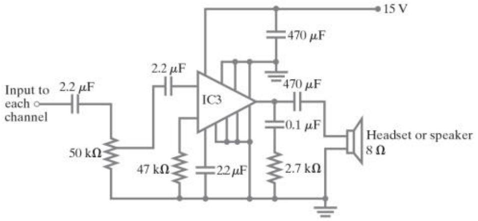
DC CIRCUITS

Chapter 26



Alessandro Bombelli

Operations & Environment

Faculty of Aerospace Engineering

Structure of the lecture

1. ElectroMotive Force (EMF) and Terminal Voltage
2. Resistors in Series and Parallel
3. Kirchhoff’s Rules
4. EMFs in Series and Parallel: Charging a Battery
5. RC Circuits
6. Ammeters and Voltmeters

Learning objectives for today's lecture

After this lecture you should be able to:

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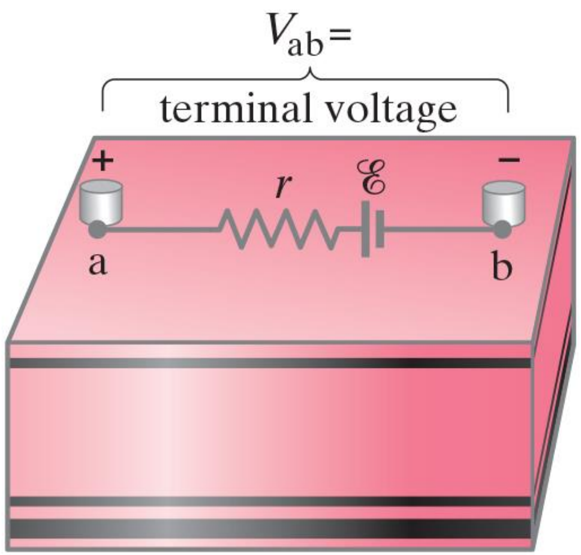
- Understand the **transient dynamics** of RC circuits
- Understand the **functioning** of ammeters and voltmeters

26.1 – EMF and Terminal Voltage



A battery transforms chemical energy into electric one. Such a device is called a source of ElectroMotive Force (EMF). The symbol ε is generally used to define an EMF, and this is equal to the potential difference across the terminals when no current flows.

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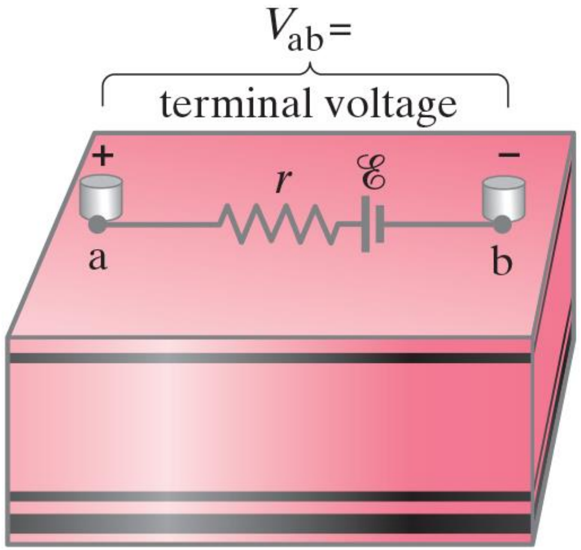

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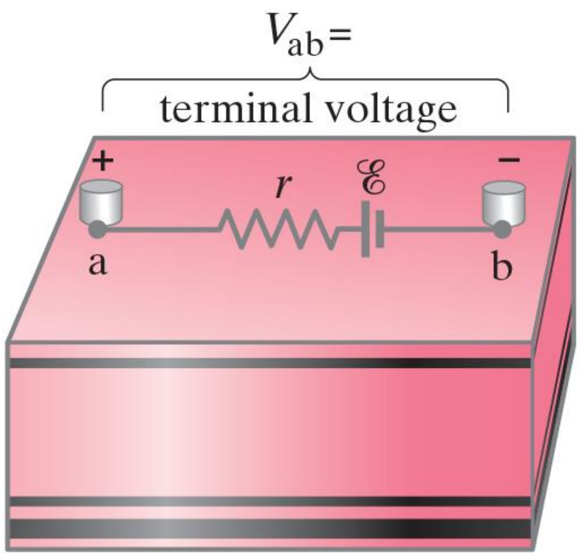

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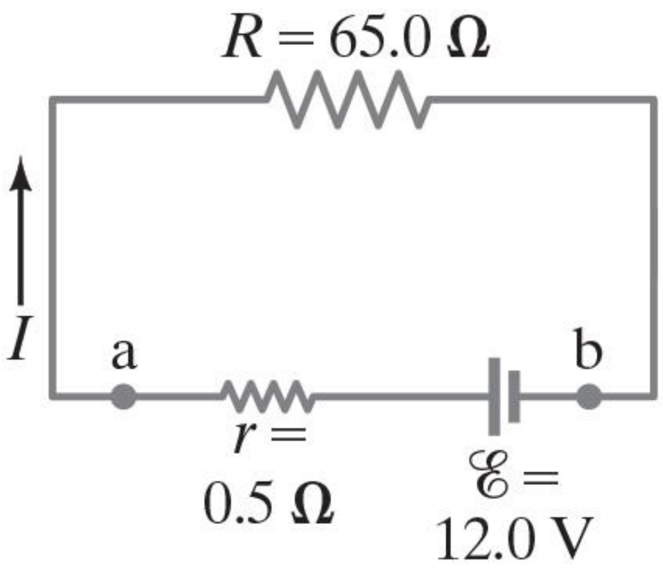
The internal resistance behaves as if it were in series with the EMF (more to follow), and increases with the battery age.

26.1 – Battery with internal resistance



A $65.0\ \Omega$ resistor is connected to the terminals of a battery whose emf is $12.0\ \mathrm{V}$ and whose internal resistance is $0.5\ \Omega$. We need to determine the current in the circuit, the voltage V_{ba}, and the power dissipated by the internal resistance.

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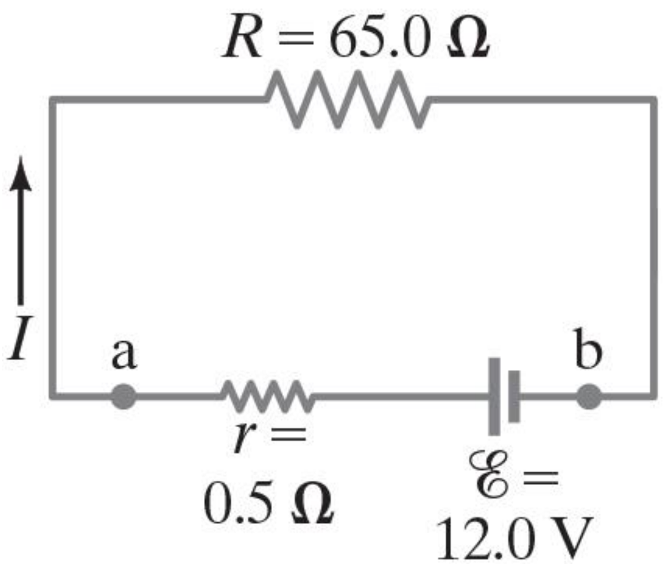

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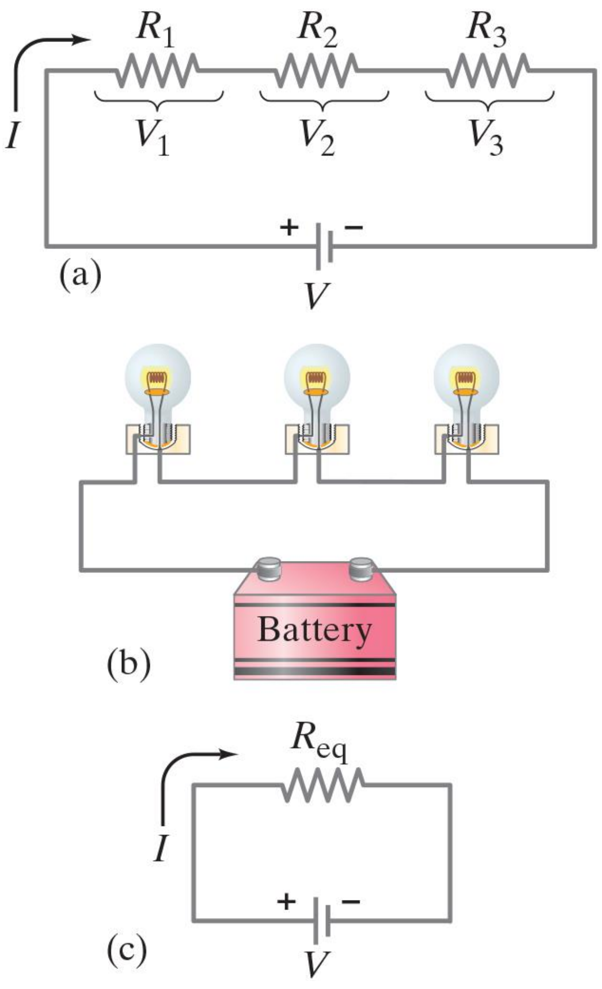
$$I = \frac{\varepsilon}{R + r} = 0.183\ A$$

$$V_{ba} = \varepsilon - Ir = 11.9\ V$$

$$P = I^2 r = 2.18\ W$$

26.2 – Resistors in Series and Parallel



Similarly to capacitors, we want to compute the equivalent resistance R_{eq} of a set of resistances in series and parallel, so that later we can tackle combinations as well.

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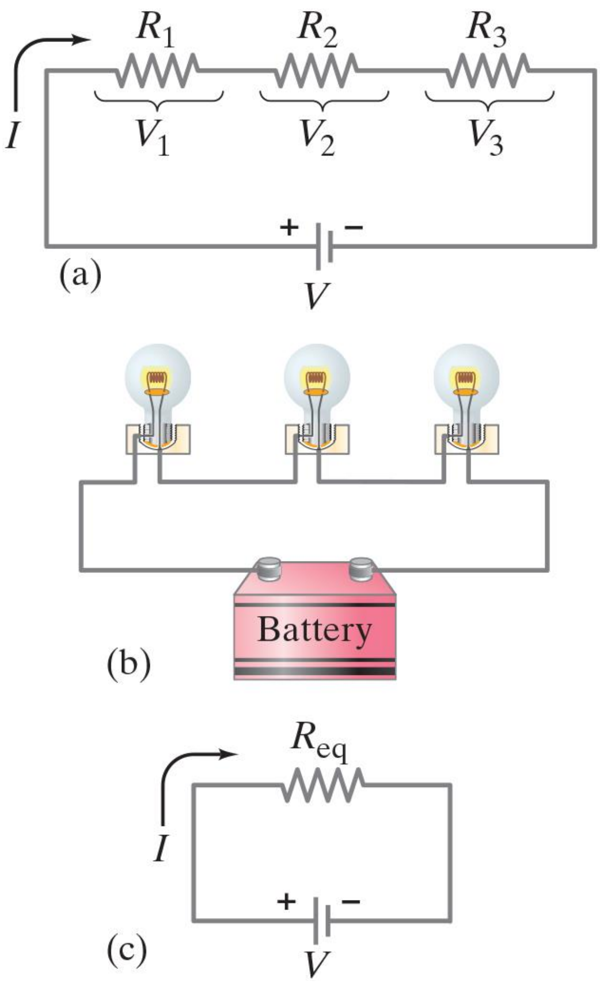

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Resistors in series see the same current I, while the sum of the drops in potential across all resistors is equivalent to the provided potential difference V across them.

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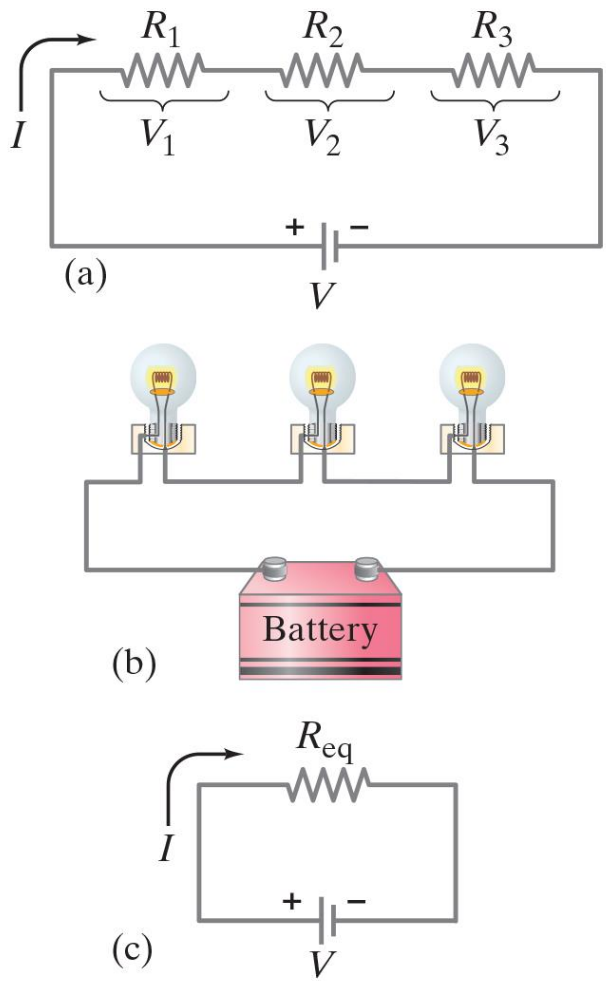

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$$\boldsymbol{V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3) = IR_{eq}}$$

For resistances in series, the equivalent resistance is the sum of them

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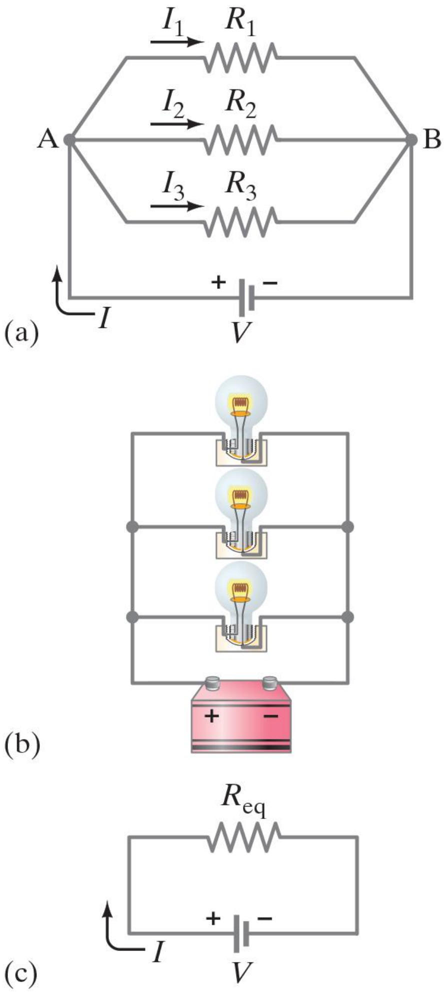

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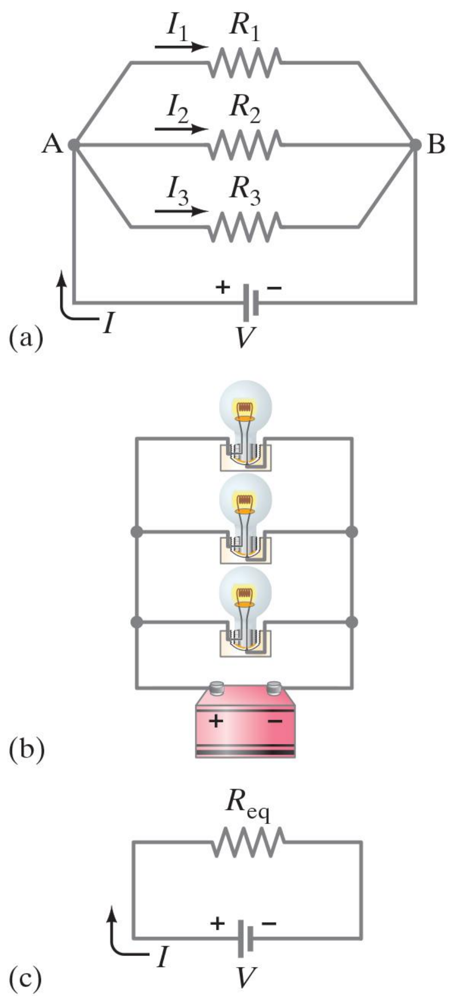

