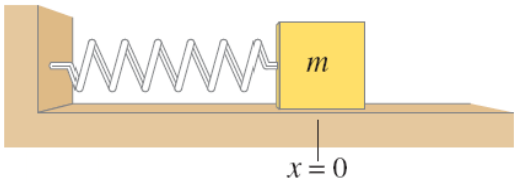
14.4 – Practical exercise

A 10-kg mass is attached to a spring with a stiffness constant k. It follows a simple harmonic motion given by the following equation:

$$x = 0.6\cos(31t + 0.25)$$



1. What is the stiffness constant k of the spring?
2. What is the oscillation period?
3. What is the maximum value of the mass' velocity and where does it occur (position in x)?
4. What is the maximum value of the mass' acceleration and where does it occur (position in x)?
5. What is the total energy of the simple harmonic oscillator?

14.4 – Practical exercise

A 10-kg mass is attached to a spring with a stiffness constant k. It follows a simple harmonic motion given by the following equation:

$$x = 0.6\cos(31t + 0.25)$$

1. What is the stiffness constant k of the spring?

$$\omega = 31\ (1/s)\ , \quad \omega = \sqrt{\frac{k}{m}}\,, \qquad k = \omega^2 m = 9610\ \mathrm{N/m}$$

2. What is the oscillation period?

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.2\ \mathrm{s}$$

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14.4 – Practical exercise

3. What is the maximum value of the mass' velocity and where does it occur (position in x)?

$$v = \frac{\partial x}{\partial t} = -18.6 \sin(31t + 0.25)$$

$\max(v) = \pm 18.6 \text{ m/s}$ and occurs at x = 0 m

4. What is the maximum value of the mass' acceleration and where does it occur (position in x)?

$$a = \frac{\partial v}{\partial t} = -576.6 \cos(31t + 0.25)$$

$\max(a) = \pm 576.6 \text{ m/s}^2$ (technically the absolute value), and occurs at x = ∓ 0.6 m, respectively

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14.4 – Practical exercise

5. What is the total energy of the simple harmonic oscillator?

$$E = \frac{1}{2}kA^2 = 1729.8\ J$$

$$E = \frac{1}{2}mv_{max}^2 = 1729.8\ J$$

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14.5 – Practical exercise

An 80-kg climber is attached to a long rope to the end of the route. If we neglect the stretching of the rope, if the climber drops:

1. What is the oscillation period if the length of the rope is 10 m?

2. If he drops when he is about to finish (rope length = 1 m), what would be the oscillation period now?

3. If the climber was very hungry before climbing and ate 2 kg of bitterballen, what would be the oscillation period now? Consider the rope length = 1 m.

14.5 – Practical exercise

An 80-kg climber is attached to a long rope to the end of the route. If we neglect the stretching of the rope, if the climber drops:

1. What is the oscillation period if the length of the rope is 10 m?

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{10}{9.8}} = 6.35\ s$$

2. If he drops when he is about to finish (rope length = 1 m), what would be the oscillation period now?

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1}{9.8}} = 2.01\ s$$

14.5 – Practical exercise

An 80-kg climber is attached to a long rope to the end of the route. If we neglect the stretching of the rope, if the climber drops:

3. If the climber was very hungry before climbing and ate 2 kg of bitterballen, what would be the oscillation period now? Consider the rope length = 1 m.

It would be the **same** (2.01 s) since the period of oscillation is not dependent on the mass nor amplitude of the motion.

14.7 – Practical exercise

A system moving in simple harmonic motion consists of a mass m of 120 kg and a spring with a stiffness constant k of 230 N/m.

1. Calculate the value of the damping constant, b, required to obtain critical damping.

2. For the same damping constant and an initial displacement amplitude of 0.5 m and an initial velocity of -0.69 m/s, provide the expression of the displacement x as a function of time and the value of the damped angular oscillation frequency ω'.

3. For the same conditions, provide the expressions for the kinetic and potential energy over time.

4. Calculate the instant in which the total energy of the system is reduced to half of the initial energy.

14.7 – Practical exercise

A system moving in simple harmonic motion consists of a mass m of 120 kg and a spring with a stiffness constant k of 230 N/m.

1. Calculate the value of the damping constant, b, required to obtain critical damping.

$$b = 2\sqrt{mk} = 332\ kg/s$$

2. For the same damping constant and an initial displacement amplitude of 0.5 m and an initial velocity of -0.69 m/s, provide the expression of the displacement x as a function of time and the value of the damped angular oscillation frequency ω'.

