ELECTRIC CURRENT AND RESISTANCE

Chapter 25

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Operations & Environment

Faculty of Aerospace Engineering

Structure of the lecture

1. Electric battery and Electric Current
2. Ohm’s Law: Resistance and Resistors
3. Resistivity
4. Electric Power
5. Alternating Current
6. Microscopic view of Electric Current

Learning objectives for today's lecture

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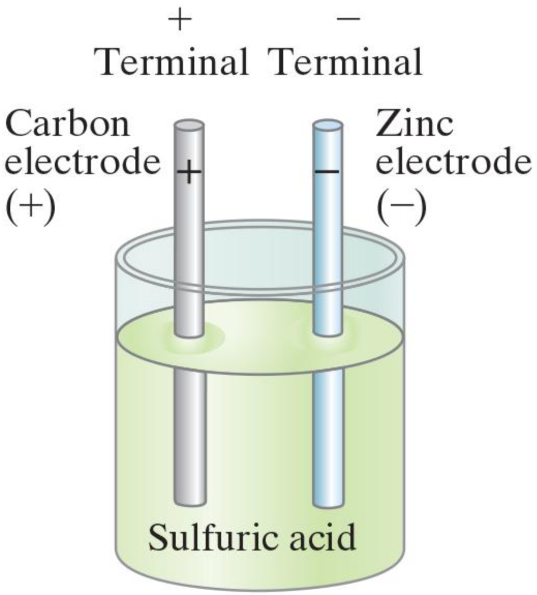
- Understand the general **characteristics** of electric current
- Understand the **Ohm's Law** and the relationship between voltage, current, and resistance and determine the **resistance** of a wire or other conducting mean given its geometric and material properties
- Understand the basics of **alternate current**
- Understand the basics of **electric current at the microscopic level**

25.1 – The Electric Battery



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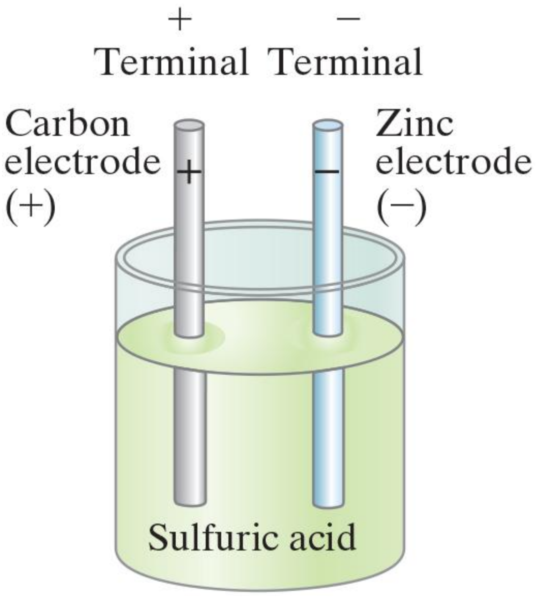

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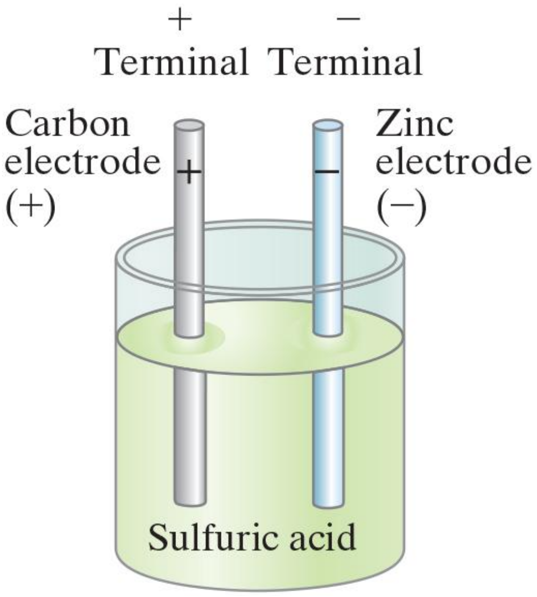

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A battery produces electricity by transforming chemical energy into electrical energy. The simplest version of a battery consists of two electrodes (two metal plates or rods) immersed into a solution, namely the aforementioned electrolyte.

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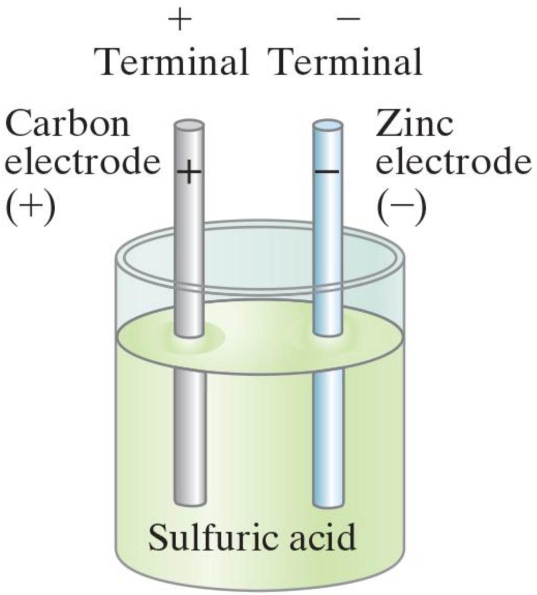

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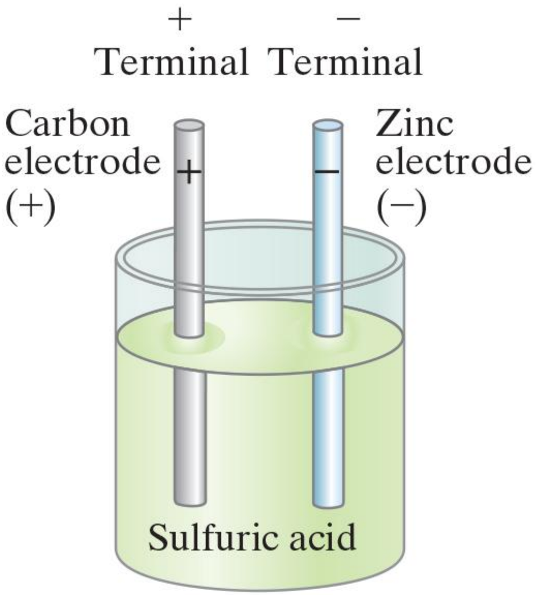

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The electrolyte becomes positively charged and pulls electrons from the carbon electrode, which charges positively. The carbon electrode is the anode and the zinc one the cathode of the battery.

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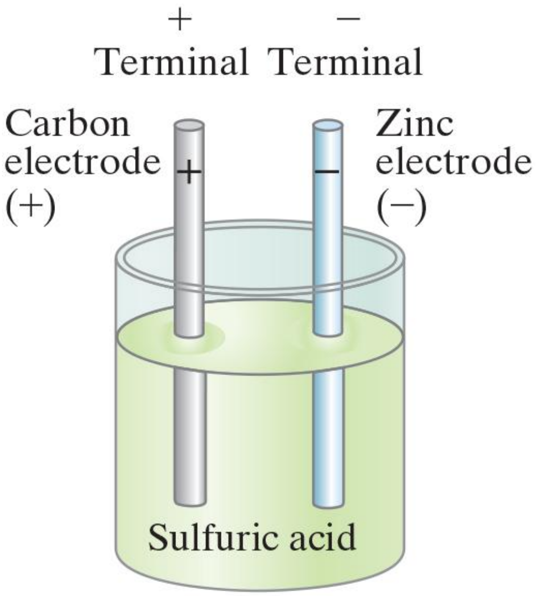

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Once one of the two electrodes has run out of electrons, the battery is dead. The advantage of such a battery, when the electrodes are connected to an electricity-demanding device, is that the provided voltage is basically constant.

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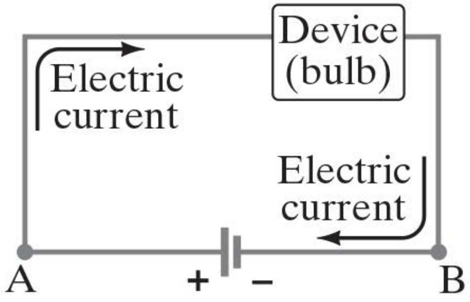
25.2 – Electric Current

(a)



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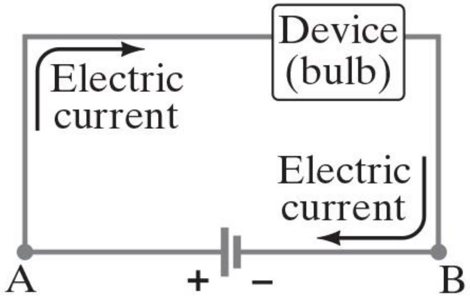

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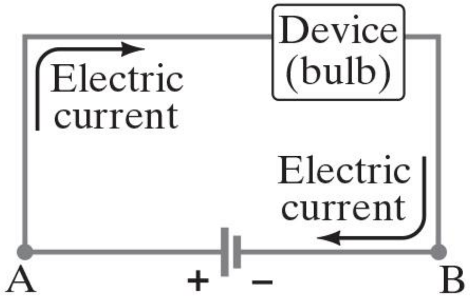

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Electric current is measured in Amperes A, being 1 Ampere $1\frac{C}{s}$.

