Chapter 34: The Wave Nature of Light; Interference

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Chapter 34: The Wave Nature of Light; Interference

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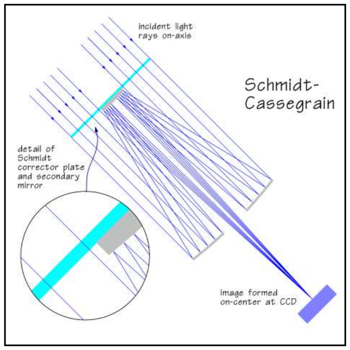
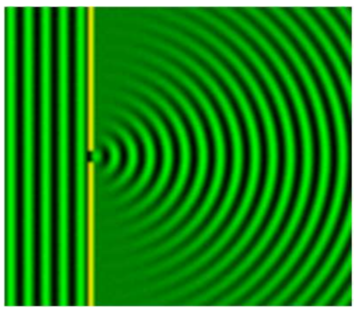
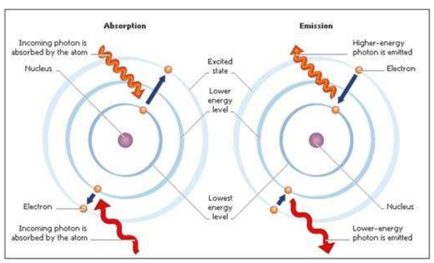
Optics: a game with light







- Designing an optical instrument doesn’t only mean the use the geometrical optics.
- Light can be considered as:
  - **a ray** → geometrical optics limits
    - Reflective optics
    - Refractive optics
  - **a wave** → diffractive optics
    - Grating description
    - Effect of the pupil
    - Interferometry
    - Scattering
  - **A particle**/ energy quanta
    - Interaction light-matter
    - Spectroscopic process
    - Interaction light → detector (photon → electron)

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34-1 Waves versus Particles; Huygens' Principle and Diffraction

- Light carries energy
  - Particles? Ray? Wave?
- Until now we've seen that light is travelling in straight lines

- In chapter 31:
  - Mawell equations ➔ A right combination of these equations gives a **wave equation**
    - Leads to Electromagnetic waves (ELM)
    - Light is an ELM wave
  - **Superposition principle** (for any kind of wave):

« ***The resultant disturbance at any point in a medium is the algebric sum of the separate constituent waves*** » [Optics – Hecht]

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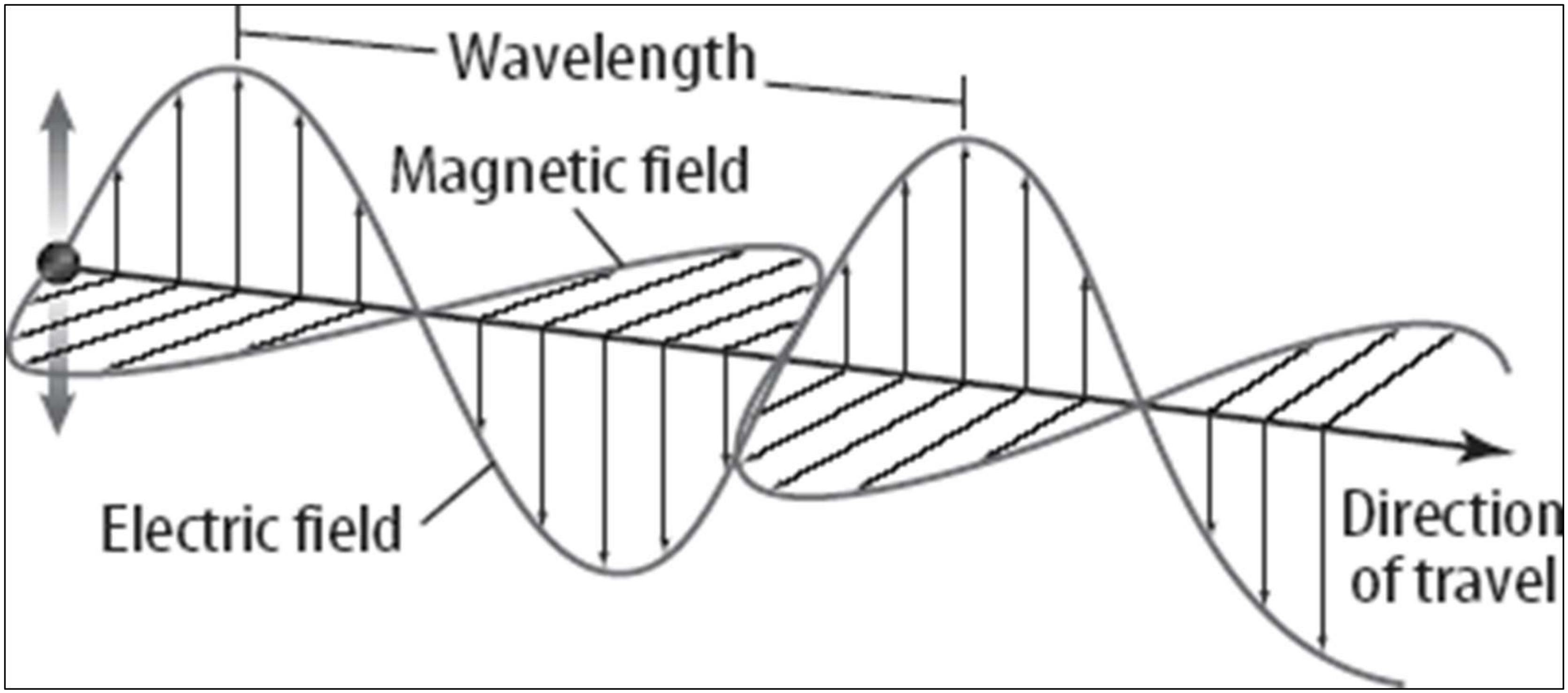
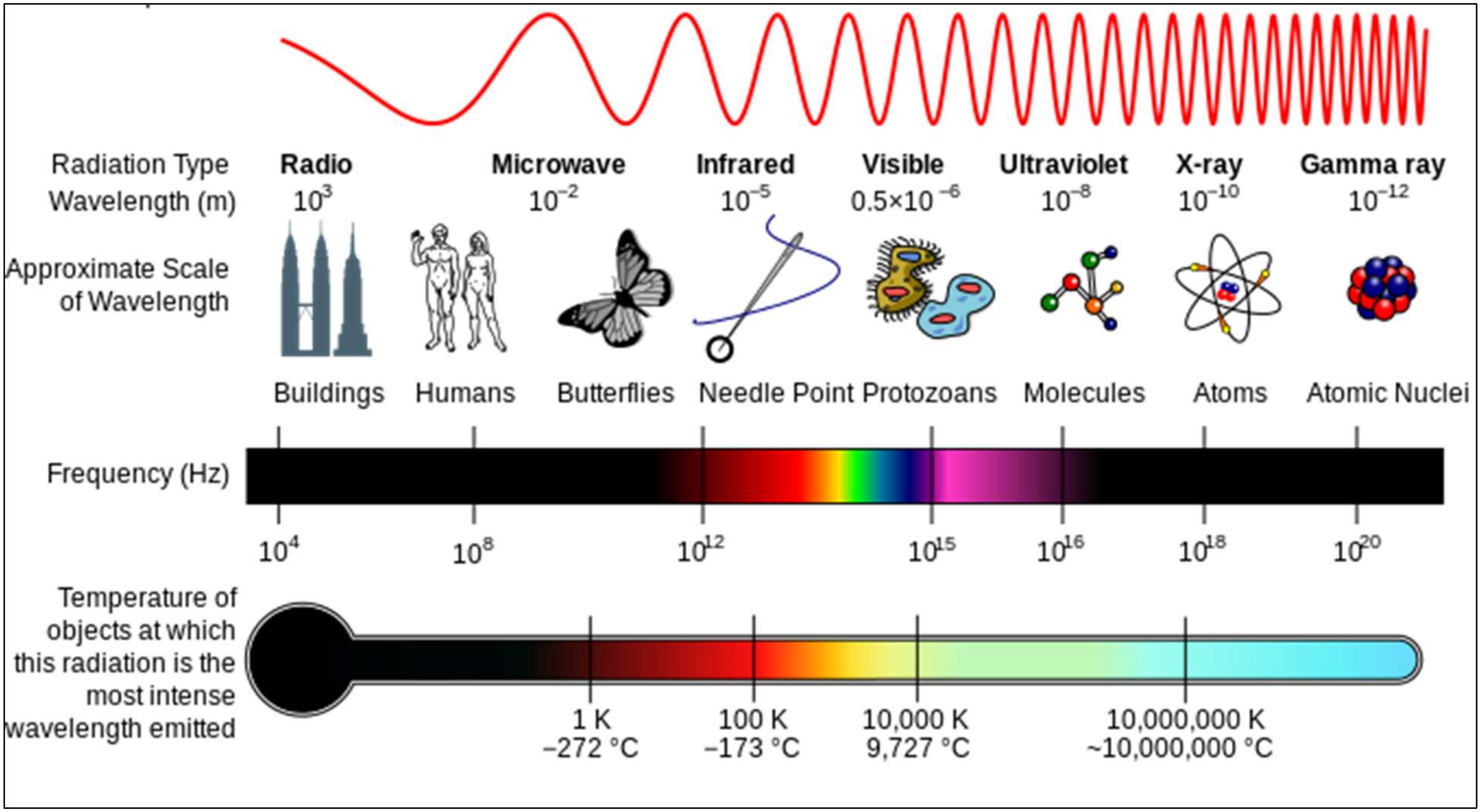
34-1 Waves versus Particles; Huygens' Principle and Diffraction



Electromagnetic spectrum



34-1 Waves versus Particles; Huygens' Principle and Diffraction

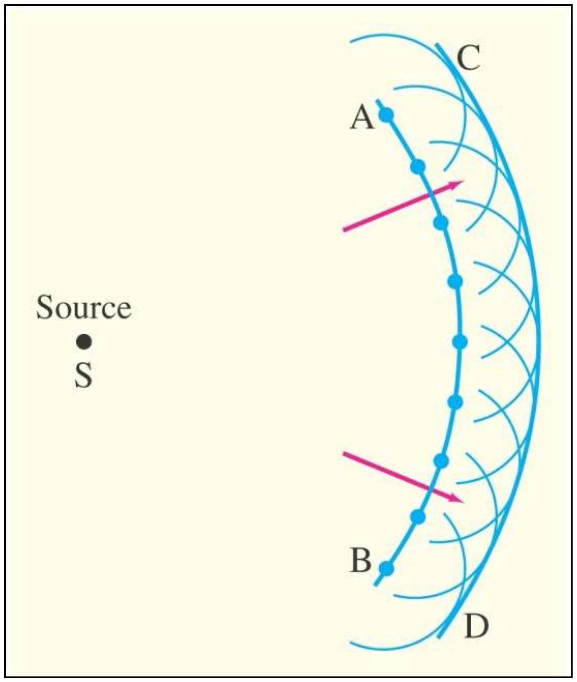
- **Huygens' principle:**

Every point on a wave front acts as a point source; the wavefront as it develops is tangent to all the wavelets. The wavelets spread out in the ***forward direction*** *at the speed of the wave itself.*

- Let's consider the wavefront AB
- The medium is considered as isotropic

  ⇒ the velocity of the wave $v = cst$ and the same in every direction

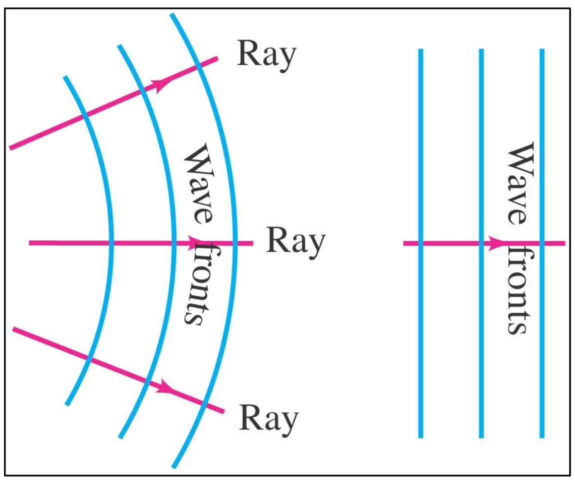
- **Each point source** creates a spherical wave (circle in 2D) of a radius $r = v\,t$. Its center is represented by the blue dots, on the wavefront AB.



