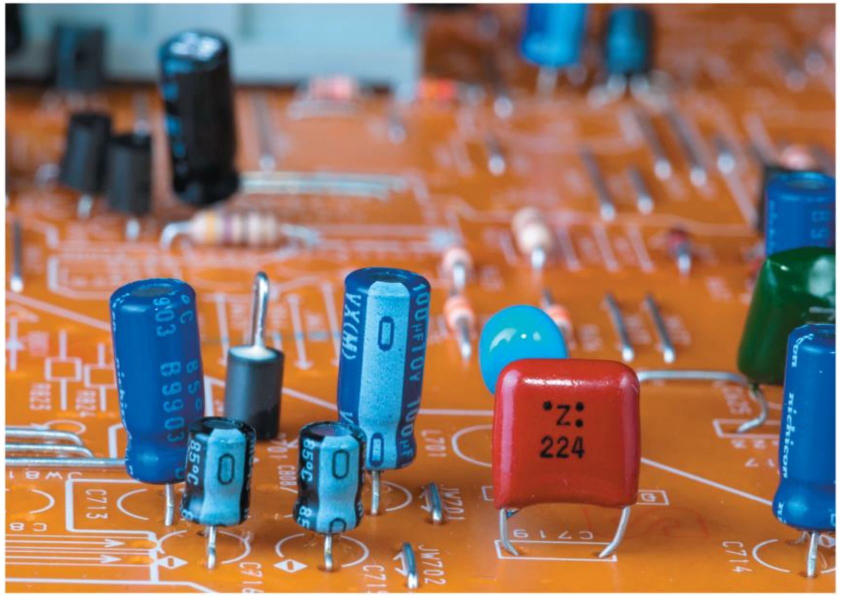
CAPACITORS AND ENERGY STORAGE

Chapter 24



Alessandro Bombelli

Operations & Environment

Faculty of Aerospace Engineering

TU Delft

Structure of the lecture

1. Capacitors
2. Determining Capacitance
3. Capacitors in Series and Parallel
4. Storage of Electric Energy
5. Dielectrics

Learning objectives for today's lecture

After this lecture you should be able to:

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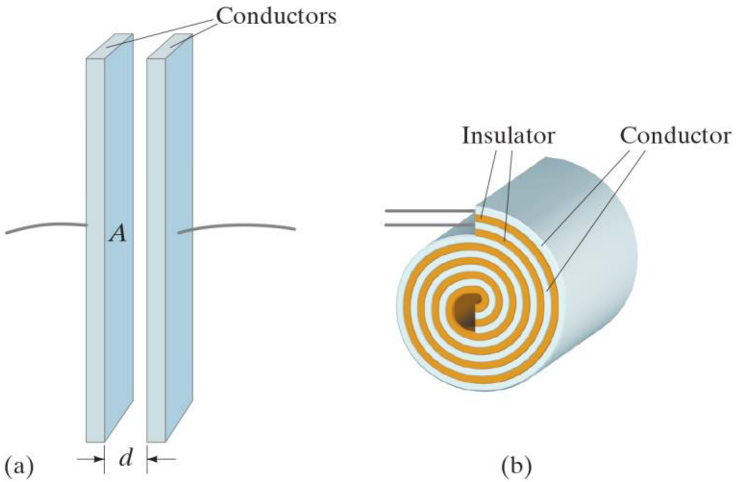
- Determine the **maximum energy** that a capacitor can store

24.1 – Capacitors



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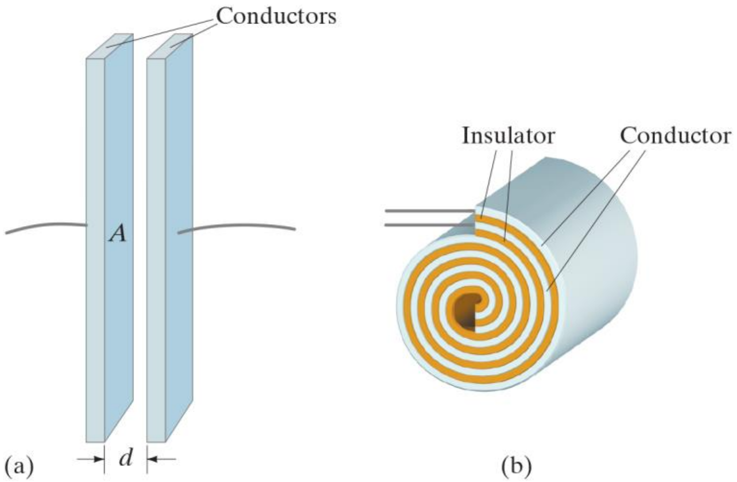

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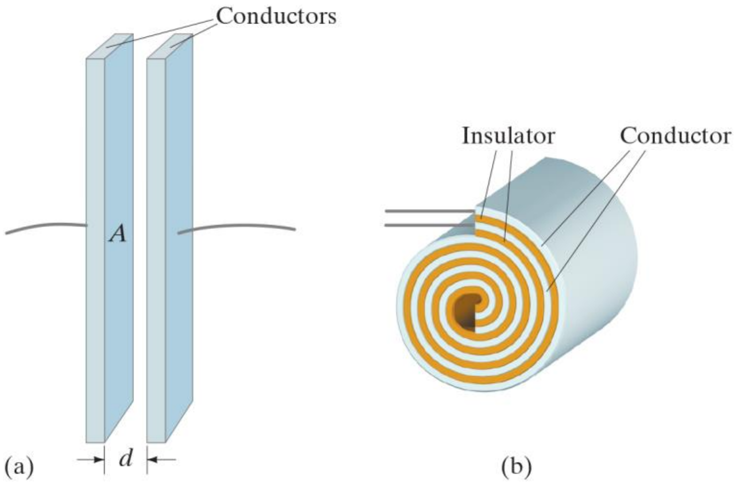

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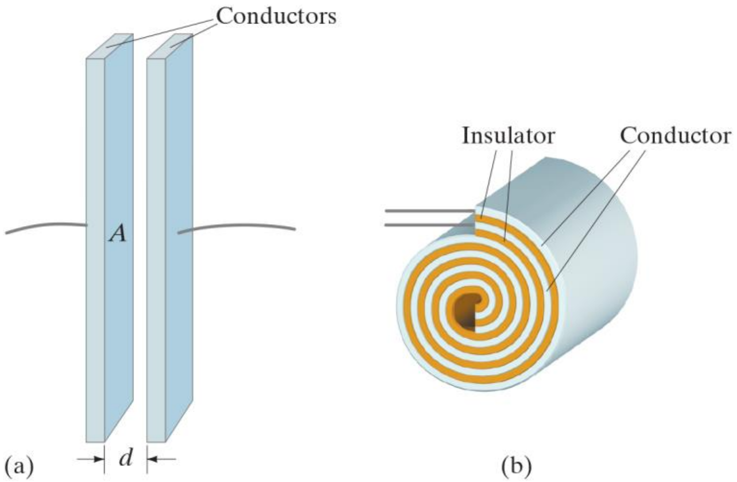

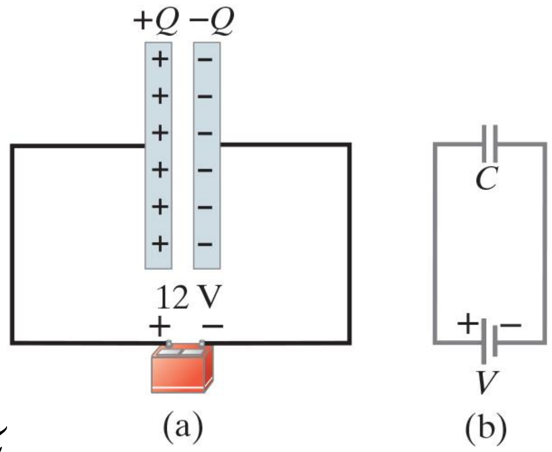

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They have a wide variety of applications, such as energy backups, power surge protection, etc.

The two plates can be flat or folded to increase the available surface per unit volume.

