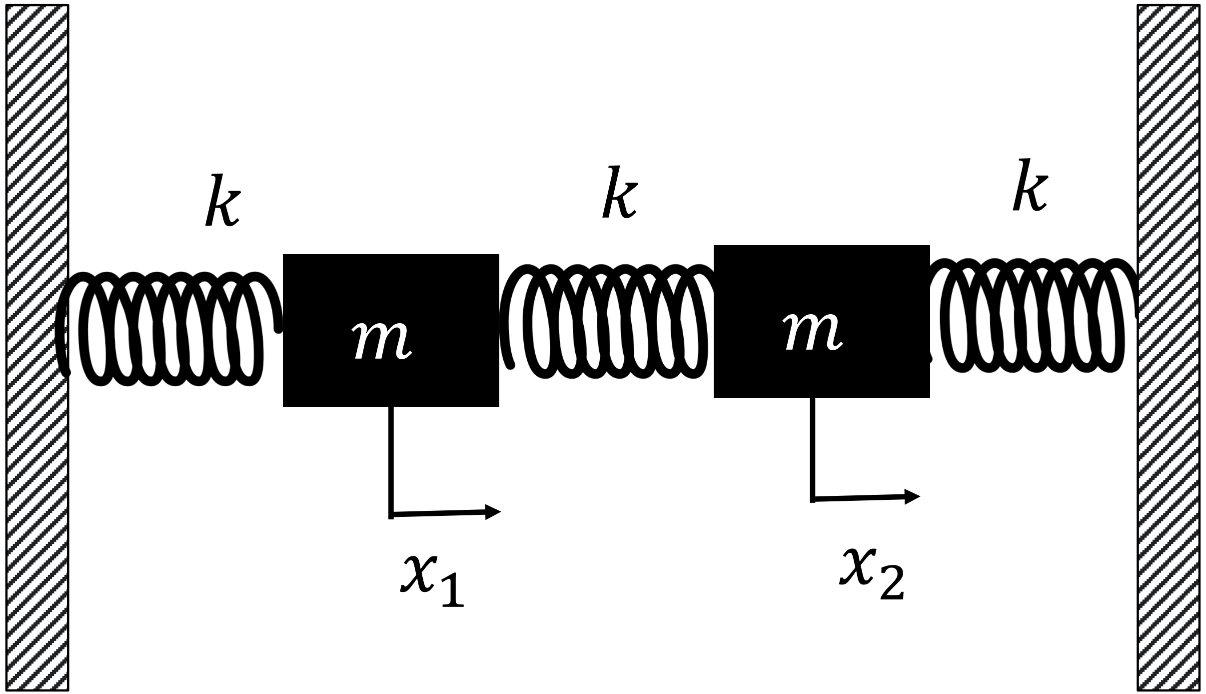
$$\theta = \theta_{max}\cos(\omega t + \phi) \qquad T = 2\pi\sqrt{\frac{L}{g}}$$

For next lecture

- 14.7 - Damped harmonic motion
- 14.8 - Forced oscillations. Resonance

- 15.1 – Characteristics of wave motion
- 15.2 – Types of waves: Transverse and longitudinal
- 15.3 – Energy transported by waves
- 15.4 – Mathematical representation of a traveling wave
- 15.5 – The wave equation

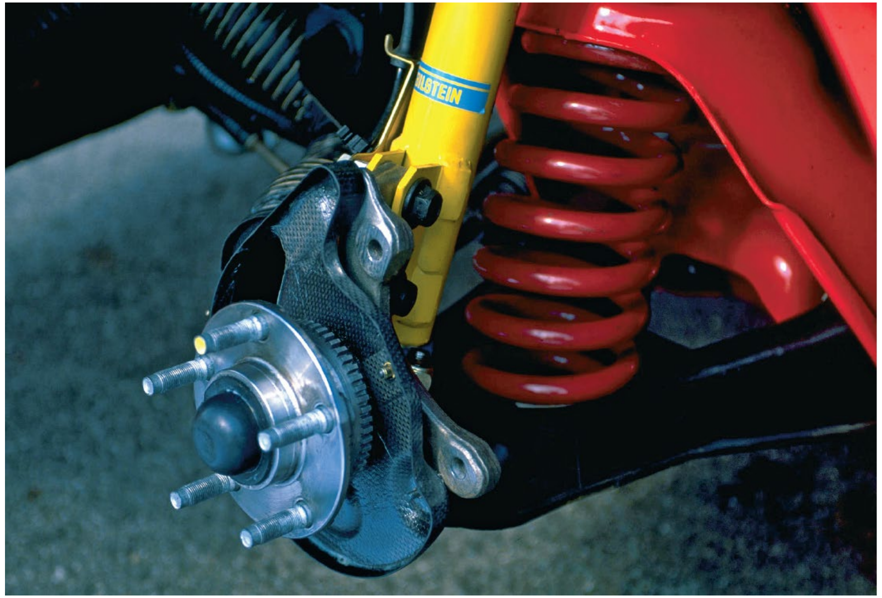
Brain teaser for next lecture



Try to write the differential equations to determine the displacements of the individual masses x_1 and x_2 over time.