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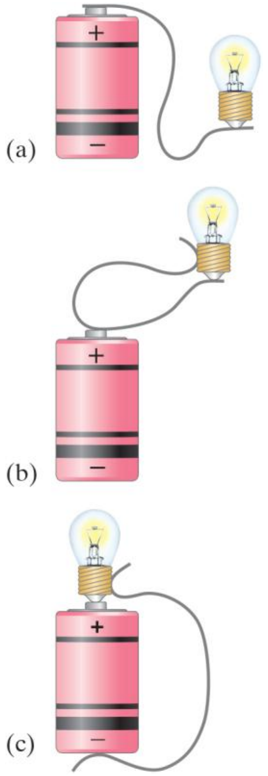

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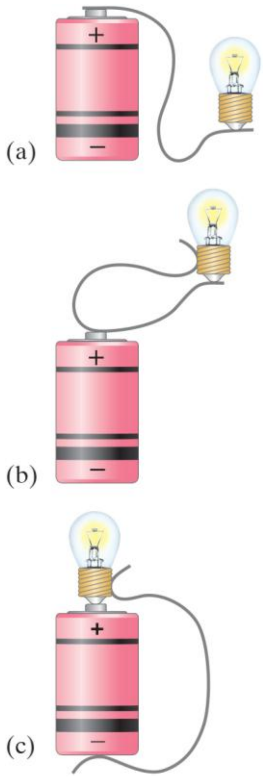

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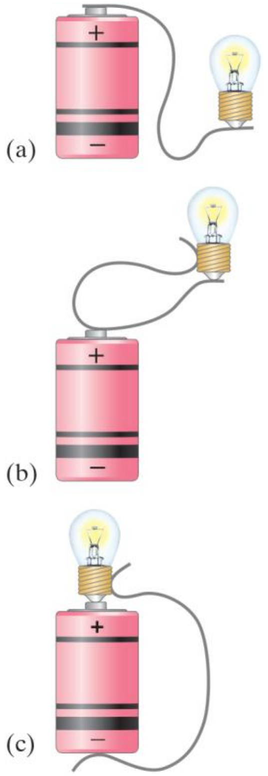

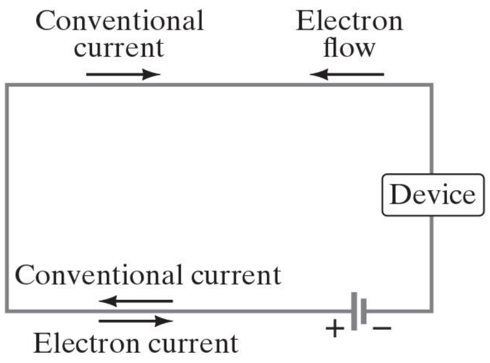

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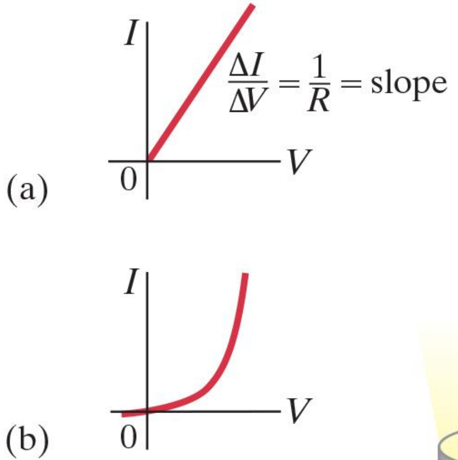
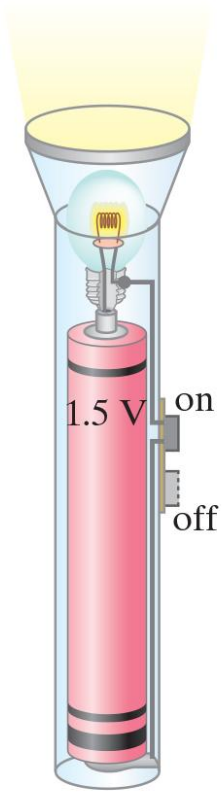
In a circuit, electrons move from the negative to the positive terminal of a battery (electron flow). When electricity was discovered, it was thought that positive charges flowed in a wire. We still keep this convention, assuming that electric current flows from the positive to the negative terminal

25.3 – Ohm's Law: Resistance and Resistors





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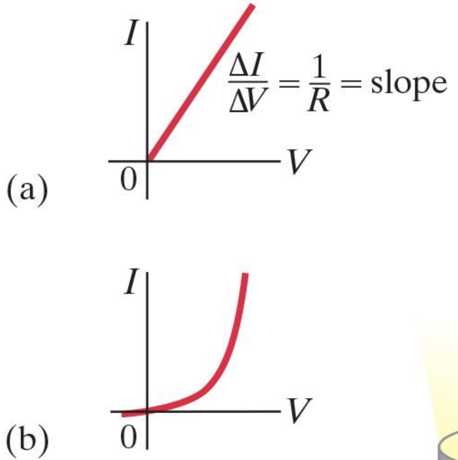

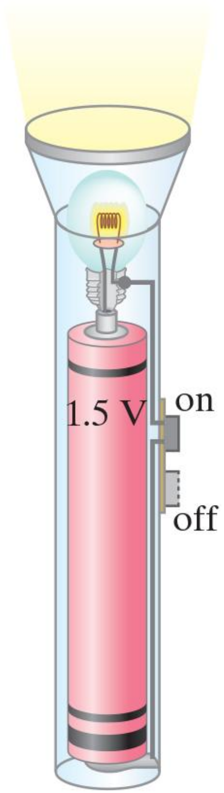

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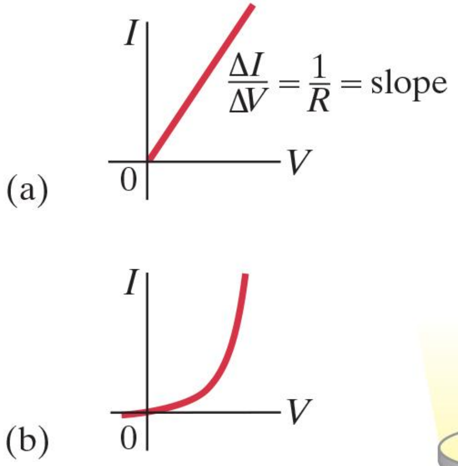

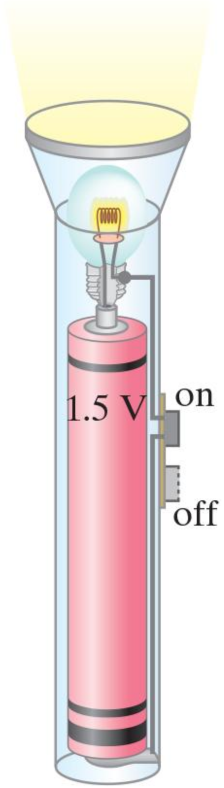

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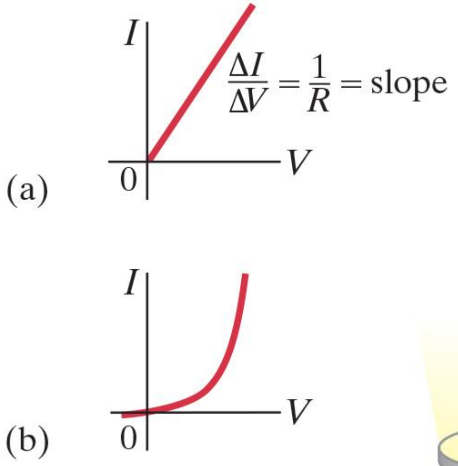

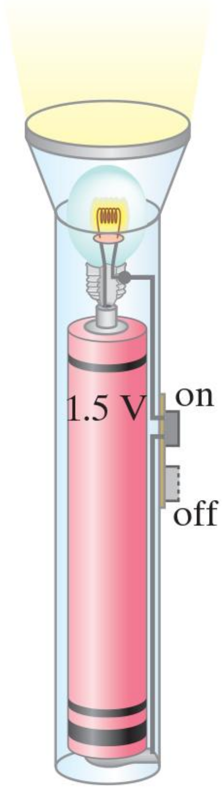

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Note that, even for metals, the resistance is not constant if the material is subject to severe changes in temperature. Materials for which Ohm’s law applies (resistance is a constant) are called ohmic materials. The unit is the Ohm $\Omega \rightarrow \boldsymbol{1\ \Omega = \frac{1V}{1A}}$