If a voltage is applied to the two plates, one plate is charged positively and the other negatively: $\boldsymbol{Q = CV}$ where C is the capacitance of the capacitor (expressed in Farad F)

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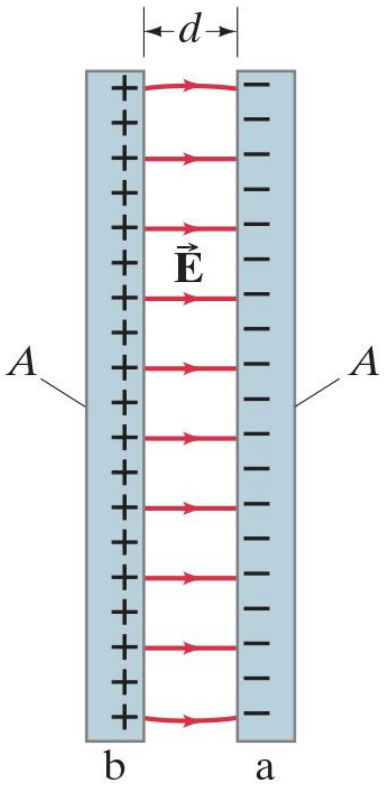
Capacitance can be determined experimentally by measuring the charge Q that forms on the plates of a capacitor for a given applied voltage V. For simple geometries, capacitance can be determined analytically.



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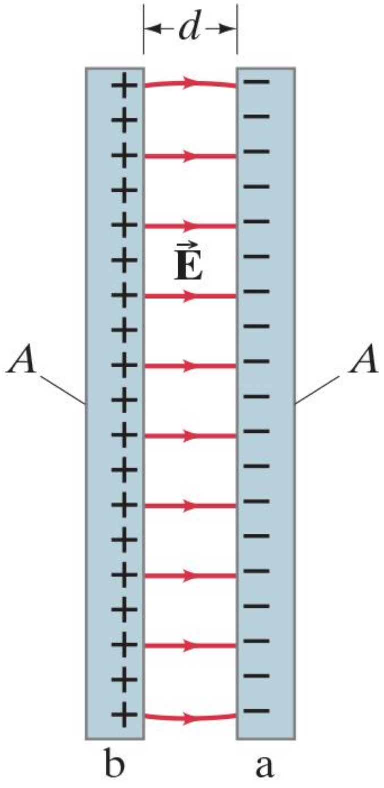

Let us take the example of a capacitor with parallel plates of area A. We determined in previous lectures that the electric field across the plates is

$$|\vec{E}| = \frac{Q}{\epsilon_0 A}$$

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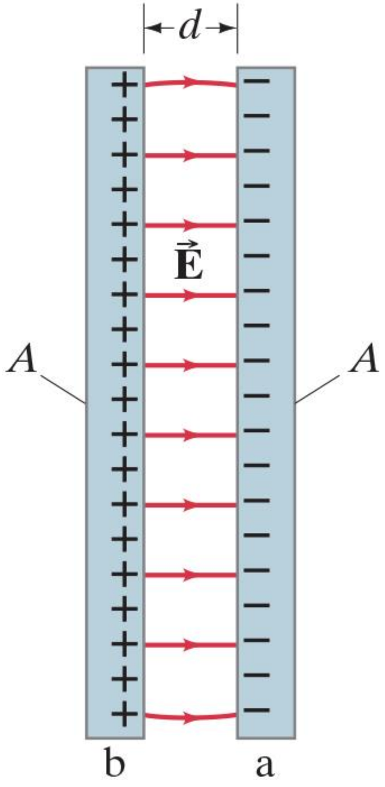

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and is perpendicular to the plates (aside from some curvature at the two edges). To compute the capacitance, we can write (note that b is the left plate and a the right one)

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = -\int_a^b \frac{Q}{\epsilon_0 A} dl \cos \pi = \frac{Qd}{\epsilon_0 A}$$

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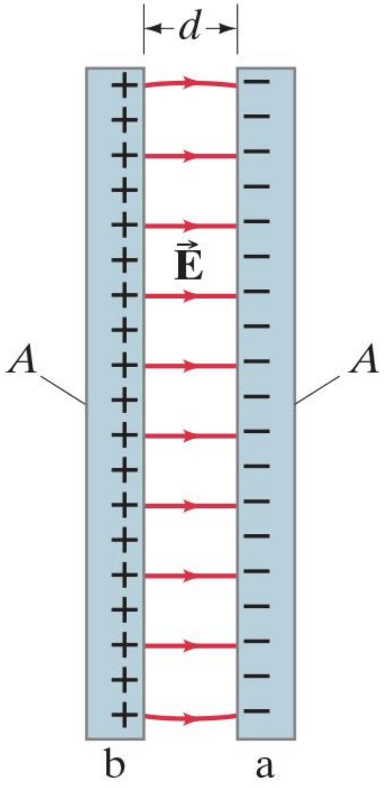

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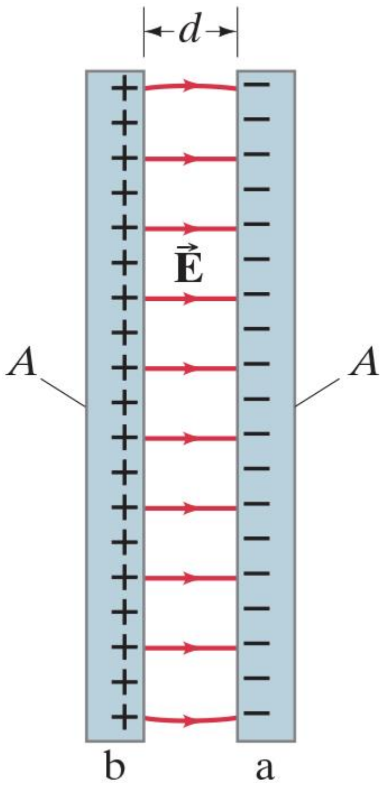

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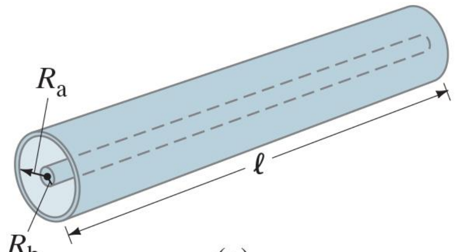
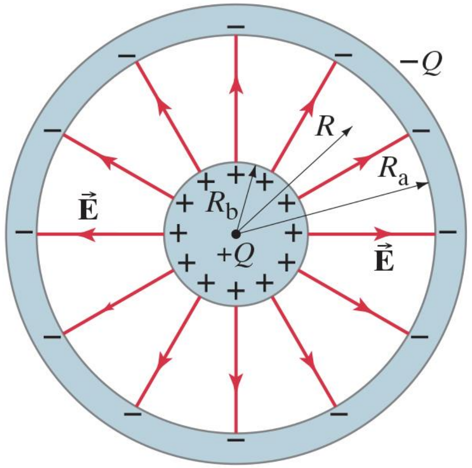
In today's operations, capacitances in the range of $1 - 2\ F$ are possible, and mostly used for power backups. The advantage over batteries is that they endure more cycles with less degradation

24.2 – Capacitance of a cylindrical capacitor



(a)



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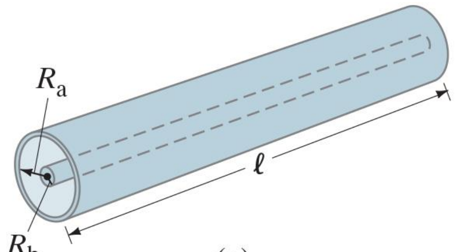

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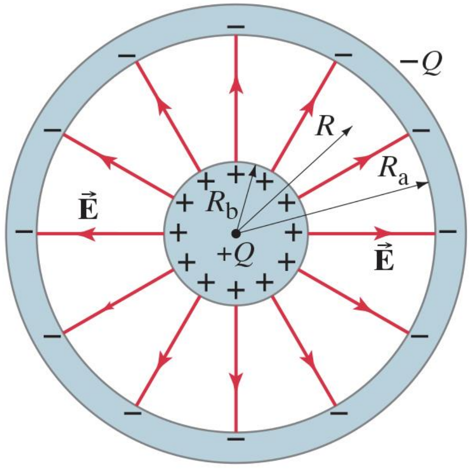