OSCILLATIONS

Chapter 14



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Position in the syllabus

14. **Oscillations**
15. Waves
16. Sound
17. Temperature and the ideal gas law
18. Thermodynamics
19. Electricity and circuits
20. Electromagnetism
21. Optics

Structure of the lecture

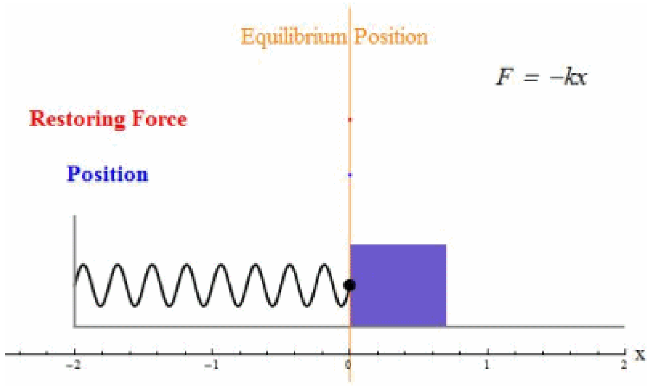
1. Oscillations of a spring
2. Simple harmonic motion (SHM)
3. Energy in the simple harmonic oscillator (SHO)
4. Simple harmonic motion related to uniform circular motion
5. The simple pendulum
6. The physical pendulum and the torsion pendulum
7. Damped harmonic motion
8. Forced oscillations. Resonance

Quick reminder or last lecture

Simple Harmonic Motion:

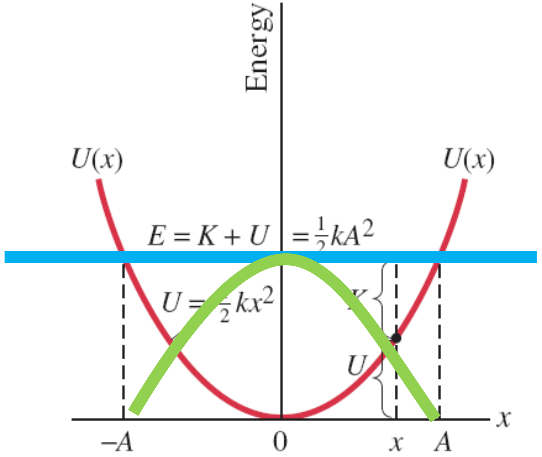
$$F = -kx \qquad x = A\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$



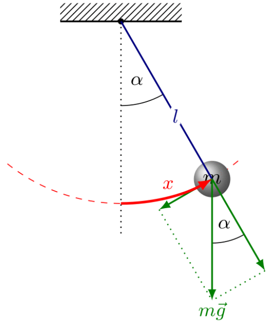
Energy:

$$E = U + K = \frac{1}{2}kx^2 + \frac{1}{2}m\omega^2(A^2 - x^2)$$



Pendulum:

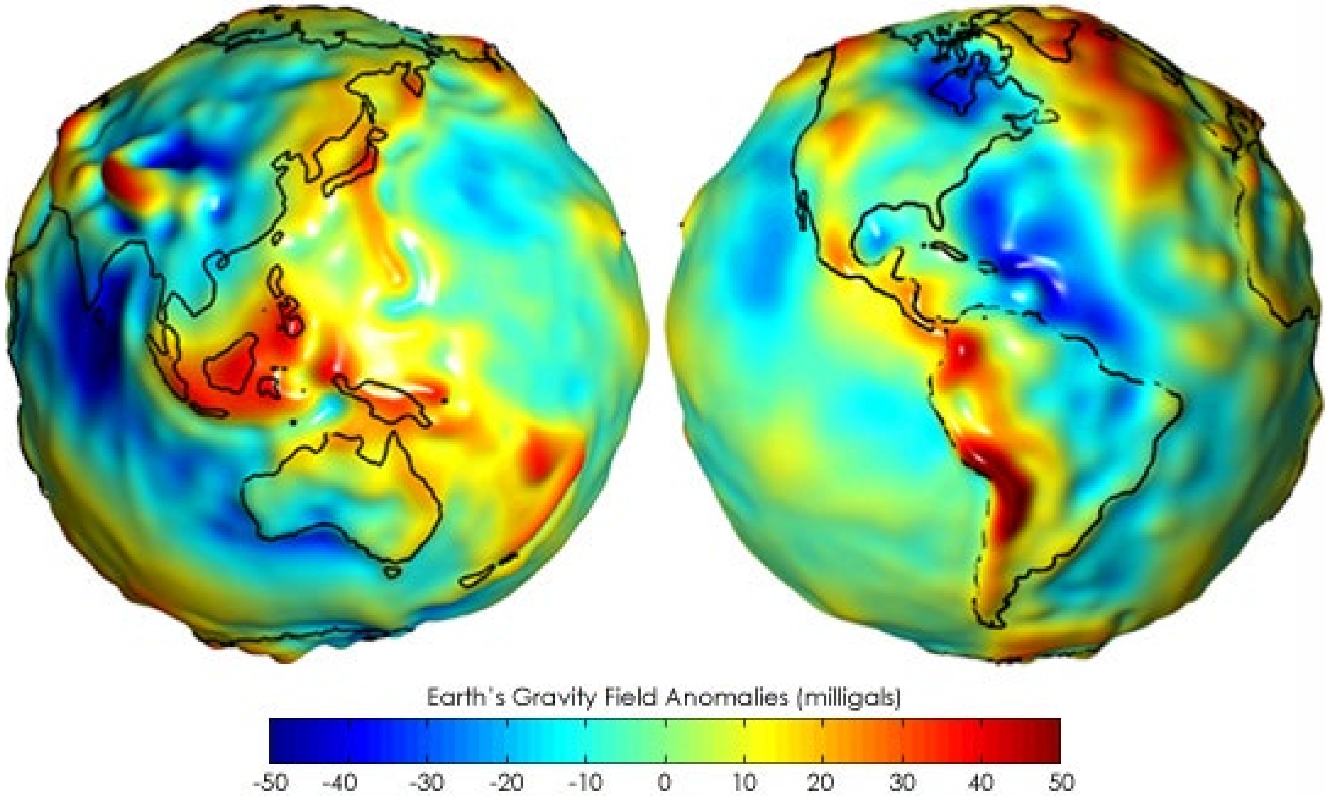
$$\theta = \theta_{max}\cos(\omega t + \phi) \qquad T = 2\pi\sqrt{\frac{L}{g}}$$



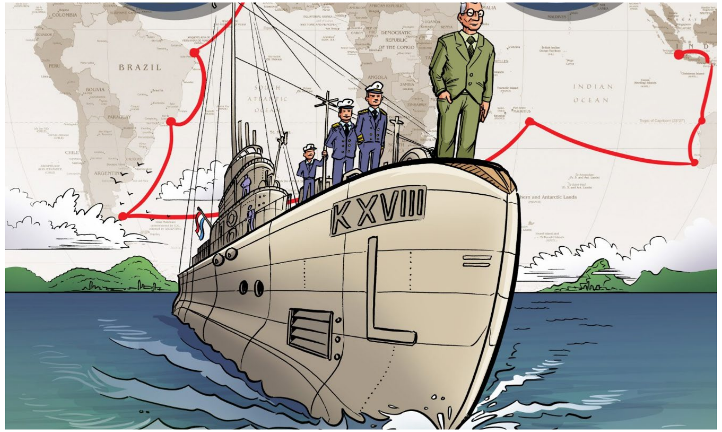
Simple pendulum: What if we don't know the gravity?

Huygens' pendulum equation:

$$T = 2\pi\sqrt{\frac{L}{g}}$$



Using pendulums to determine gravity



[The story of Prof. Vening Meinesz in the 1930s and the “Gouden Kalf”.]

Learning objectives for today's lecture

After this lecture you should be able to:

- Explain the fundamentals of **damped harmonic motion**.
- Calculate the motion of a system subject to **forced oscillations**.
- Explain the concept of **resonance** and estimate the resonance frequency.

