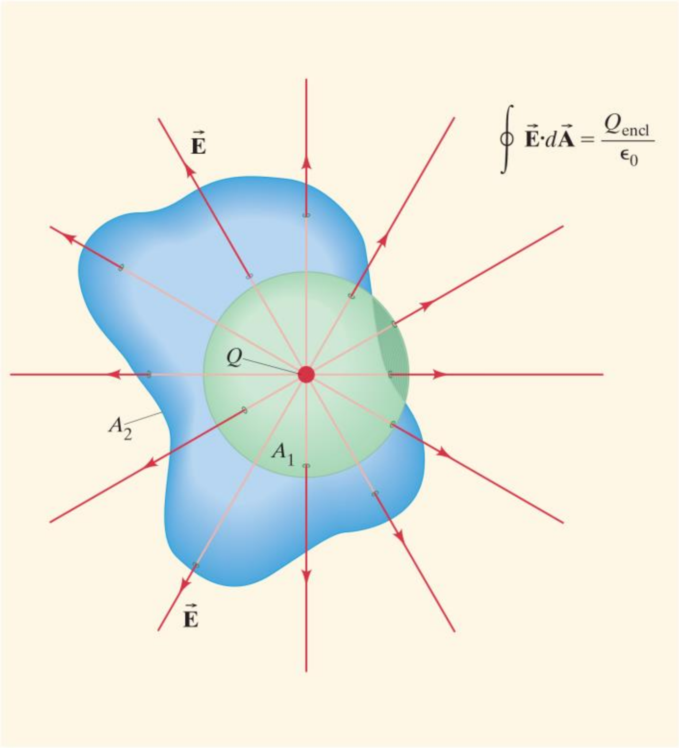
GAUSS'S LAW

Chapter 22



Alessandro Bombelli

Operations & Environment

Faculty of Aerospace Engineering

Structure of the lecture

1. Electric flux
2. Gauss's Law
3. Applications of Gauss's Law

Learning objectives for today's lecture

After this lecture you should be able to:

- Compute the **electric flux** given an electric field and a pre-defined surface

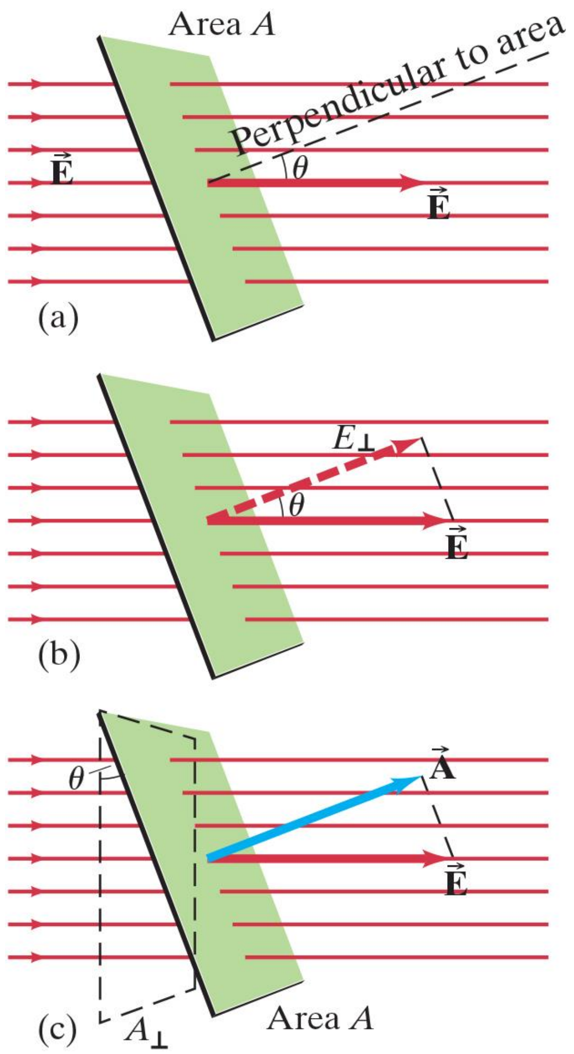
Learning objectives for today's lecture

After this lecture you should be able to:

- Compute the **electric flux** given an electric field and a pre-defined surface

- Use such electric flux to apply **Gauss's law** to determine the magnitude of the electric field in a given point of space

22.1 – Electric Flux



The electric flux Φ_E of a constant electric field E passing through a surface A is defined as

$$\boldsymbol{\Phi_E = EA\cos\theta}$$

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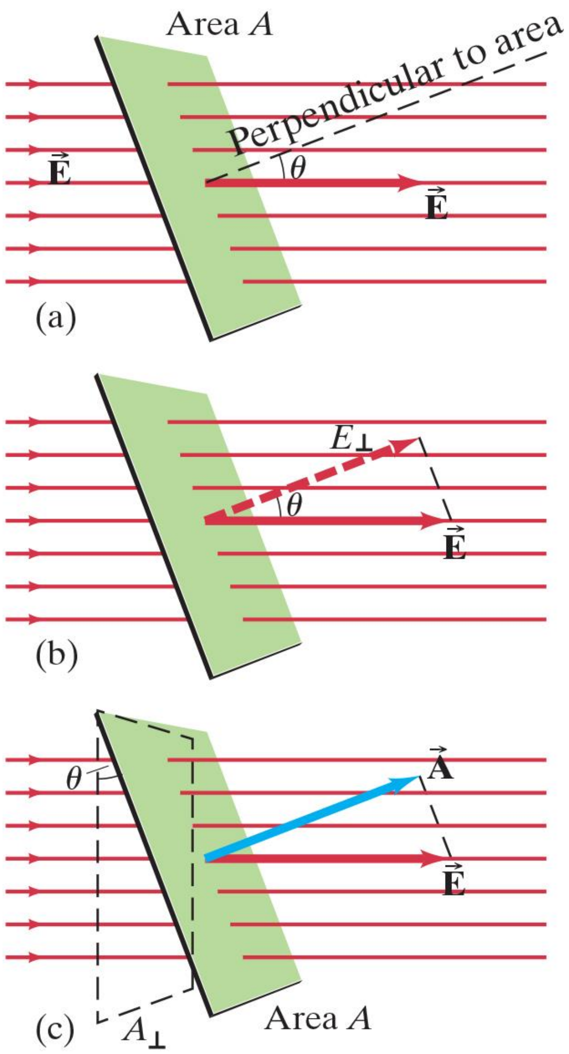

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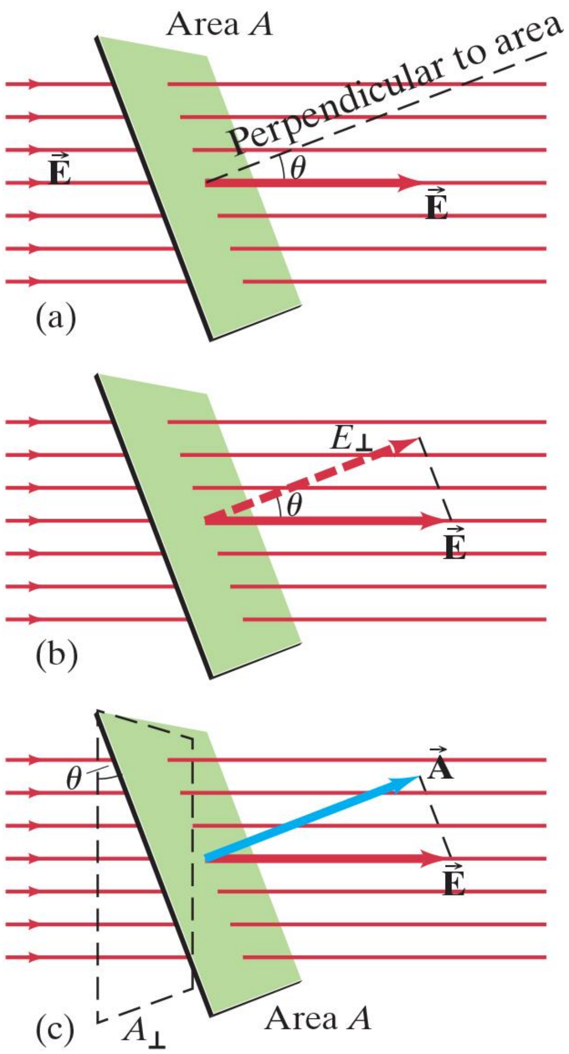

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where θ is the angle between the surface and the electric field lines, i.e., we are considering the perpendicular component of the electric field to the surface.

We can replace the scalar A with the **vector $\vec{A}$** with magnitude A and orientation perpendicular to the actual surface, so that

$$\boldsymbol{\Phi_E = \vec{E} \cdot \vec{A}}$$

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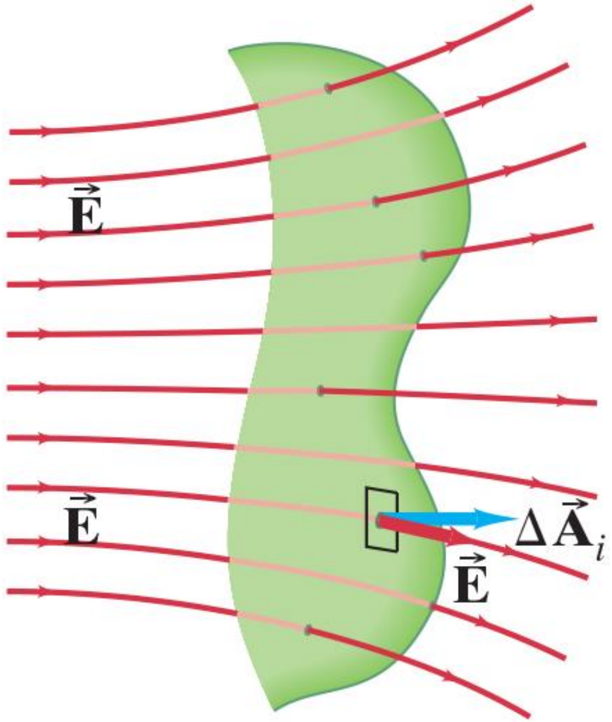

What if the electric field is not uniform and/or the surface is not flat (hence without a constant orientation)?

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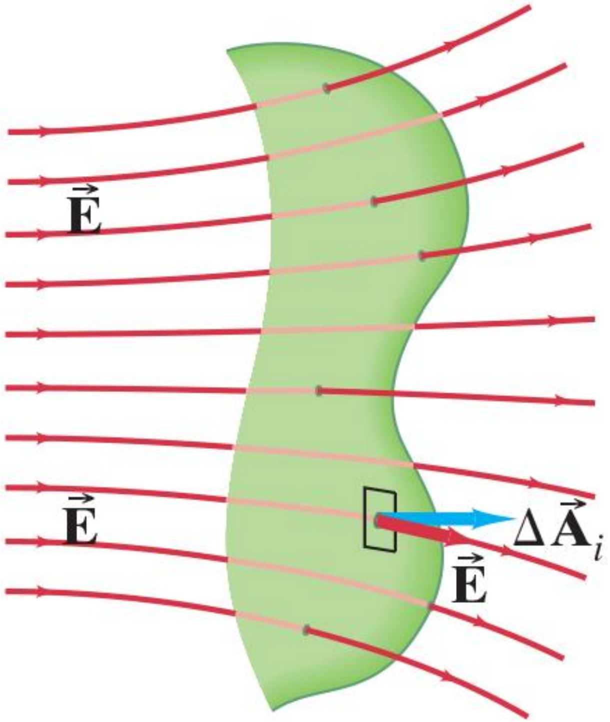

What if the electric field is not uniform and/or the surface is not flat (hence without a constant orientation)?

