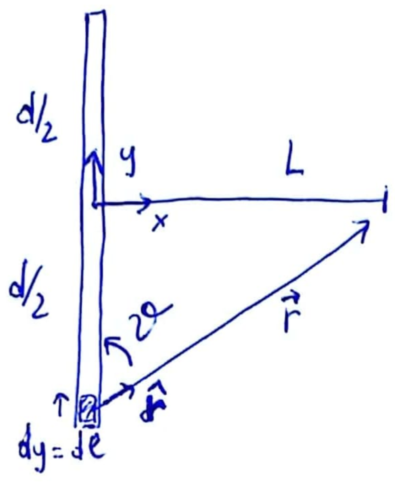
OPEN QUESTION 1

We will have to use Biot-Savart. Let us start with some geom./trigon. definitions.



a) $y = -\dfrac{L}{\tan\vartheta} \rightarrow dy = \dfrac{L}{\sin^2\vartheta} d\vartheta\, \hat{j}$

$|dy| = \dfrac{R}{\sin^2\vartheta} d\vartheta$

$|\hat{r}| = 1$

The angle between $\vec{dy}$ and $\hat{r}$ is ϑ

We can also define $|\vec{r}| = \sqrt{y^2 + L^2}$ and

$\cos\vartheta = -\dfrac{y}{\sqrt{y^2+L^2}} \qquad \sin\vartheta = \dfrac{L}{\sqrt{y^2+L^2}} \quad (*)$

From Biot-Savart we have

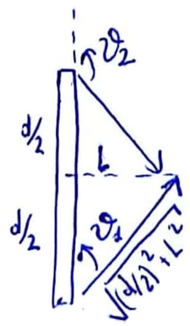
$$|dB| = \frac{\mu_0 I}{4\pi} \frac{|d\vec{\ell} \times \hat{r}|}{r^2} = \frac{\frac{\mu_0 I}{4\pi} \cdot \frac{L}{\sin^2\vartheta} d\vartheta \; 1 \cdot \sin\vartheta}{y^2 + L^2}$$

$$|dB| = \frac{\mu_0 I \, d\vartheta \, L}{4\pi \sin\vartheta (y^2 + L^2)}$$

and we can use the fact from $(*)$ that $\dfrac{1}{(y^2+L^2)} = \dfrac{\sin^2\vartheta}{L^2}$. Hence

$$|dB| = \frac{\mu_0 I}{4\pi L} \sin\vartheta \, d\vartheta$$

which should be integrated between ϑ_1 and ϑ_2



where $\cos\vartheta_1 = \dfrac{d/2}{\sqrt{(d/2)^2 + L^2}}$

and $\cos\vartheta_2 = -\dfrac{d/2}{\sqrt{(d/2)^2+L^2}}$ as $\vartheta_2 = \pi - \vartheta_1$

We get

$$|B| = \frac{\mu_0 I}{4\pi L}\int_{\vartheta_1}^{\vartheta_2} \sin\vartheta\, d\vartheta = \frac{\mu_0 I}{4\pi L}\left[\cos\vartheta_1 - \cos\vartheta_2\right]$$

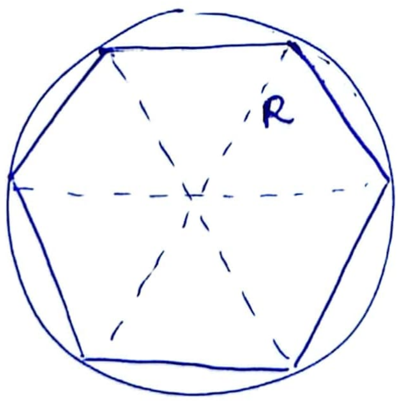
$$|B| = \frac{\mu_0 I}{4\pi L}\frac{d}{\sqrt{(d/2)^2 + L^2}} = \frac{\mu_0 I}{4\pi L}\frac{d}{\sqrt{\frac{d^2+4L^2}{4}}} = \frac{\mu_0 I}{2\pi L}\frac{d}{\sqrt{d^2+4L^2}}$$