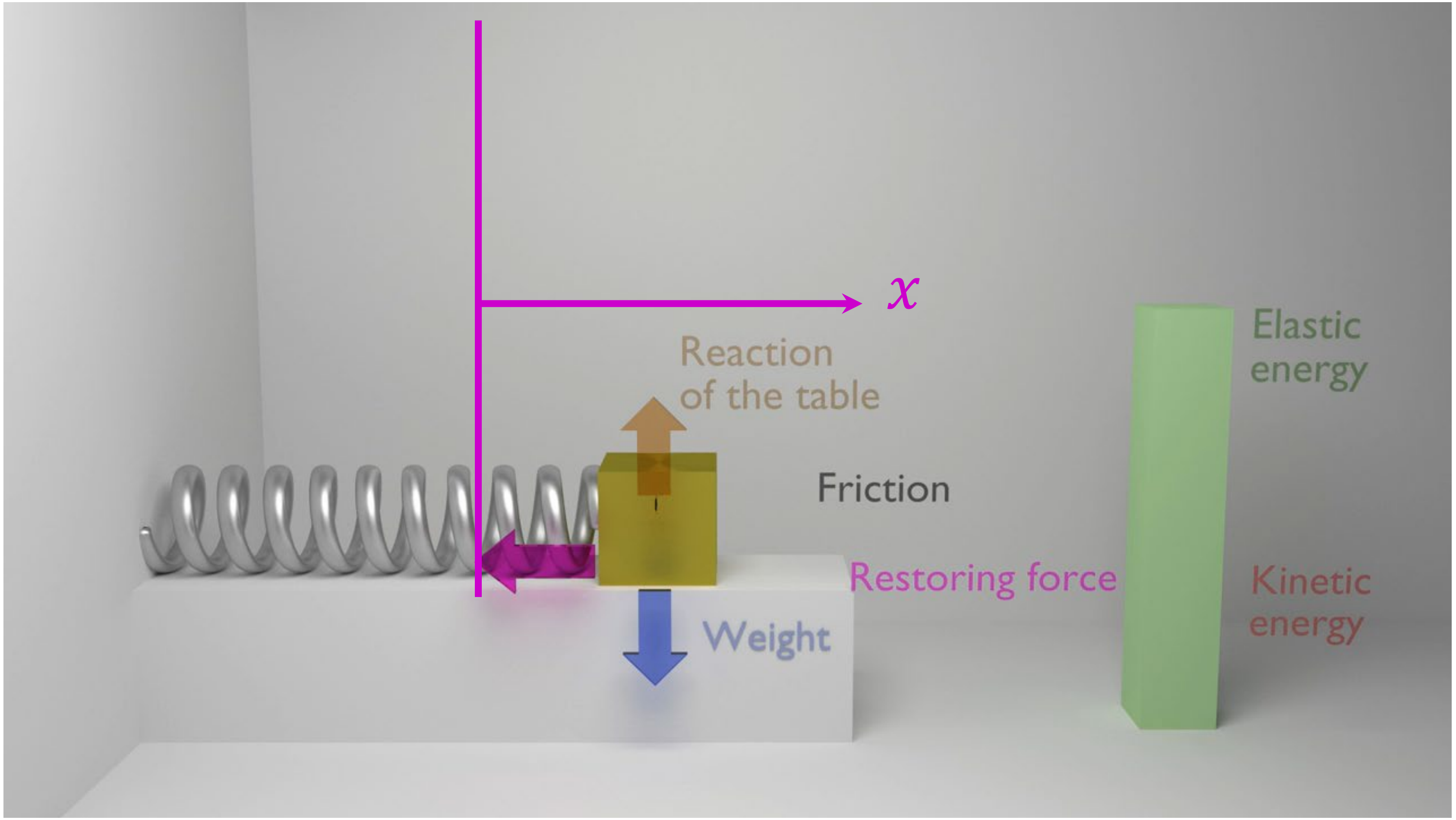
14.7 – Damped harmonic motion

In a **more realistic** scenario, damped harmonic motion includes a **frictional** or **drag** force that gradually attenuates the oscillations.

We can represent this with a **velocity-dependent damping term** that depends on a damping constant b, measured in [kg/s]:

$$\boxed{F_{damping} = -bv}$$

14.7 – Damped harmonic motion



$$F_{restoring} = -kx$$

$$F_{friction} = -bv$$

The energy within a damped harmonic oscillator is gradually dissipated by the friction.

[Link to video]

14.7 – Damped harmonic motion

$$ma = -kx - bv$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

Following the same procedure as for the SHM and assuming that b is small:

$$\boxed{x = Ae^{-\gamma t}\cos\omega' t}$$

$$\boxed{\gamma = \frac{b}{2m}}$$

$$\boxed{\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$

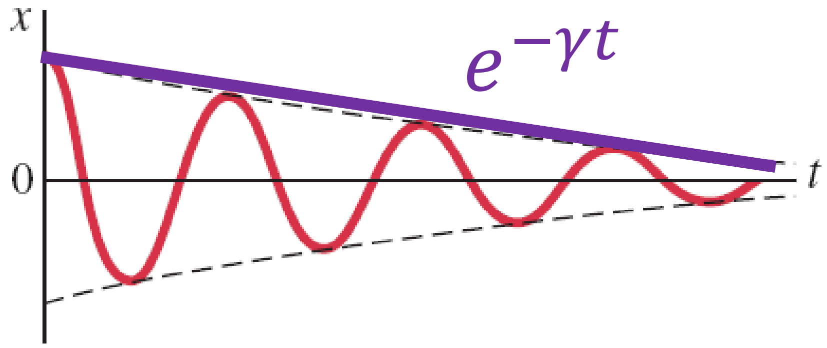
Notice that for $\mathrm{b} = \gamma = 0$ we recover the SHM.