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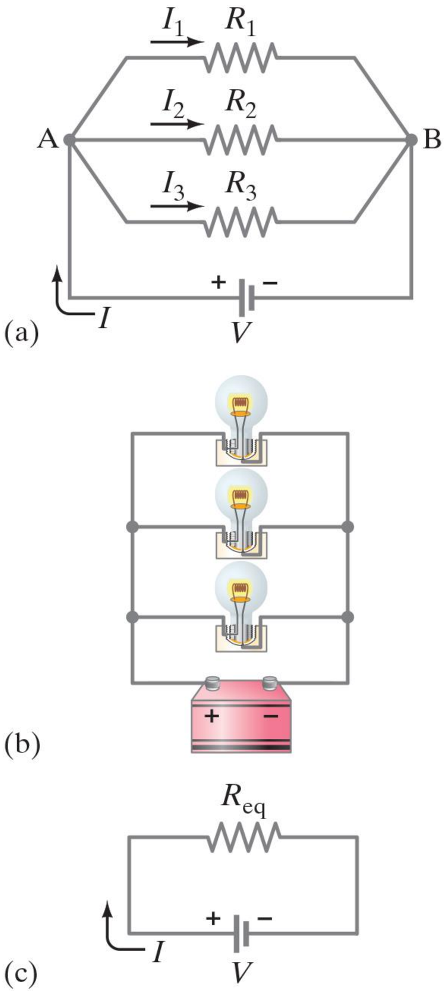

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For resistances in parallel, the equivalent resistance is the inverse of the summation of the reciprocals of each individual resistance

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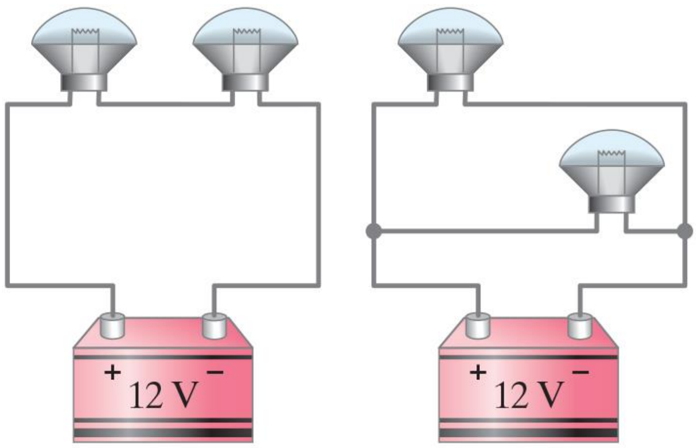
26.2 – Resistors in Series and Parallel?



(1) Series (2) Parallel

The lightbulbs in the figure are identical. Which configuration produces more light? Which way do you think the headlights of a car are wired? Ignore change of filament resistance R with current.

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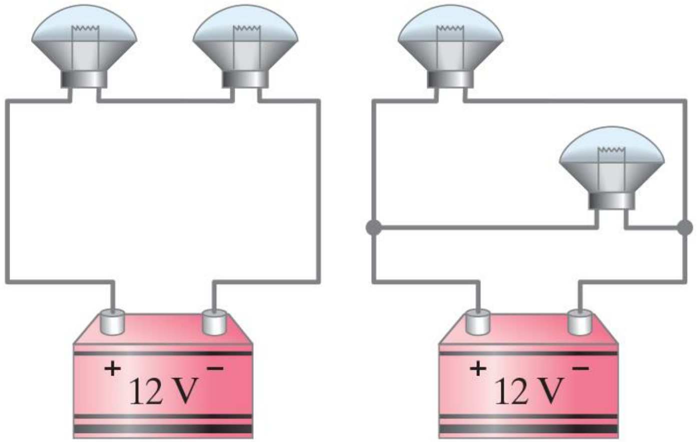

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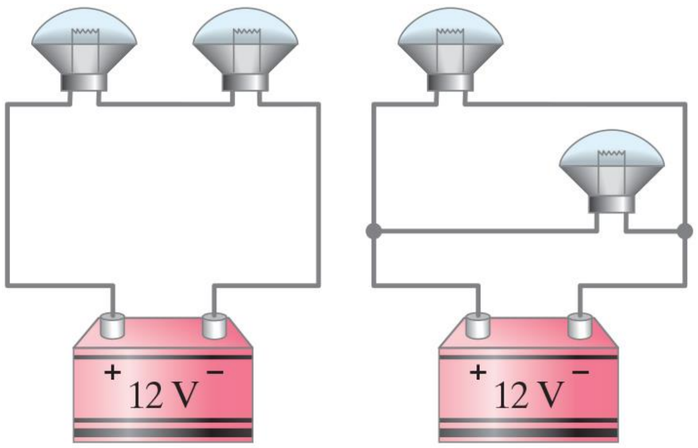

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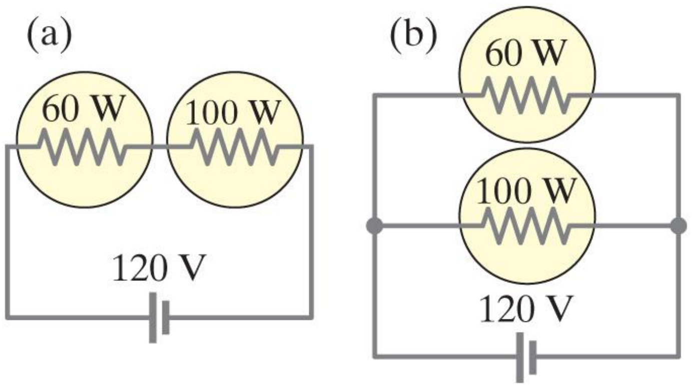
They are connected in parallel so that if one breaks down, we do not get an open circuit and the other one can still operate

26.2 – Lightbulbs in Series and Parallel (aka an Illuminating Surprise)



A $100\,W$ lightbulb and a $60\,W$ lightbulb are connected in two different ways as shown. In each case, which bulb glows more brightly? Ignore change of filament resistance with current (and temperature).

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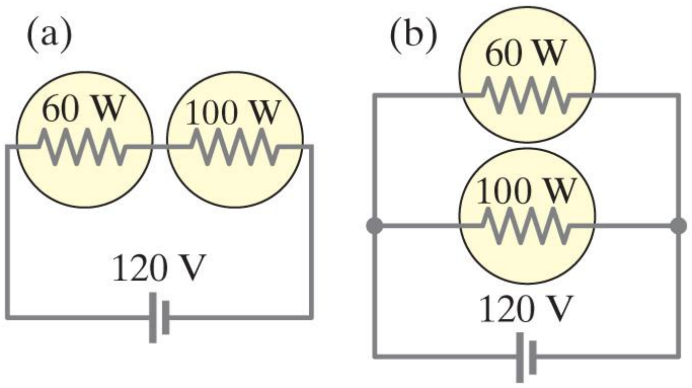

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We can determine the resistance of each lightbulb as $R = \frac{V^2}{P}$, hence the $100\,W$ lightbulb has a lower resistance than the $60\,W$ one (largest denominator).

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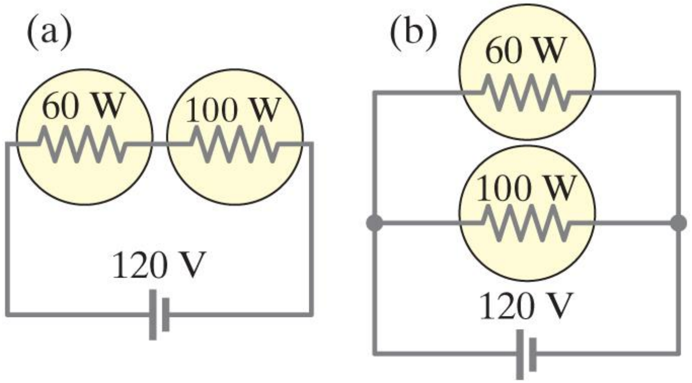

In parallel, we can use $P = \frac{V^2}{R}$ and the lightbulb dissipating more power (and hence glowing brighter) is the one with the smallest resistance, namely the $100\,W$ one.

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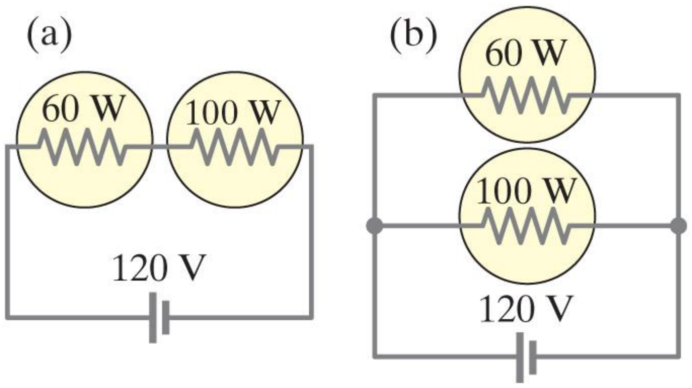

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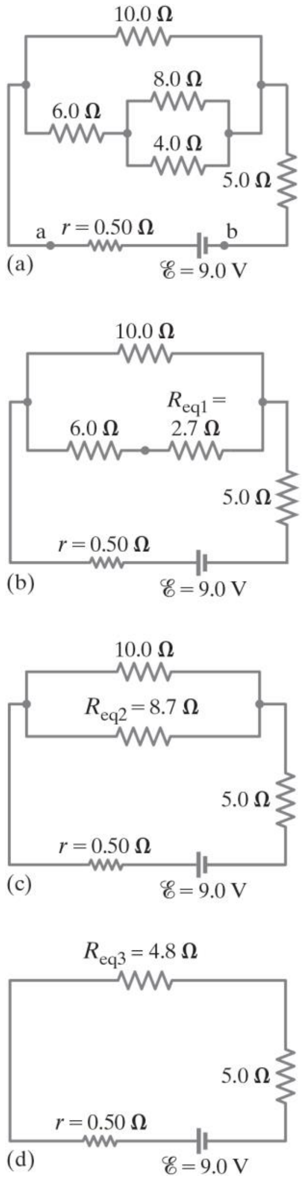
In series, we can use $P = I^2 R$, and the lightbulb dissipating more power (and hence glowing brighter) is the one with the highest resistance, namely the $60\ W$ one.

26.2 – Analyzing a Circuit



A $9.0\ V$ battery whose internal resistance r is $0.50\ \Omega$ is connected in the circuit shown. How much current is drawn from the battery? What is the terminal voltage of the battery? What is the current in the $6.0\ \Omega$ resistor?

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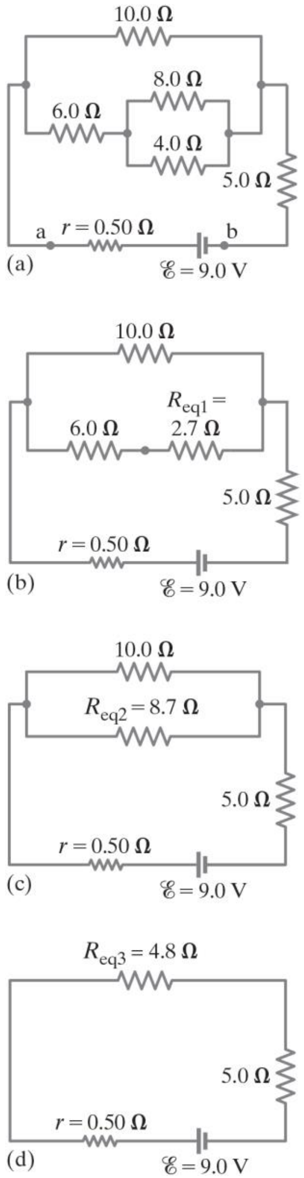

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The final equivalent resistance is $R_{eq} = (4.8 + 5.0 + 0.5)\ \Omega = 10.3\ \Omega$. Hence the current is

$$\boldsymbol{I = \frac{\varepsilon}{R_{eq}} = 0.87\ A}$$

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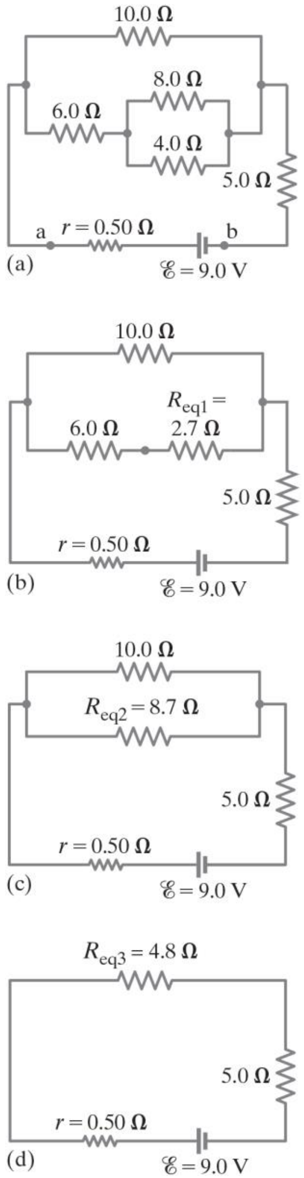

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For the terminal voltage of the battery, we need to deduct the loss due to the internal resistance

$$\boldsymbol{V_{ba} = \varepsilon - rI = 8.6\ V}$$

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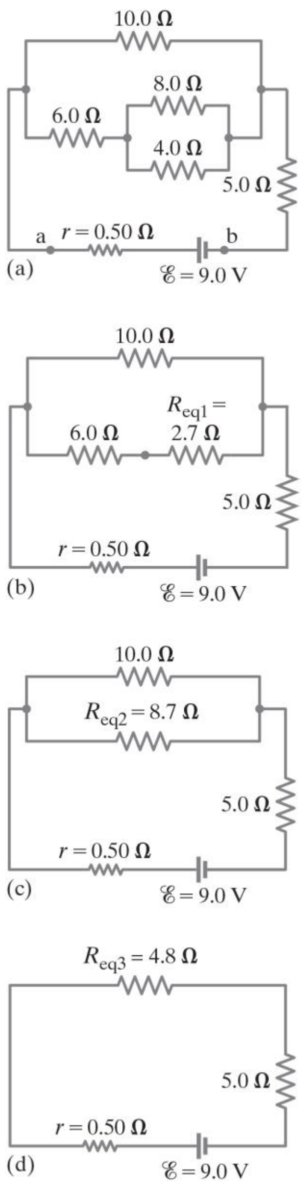

To determine the current in the specified resistance, we acknowledge it should be the same flowing in the lower branch of the parallel block in figure (c).

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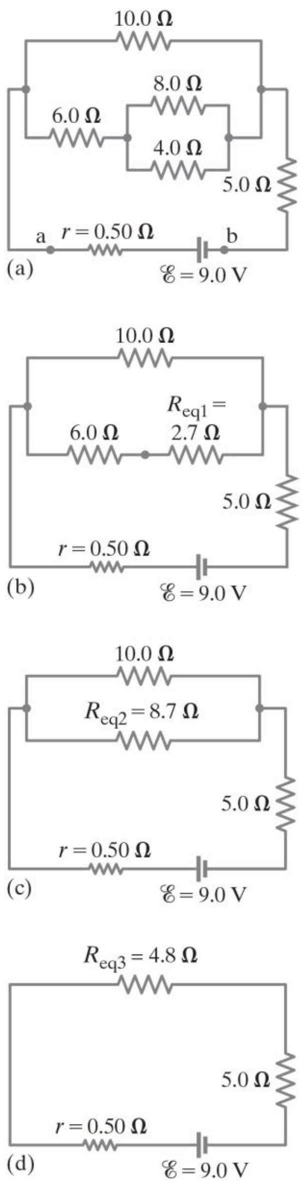

To determine the current in the specified resistance, we acknowledge it should be the same flowing in the lower branch of the parallel block in figure (c). The voltage across such parallel block is the original EMF minus the drop due to r and due to the $5.0\,\Omega$ resistor (this is a spoiler of the next section). Hence

$$\boldsymbol{I' = \frac{9.0\,V\ - (0.87\,A)(5.50\,\Omega)}{8.7\,\Omega} = 0.48\,A}$$

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26.3 – Kirchhoff’s Rules

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Junction rule: at any junction point, the sum of all currents entering the junction is equal to the sum of the exiting currents

Loop rule: the sum of changes in potential across a closed loop in a circuit is zero

For the loop rule, a couple of conventions on signs are as follows

26.3 – Kirchhoff's Rules

For a resistor, apply Ohm's law; the potential difference is negative (a decrease) if your chosen loop direction is the same as the chosen current direction through that resistor; the potential difference is positive (an increase) if your chosen loop direction is opposite to the chosen current direction. Note that there is always a drop in potential across a resistor. The second case still satisfies the condition.