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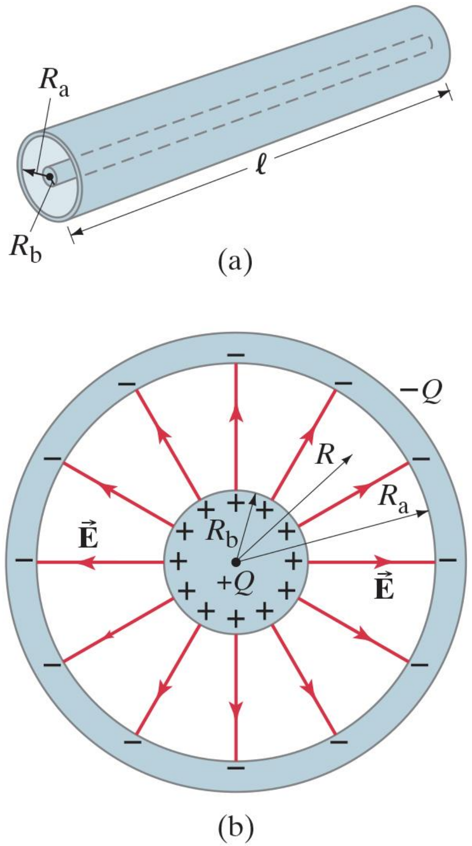

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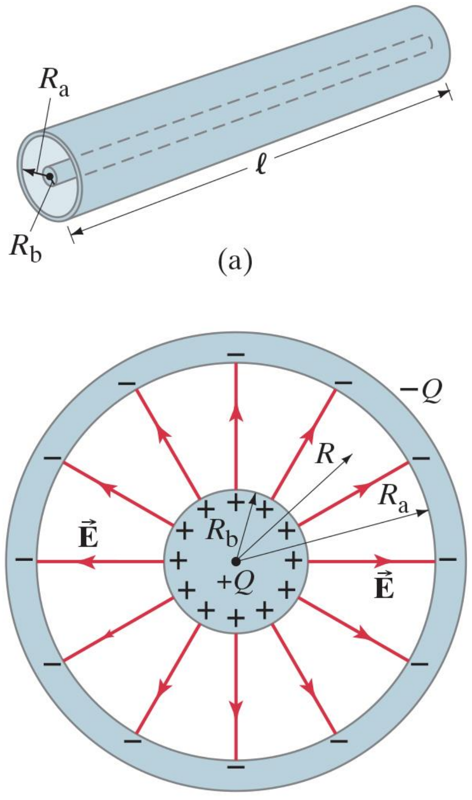

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If $R_a - R_b = \Delta R$ is very small, we have $C = \frac{2\pi\epsilon_0 l R_b}{\Delta R} = \frac{\epsilon_0 A}{\Delta R}$ as for the parallel plates

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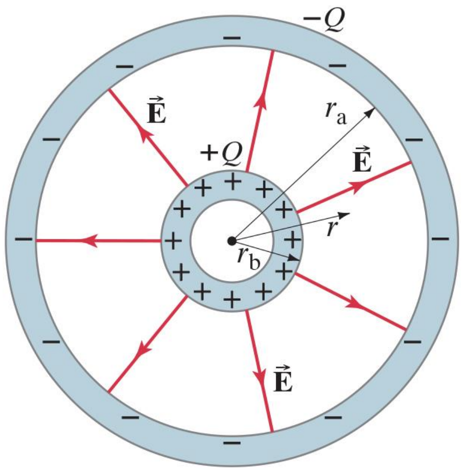

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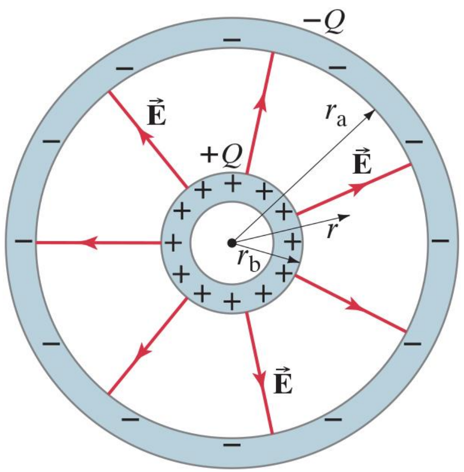

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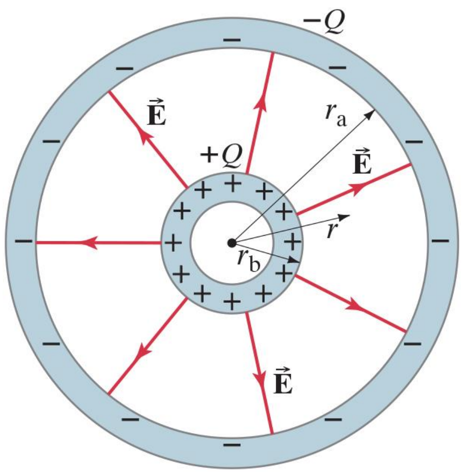

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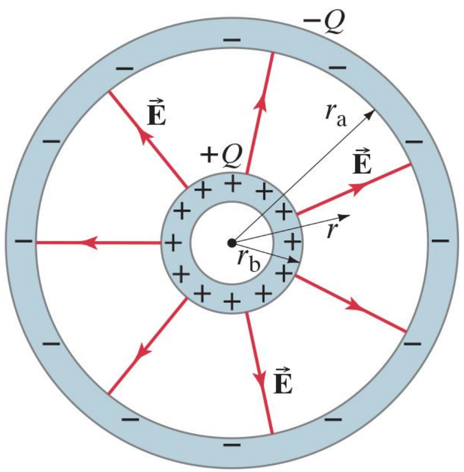

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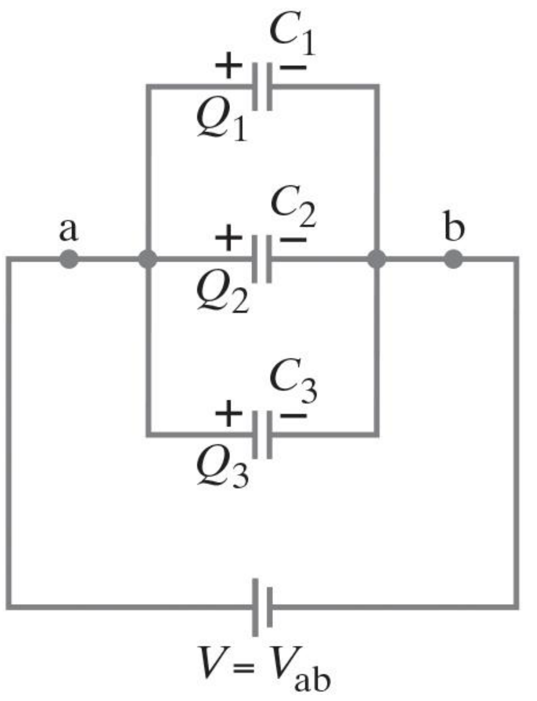
On top of being able to determine the capacitance of a single capacitor of (almost) any given shape, it is also important to understand how capacitors is series and parallel work and how an equivalent capacitance can be computed.



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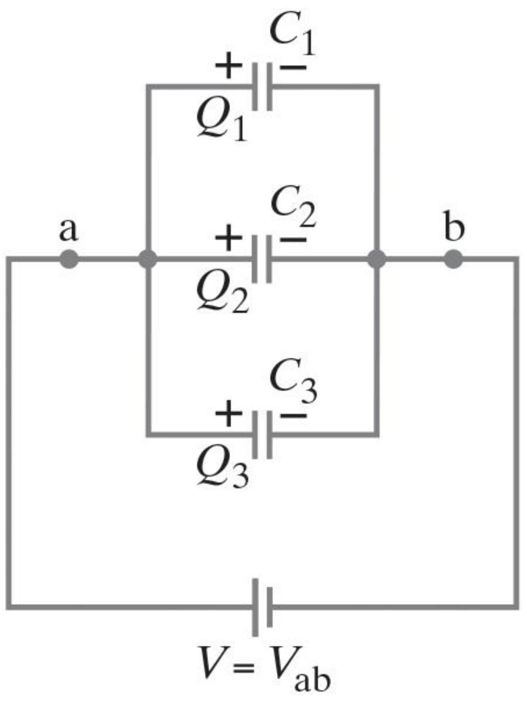

When placing capacitors in parallel (see Figure on the left), the potential difference across all 3 sets of plates is the same. This is because all left plates gave a potential V_a and all right plates a potential V_b, so that $V = V_a - V_b$.

