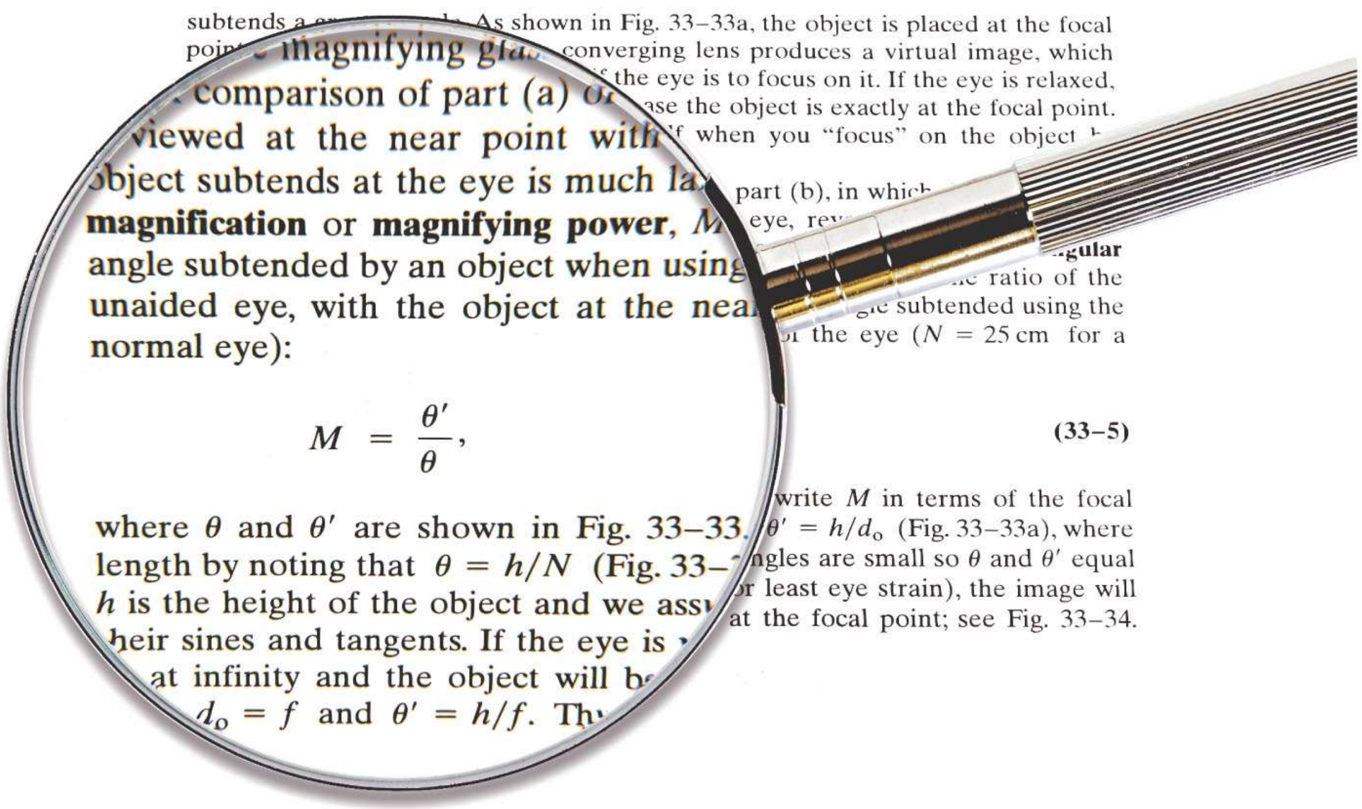
Chapter 33: Lenses and Optical Instruments



Chapter 33: Lenses and Optical Instruments

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  - Spherical aberration
  - Chromatic aberration