34-1 Waves versus Particles; Huygens' Principle and Diffraction

reminder:

- A **wavefront** is a set or locus of all points at a particular instant of time, having the **same phase**. Wavefronts are imaginary surfaces (curves in 2D) to represent how waves move.
- Rays are perpendicular to the wavefront



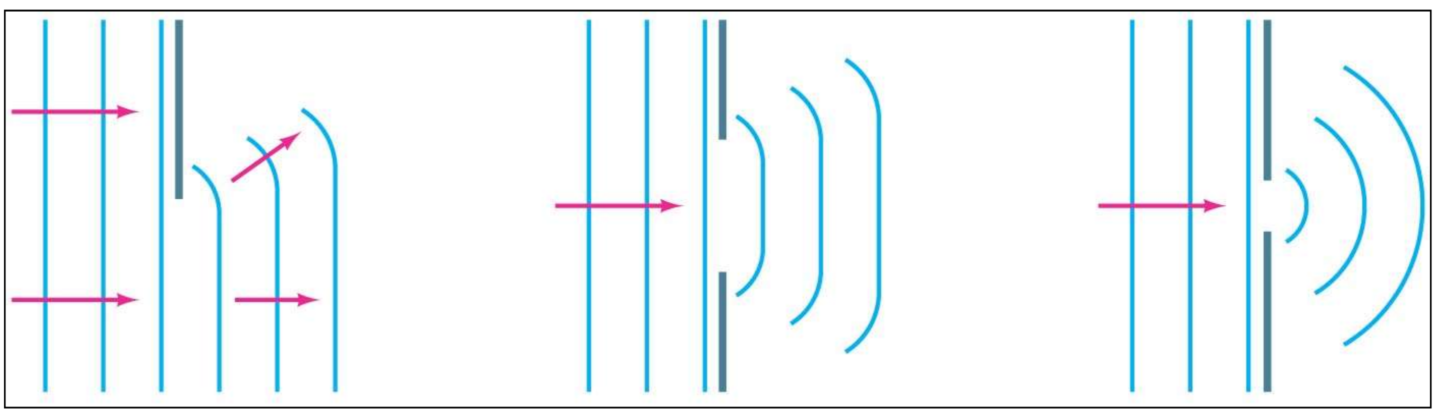
34-1 Waves versus Particles; Huygens' Principle and Diffraction

- Huygens' principle is consistent with **diffraction**

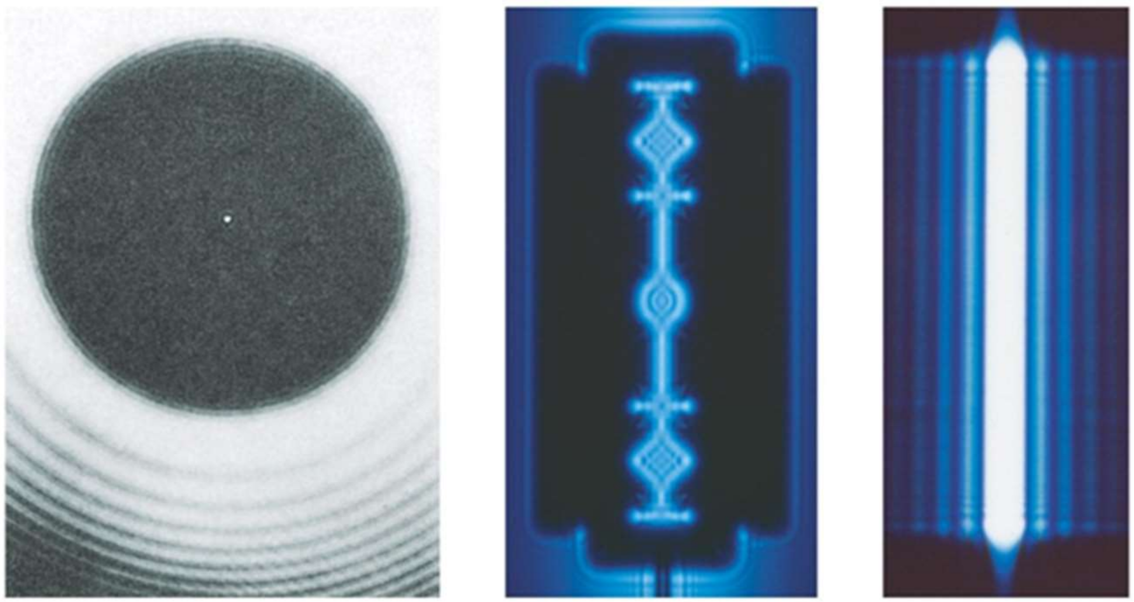
(bending of waves behind an obstacle):

'large' hole

'small' hole

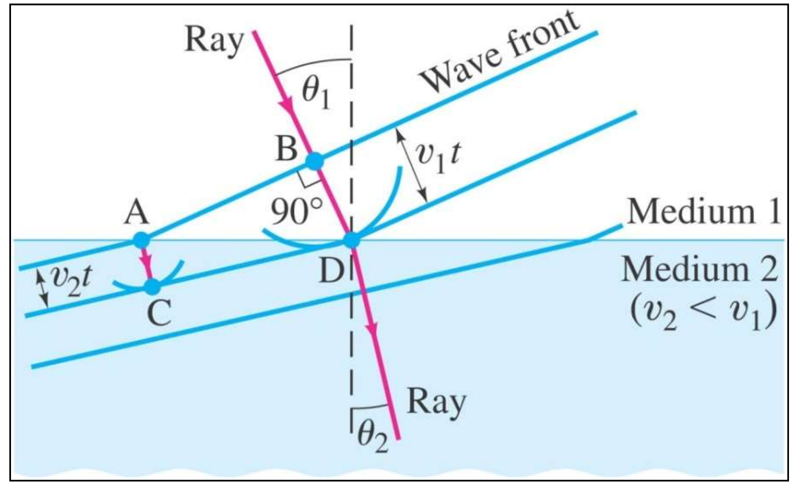


34-1 Waves versus Particles; Huygens’ Principle and Diffraction



34-2 Huygens' Principle and the Law of Refraction

- Huygens' principle can also explain **Snell's law of refraction**.
- As the wavelets propagate from each point, they propagate more **slowly** in the medium of higher index of refraction.
- This leads to a '**bend**' (no diffraction!) in the wave front and therefore in the ray.



$$\left.\begin{aligned} \sin(\theta_1) &= \frac{v_1 t}{AD} \\ \sin(\theta_2) &= \frac{v_2 t}{AD} \end{aligned}\right\} \quad \frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

- The **frequency of the light does not change**, but the wavelength does as it travels into a new medium:

$$\boxed{\frac{\lambda_2}{\lambda_1} = \frac{v_2 t}{v_1 t} = \frac{v_2}{v_1} = \frac{n_1}{n_2}.}$$

$$\boxed{\lambda_n = \frac{\lambda}{n}.}$$

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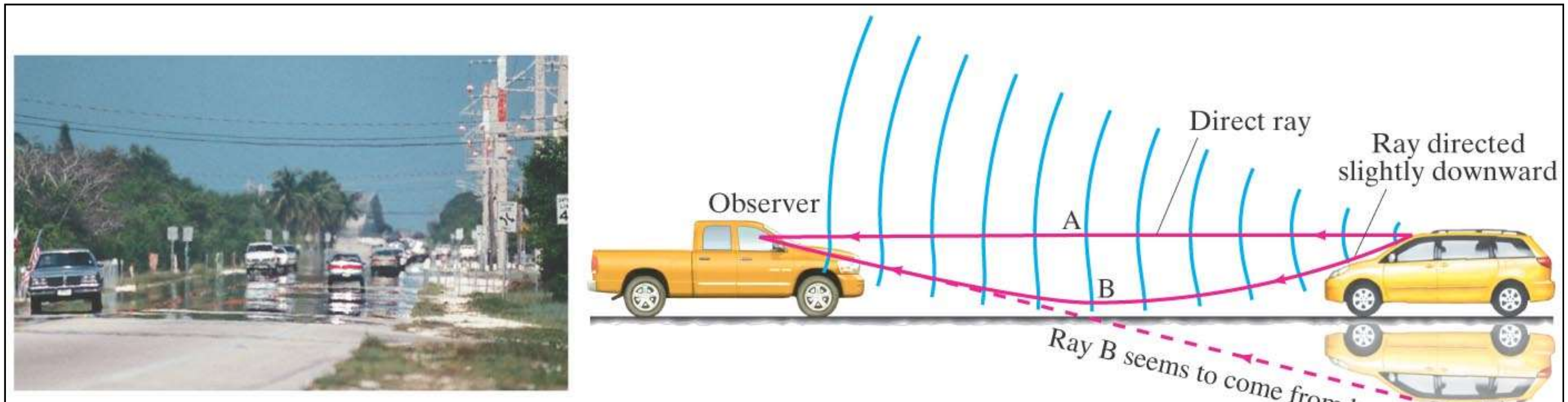
34-2 Huygens' Principle and the Law of Refraction

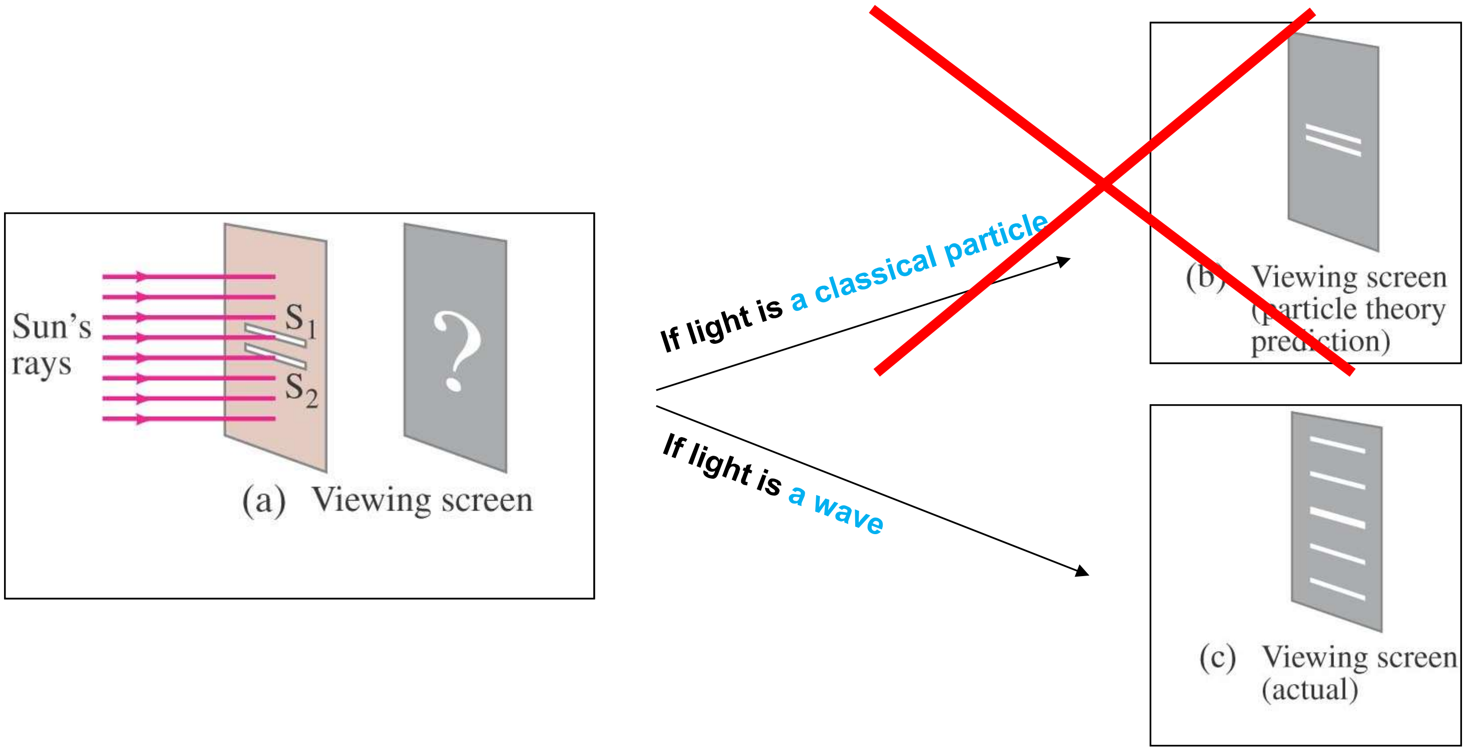


(a) (b)

- Highway mirages are due to a gradually changing index of refraction in **heated air**.