14.7 – Damped harmonic motion

$$\boxed{x = Ae^{-\gamma t}\cos\omega' t}$$

$$\boxed{\gamma = \frac{b}{2m}}$$

$$\boxed{\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$



We can treat this damping as an **envelope** that modifies the undamped oscillation of the SHM.

The damping constant b influences this envelope, as well as the (damped) oscillation frequency ω'.

14.7 – Types of damped harmonic motion

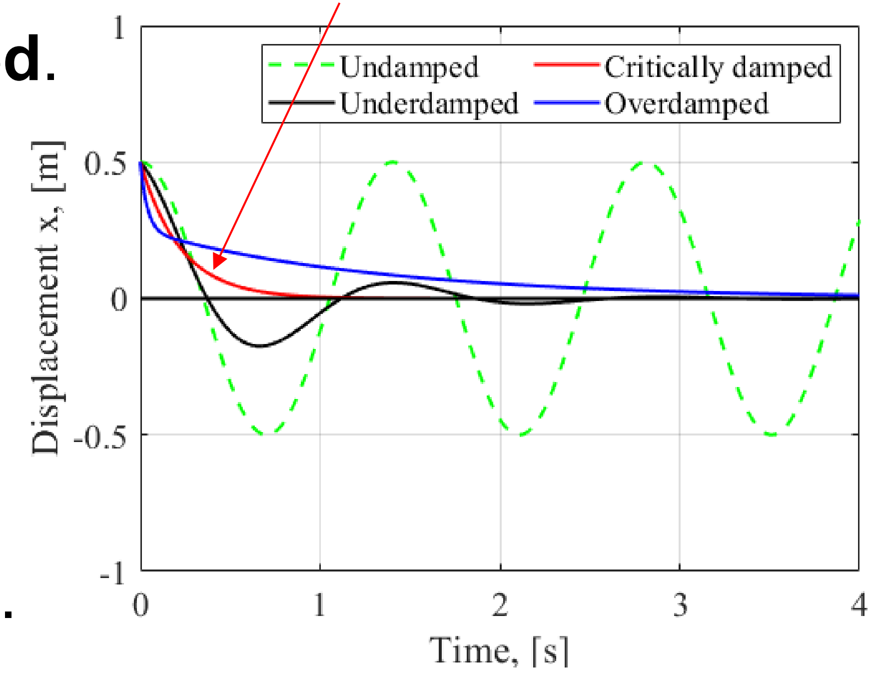
Error in the book!

- For $b^2 < 4mk$, $\omega'^2 > 0$ the system is **underdamped**.

- For $b^2 = 4mk$, $\omega'^2 = 0$ the system is **critically damped**. This is the case in which the system reaches equilibrium in the **shortest time**.

$$x = Ae^{-\gamma t}$$

- For $b^2 > 4mk$, $\omega'^2 < 0$ the system is **overdamped**. The oscillation frequency ω' becomes **imaginary**.



$$\boxed{x = Ae^{-\gamma t}\cos\omega' t}$$

$$\boxed{\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$

See MATLAB script in:
https://github.com/rmerinomartinez/physics_lectures

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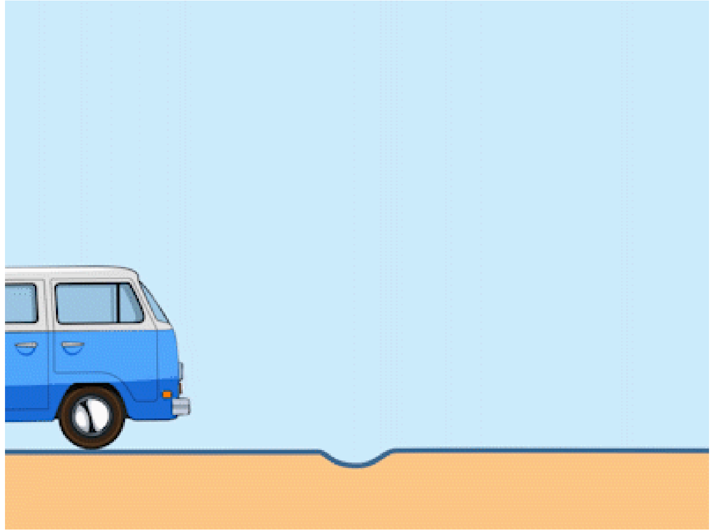
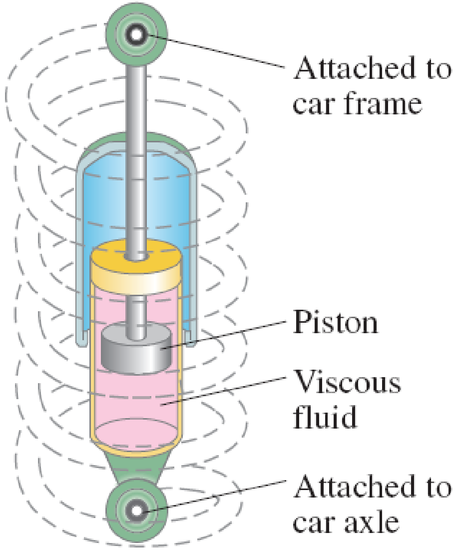
14.7 – Damped harmonic motion - Examples