We resort to numerical integration (outside the scope of this course)

$$\Phi_E = \sum_{i=1}^{n} \vec{E_i} \cdot \Delta\vec{A_i}$$

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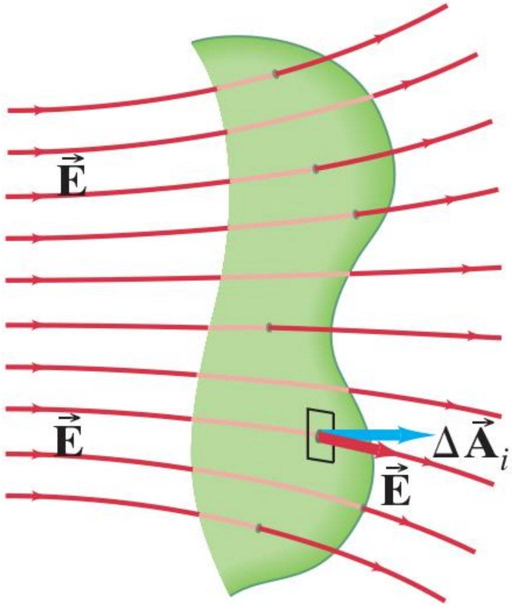

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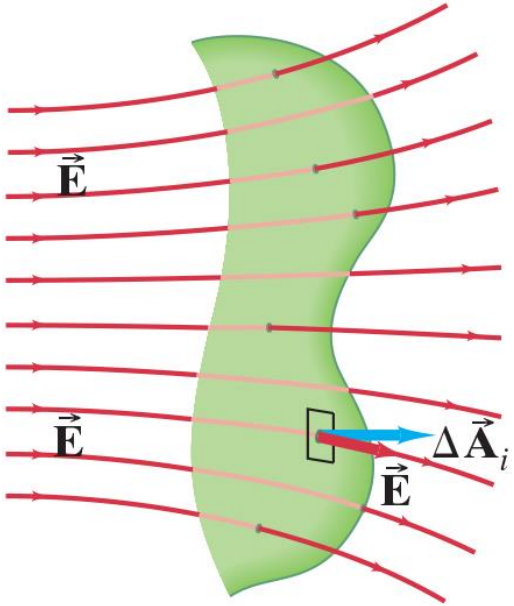

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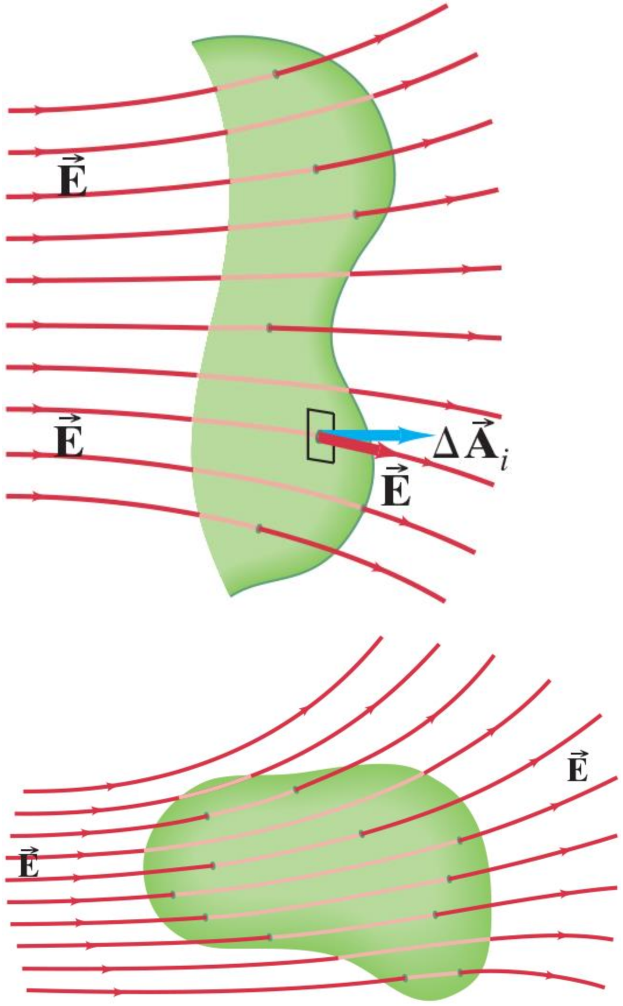

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In the case of a closed surface (quite common), we are interested in the net electric flux computed using a closed integral

$$\boldsymbol{\Phi_E = \oint \vec{E} \cdot d\vec{A}}$$

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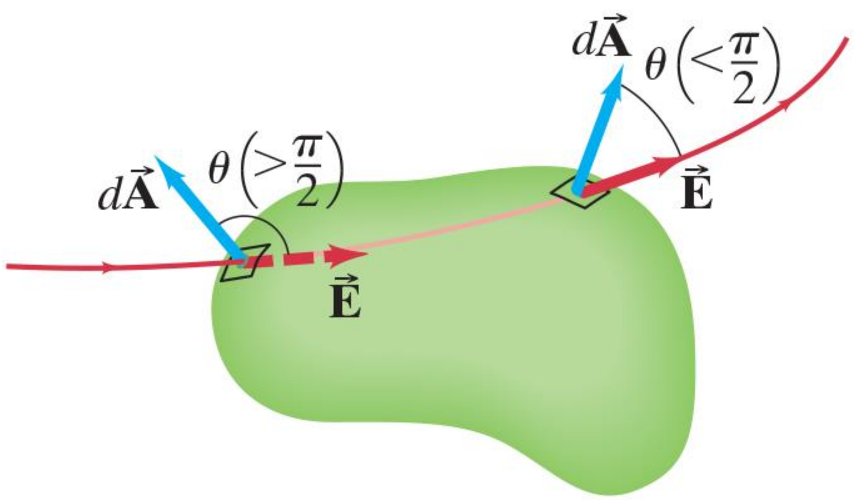
22.1 – Electric Flux: direction of $\vec{A}$



The direction of $\vec{A}$, especially for an open surface, might be set arbitrarily. The same might be argued for a closed surface, but it is a common choice to point it outward from the enclosed volume.

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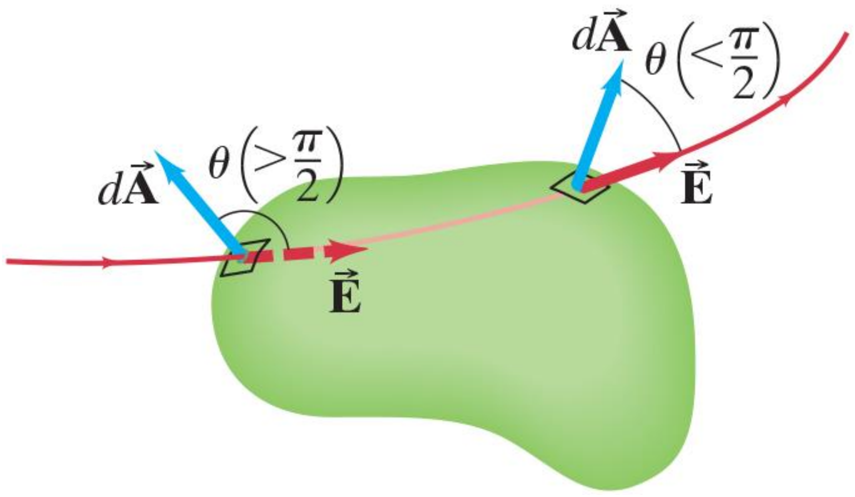
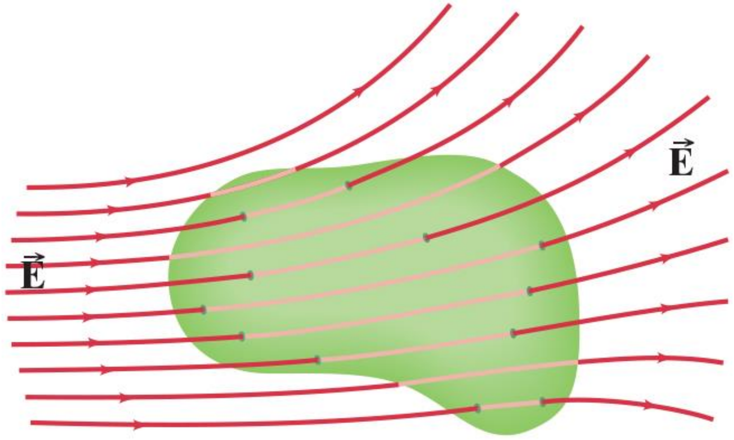


The direction of $\vec{A}$, especially for an open surface, might be set arbitrarily. The same might be argued for a closed surface, but it is a common choice to point it outward from the enclosed volume.

In this way, flux entering the volume will have a negative contribution and flux exiting the volume will have a positive contribution

22.1 – Electric Flux: when it is non-zero



If a closed surface is crossed by electric field lines, the cumulative positive contribution to the flux is perfectly balanced by the negative contribution, hence there is no net flux

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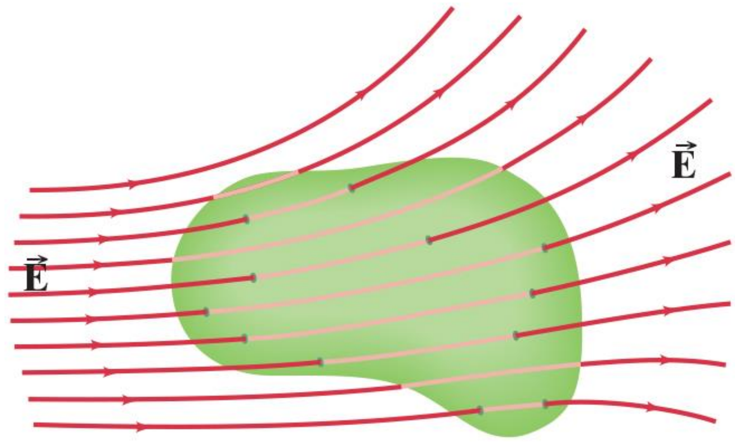

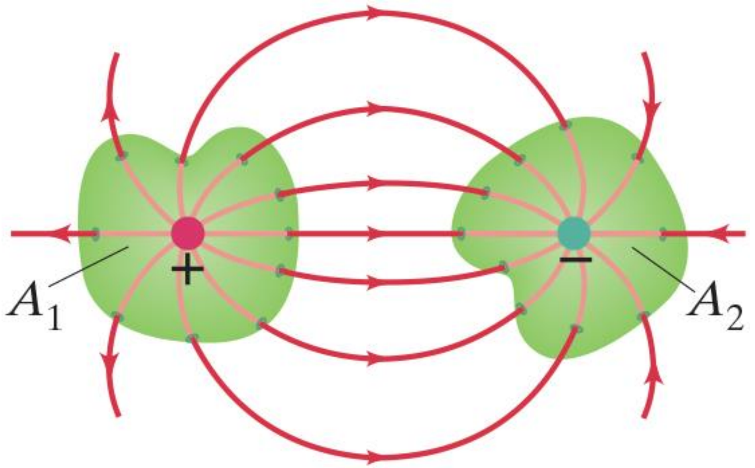

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The only way to have a non-zero net flux is to surround a charged particle with a closed surface. The flux will be positive (exiting the surface) for a positive charge and negative (entering the surface) for a negative one

