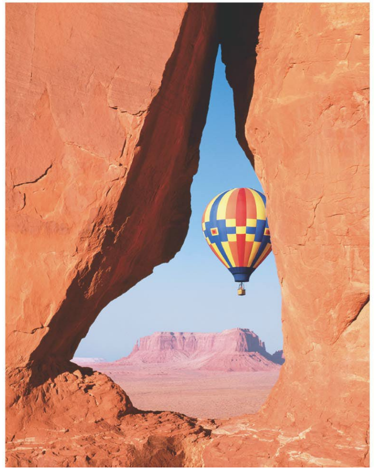
Temperature, thermal expansion and the ideal gas law

Chapter 17

Dr. Roberto Merino-Martinez

Operations & Environment section

Faculty of Aerospace Engineering

TU Delft

Quick reminder or last lecture (sound)

Adding SPLs of incoherent sound sources:

For coherent sources (e.g. 2 sine waves of the same frequency) it depends on the relative phase (interference)

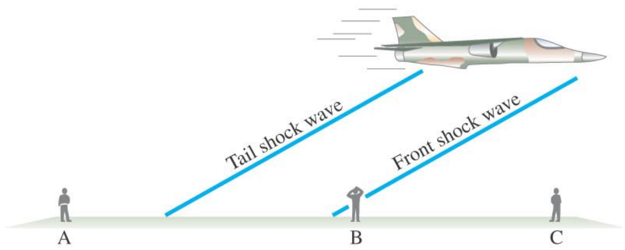
$$SPL_1 + SPL_2 + \cdots + SPL_N = 10\log\left(10^{\frac{SPL_1}{10}} + 10^{\frac{SPL_2}{10}} + \cdots + 10^{\frac{SPL_N}{10}}\right)$$

Beats:

Small typo (2π) corrected on slide 80 in chapter 16, thanks for noticing! ☺

$$f_{beat} = f_1 - f_2$$

$$D_1 + D_2 = 2A\cos\left[2\pi\left(\frac{f_1 - f_2}{2}\right)t\right]\sin\left[\boldsymbol{2\pi}\left(\frac{f_1 + f_2}{2}\right)t\right]$$



Doppler effect:

$$f' = \frac{f}{1 - \|\boldsymbol{M}_s\|\cos\theta} \qquad \sin\theta = \frac{1}{M}$$

The shock wave moves with the aircraft

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Visit to the anechoic chamber of applied sciences!

We can meet on Monday 3rd of March at 12:45 at the service point of Echo and walk together there! ☺

Please register below to have a headcount:

Visit to the anechoic chamber of the faculty of Applied Sciences (old building)

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Position in the syllabus

14. ~~Oscillations~~
15. ~~Waves~~
16. ~~Sound~~
17. **Temperature and the ideal gas law**
18. Thermodynamics
19. Electricity and circuits
20. Electromagnetism
21. Optics

Stepping stone

Structure of the lecture

1. Atomic Theory of Matter
2. Temperature and Thermometers
3. Thermal Equilibrium and the Zeroth Law of Thermodynamics
4. Thermal Expansion
5. Thermal Stress
6. The Gas Laws and Absolute Temperature
7. The Ideal Gas Law (**extended**)

Learning objectives for today's lecture

After this lecture you should be able to:

- Explain the concepts of **temperature** and **thermal equilibrium** ($\mathbf{zero^{th}}$ **law of thermodynamics**).

- Calculate **thermal expansion** and **thermal stress** in different materials.

- Explain and use the **ideal gas law.**

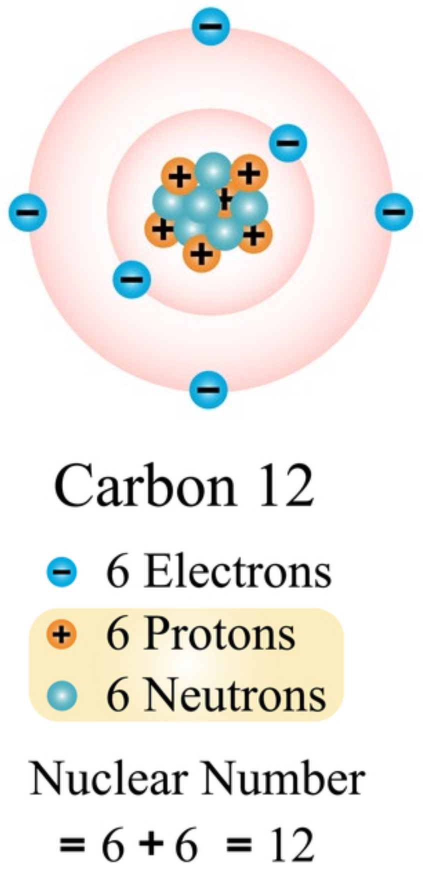
Assumed prior knowledge

- Basic trigonometry (cosine, etc.)
- Basic math (**logarithm**)
- Basic mechanics and kinematics (Newton’s laws, etc.)
- Differential equations

17.1 – Atomic theory of matter

The **atom** is considered as the smallest piece of a substance.

However, it is **<u>not indivisible</u>**. It consist of a central nucleus (**protons** and **neutrons**) surrounded by **electrons**.



Atomic and molecular masses are measured in **unified atomic mass units** (u). This unit is defined so that the carbon-12 atom has a mass of exactly 12.0000 u.

Expressed in kilograms:

$$1\ \text{u} = 1.6605 \times 10^{-27}\,\text{kg}$$

For example, a hydrogen atom has a mass of 1.0078 u.