In many systems (ground vehicles, earthquake protections), it is desired to damp oscillations as quickly as possible, so the damping is designed to be as close to the **critical damping** as possible.





On the other hand, in other systems, such as clocks, watches, and musical instruments, damping in unwanted.

14.8 – Forced oscillations and resonance

Forced oscillations occur when there is a **periodic driving force**.

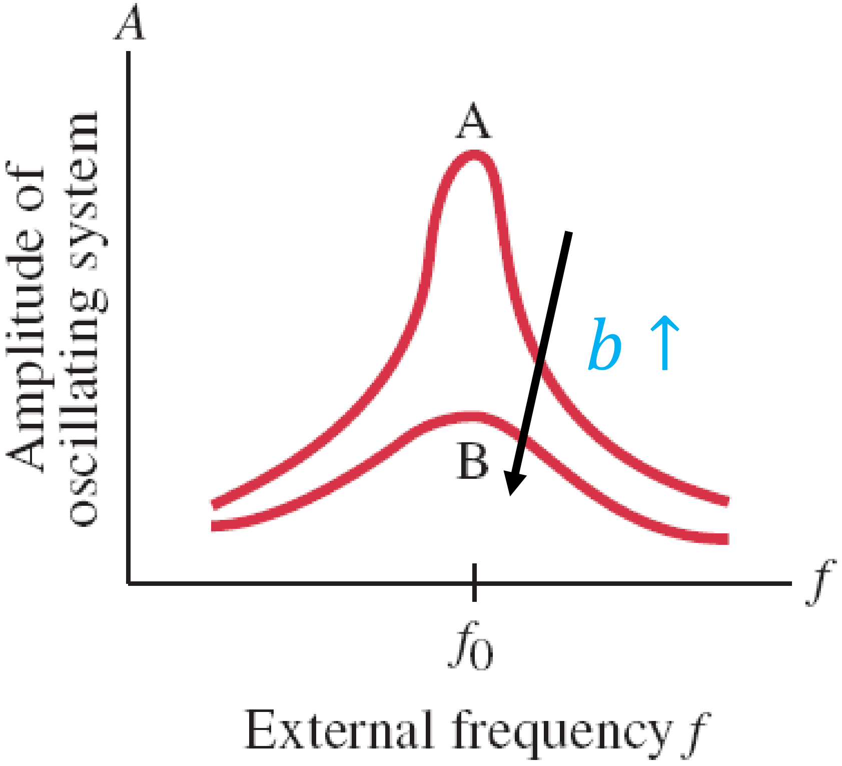
The frequency ω of this periodic driving force might not be the same as the **natural frequency** of the system ($\omega_0 = \sqrt{k/m}$).

In case it is ($\omega = \omega_0$), the oscillation amplitude can become very large. This phenomenon is called **resonance** and can be quite destructive.

Tacoma Narrows Bridge (wind gusts), 1940

Freeway collapse in California (earthquake), 1989

14.8 – Forced oscillations and resonance



The sharpness of the resonant peak depends on the **damping** b:

- If the damping is small (A) it can be quite sharp.
- If the damping is larger (B) it is less sharp.

f is the frequency of the driving force

f_0 is the natural frequency of our system $\left(f_0 = \frac{\sqrt{k/m}}{2\pi}\right)$

Like with damping, resonance can be **wanted** or **unwanted**. Musical instruments and TV/radio receivers depend on it (see also chapter 30).