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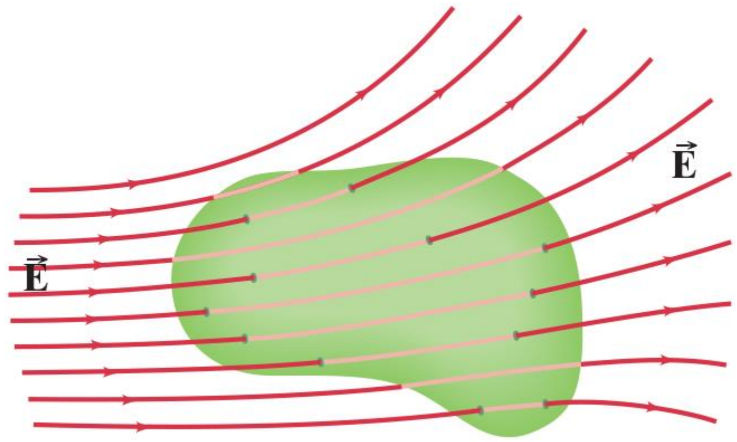

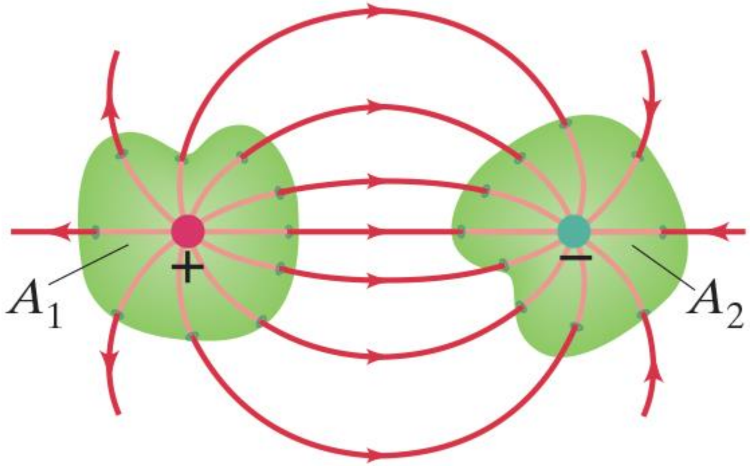

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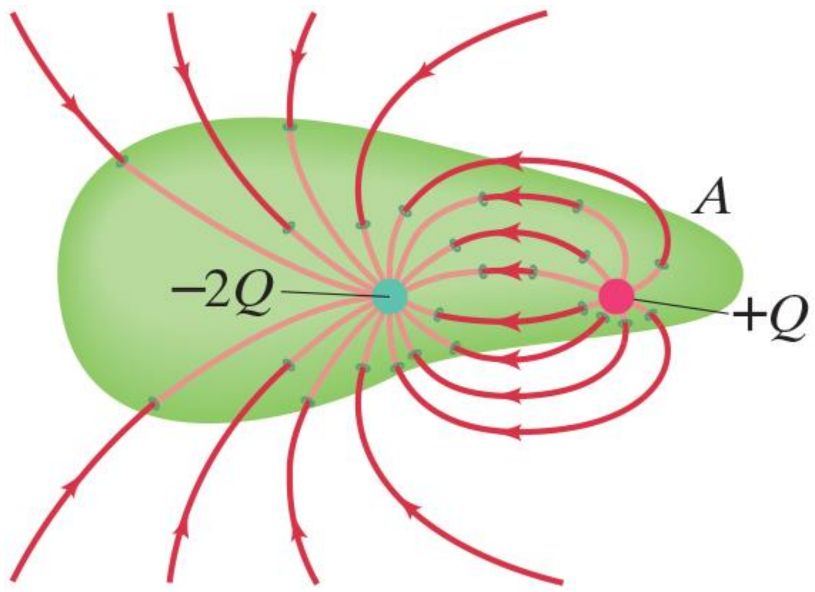
This is already a spoiler of Gauss's Law

22.2 – Gauss's Law



Gauss's Law defines a relation between the electric flux through a closed surface and the net charge Q_{encl} enclosed by that surface

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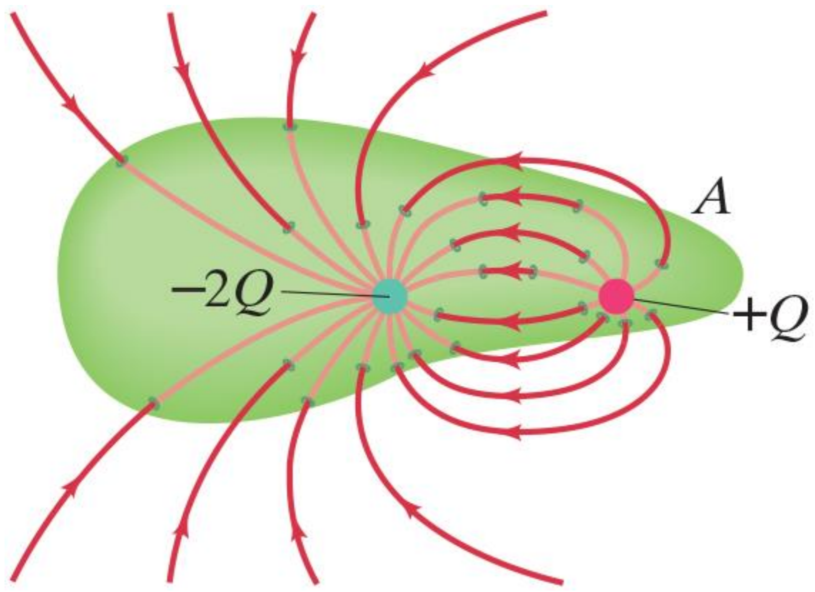

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A few important considerations:

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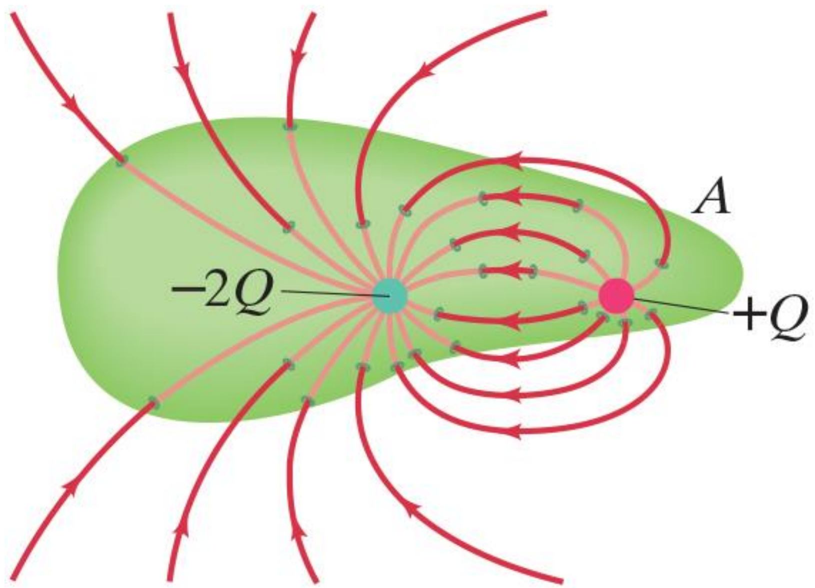

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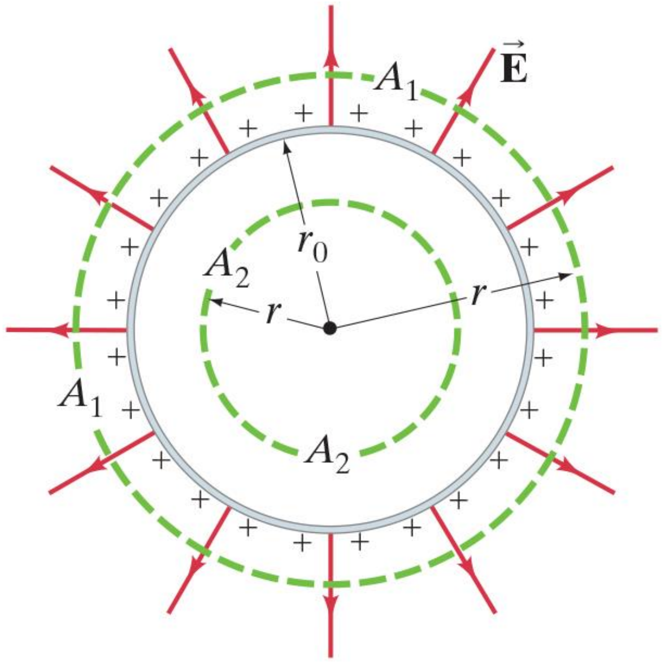
A few important considerations:

- the same net charge can be enclosed by many different closed surfaces. Choosing a proper surface is key in simplifying the integral
- The electric field can be moved outside of the integral if and only if constant across the full integral!

22.3 – Applications: charged spherical conducting shell



Because the shell is spherical and the charge is uniformly distributed on it (the overall charge being Q), the magnitude of the electric field only depends on the distance from the center of the shell and is constant for every orientation at a given distance.

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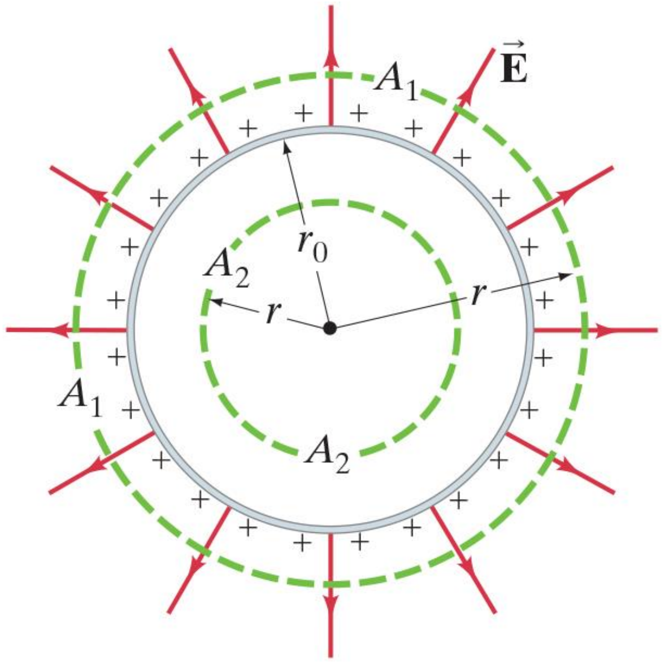

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Hence, if we enclose the shell with a Gaussian surface at a generic distance $\mathrm{r} > r_0$, we get

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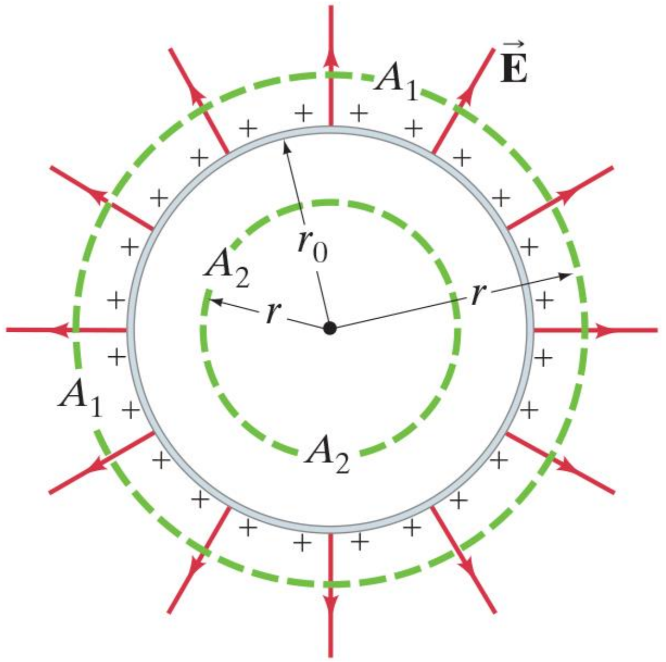