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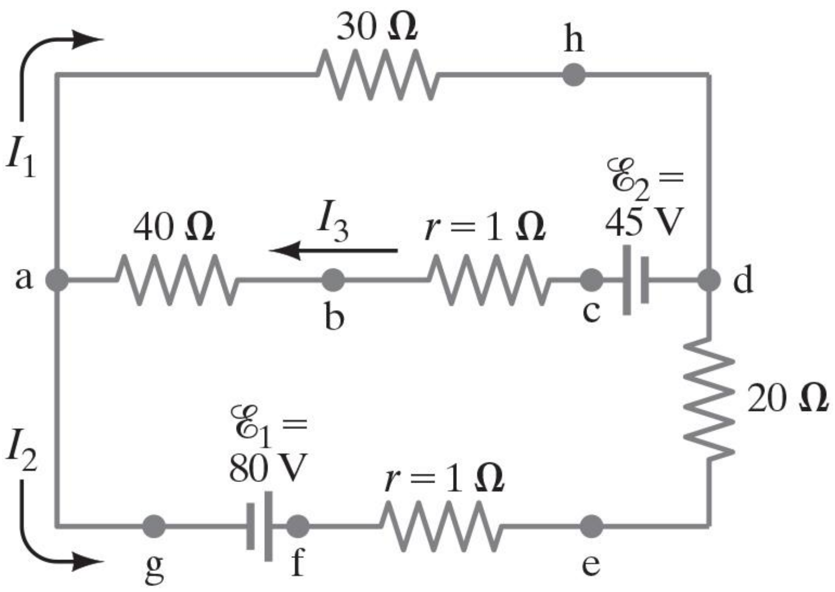
For a battery, the potential difference is positive if your chosen loop direction is from the negative terminal toward the positive terminal; the potential difference is negative if the loop direction is from the positive terminal toward the negative terminal. Similar consideration as above.

26.3 – Kirchhoff’s Rules

To fully solve a circuit, you need to:

1. Label each current, including its direction
2. Identify unknowns
3. Apply junction and loop rules; you will need as many independent equations as there are unknowns
4. Solve the equations, being careful with signs: if the solution for a current is negative, that current is in the opposite direction from the one you have chosen.

26.3 – Kirchhoff's Rules: an Example



We want to identify the currents in this circuit.

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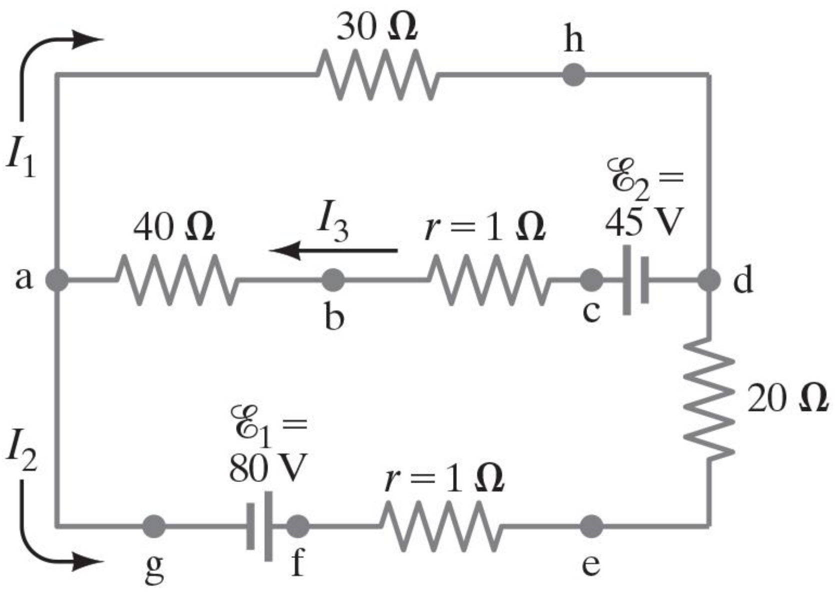

We want to identify the currents in this circuit.

We use the junction rule in a and the loop rule considering the upper loop and the outer loop.

$$\boldsymbol{I_3 = I_1 + I_2}$$
$$\boldsymbol{45 - 1I_3 - 40I_3 - 30I_1 = 0}$$
$$\boldsymbol{-80 - 30I_1 + 20I_2 + 1I_2 = 0}$$

We can solve such a system by substitution or using matrix inversion

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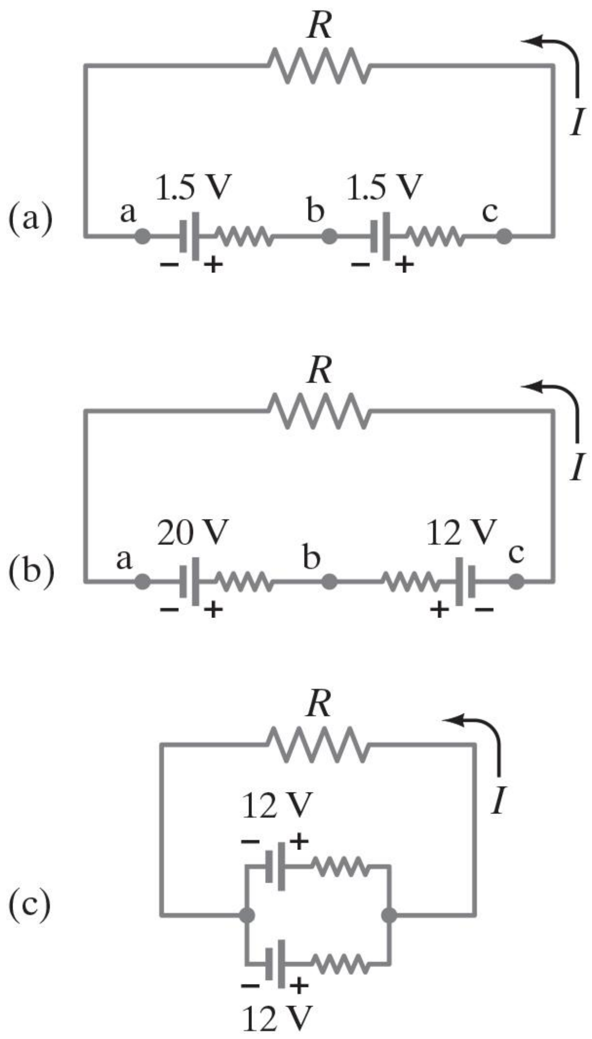
26.4 – EMFs in Series and Parallel



When two EMFs are connected in series, then the total provided voltage is the algebraic sum of their voltages.

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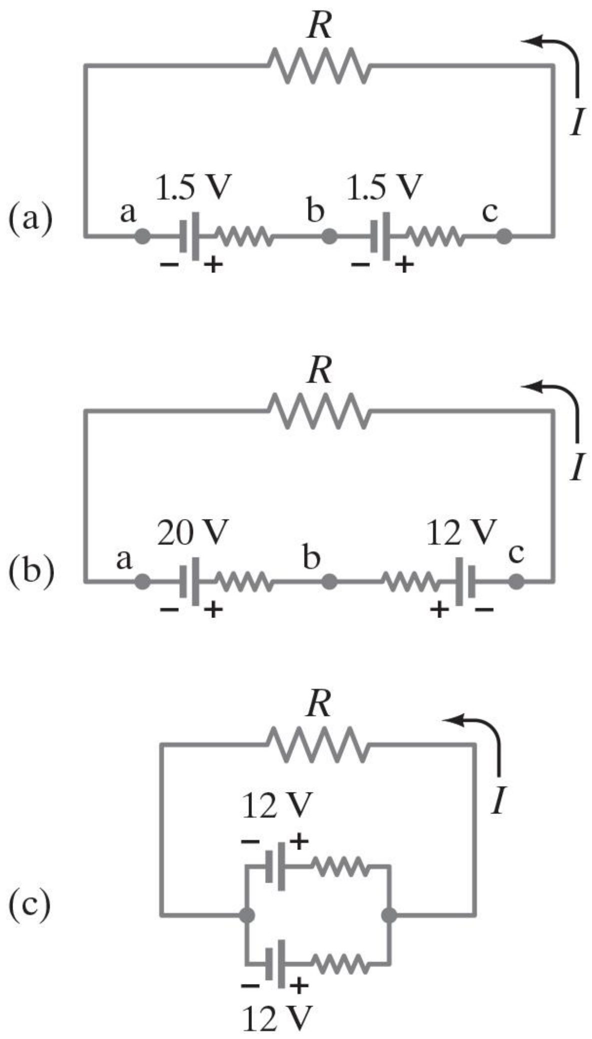

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For example, in figure (a) the overall provided voltage (neglecting internal resistances) is $3\,V$.

In figure (b), the difference in potential between c and a is $8\,V$ as the gain of $20\,V$ in b is offset by the second $12\,V$ battery.

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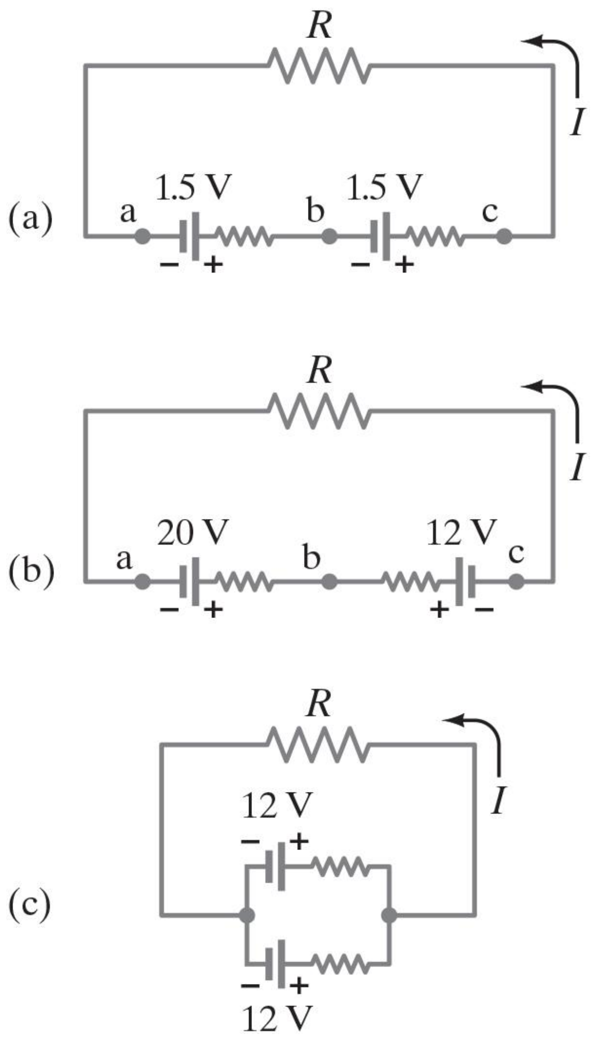

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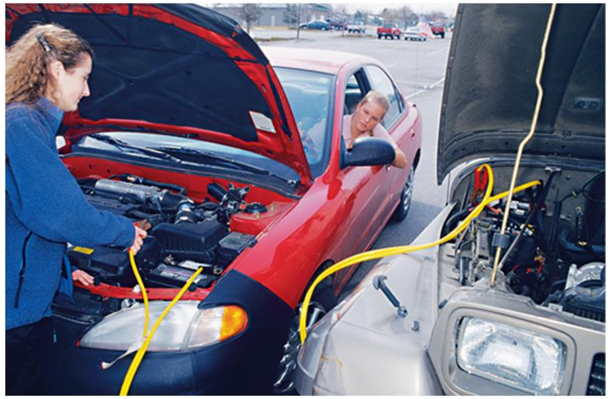
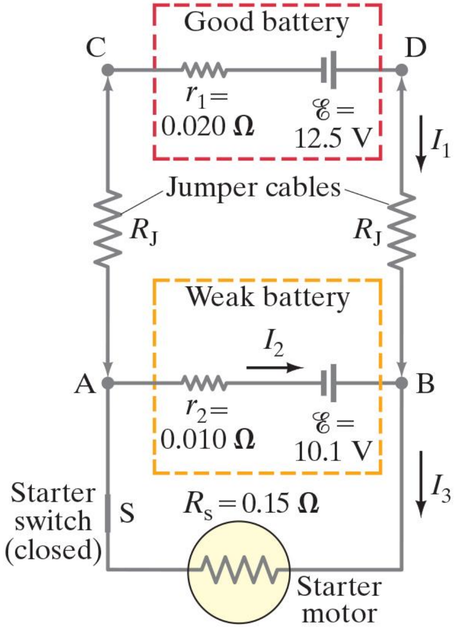
EMFs, when equivalent, can also be arranged in parallel (figure (c)), so that they can provide more energy when large currents are needed. Each EMF must only provide a portion of the total current, so that batteries can last longer.

26.4 – Jump starting a car





A good car battery is used to jump start a car with a weak battery (see figure to the left). The two cables connecting the good battery to the weak one have a length of $3\ m$, a diameter of $0.5\ cm$, and are made of copper. The other data are shown in the figure instead.

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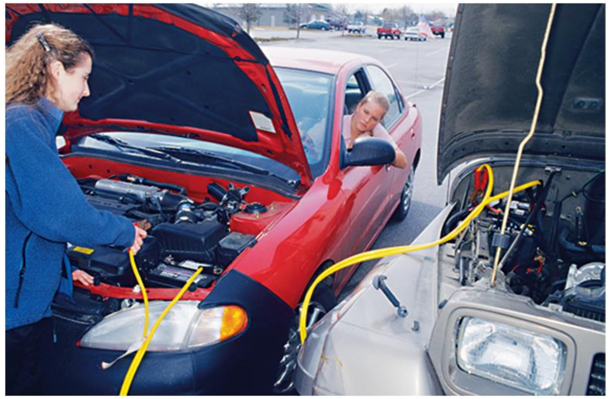

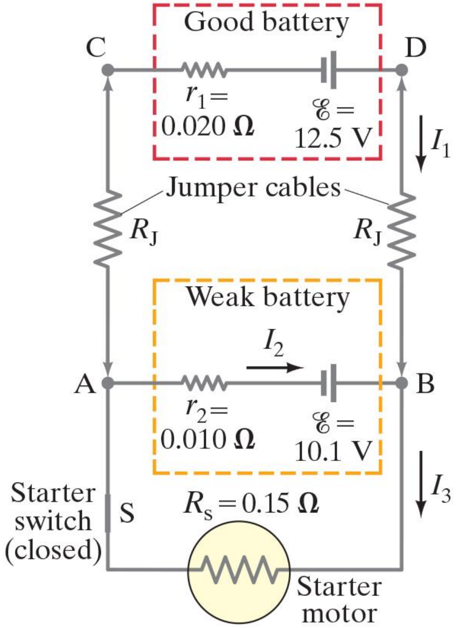

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We should determine the current passing through the starter motor without and with the good battery attached.