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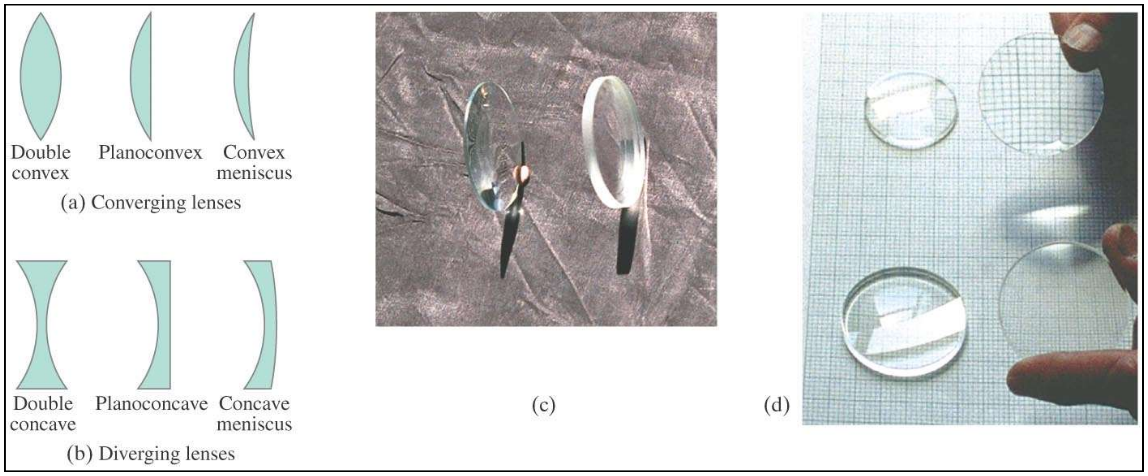
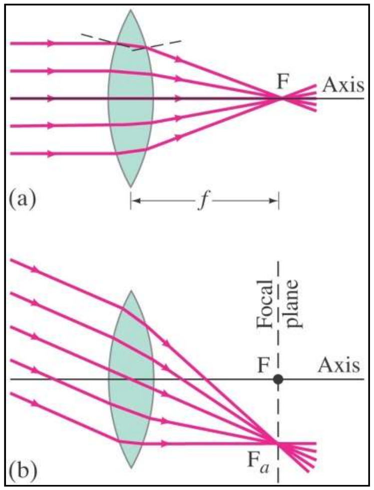
33-1 Thin Lenses; Ray Tracing

- Thin lenses are those whose diameter is small compared to their radius of curvature.
  - They may be either converging (a) or diverging (b).

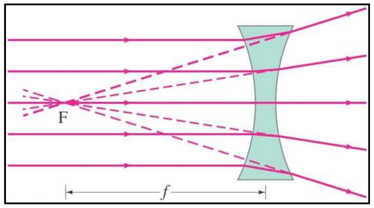


33-1 Thin Lenses; Ray Tracing

- Rays are brought to a focus by a converging lens
  - one that is thicker in the center than it is at the edge.