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Note: given that we deal with a perfect sphere, $\vec{E} \cdot d\vec{A} = |E||dA| \cos 0 = |E||dA|$, and because of axialsymmetry $|E|$ is constant, and can hence be brought outside the integral

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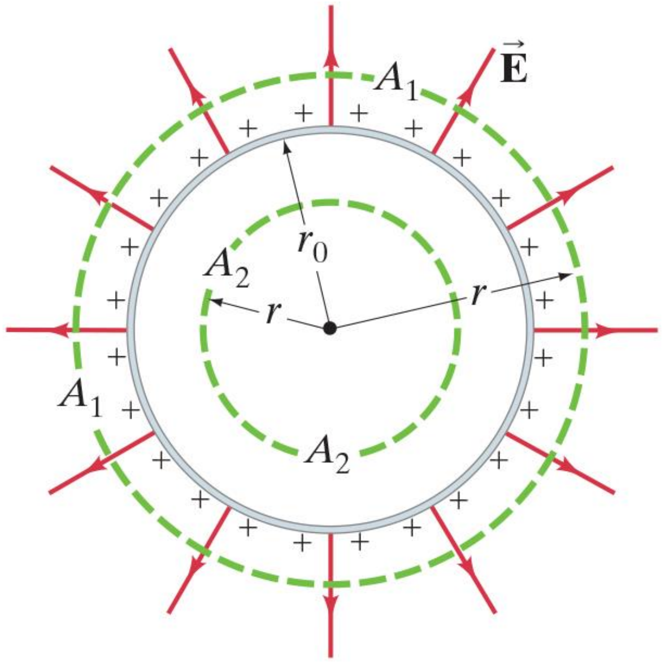

How about a point inside the shell, i.e., where $r < r_0$?

As $Q_{encl} = 0$ here, there is no electric field

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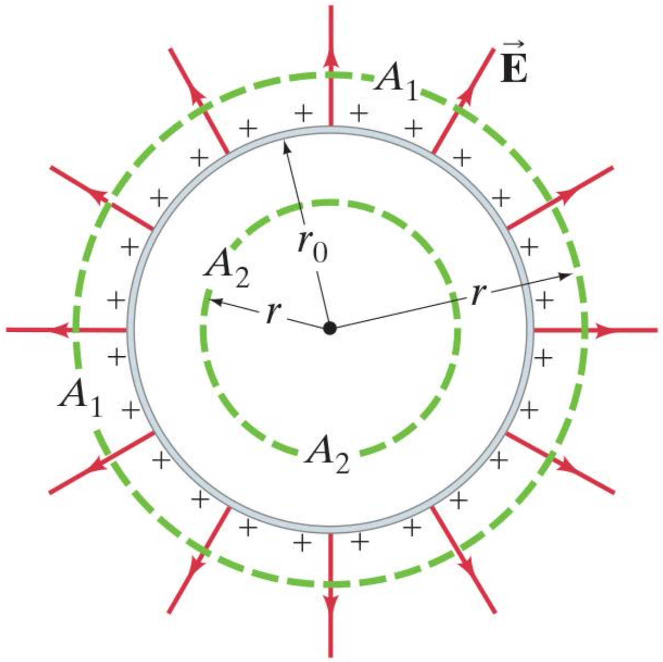

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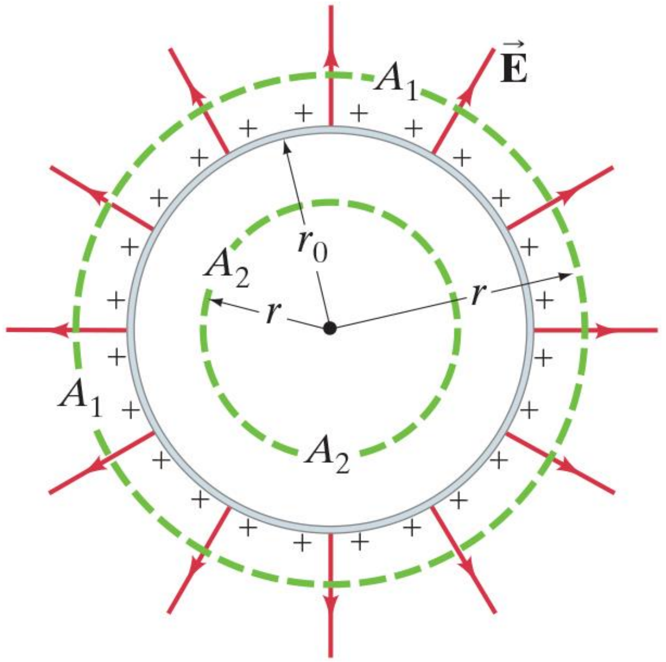

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Unlike the shell, where Gauss's Law directly gives zero field, a ring does not allow a simple Gaussian surface to enclose zero net charge in a way that forces the field to be zero. Computing the field inside a ring requires integrating contributions from different charge elements, and the result shows that the field is not zero except at the center.

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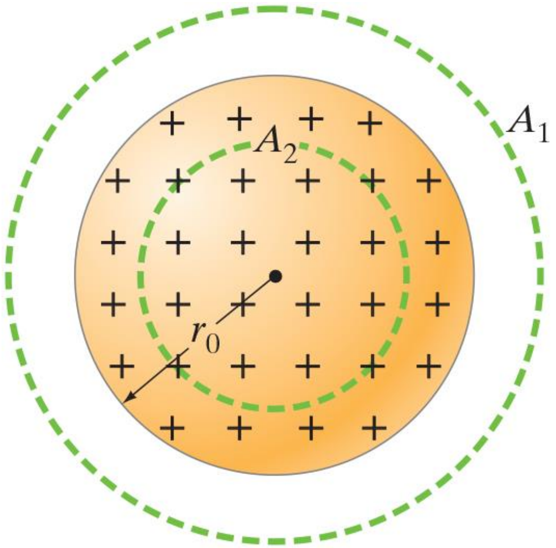
22.3 – Applications: solid sphere of charge



Some preliminary considerations:

- ρ_e is the (constant) charge density per unit volume, with Q being the overall charge carried inside the sphere

Similarly to the previous example, there is axialsymmetry to be exploited. We distinguish two cases: $r > r_0$ (outside the sphere) and $r \leq r_0$ (inside the sphere)

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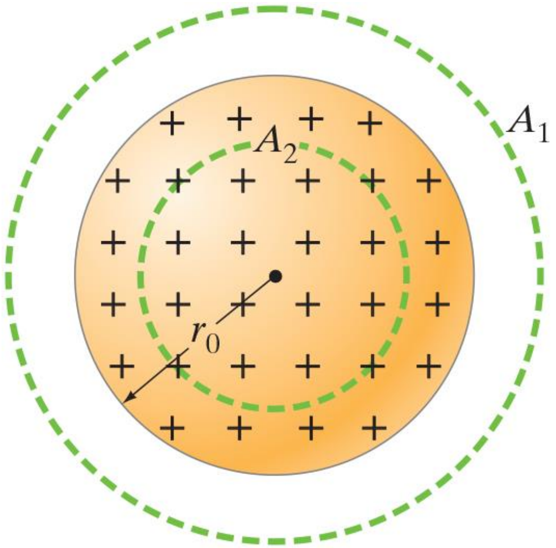

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The first one is slightly easier, as we can draw a gaussian surface outside the sphere at a generic distance r and the full charge Q will be contained no matter what

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0} \rightarrow E4\pi r^2 = \frac{Q}{\epsilon_0} \rightarrow E(r) = \frac{Q}{4\pi r^2 \epsilon_0}$$

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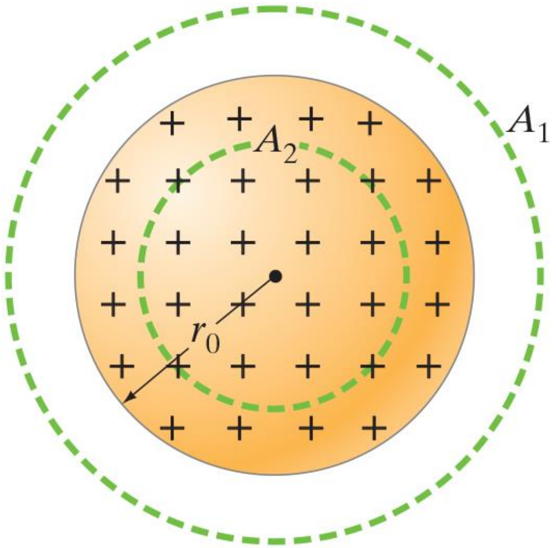

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Decrease proportional to inverse of r^2: same as if the full charge was concentrated in the center (point charge)

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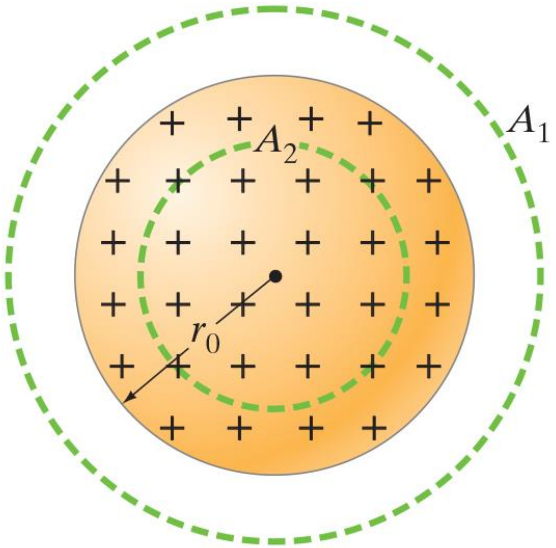

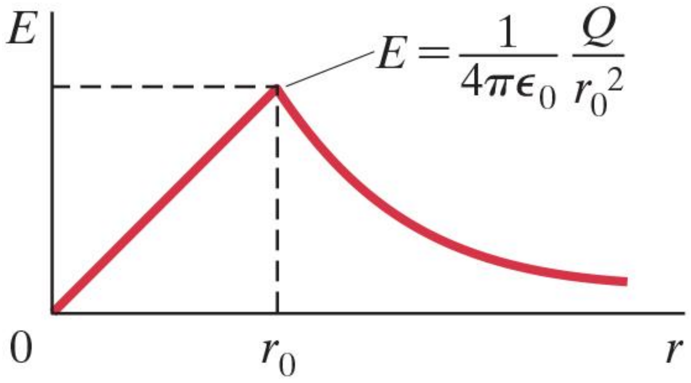

In the second case, we need to be a bit more careful with how we compute Q_{encl}, as it is not constant any longer, but depends on the radius r.