34-3 Interference – Young's Double-Slit Experiment

- If the light is a wave, what happens when the light passes through two slits or two holes?



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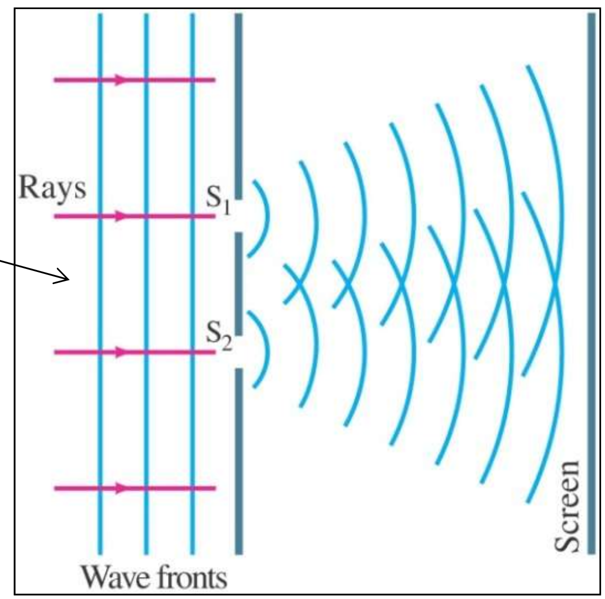
34-3 Interference – Young's Double-Slit Experiment

- If light is a wave, there should be an **interference pattern**.
- Interference pattern only observed for **coherent sources** (as in this case).
  - With e.g. two tiny light bulbs (two different sources) no interference pattern is seen.
  - It needs to have the same source split in two.

Monochromatic plane waves of light

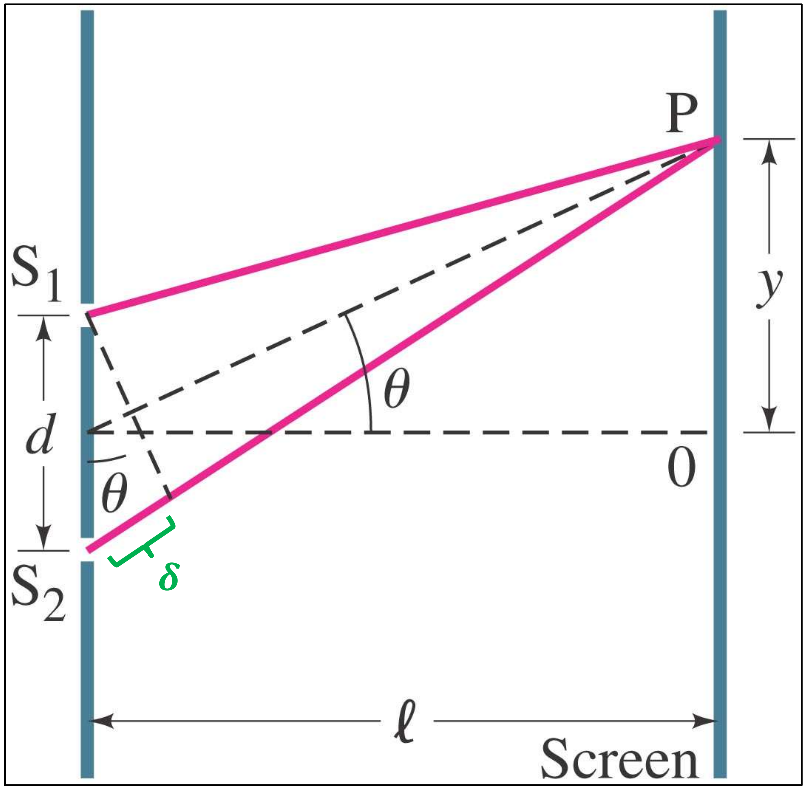


S1 and S2 are coherent if they have the same frequency and a constant phase difference

34-3 Interference – Young's Double-Slit Experiment

- The electric fields at the point P from the two slits are given by

  - $E_1 = E_{10}\sin(\omega t)$
  - $E_2 = E_{20}\sin(\omega t + \boldsymbol{\delta})$
  - $E_{tot} = E_1 + E_2$ (superposition principle)



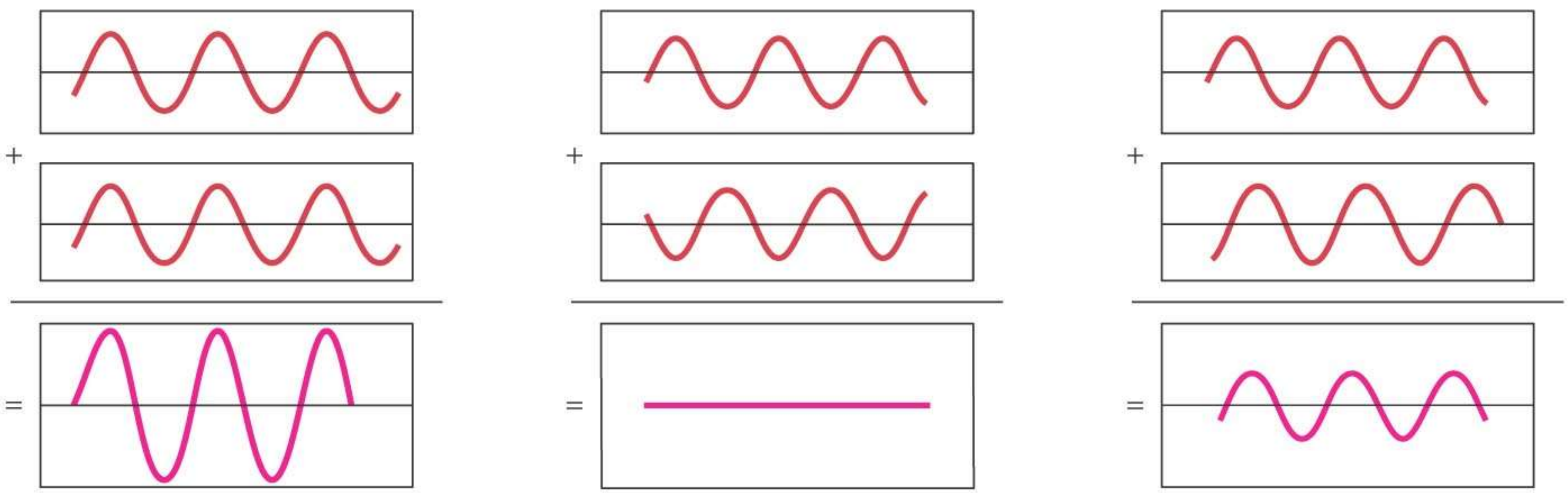
Light intensity $\Rightarrow I = |E_{tot}|^2 = |E_1 + E_2|^2$

Where $\boldsymbol{\delta} = \dfrac{\mathbf{2\pi}}{\boldsymbol{\lambda}} \boldsymbol{d}\,\mathbf{sin}(\boldsymbol{\theta})$ (optical phase difference)

$\boldsymbol{k} = \mathbf{2\pi}/\boldsymbol{\lambda}$ converts distance into phase related to the wavelength of interest.

34-3 Interference – Young's Double-Slit Experiment

- **From chapter 15:**
  - **These graphs show the sum of two waves. In (a) they add constructively; in (b) they add destructively; and in (c) they add partially destructively.**



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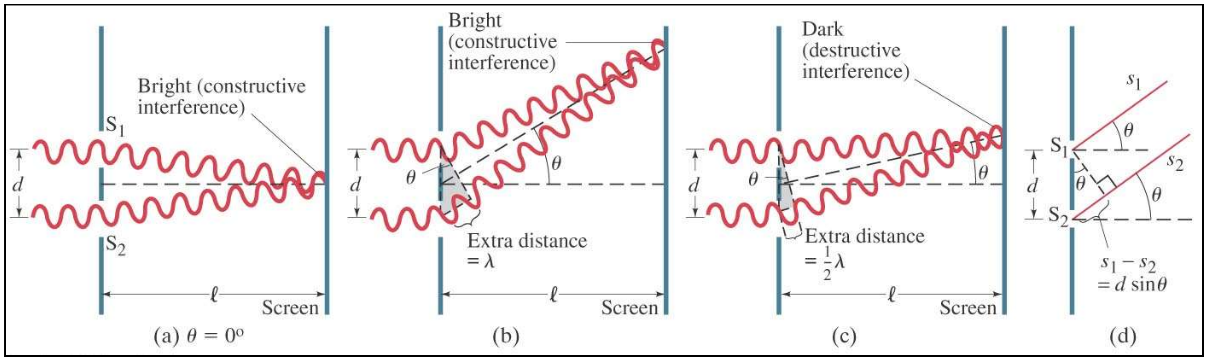
34-3 Interference – Young's Double-Slit Experiment

- The interference occurs because each point on the screen is not the at same distance from both slits. Depending on the path length difference, the wave can interfere constructively (**bright spot**) or destructively (**dark spot**). (superposition principle)



- We use **simple geometry** and the **superposition principle** to find the conditions for **constructive** and **destructive** interference:

$$d \sin\theta = m\lambda, \qquad m = 0, 1, 2, \cdots. \qquad \left[\begin{array}{c}\text{constructive}\\ \text{interference}\\ \text{(bright)}\end{array}\right]$$

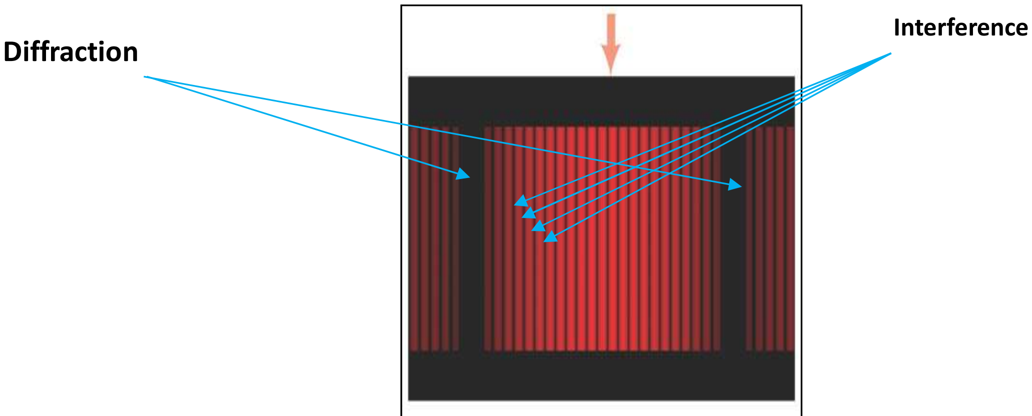
And:

$$d \sin\theta = (m + \tfrac{1}{2})\lambda, \qquad m = 0, 1, 2, \cdots. \qquad \left[\begin{array}{c}\text{destructive}\\ \text{interference}\\ \text{(dark)}\end{array}\right]$$

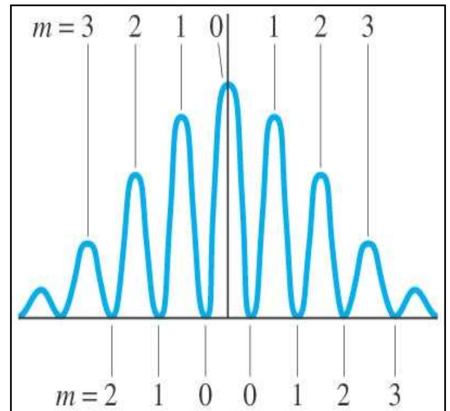
See chapter 15 for the details

34-3 Interference – Young's Double-Slit Experiment

- Between the maxima and the minima, the interference varies smoothly.