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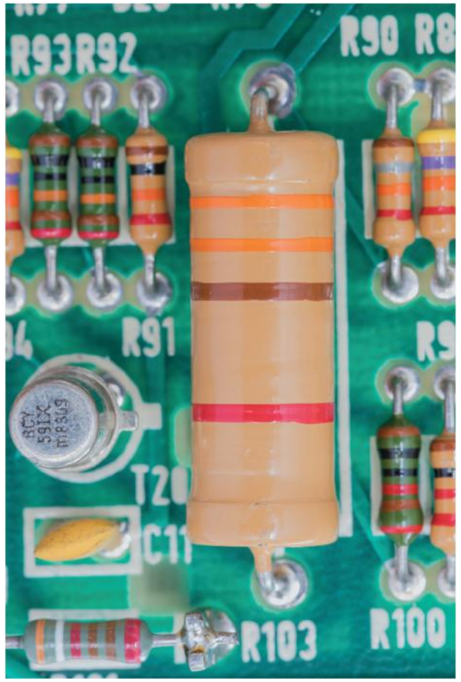

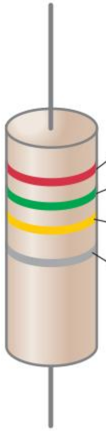

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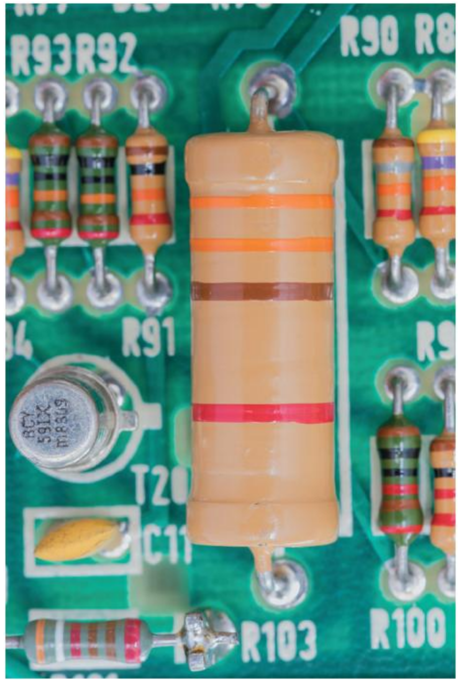

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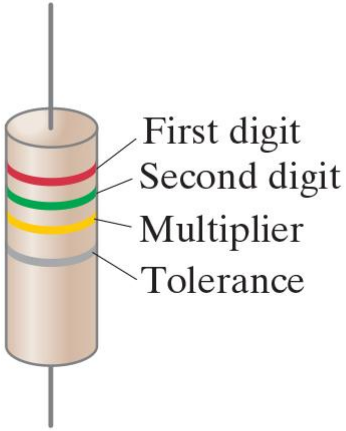
Resistors have values of resistance ranging from less than 1 Ohm to millions of Ohms. Color codes are used on resistors to univocally define their resistance value.



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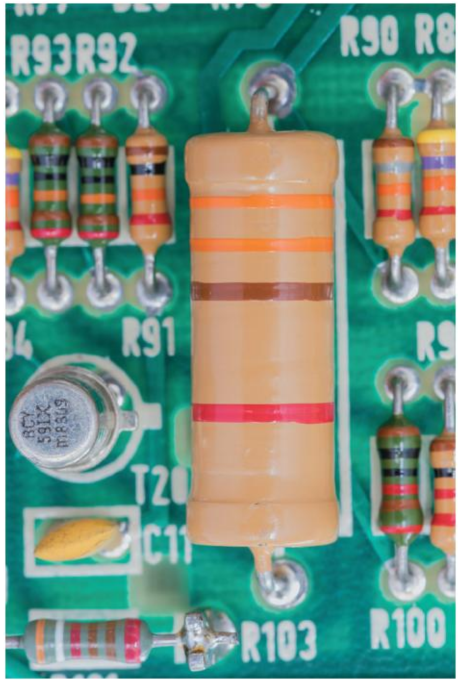

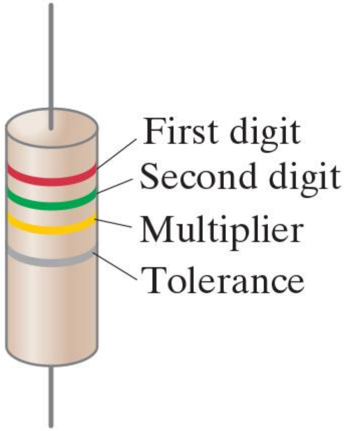

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Resistances always dissipate energy. Hence, a drop in potential occurs between the point before and after the resistance itself.

25.4 – Resistivity

It is found experimentally that the resistance R of a wire is directly proportional to its length l and inversely proportional to its cross-sectional area A. Hence

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α is generally positive, but some materials (semiconductors) have it negative.

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$$V_1 = l_1 A_1, V_2 = 2l_1 A_2 = V_1 \rightarrow A_2 = \frac{l_1 A_1}{2l_1} = \frac{A_1}{2}$$

$$R_{new} = \rho \frac{l_2}{A_2} = \rho \frac{2l_1}{\frac{A_1}{2}} = 4\rho \frac{l_1}{A_1} = 4R$$

25.4 – Resistivity

The variation in electrical resistance with temperature can be used to make precise temperature measurements. Platinum is commonly used since it is relatively free from corrosive effects and has a high melting point. Suppose at $20.0°C$ the resistance of a platinum resistance thermometer is $164.2\ \Omega$. When placed in a particular solution, the resistance is $187.4\ \Omega$. What is the temperature of this solution?

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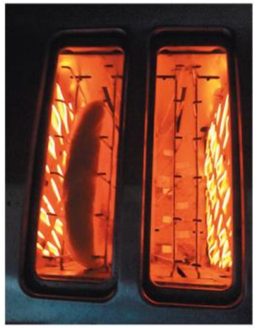
$$\boldsymbol{R = R_0[1 + \alpha(T - T_0)] \rightarrow T = T_0 + \frac{\left(\frac{R}{R_0} - 1\right)}{\alpha} = 56℃}$$

where the value for α has been retrieved from a table

25.5 – Electric Power

(a)



(b)

Power, as in kinematics, is the energy transformed by a device per unit time. Here we are still dealing with a (difference of) potential and charges being moved, hence we can use $dU = Vdq$:

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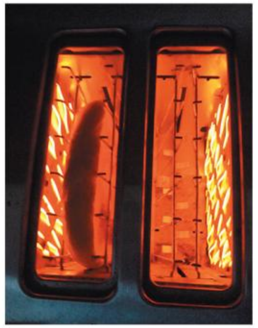
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for ohmic materials

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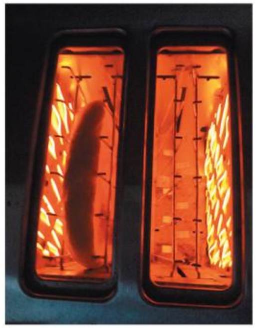
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When using electricity, what matters is the energy we use, hence the power times the time of utilization. In general, we use Joules J for energy, but in this context kilowatt-hours kWh are used:

$$\boldsymbol{1\ kWh = (1000\ W) \times (3600\ s) = 3.6 \times 10^6\ J}$$

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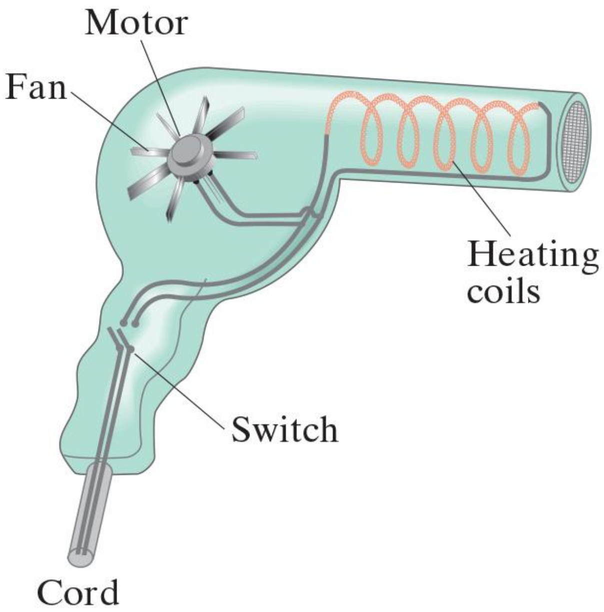

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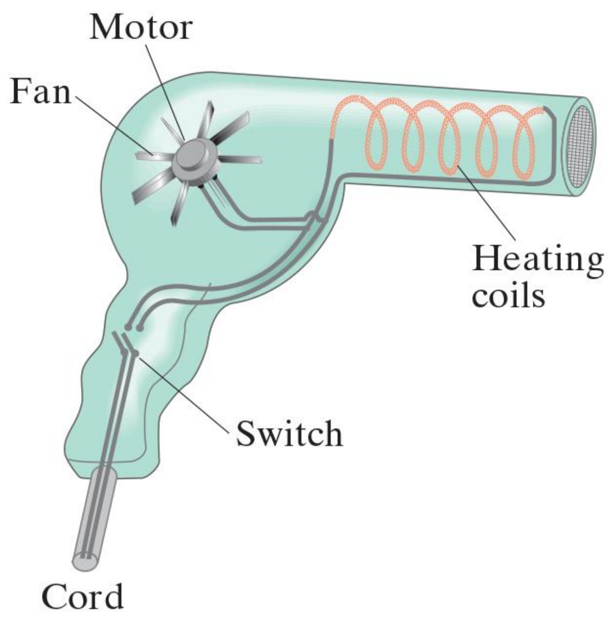

