Open Question 1

a) The intensity of 4 turbofans is 4 times as large as the intensity of a single turbofan. Let the sound level of one turbofan be given by $10\log\frac{I}{I_0}$, then the sound level of 4 turbofans is equal to $10\log\frac{4I}{I_0} = 10\log\frac{I}{I_0} + 10\log 4 = 110 + 6.0 = 116\ \mathrm{dB}$

b) The source is moving towards a steady observer. This gives a Doppler shift in frequency of $f' = \frac{f}{1-\frac{v_{source}}{v_{sound}}}$. Rewriting gives $v_{source} = v_{sound}\frac{f'-f}{f'} = 343\cdot\frac{2400-1800}{2400} = 85.75\frac{m}{s} = 309\frac{km}{h}$.

c) The Mach angle is given by $\sin\theta = \frac{1}{M}$. For $M = 2$ this gives $\theta = 30^{\circ}$. The tangent of this angle is given by $\tan\theta = \frac{h}{v_{aircraft}\Delta t}$. Rewriting gives $h = \tan\theta\cdot v_{aircraft}\Delta t = \tan\theta\cdot M\cdot v_{sound}\cdot\Delta t$. Filling in the numbers gives the height, $h = 171.5\ \mathrm{m}$.

Open Question 2 (See Example 19-15 from the book)

a) An isothermal process is a process which is executed at constant temperature. Since the internal energy is a function of the temperature only, the change in internal energy during the process is zero. The heat added to the system equals therefore the work applied to the gas.

b) An adiabatic process is a process which is executed without heat added or taken from the system during the process. As a consequence the change in internal energy equals the work applied to the gas.

c) For an adiabatic process we have that $pV^{\gamma} = const$. For a monatomic gas the ratio of specific heats $\gamma = \frac{5}{3}$, so we have $p_A V_A^{\gamma} = p_B V_B^{\gamma}$, therefore $V_B = V_A\left(\frac{p_A}{p_B}\right)^{\frac{1}{\gamma}} = 1.0\cdot 2^{-\frac{3}{5}} = 0.66\ \mathrm{m}^3$.

d) The pressure during the adiabatic process is related to the volume by $P = P_A V_A^{\gamma} V^{-\gamma}$, so the work done on the gas in the branch *AB* is given by $W_{AB} = -\int_A^B p\,dV = -P_A V_A^{\gamma}\int_A^B V^{-\gamma}dV = -P_A V_A^{\gamma}\cdot\left[\frac{1}{1-\gamma}\right]\left(V_B^{1-\gamma} - V_A^{1-\gamma}\right)$. Filling in the numbers gives $W_{AB} = 48\ \mathrm{kJ}$. In the branch BC the process is isothermal, therefore the pressure during the process is related to the volume by $p = \frac{nRT_B}{V}$. Therefore the work on the isothermal branch is $W_{BC} = -\int_B^C p\,dV = -nRT_B\int_B^C\frac{dV}{V} = -P_B V_B\ln\frac{V_C}{V_B}$. Once again, filling in the numbers gives $W_{BC} = 37\ \mathrm{kJ}$. The total work done on the gas in the process of going from A to C is $W_{AC} = W_{AB} + W_{BC} = 48 + 37 = 85\ \mathrm{kJ}$.

Open Question 3

a) An electric compressor forces the gas in the cooling system to a very high pressure. Through a heat exchanger on the rear, outside the freezer and amount of heat $Q_H$is given off to the environment and the gas cools to become a liquid. The liquid passes from a high pressure region, through an expansion valve to a low pressure tubes on the inside of the freezer. The liquid evaporates at this low pressure and absorbs heat Q_L from the inside of the freezer. The fluid returns to the compressor where the cycle starts again.

b) To cool 500 g of water from 15 °C to 0 °C requires $4186\cdot 0.5\cdot 15 = 31395$ J, where 4186 is the specific heat of water at 15 °C. To turn half the amount of water into ice requires $333\cdot 10^3\cdot$

$0.25 = 83250$ J, where $333 \cdot 10^3$ is the heat of fusion for water. So the total amount of energy required is 31395 + 83250 = 114645 = 115 kJ.

c) The COP of the freezer is defined as the amount of heat taken from the freezer divided by the amount of energy inserted in this process. In part b) the amount of heat taken from the freezer was calculated. The amount of energy required to do so is the power *P* times the duration, i.e. $E = P \cdot \Delta t = 200 \cdot 5 \cdot 60 = 60$ kJ, so the Coefficient of Performance is $COP = \frac{Q_L}{E} = \frac{115 \cdot 10^3}{60 \cdot 10^3} = 1.9$.