Constructive interference



Destructive interference

Combination of diffraction pattern (see next chapter) and interference pattern

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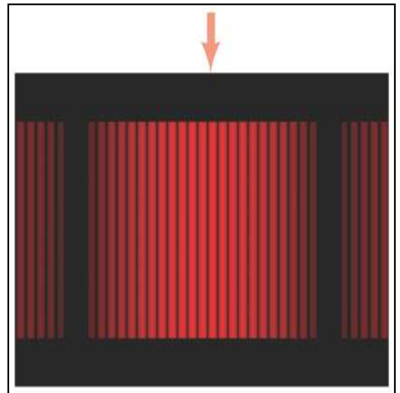
34-3 Interference – Young's Double-Slit Experiment

Conceptual Example 34-1: Interference pattern lines.

(a) Will there be an **infinite number of points (fringes)** on the viewing screen where constructive and destructive interference occur, or only a **finite number of points**?

(b) Are neighboring points of constructive interference **uniformly spaced**, or is the spacing between neighboring points of constructive interference **not uniform**?


Constructive interference

(a) Finite. → $sin(\theta_m) = \frac{m\lambda}{d} \leq 1$

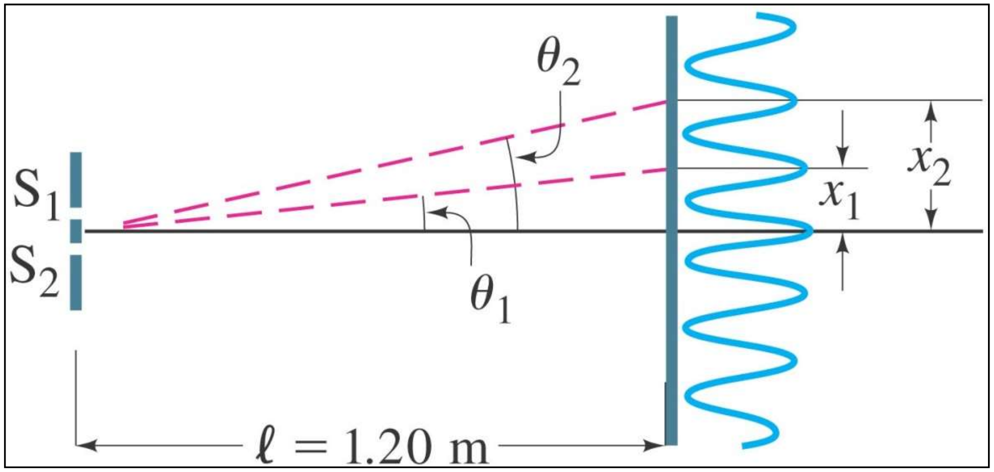
(b) Not uniform. → $sin(\theta)$ is a non-linear function

but close to $\theta = 0$ the spacing is nearly uniform as we consider $\sin(\theta) \approx \theta$

34-3 Interference – Young's Double-Slit Experiment

Example 34-2: Line spacing for double-slit interference.

– A screen containing two slits 0.100 mm apart is 1.20 m from the viewing screen. Light of wavelength λ = 500 nm falls on the slits from a distant source. Approximately how far apart will adjacent bright interference fringes be on the screen?



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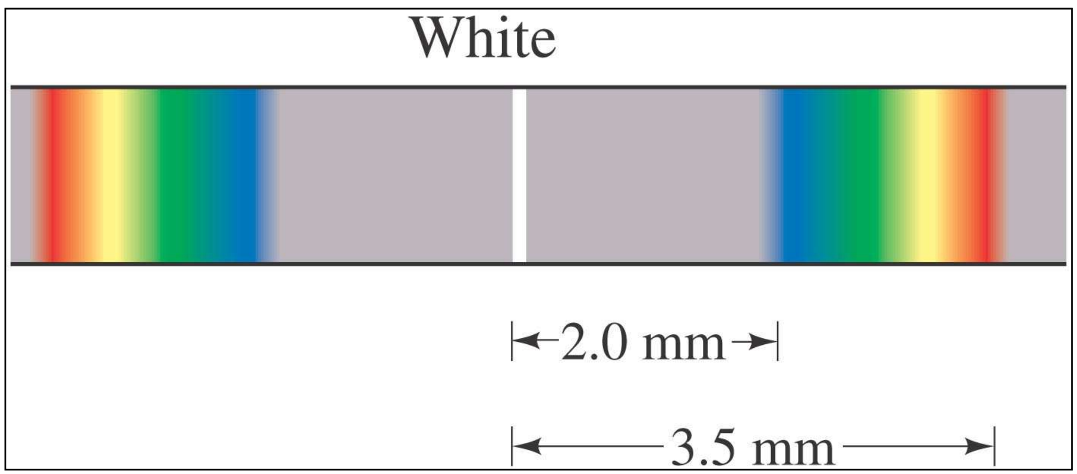
34-3 Interference – Young's Double-Slit Experiment

- What happens to the interference pattern in the previous example if the incident light (500 nm) is replaced by light of wavelength 700 nm?

- What happens instead if the wavelength stays at 500 nm but the slits are moved farther apart?

34-3 Interference – Young's Double-Slit Experiment

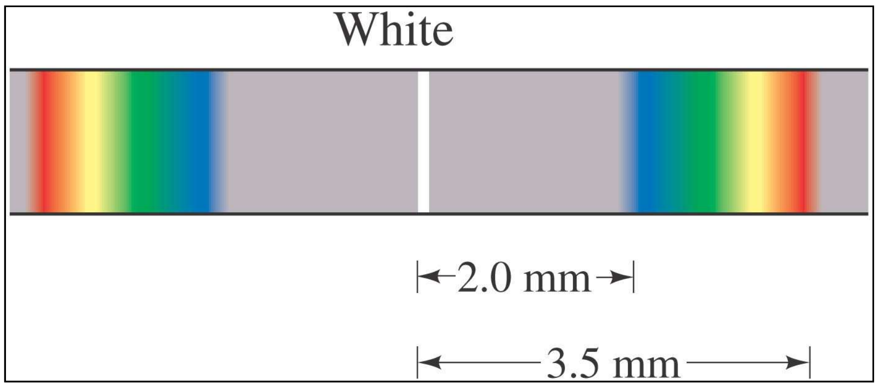
- Since the position of the maxima (except the central one) depends on wavelength, the first- and higher-order fringes contain a spectrum of colors.



34-3 Interference – Young’s Double-Slit Experiment

- Example 34-4: Wavelengths from double-slit interference.

  – White light passes through two slits 0.50 mm apart, and an interference pattern is observed on a screen 2.5 m away. The first-order fringe resembles a rainbow with violet and red light at opposite ends. The violet light is about 2.0 mm and the red 3.5 mm from the center of the central white fringe. Estimate the wavelengths for the violet and red light.



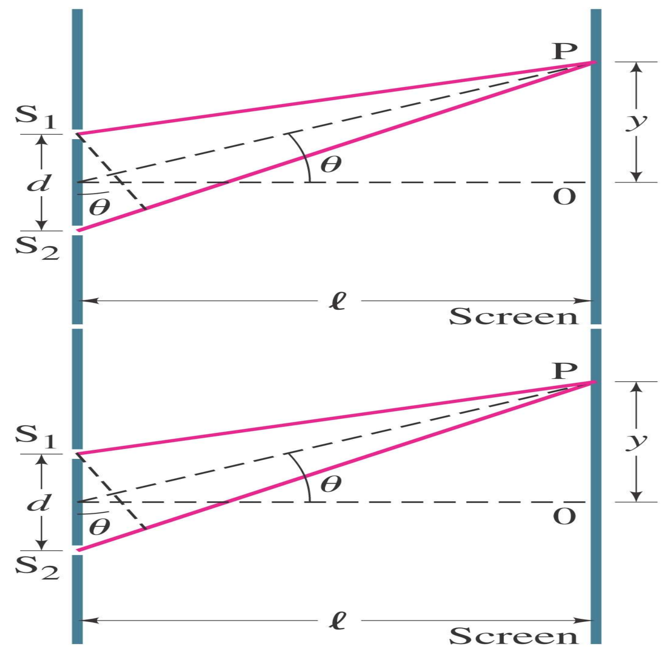
34-4 Intensity in the Double slit Interference pattern

- The electric fields at the point P from the two slits are given by

$$
\begin{aligned}
E_1 &= E_{10} \sin \omega t \\
E_2 &= E_{20} \sin(\omega t + \delta)
\end{aligned}
$$

- where

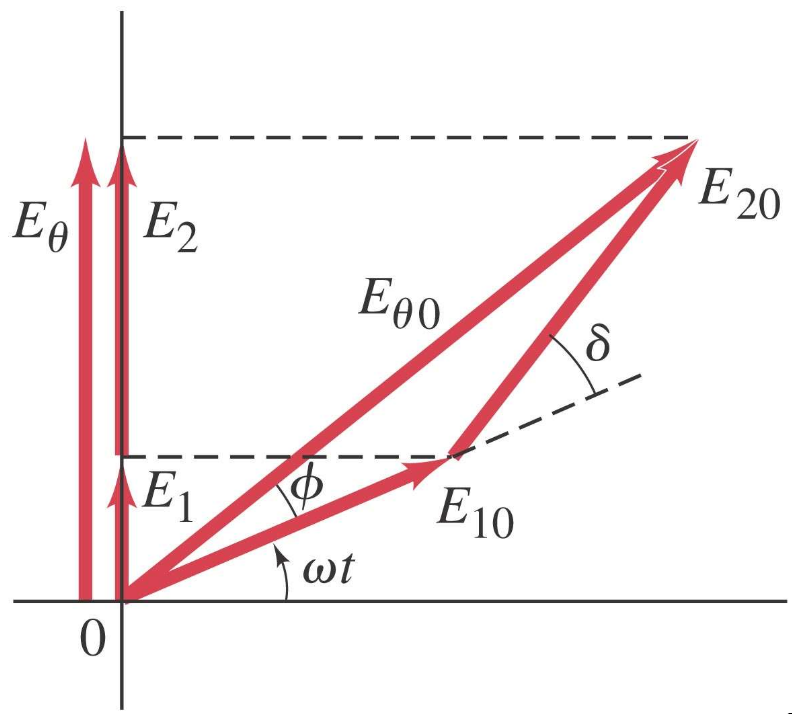
$$\delta = \frac{2\pi}{\lambda} d \sin\theta.$$