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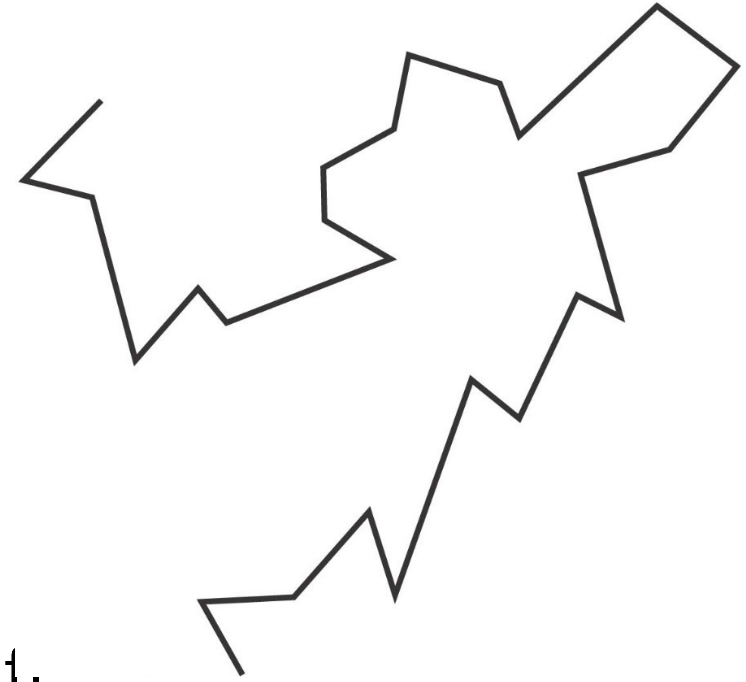
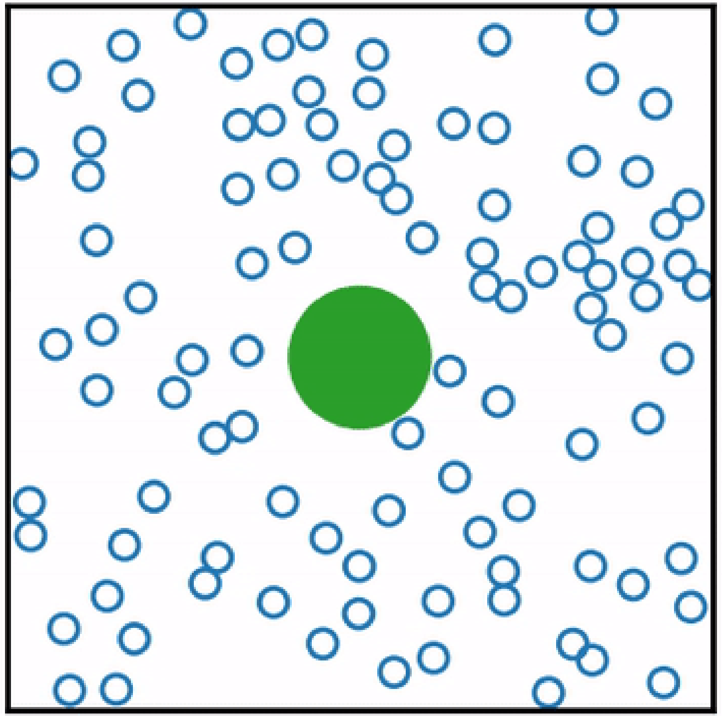
17.1 – Brownian motion

As evidence for the atomic theory, **Brownian motion** is the jittery motion of tiny particles (e.g. pollen grains) suspended in water or any fluid; these are the result of **collisions with individual water molecules**.



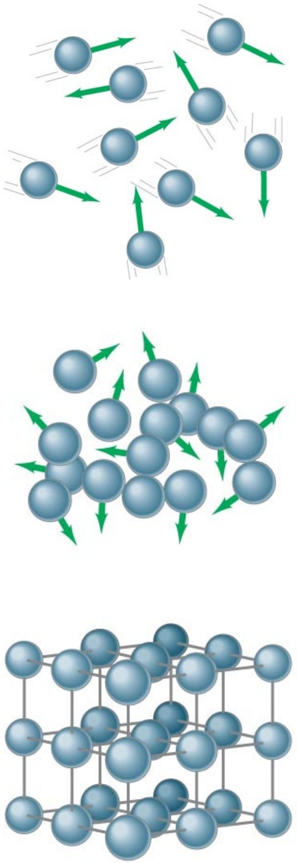


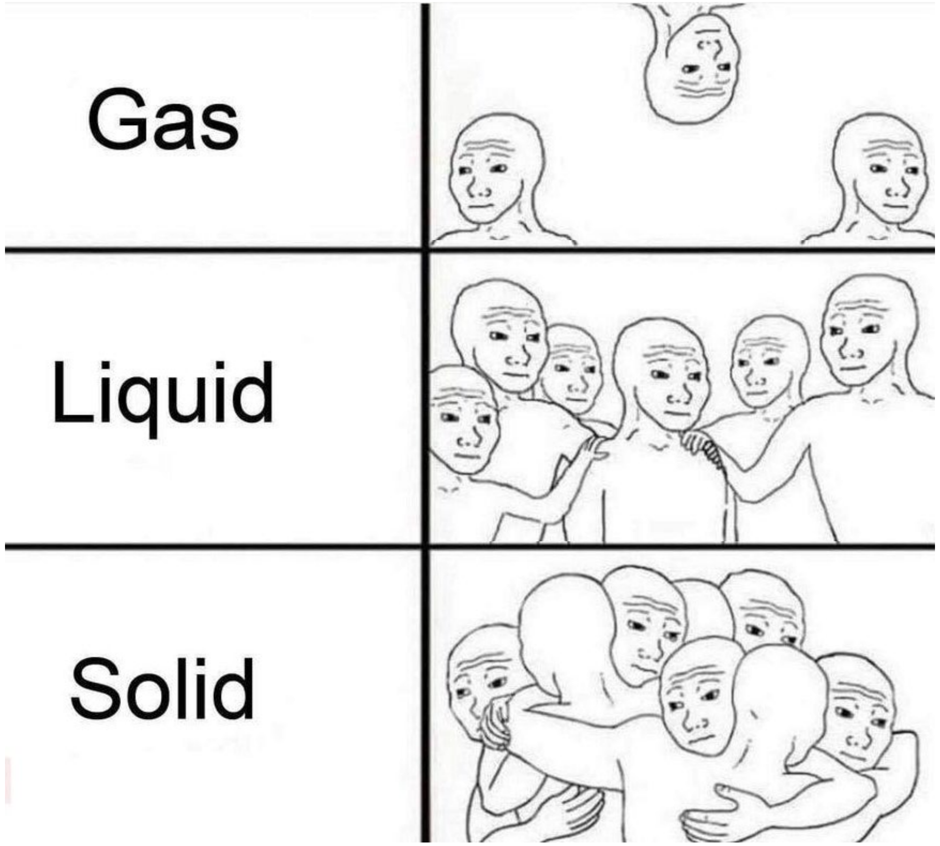
17.1 – Brownian motion

Link to full video

17.1 – Atomic theory of matter

On a microscopic scale, the arrangements of molecules in solids, liquids, and gases are different and depend on their **attractive forces** and **atoms motion**.