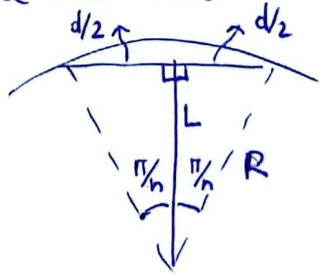
b) If $d \to \infty$, then $\frac{d}{\sqrt{d^2+4L^2}} \simeq \frac{d}{\sqrt{d^2}} = 1$ and we get

$$|B| = \frac{\mu_0 I}{2\pi L}$$

as expected

c) We start by defining proper expressions for d and L as a function of R (radius of circle) and n (# sides of polygon). We take $n = 6$ as example, but results are general. Focusing on one of the 6 triangles on the left, we can write





$$\begin{cases} L = R\cos \pi/n \\ d = 2R\sin \pi/n \end{cases}$$

where we use the fact the central angle of each triangle is $\frac{2\pi}{n}$.

Hence, we can write

$$|B| = \frac{\mu_0 I}{2\pi R\cos \pi/n}\frac{2R\sin \pi/n}{\sqrt{4R^2\sin^2 \pi/n + 4R^2\cos^2 \pi/n}} = \frac{\mu_0 I}{\pi}\cdot\tan\frac{\pi}{n}\cdot\frac{1}{\sqrt{4R^2(\sin^2 \pi/n + \cos^2 \pi/n)}}$$

$$|B| = \frac{\mu_0 I}{2\pi R}\tan \pi/n$$

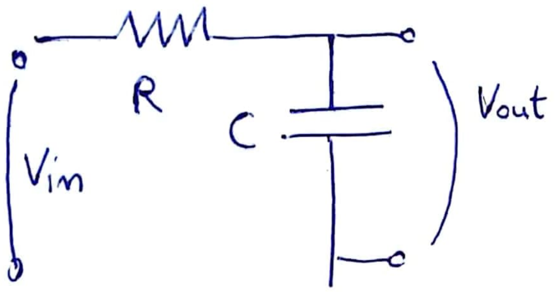
That is the contribution of a single side. Because of superimposition of effects

$$|B|_{TOT} = n \cdot \frac{\mu_0 I}{2\pi R} \tan\frac{\pi}{n}$$

d) If $n \to \infty$, then $\tan\frac{\pi}{n} \simeq \frac{\pi}{n}$

$$|B|_{TOT} = n \cdot \frac{\mu_0 I}{2\pi R} \cdot \frac{\pi}{n} = \frac{\mu_0 I}{2R}$$

OPEN QUESTION 2



a) For a low-pass filter, we have

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}}$$

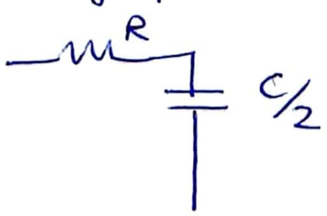
For the cutoff frequency $f_{co} = \frac{\omega_{co}}{2\pi}$, we have that

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{1}{\sqrt{2}} \to \omega^2 R^2 C^2 = 1$$

$$C = \frac{1}{2\pi f_{co} R} = 7.96 \cdot 10^{-7}\ F$$

b) Because $f_{co} = \frac{1}{2\pi RC}$, we need a lower equivalent capacitance so that the denominator decreases and hence f_{co} increases. For 2 capacitors in series, it applies that $\frac{1}{C_{EQ}} = \frac{1}{C_1} + \frac{1}{C_2}$ and, if they are the same $\frac{1}{C_{EQ}} = \frac{1}{C} + \frac{1}{C} \to C_{EQ} = \frac{C}{2}$

c) Following up from b), we have this new low-pass filter



where $\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{1}{\sqrt{1+\frac{\omega^2 R^2 C^2}{4}}}$

The new cutoff frequency is

$$\frac{\omega_{co}^2 R^2 C^2}{4} = 1 \rightarrow \omega_{co} RC = 2 \rightarrow \omega_{co} = \frac{2}{RC}$$

$$\rightarrow f_{co} = 2 \cdot \frac{1}{2\pi RC}$$ twice as large as the original

d) If the original resistor and capacitor are swapped, then

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{\omega RC}{\sqrt{1+\omega^2 R^2 C^2}}$$