34-4 Intensity in the Double slit Interference pattern

- **The two waves can be added using phasors, to take the phase difference into account:**

$$E_\theta = 2E_0 \cos\frac{\delta}{2}\sin\left(\omega t + \frac{\delta}{2}\right).$$



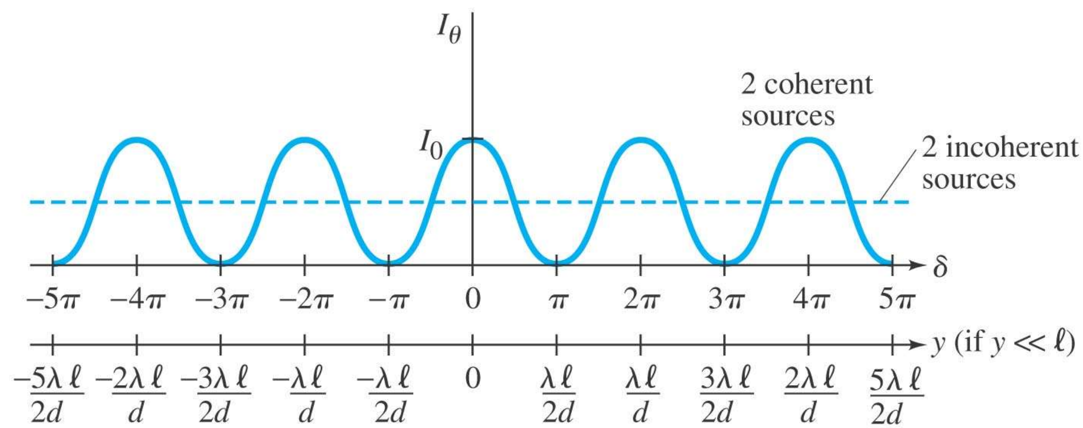
More about phasor – see chapter 30-8

34-4 Intensity in the Double slit Interference pattern

- **The time-averaged intensity is proportional to the square of the field:**

$$I_\theta = I_0 \cos^2 \frac{\delta}{2}$$

$$= I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$



$$I_\theta = I_0\left[\cos\left(\frac{\pi d}{\lambda\ell}y\right)\right]^2. \qquad [y \ll \ell,\ d \ll \ell]$$

34-5 Interference in Thin Films

- Another way path lengths can differ, and waves interfere, is if they travel through different media. If there is a very thin film of material – a few wavelengths thick – light will reflect from both the bottom and the top of the layer, causing interference. This can be seen in soap bubbles and oil slicks.

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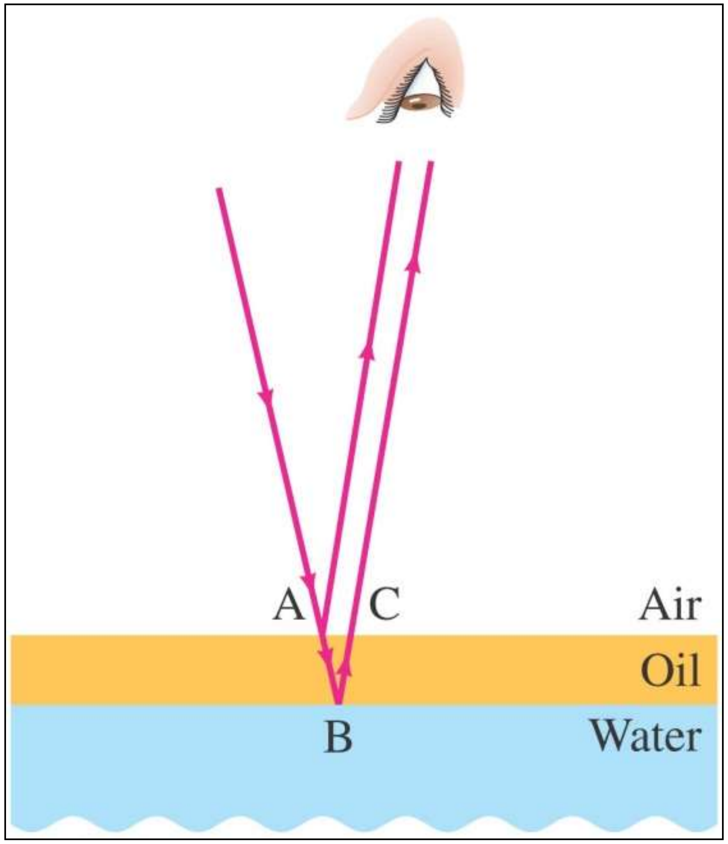
34-5 Interference in Thin Films

- The wavelength of the light will be different in the oil and the air,
- and the reflections at points A and B may or may not involve phase changes.



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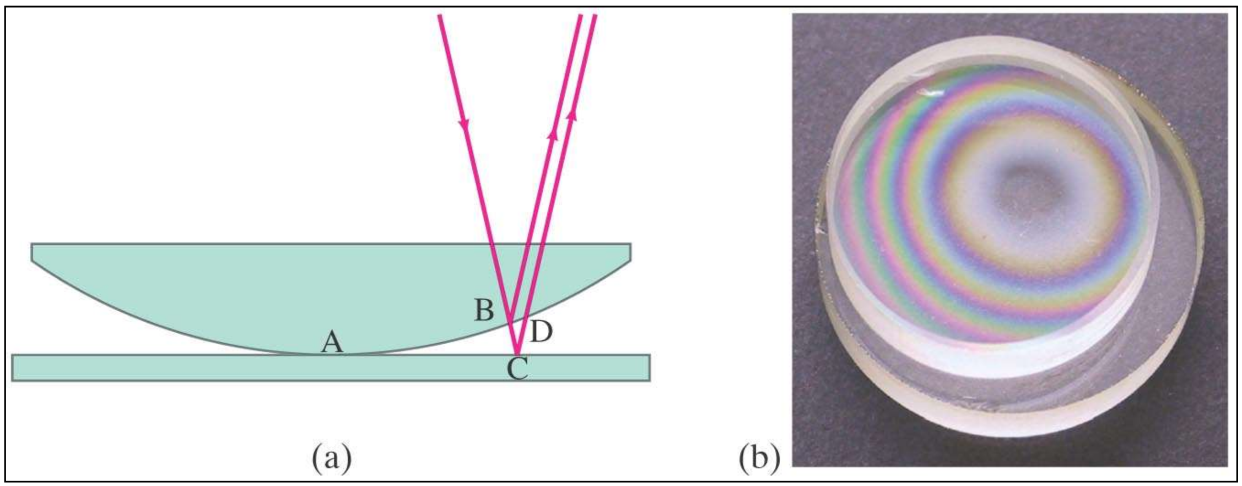
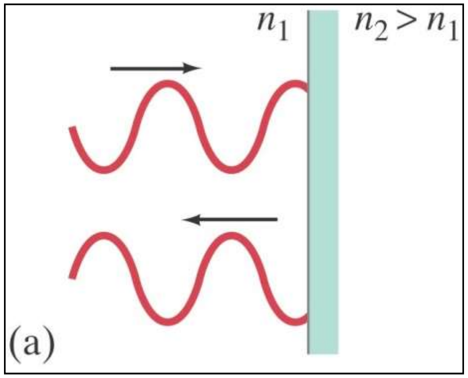
34-5 Interference in Thin Films

- A similar effect takes place when a shallowly curved piece of glass is placed on a flat one. When viewed from above, concentric circles appear that are called Newton's rings.

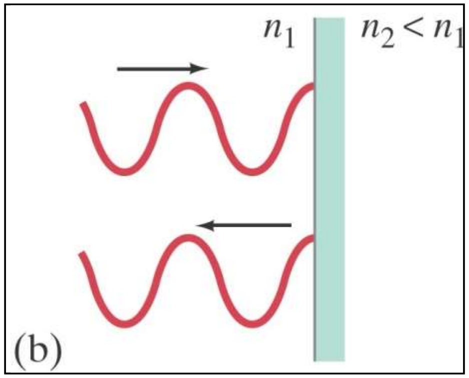


34-5 Interference in Thin Films

- Rule:
  - A beam of light reflected by a material with index of refraction greater than that of the material in which it is traveling, changes phase by 180° or ½ cycle or π.



Reflected ray experiences a $\pi - shift$

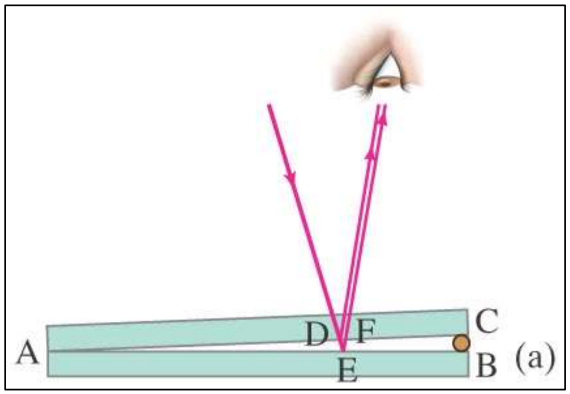


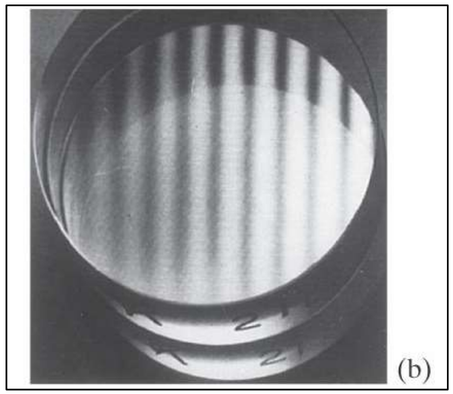
Reflected ray experiences a no phase shift

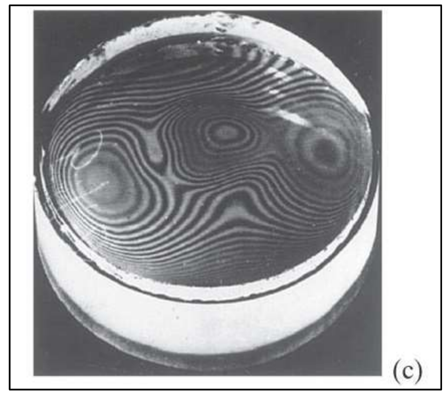
Remark: in advanced optical course we will see that this phase shift is dependent of the angle of incidence and the polarisation state of light

34-5 Interference in Thin Films

- Example 34-6: Thin film of air, wedge-shaped.







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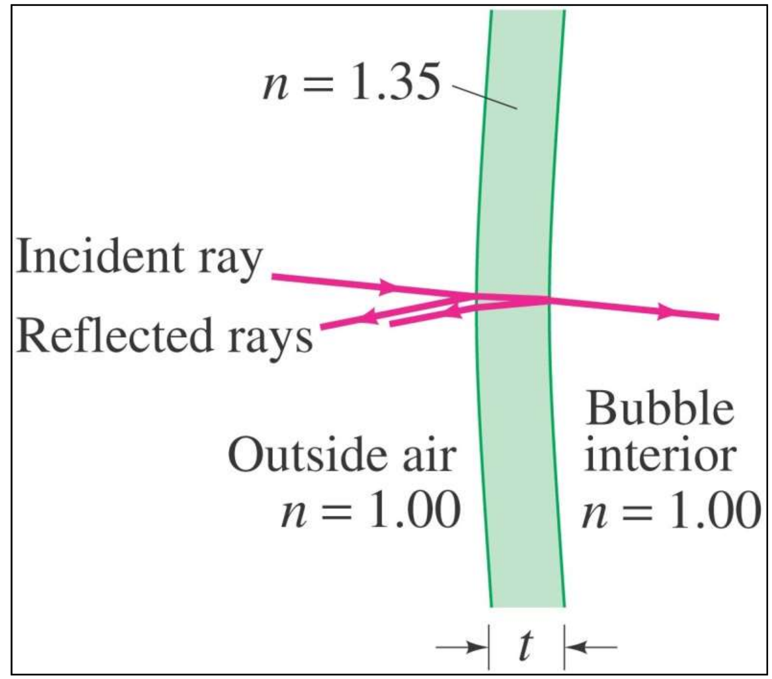
34-5 Interference in Thin Films

Example 34-7: Thickness of soap bubble skin.