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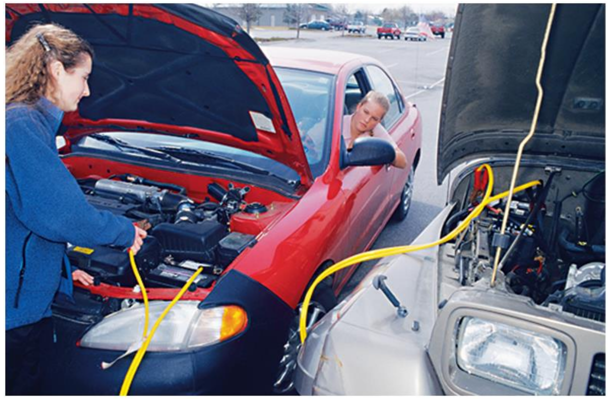

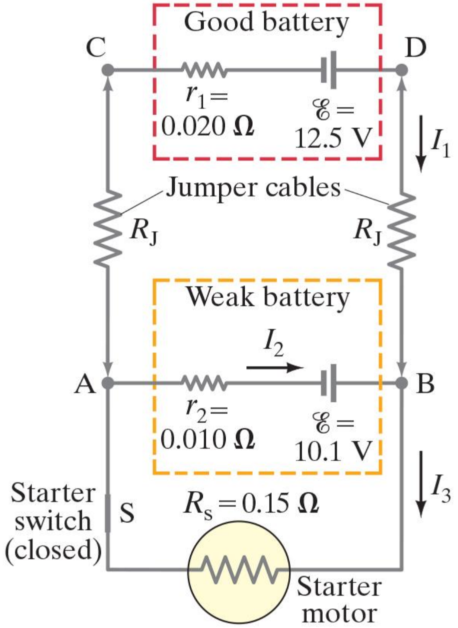

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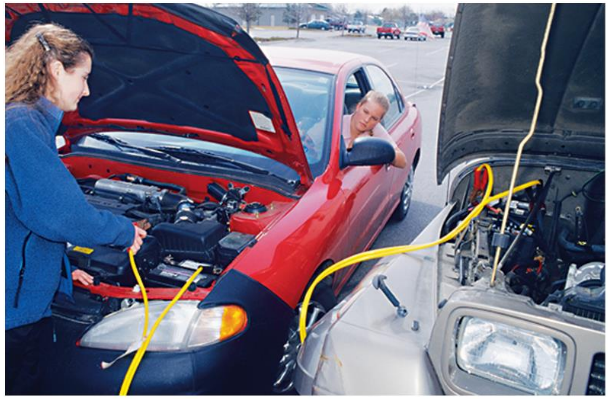
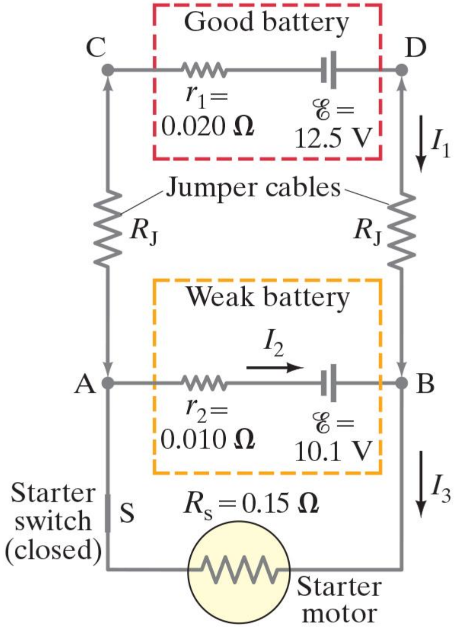
Without the good battery attached, we use Kirchhoff's loop rule directly and write

$$\boldsymbol{\varepsilon_2 - IR_S - Ir_2 = 0 \rightarrow I = \frac{10.1\ V}{0.25\ \Omega} = 40\ A}$$

26.4 – Jump starting a car





A good car battery is used to jump start a car with a weak battery (see figure to the left). The two cables connecting the good battery to the weak one have a length of $3\ m$, a diameter of $0.5\ cm$, and are made of copper. The other data are shown in the figure instead.

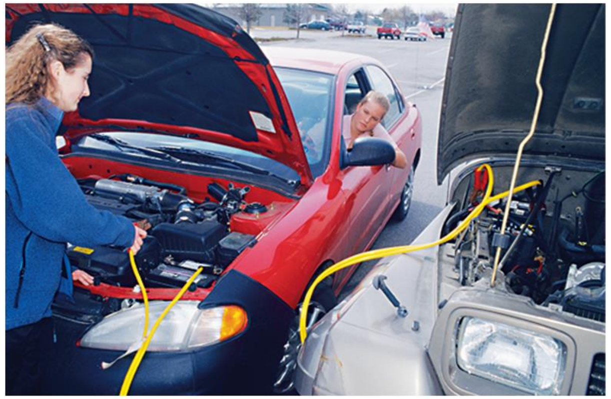
We should determine the current passing through the starter motor without and with the good battery attached.

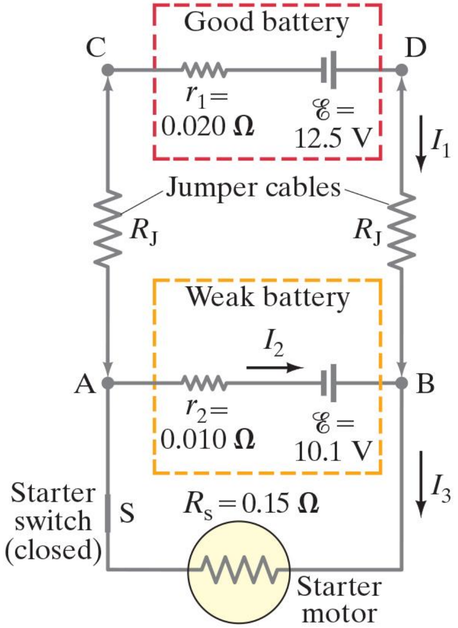
Without the good battery attached, we use Kirchhoff's loop rule directly and write

$$\boldsymbol{\varepsilon_2 - IR_S - Ir_2 = 0 \rightarrow I = \frac{10.1\ V}{0.25\ \Omega} = 40\ A}$$

Note that there is a typo in the figure, where $r_2 = 0.010\ \Omega$, but the solution entails a resistance ten time higher

26.4 – Jump starting a car





With the good battery attached, the circuit is slightly more complicated. First, we need to determine

$$R_J = \rho \frac{l}{A} = 0.0026\ \Omega$$

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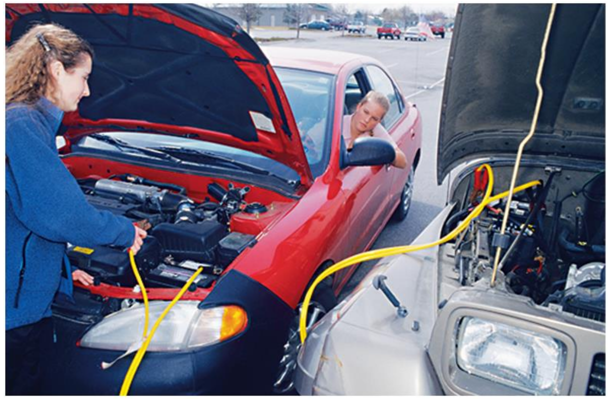
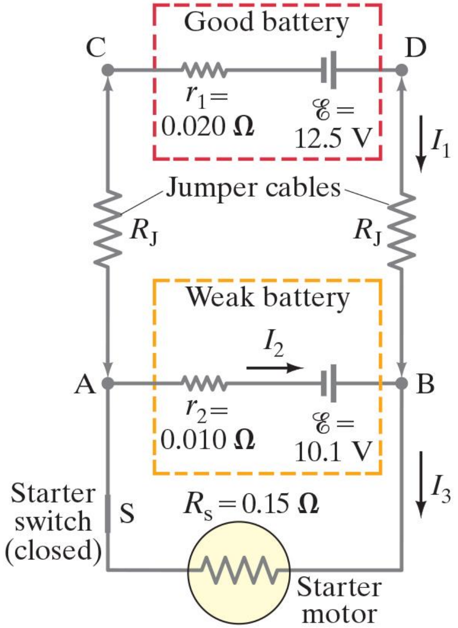
26.4 – Jump starting a car





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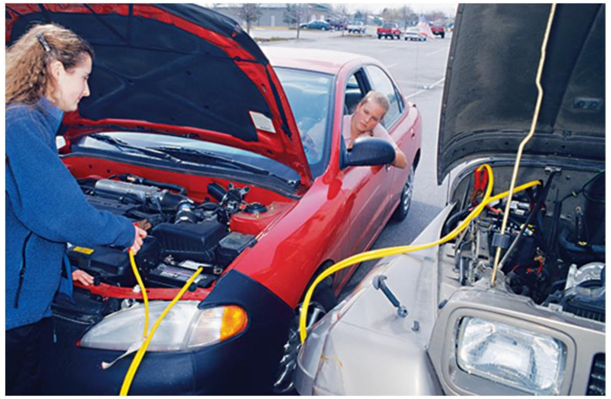
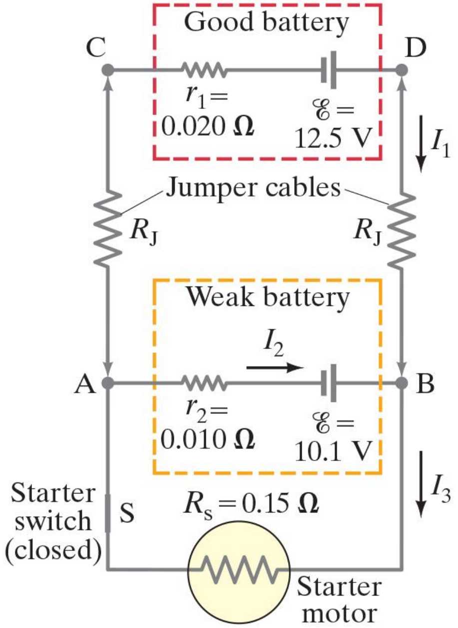
We then use Kirchhoff's junction rule in node B (or A, it would be equivalent) and write

$$I_3 = I_1 + I_2$$

26.4 – Jump starting a car





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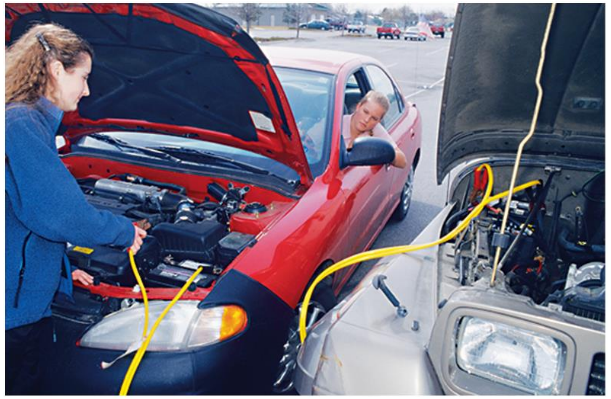
$$I_3 = I_1 + I_2$$

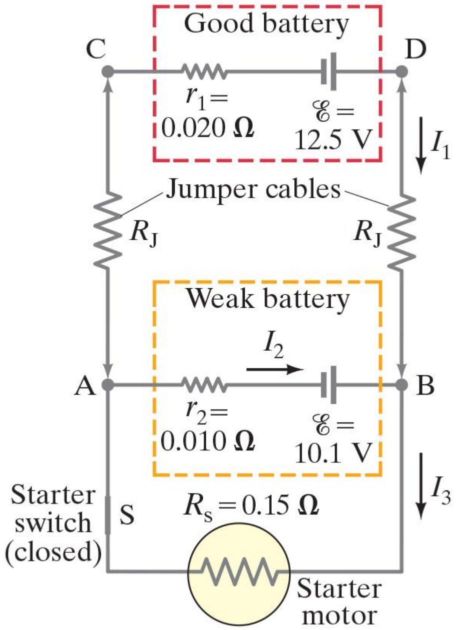
and then use Kirchhoff's loop rule for the outer loop and lower loop

$$\varepsilon_1 - I_1 R_J - I_3 R_S - I_1 R_J - I_1 r_1 = 0$$

$$\varepsilon_2 - I_3 R_S - I_2 r_2 = 0$$

26.4 – Jump starting a car





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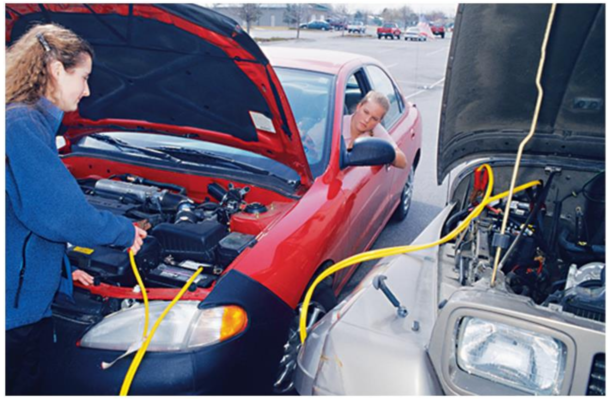
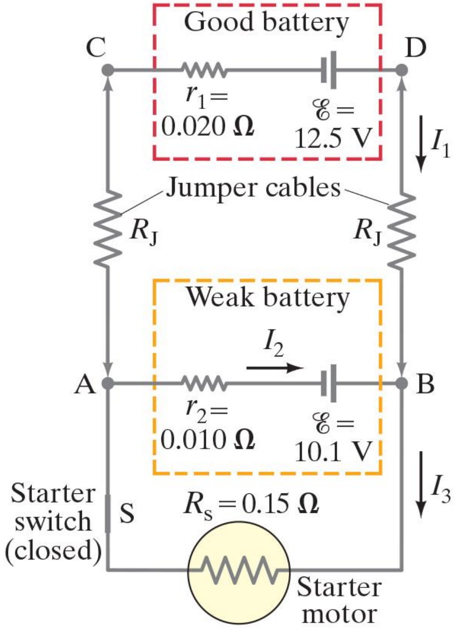
$$\boldsymbol{\varepsilon_2 - I_3 R_S - I_2 r_2 = 0}$$

Solving for the three current, we get $I_1 = 76, I_2 = -5, I_3 = 71\ A$

The current in the motor is much higher now.

26.4 – Jump starting a car

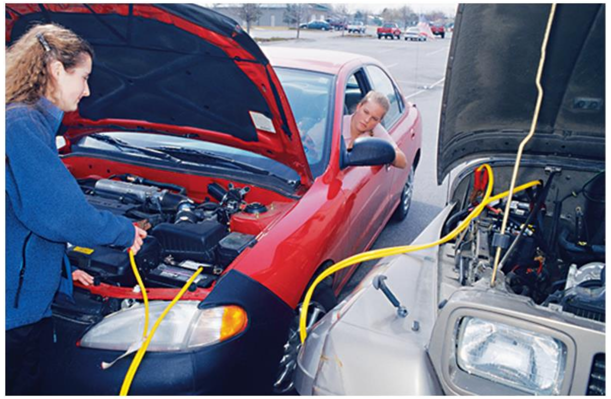


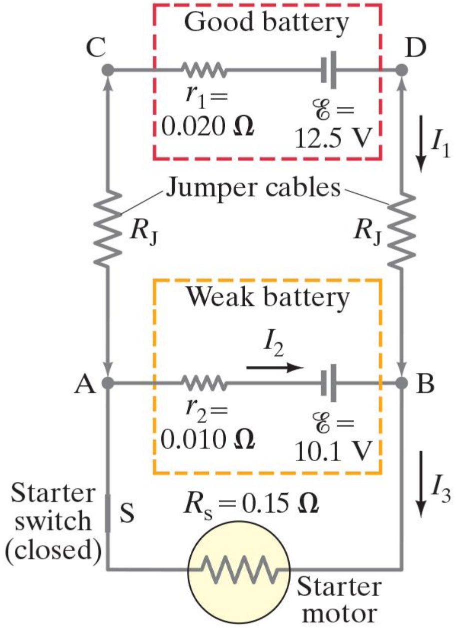


Note that $I_2 < 0$, hence its actual direction is from node B to node A. This entails that the potential provided by the weak battery between nodes B and A (when connected with the good battery) is

$$\boldsymbol{V_{BA} = \varepsilon_2 - I_2 r_2 = 10.1\ V - (-5.0\ A)(0.1\ \Omega)}$$
$$\boldsymbol{= 10.6\ V}$$