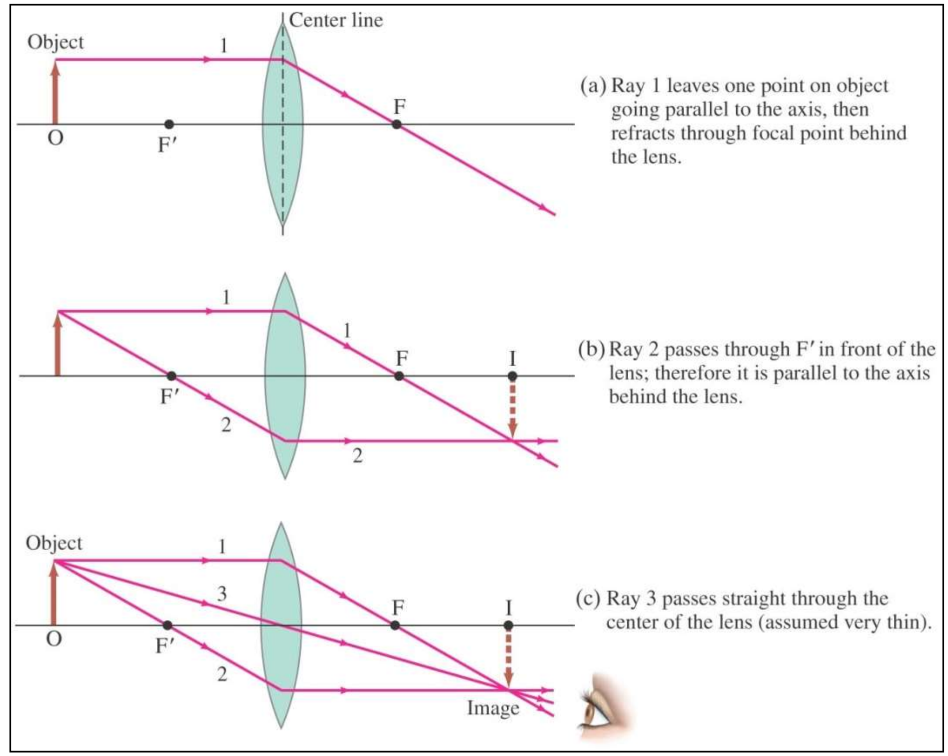


- A diverging lens (thicker at the edge than in the center) makes parallel light diverge;
  - the focal point is that point where the diverging rays would converge if projected back.



33-1 Thin Lenses; Ray Tracing

- Ray tracing for thin lenses is similar to that for mirrors. We have **three key rays**:
  - This ray comes in **parallel to the axis** and **exits** through the **focal point**. **Ray 1**
  - This ray comes in through the **focal point** and **exits parallel** to the axis. **Ray 2**
  - This ray goes through the **center** of the lens and is **undeflected**. **Ray 3**



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33-1 Thin Lenses; Ray Tracing

- The power of a lens is the inverse of its focal length:

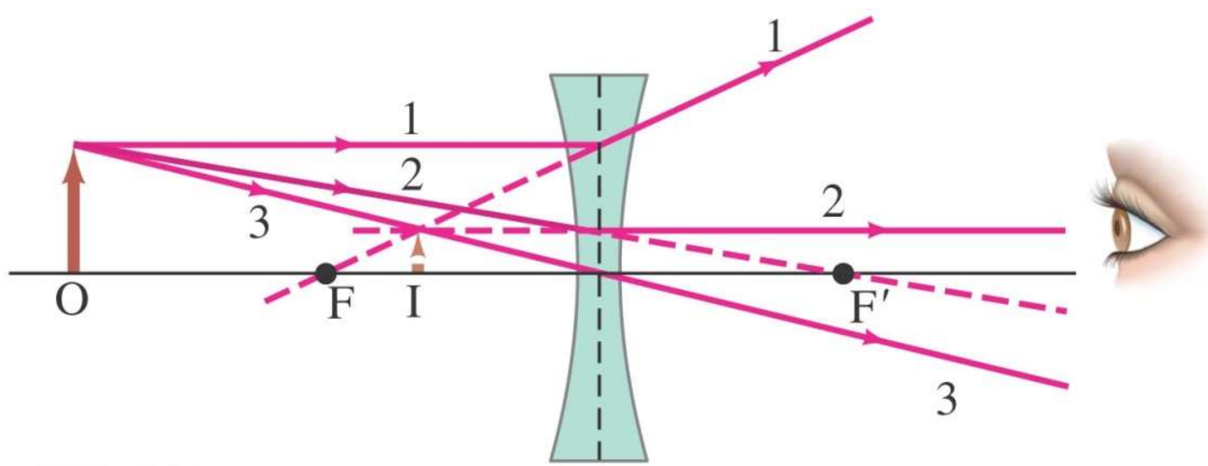
$$\boxed{P = \frac{1}{f}}$$

  - Lens power is measured in diopters, D:

$$\boxed{1D = m^{-1}}$$

33-1 Thin Lenses; Ray Tracing

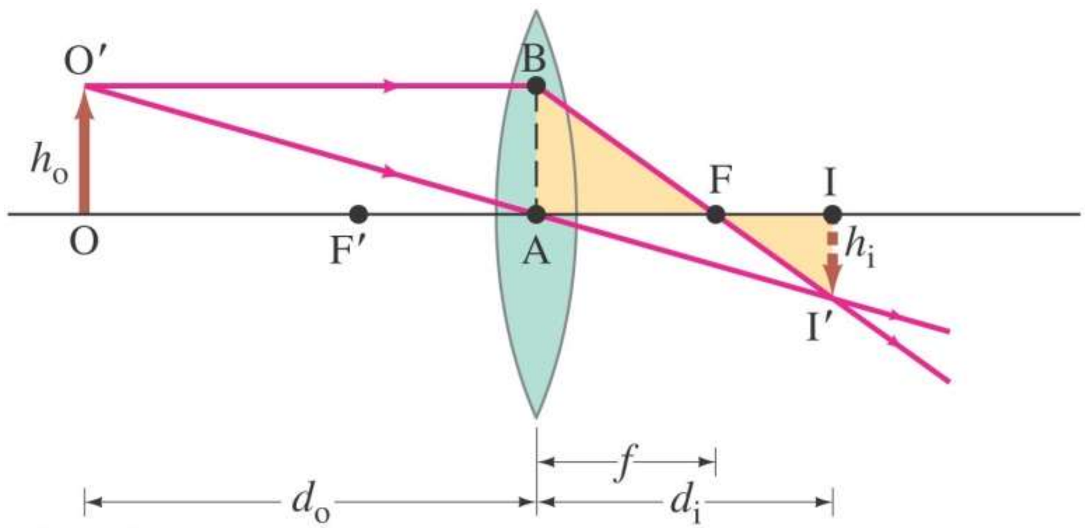
- For a diverging lens,
  - we can use the same three rays;
  - the image is upright and virtual.
- This virtual image can be seen!



33-2 The Thin Lens Equation; Magnification

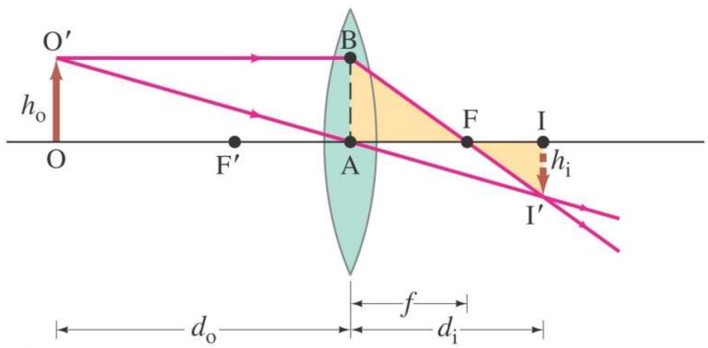
- The thin lens equation is similar to the mirror equation:

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$$



33-2 The Thin Lens Equation; Magnification

- The <u>sign conventions</u> are slightly different from those for mirrors:
  - The **focal length is positive for converging lenses** and negative for diverging.
  - The **object distance** is **positive** when the object is on the **same side** as the light entering the lens; otherwise it is negative.
  - The **image distance is positive** if the image is on the **opposite side of the light entering** the lens; otherwise, it is negative.
  - The **height of the image** is **positive** if the image **is upright** and negative otherwise.

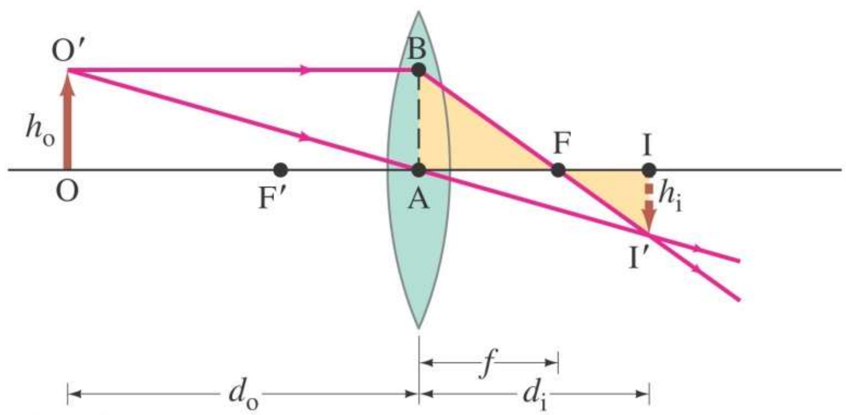


- IMPORTANT remark:
  - If the object is at the infinite, then $\frac{1}{d_0} = 0 \Rightarrow d_i = f$

33-2 The Thin Lens Equation; Magnification

- The magnification formula is also the same as that for a mirror:

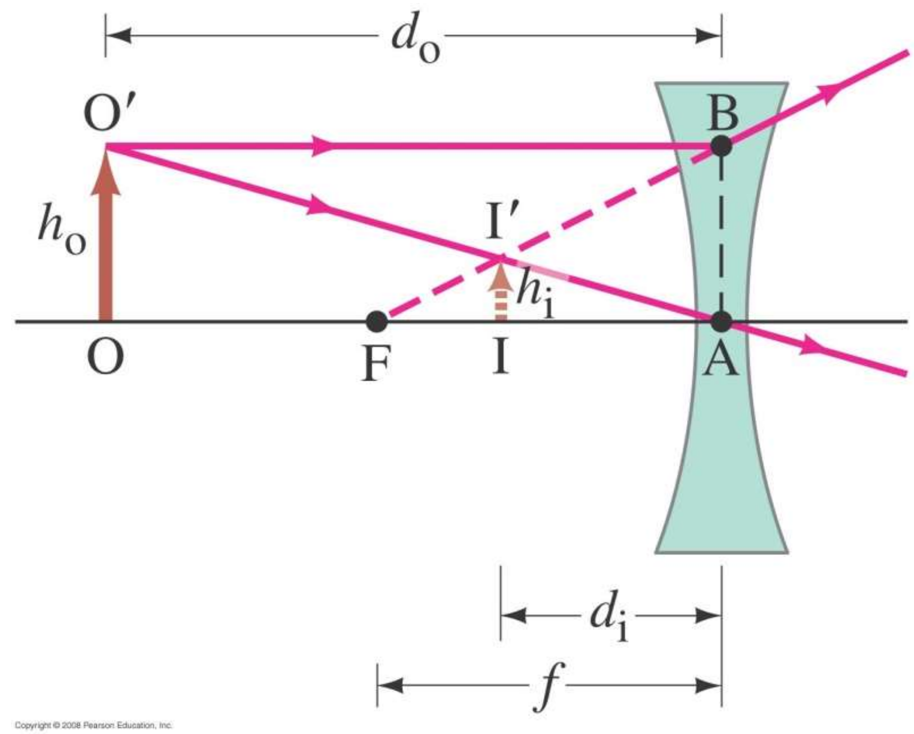
$$\boxed{m = \frac{h_i}{h_0} = -\frac{d_i}{d_0}}$$



33-2 The Thin Lens Equation; Magnification

- For a diverging lens:

$$\frac{1}{d_0} - \frac{1}{d_i} = -\frac{1}{f}$$



- Make d_i and f negative and we obtain again:

$$\frac{1}{d_0} + \frac{1}{d_i} = +\frac{1}{f}$$

33-2 The Thin Lens Equation; Magnification

- The power of a lens is
  - positive if it is converging and
  - negative if it is diverging.

- Problem Solving: Thin Lenses

  1. Draw a ray diagram.

     The image is located where the key rays intersect.

  2. Solve for unknowns using the thin lens equation.

  3. Follow the sign conventions.

  4. Check that your answers are consistent with the ray diagram.

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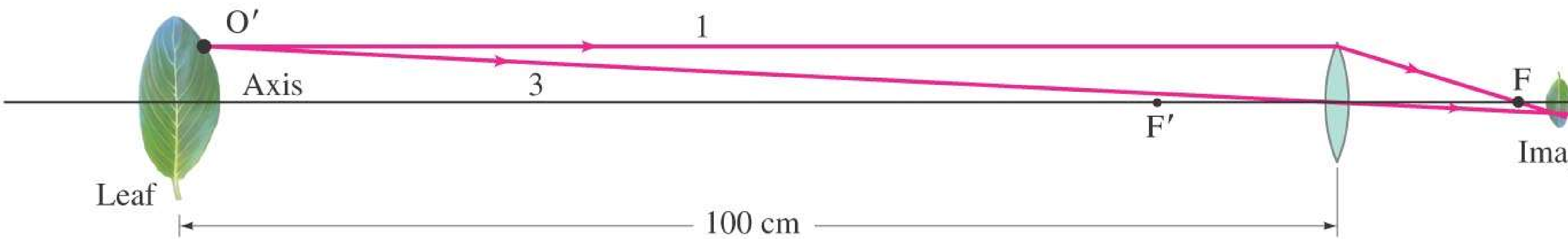
33-2 The Thin Lens Equation; Magnification

Example 33-2: Image formed by converging lens.

- What are
  - the position of the image,
  - and the size of the image of a 7.6-cm-high leaf placed 1.00 m from a +50.0-mm-focal-length camera lens?



33-2 The Thin Lens Equation; Magnification

Example 33-2: Image formed by converging lens.