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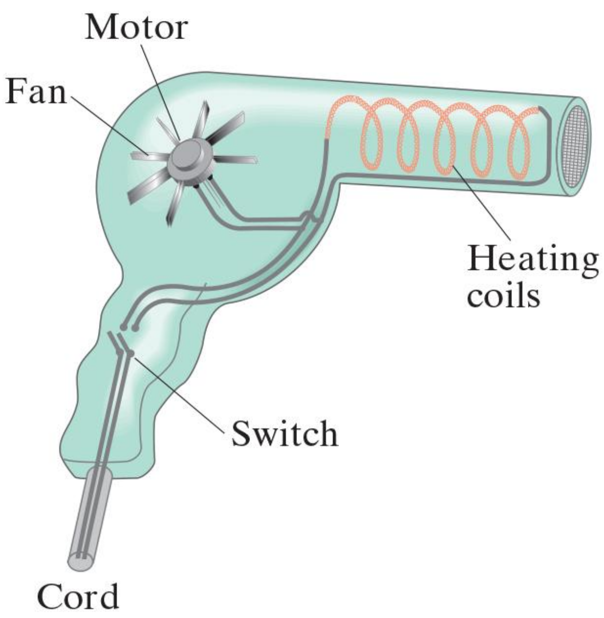

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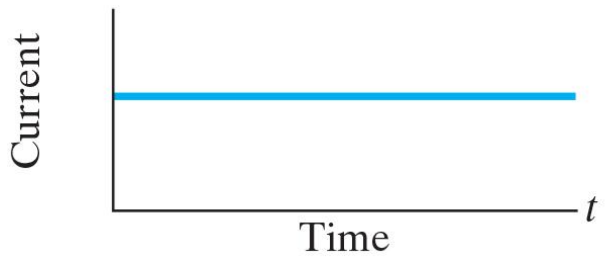
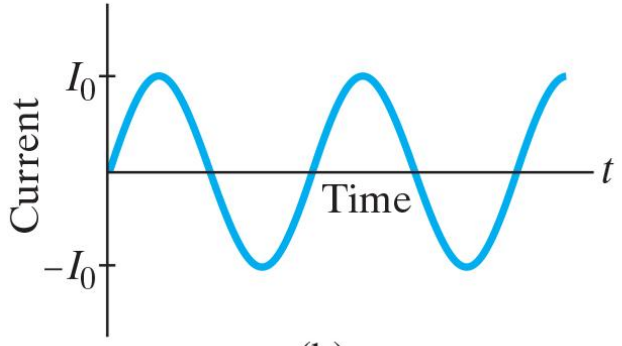
Assuming resistance does not change, we have

$$\bar{P} = \frac{V_{rms}^2}{R} = 4{,}000\ W$$

25.7 – Alternating Current



(a) dc



(b) ac

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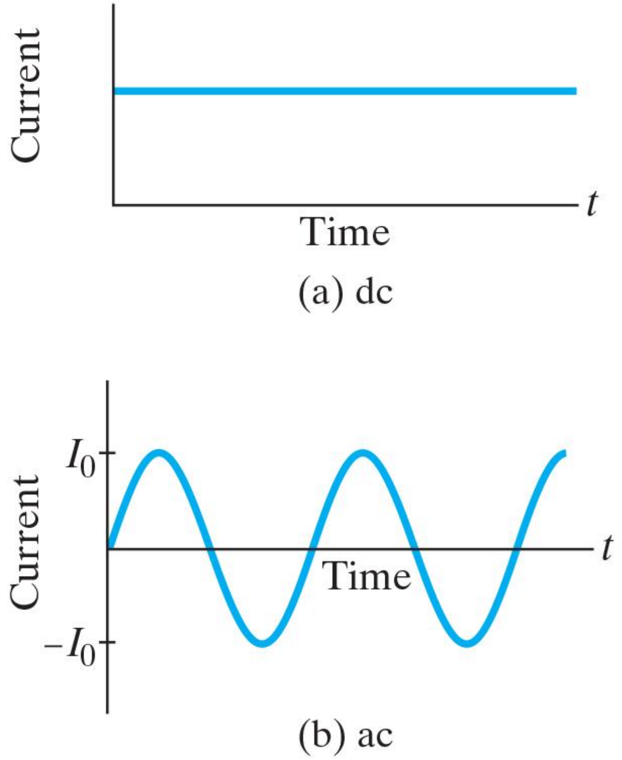

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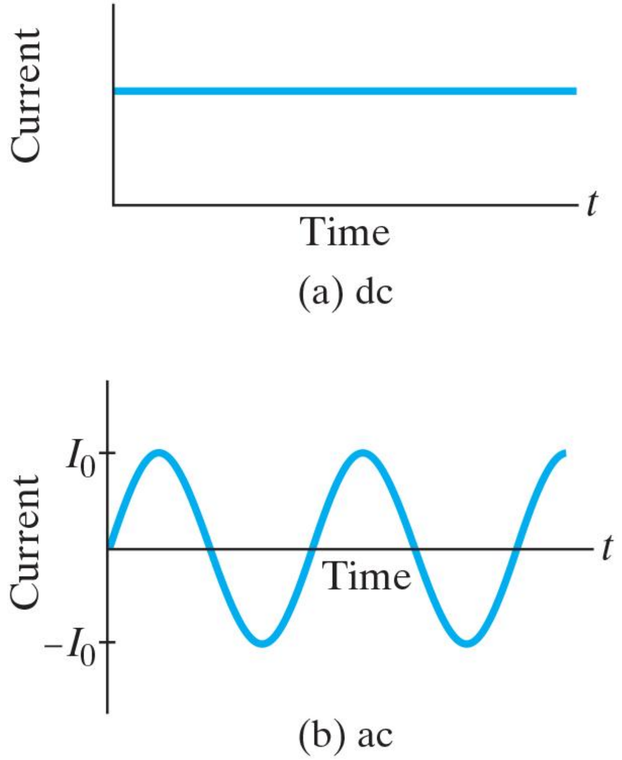
While the details of why this is the case (spoiler: it has to do with the functioning behind electric generators) will be revealed later in the course, we discuss some basics here given the relevance of the topic.

25.7 – Alternating Current



While a battery provides a current that is constant and moves steadily in one direction (left, top figure), power plants generate alternate current (left, bottom figure) that changes direction many times per second and is generally sinusoidal.

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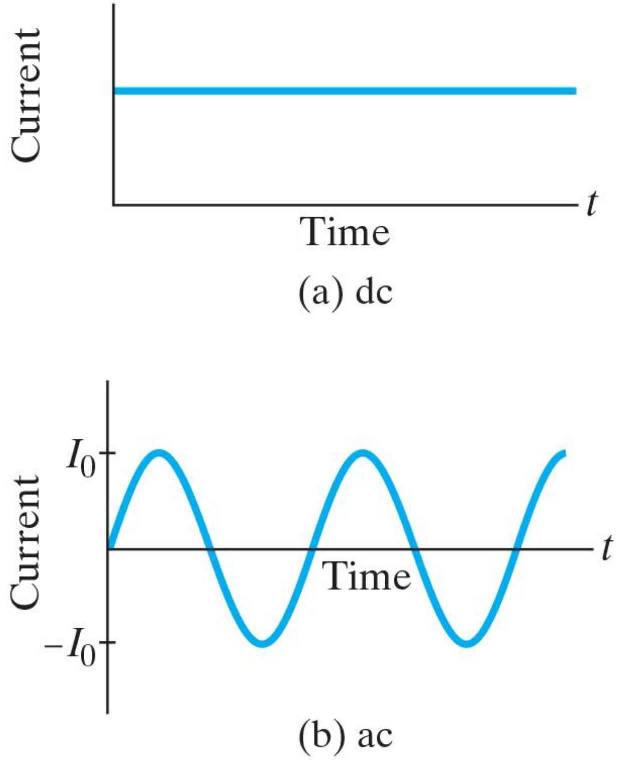
TUDelft

25.7 – Alternating Current



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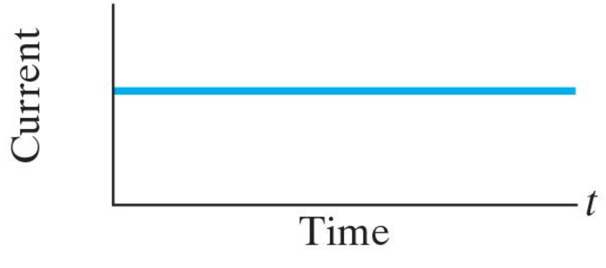
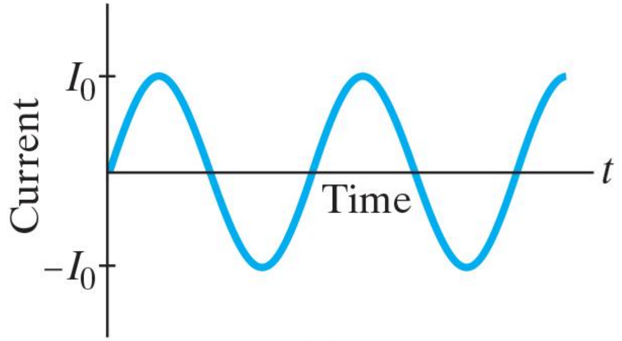
where ω is the angular frequency of the voltage signal and V_0 is referred to as the peak voltage.