Since $\omega' = 0$, for the critically damped case, we have:

$$x(t) = Ae^{-\gamma t} \qquad v(t) = -\gamma A e^{-\gamma t}$$

$$\gamma = \frac{b}{2m} \sim 1.4 \qquad x(t) = 0.5e^{-1.4t} \qquad v(t) = -0.69e^{-1.4t}$$

Check that initial conditions ($t = 0$) are satisfied: $x(0) = 0.5$ $\qquad v(0) = -0.69\ \mathrm{m/s}$

14.7 – Practical exercise

A system moving in simple harmonic motion consists of a mass m of 120 kg and a spring with a stiffness constant k of 230 N/m.

3. For the same conditions, provide the expressions for the kinetic and potential energy over time.

$$K(t) = \frac{1}{2}mv(t)^2 \qquad v(t) = -0.69e^{-1.4t} \qquad K(t) = 29e^{-2.8t}$$

$$U(t) = \frac{1}{2}kx^2 = 29e^{-2.8t} = K(t)$$

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14.7 – Practical exercise

A system moving in simple harmonic motion consists of a mass m of 120 kg and a spring with a stiffness constant k of 230 N/m.

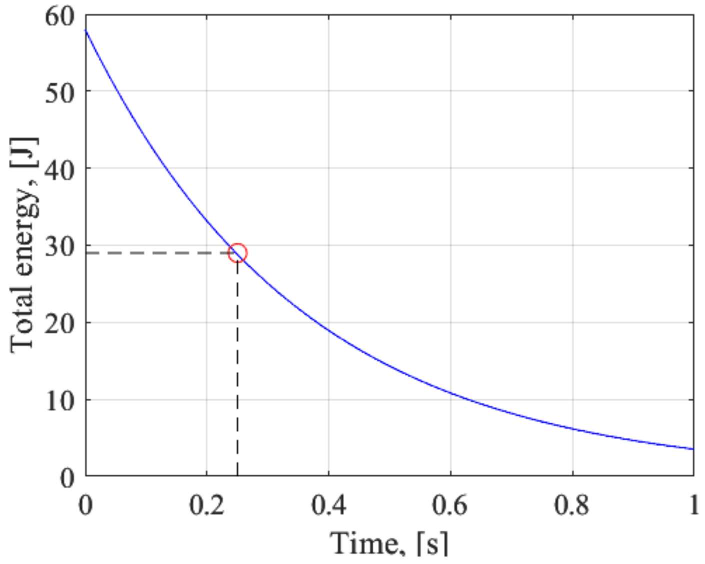
4. Calculate the instant in which the total energy of the system is reduced to half of the initial energy.

$$E(t) = K(t) + U(t) = 58e^{-2.8t}$$

The energy is half of the initial value when $e^{-2.8t} = \frac{1}{2}$, which gives:

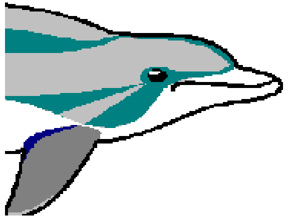
$$-2.8t = -\ln 2$$

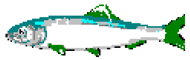
$$t = \frac{1}{2.8}\ln 2 = 0.25\ s$$



15.2 – Practical exercise

Some animals (e.g. bats, dolphins, whales) use echolocation as a form of perception. The animal emits a pulse of sound which, after being reflected on an object, returns back, and is detected by the animal.





$$f = 100{,}000 \text{ Hz}, \; B = 2 \times 10^9 \text{ N/m}^2, \; \rho = 1.025 \times 10^3 \text{ kg/m}^3$$

1. What is the wavelength?
2. How long does it take to the animal to detect an obstacle 100 m away?

15.2 – Practical exercise

$f = 100{,}000$ Hz, $B = 2 \times 10^9$ N/m^2, $\rho = 1.025 \times 10^3$ kg/m^3

1. What is the wavelength?

$v = \lambda f, \qquad \lambda = \frac{v}{f}$

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2\times 10^9}{1.025\times 10^3}} = 1.4 \times 10^3 \text{ m/s}$$

$$\lambda = \frac{v}{f} = \frac{1.4 \times 10^3}{100{,}000} = 14 \text{ mm}$$

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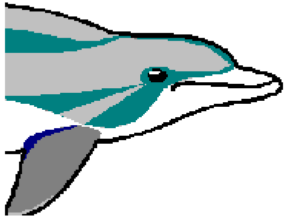
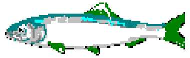
15.2 – Practical exercise