The ratio $\frac{Q_{encl}}{Q}$ is equivalent to the ratio between the contained (by the gaussian surface) and total charge, and can be expressed as

$$\frac{\boldsymbol{Q_{encl}}}{\boldsymbol{Q}} = \frac{\frac{4}{3}\pi r^3 \rho_E}{\frac{4}{3}\pi r_0^3 \rho_E}$$

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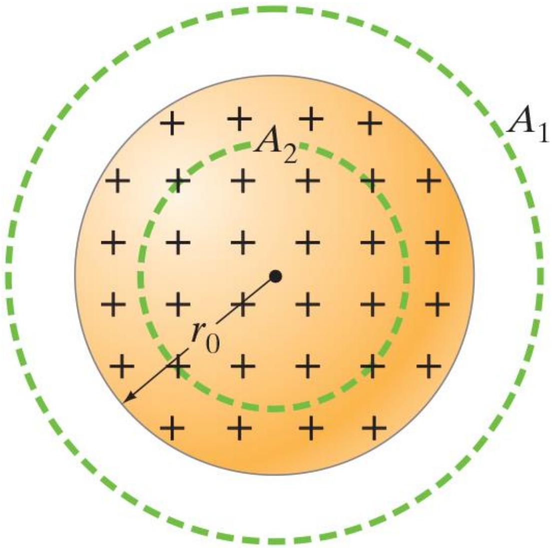

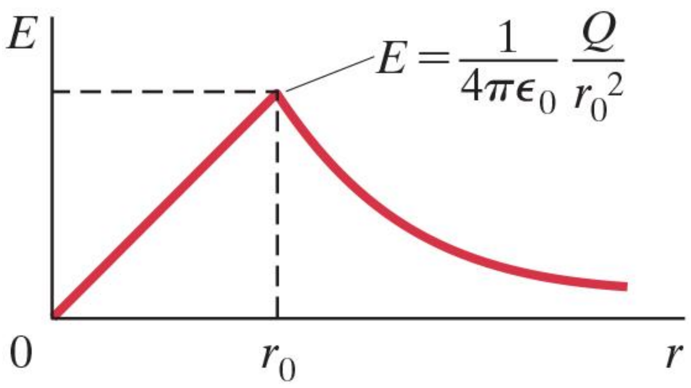

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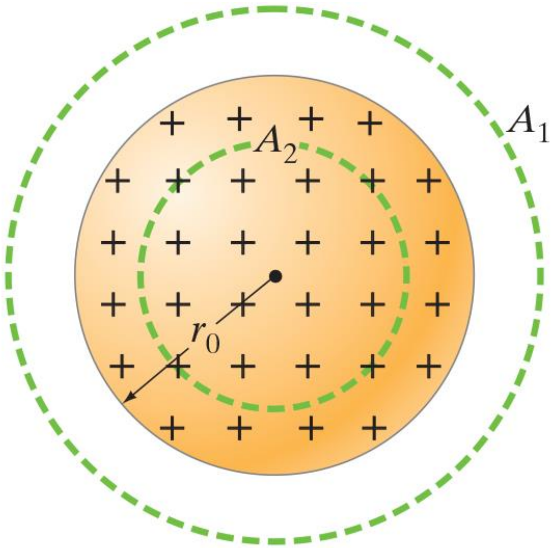

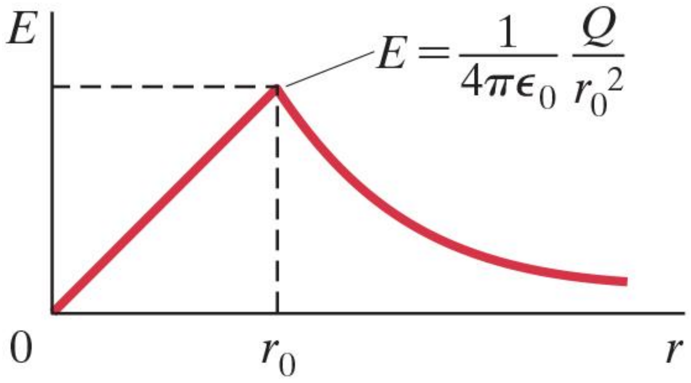

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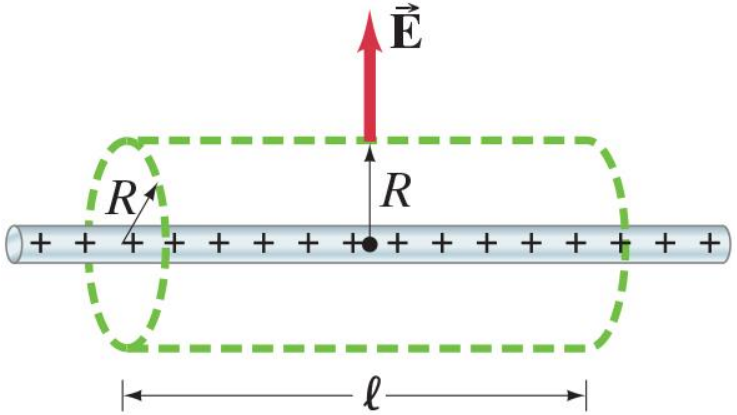
Linear increase between the center and the edge of the sphere

22.3 – Applications: infinite wire



This is the same example from last lecture, with λ being the uniform charge per unit length and $l \gg R$ (we used x in the previous lecture).

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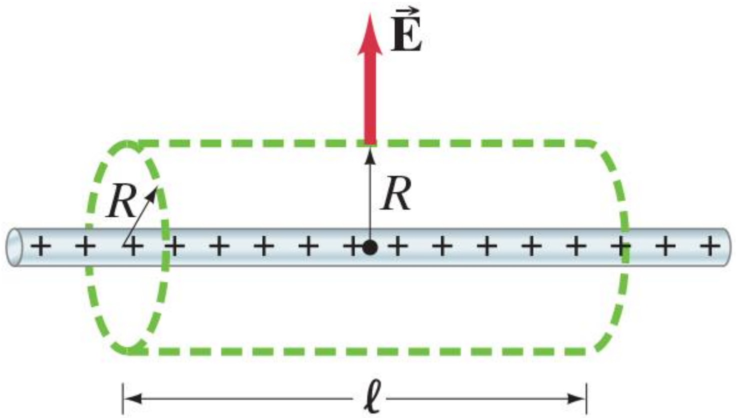

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Because of $l \gg R$, for axialsymmetry the electric field points away and is perpendicular to the wire everywhere. We can define a closed gaussian surface that is a cylinder as shown to the left.

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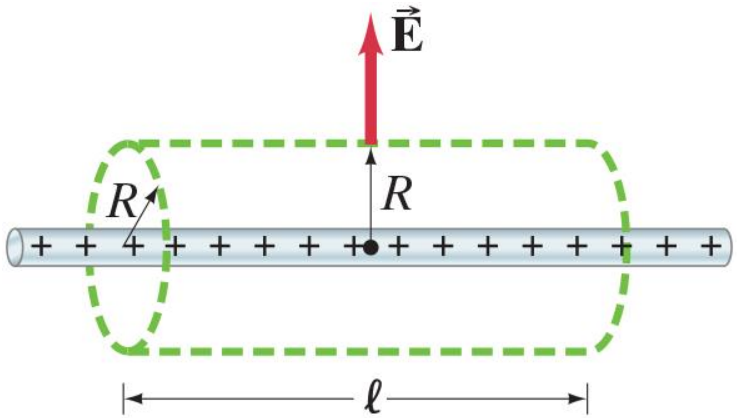

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On the lateral area $\vec{E}$ and $d\vec{A}$ are parallel, on the two bases they are perpendicular. Hence the integral is only non-zero along the lateral area. In addition, $Q_{encl} = \lambda l$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \rightarrow E 2\pi R l = \frac{\lambda l}{\epsilon_0} \rightarrow E = \frac{\lambda}{2\pi\epsilon_0 R}$$

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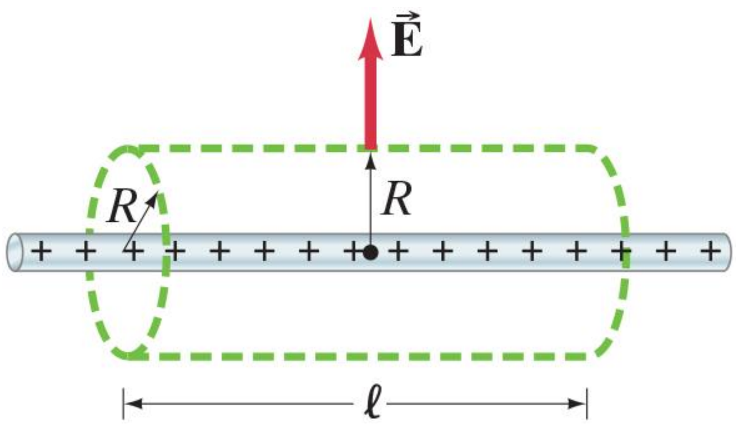

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Simpler approach than the $\int dE$ one. But only the latter one can be employed for a non-infinite wire as the axialsymmetry is lost

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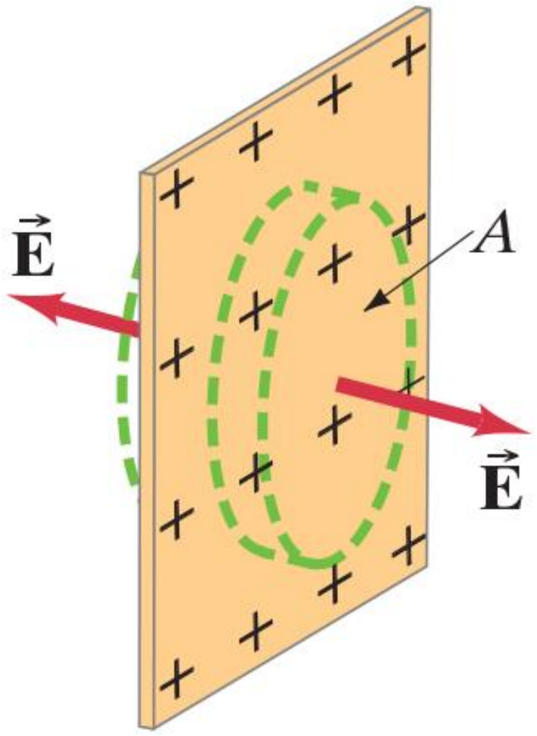
22.3 – Applications: infinite plane of charge



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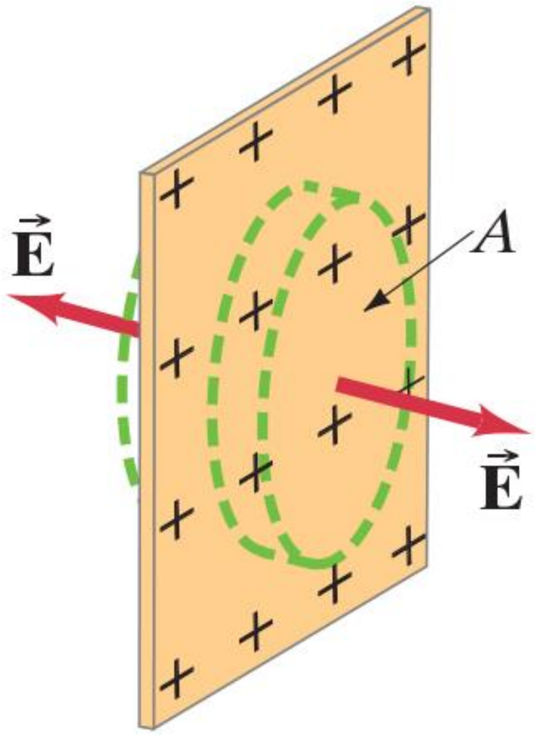