d) $\Delta S = \frac{Q_L}{T} = \frac{115 \cdot 10^3}{20+273} = 392$ J/°C

Open Question 4

a) The potential due to ring with radius *r* total charge dq is $dV = \frac{dq}{4\pi\epsilon_0\sqrt{r^2+x^2}}$, see Example 23-9 from the book. In the book, Example 23-10, a charge distribution is integrated over a disk. Here we integrate over a ring. We know that

$$\frac{dq}{Q} = \frac{2\pi r\, dr}{\pi(R_2^2 - R_1^2)} = \frac{2r\, dr}{(R_2^2 - R_1^2)}$$

Therefore we have that

$$V = \int dV = \int_{R_1}^{R_2} \frac{2Qr\, dr}{4\pi\epsilon_0(R_2^2 - R_1^2)\sqrt{r^2+x^2}} = \frac{Q}{2\pi\epsilon_0(R_2^2 - R_1^2)}\left[\sqrt{R_2^2 + x^2} - \sqrt{R_1^2 + x^2}\right]$$

b) Due to symmetry $E_y = E_z = 0$ and $E_x = -\frac{\partial V}{\partial x} = \frac{-Q}{2\pi\epsilon_0(R_2^2 - R_1^2)}\left[\frac{x}{\sqrt{R_2^2+x^2}} - \frac{x}{\sqrt{R_1^2+x^2}}\right]$

c) If $x \gg R_2$ then also $x \gg R_1$. We can then write

$$E_x = \frac{-Q}{2\pi\epsilon_0(R_2^2 - R_1^2)}\left[\frac{1}{\sqrt{1+\left(\frac{R_2}{x}\right)^2}} - \frac{1}{\sqrt{1+\left(\frac{R_1}{x}\right)^2}}\right] \approx \frac{Q}{2\pi\epsilon_0(R_2^2 - R_1^2)}\left[1 - \frac{1}{2}\left(\frac{R_2}{x}\right)^2 - \left(1 - \frac{1}{2}\left(\frac{R_1}{x}\right)^2\right)\right]$$

Here we used the a Taylor series expansion $\frac{1}{\sqrt{1+a}} \approx 1 - \frac{1}{2}a$ for $a \ll 1$. Simplifying gives

$$E_x = \frac{-Q}{4\pi\epsilon_0(R_2^2 - R_1^2)}\left(\frac{R_1^2 - R_2^2}{x^2}\right) = \frac{Q}{4\pi\epsilon_0 x^2}$$

This expression makes sense, because very far away from the disk, it looks as if all charge is located at the origin. This conforms with the expression for E_x because this is the electric field component for a charge *Q* located at the origin.

d) If $x \ll R_1$ then also $x \ll R_2$ and we can simplify the expression for E_x using $\frac{x}{\sqrt{R^2+x^2}} = \frac{x}{R\sqrt{1+\left(\frac{x}{R}\right)^2}} \approx \frac{x}{R}$. This gives

$$E_x \approx \frac{-Q}{2\pi\epsilon_0(R_2^2 - R_1^2)}\left[\frac{x}{R_2} - \frac{x}{R_1}\right] = \frac{-Qx}{2\pi\epsilon_0(R_2^2 - R_1^2)}\frac{R_1 - R_2}{R_1 R_2} = \frac{Qx}{2\pi\epsilon_0(R_1 + R_2)R_1 R_2}$$

e) For a negative charge the force is $-q$

$$F_x \approx \frac{-qQx}{2\pi\epsilon_0(R_1 + R_2)R_1 R_2}$$

The other two force components are zero. This is a linear restoring force. Applying Newton's second law then gives

$$m\frac{d^2x}{dt^2} = \frac{-qQx}{2\pi\epsilon_0(R_1 + R_2)R_1 R_2}$$

Therefore the angular frequency is

$$\omega = \sqrt{\frac{qQ}{2\pi\epsilon_0 m(R_1 + R_2)R_1 R_2}}$$

And therefore the period is given by

$$T = 2\pi\sqrt{\frac{2\pi\epsilon_0 m(R_1 + R_2)R_1 R_2}{qQ}}$$

Question 1 The period of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. This period is independent of the mass, so if the mass changes, the period remains the same. Correct answer: D

Question 2 For a damped oscillation we have $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$. If the expression underneath the square root is positive, we have (damped) oscillations. If this expression is negative, there will be no oscillations and the harmonic motion is over-damped. If $\frac{k}{m} - \frac{b^2}{4m^2} = 0$ the motion is critically damped. In this case $b = 2\sqrt{km}$. Correct answer: C

Question 3 The displacement of a traveling wave is given by $D(x,t) = A\sin(kx - \omega t + \phi)$, see page 405 of the book. If we compare this with the traveling wave that is given, we see that $\omega = 12.57$. Then $T = \frac{2\pi}{\omega} = 0.5$ s. Correct answer: D