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14.8 – Forced oscillations and resonance

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \cos\omega t$$

ω here is the frequency of the driving force!

$$\boxed{x = A_0 \sin(\omega t + \phi_0)}$$

ω_0 here is the natural frequency of our system: $\sqrt{k/m}$

$$A_0 = \frac{F_0}{m\sqrt{(\omega^2 - \omega_0^2)^2 + \frac{b^2\omega^2}{m^2}}}$$

$$\phi_0 = \tan^{-1}\left(\frac{\omega_0^2 - \omega^2}{\frac{\omega b}{m}}\right)$$

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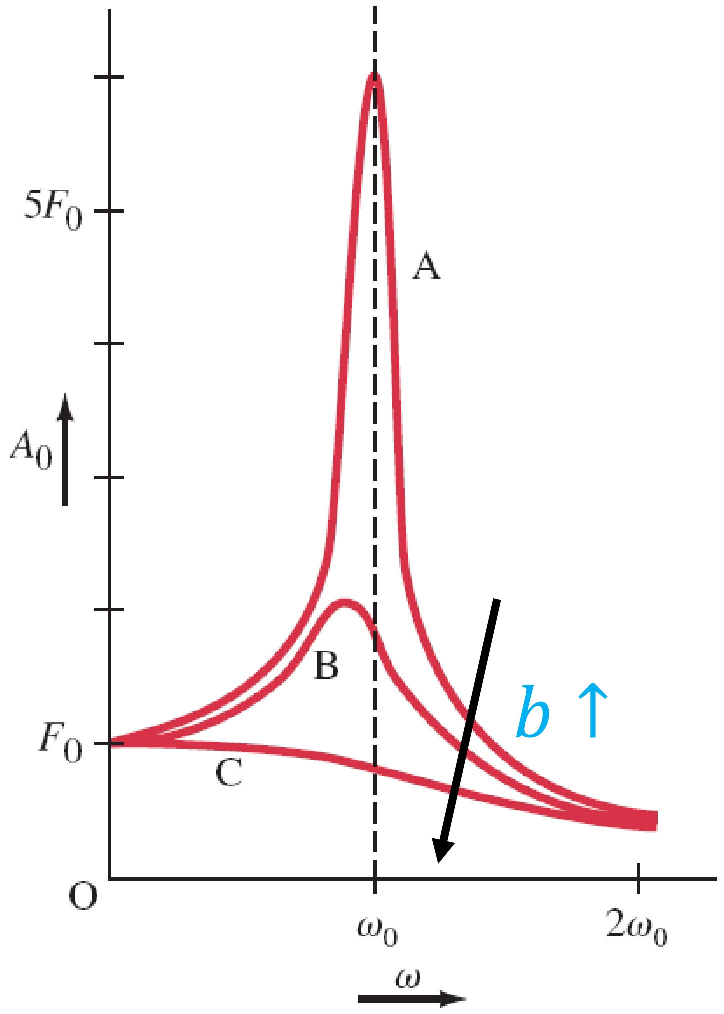
14.8 – Forced oscillations and resonance

$$A_0 = \frac{F_0}{m\sqrt{\cancel{(\omega^2 - \omega_0^2)^2} + \frac{b^2\omega^2}{m^2}}}$$



At which frequency ω we get the maximum amplitude?

$$\omega = \frac{\omega_0}{\sqrt{1+\frac{b^2}{m^2}}} \sim \omega_0$$

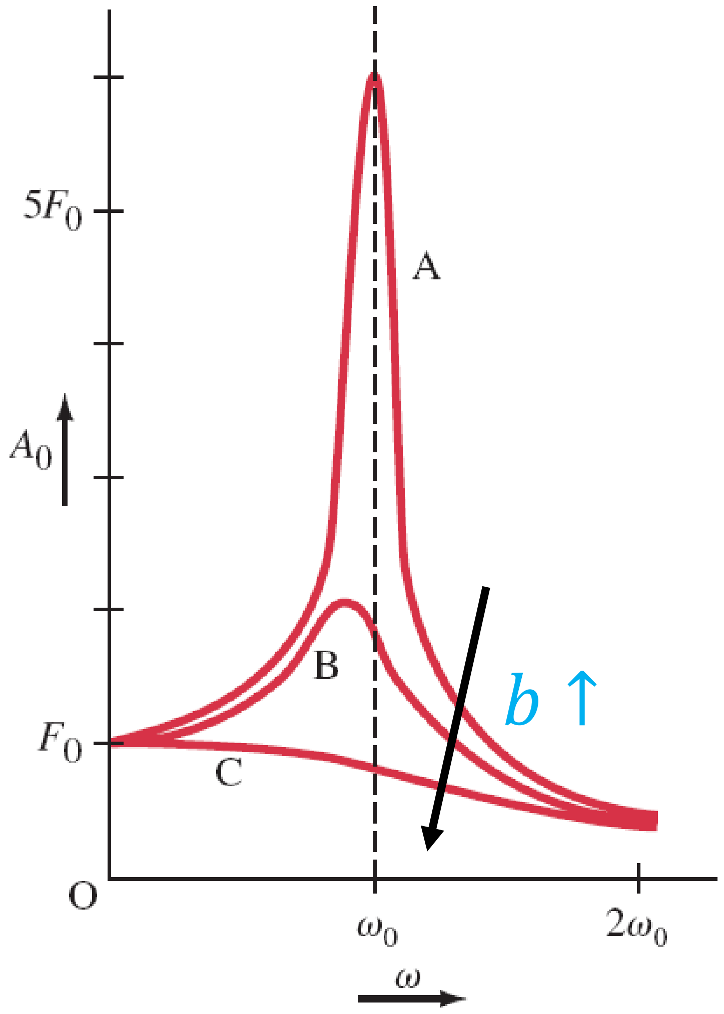
14.8 – Forced oscillations and resonance