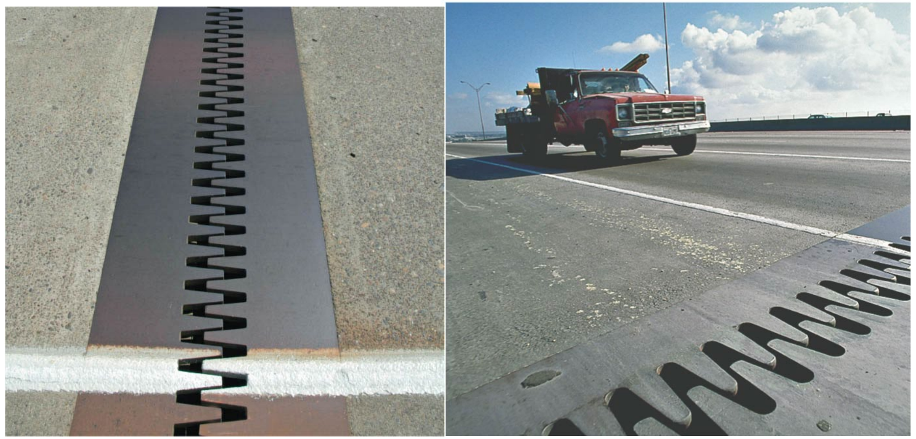


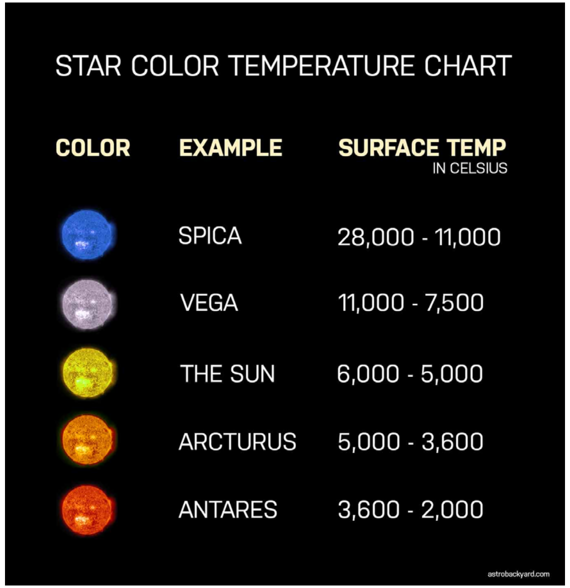
17.2 – Temperature

In essence, temperature is a measure of “*how hot or cold something is*”.

It reflects the **average kinetic energy** of the vibrating and colliding atoms that make up a substance.

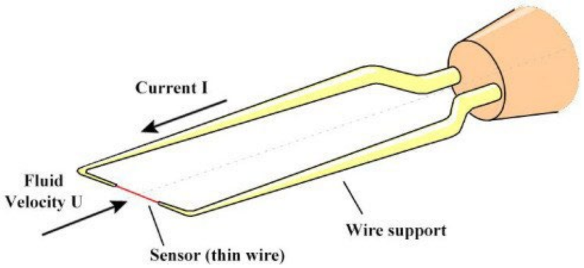
Many **properties of matter change with temperature**, such as the electric resistance, the radiated color (e.g. stars), etc. For example, most materials expand when they are heated.





17.2 – Example: Hot-wire anemometry

For example, for wind-tunnel testing, we use the fact that electric resistance changes with temperature to perform **hot-wire anemometry (HWA)** to measure flow velocity.



An electric current is sent through a very thin wire ($\sim 1 \mu m$) causing it to become hot.

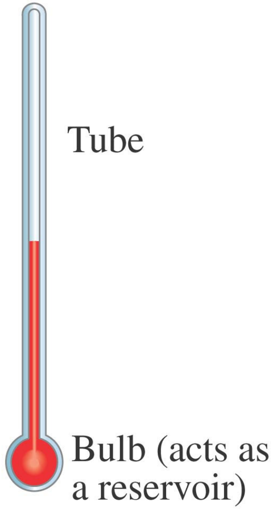
As the fluid (here, air) flows around it, it cools it down, changing its electric resistance. The higher the flow velocity, the higher the cooling.

Therefore, with some considerations and calculations, we can obtain the **flow velocity evolution** very rapidly over time to calculate parameters like **turbulence intensity**.

17.2 – Thermometers

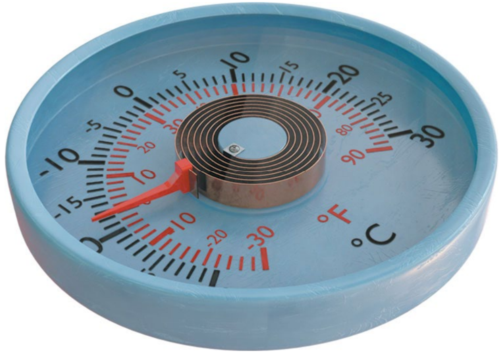
Thermometers are instruments designed to measure temperature.

In order to do this, they take advantage of some **property of matter that changes with temperature**.



Mercury or alcohol thermometer

Bimetallic strip



Bimetallic coil

17.2 – Temperature scales

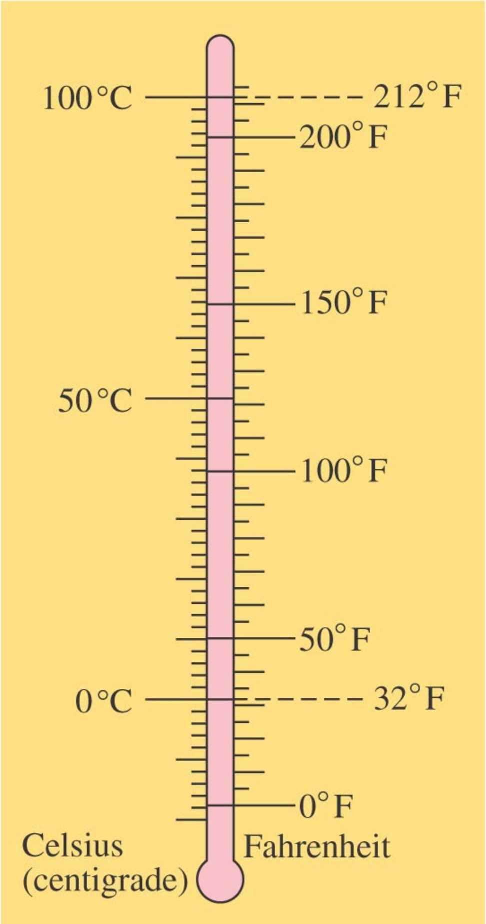
Temperature scales are normally based on the **freezing** and **boiling** point of water.