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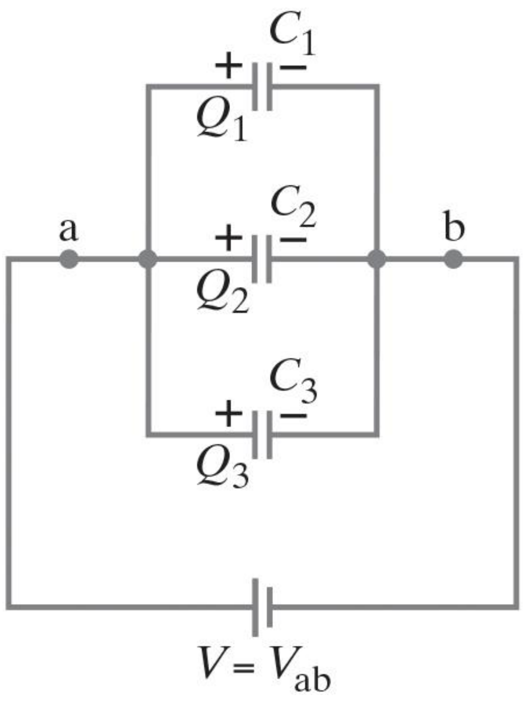

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$$\boldsymbol{Q_1 = C_1 V, Q_2 = C_2 V, Q_3 = C_3 V}$$

where $Q = Q_1 + Q_2 + Q_3$ is the total charge that has left the battery providing the potential difference.

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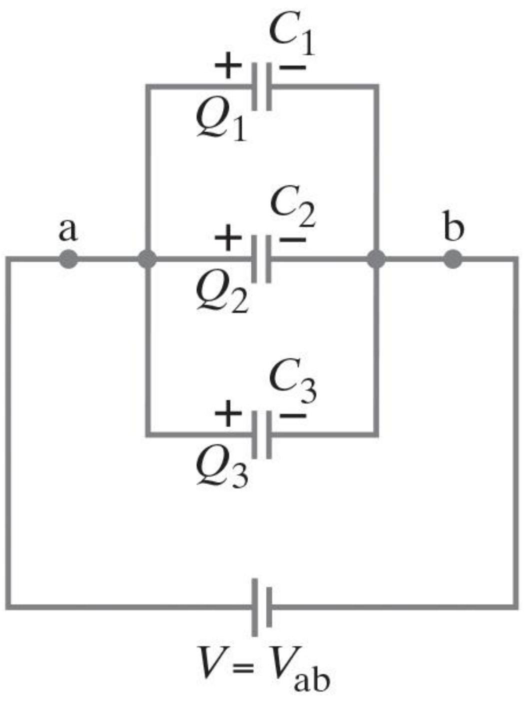

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$$\boldsymbol{Q = (C_1 + C_2 + C_3) V}$$

defines the equivalent system with $\boldsymbol{C_{eq} = C_1 + C_2 + C_3}$

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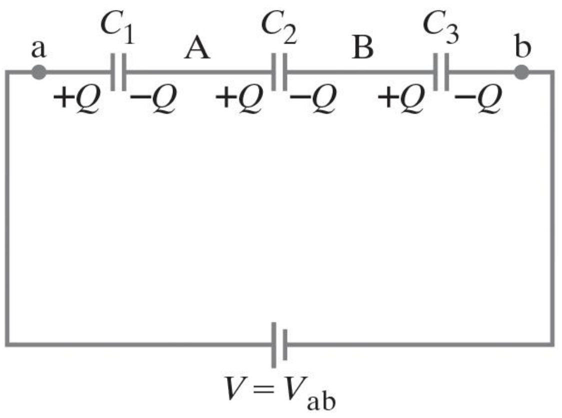

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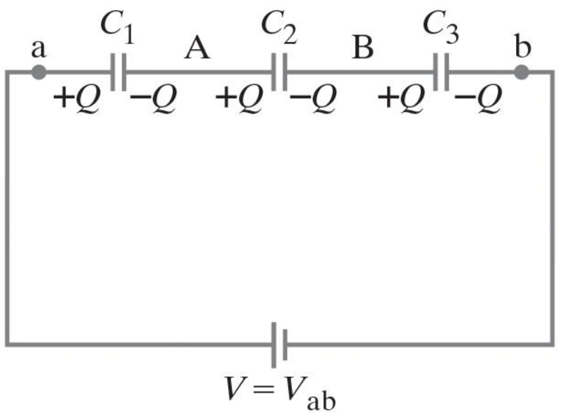

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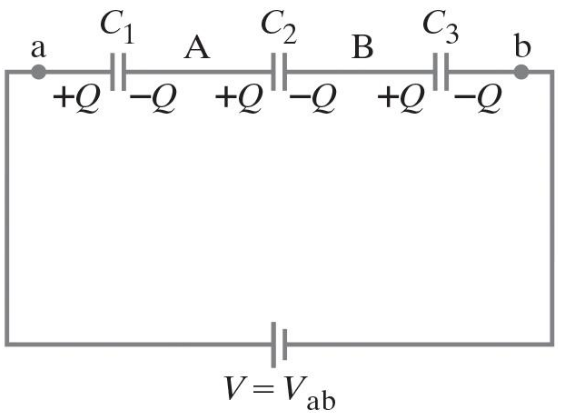

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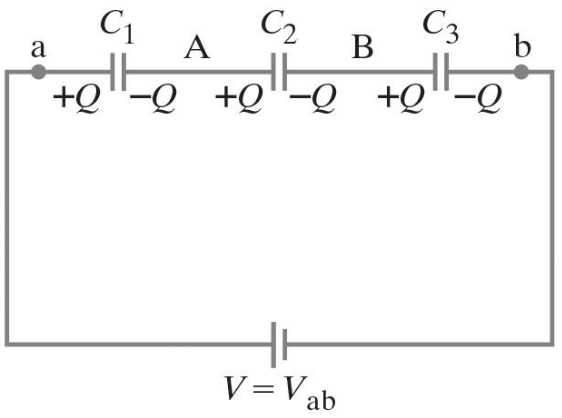

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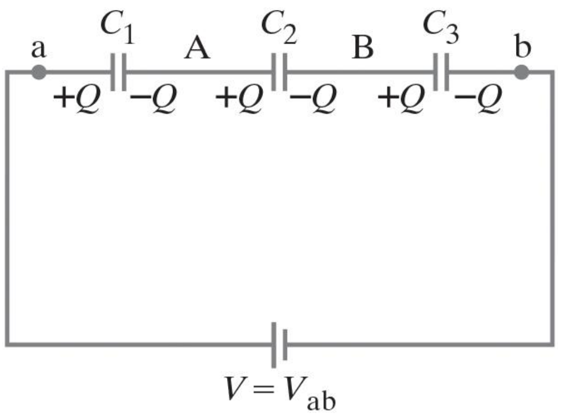

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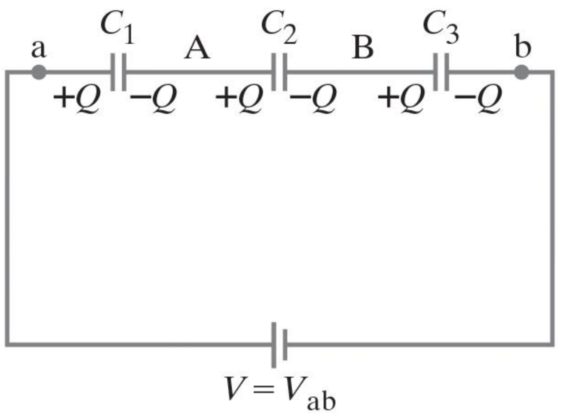

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We can determine the overall work carried out by the battery, and hence the total energy stored in the capacitor once fully charged (electrostatic case) as

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Equivalent expressions are $U = \frac{1}{2}CV^2$ or $U = \frac{1}{2}QV$

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- they keep the two plates separate even if they are extremely close, hence increasing the capacitance as smaller values of d can be achieved

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- higher charges can be applied across the plates without allowing movement of charges
- they keep the two plates separate even if they are extremely close, hence increasing the capacitance as smaller values of d can be achieved
- more in general, they can naturally increase the capacitance (according to the specific material) by a factor K, known as the dielectric constant: $C = KC_0$ with C_0 being the capacitance with air (or vacuum) between plates

24.5 – Dielectrics

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For example, porcelain can increase capacitance by 6-8 times, while water by roughly 80 times.