For the cutoff frequency it holds that

$$\frac{\omega_{co} RC}{\sqrt{1+\omega_{co}^2 R^2 C^2}} = \frac{\sqrt{2}}{2} \rightarrow \frac{\omega_{co}^2 R^2 C^2}{1+\omega_{co}^2 R^2 C^2} = \frac{1}{2} \rightarrow 2\omega_{co}^2 R^2 C^2 = 1 + \omega_{co}^2 R^2 C^2$$

$$\rightarrow \omega_{co}^2 R^2 C^2 = 1 \rightarrow \omega_{co} = \frac{1}{RC} \rightarrow f_{co} = \frac{1}{2\pi RC}$$ (same as in the low-pass filter case)

e) In the low-pass filter, for $\omega >> \omega_{co}$ we have

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}} \simeq \frac{1}{\sqrt{\omega^2 R^2 C^2}} = \frac{1}{\omega RC}$$

Let us now consider ω_1 and $\omega_2 = 10\,\omega_1$

$$20 \log_{10}\left(\left|\frac{V_{OUT}}{V_{IN}}\right|_{\omega_1}\right) = 20 \log_{10} \frac{1}{\omega_1 RC} \quad [dB]$$

$$20 \log_{10}\left(\left|\frac{V_{OUT}}{V_{IN}}\right|_{\omega_2}\right) = 20 \log_{10} \frac{1}{10\omega_1 RC} = 20 \log_{10} \frac{1}{\omega_1 RC} - 20 \log_{10} 10$$

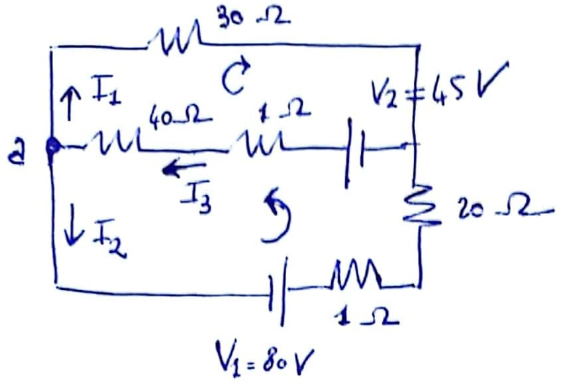
$$= 20 \log_{10} \frac{1}{\omega_1 RC} - 20 \quad [dB]$$

Hence, for $\omega = \omega_2$ we get an attenuation 20 dB stronger than for $\omega = \omega_1$

OPEN QUESTIONS 3 AND 4

The two derivations are directly taken from the book.

MULTIPLE CHOICE 1



We use conservation of current in node a:

$$I_3 = I_1 + I_2$$

Then, we use conservation of voltage in the lower loop using a counterclockwise loop and in the upper loop using a clockwise loop.

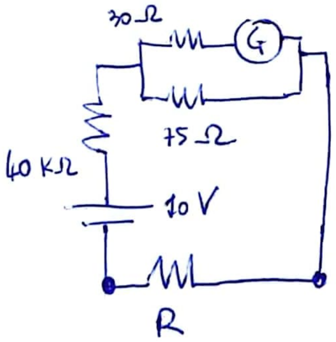
$$\begin{cases} 80 - 1 \cdot I_2 - 20 I_2 + 45 - 1 I_3 - 40 I_3 = 0 \\ 45 - 1 I_3 - 40 I_3 - 30 I_1 = 0 \end{cases}$$

$$\rightarrow \begin{cases} I_1 + I_2 - I_3 = 0 \\ 21 I_2 + 41 I_3 = 125 \\ 30 I_1 + 41 I_3 = 45 \end{cases}$$

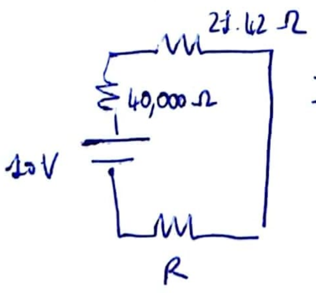
Solving the system we obtain $I_3 = 1.7$ A

ANSWER A

MULTIPLE CHOICE 2



We need to compute R such that the current in G is exactly $I = 50\ \mu A$. We start by determining the equivalent circuit where we replace the 2 resistances in parallel with $R_{EQ} = 21.42$