- Position of the image

$$\frac{1}{di} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{0{,}05} - \frac{1}{1} \Rightarrow d_i = \frac{1}{19} = 0{,}0526\, m$$

  - The image is then 2,6 mm farther from the lens than the image for an object placed at infinity (focussing of the camera lens)

- Magnification

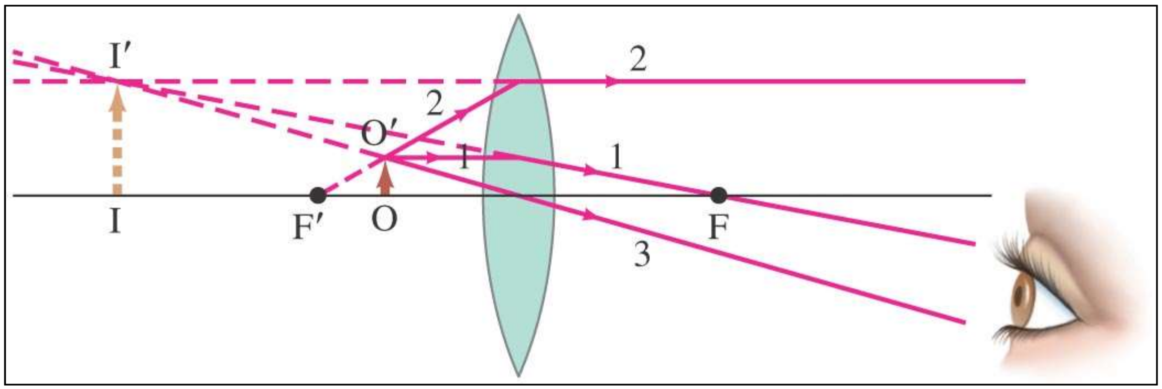
$$m = -\frac{d_i}{d_o} = -\frac{5{,}26[cm]}{100[cm]} = -\,0{,}0526$$

$$h_i = m\, h_o = -\,0{,}0526\; * 7{,}6 = -0{,}4\; [cm]$$

33-2 The Thin Lens Equation; Magnification

Example 33-3: Object close to converging lens.

- An object is placed 10 cm from a 15-cm-focal-length converging lens. Determine the image position and size (a) analytically, and (b) using a ray diagram.



$$\frac{1}{\mathrm{d_i}} = \frac{1}{0{,}15} - \frac{1}{0{,}10} = -\frac{1}{0{,}3}$$

$$m = -\frac{d_i}{d_o} = -\frac{0{,}3}{0{,}1} = 3$$

- Virtual image, three times as large as the object and it is upright.
- The lens is used here as a **magnifying glass**.

33-2 The Thin Lens Equation; Magnification.

Example 33-4: Diverging lens.

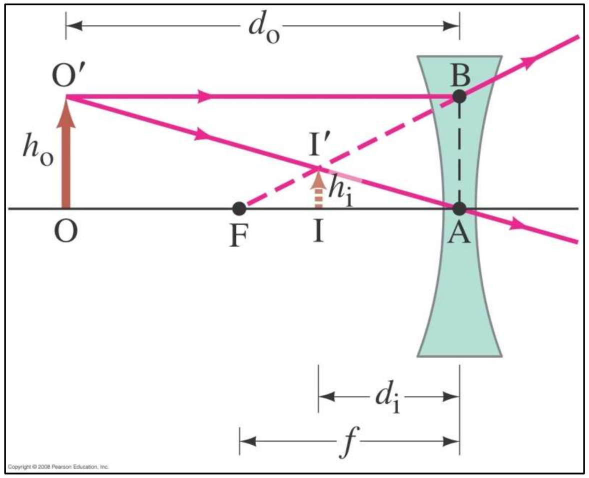
- Where must a small insect be placed if a 25-cm-focal-length diverging lens is to form a virtual image 20 cm from the lens, on the same side as the object? (an exercise in sign convention)