In Celsius: Water freezes at 0℃ and boils at 100℃

In Fahrenheit: Water freezes at 32℉ and boils at 212℉

$$℉ = \frac{9}{5}℃ + 32$$

$$℃ = \frac{5}{9}(℉ - 32)$$



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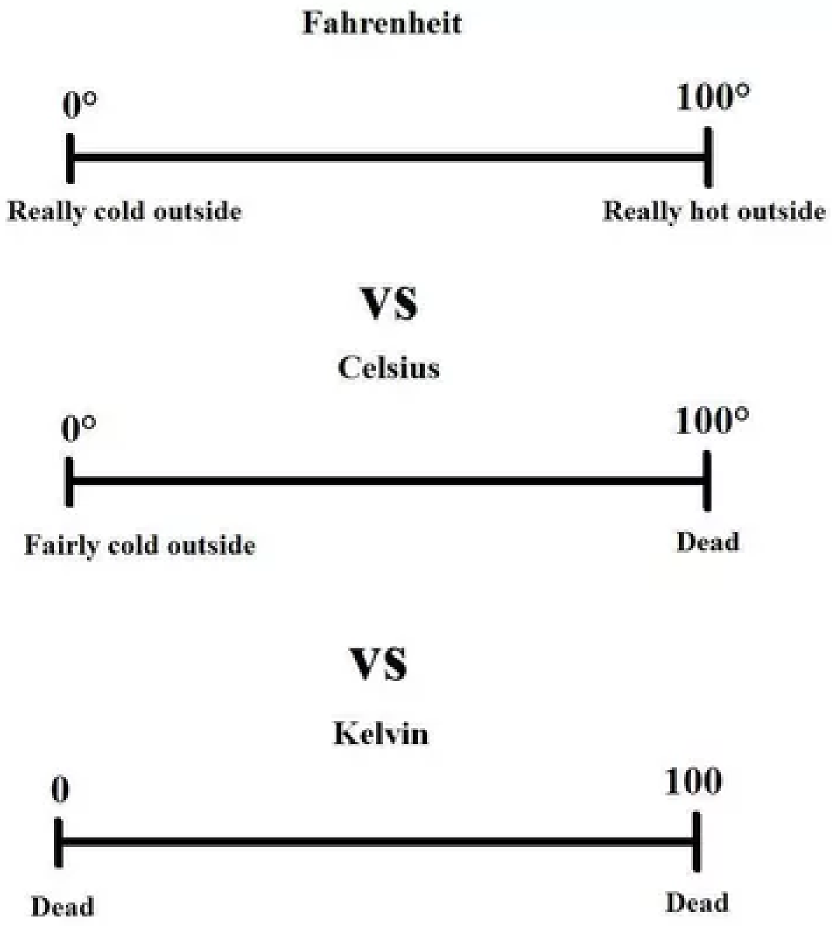
17.2 – Temperature scales

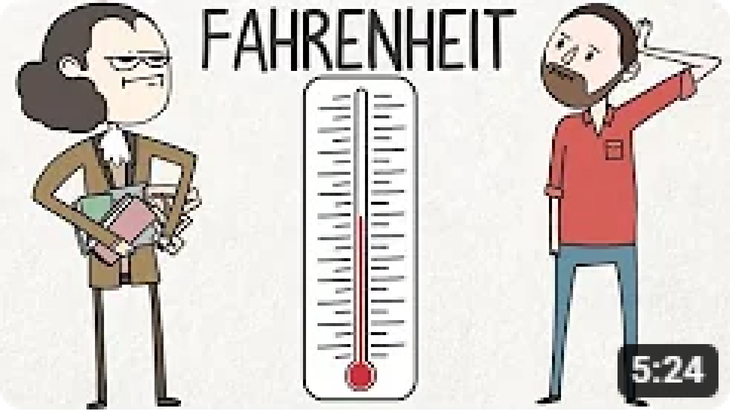
For thermodynamics, engineering, and physical sciences, we normally use the **Kelvin scale** (one of the **seven base units** in the international system of units (SI)).

$$K = °C + 273.15$$

The zero value in this scale is the lowest temperature possible (**absolute zero**).



17.2 – Temperature scales



Two videos about the history behind the Celsius and Fahrenheit temperature scales are in the Veritasium YouTube channel under the names:

[“Celsius made his thermometer upside down”]

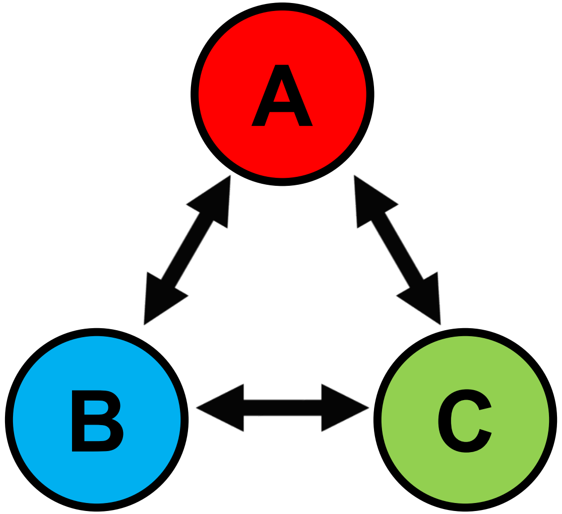
[“What the Fahrenheit?!”]

17.3 – Thermal equilibrium and the zeroth law of thermodynamics

Two objects placed in thermal contact will eventually come to the **same temperature**. We then say that they are in **thermal equilibrium** and there is **no net energy flow** between them, and the temperature remains the same.

The **zeroth law of thermodynamics** states that:

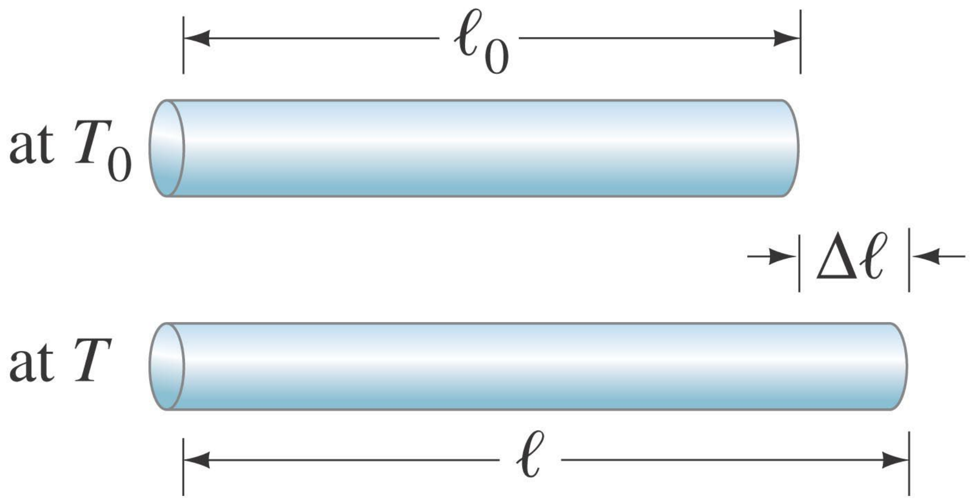
"Two systems (A) and (B) in thermal equilibrium with a third system (C) are also in thermal equilibrium with each other."