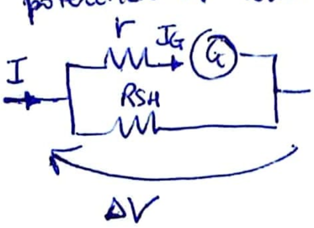


$$I = \frac{10}{40{,}021.42 + R} \rightarrow$$

We now determine the drop in potential across R_{EQ}:

$$\Delta V = I \cdot R_{EQ} = \frac{214.2}{40{,}021.42 + R}$$

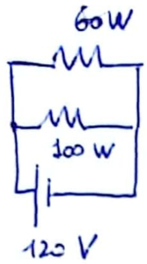
Now we consider again the 2 original resistances, as they "see" the same potential ΔV and compute the current in G

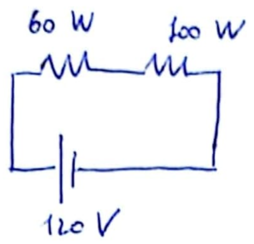


$$I_G = \frac{\Delta V}{r} = \frac{7.143}{40{,}021.42 + R} = 50 \cdot 10^{-6} \rightarrow R = 1.02 \cdot 10^{5}\ \Omega$$

$$\rightarrow R \simeq 1.0 \cdot 10^{2}\ K\Omega \qquad \text{ANSWER A}$$

MULTIPLE CHOICE 3





The resistance of a lightbulb can be computed as

$$R = \frac{V_{NOM}^2}{P}$$

where V_{NOM} is the "nominal" voltage the lightbulb is designed for (not necessarily the actual voltage it receives) and P is the specified power consumption. Hence

$$R_{60W} = \frac{120^2}{60} = 240\ \Omega \qquad \rightarrow \qquad R_{60W} > R_{100W}$$

$$R_{100W} = \frac{120^2}{100} = 144\ \Omega$$

When in parallel, the two lightbulbs receive the same voltage. Because brightness is proportional to the dissipated power, we write

$$P_{DISSIP} = \frac{V^2}{R} \rightarrow$$ the lightbulb glowing more is the one with the smallest resistance, i.e., the one with 100 W

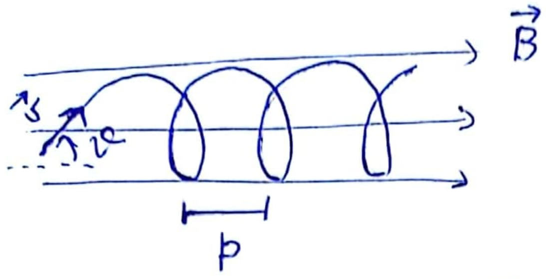
When in series, the two lightbulbs receive the same current, hence for power dissipation we use

$$P_{DISSIP} = I^2 R \rightarrow$$ the lightbulb glowing more is the one with the largest resistance, i.e., the one with 60 W

To summarize, the 60 W bulb glows more when in series and the 100 W when in parallel

ANSWER C

MULTIPLE CHOICE 4



We split velocity into

$$\begin{cases} v_{\parallel} = v\cos\vartheta \\ v_{\perp} = v\sin\vartheta \end{cases}$$

We use $v_\perp$ to determine the radius of the helix

$$\frac{m v_\perp^2}{r} = q v_\perp B \rightarrow r = \frac{m v_\perp}{qB}$$

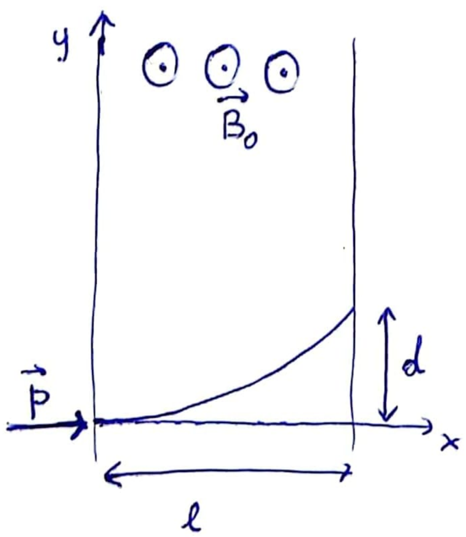
and then compute the period of one rotation as

$$T = \frac{2\pi r}{v_\perp}$$

Finally, $p = T \cdot v_{\parallel}$

In our case $\vartheta = \pi/4$ and using the mass and charge (in abs. value) of an electron we get $p = 2.7 \cdot 10^{-4}$ m ANSWER B

MULTIPLE CHOICE 5

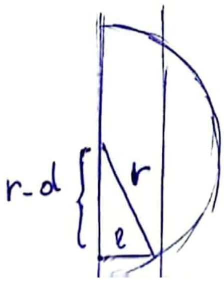


The momentum $\vec{p}$ is $m\vec{v}$, hence its magnitude is mv. On a side note, the particle is negatively charged, because otherwise the deflection would be downward. In our calculations we use the absolute value of the charge, as the positive/negative value only influences the direction of the deflection, not the actual radius.