26.4 – Jump starting a car





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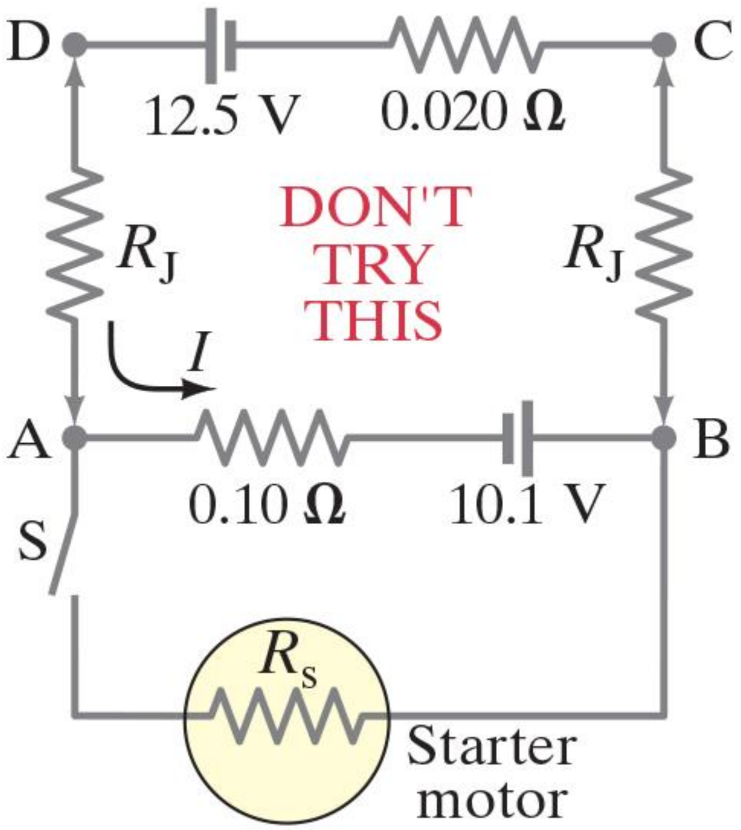
$$\begin{aligned} V_{BA} &= \varepsilon_2 - I_2 r_2 = 10.1\ V - (-5.0\ A)(0.1\ \Omega) \\ &= 10.6\ V \end{aligned}$$

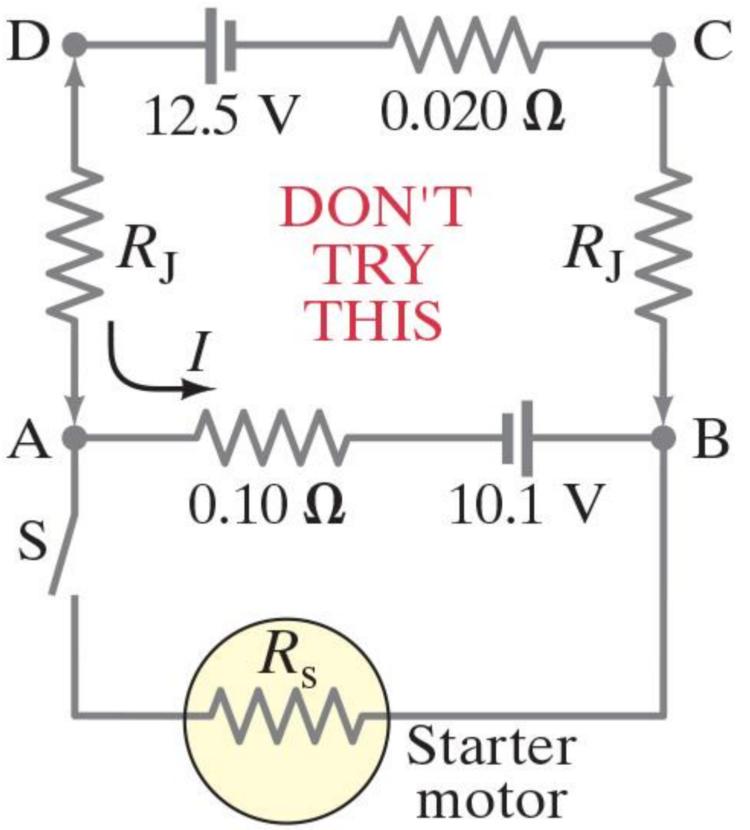
The good battery is boosting up the weak one.

26.4 – Jump starting a car



It is important to connect the good and the weak battery so that the two positive and two negative terminals are connected. What if we make a mistake and connect them the other way around (as in the figure to the left)? Let us assume the starter motor is disengaged (switch S open)

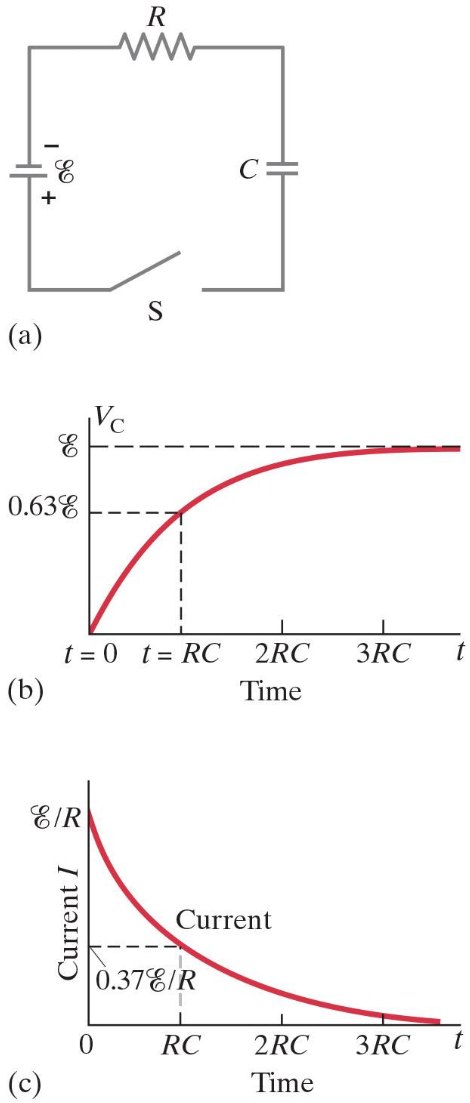
26.4 – Jump starting a car



It is important to connect the good and the weak battery so that the two positive and two negative terminals are connected. What if we make a mistake and connect them the other way around (as in the figure to the left)? Let us assume the starter motor is disengaged (switch S open)

$$\boldsymbol{\varepsilon_1 - IR_J - Ir_2 + \varepsilon_2 - IR_J - Ir_1 = 0 \rightarrow I = \frac{\varepsilon_1 + \varepsilon_2}{2R_J + r_1 + r_2} = \frac{22.6\,V}{0.1252\,\Omega} = 180.5\,A}$$

26.5 – RC Circuits: Resistor and Capacitor in Series



We now analyze the transient phase of a system comprising a battery, a resistance, and a capacitor (RC). In previous chapters, we analyzed the steady-state configuration of a capacitor. Here, we analyze what comes before.

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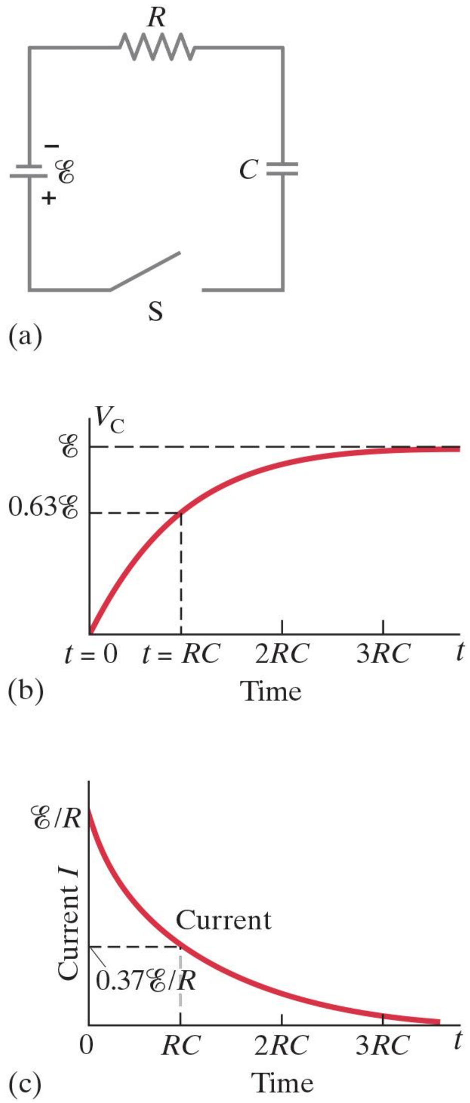

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When we close the switch, charges will start flowing on one plate of the capacitor, slightly charging it. As this happens, the voltage across the plates will increase: $V_c(t) = \frac{Q(t)}{C}$

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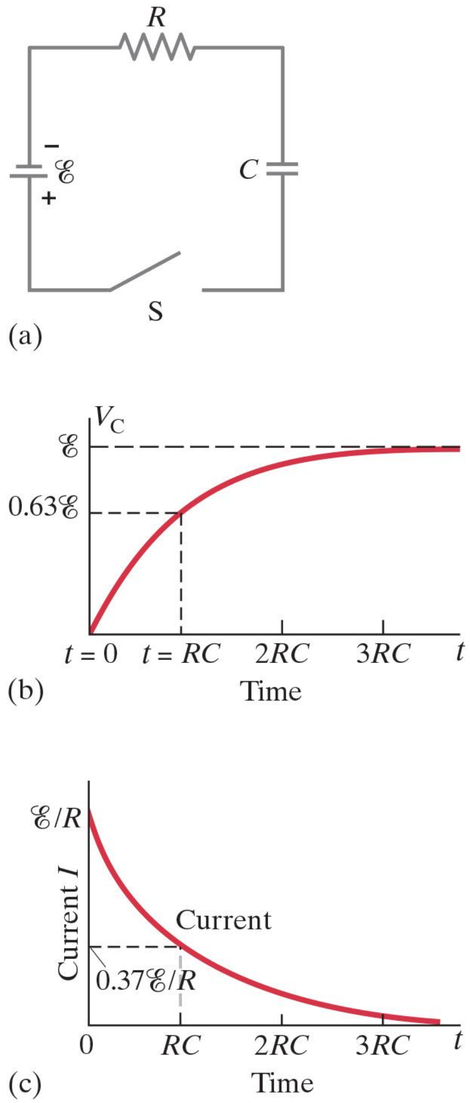
26.5 – RC Circuits: Resistor and Capacitor in Series



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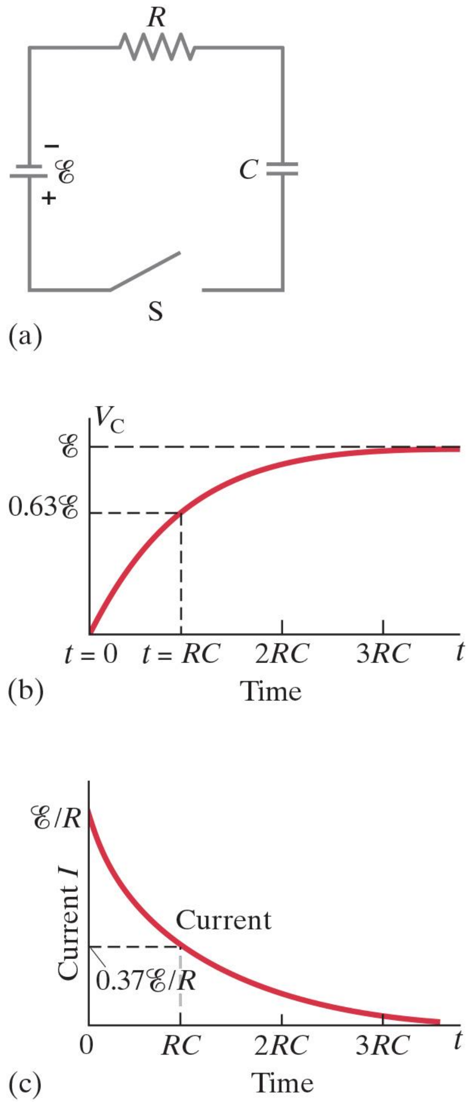
We can also use Kirchhoff's loop rule to write

$$\boldsymbol{\varepsilon - I(t)R - \frac{Q(t)}{C} = 0}$$

26.5 – RC Circuits: Resistor and Capacitor in Series



Finally, we can write that $\boldsymbol{I = \frac{dQ}{dt}}$ as the charge is increasing over time ($\frac{dQ}{dt} > 0$). Putting all together

$$\boldsymbol{\varepsilon = R\frac{dQ}{dt} + \frac{1}{C}Q}$$

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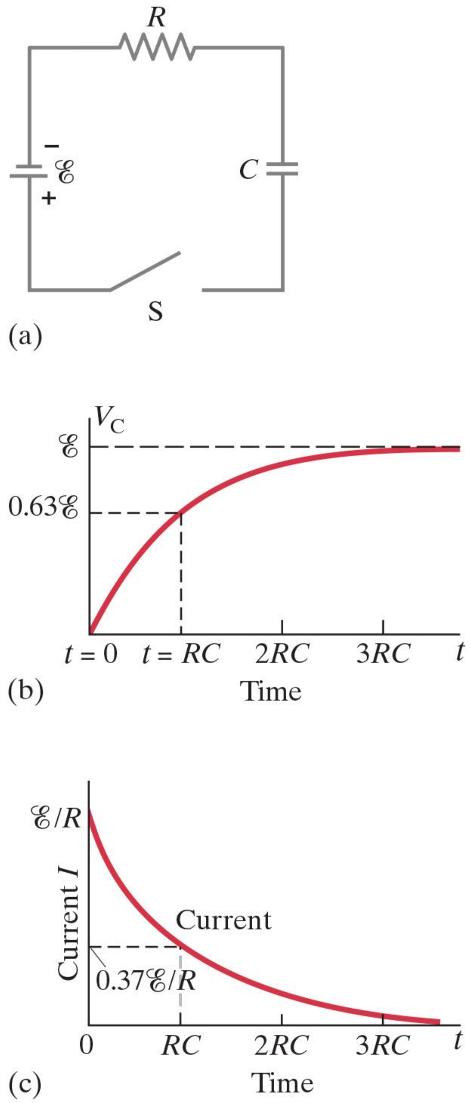

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or, rearranging the terms

$$\boldsymbol{\frac{dQ}{C\varepsilon - Q} = \frac{dt}{RC}}$$

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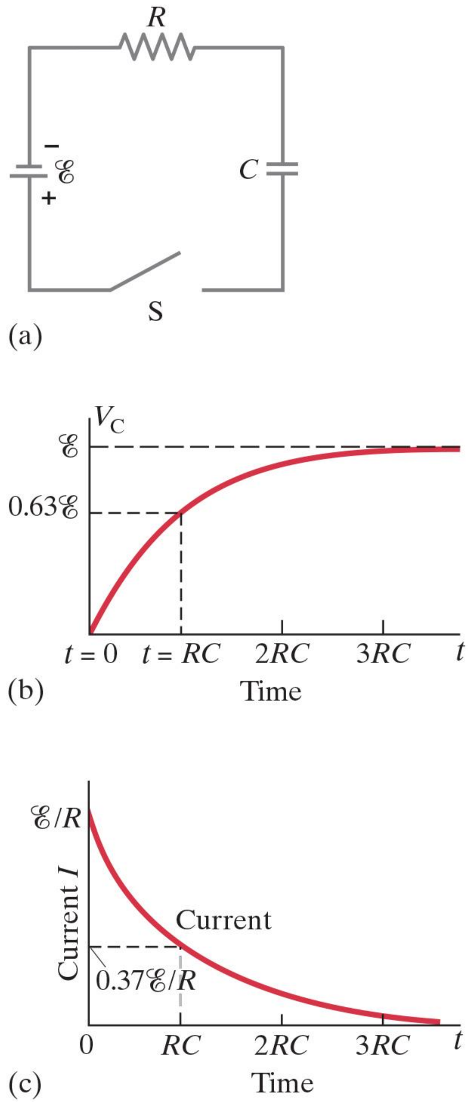