17.4 – Thermal expansion

Most substances **expand when heated** (because their molecules are a bit farther apart on average) and **contract when cooled.**

For a one-dimensional approximation, we consider **linear expansion**:



$$\Delta l = l - l_0 = \alpha l_0 \Delta T$$

$$l = l_0(1 + \alpha\Delta T)$$

$T > T_0$

Here α [$°\mathrm{C}^{-1}$] is the coefficient of linear expansion. This coefficient is different per material and it also varies with temperature.

17.4 – Thermal expansion

Extending this concept to the 3D case (relevant for **fluids** too), we have volume changes:

$$\Delta V = \beta V_0 \Delta T$$

Here β [$°\mathrm{C}^{-1}$] is the coefficient of volume expansion.

For uniform solids:

$$\beta \approx 3\alpha$$

TABLE 17–1 Coefficients of Expansion, near 20°C

Material	Coefficient of Linear Expansion, α $(C°)^{-1}$	Coefficient of Volume Expansion, β $(C°)^{-1}$
Solids		
Aluminum	25×10^{-6}	75×10^{-6}
Brass	19×10^{-6}	56×10^{-6}
Copper	17×10^{-6}	50×10^{-6}
Gold	14×10^{-6}	42×10^{-6}
Iron or steel	12×10^{-6}	35×10^{-6}
Lead	29×10^{-6}	87×10^{-6}
Glass (Pyrex®)	3×10^{-6}	9×10^{-6}
Glass (ordinary)	9×10^{-6}	27×10^{-6}
Quartz	0.4×10^{-6}	1×10^{-6}
Concrete and brick	$\approx 12 \times 10^{-6}$	$\approx 36 \times 10^{-6}$
Marble	$1.4\text{–}3.5 \times 10^{-6}$	$4\text{–}10 \times 10^{-6}$
Liquids		
Gasoline		950×10^{-6}
Mercury		180×10^{-6}
Ethyl alcohol		1100×10^{-6}
Glycerin		500×10^{-6}
Water		210×10^{-6}
Gases		
Air (and most other gases at atmospheric pressure)		3400×10^{-6}

$\sim \times 3$

17.4 – Expansion joints

[Link to the full video]

STREET RAILWAYS
AND
THE THERMIT PROCESS

THE Thermit Process enables you to install the most efficient rail bond that there is—the THERMIT WELDED RAIL JOINT. This joint gives a permanent bond equal in electrical conductivity to the rail itself. Time causes no deterioration in this bond and by adopting it you not only cut down the resistance of the return circuit, but avoid losses due to electrolysis with their consequent damages.

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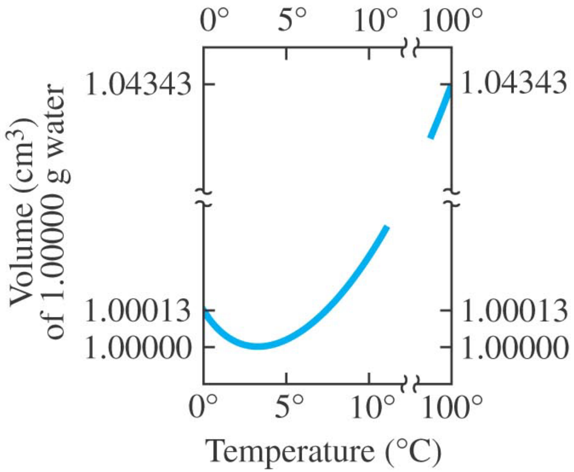
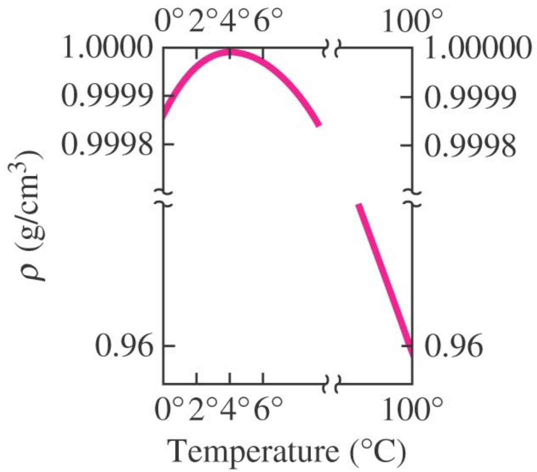
17.4 – Thermal expansion: water

Water is a special case. Unlike most materials, it **expands when it freezes**.

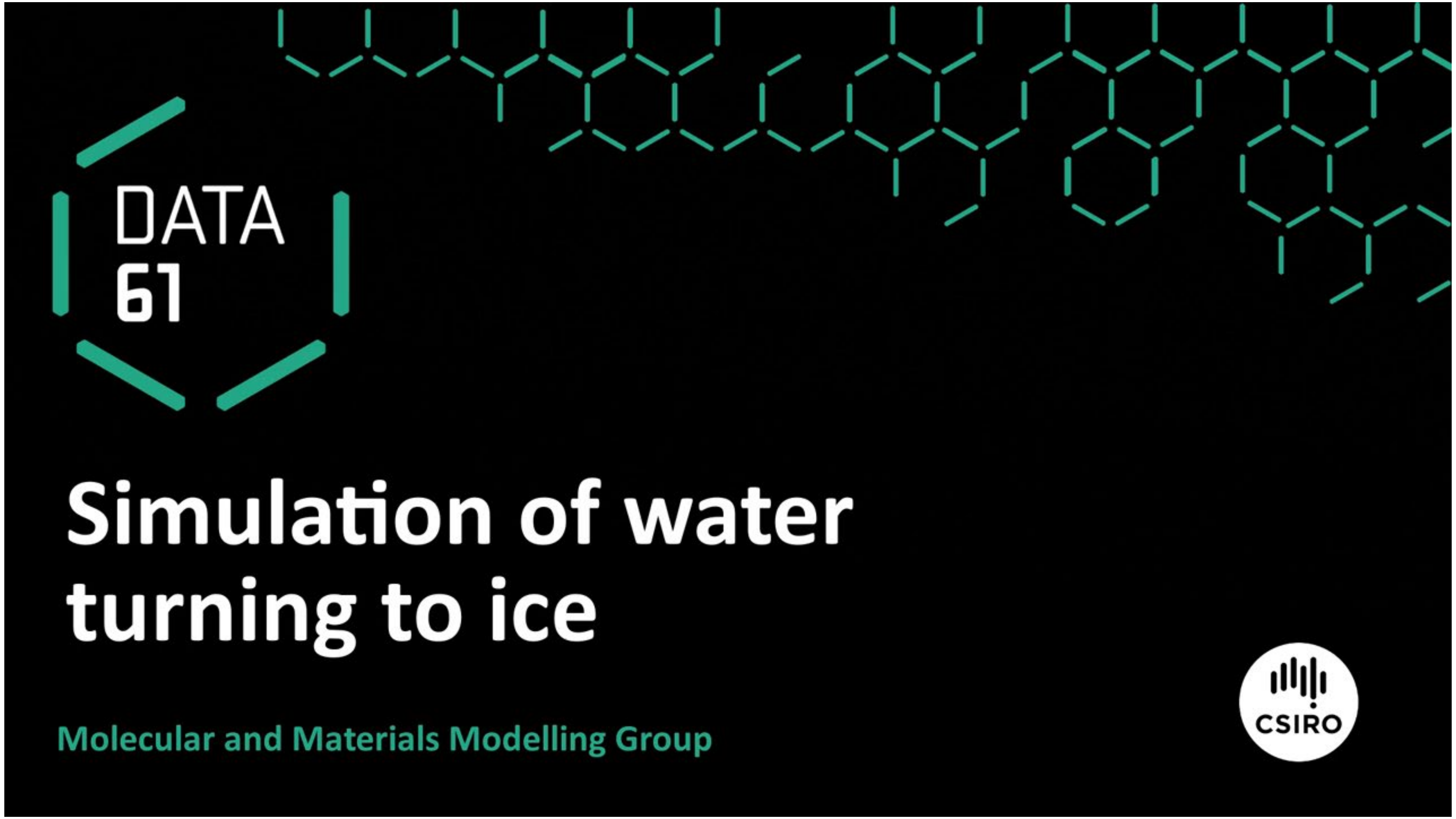
In fact, its minimum volume (maximum density) occurs around **4**℃.