$f = 100{,}000$ Hz, $B = 2 \times 10^9$ N/m^2, $\rho = 1.025 \times 10^3$ kg/m^3

2. How long does it take to the animal to detect an obstacle 100 m away?

$$t = \frac{distance}{v} = \frac{2\times100}{1.4\times10^3} = 0.14 \text{ s}$$





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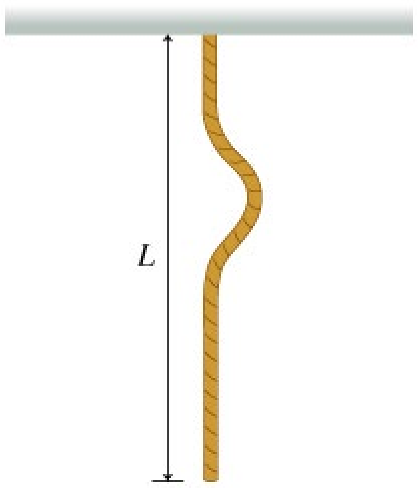
15.5 – Brain teaser from previous lecture

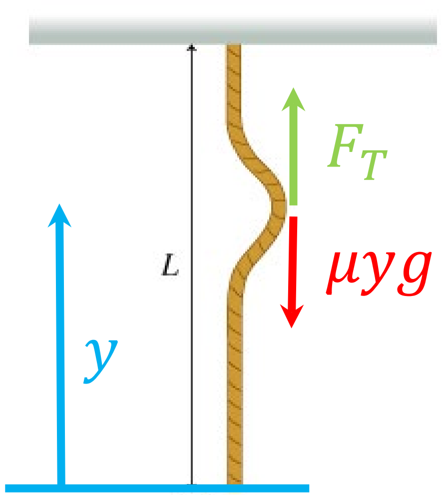
A uniform rope of length L, mass per unit length μ, and negligible stiffness hangs from a solid fixture in the ceiling. The free lower end of the rope is struck sharply at time t=0.



What is the time t it takes the resulting wave on the rope to travel to the ceiling, be reflected, and return to the lower end of the rope?

15.5 – Practical exercise

What is the time t it takes the resulting wave on the rope to travel to the ceiling, be reflected, and return to the lower end of the rope?



At a given y position along the rope, the rope mass below that position is μy and, therefore, the gravity force pulling downward at that position is $\mu y g$.

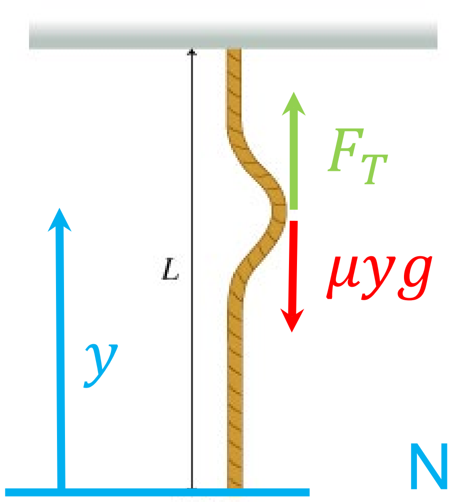
Since there is no displacement of the rope in the y direction, the downward force is compensated by an upward tension force F_T:

$$F_T = \mu y g$$

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{\mu y g}{\mu}} = \sqrt{yg}$$

15.5 – Practical exercise

What is the time t it takes the resulting wave on the rope to travel to the ceiling, be reflected, and return to the lower end of the rope?