25.7 – Alternating Current



(a) dc



(b) ac

Using Ohm's law, we can express the current as well as a sinusoidal wave

$$\boldsymbol{I = \frac{V}{R} = \frac{V_0}{R}\sin\omega t = I_0 \sin\omega t}$$

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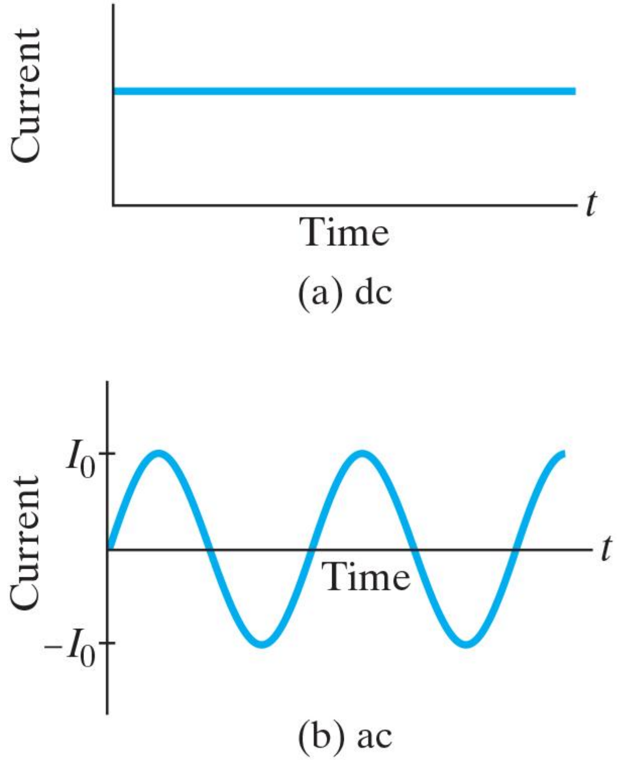
25.7 – Alternating Current



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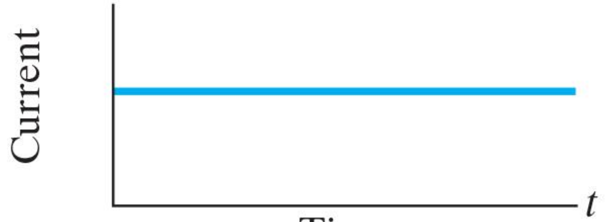
TU Delft

25.7 – Alternating Current



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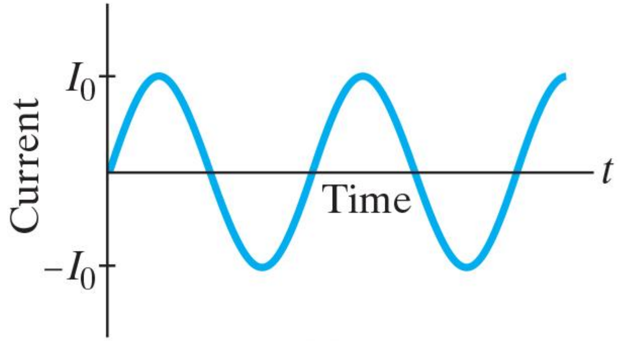

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$$\boldsymbol{P = I^2 R = I_0^2 R \sin^2 \omega t}$$

is always non-negative. The integral over one period of $\sin^2 \omega t$ is $\int_0^T \sin^2 \omega t \, dt = \frac{1}{2}$, hence the average power transformed (by a resistance R) is

TUDelft

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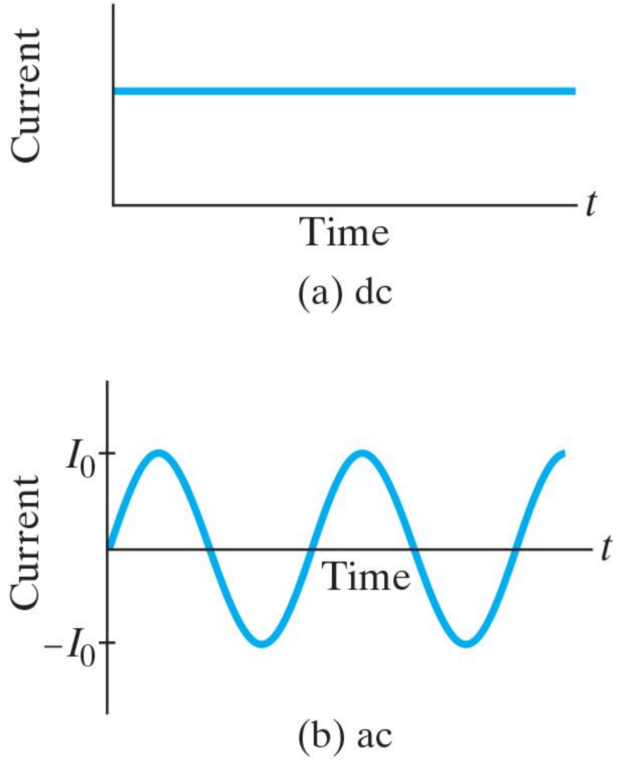

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$$\overline{P} = I_0^2 R \frac{1}{T} \int_0^T \sin^2 \omega t \, dt = \frac{1}{2} I_0^2 R = \frac{1}{2} \frac{V_0^2}{R}$$

TU Delft

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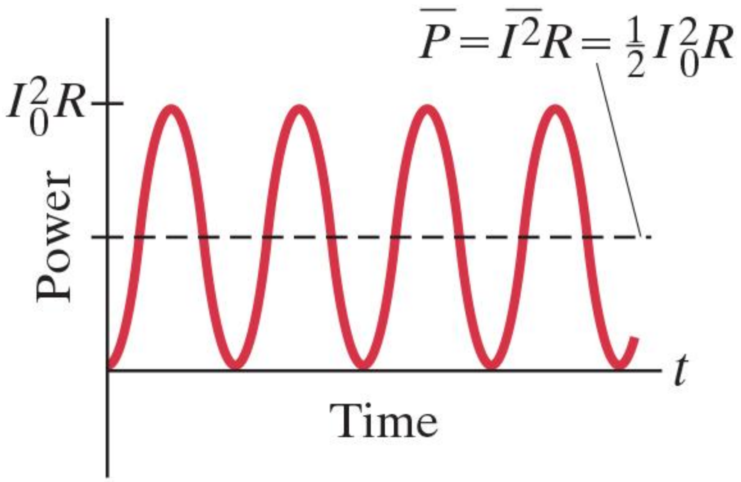

Given that we have been using $P = \frac{V^2}{R} = I^2R$ before, we want to use a similar expression now.

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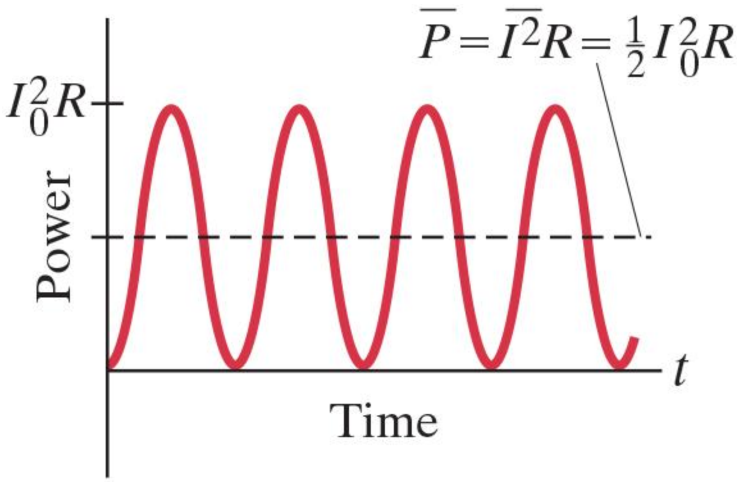

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$$\boldsymbol{I_{rms} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}}, V_{rms} = \sqrt{\overline{V^2}} = \frac{V_0}{\sqrt{2}}}$$

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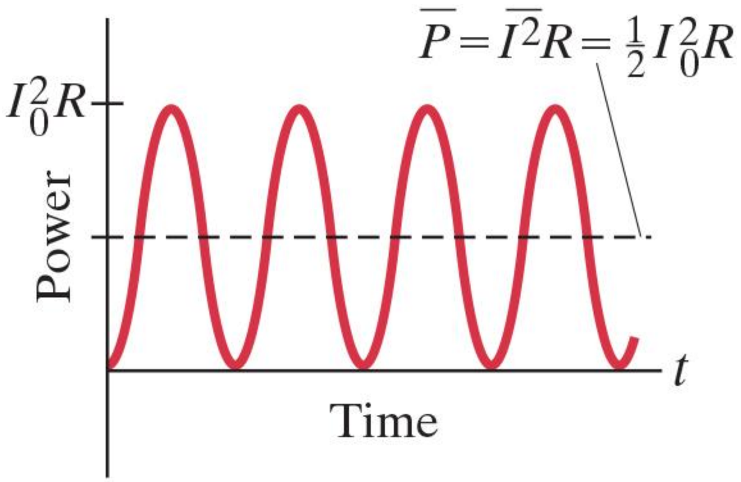