This fact is crucial for the survival of **aquatic life**, since water cooled down below 4℃ (and ice) is less dense and will then float.

17.4 – Thermal expansion: water



This is due to the **hydrogen bond** between water molecules and its restructuring into a larger lattice when it becomes solid (freezing)

Link to full video

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17.5 – Thermal stresses

If a material is fixed at its ends (and hence unable to expand or contract when the temperature changes), it will experience **thermal stresses** (compressive or tensile, respectively).

From the definition of Young's modulus:

$$E = \frac{\text{uniaxial stress}}{\text{strain (proportional deformation)}} = \frac{\frac{F}{A}}{\frac{\Delta l}{l_0}}$$

$$\Delta l = \alpha l_0 \Delta T$$

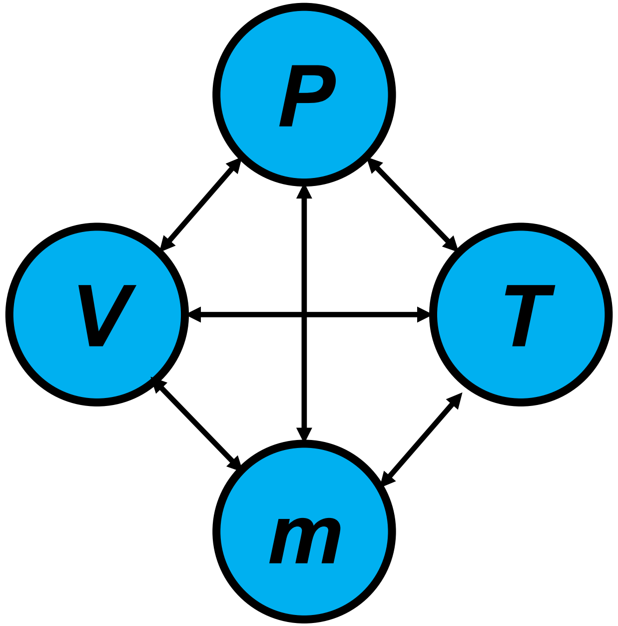
$$\frac{\Delta l}{l_0} = \alpha \Delta T$$

Therefore the **thermal stress** is:

$$\boxed{\frac{F}{A} = \alpha E \Delta T}$$

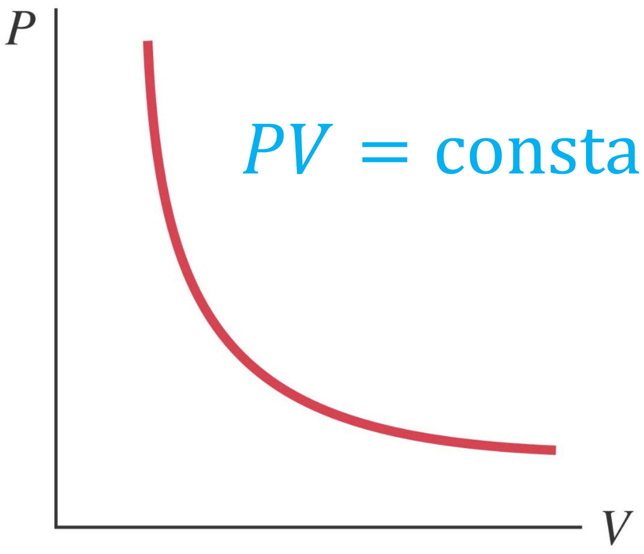
17.6 – The gas laws and absolute temperature

A relationship between the volume, pressure, temperature, and mass of a gas is called an **equation of state**.



For now, we consider gases that are not too dense, not close to the liquefaction point, and with a pressure below 1 atm.

17.6 – Boyle -Mariotte's law



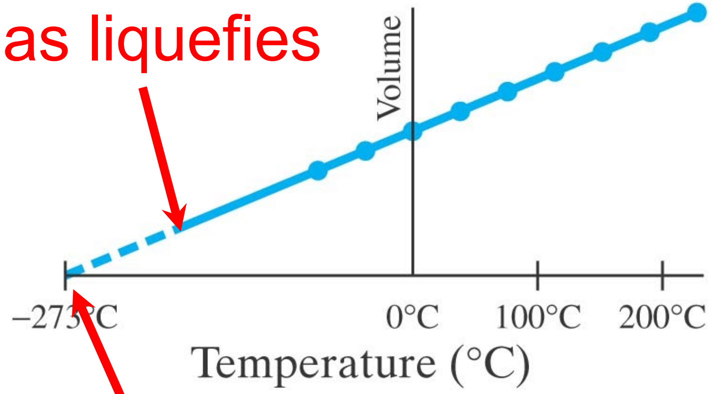
$$PV = \text{constant}$$

Boyle-Mariotte's law states that the **volume** of a given amount of gas is inversely proportional to the **pressure** (as long as the temperature is constant)

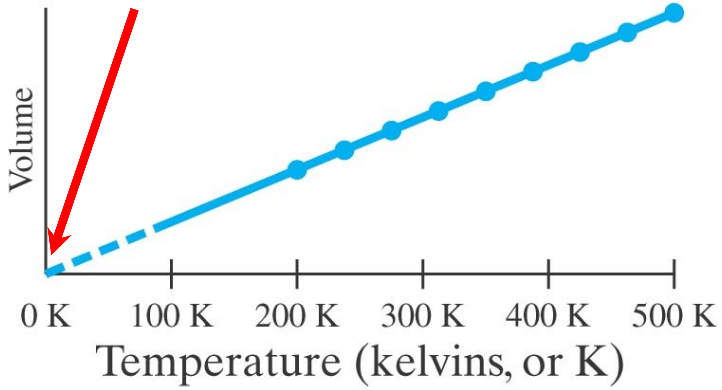
$$V \propto \frac{1}{P} \qquad T = \text{constant}$$