24.5 – Dielectrics

For a parallel plate capacitor we can hence write

$$C = K\epsilon_0 \frac{A}{d}$$

24.5 – Dielectrics

For a parallel plate capacitor we can hence write

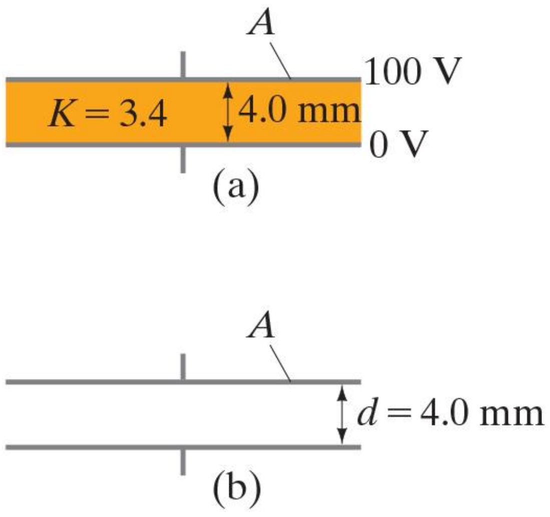
$$\boldsymbol{C = K\epsilon_0 \frac{A}{d}}$$

or, equivalently, that

$$\boldsymbol{\epsilon = K\epsilon_0}$$

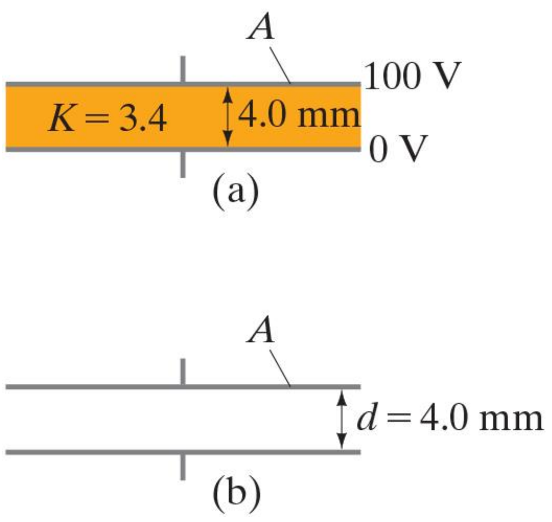
is the permittivity of the dielectric under scrutiny.

24.5 – Example: removal of dielectrics



In the example to the left, the area of the parallel plate capacitor is $A = 4m^2$. We are asked to find the capacitance, charge, electric field magnitude, and stored energy in the capacitor. Then, the dielectric is removed and we are asked to find the new values of capacitance, electric field magnitude, voltage across plates, and energy stored.

24.5 – Example: removal of dielectrics



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With dielectric:

$$C = K\epsilon_0 \frac{A}{d} = 30\, nF$$

$$Q = CV = 3.0 \times 10^{-6} C$$

$$E = \frac{V}{d} = 25 \frac{kV}{m}$$

$$U = \frac{1}{2} CV^2 = 1.5 \times 10^{-4} J$$

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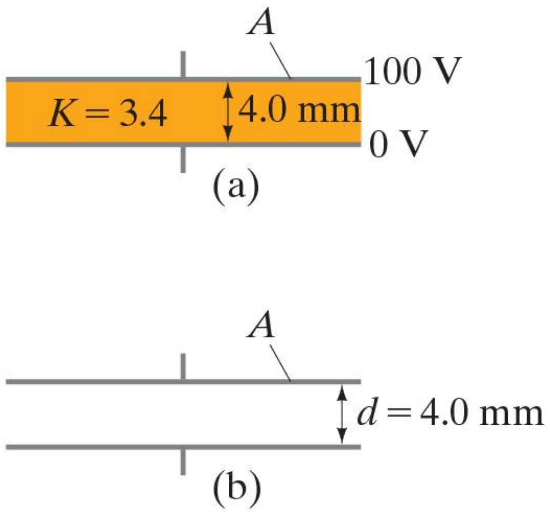
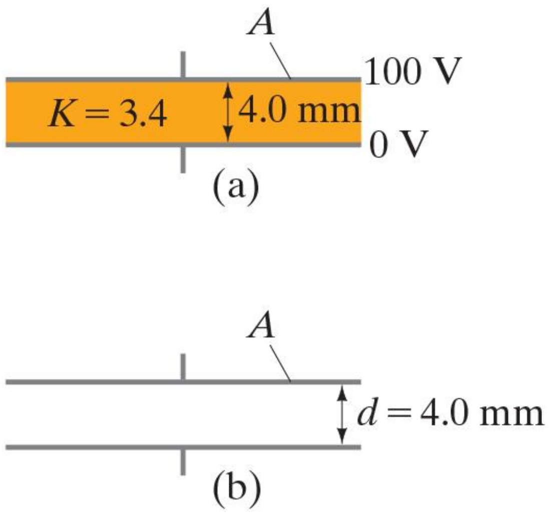
24.5 – Example: removal of dielectrics



Without the dielectric, it is important to realize that the charge stays unchanged as it has nowhere else to go/move.

24.5 – Example: removal of dielectrics



Without the dielectric, it is important to realize that the charge stays unchanged as it has nowhere else to go/move. The other values will change accordingly

$$C = \epsilon_0 \frac{A}{d} = 8.8\, nF$$

$$V = \frac{Q}{C} = 340V$$

$$E = \frac{V}{d} = 85 \frac{kV}{m}$$

$$U = \frac{1}{2} CV^2 = 5.1 \times 10^{-4} J$$

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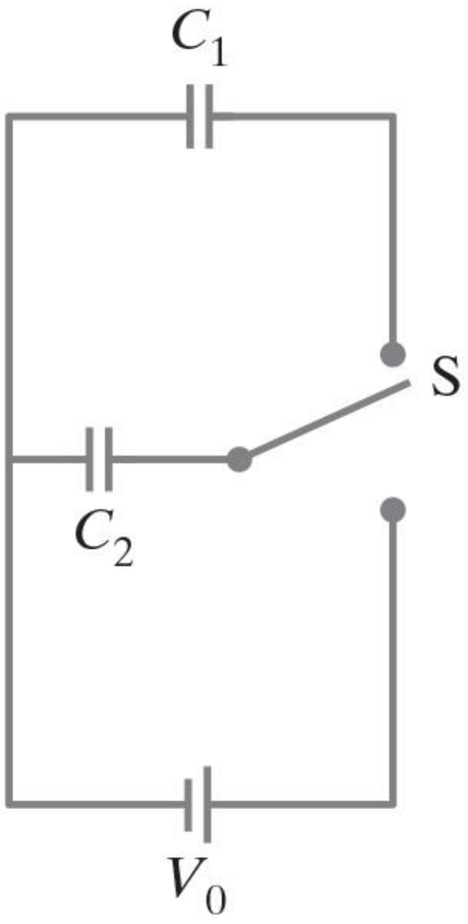
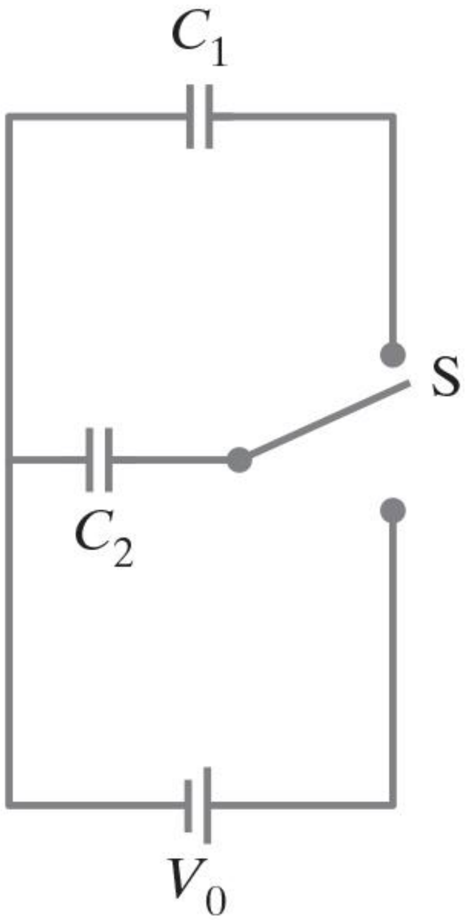
Exercise: circuit with a switch



The switch in the circuit shown on the left is initially placed downward, so capacitor C_2 is charged due to the effect of the battery. Then, the switch is moved upward. What is the final charge on each capacitor?