The resonant peak is characterized by the so-called *Q-factor*:

$$Q = \frac{m\omega_0}{b}$$

The relative width of the resonant peak ($\Delta\omega/\omega_0$) is the inverse of the Q factor:

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{Q}$$



$$b \uparrow \rightarrow A_0 \downarrow, Q \downarrow, \Delta\omega/\omega_0 \uparrow$$

14.8 – Forced oscillations and resonance

Example of (non predicted) **lateral** oscillations in the Millenium bridge in London.

[Link to video]

14.8 – Forced oscillations and resonance

Fatigue testing for a large wind turbine blade (LM wind power).

Link to video

14.8 – Forced oscillations and resonance

Japan's earthquake simulator:

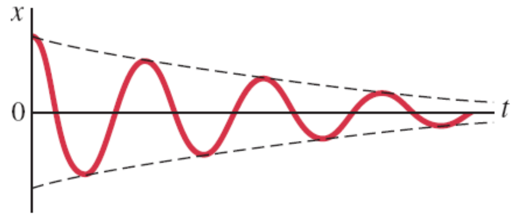
Link to video

Wrap-up: revisit learning objectives

After this lecture you should be able to:

- Explain the fundamentals of **damped harmonic motion**.

$$x = Ae^{-\gamma t}\cos\omega' t \qquad \gamma = \frac{b}{2m} \qquad \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$



- Calculate the motion of a system subject to **forced oscillations**.

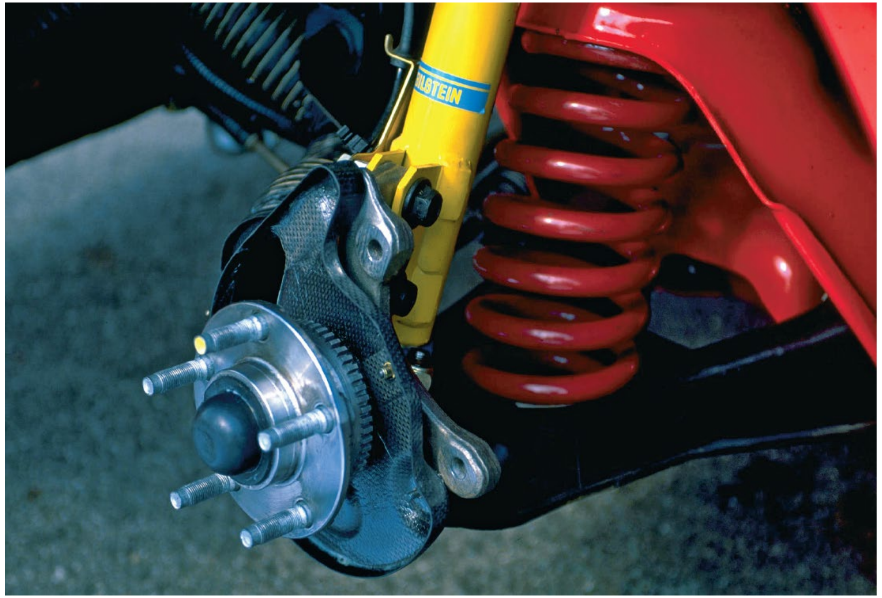
$$x = A_0 \sin(\omega t + \phi_0) \qquad A_0 = \frac{F_0}{m\sqrt{(\omega^2 - \omega_0^2)^2 + \frac{b^2\omega^2}{m^2}}}$$

- Explain the concept of **resonance** and estimate the resonance frequency.

$$\omega \sim \omega_0 = \sqrt{k/m} \qquad \frac{\Delta\omega}{\omega_0} = \frac{1}{Q} \qquad Q = \frac{m\omega_0}{b}$$

OSCILLATIONS

Chapter 14



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