- A soap bubble appears green (λ = 540 nm) at the point on its front surface nearest the viewer. What is the smallest thickness the soap bubble film could have? Assume n = 1.35.



- Green light is bright when minimum path difference is:

$$2t = \frac{1}{2}\lambda_n = \frac{\lambda}{2n}$$

- t = 540 nm/(4x1.35) = 100 nm

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34-5 Interference in Thin Films

- Problem Solving: Interference
- Interference occurs when two or more waves arrive simultaneously at the same point in space.
- Constructive interference occurs when the waves are in phase.
- Destructive interference occurs when the waves are out of phase.
- An extra half-wavelength shift occurs when light reflects from a medium with higher refractive index.

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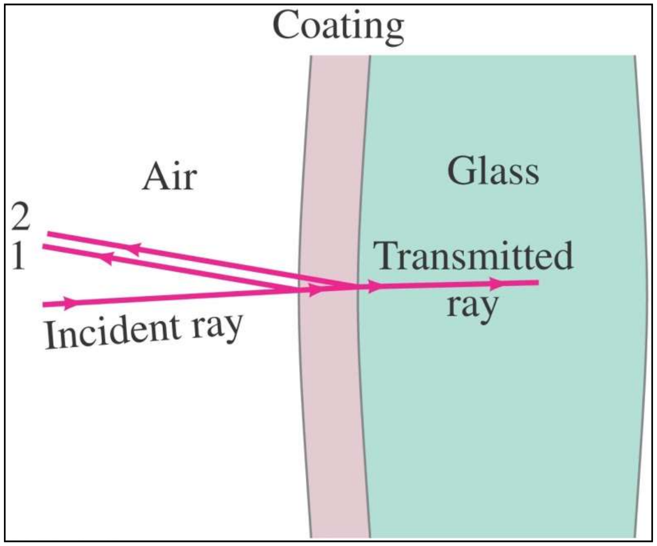
34-5 Interference in Thin Films

Example 34-8: Non-reflective coating (important application)

- What is the thickness of an optical coating of MgF_2 whose index of refraction is n = 1.38 and which is designed to eliminate reflected light at wavelengths (in air) around 550 nm when incident normally on glass for which n = 1.50?



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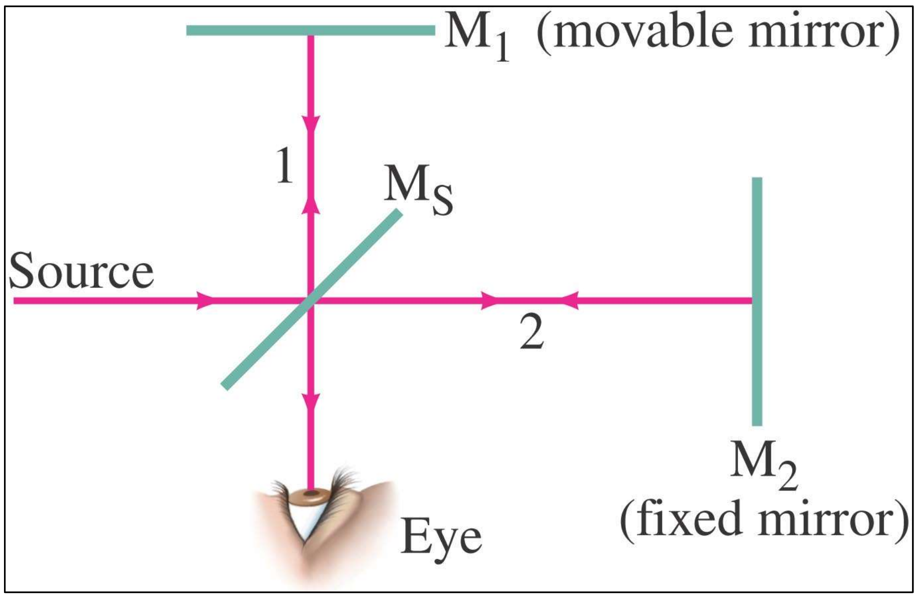
34-6 Michelson Interferometer

- The Michelson interferometer is centered around a beam splitter, which transmits about half the light hitting it and reflects the rest. It can be a very sensitive measure of length.



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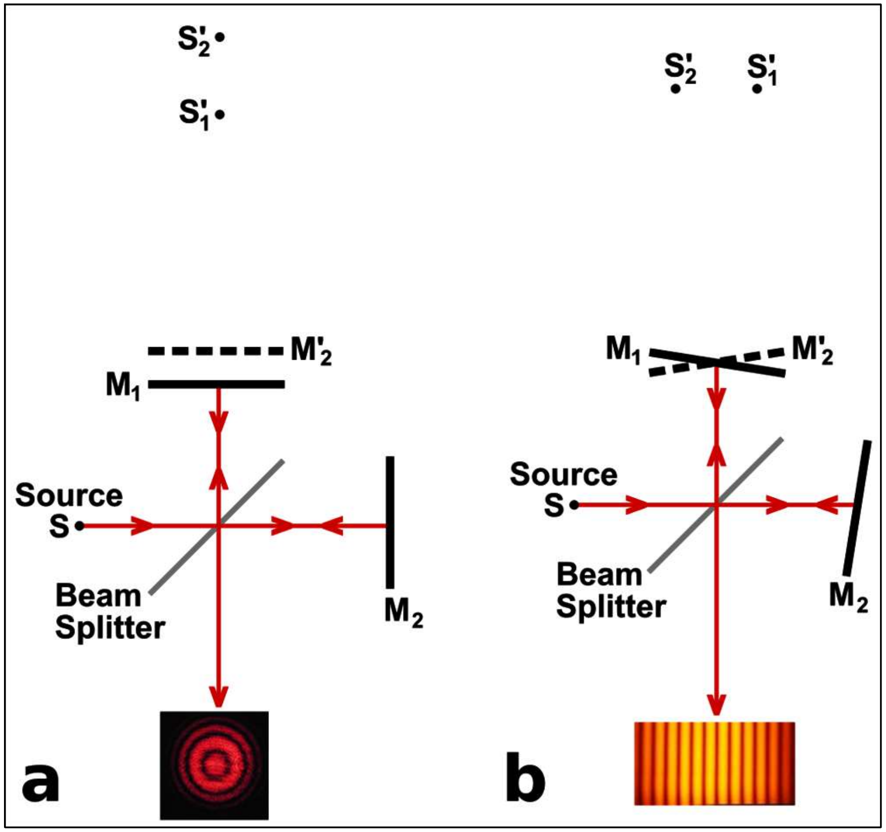
34-6 Michelson Interferometer



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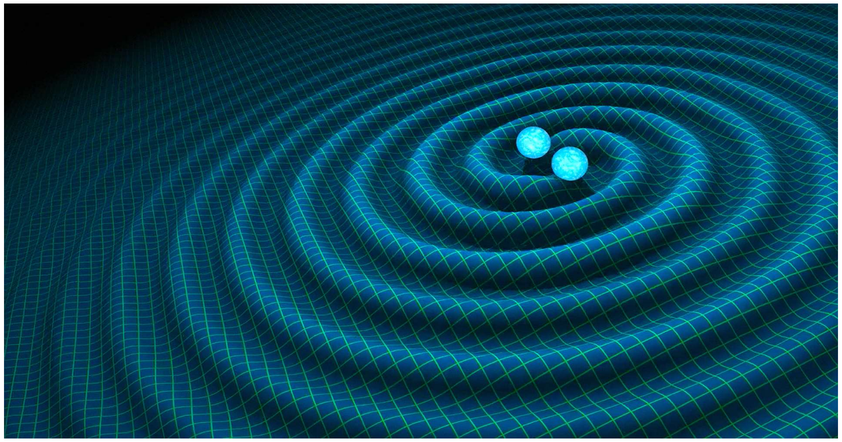
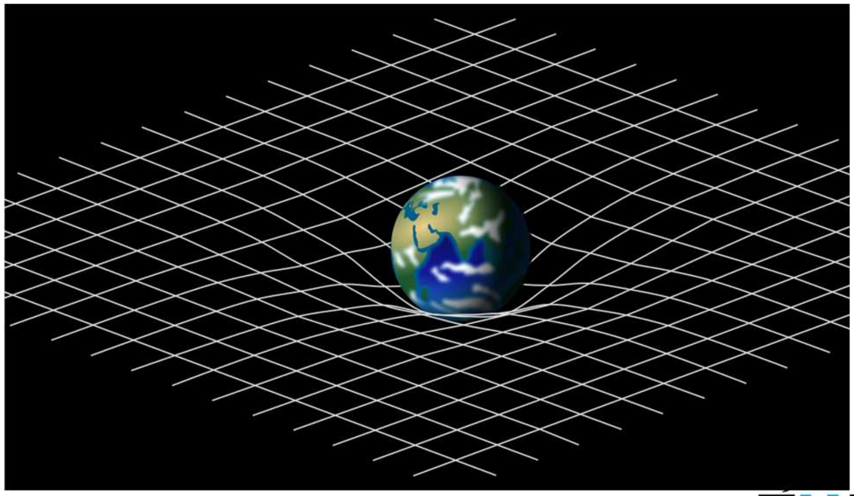
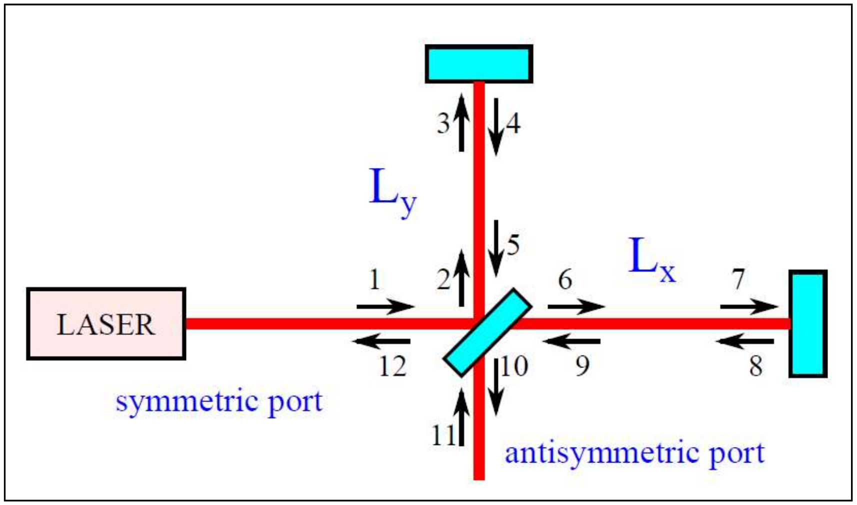
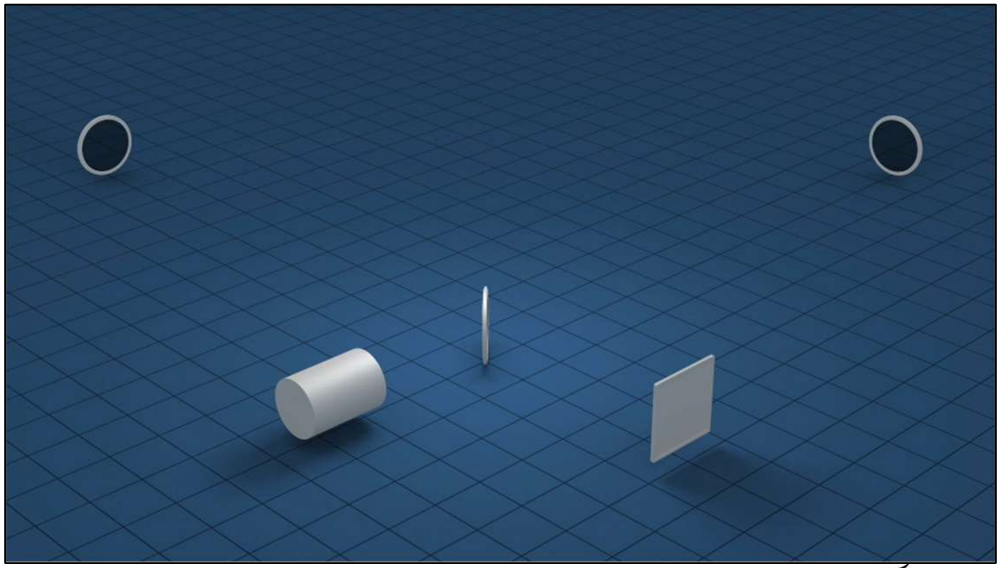
34-6 Michelson Interferometer & Gravitational waves