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$$\boldsymbol{I_{rms} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}}, V_{rms} = \sqrt{\overline{V^2}} = \frac{V_0}{\sqrt{2}}}$$

These rms values are called effective values, and they can be substituted in the power formulas to get the average power

$$\boldsymbol{\overline{P} = I_{rms}V_{rms} = \frac{1}{2}I_0^2R = I_{rms}^2R = \frac{1}{2}\frac{V_0^2}{R} = \frac{V_{rms}^2}{R}}$$

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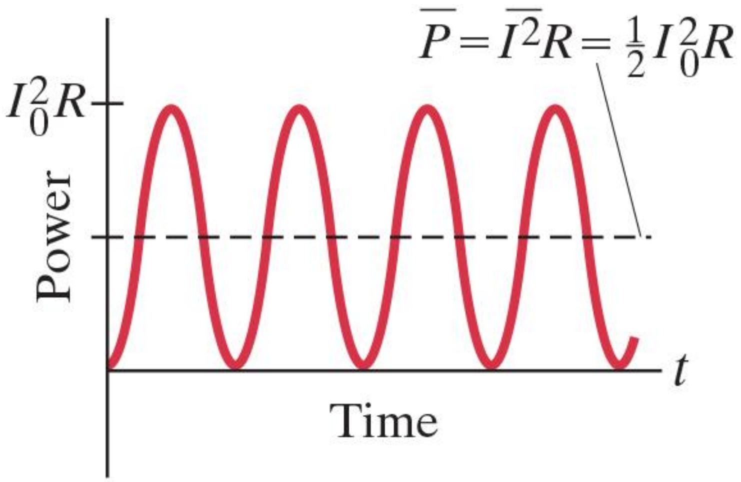
25.7 – Alternating Current



Thus, a direct current whose values of I and V are equal to the rms values of I and V for an alternate current, will produce the same power.

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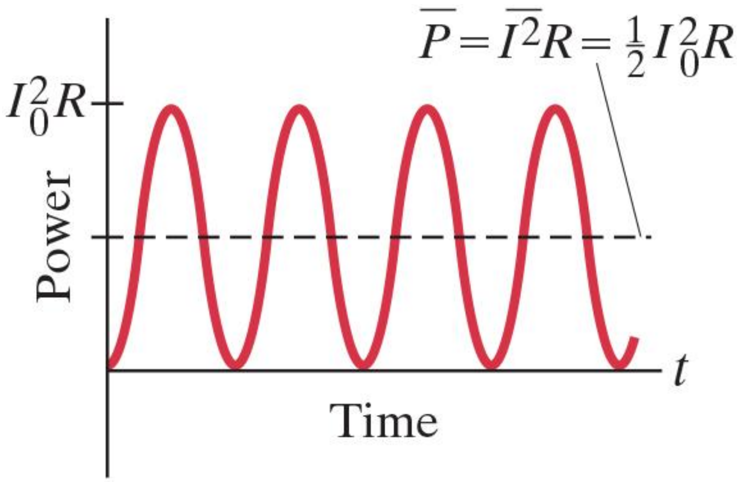

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TUDelft

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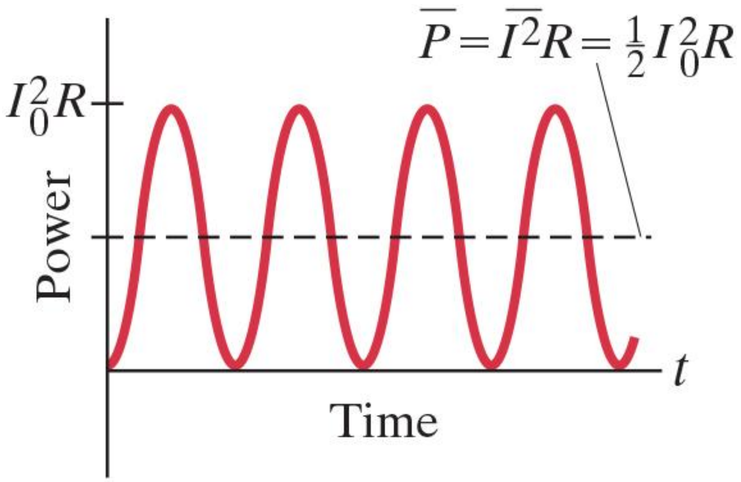

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In much of the world, the rms voltage is $240\ V$.

The frequency of the voltage signal is either 50 or 60 Hz.

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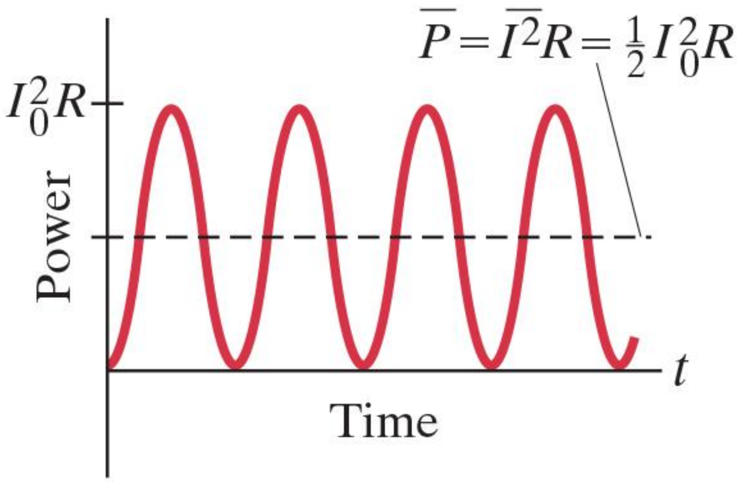

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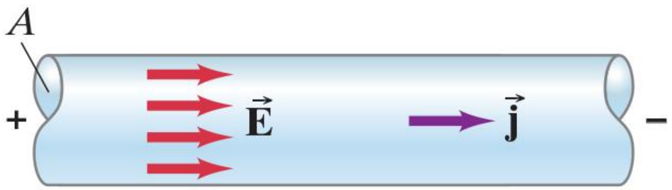
The specified power required by a device is $\bar{P}$, with the actual power required ranging between 0 and $\overline{2P}$ (see figure to the left).

25.8 – Microscopic View of Electric Current



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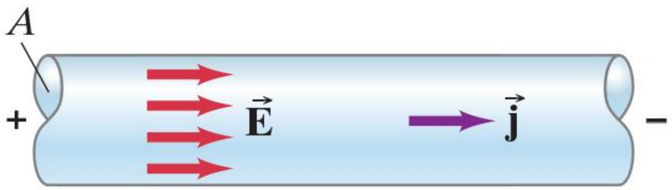

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When a voltage is applied at the extremes of a wire of uniform cross-section, the direction of the electric field $\vec{E}$ is parallel to the walls of the wire. Note that this is consistent with what seen in previous chapters about conductors and electric fields, as we now deal with moving charges and not with the electrostatic case.

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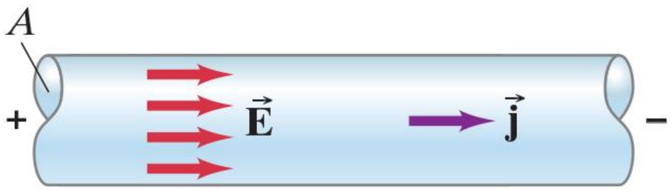

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We now define the current density $\vec{j}$ as the electric current per unit cross-sectional area at any point in space.