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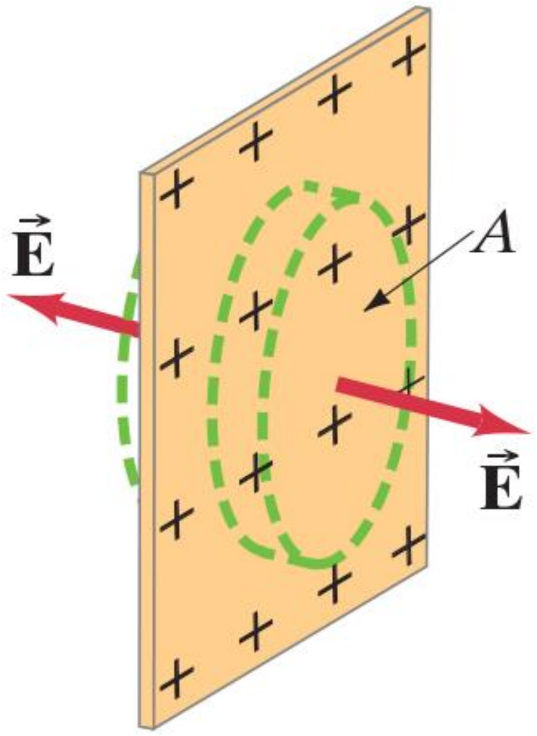

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On the two base areas $\vec{E}$ and $d\vec{A}$ are parallel, on the side area they are perpendicular. Hence the integral is only non-zero along the two bases. In addition, $Q_{encl} = \sigma A$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \rightarrow E2A = \frac{\sigma A}{\epsilon_0} \rightarrow E = \frac{\sigma}{2\epsilon_0}$$

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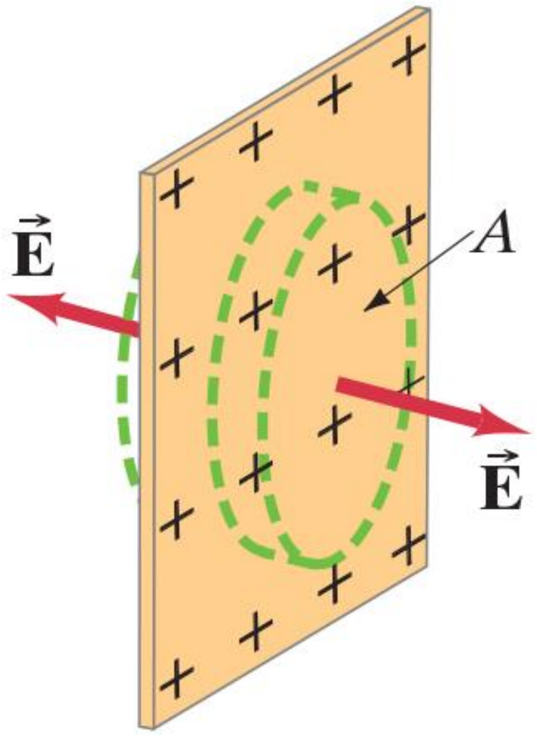

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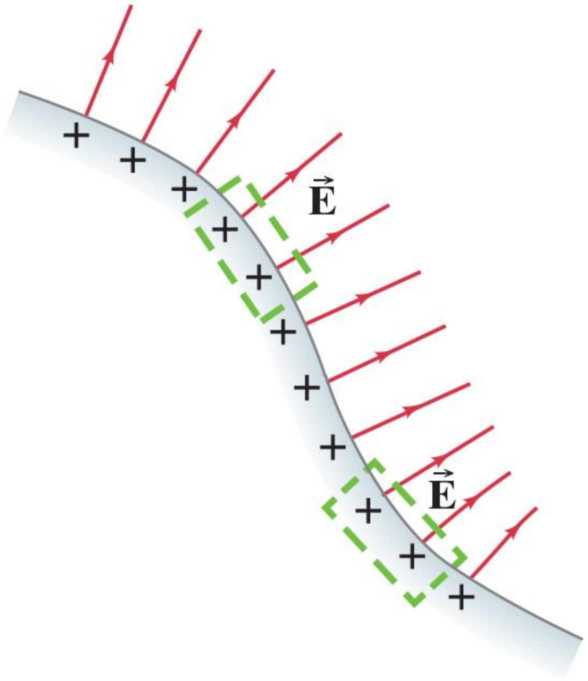
Simpler approach than the $\int dE$ one again

22.3 – Applications: electric field near conducting surface



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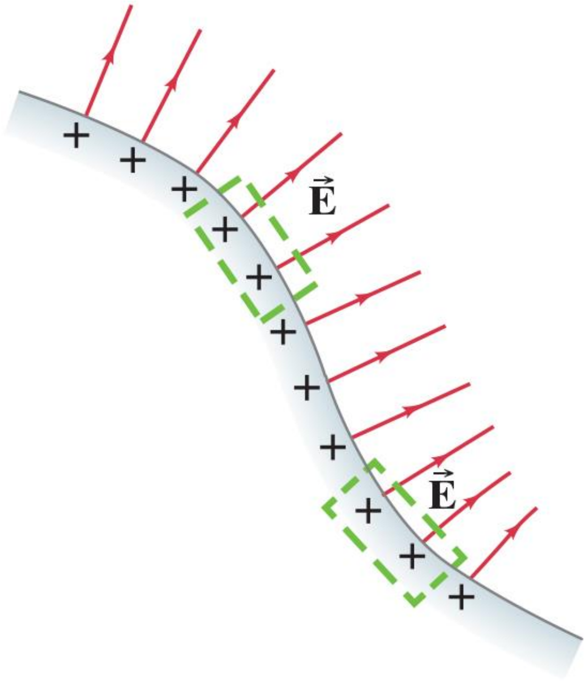
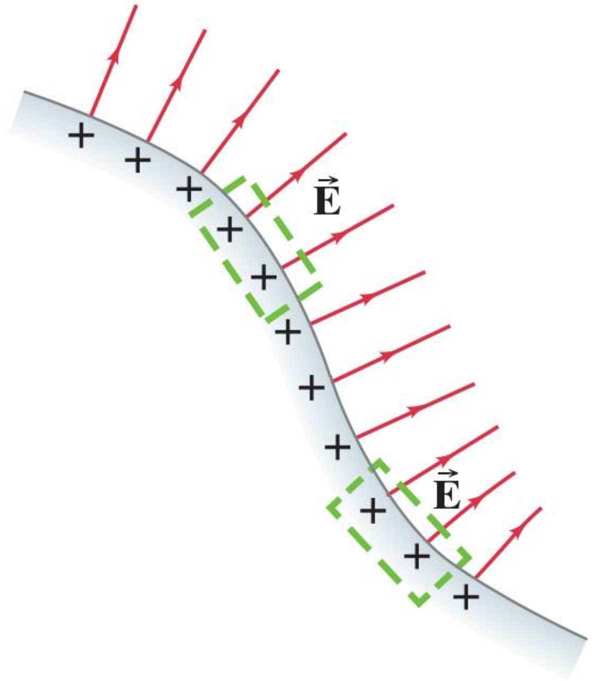


In a conducting material, we already discussed that the charge is positioned on the surface and that the electric field lines are always perpendicular to the surface. Hence, we take a small closed gaussian surface that is a cylinder of area A (in the figure, only the side area is shown) and that follows the curvature (if any) of the surface, there will be flux only through one of the two base areas (and none through the other base area and the side area)

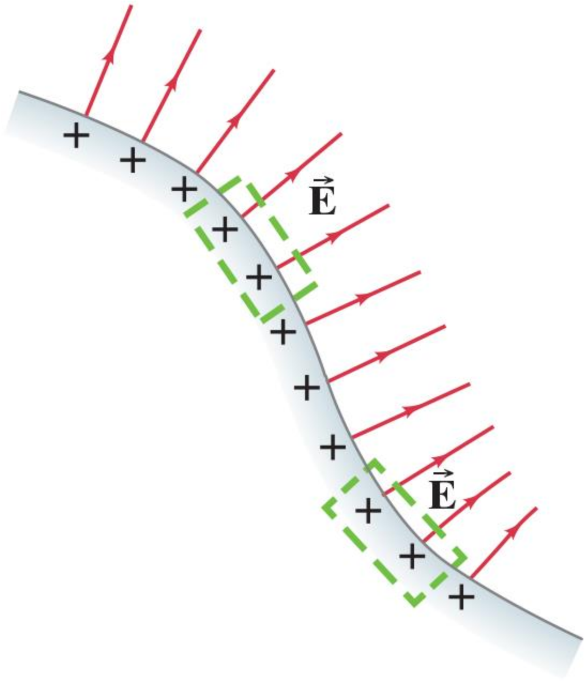
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \rightarrow EA = \frac{\sigma A}{\epsilon_0} \rightarrow E = \frac{\sigma}{\epsilon_0}$$

22.3 – Applications: electric field near conducting surface



This result is quite similar to the one obtained for the infinite plate, but without the $\frac{1}{2}$ factor. Why is that? For the non-conducting infinite plate, the charge is uniformly distributed throughout the volume (although we only consider the effect on the two base areas, as that is the only non-zero one).

22.3 – Applications: electric field near conducting surface



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In other words, the electric field for the non-conducting plate appears on both sides of the plate, whereas for the conducting surface it only appears on one side.

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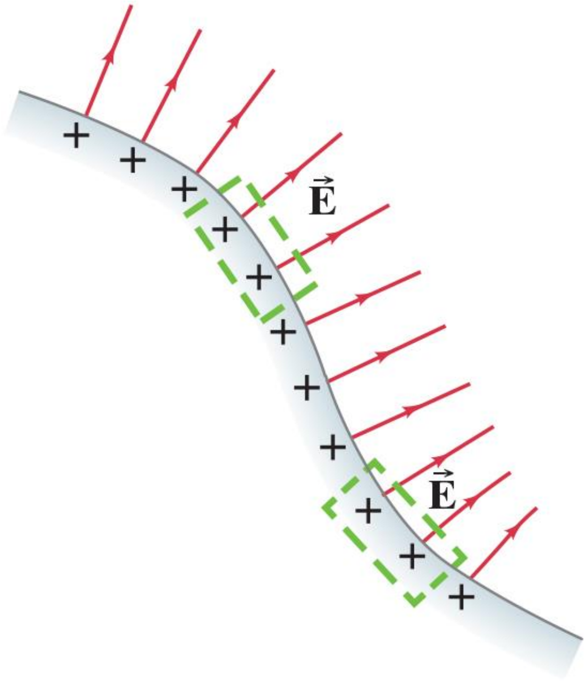