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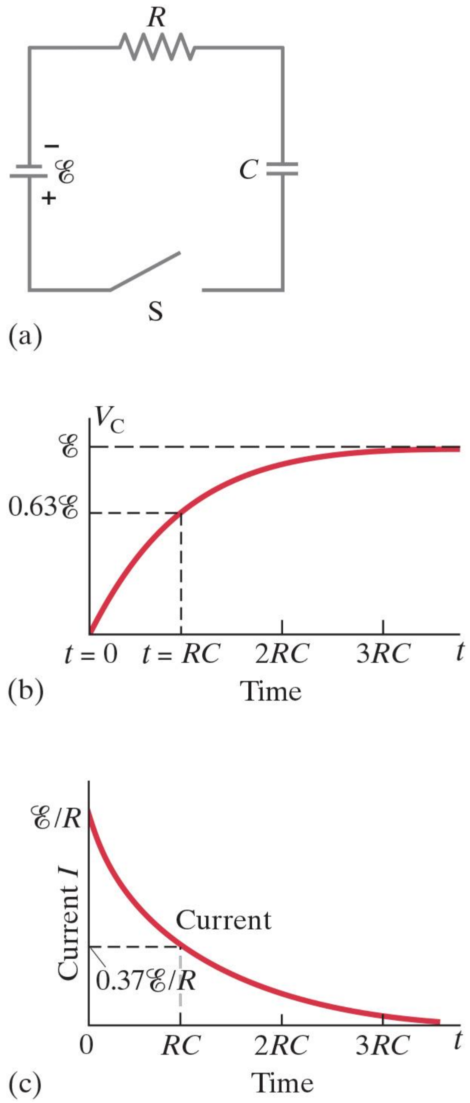
which can be integrated by separation of variables between $t_0 = 0$ and a generic t (right side) and $Q(t_0) = 0$ and a generic $Q(t)$

$$\int_0^Q \frac{d\,\bar{Q}}{C\varepsilon - \bar{Q}} = \int_0^t \frac{d\bar{t}}{RC}$$

26.5 – RC Circuits: Resistor and Capacitor in Series



$$-\ln(C\varepsilon - \bar{Q})\Big|_0^Q = \frac{\bar{t}}{RC}\Big|_0^t$$

$$\ln\left(1 - \frac{Q}{C\varepsilon}\right) = -\frac{t}{RC}$$

$$Q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = Q_0\left(1 - e^{-\frac{t}{RC}}\right)$$

TUDelft

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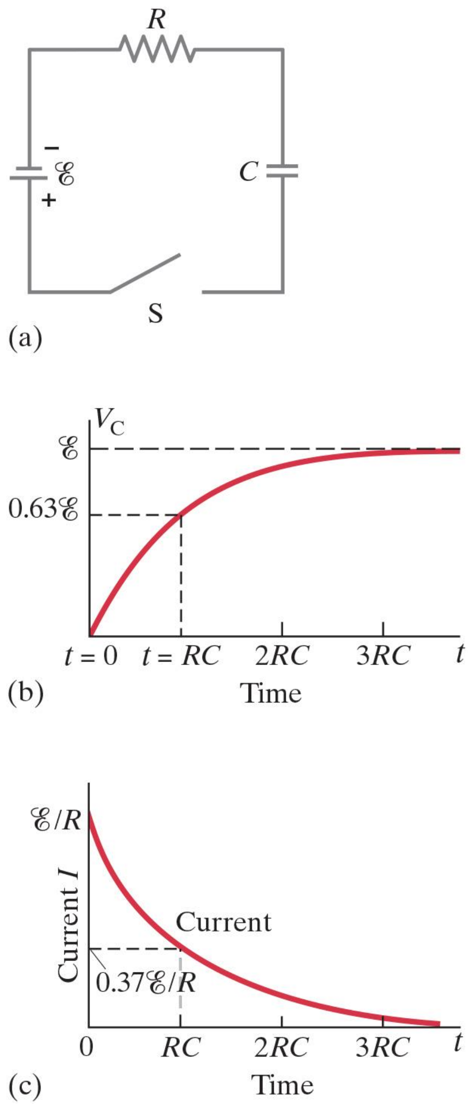

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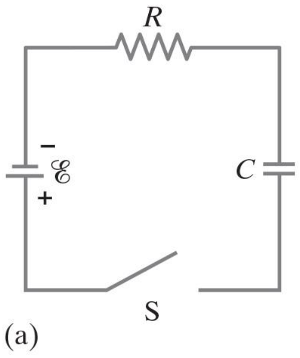
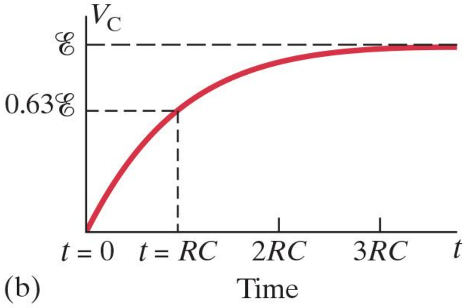
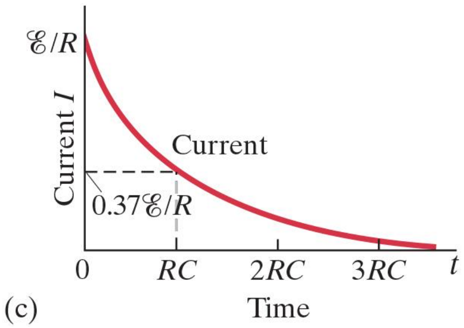
We can express the potential across the plates as

$$V_c(t) = \frac{Q(t)}{C} = \varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

26.5 – RC Circuits: Resistor and Capacitor in Series







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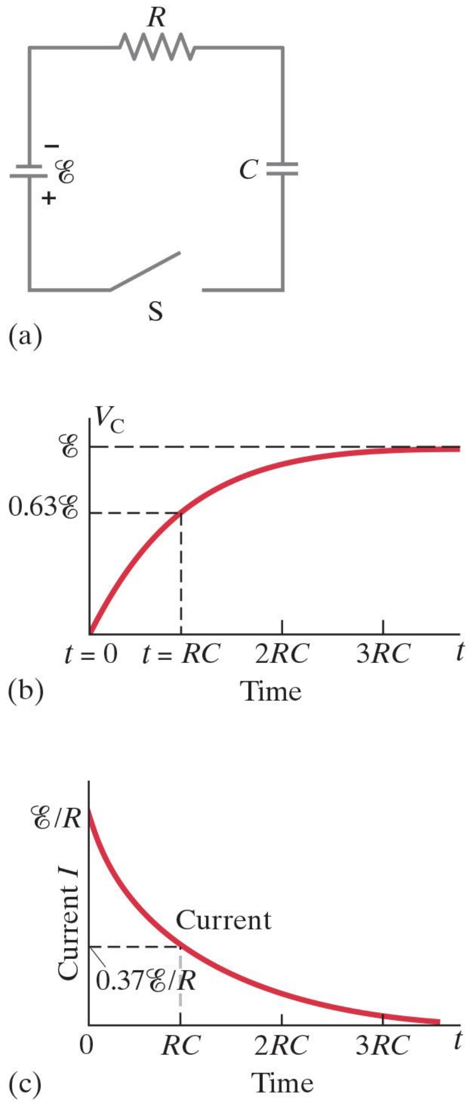
$\tau = RC$ is generally called the time constant and is useful to determine the state of charge (w.r.t. Q_0) given the specs of the resistor and capacitor. After one time constant, the state of charge (and voltage) is 63% ($1 - e^{-1} = 0.63$)

26.5 – RC Circuits: Resistor and Capacitor in Series



Finally, we can express the current as

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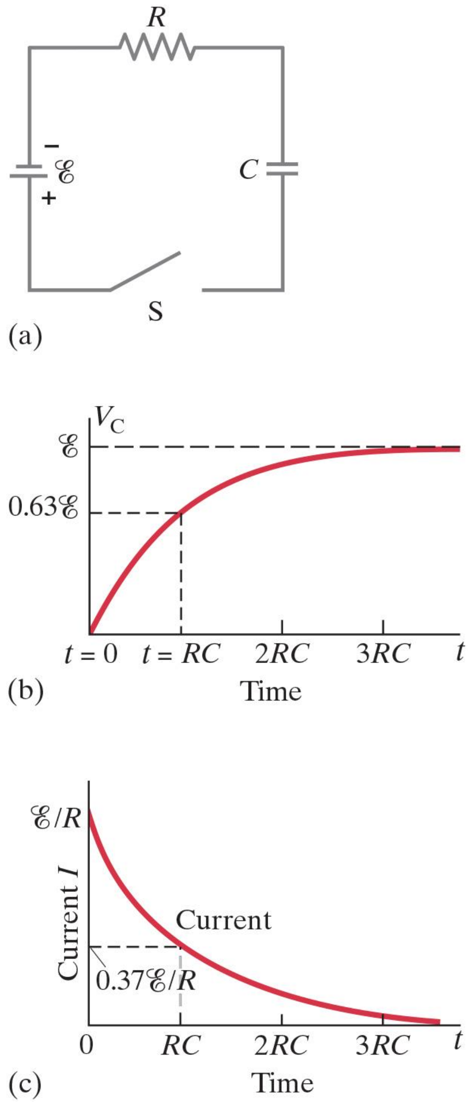
TUDelft

26.5 – RC Circuits: Resistor and Capacitor in Series

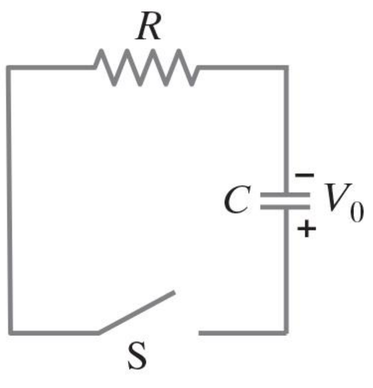


Finally, we can express the current as

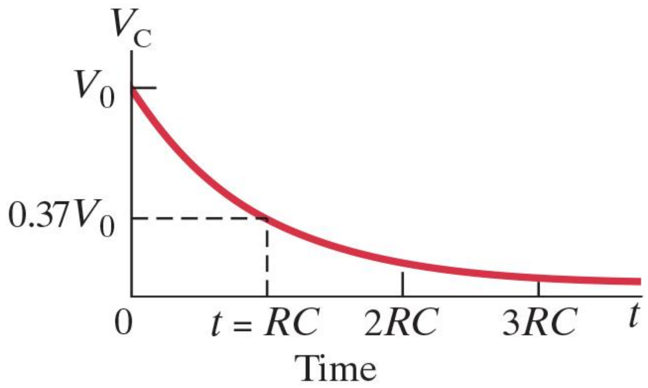
$$I(t) = \frac{dQ(t)}{dt} = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

which decreases over time and goes to zero once the capacitor is fully charged. This final result is what we already used in Chapter 24 when dealing with the electrostatic case.

26.5 – RC Circuits: Resistor and Capacitor in Series



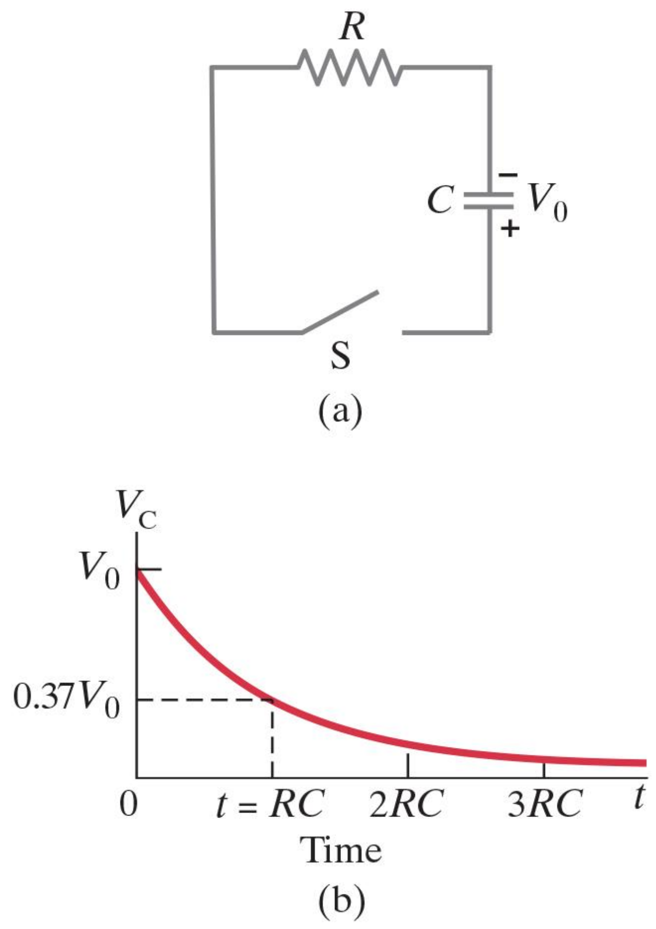
(a)



(b)

How about a discharging capacitor in series with a resistance (no battery any longer)?
Once the switch is closed, charges flow from one side of the capacitor through the resistance to reach the other plate.

26.5 – RC Circuits: Resistor and Capacitor in Series



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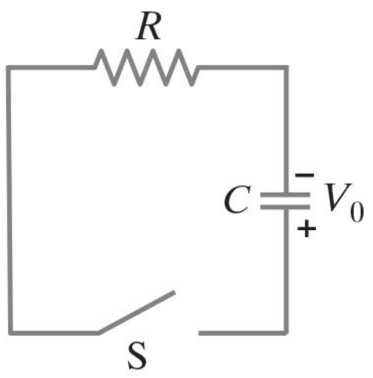
$$I(t)R = \frac{Q(t)}{C}$$

together with $I(t) = -\frac{Q(t)}{dt}$. Note that now there is a minus as the charge is decreasing over time.

26.5 – RC Circuits: Resistor and Capacitor in Series



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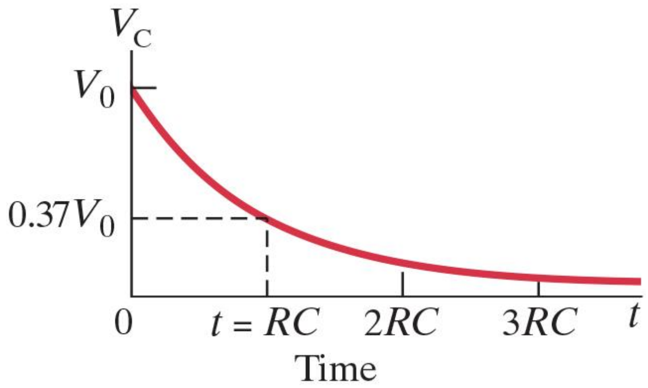

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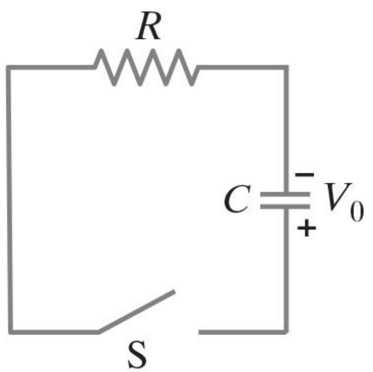
Putting all together

$$\frac{dQ}{Q(t)} = -\frac{dt}{RC}$$

26.5 – RC Circuits: Resistor and Capacitor in Series



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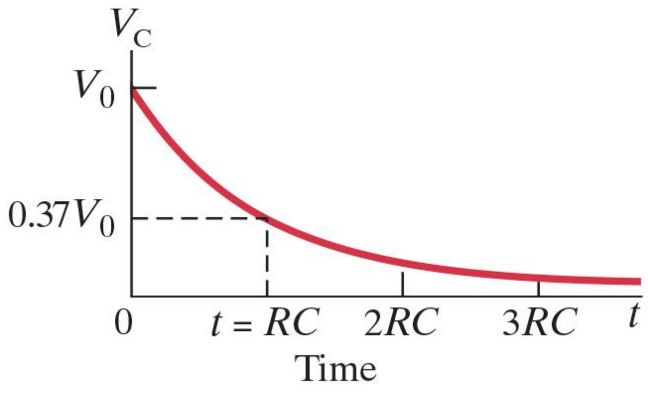
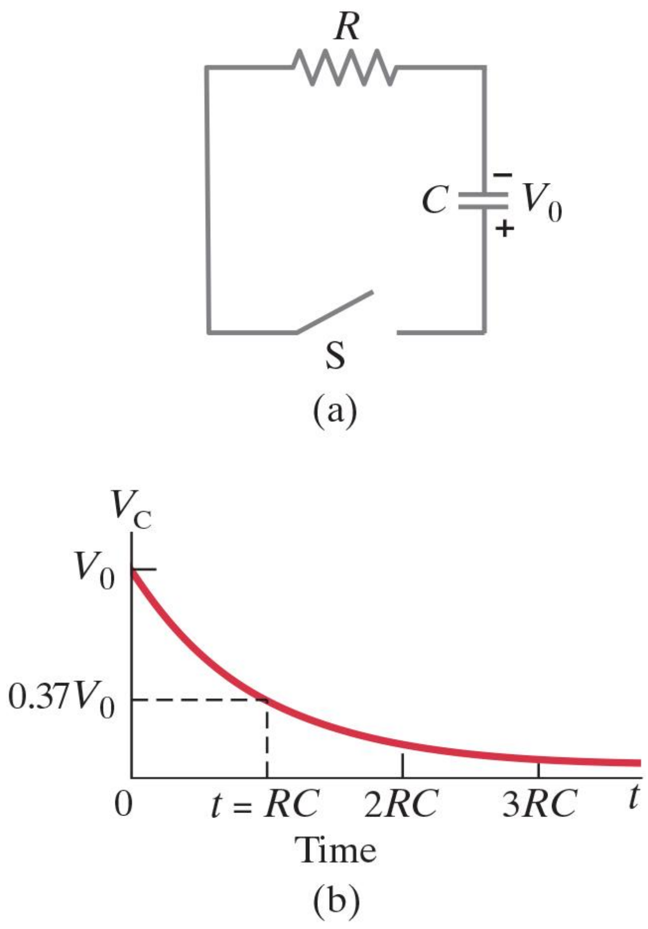