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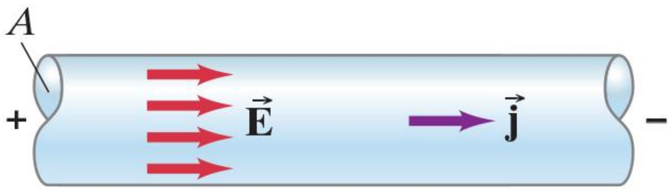

If the current density in a wire of cross-section A is uniform, then $j = \frac{I}{A} \rightarrow I = jA$. If not, we need to use an integral notation

$$\boldsymbol{I = \int \vec{j} \overrightarrow{d A}}$$

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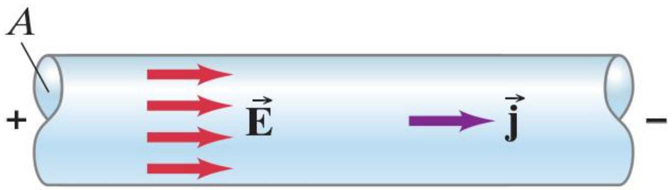

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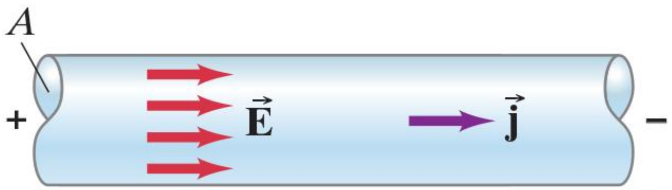

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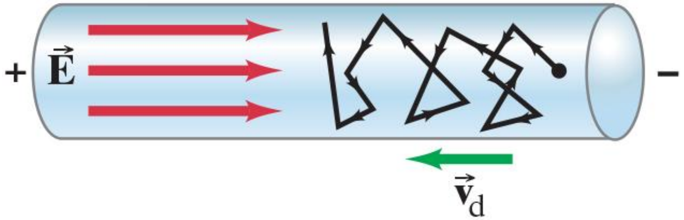
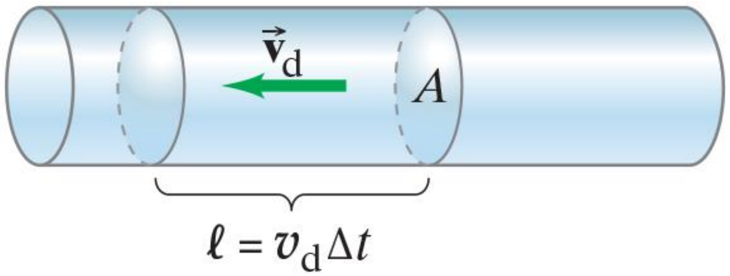


Free electrons initially move randomly at high speeds in a wire bouncing off the metal atoms of the wire, then they will eventually reach an almost steady average speed, known as drift speed v_d.

25.8 – Microscopic View of Electric Current



We can express the drift speed (which is much smaller than the thermal speed!) as follows.
In a time Δt, electrons travel a distance $l = v_d \Delta t$ and hence occupy a volume $V = A v_d \Delta t$.

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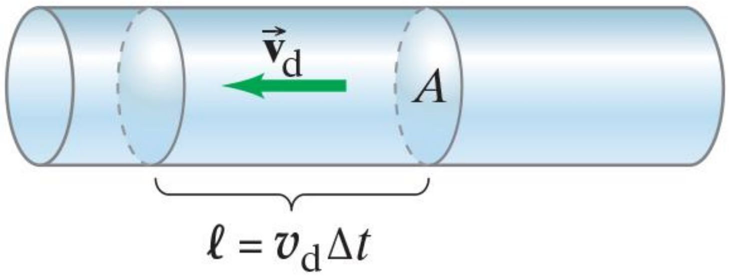

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In a time Δt, electrons travel a distance $l = v_d \Delta t$ and hence occupy a volume $V = A v_d \Delta t$. Assuming n electrons, each of charge $-e$, per unit volume, then the overall number of electrons is $N = nV = nAv_d \Delta t$ and the total charge passing through the area in the given time is

$$\boldsymbol{\Delta Q = (nV) \times (-e) = -nAv_d \Delta t e}$$

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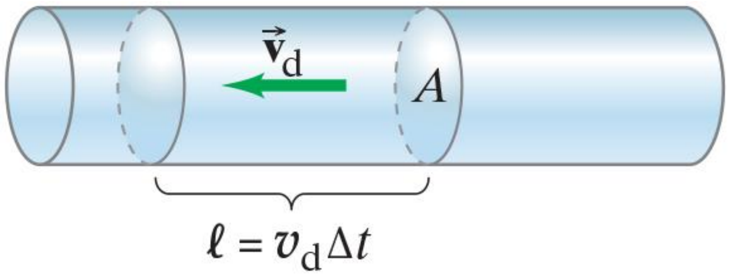

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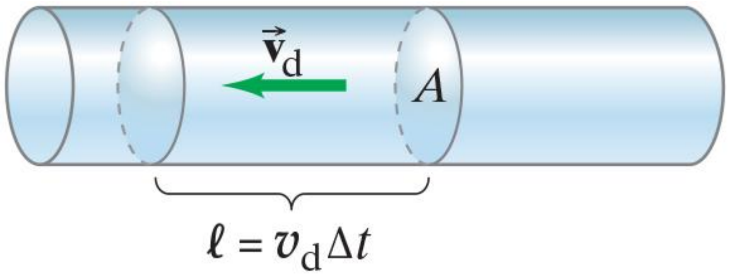

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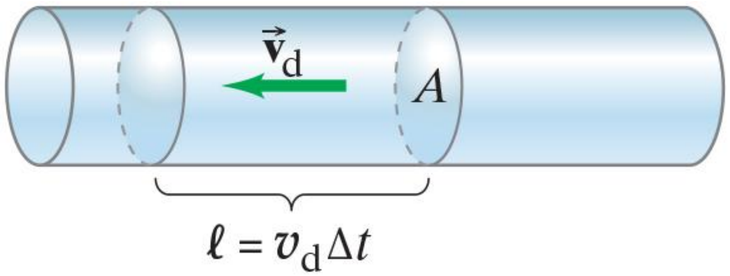

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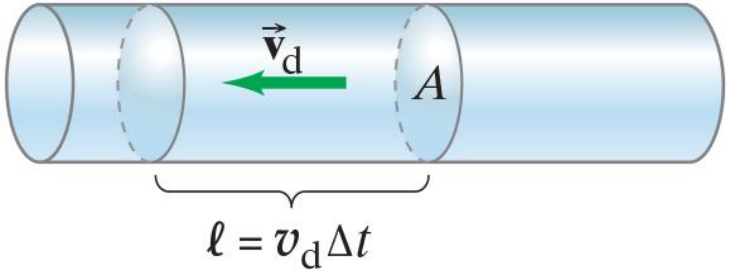

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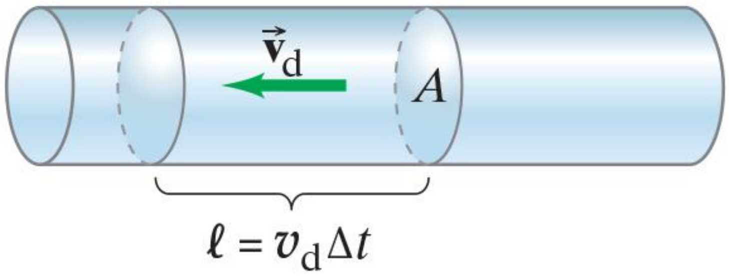
25.8 – Microscopic View of Electric Current



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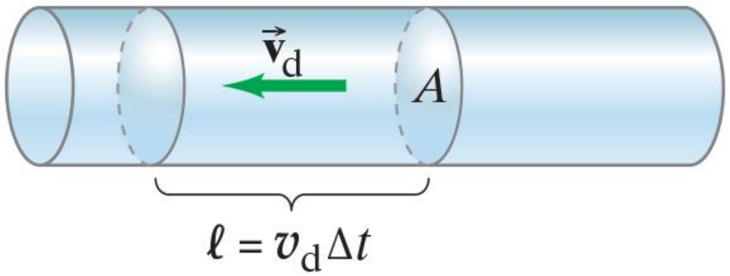
We wrap up this section by determining the electric field inside a wire of length l using a mix of the macroscopic and microscopic properties unfolded so far.

The resistance of such a wire is

$$\boldsymbol{R = \rho\frac{l}{A}}$$

25.8 – Microscopic View of Electric Current



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$$I = jA$$

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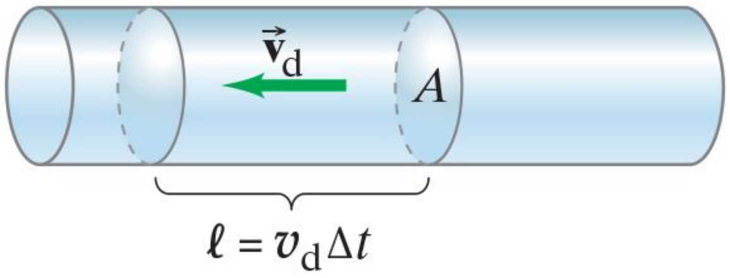

while the current is

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where we assume a constant electric field across the wire and a potential V across it.

$$V = IR \rightarrow El = \frac{jA\rho l}{A} = j\rho l$$

$$j = \frac{1}{\rho} E = \sigma E$$

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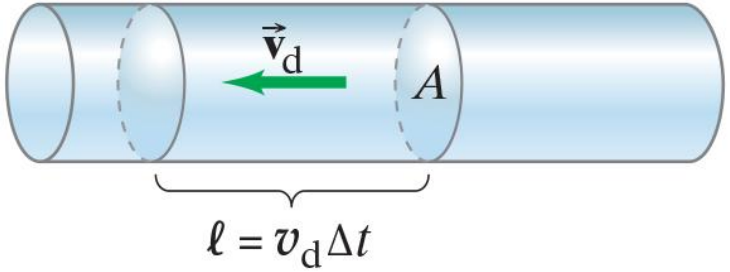

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For metal conductors, ρ and σ do not depend on V (and hence on E), hence the current density is proportional (vectorially) to the electric field:

$$\vec{j} = \sigma \vec{E} = \frac{1}{\rho} \vec{E}$$

25.8 – Microscopic View of Electric Current

A copper wire $3.2\,mm$ in diameter carries a $5.0\,A$ current. Determine (a) the current density in the wire, and (b) the drift velocity of the free electrons. (c) Estimate the *rms* speed of electrons assuming they behave like an ideal gas at $20℃$. Assume that *one electron per Cu atom* is free to move (the others remain bound to the atom).