Exercise: circuit with a switch



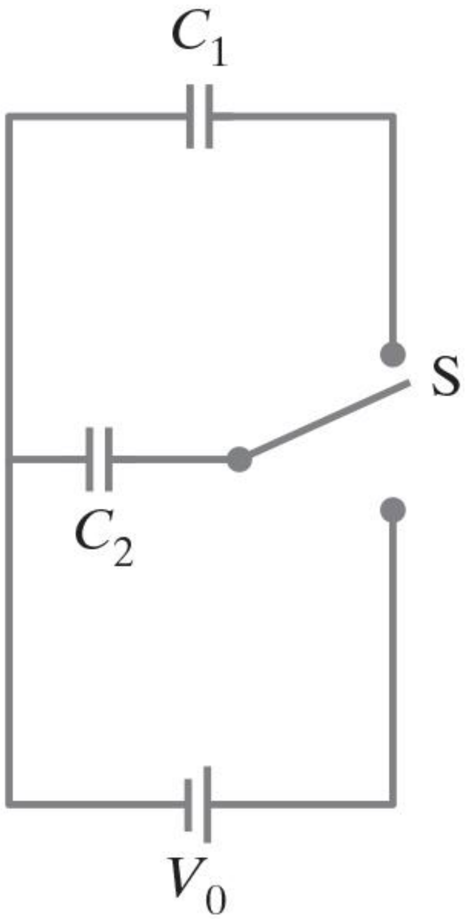
The switch in the circuit shown on the left is initially placed downward, so capacitor C_2 is charged due to the effect of the battery. Then, the switch is moved upward. What is the final charge on each capacitor?

When the switch is down, after a sufficiently long time we have

$$\boldsymbol{Q_2 = C_2 V_0}$$

When the switch is moved up, some charge will move from the second to the first capacitor until they reach the same potential.

Exercise: circuit with a switch



Hence

$$V = \frac{Q_2'}{C_2} = \frac{Q_1'}{C_1}$$

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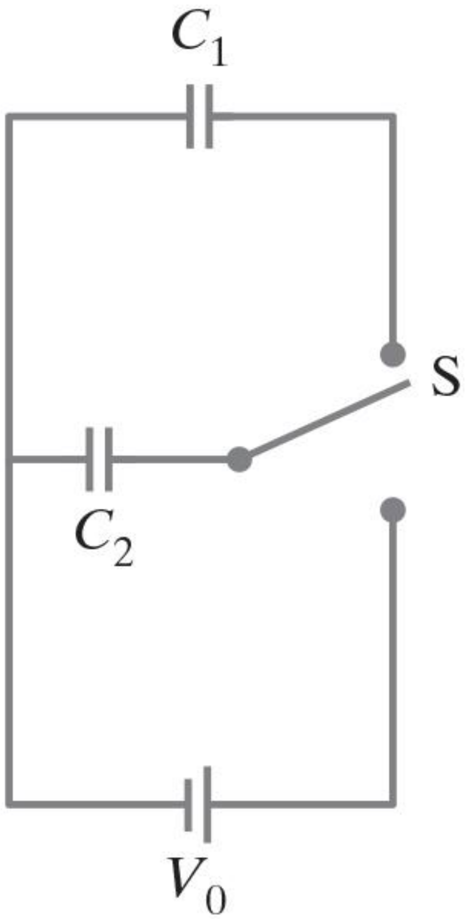

Hence

$$V = \frac{Q_2'}{C_2} = \frac{Q_1'}{C_1}$$

In addition, we have that

$$Q_2 = Q_1' + Q_2'$$

as charge can only redistribute:

$$Q_2 = C_2 V_0 = Q_1' + \frac{C_2}{C_1} Q_1' = \left(\frac{C_1 + C_2}{C_1}\right) Q_1'$$

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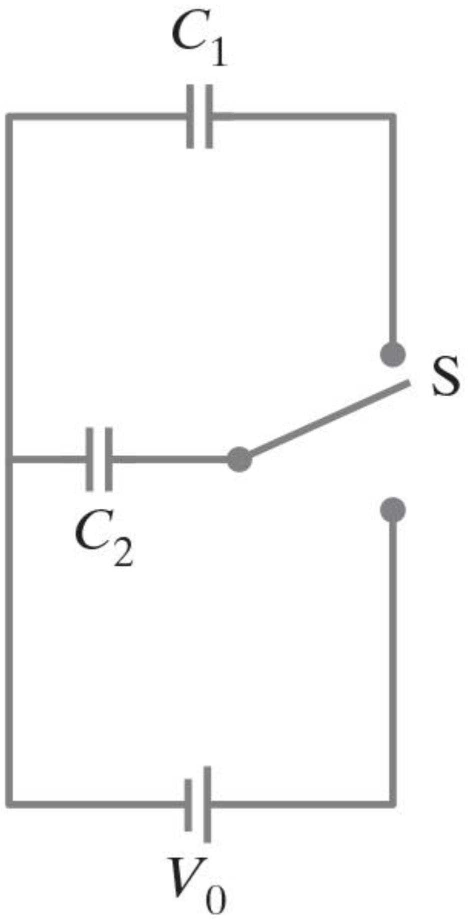

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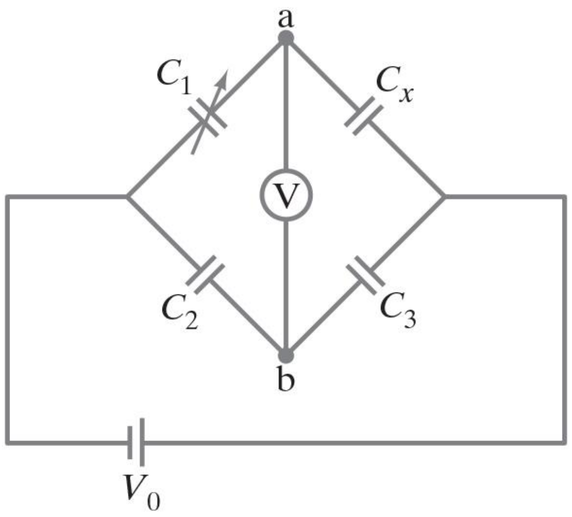
We can now express the two final charge values

$$Q_1' = \left(\frac{C_1 C_2}{C_1 + C_2}\right) V_0$$

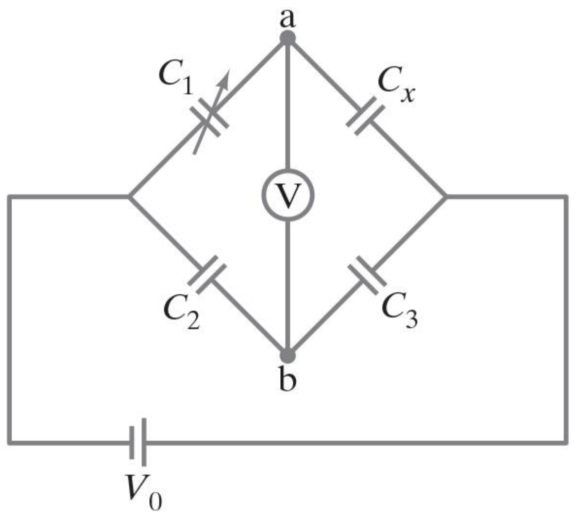
$$Q_2' = \left(\frac{C_2^2}{C_1 + C_2}\right) V_0$$

Exercise: capacitance bridge



In this capacitance bridge, capacitance C_1 is adjusted so that there is no difference in voltage between points a and b. We need to compute C_x knowing that $C_1 = 8.9\ \mu F$, $C_2 = 16.90\ \mu F$, and $C_3 = 4.8\ \mu F$. No charge flows through the voltmeter when it reads a zero potential difference.