$$v = \sqrt{yg} \qquad v = \frac{dy}{dt} \qquad dt = \frac{dy}{v}$$

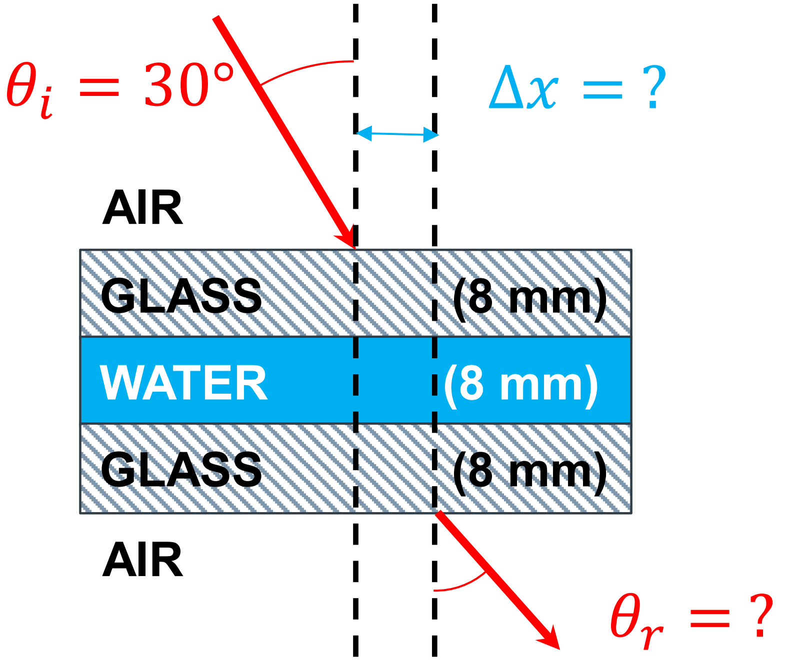
$$t = \int dt = \int_0^L \frac{dy}{v} = \int_0^L \frac{dy}{\sqrt{yg}} = 2\sqrt{\frac{L}{g}}$$

Note that this is just the time it takes for the wave to travel from the bottom of the rope to the ceiling. To travel back, it takes an equal amount of time. Therefore the total time is:

$$t_{total} = 4\sqrt{\frac{L}{g}}$$

15.5 – Practical exercise

A laser pointer reaches a double-glazed window at an incidence angle of 30° with respect to the vertical.



For light:

$$v_{air} \approx 299{,}702 \text{ km/s}$$

$$v_{glass} \approx 200{,}000 \text{ km/s}$$

$$v_{water} \approx 225{,}000 \text{ km/s}$$

1. What is the final refracted angle θ_r?
2. What is the horizontal distance Δx traveled by the ray within the glass?

15.5 – Practical exercise

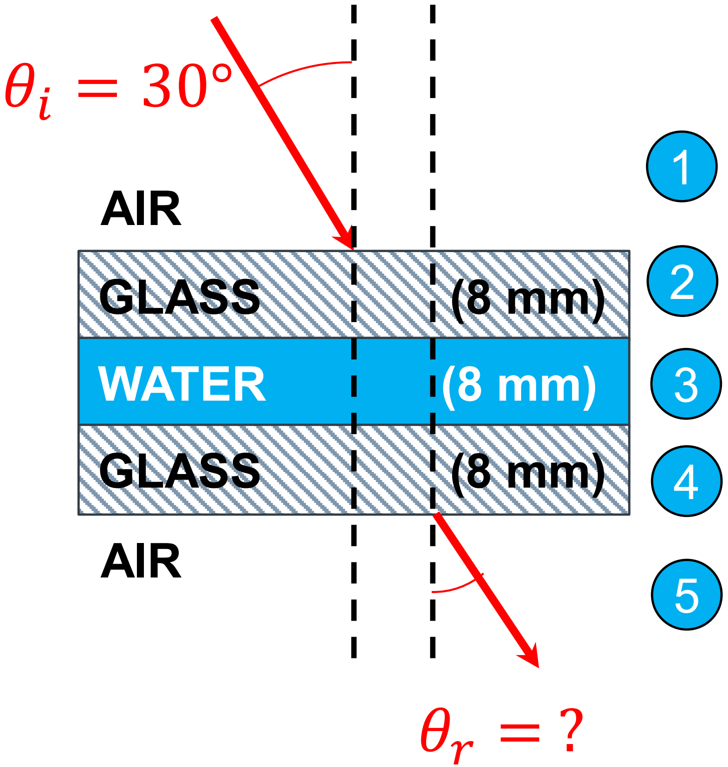
1. What is the final refracted angle θ_r?