17.6 – Charles' law

Gas liquefies



Lowest possible temperature



Charles' law states that the **volume** of a given amount of gas is linearly proportional to the **temperature** (as long as the pressure is constant – and not too high)

$$V \propto T \qquad P = \text{constant}$$

By extrapolating this law, the volume would become zero at a temperature of -273.15℃ (**absolute zero**, 0 K)

$$K = ℃ + 273.15$$

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17.6 – Gay-Lussac's law

Gay-Lussac's law states that the **volume** of a given amount of gas is linearly proportional to the **temperature** (as long as the volume is constant – and not too high)

$$P \propto T \qquad V = \text{constant}$$

These three gas laws are only accurate in practice as long as the pressure and density of the gas are not too high and the gas is not close to liquefaction (condensation).

Therefore, these are not really laws but only **approximations**.

17.7 – The ideal gas law

We can combine these three relations into a **single expression**:

$$PV \propto T$$

If we keep temperature and the pressure constant, then the volume is proportional to the **mass of the gas**:

$$PV \propto mT$$

17.7 – The concept of mole – Avogadro's number

When discussing mass at the molecular level, it is quite common to use the concept of **mole** [mol].

One mole is defined as the number of atoms of a substance that is numerically equal to the **atomic/molecular mass** of a substance.

- 1 mol of H_2 has a mass of 2 x 1 = 2 g
- 1 mol of CO_2 has a mass of 12 + 2 x 16 = 44 g
- 1 mol of Ne has a mass of 20 g

It is another one of the **7 official SI units** (together with the Kelvin).

17.7 – The concept of mole – Avogadro's number

The number of moles in a certain mass of material is given by:

$$n[\text{mole}] = \frac{m\ [\text{g}]}{M\ \left[\frac{\text{g}}{\text{mol}}\right]}$$

Where M is the molecular mass in g/mol.

In terms of number of particles:

$\mathbf{1\ \boldsymbol{mol} = 6.02214 \times 10^{23}}$ **atoms** (Avogadro's number, N_A)

This is the latest definition (from 20th of May 2019)

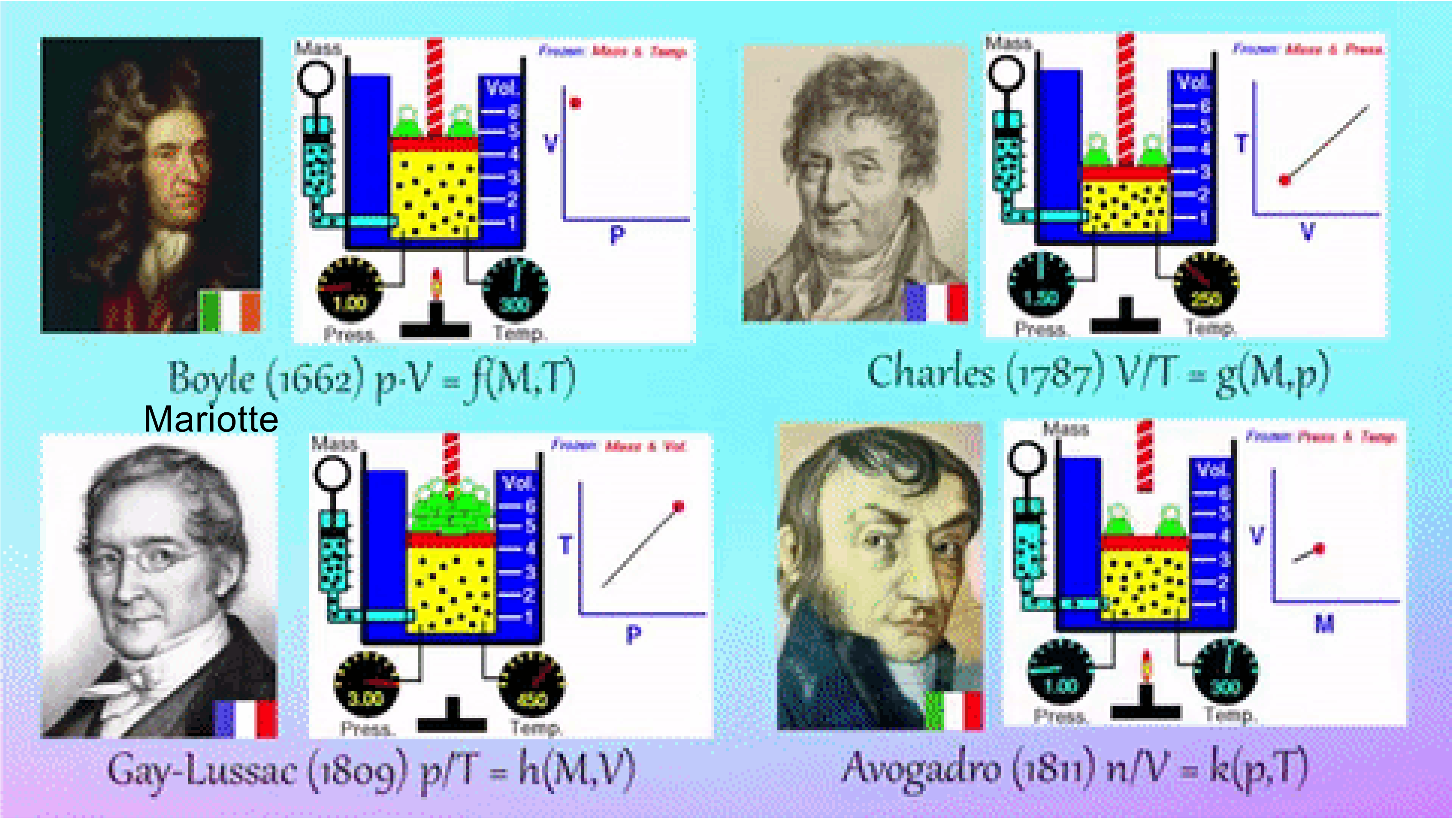
17.7 – The ideal gas law

$$V \propto \frac{1}{P}$$

T constant

$$V \propto T$$

P constant



Mariotte

$$P \propto T$$

V constant

$$M \propto V$$

P, T constant

https://giphy.com/

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17.7 – The ideal gas law

We can now finally write the ideal gas law (equation of state for an ideal gas) as:

$$\boxed{PV = nRT}$$

Where R is the **universal gas constant** (the same for all gases!):

$$R = 8.314\ J/mol \cdot K$$

$$R = 0.0821\ L \cdot atm/mol \cdot K$$

$$R = 1.99\ calories/mol \cdot K$$

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17.7 – The ideal gas law

Absolute pressure $[\mathrm{N/m^2}]$

Absolute temperature $[\mathrm{K}]$

$$\boxed{PV = nRT}$$

Volume $[\mathrm{m^3}]$

Number of moles [mol]