Question 4 We have that $\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$. Therefore $\sin\theta_2 = \sin\theta_1\frac{v_2}{v_1} = \frac{1}{2}\sqrt{2}\frac{6}{10} = 0.42$. From which $\theta_2 = 25^\circ$. Correct answer: D

Question 5 At 1 m we have the sound level $\beta_1 = 10\log\frac{I}{I_0}$. At 100 m the intensity becomes 100^2 smaller, therefore $\beta_{100} = 10\log\frac{I/100^2}{I_0} = 10\log\frac{I}{I_0} - 10\log 10^4 = \beta_1 - 40$. $\beta_1 = 135$ dB, therefore $\beta_{100} = 135 - 40 = 95$ dB. Correct answer: C

Question 6 Take the *y*-axis from the bottom of the rope upwards. At a certain position *y* along the rope, the amount of mass of the rope below this position is $y\mu$, where μ is the mass density of the rope per unit length. Therefore the gravity force pulling downward at this position *y* is equal to $y\mu g$. Since there is no displacement of the rope in the *y*-direction, this downward force is compensated by an upward tension force equal to $F_T = y\mu g$. The wave velocity is given by $v = \sqrt{\frac{F_T}{\mu}} = \sqrt{gy}$, see equation (15-2) in the book. Now we use the fact that $v = \frac{dy}{dt}$, which gives us $t = \int dt = \int_0^L \frac{dy}{v} = \int_0^L \frac{dy}{\sqrt{gy}} = 2\sqrt{\frac{L}{g}}$. Note that this is the time it takes for the wave to travel from the bottom of the rope to the ceiling. To travel back, it takes an equal amount of time, therefor the time it takes to travel back and forth is $4\sqrt{\frac{L}{g}}$. Correct answer: C

Question 7 We use $\Delta l = \alpha l_0 \Delta T$ this gives $\Delta l = 12 \cdot 10^{-6} \cdot 100 \cdot (60 - 20) = 0.048$ m = 4.8 cm. Correct answer: A

Question 8 The root mean square velocity is given by $v_{rms} = \sqrt{\frac{3kT}{m}}$. The temperature is the same. The mass of oxygen molecules is higher than the mass of nitrogen molecules, therefore the root mean square velocity of the nitrogen is higher than the average velocity of oxygen. Correct answer: B

Question 9 See section 18-4-3 on Boiling in the book. Mexico City is at higher elevation than Singapore and therefore the atmospheric pressure is lower. The boiling temperature in Mexico City is lower and it will therefore take longer to cook the food. Correct answer: A

Question 10 The saturation pressure of water at 20 °C is $2.33 \cdot 10^3$Pa. The volume of the room is 45 m^3 and the temperature is 293 K. The number of moles water in the room is $n = \frac{p_{sat}V}{RT} = 774$ g. This would be the amount of mass when fully saturated, but now the humidity is 60%, so the mass of water in the room is $775 \times 0.6 = 464$ g. Probably different rounding off. Correct answer: A

Question 11 Use the Stefan-Boltzmann equation given in the exam. $\frac{Q}{t} = \epsilon\sigma A(T_{skin}^4 - T_{room}^4) = 0.9 \cdot 5.67 \cdot 10^{-8} \cdot 1.5 \cdot (310^4 - 290^4) = 166$ W. Correct answer: C

Question 12 In an adiabatic process no heat is added or taken from the system, therefore $dQ = 0$, therefore the entropy change is also zero. Correct answer: D

Question 13 A Carnot cycle does not contain an iso-volumetric branch, therefore answer A is wrong. The efficiency of the Carnot cycle is optimal efficiency that can be achieved and the typical efficiency is always lower, so answer B is wrong. The efficiency of the Carnot cycle is given by $e = 1 - \frac{T_L}{T_H}$. If we lower T_L by a small amount ΔT, the efficiency becomes $e_1 = 1 - \frac{T_L - \Delta T}{T_H}$, so the efficiency increases by $\frac{\Delta T}{T_H}$. If we increase T_H by ΔT the efficiency is $e_2 = 1 - \frac{T_L}{T_H + \Delta T} \approx 1 - \frac{T_L}{T_H} + \frac{T_L}{T_H^2}\Delta T$. Because $\frac{T_L}{T_H^2} < \frac{1}{T_H}$ it is more efficient to reduce the lower temperature by an amount ΔT than to increase the higher temperature by ΔT. Correct answer: D

Question 14 We can decompose the tension force in the string in a component in the *x*- and *y*-direction as $F_T \sin\theta$ and $F_T \cos\theta$, respectively. For equilibrium we need to have

$$F_T \cos\theta = mg \quad and \quad F_T \sin\theta = \frac{\left({}^{Q}/_{2}\right)^2}{4\pi\epsilon_0 d^2}$$