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

$v_{air} \approx 299{,}702 \text{ km/s}$

$v_{glass} \approx 200{,}000 \text{ km/s}$

$v_{water} \approx 225{,}000 \text{ km/s}$



$$\sin\theta_2 = \sin\theta_1 \frac{v_2}{v_1} = \sin 30 \frac{200{,}000}{299{,}702} = 0.3337$$

$$\sin\theta_3 = \sin\theta_2 \frac{v_3}{v_2} = 0.3337 \frac{225{,}000}{200{,}000} = 0.3754$$

$$\sin\theta_4 = \sin\theta_3 \frac{v_4}{v_3} = 0.3754 \frac{200{,}000}{225{,}000} = 0.3337 = \sin\theta_2$$

$$\sin\theta_5 = \sin\theta_4 \frac{v_5}{v_4} = 0.3337 \frac{299{,}702}{200{,}000} = 0.5 = \sin\theta_1$$

$$\boxed{\theta_5 = 30°}$$

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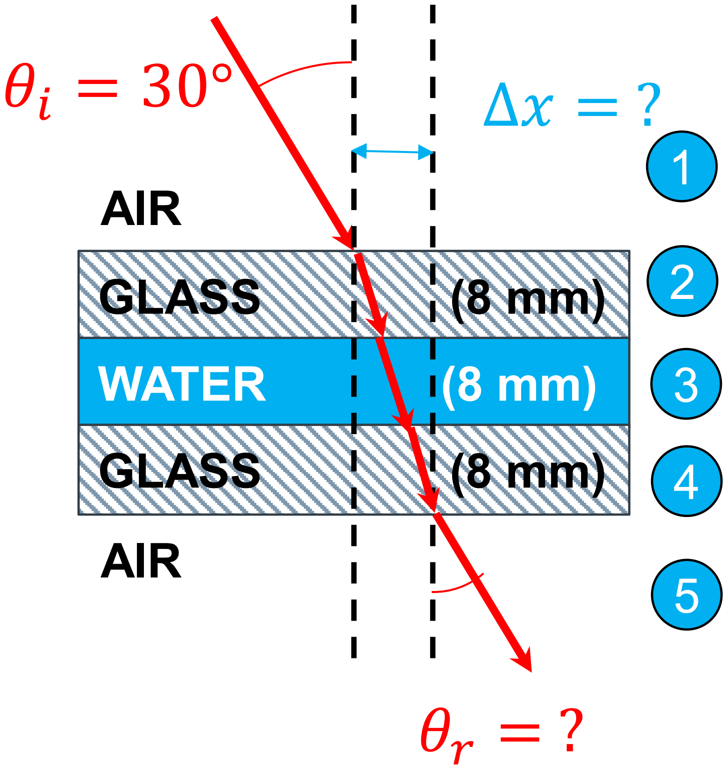
15.5 – Practical exercise

1. What is the horizontal distance Δx traveled by the ray within the glass?



$$\tan\theta_i = \frac{\Delta x_i}{d_i}$$

$\theta_1 = 30°$

$\theta_2 = 19.49°$

$\theta_3 = 22.05°$

$\theta_4 = 19.49°$

$\theta_5 = 30°$

$$\Delta x_2 = \tan\theta_2\, d_2 = \tan 19.49 \cdot 8 = 2.83\ \mathrm{mm}$$

$$\Delta x_3 = \tan\theta_3\, d_3\ \tan 22.05 \cdot 8 = 3.24\ \mathrm{mm}$$

$$\Delta x_4 = \tan\theta_4\, d_4 = \tan 19.49 \cdot 8 = 2.83\ \mathrm{mm}$$

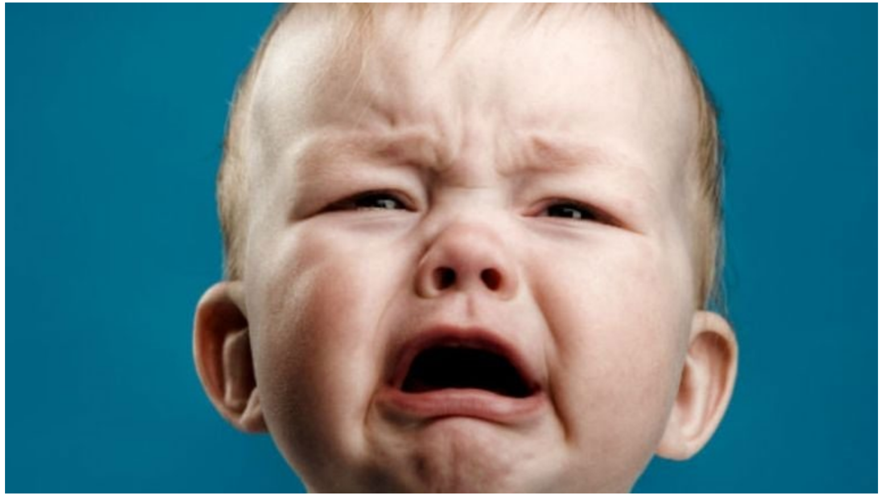
$$\boxed{\Delta x_{total} = \Delta x_2 + \Delta x_3 + \Delta x_4 = 2.83 + 3.24 + 2.83 = 8.9\ \mathrm{mm}}$$

16.3 – Practical exercise

A baby crying generates a SPL of 80 dB.