We write

$$\frac{mv^2}{r} = qvB_0 \rightarrow mv = qB_0 r$$

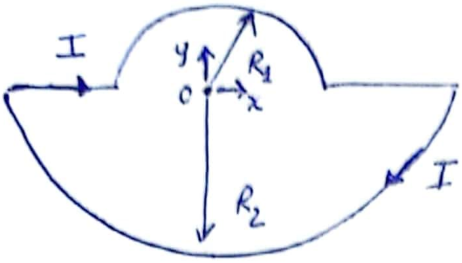
and we now need to express r as a function of l and d.



$$r^2 = (r-d)^2 + l^2 \rightarrow r^2 = r^2 - 2rd + d^2 + l^2 \rightarrow r = \frac{d^2 + l^2}{2d}$$

$$mv = qB_0 \frac{d^2 + l^2}{2d}$$ ANSWER D

MULTIPLE CHOICE 6



We need to use Biot-Savart here. We will apply it just to the 2 semi-circles, as

$$|dB| = \frac{\mu_0 I}{4\pi} \frac{|\vec{dl} \times \hat{r}|}{r^2}$$

and $\vec{dl} \times \hat{r} = 0$ in the two horizontal segments.

If we consider the R_1 semi-circle, the resulting $\vec{B}$ in point O "enters" the page with orientation $-\hat{k}$. The same applies to the R_2 semi-circle as the current goes now towards the left. Because we are interested in the magnitude of $\vec{B}$, we will sum the absolute values of the 2 contributions.

$$|B_{R_1}| = \frac{\mu_0 I}{4\pi} \int_0^{\pi} \frac{R_1 \, d\vartheta}{R_1^2} = \frac{\mu_0 I}{4\pi R_1} \int_0^{\pi} d\vartheta = \frac{\mu_0 I}{4 R_1}$$

$$|B_{R_2}| = \frac{\mu_0 I}{4\pi R_2} \int_0^{\pi} d\vartheta = \frac{\mu_0 I}{4 R_2}$$

$$|B| = |B_{R_1}| + |B_{R_2}| = \frac{\mu_0 I}{4} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$ ANSWER A

MULTIPLE CHOICE 7

Ampere's law states that

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{ENCL}$$

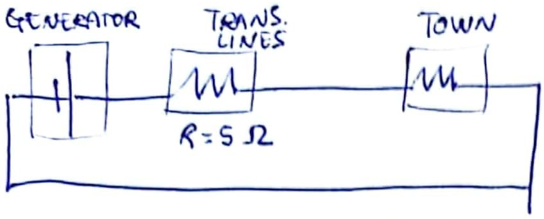
while we are given

$$\oint \vec{B} \cdot \vec{dl} = 0$$

Hence, given the closed path defined by the left-hand side, there is balance between the current "entering" and "exiting" the surface enclosed by that path so that $I_{ENCL} = 0$

ANSWER C

MULTIPLE CHOICE 8



Out of the 51 KV produced by the generator, only 45 KV get to the town.
We can compute the drop in potential across the transmission lines as

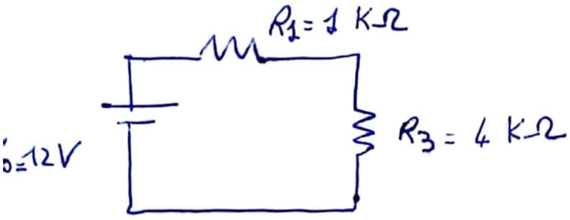
$$\Delta V = 51\ KV - 45\ KV = 6000\ V = IR \rightarrow 6000\ V = I \cdot 5\ \Omega \rightarrow I = 1200\ A$$