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$$\boldsymbol{j = \frac{I}{A} = 5.0\,\frac{A}{\pi(0.0016\,m)^2} = 6.2 \times 10^5\,\frac{A}{m^2}}$$

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For the drift velocity, we first need to determine the number of free electrons per unit volume

$$\boldsymbol{n = \frac{6.02\times 10^{23}\,\frac{electrons}{mol}}{\frac{63.5\times 10^{-3}\,\frac{kg}{mol}}{8.9\times 10^{3}\,\frac{kg}{m^3}}} = 8.4\times 10^{28}\,\frac{electrons}{m^3}}$$

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$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23}\,\frac{J}{K})(293\,K)}{9.11 \times 10^{-31}\,kg}} = 1.2 \times 10^{5}\,\frac{m}{s} \rightarrow v_{rms} \gg v_d$$

Exercise: car with lights on

A person accidentally leaves a car with the lights on. If each of the two headlights uses $40\,W$ and each of the two tail-lights $6\,W$, for a total of $92\,W$, how long will a fresh $12\,V$ battery last if it is rated at $85\,Ah$? Assume the full $12\,V$ appears across each bulb.

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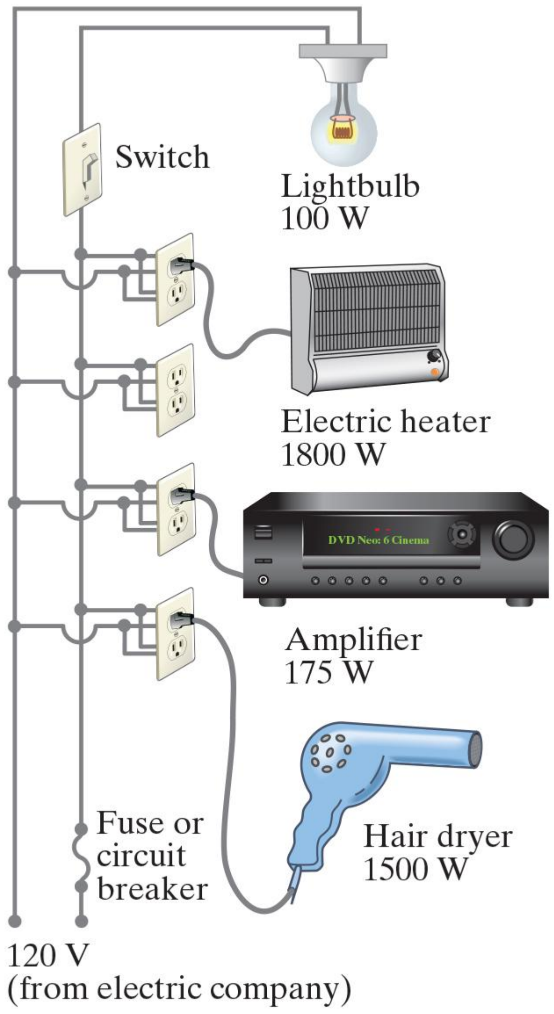
The full energy supplied by the battery is consumed by the lights:

$$\boldsymbol{QV = P\Delta t}$$

$$\boldsymbol{\Delta t = \frac{QV}{P} = \frac{(85\,Ah)\left(3,600\,\frac{s}{h}\right)(12\,V)}{92\,W} = 39,913\,s \simeq 11\,h}$$

Exercise: household current consumption



Determine the total current needed by the appliances showed in the figure.

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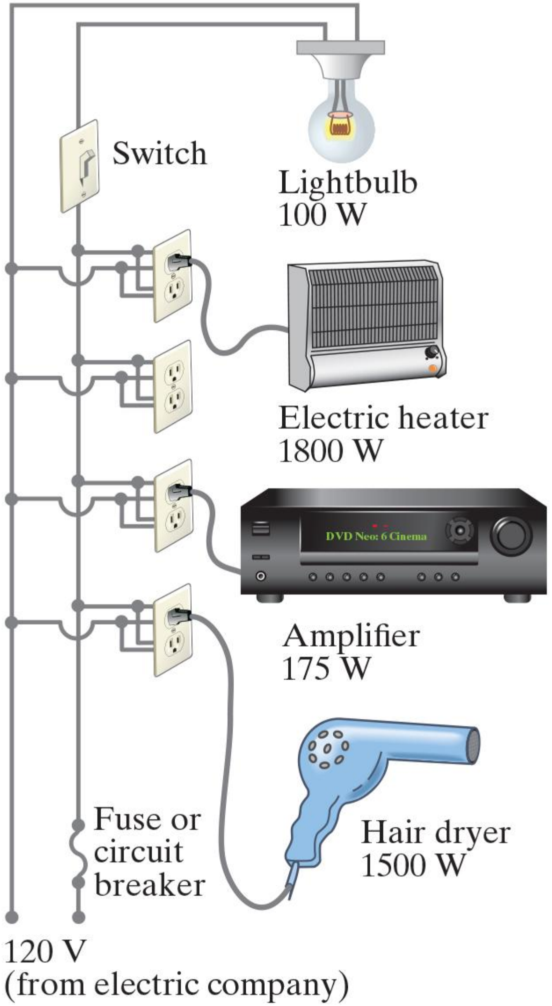

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Each appliance requires a current $I_{rms} = \frac{\bar{P}}{V_{rms}}$, where all power/voltage values in the figure are average/rms. In addition, each device sees the same voltage. Hence:

$$\boldsymbol{I = \frac{(100 + 1,800 + 175 + 1,500)W}{120\,V} = 29.8\,A}$$

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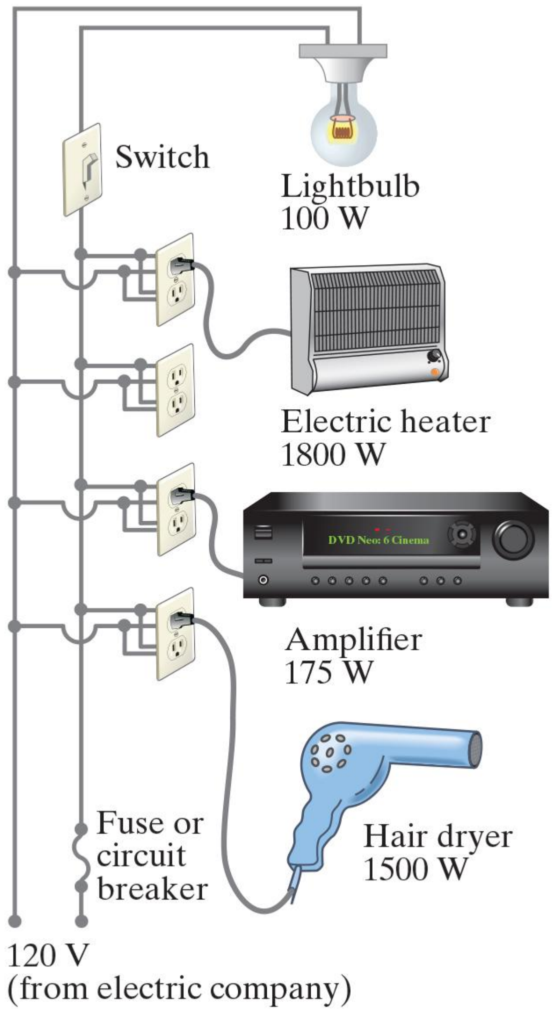

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The fuse in the household should be designed to withstand currents greater than $29.8\,A$. If not, the fuse should blow to open the circuit and prevent excessive heating that might cause fires

TUDelft

Exercise: air conditioner

An air conditioner draws $14\ A$ at $200\ V$ ac. The connecting cord is copper wire with a diameter of $1.628\ mm$.

- How much power does the air conditioner draw?
- If the total length of wire is $15\ m$, how much power is dissipated in the wiring?
- If a no. 12 wire with a diameter of $2.053\ mm$, was used instead, how much power would be dissipated in the wiring?
- Assuming that the air conditioner is run $12\ h$ per day, how much money per month (30 days) would be saved by using no. 12 wire?

Assume that the cost of electricity is $12\,\frac{cents}{kWh}$.

TUDelft

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The power drawn is $\boldsymbol{P = IV \simeq 3,100\,W}$ (we dropped all the additional symbols/subscripts for simplicity).

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The savings S are due to the smaller dissipation of power within the wire:

$$\boldsymbol{S = (24 - 15\,W)(30\,days)\left(12\frac{h}{days}\right)\left(0.00012\frac{\$}{W}\right) = 0.038\$}$$

Wrap-up: revisiting Learning objectives

After this lecture you should be able to:

- Understand the general **characteristics** of electric current
- Understand the **Ohm's Law** and the relationship between voltage, current, and resistance and determine the **resistance** of a wire or other conducting mean given its geometric and material properties
- Understand the basics of **alternate current**
- Understand the basics of **electric current at the microscopic level**

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$$R = \rho \frac{l}{A},$$

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

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