(b)

$$Q(t) = Q_0 e^{-\frac{t}{RC}}$$

26.5 – RC Circuits: Resistor and Capacitor in Series

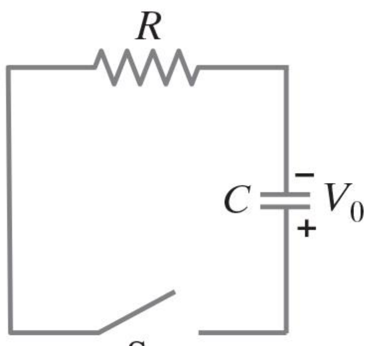


$$Q(t) = Q_0 e^{-\frac{t}{RC}}$$

and the voltage across the capacitor is

$$V(t) = \frac{Q_0}{C} e^{-\frac{t}{RC}} = V_0 e^{-\frac{t}{RC}}$$

26.5 – RC Circuits: Resistor and Capacitor in Series



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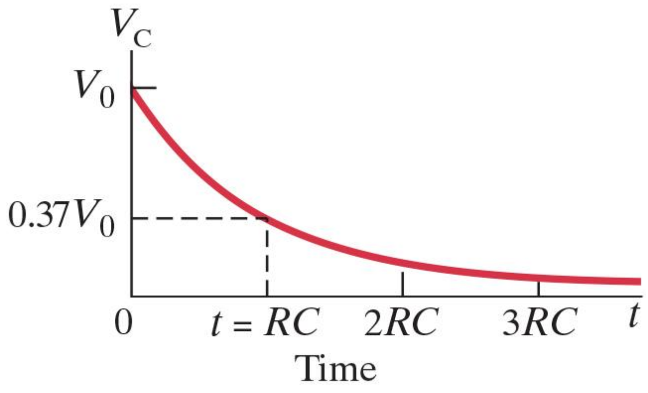

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and the voltage across the capacitor is

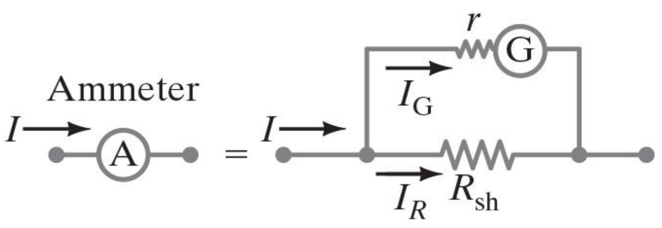
$$V(t) = \frac{Q_0}{C} e^{-\frac{t}{RC}} = V_0 e^{-\frac{t}{RC}}$$

while the current is

$$I(t) = I_0 e^{-\frac{t}{RC}}$$

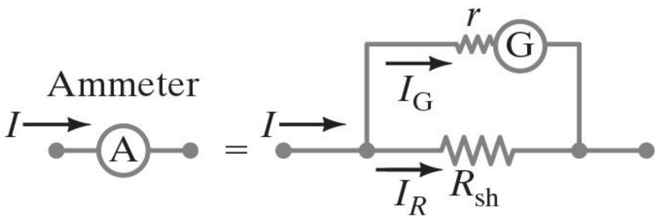
All three quantities decrease over time and (asymptotically) reach zero values. This is consistent with the fact we are not relying on any voltage-providing device (e.g., a battery).

26.7 – Ammeters and Voltmeters



Both devices are used to measure an unknown current (resp. voltage) in a specific portion of an electric circuit. When analog, they are based on a galvanometer, a device based on the interaction of magnetic field and current to that the displacement of a needle is linearly proportional to the measured current.

26.7 – Ammeters and Voltmeters



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In general, the full-scale current sensitivity (when the needle is fully deflected) I_G is around $50\ \mu A$.

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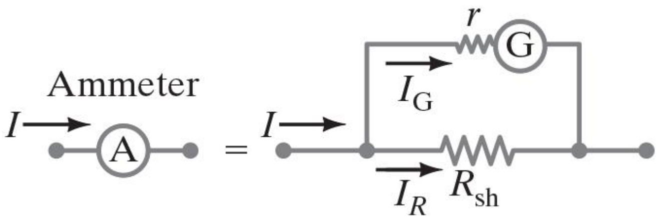

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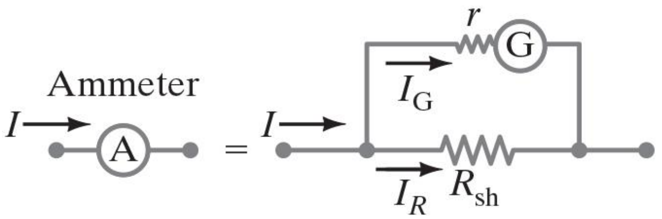
An analog ammeter consists of a galvanometer in parallel with a resistor called the shunt resistor. The shunt resistance is R_{sh} while the resistance in the galvanometer coil is r. R_{sh} is chosen according to the full-scale deflection needed, and is generally very small, so that (almost) all current flows through it.

26.7 – Ammeters and Voltmeters



An ammeter is placed in series with the portion of circuit where unknown current I must be measure.

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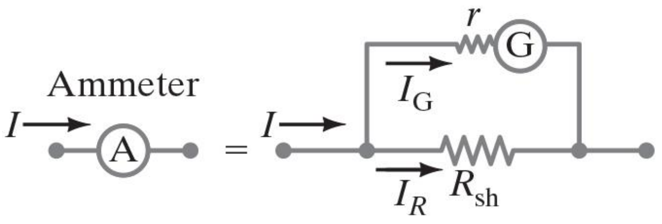

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In such a setting, we have that

$$I = I_G + I_R$$

as the current splits between the galvanometer and the shunt. We can also write

$$I_R R_{sh} = I_G r$$

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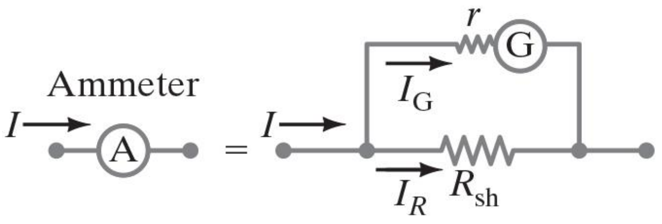

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In such a setting, we have that

$$I = I_G + I_R$$

as the current splits between the galvanometer and the shunt. We can also write

$$I_R R_{sh} = I_G r$$

as the potential across the galvanometer and the shunt is the same. Hence

$$(I - I_G) R_{sh} = I_G r \rightarrow R_{sh} = \frac{I_G r}{I - I_G}$$

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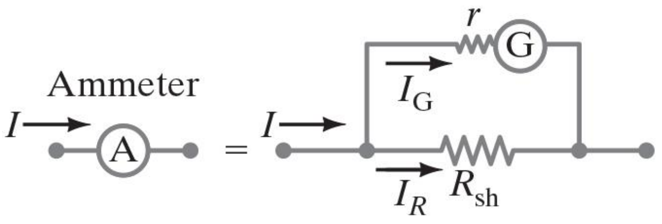

If we want to design an ammeter that reads $1.0\,A$ at full-scale using a galvanometer that has a full-scale sensitivity of $50\,\mu A$ and a resistance $r = 30\,\Omega$, what should the shunt resistance R_{sh} be?

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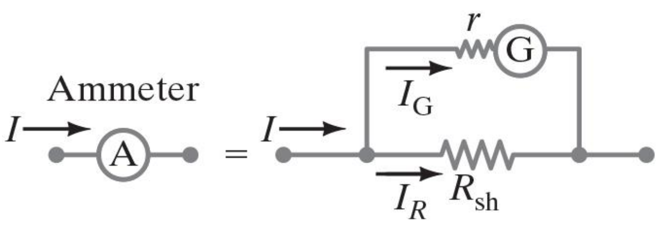

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At full-scale, we have $I_G = 0.00005\,A$ and hence $I_R = 1 - 0.00005 = 0.999950\,A$. Hence

$$R_{sh} = \frac{(5.0 \times 10^{-5}\,A)(30\,\Omega)}{0.999950\,A} = 1.5 \times 10^{-3}\Omega$$

As anticipated, the shunt resistance must be very small.

26.7 – Ammeters and Voltmeters

A voltmeter is a galvanometer with a resistor R_{ser} is series, and is attached in parallel to the portion of the circuit where we need to determine the unknown voltage.

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with the resistance and the galvanometer seeing the same current. Hence

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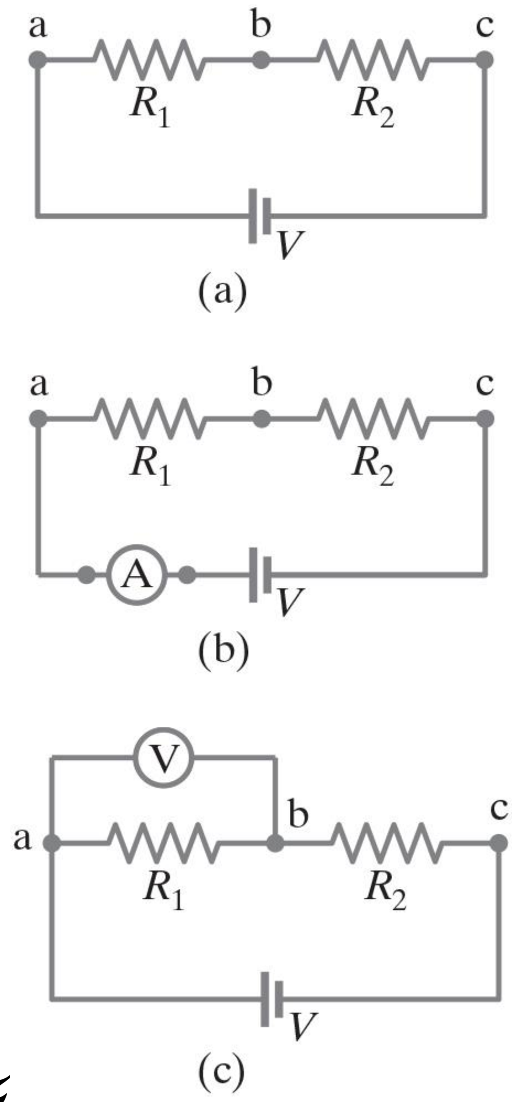
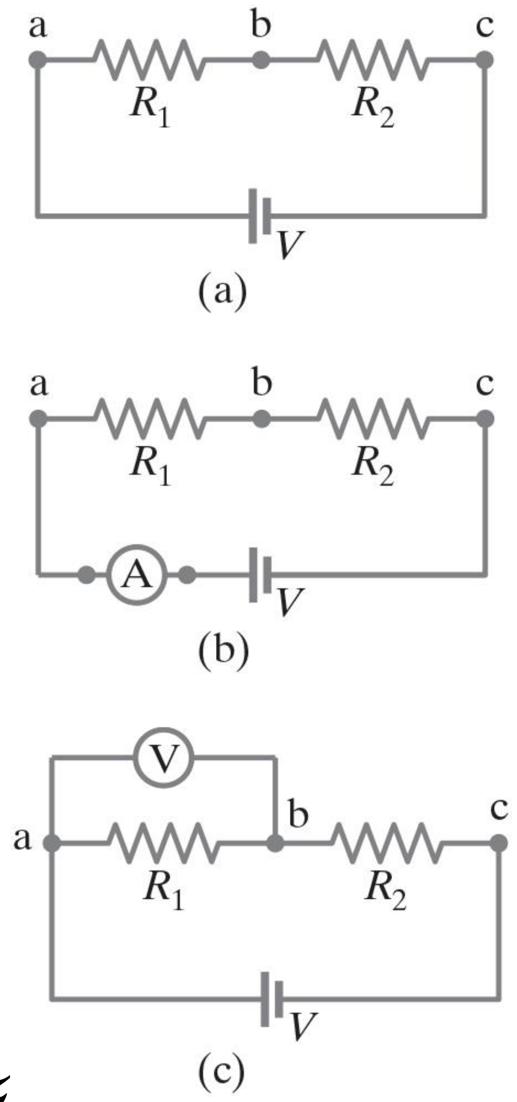
If we want to design a voltmeter that reads $15.0\,V$ at full-scale using a galvanometer that has a full-scale sensitivity of $50\,\mu A$ and a resistance $r = 30\,\Omega$, what should the resistance R_{ser} be? $\boldsymbol{R_{ser} = \frac{15.0\,V}{0.00005\,A} - 30\,\Omega \simeq 300\,k\Omega}$

26.7 – Ammeters and Voltmeters



Why are we connecting an ammeter and voltmeter differently?

26.7 – Ammeters and Voltmeters



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An ammeter measures current, hence it must be inserted directly into the circuit (in series) where such a current must be measured.

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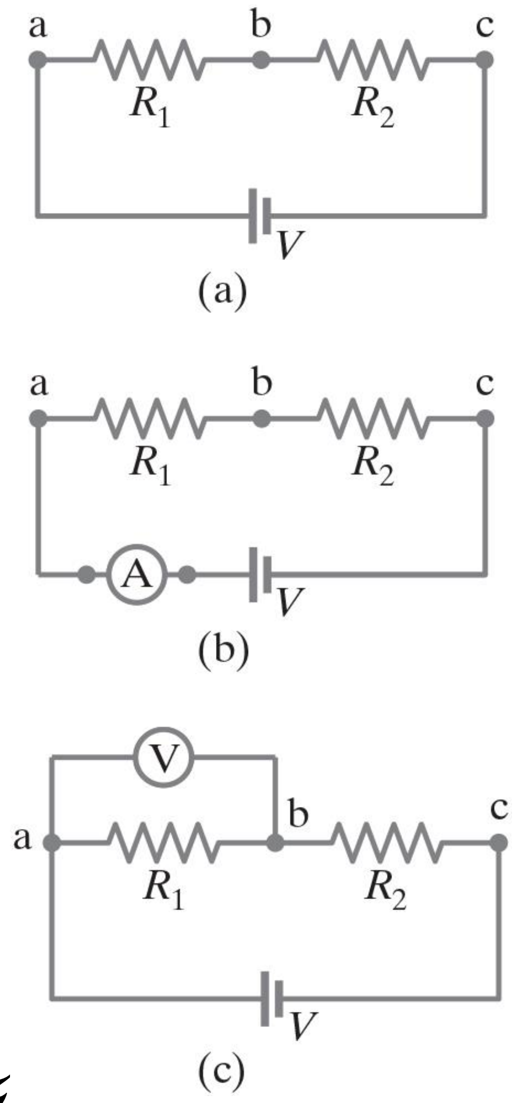

Why are we connecting an ammeter and voltmeter differently?

An ammeter measures current, hence it must be inserted directly into the circuit (in series) where such a current must be measured.

Conversely, a voltmeter must be connected "externally" in parallel with the circuit element of which the unknown voltage must be measured.