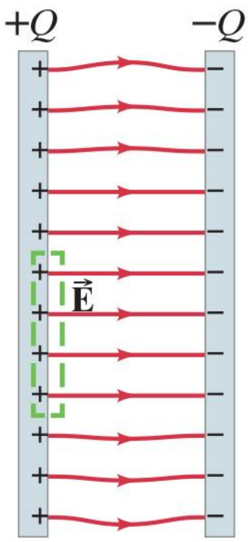

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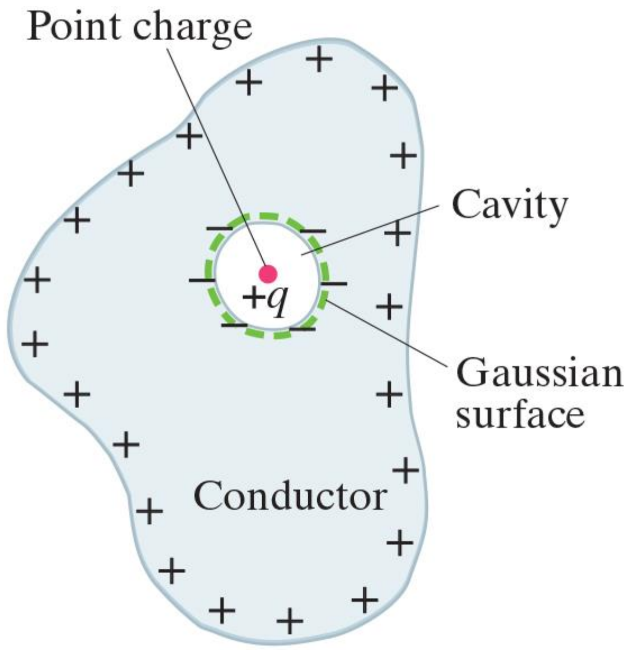
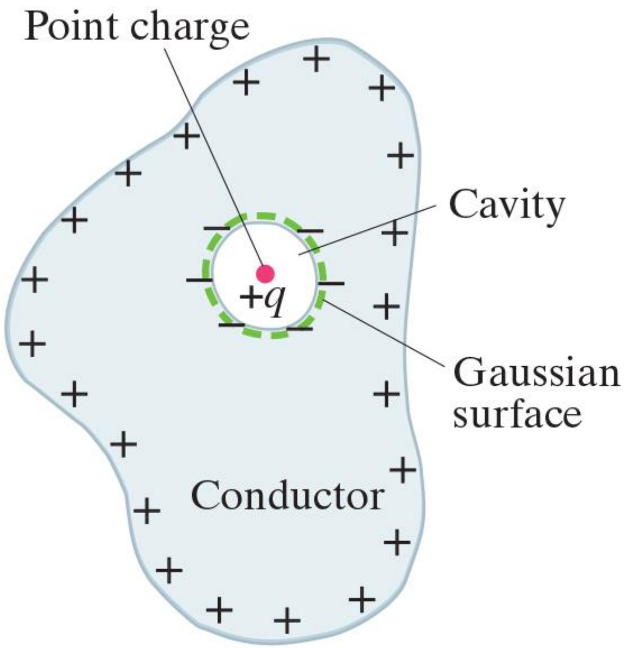
We could use this example with two flat surfaces to re-confirm the value of the electric field inside a capacitor.

22.3 – Applications: conductor with charge inside cavity



Suppose a conductor carries a net charge $+Q$ and contains a cavity. Inside the cavity resides a point charge $+q$. What can be said about the charges on the inner and outer surfaces of the conductor?

22.3 – Applications: conductor with charge inside cavity



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As the charge is concentrated on the boundary of the conductor, there should be non net charge inside the conductor. Hence, on the inner surface there is an overall charge equal to $-q$ that cancels the effect of the point charge.

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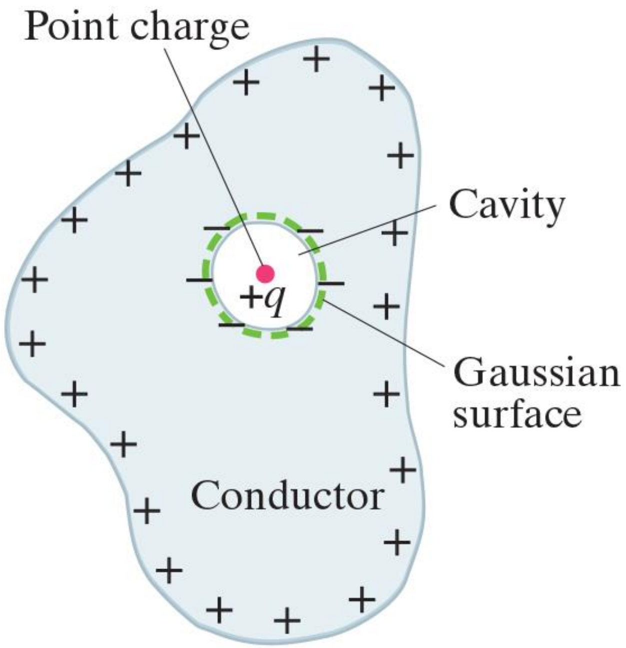

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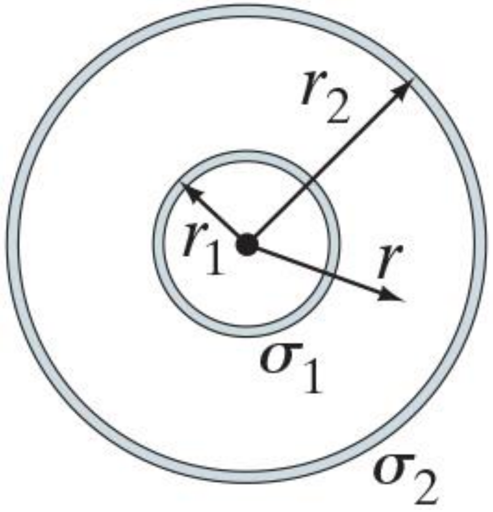
$$\boldsymbol{+(Q+q)-q=+Q}$$

22.3 – Applications: two thin concentric shells



Two concentric shells of radius and uniform surface charge r_1, σ_1 and r_2, σ_2 are shown in the Figure to the left.

What is the electric field for points inside the first shell, points in between shells, and points outside the second shell?

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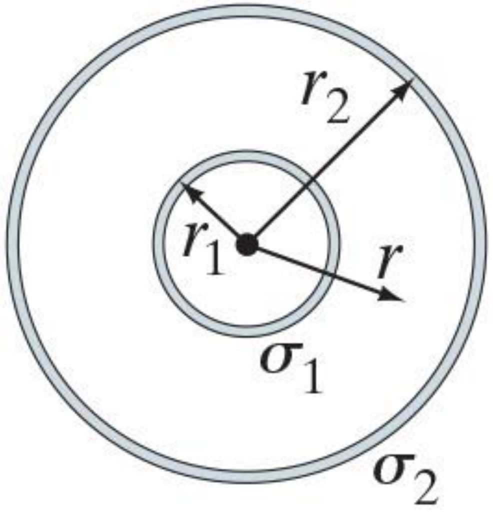

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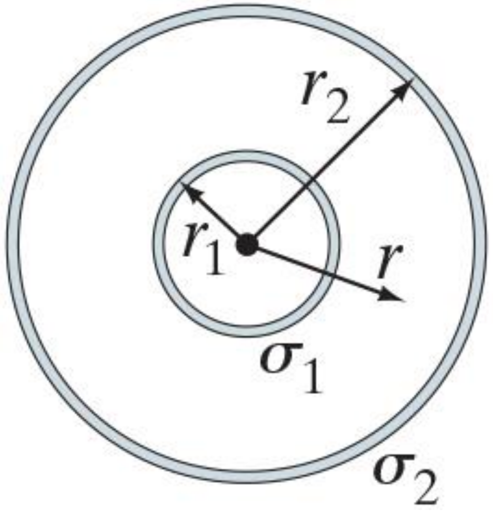

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In the second case, the charge enclosed is $Q_{encl} = 4\pi r_1^2 \sigma_1$ and the field is axialsymmetric, hence

$$\boldsymbol{E 4\pi r^2 = \frac{4\pi r_1^2 \sigma_1}{\epsilon_0} \rightarrow E(r) = \frac{r_1^2 \sigma_1}{\epsilon_0} \frac{1}{r^2}}$$

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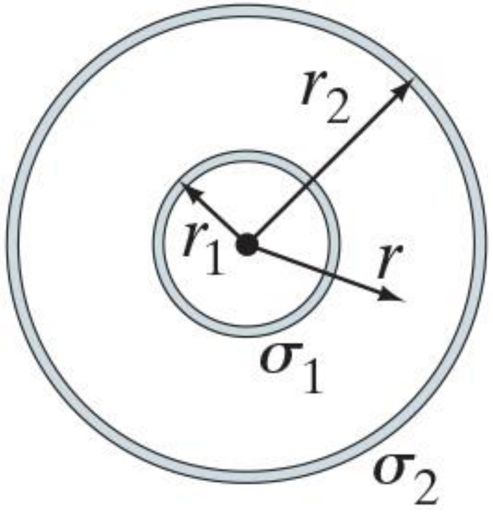

In the third case, the charge enclosed is $Q_{encl} = 4\pi r_1^2\sigma_1 + 4\pi r_2^2\sigma_2$ and the field is still axialsymmetric, hence

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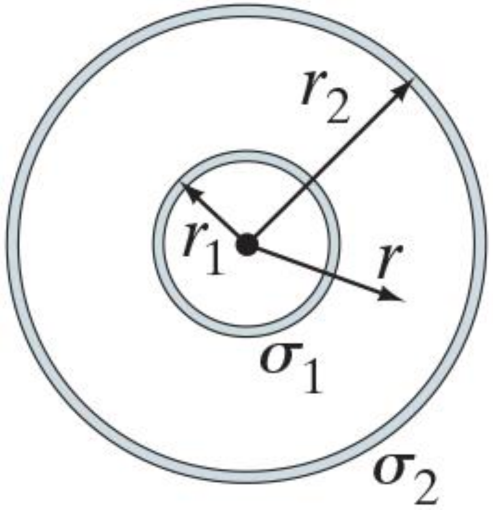

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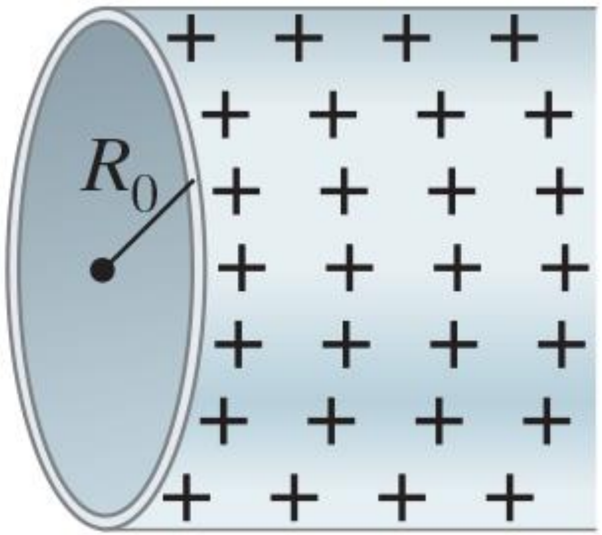
The field can be zero outside the second shell is one of the two is positively charged and the one is negatively charged (it does not matter the order) such that

$$\sigma_1 = -\frac{r_2^2}{r_1^2}\sigma_2$$

22.3 – Applications: long cylindrical shell



R_0 is much smaller than the length of the cylindrical shell. In addition, σ is the uniform charge per unit area.
We want to determine the electric field inside the shell and outside the shell. We can neglect boundary effects (i.e., we are far enough from the boundaries of the shell).