Universal gas constant
(same for all gases)
8.314 [J/(mol K)]

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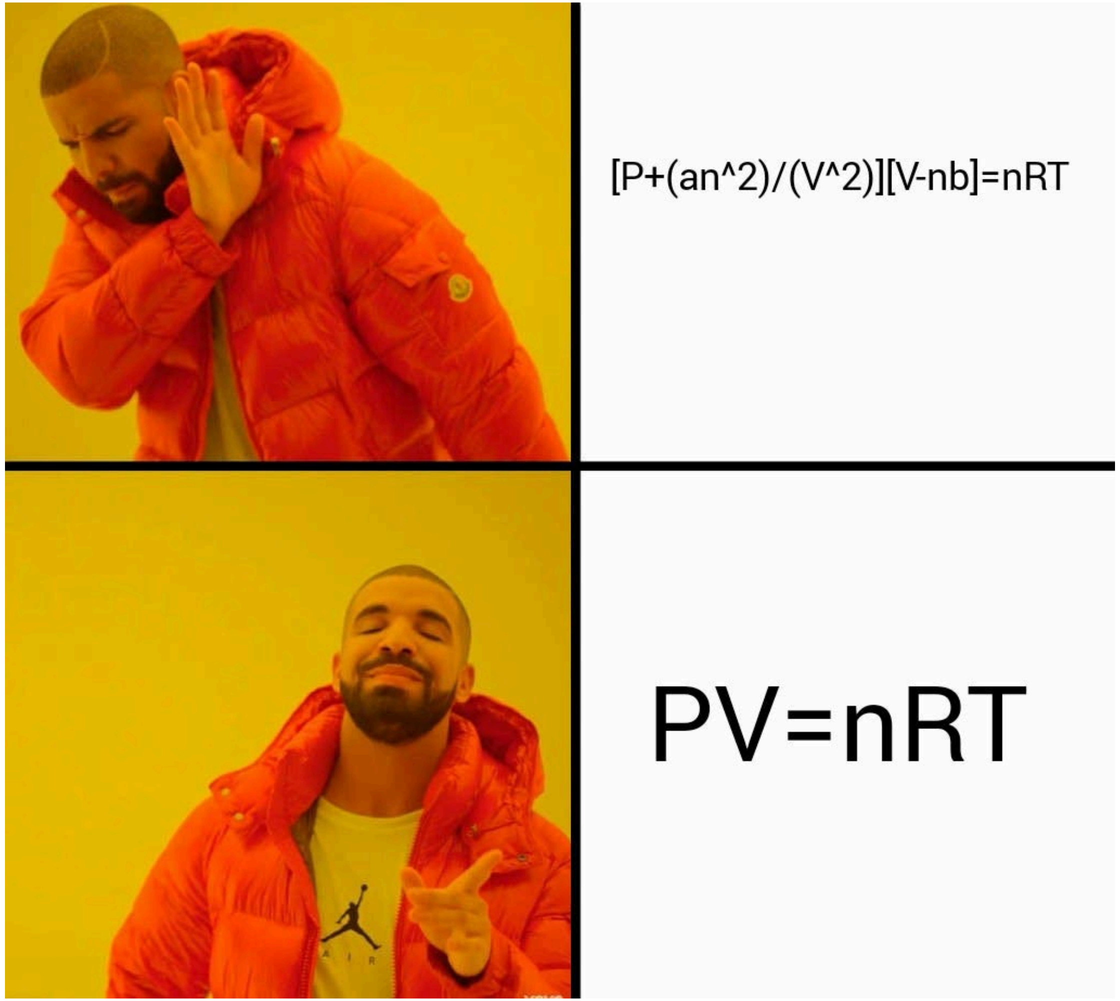
17.7 – The ideal gas law – some considerations

Remember that this is just an approximation for **ideal gases**!

For **real gases**, things complicate a bit more (see Chapter 18).



https://www.reddit.com

17.7 – The ideal gas law – some considerations

Pay attention to the **units**!

In thermodynamics always use the **Kelvin scale**, unless specifically told not to.

https://quizizz.com/

17.7 – The ideal gas law rewritten

We can rewrite the ideal gas law as:

$$PV = nRT = \frac{N}{N_A} RT = N \frac{R}{N_A} T = NkT$$

Where N is the total number of molecules ($N = nN_A$), and k is **Boltzmann's constant**:

$$k = \frac{R}{N_A} = \frac{8.314\ \mathrm{J/mol \cdot K}}{6.02214 \times 10^{23}\ \text{atoms/mol}} = 1.38 \times 10^{-23} \mathrm{J/K}$$

WRAP-UP

Wrap-up: revisit learning objectives

After this lecture you should be able to:

- Explain the concepts of **temperature** and **thermal equilibrium** (**zero$^{\text{th}}$ law of thermodynamics**)

$$°\mathrm{F} = \frac{9}{5}\,°\mathrm{C} + 32 \qquad K = °\mathrm{C} + 273.15$$

- Calculate **thermal expansion** and **thermal stress** in different materials.

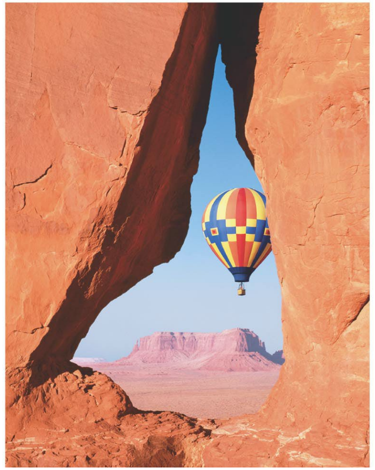
$$\Delta l = \alpha l_0 \Delta T \qquad \frac{F}{A} = \alpha E \Delta T$$

- Explain and use the **ideal gas law**

$$PV = nRT = NkT$$

Temperature, thermal expansion and the ideal gas law

Chapter 17



Dr. Roberto Merino-Martinez

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Faculty of Aerospace Engineering

TU Delft

THANK YOU FOR YOUR ATTENTION THESE LAST WEEKS ☺

Help me improve with your feedback, thanks! ☺

Feedback for AE1241 Physics course - Roberto Merino-Martinez

AND… THE RESULTS OF THE ANNOYANCE COMPETITION!