Now, we obtain the generated power as

$$P = VI = 51\ KV \cdot 1200\ A = 61.2\ MW$$ ANSWER D

MULTIPLE CHOICE 9

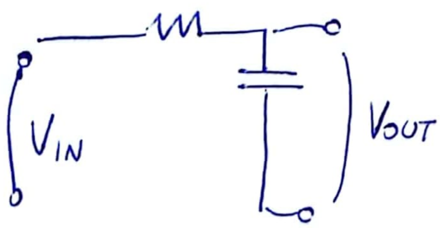
After a sufficiently long time, the capacitor does not allow current to go through it and the solenoid generates no drop in potential (recall that $\mathcal{E} = -L\, dI/dt$ and after a "long time" I is constant).
Hence the "new" circuit is



$$I = \frac{V_0}{R_1 + R_3} = \frac{12\ V}{5000\ \Omega} = 2.4\ mA$$

ANSWER A

MULTIPLE CHOICE 10



This is a low-pass filter where

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{1}{\sqrt{1+\omega^2R^2C^2}}$$

Hence we write $\frac{1}{\sqrt{1+\omega^2R^2C^2}} = 0.9 \rightarrow 1 + \omega^2R^2C^2 = 1.234 \rightarrow \omega = \frac{0.484}{RC}$

$$\omega = \frac{0.484}{850 \cdot 1 \cdot 10^{-6}} = 569\ rad/s \rightarrow f = \frac{\omega}{2\pi} = 91\ Hz$$ ANSWER A

MULTIPLE CHOICE 11

We should recall that $V_{PEAK} = NABw$, while $V_{RMS} = \frac{\sqrt{2}}{2} NABw$ Hence

$$V_{RMS} = \frac{\sqrt{2}}{2} NAB\, 2\pi f \rightarrow A = \frac{V_{RMS}}{\frac{\sqrt{2}}{2} NB 2\pi f} = 0.011 \ m^2 \quad \text{ANSWER B}$$

MULTIPLE CHOICE 12

We can re-write the only non-zero component of the electric field as

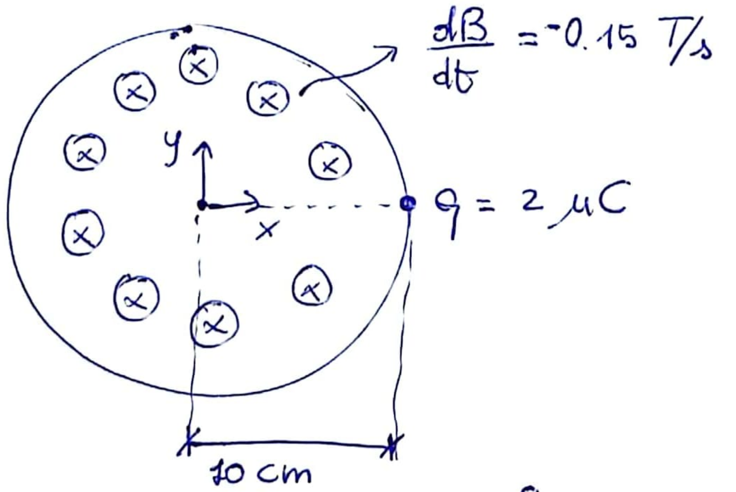
$$E_y = E_0 e^{-[(ax-bt)^2]} = E_0 e^{-[a(x-b/a\, t)]^2}$$

where $\frac{b}{a}$ is a velocity (speed of light) with $b = [1/s]$ and $a = [1/m]$

and the expression $a(x - b/a\, t)$ is adimensional (as it should be)

ANSWER D

MULTIPLE CHOICE 13



We can use Faraday's law $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ realizing that $\vec{E}$ is axialsymmetric along a circle. We use a closed path (circle) of radius 10 cm in our calculation.

$$E \cdot 2\pi r = -\pi r^2 \frac{dB}{dt} \rightarrow E = -\frac{r}{2}\frac{dB}{dt} = 7.5 \cdot 10^{-3} \ N/C$$

Then

$$F = QE = 15 \ nN \quad \text{ANSWER A}$$

MULTIPLE CHOICE 14-20:

14) D
15) C
16) B
17) A
18) C
19) D
20) C