How many crying babies do we need to generate the SPL of a jet fighter taking off (140 dB)?

= X



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16.3 – Practical exercise

A baby crying generates a SPL of 80 dB.

How many crying babies do we need to generate the SPL of a jet fighter taking off (140 dB)?

$$140 = 80 + 10 \log N_{babies}$$

$$N_{babies} = 10^{(140-80)/10} = 10^{6} \text{ babies}$$

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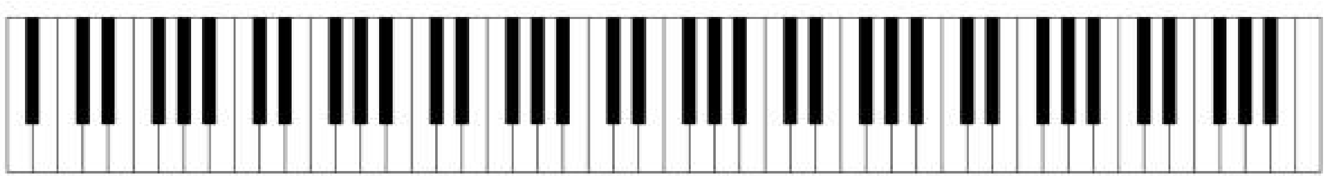
16.4 – Practical exercise

A full piano has 88 keys. The leftmost key has a fundamental frequency of approximately 27.5 Hz, whereas for the rightmost key it is 4186 Hz. Each key has a frequency about $\mathbf{2^{1/12}}$ **times** (~1.059 times) higher than the one to the left, meaning that for every 12 keys, the frequency roughly doubles.

Leftmost key

$f \sim 27\ \mathrm{Hz}$



Rightmost key

$f \sim 4186\ \mathrm{Hz}$

1. How many keys would a piano have to be suitable to create songs for entertaining elephants ($f \sim 15\ \mathrm{Hz}$), as well as bats ($f \sim 150\ \mathrm{kHz}$)?

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16.4 – Practical exercise

1. How many keys would a piano suitable to create songs for entertaining elephants ($f \sim 15\ \mathrm{Hz}$), as well as bats ($f \sim 150\ \mathrm{kHz}$)?

The number of extra keys n is given by:

$$\log_2 x = \frac{\log_n x}{\log_n 2}$$

$$f_{right} = f_{left}\left(2^{\frac{1}{12}}\right)^n \qquad \frac{f_{right}}{f_{left}} = 2^{\frac{n}{12}} \qquad \frac{n}{12} = \log_2\left(\frac{f_{right}}{f_{left}}\right)$$

$$n = 12\log_2\left(\frac{f_{right}}{f_{left}}\right)$$

For high frequencies: $n = 12\log_2\left(\frac{150000}{4186}\right) \approx 62$

For low frequencies: $n = 12\log_2\left(\frac{27.5}{15}\right) \approx 10$

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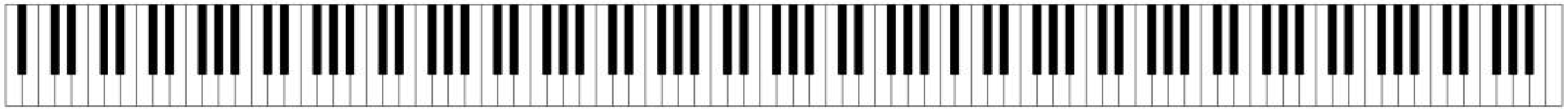
16.4 – Practical exercise

1. How many keys would a piano suitable to create songs for entertaining elephants ($f\sim 15\ \mathrm{Hz}$), as well as bats ($f\sim 150\ \mathrm{kHz}$)?

The total number of keys would therefore be:

$$N_{total} = 10\ (\text{low freq}) + 88\ (\text{normal piano}) + 62\ (\text{high freq}) = \mathbf{160\ keys}$$



16.4 – Practical exercise

1. A certain turbofan engine emits a sound pressure level of 110 dB. What would be the sound pressure level if four turbofan engines are present?