Exercise: capacitance bridge



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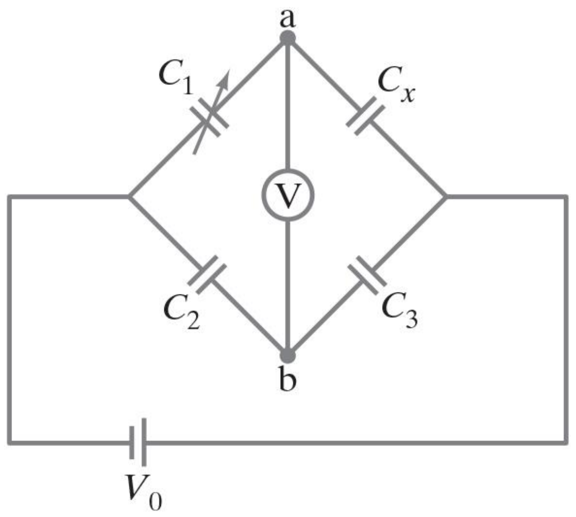
Because points a and b have the same potential, then capacitors C_1 and C_2 are in parallel, and so are C_x and C_3. Hence

$$\frac{\boldsymbol{Q_1}}{\boldsymbol{C_1}} = \frac{\boldsymbol{Q_2}}{\boldsymbol{C_2}}, \frac{\boldsymbol{Q_x}}{\boldsymbol{C_x}} = \frac{\boldsymbol{Q_3}}{\boldsymbol{C_3}}$$

Exercise: capacitance bridge



Because no charge flows across the voltmeter, then C_1 and C_x are in series and so are C_2 and C_3. Hence

$$\boldsymbol{Q_1 = Q_x, Q_2 = Q_3}$$

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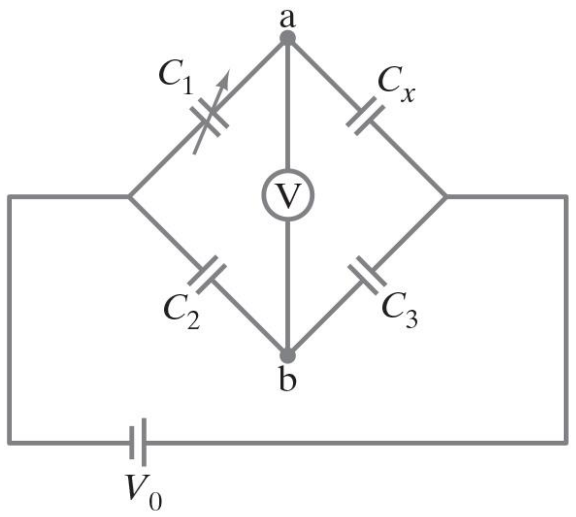
Exercise: capacitance bridge



Because no charge flows across the voltmeter, then C_1 and C_x are in series and so are C_2 and C_3. Hence

$$\boldsymbol{Q_1 = Q_x, Q_2 = Q_3}$$

$$\boldsymbol{C_x = \frac{Q_x C_3}{Q_3} = \frac{Q_1 C_3}{Q_2} = \frac{C_1 C_3}{C_2} = 2.4\ \mu F}$$

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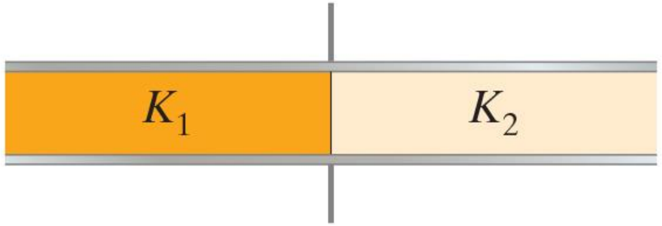
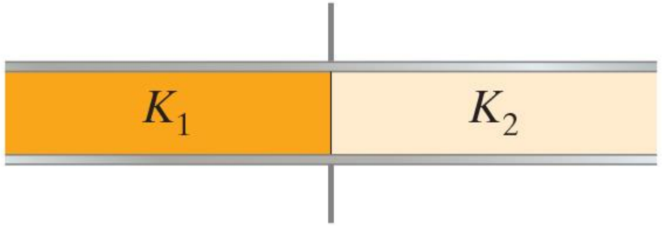
Exercise: different dielectrics



Two different dielectrics of capacitances K_1 and K_2 are placed each in half of the space between the two plates of a flat capacitor of area A and distance between plates d. We are asked to determine the equivalent capacitance of the system.

Exercise: different dielectrics

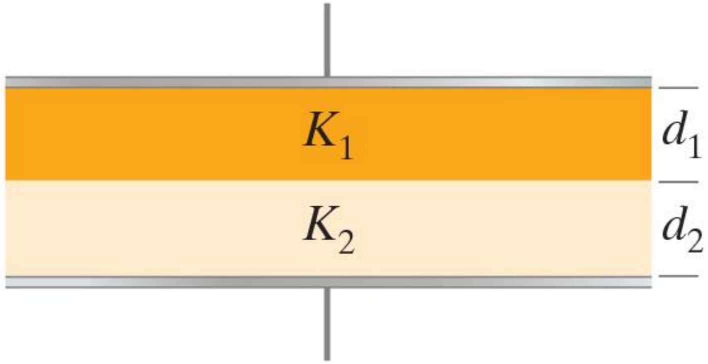


Two different dielectrics of capacitances K_1 and K_2 are placed each in half of the space between the two plates of a flat capacitor of area A and distance between plates d. We are asked to determine the equivalent capacitance of the system.

The potential is the same on each half of the capacitor, hence the two dielectrics “see” the same V and are hence in parallel:

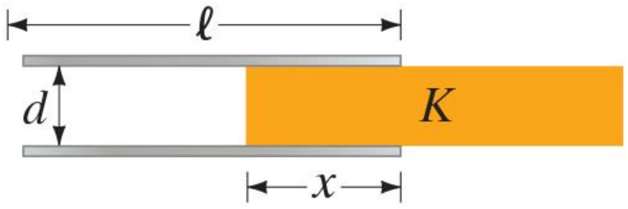
$$\boldsymbol{C = C_1 + C_2 = K_1 \epsilon_0 \frac{\frac{A}{2}}{d} + K_2 \epsilon_0 \frac{\frac{A}{2}}{d} = \frac{1}{2}(K_1 + K_2)\epsilon_0 \frac{A}{d}}$$

Exercise: different dielectrics



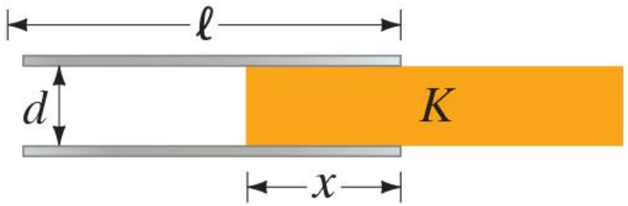
What if the dielectrics were positioned now like this?