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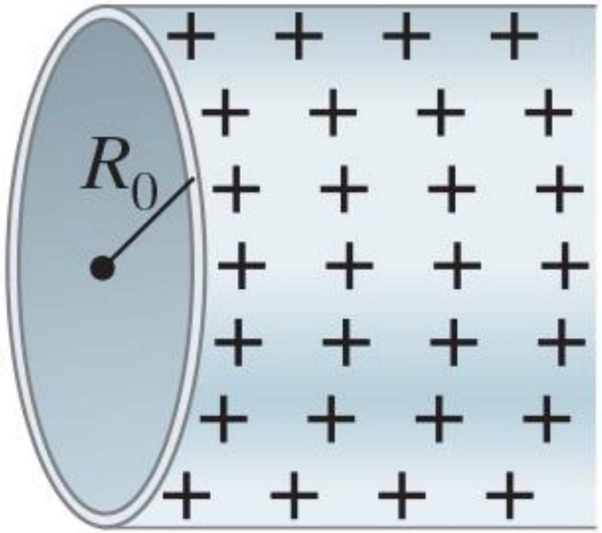

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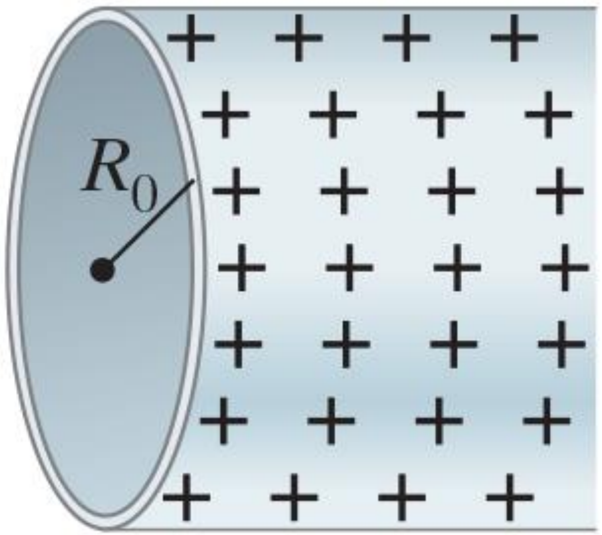

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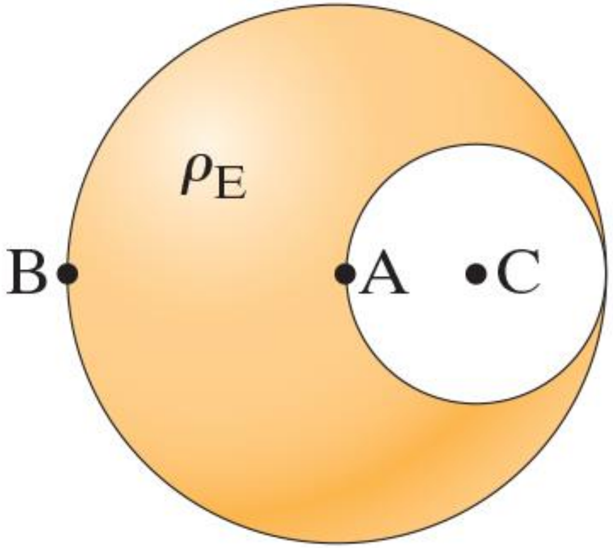
Inside the shell, there is no charge, hence no electric field. Outside the shell we have (the integral is non-zero only along the side area)

$$\boldsymbol{E2\pi rl = \frac{2\pi R_0 l\sigma}{\epsilon_0} \rightarrow E(r) = \frac{R_0\sigma}{\epsilon_0}\frac{1}{r}}$$

22.3 – Applications: irregular sphere



A sphere with uniform charge volume distribution ρ_E and radius r_0 has a smaller sphere of radius $\frac{r_0}{2}$ being removed, as shown in the Figure to the left. What is the value of the electric field in points A, B, and C?

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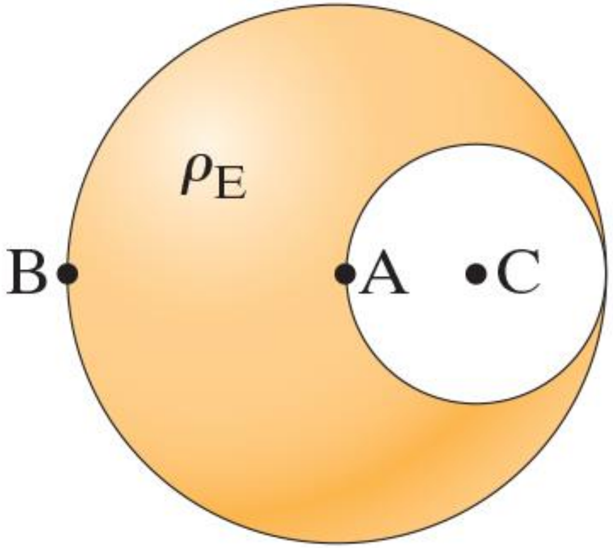

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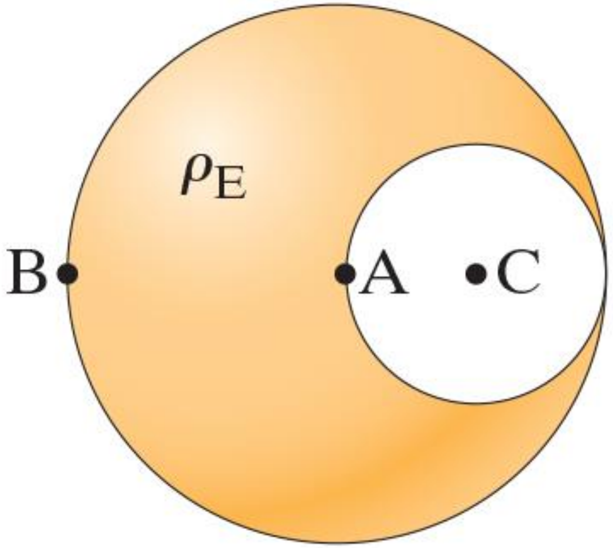

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Let us start with the full sphere with density ρ_E. We can define the electric field at a point distant r from the center as

$$E 4\pi r^2 = \frac{4}{3}\pi r_0^3 \rho_E \frac{r^3}{r_0^3}\frac{1}{\epsilon_0} \rightarrow E(r) = \frac{1}{3\epsilon_0}\rho_E r$$

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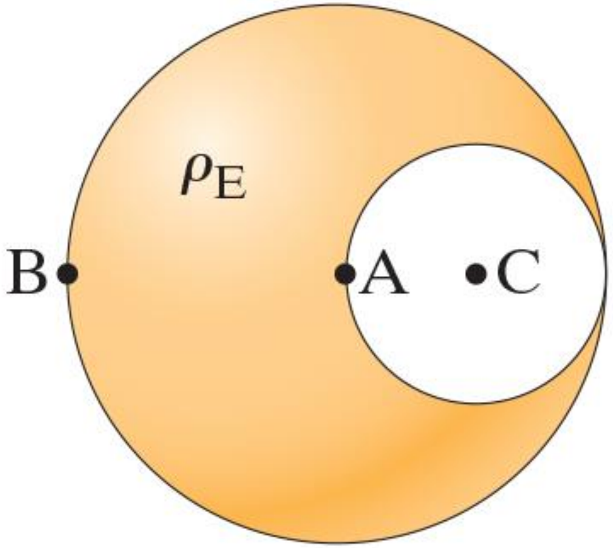

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$E(A) = 0, E(B) = \frac{1}{3\epsilon_0}\rho_E r_0$ (pointing left), $E(C) = \frac{1}{3\epsilon_0}\rho_E \frac{r_0}{2}$ (pointing right)

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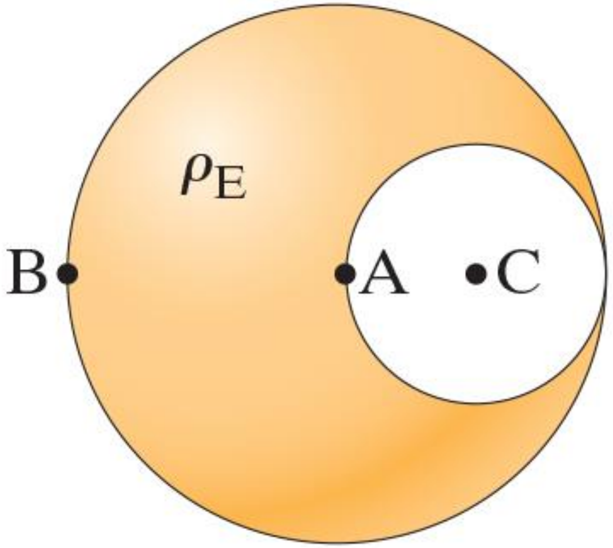

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Let us now consider the smaller sphere with negative charge per unit volume

$$E4\pi r^2 = \frac{4}{24}\pi\, r_0^3\, \rho_E \frac{r^3}{r_0^3}\frac{1}{\epsilon_0} \rightarrow E(r) = \frac{1}{24\epsilon_0}\rho_E r$$

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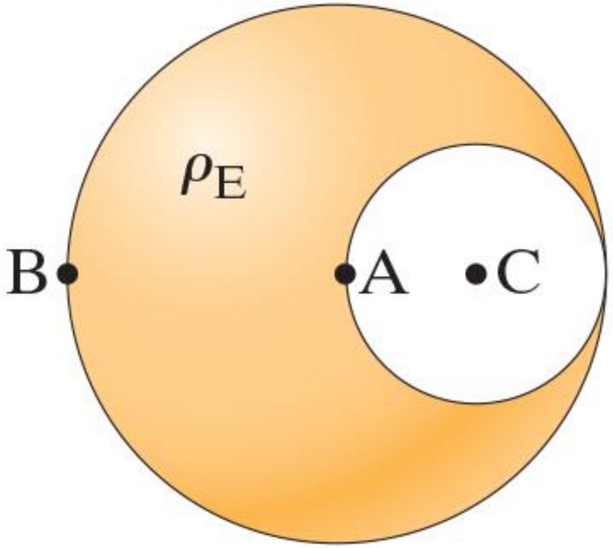

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$E(A) = \frac{1}{48\epsilon_0}\rho_E r_0$ (pointing right), $E(B) = \frac{1}{16\epsilon_0}\rho_E r_0$ (pointing right), $E(C) = 0$

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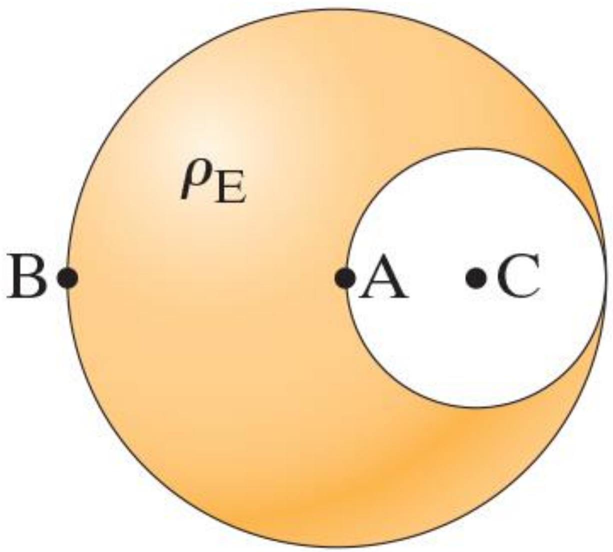

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Vectorially summing the two contributions per point, we obtain the desired results

Wrap-up: revisiting Learning objectives

After this lecture you should be able to:

- Compute the **electric flux** given an electric field and a pre-defined surface

- Use such electric flux to apply **Gauss's law** to determine the magnitude of the electric field in a given point of space

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$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Exploit symmetries. Defining the proper Gaussian surface is key