2. An aircraft equipped with these turbofan engines emits a tonal sound of 1800 Hz. The aircraft is moving towards an observer on the ground, who perceives a frequency of 2400 Hz because of the Doppler effect. What is the velocity of the aircraft in m/s? Take for the speed of sound 343 m/s.

3. If the aircraft flies at a Mach number of 2 and an observer on the ground perceives the shock wave 0.433 s after passing directly overhead the observer, at what altitude is the aircraft flying? Take for the speed of sound 343 m/s.

16.4 – Practical exercise

1. A certain turbofan engine emits a sound pressure level of 110 dB. What would be the sound pressure level if four turbofan engines are present?

$$SPL_{4engines} = SPL_{1engine} + 10\log N_{engines}$$

$$SPL_{4engines} = 110 + 10\log 4$$

$$SPL_{4engines} = 110 + 6$$

$$SPL_{4engines} = 116\text{ dB}$$

16.4 – Practical exercise

2. An aircraft equipped with these turbofan engines emits a tonal sound of 1800 Hz. The aircraft is moving towards an observer on the ground, who perceives a frequency of 2400 Hz because of the Doppler effect. What is the velocity of the aircraft in m/s? Take for the speed of sound 343 m/s.

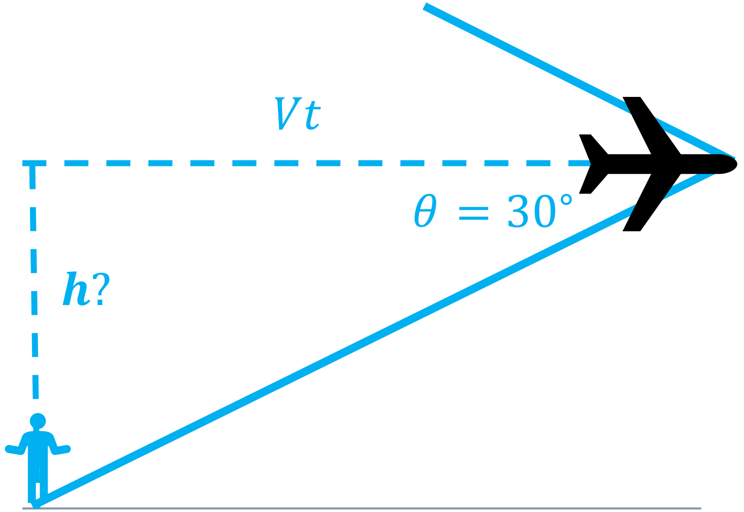
$$f' = \frac{f}{1 - M_s}$$

$$M_s = 1 - \frac{f}{f'} = 1 - \frac{1800}{2400} = 0.25$$

$$V_s = M_s v_{sound} = 0.25 \cdot 343 = 85.75 \text{ m/s}$$

16.4 – Practical exercise

3. If the aircraft flies at a Mach number of 2 and an observer on the ground perceives the shock wave 0.433 s after passing directly overhead the observer, at what altitude is the aircraft flying? Take for the speed of sound 343 m/s.



$$\sin\theta = \frac{1}{M} = \frac{1}{2} = 0.5 \qquad \theta = 30°$$

$$\tan 30 = \frac{h}{Vt} = \frac{h}{M v_{sound} t}$$

$$h = M v_{sound} t \tan 30 = 2 \cdot 343 \cdot 0.433 \tan 30$$

$$h = 171.5 \text{ m}$$

17.4 – Practical exercise

A car is equipped with a gas tank of 70 liters. Imagine that we fill up the tank to its top with gasoline when the weather outside is 10℃.
Surprisingly, it is an unusually warm day in the Netherlands and the temperature reaches 40℃. How much gasoline from the tank would overflow?

$\beta_{gasoline} = 950 \times 10^{-6}/℃ \quad \beta_{steel} = 35 \times 10^{-6}/℃$

17.4 – Practical exercise

A car is equipped with a gas tank of 70 liters. Imagine that we fill up the tank to its top with gasoline when the weather outside is 10℃.
Surprisingly, it is an unusually warm day in the Netherlands and the temperature reaches 40℃. How much gasoline from the tank would overflow?