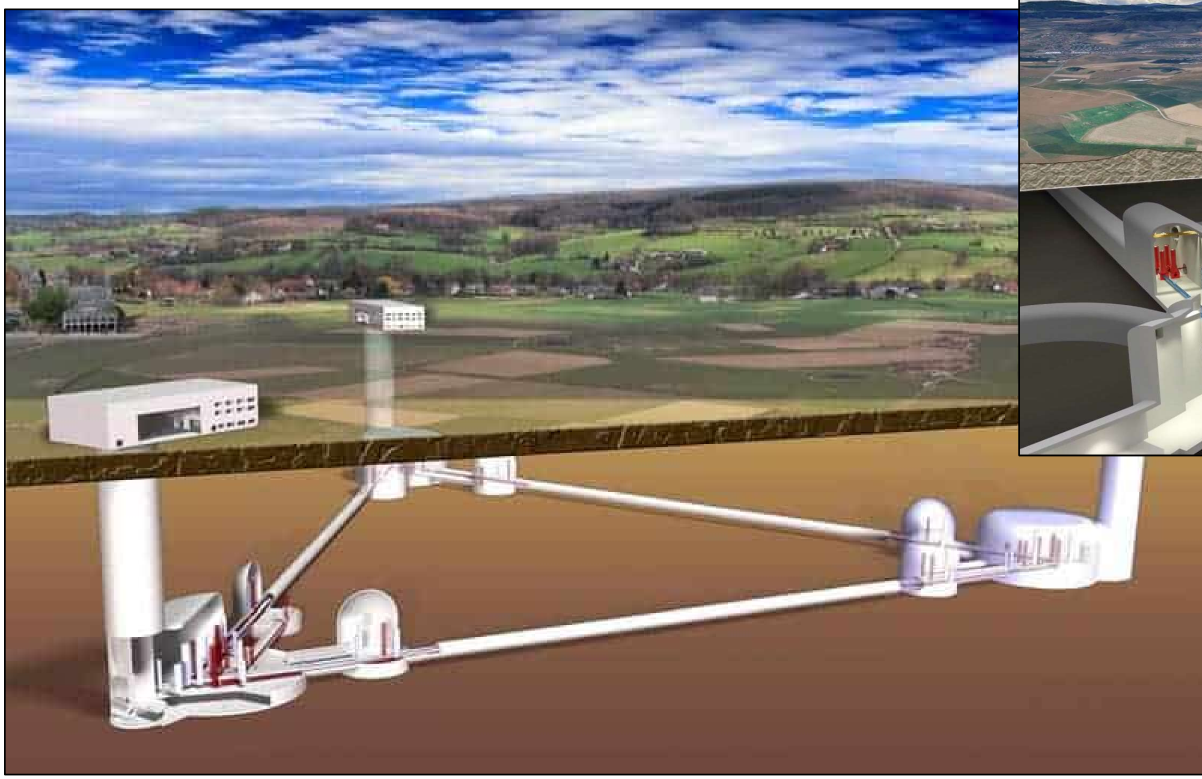
34-6 Michelson Interferometer & Gravitational waves

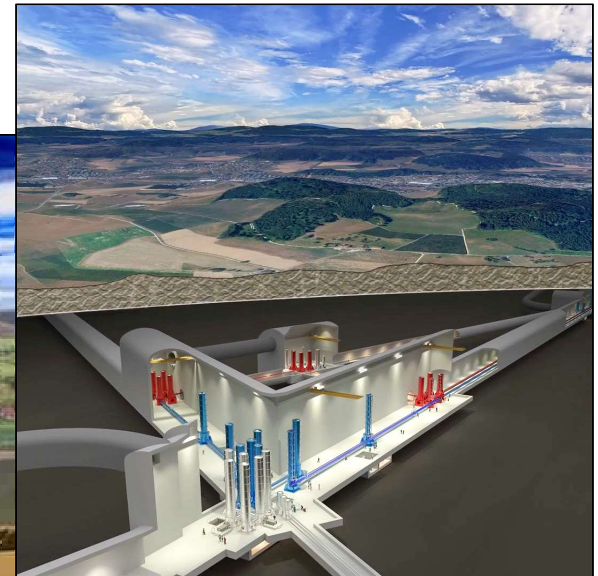




34-6 Michelson Interferometer & Gravitational waves

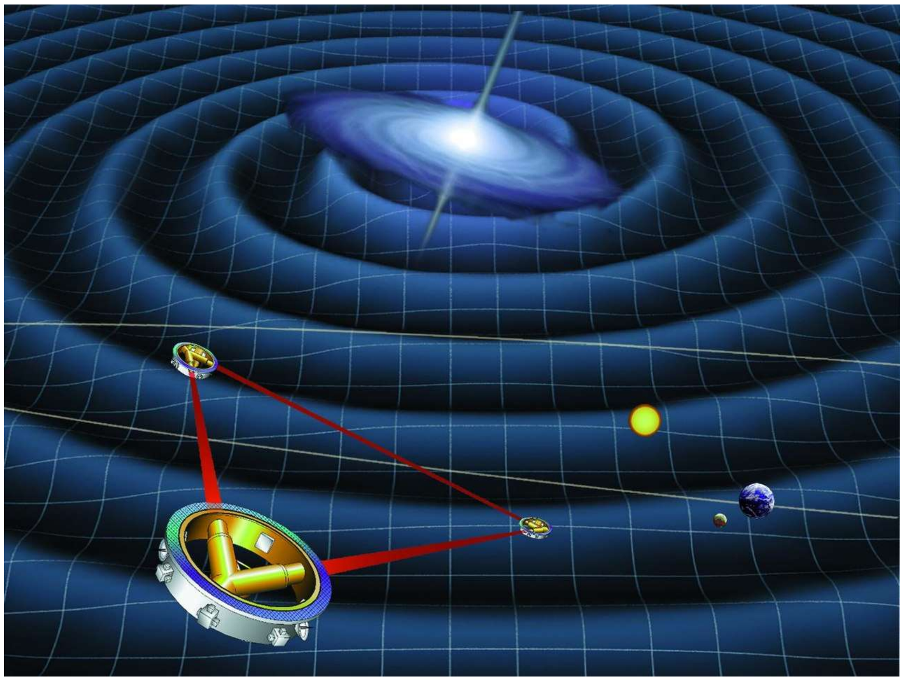
- Einstein telescope

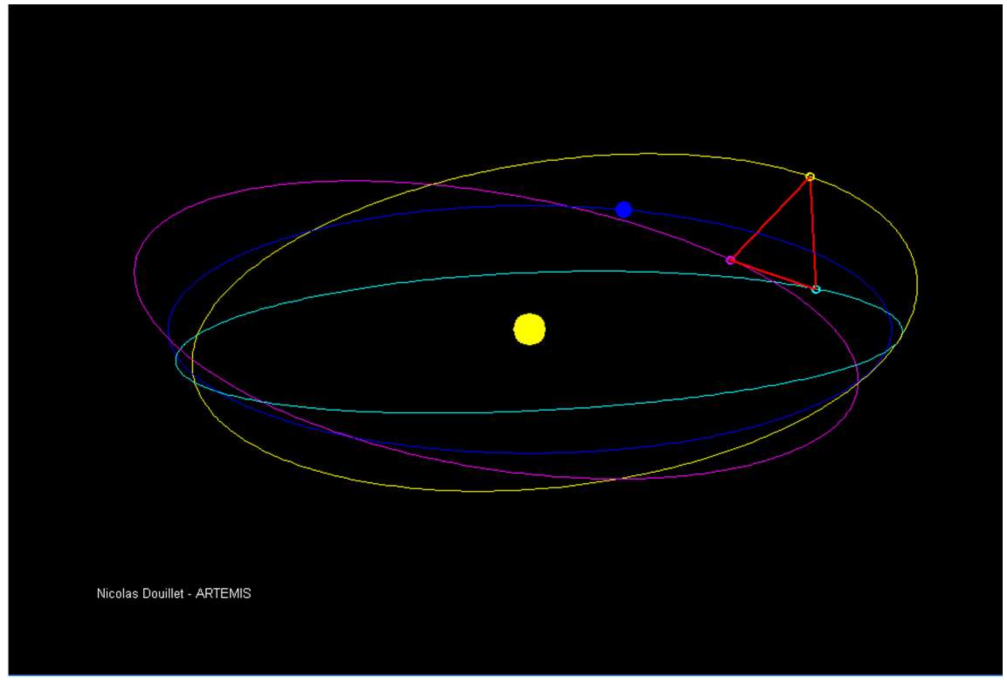




34-6 Michelson Interferometer & Graviational waves

- LISA (Laser Interferometer Space Antenna)



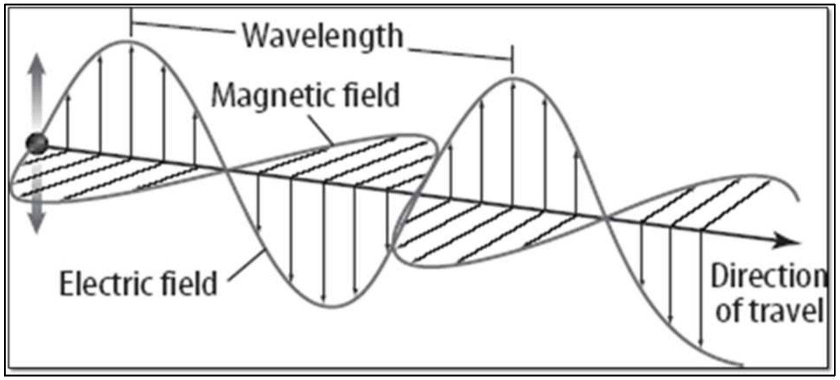


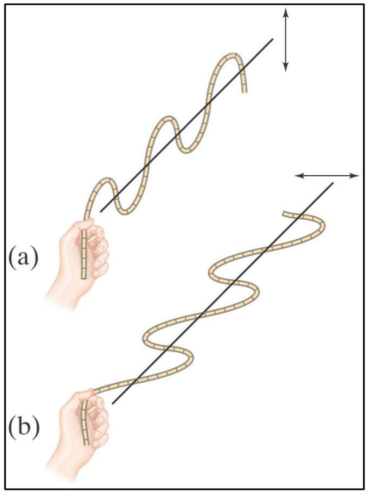
[ESA]