Eliminating the tensor force from the two equations gives

$$\tan\theta = \frac{\left({}^{Q}/_{2}\right)^2}{4\pi\epsilon_0 d^2 mg}$$

Here $d = 2l\ \sin\theta$. Using these relations we have

$$Q^2 = 64\pi\epsilon_0 mg \tan\theta\,(l\sin\theta)^2 = 1.85\cdot 10^{-11}\ \mathrm{C}^2$$

This gives $Q = 4.3\cdot 10^{-6}$ C $= 4.3$ µC. Correct answer: D

Question 15 This is a combination of Example 24-5 and Problem 31 from the book. We know that equivalent capacitance of the three equal capacitors on top is equal to $C_{eq} = \frac{2}{3}C$. Furthermore, from Problem 31, also discussed in the lecture, we have that the charge on the equivalent capacitor is equal to $Q_{eq} = \frac{C_{eq}C_0}{C_{eq}+C_0}V_0$. Using the value for the equivalent capacitance we get $Q_{eq} = \frac{\frac{2}{3}CC_0}{\frac{2}{3}C+C_0}V_0$. Correct solution: A

Question 16 The capacitance of a cylindrical capacitor of length *l* is given by $C = \frac{2\pi\epsilon_0 l}{\ln {}^{R_A}/_{R_b}}$. If the cylinder is partially filled with a dielectric material, we essentially have two cylindrical capacitors in parallel. The capacitance is then given by

$$C = K_{liq}\frac{2\pi\epsilon_0 h}{\ln {}^{R_A}/_{R_b}} + K_V\frac{2\pi\epsilon_0 (l-h)}{\ln {}^{R_A}/_{R_b}}$$

Solving for *h/l* gives

$$\frac{h}{l} = \frac{C}{l}\frac{\ln {}^{R_A}/_{R_b}}{2\pi\epsilon_0\left(K_{liq}-K_V\right)} - \frac{K_V}{K_{liq}-K_V}$$

Filling in the numbers given gives $\frac{h}{l} = 0.326$ so the percentage of fuel in the tank is 33%. Correct answer: B

Question 17 The electric potential for a point source *Q* is $V = \frac{Q}{4\pi\epsilon_0 d}$, where *d* is the distance from the position where the charge *Q* is located. We set the potential to zero at infinity, but since we look at a potential differential any other convention will give the same solution. Initially the distance is $(L-h)$, so $V_a = \frac{Q}{4\pi\epsilon_0 (L-h)}$. The distance from the point charge to the origin is *h*, so there the potential is $V_b = \frac{Q}{4\pi\epsilon_0 h}$. So the potential difference is

$$\Delta V = V_b - V_a = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{h} - \frac{1}{L-h}\right] = \frac{Q}{4\pi\epsilon_0}\frac{L-2h}{h(L-h)}$$

The change in electric potential energy is then $\Delta U = q\Delta V = \frac{qQ}{4\pi\epsilon_0}\frac{L-2h}{h(L-h)} = \frac{qQ}{4\pi\epsilon_0}\frac{2h-L}{h(h-L)}$. This difference is independent of the path taken. Correct answer: C

Question 18 Let the charges be given by q_1 and q_2. Both charges have opposite sign, because they attract each other. The sum of the two charges is $q_1 + q_2 = 90$ µC. The electric force between them is

$$\frac{q_1 q_2}{4\pi\epsilon_0 d^2} = -12\ \mathrm{N}$$

Where $d = 1.16$ m. We have a minus sign in front of the force, because the charges attract each other. Here we have two equations with two unknowns, which we can solve. $q_2 = Q - q_1$. Insert this in the force equation gives the quadratic equation

$$q_1^2 - Qq_1 = 48\pi\epsilon_0 d^2$$

Which we can solve to find the charges $106.8 \cdot 10^{-6}$ C, $-16.8 \cdot 10^{-6}$ C. Converting to micro Coulomb gives 107 µC and -16.8 µC. Correct answer: D

Question 19 For $V(x, y, z) = -Kxy$, we find the electric field, $E_x = Ky$, $E_y = Kx$ and $E_z = 0$. The divergence of this electric field is zero, so the charge density is zero. No charge inside the box. One can also use Gauss, but then one will notice that the electric flux on opposite sides of the cube cancel. Correct answer: A

Question 20 One can close of the hemisphere with a circular disk of radius *r.* Since there are no charges present, Gauss states that the total electric flux through the surface consisting of the disk and hemisphere is equal to zero. Since the disk is oriented perpendicular to the electric field, the electric flux through the disk is $\pi r^2 E$, where E denotes the magnitude of the (constant) electric field. The same flux needs to pass through the hemisphere, since the total flux is zero. Correct answer: A