$\beta_{gasoline} = 950 \times 10^{-6}/℃ \quad \beta_{steel} = 35 \times 10^{-6}/℃$

$$\Delta V_{gasoline} = \beta V_0 \Delta T = 950 \times \frac{10^{-6}}{℃} \times 70 \times (40 - 10) = 1.995\ l$$

$$\Delta V_{steel} = \beta V_0 \Delta T = 35 \times \frac{10^{-6}}{℃} \times 70 \times (40 - 10) = 0.0735\ l$$

$$V_{overflow} = \Delta V_{gasoline} - \Delta V_{steel} = 1.995 - 0.0735 = 1.92 l$$

17.7 – Practical exercise

1. How many particles (order of magnitude) are in this room at the moment? Use your engineering instincts for approximating the necessary parameters ☺.

2. Assuming I can talk forever (**which I can**), how long can we all keep breathing inside of the room? Consider the following:
   - The room is hermetically sealed (no air comes in or out).
   - The pressure and temperature remain the same as in point 1.
   - There is about 21% of oxygen in the air and below a concentration of 19% we faint.
   - Say that we are 300 people.
   - We breathe every ~5 seconds and each breath takes about 0.5 l of air (and same amount of oxygen particles).

17.7 – Practical exercise

1. How many particles (order of magnitude) are in this room at the moment?

Use your engineering instincts for approximating the necessary parameters ☺.

$$PV = nRT$$

$$n = \frac{PV}{RT} = \frac{101325\,\frac{\mathrm{N}}{\mathrm{m}^2} \cdot (5\ \mathrm{m} \times 20\ \mathrm{m} \times 40\ \mathrm{m})}{8.314\,\frac{\mathrm{J}}{\mathrm{mol}} \cdot \mathrm{K} \ \cdot \ (273.15 + 27)\ \mathrm{K}} \approx 162{,}000\ \mathrm{moles} = 1.62 \times\ 10^5\ \mathrm{moles}$$

$$N = nN_A = 1.62 \times\ 10^5\ \mathrm{moles} \times 6.02214 \times\ 10^{23}\,\frac{\mathrm{particles}}{\mathrm{mol}} \approx 9.76 \times\ 10^{28}\ \text{particles}$$

$$N \approx 10^{29}\ \text{particles}$$

17.7 – Practical exercise

2. Assuming I can talk forever (which I can), how long can we all keep breathing inside of the room? Consider the following:

$N_{air} \approx 9.76 \times\ 10^{28}$ particles

$N_{O_2} \approx 0.21 N_{air} = 2.05 \times\ 10^{28}$ molecules

$N_{O_{2,min}} \approx 0.19 N_{air} = 1.85 \times\ 10^{28}$ molecules

$N_{O_{2,available}} = N_{O_2} - N_{O_{2,min}} = 2.05 \times\ 10^{28} - 1.85 \times\ 10^{28} = 2 \times\ 10^{27}$ molecules

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17.7 – Practical exercise

2. Assuming I can talk forever (which I can), how long can we all keep breathing inside of the room? Consider the following:

In one breathe, we take (collectively): $300 \times 0.5l = 150\ l$ of air = 0.15 m^3.

$$n_{breath} = \frac{PV}{RT} = \frac{101325\,\frac{\mathrm{N}}{\mathrm{m}^2}\cdot(0.15\ \mathrm{m}^3)}{8.314\,\frac{\mathrm{J}}{\mathrm{mol}}\cdot\mathrm{K}\ \cdot\ (273.15+27)\ \mathrm{K}} = 6.09\ \text{moles of air}$$

$$N = nN_A = 6.09\ \text{moles} \times 6.02214 \times\ 10^{23}\,\frac{\text{particles}}{\text{mol}} \approx 3.67 \times\ 10^{24}\ \text{particles}$$

$$N_{O_{2,breath}} = 0.21N \approx 7.7 \times\ 10^{23}$$ oxygen molecules (<u>assumed constant</u>)

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17.7 – Practical exercise

2. Assuming I can talk forever (which I can), how long can we all keep breathing inside of the room? Consider the following:

$N_{O_2,available} \approx 2 \times \ 10^{27}$ molecules

$N_{O_2,breath} \approx 7.7 \times \ 10^{23}$ molecules/breath

$N_{breaths} = \frac{N_{O_2,available}}{N_{O_2,breath}} = \frac{2\times 10^{27}}{7.7\times 10^{23}} \approx 2600$ (collective) breaths

$t = \frac{N_{breaths}}{\frac{1 breath}{5\ s}} = \frac{2600}{1/5} \approx 13000\ \mathrm{s} \approx 3.6\mathrm{h} < 1.5\ \mathrm{h}$ ☺ so we are fine

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