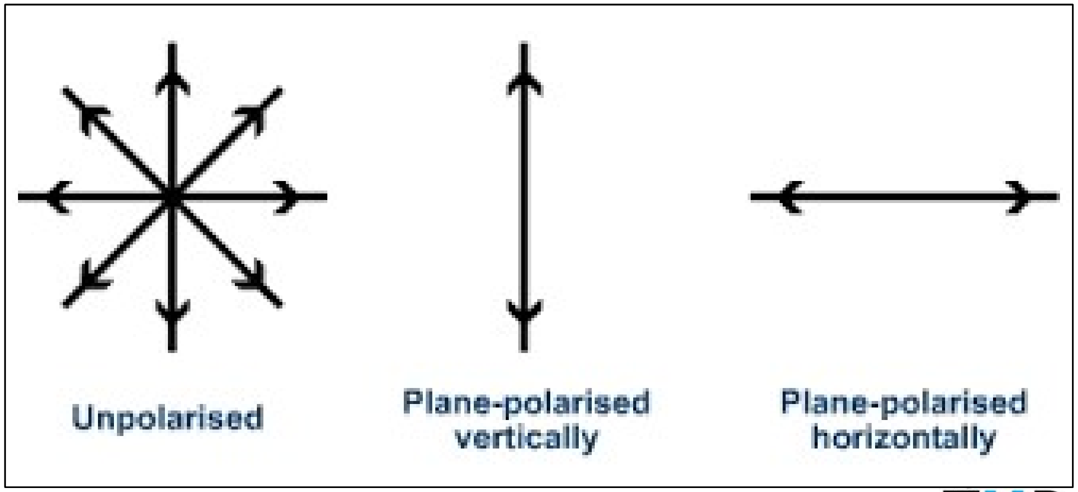
34-7 Polarization of Light

- Electric fields oscillate in a single plane, rather than in any direction perpendicular to the direction of propagation.

- When all the waves of a source oscillate in the same plane, light is said polarized

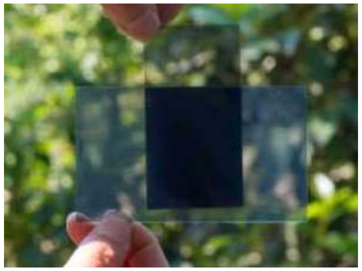


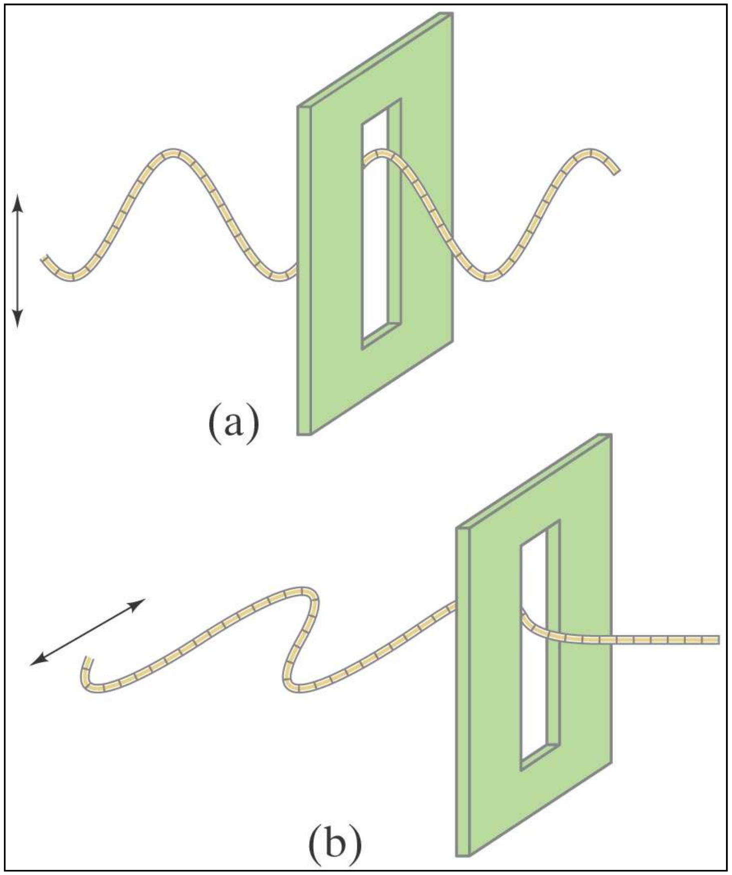




34-7 Polarization of Light

- Polarized light will not be transmitted through a polarized film whose axis is perpendicular to the polarization direction.



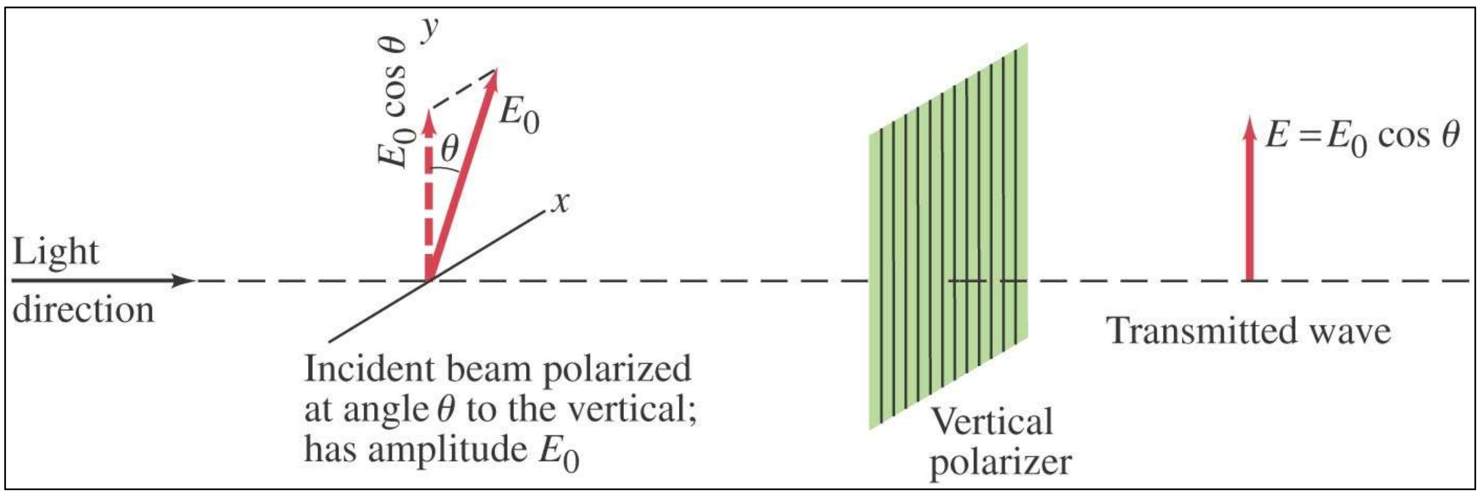


34-7 Polarization of Light

- When light passes through a polarizer, only the component parallel to the polarization axis is transmitted. If the incoming light is plane-polarized, the outgoing intensity is:

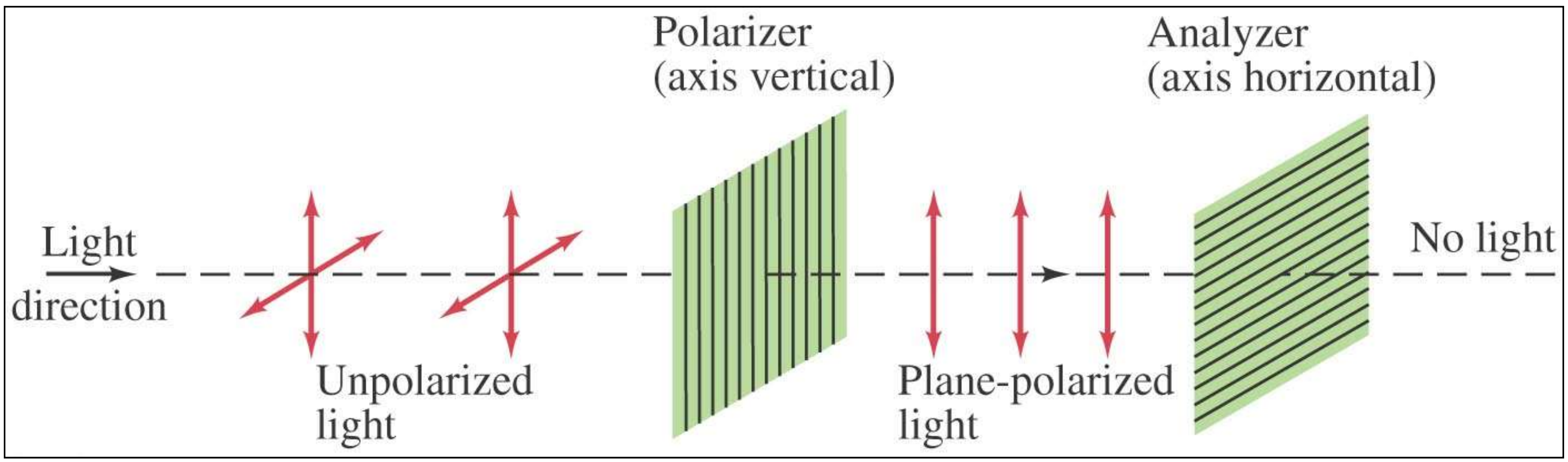
Malus law

$$\boxed{I = I_0 \cos^2 \theta,} \qquad \left[\begin{array}{c}\text{intensity of plane polarized} \\ \text{wave reduced by polarizer}\end{array}\right]$$



34-7 Polarization of Light

- This means that if initially unpolarized light passes through crossed polarizers, no light will get through the second one.

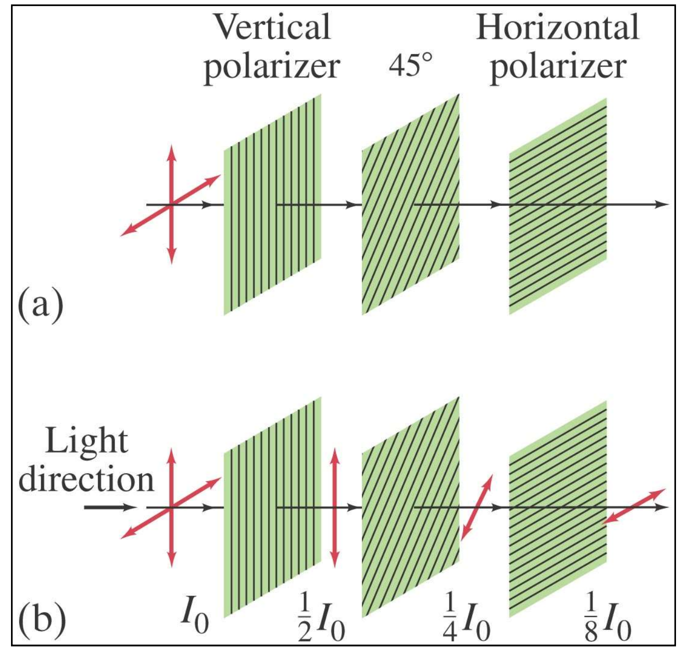


34-7 Polarization of Light

- Example 34-9: Two Polaroids at 60°.
  - Unpolarized light passes through two Polaroids; the axis of one is vertical and that of the other is at 60° to the vertical.
  - Describe the orientation and intensity of the transmitted light.

34-7 Polarization of Light

- Conceptual Example 34-10: Three Polaroids.
  - When unpolarized light falls on two crossed Polaroids (axes at 90°) no light passes through.
  - What happens if a third Polaroid, with axis at 45° to each of the other two, is placed between them?



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