Exercise: moving dielectric



Given the example to the left, we are asked to determine as a function of x (moving to the left), where the slab of dielectric is progressively inserted into the parallel plates (squares with side l), the capacitance, energy stored (given a potential V_0), and force acting on the slab

Exercise: moving dielectric



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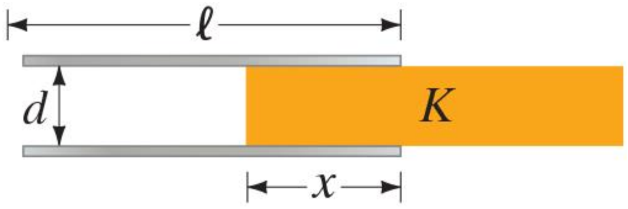
Capacitance: we consider 2 capacitors in parallel

$$\boldsymbol{C = \epsilon_0 \frac{l(l-x)}{d} + K\epsilon_0 \frac{lx}{d} = \epsilon_0 \frac{l^2}{d}\left[1 + (K-1)\frac{x}{l}\right]}$$

Exercise: moving dielectric



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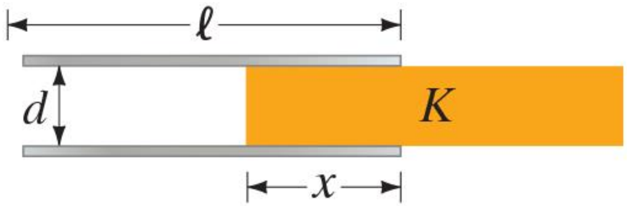
Energy: both capacitors “see” the same potential

$$\boldsymbol{U = \frac{1}{2}(C_1 + C_2)V_0^2 = \frac{1}{2}\epsilon_0 \frac{l^2}{d}\left[1 + (K-1)\frac{x}{l}\right]V_0^2}$$

Exercise: moving dielectric



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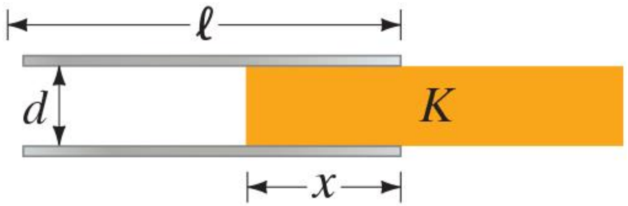
Force on the slab: as the dielectric is inserted, the battery providing the potential will lose energy and the capacitor will acquire energy.

Exercise: moving dielectric



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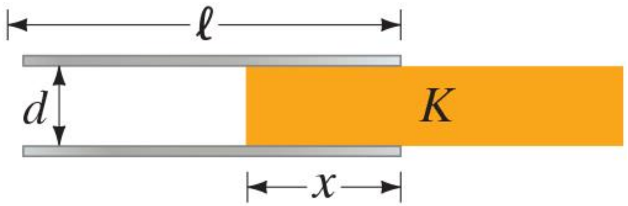
Force on the slab: as the dielectric is inserted, the battery providing the potential will lose energy and the capacitor will acquire energy. The work done to insert the dielectric is a function of both the stored energy in the capacitor and lost energy of the battery.

$$\boldsymbol{F dx = d\left(\frac{1}{2} C V_0^2\right) + d(Q_{battery} V_0)}$$

Exercise: moving dielectric



$$F = \frac{d}{dx}\left(\frac{1}{2}CV_0^2\right) + \frac{d}{dx}\left(Q_{battery}V_0\right)$$

$$F = \frac{1}{2}V_0^2\frac{dC}{dx} + V_0\frac{dQ_{battery}}{dx}$$

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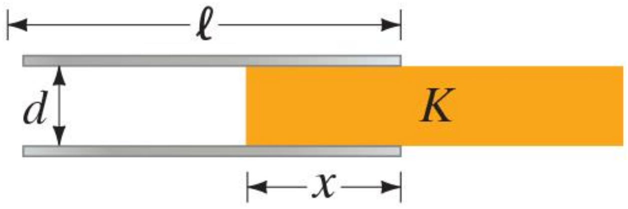

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because the lost charge by the battery is gained by the capacitor, we can write (recall $Q = VC$)

$$F = \frac{1}{2}V_0^2\frac{dC}{dx} - V_0^2\frac{dC}{dx}$$

$$F = -\frac{V_0^2\epsilon_0 l}{2d}(K-1)$$

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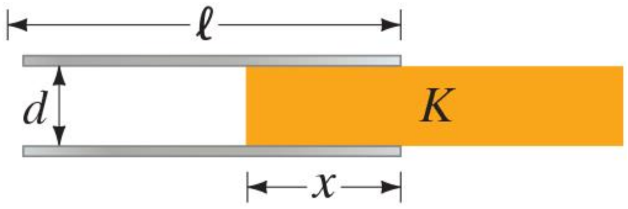

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This force is to the right (opposite to the movement of the slab) and prevents the slab from accelerating. The same force pulls the dielectric to the left and is due to the attraction of the charged plates and the induced charge on the dielectric

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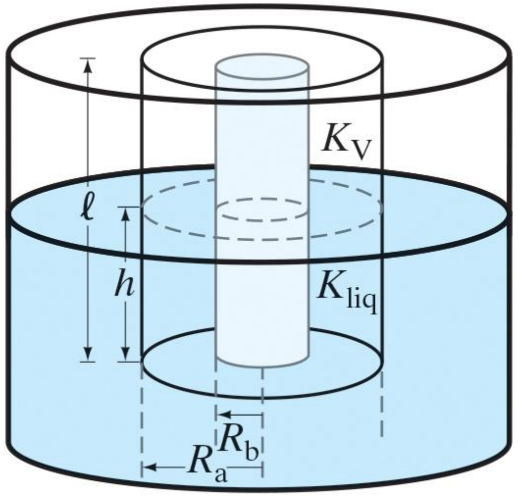
Exercise: capacitor as a tank level sensor



A cylindrical capacitor (radii R_b and R_a) can be used in a tank to measure the remaining level of fuel. When the tank is not full ($h < l$) the capacitor “”sees” two different dielectrics, the actual liquid fuel at the bottom with capacitance K_{liq} and the vapor above with capacitance K_V. Our goal is to determine a formula that highlights the percentage of height still filled with fuel, i.e., $\frac{h}{l}$.

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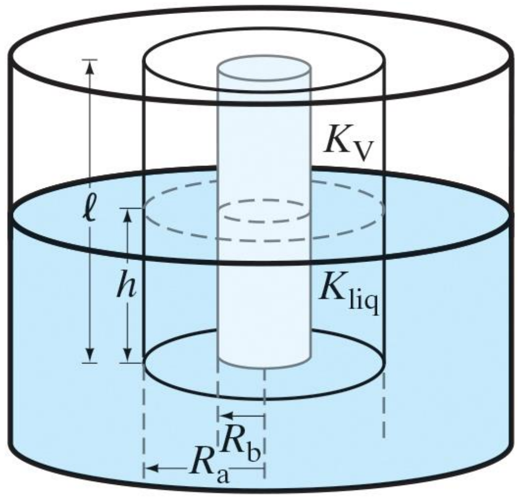

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The two capacitors are in parallel, hence

$$C = C_{liq} + C_V = \frac{2\pi\epsilon_0 K_{liq} h}{\ln\frac{R_a}{R_b}} + \frac{2\pi\epsilon_0 K_V (l-h)}{ln\frac{R_a}{R_b}} \rightarrow$$

$$\frac{h}{l} = \frac{1}{K_{liq} - K_V}\left[\frac{C\ ln\frac{R_a}{R_b}}{2\pi\epsilon_0 l} - K_V\right]$$

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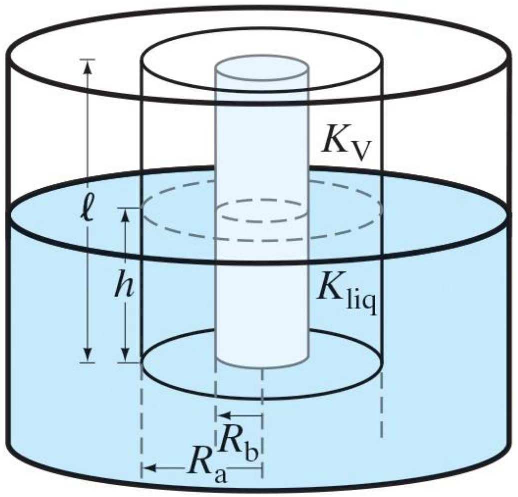

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where C might be measured by an on-board system

Wrap-up: revisiting Learning objectives

After this lecture you should be able to:

- Understand the general **characteristics** of a capacitor
- Determine the **properties of a capacitor** of (almost) any given shape
- Determine the **equivalent capacitor** given a set of capacitors in series or parallel
- Determine the **maximum energy** that a capacitor can store

Wrap-up: revisiting Learning objectives

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- Determine the **maximum energy** that a capacitor can store

$$U = \frac{1}{2}\frac{Q^2}{C}$$