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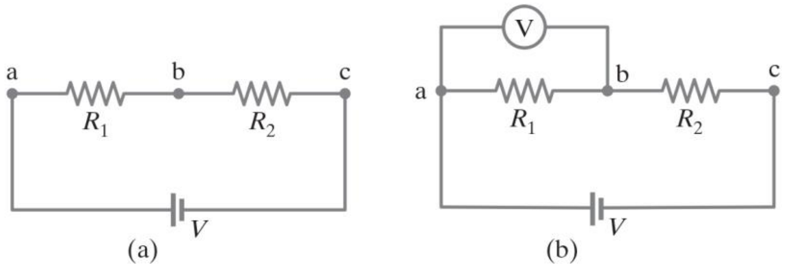
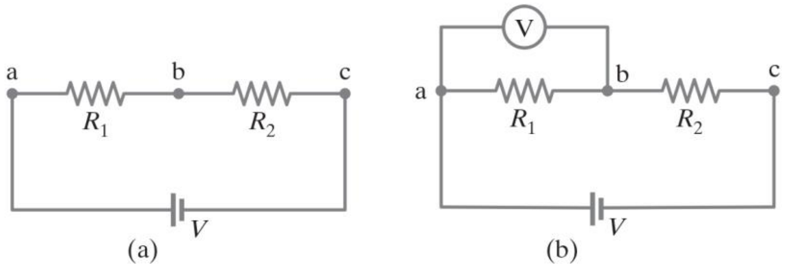


Finally, we should acknowledge that whatever we measure is not the true value, as we are using an external device that affects (albeit minimally) the system.

26.7 – Ammeters and Voltmeters



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Let us take the example shown on the left, where we have a voltmeter in parallel to R_1. The full resistance of the voltmeter, assuming $R_{ser} \gg r$, is just R_{ser}.

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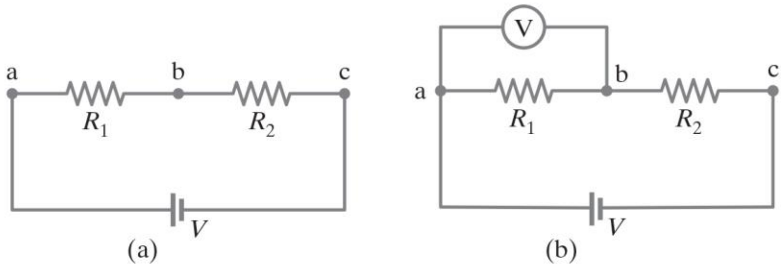

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The true voltage we should measure is $\boldsymbol{V_1 = V\frac{R_1}{R_1+R_2}}$ (we can use Kirchhoff or the equivalent resistance).

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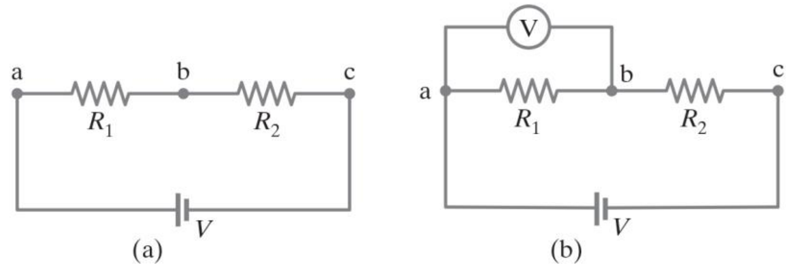

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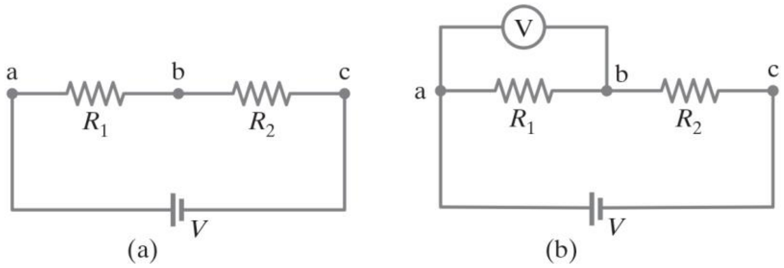
The true voltage we should measure is $\boldsymbol{V_1 = V\frac{R_1}{R_1+R_2}}$ (we can use Kirchhoff or the equivalent resistance).

The actual measured voltage can be computed as follows. First, we compute the equivalent resistance between R_1 and $R_{ser} \rightarrow R_{eq} = \frac{R_1 R_{ser}}{R_1+R_{ser}}$

26.7 – Ammeters and Voltmeters



The current flowing in the circuit is

$$I = \frac{V}{\frac{R_1 R_{ser}}{R_1 + R_{ser}} + R_2}$$

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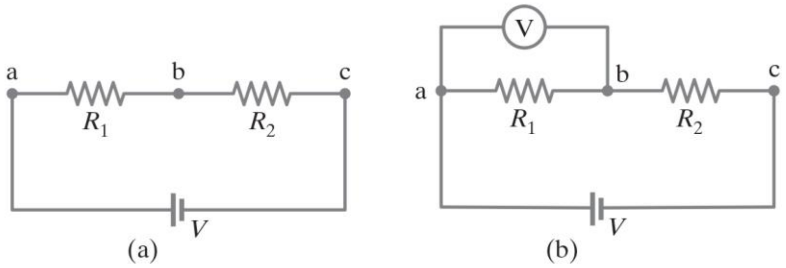

The current flowing in the circuit is

$$I = \frac{V}{\frac{R_1 R_{ser}}{R_1 + R_{ser}} + R_2}$$

and the measured potential V_1^{meas} is

$$V_1^{meas} = \frac{V}{\frac{R_1 R_{ser}}{R_1+R_{ser}}+R_2} \frac{R_1 R_{ser}}{R_1+R_{ser}} = \frac{V \frac{R_1+R_{ser}}{R_1 R_{ser}+R_1 R_2+R_2 R_{ser}} \quad R_1 R_{ser}}{R_1+R_{ser}} =$$

$$V \frac{R_1 R_{ser}}{R_1 R_{ser}+R_1 R_2+R_2 R_{ser}}$$

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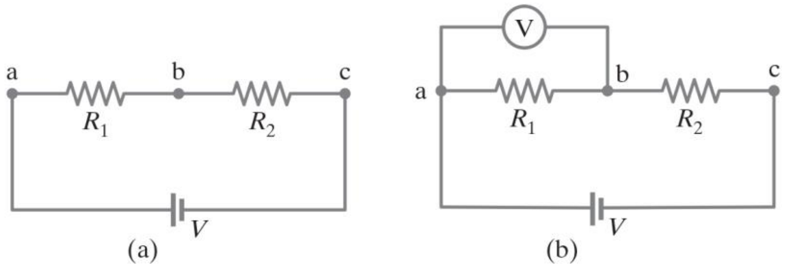

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which is not the actual voltage. If $R_{ser} \gg R_1, R_2$ then $R_1 R_2$ is negligible and we go back to $V_1^{meas} = V \frac{R_1}{R_1 + R_2}$

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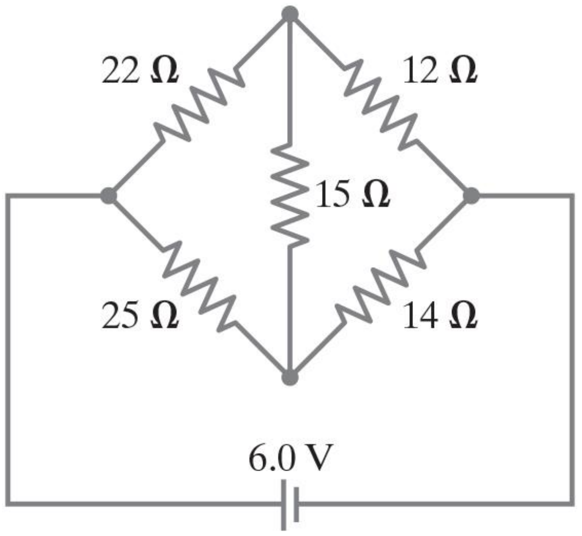
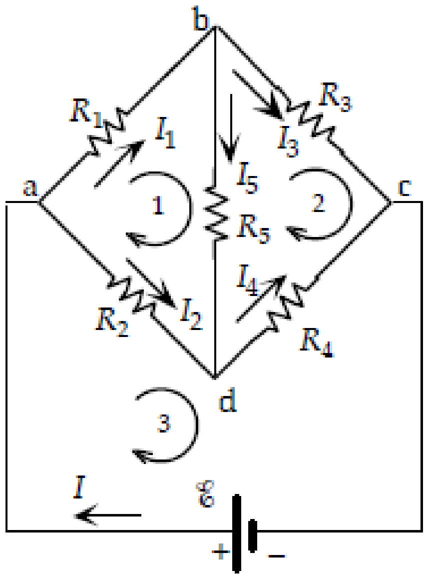
Exercise:





Determine all the currents in the circuit.

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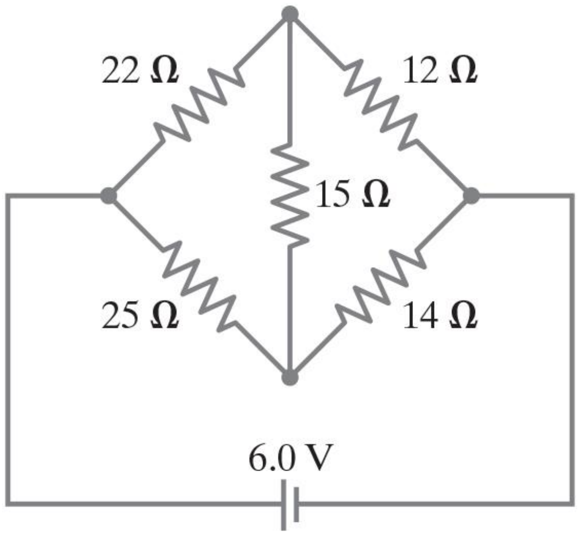
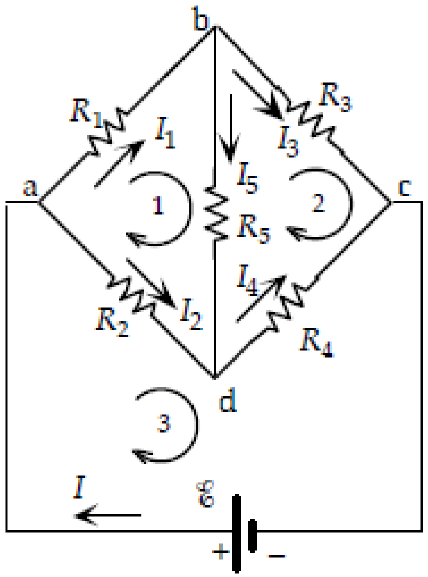


Determine all the currents in the circuit.

We use the junction law in junctions a, b, and c (we could have used d as an alternative. They are linearly dependent)

$$I = I_1 + I_2$$
$$I_1 = I_3 + I_5$$
$$I = I_3 + I_4$$



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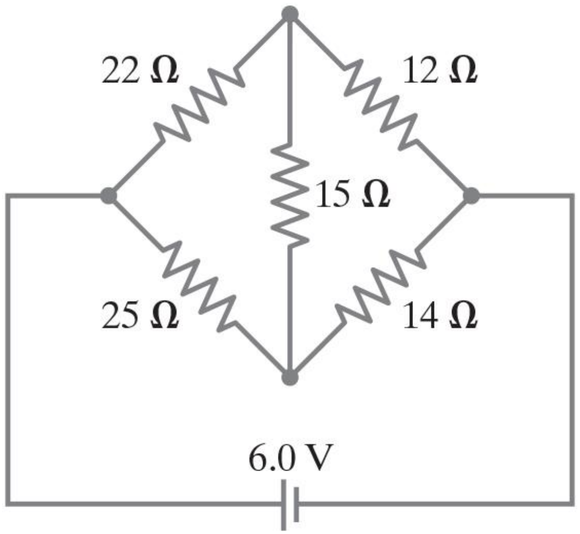

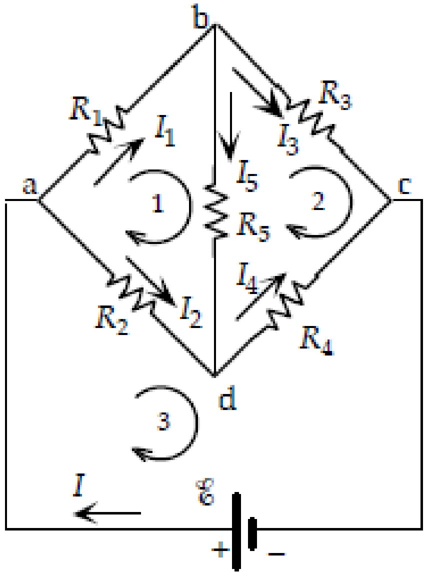

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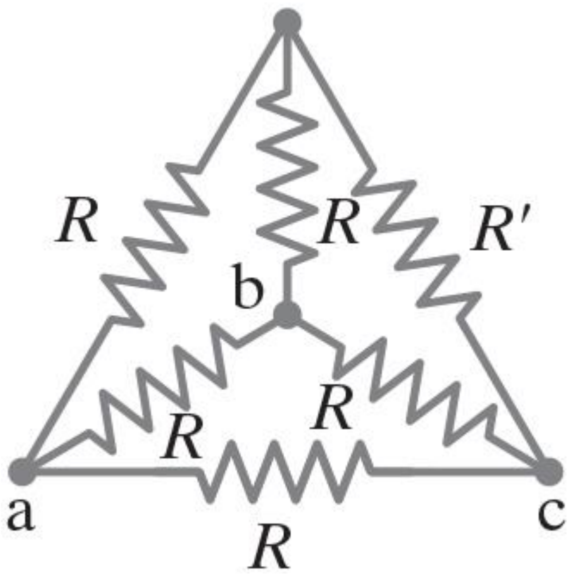
$$\boldsymbol{I = I_1 + I_2}$$
$$\boldsymbol{I_1 = I_3 + I_5}$$
$$\boldsymbol{I = I_3 + I_4}$$

and the loop rule in loops 1, 2, and 3

$$\boldsymbol{-R_1 I_1 - R_5 I_5 + R_2 I_2 = 0}$$
$$\boldsymbol{-R_3 I_3 + R_4 I_4 + R_5 I_5 = 0}$$
$$\boldsymbol{\varepsilon - R_2 I_2 - R_4 I_4 = 0}$$

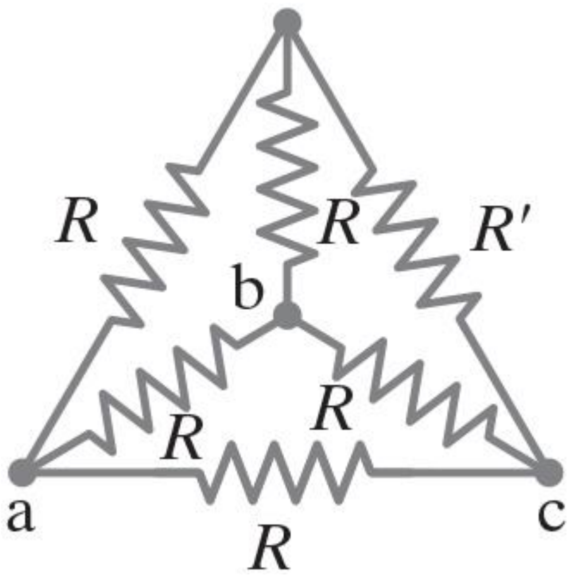
We can eliminate I by setting $I_1 + I_2 = I_3 + I_4$

Exercise:



Determine the potential V_{ba} across points a and b in the circuit to the left.

Exercise:



Determine the potential V_{ba} across points a and b in the circuit to the left.

Because of symmetry, no electric current can flow across resistance R', hence we can remove that part of the circuit. In such a case, we have two sets of resistances in series, whose $R_{eq} = 2R$ each, then in parallel with a resistance R. The overall equivalent resistance is

$$\boldsymbol{R_{eq} = \left(\frac{1}{R} + \frac{1}{2R} + \frac{1}{2R}\right)^{-1}}$$

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Wrap-up: revisiting Learning objectives

After this lecture you should be able to:

- Understand the general **characteristics** of an EMF
- Determine the **equivalent resistance** of a set of resistances in series and parallel
- Use **Kirchhoff's rules** to determine current and voltages in a circuit
- Understand the **transient dynamics** of RC circuits
- Understand the **functioning** of ammeters and voltmeters

$$\boldsymbol{V_{ab} = \varepsilon - Ir}$$

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$$\boldsymbol{V_{ab} = \varepsilon - Ir}$$

$$\boldsymbol{R_{eq} = (R_1 + R_2 + R_3), R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}}$$

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$$Q(t) = Q_0\left(1 - e^{-\frac{t}{RC}}\right)$$

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$$V(t) = V\left(1 - e^{-\frac{t}{RC}}\right)$$

$$R_{sh} = \frac{I_G r}{I - I_G}, R_{ser} = \frac{V}{I} - r$$

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