Diverging Lens

Virtual image

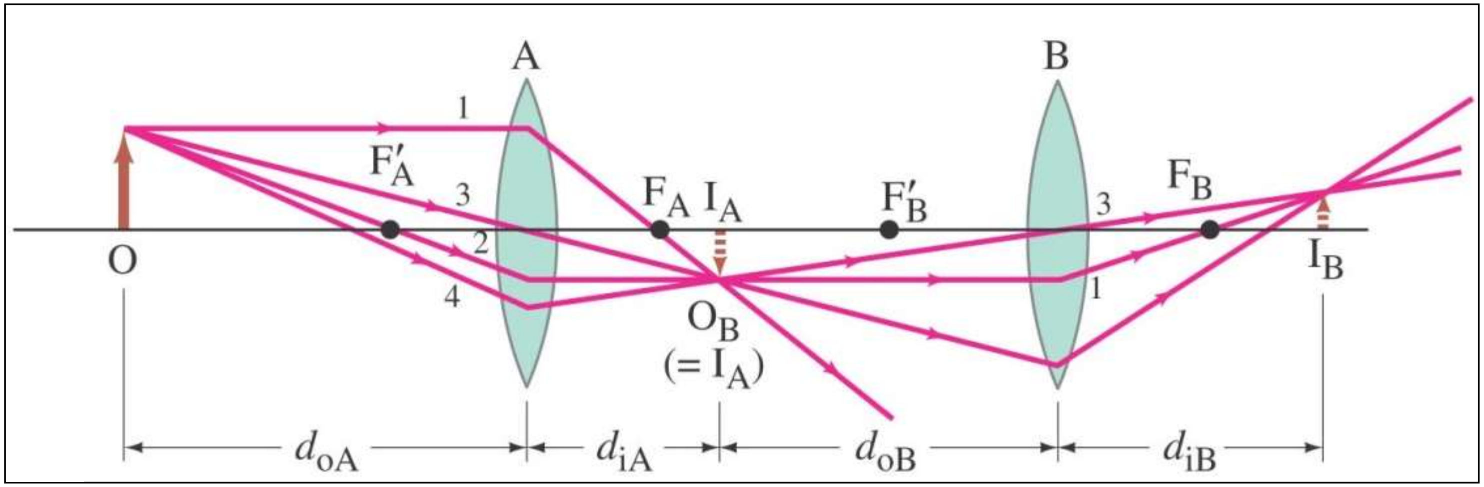
$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \left(-\frac{1}{0{,}25}\right) - \left(-\frac{1}{0{,}20}\right) = \frac{1}{1{,}00}$$

$$d_o = 1m$$



33-3 Combinations of Lenses

- In lens combinations,
  - the image formed by the first lens becomes the object for the second lens (this is where object distances may be negative sometimes).
  - The total magnification is the product of the magnification of each lens.



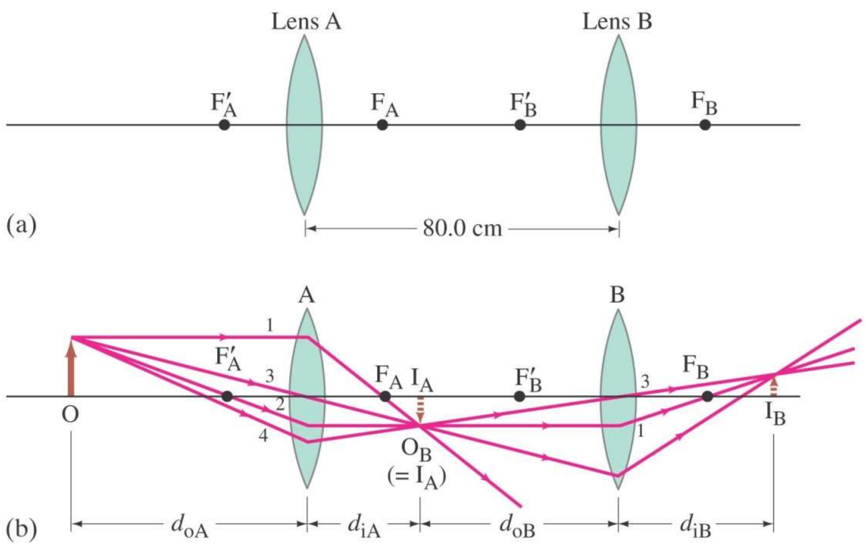
33-3 Combinations of Lenses

Example 33-5: A two-lens system.

Two converging lenses, A and B, with focal lengths $f_A = 20\ cm$ and $f_B = 25\ cm$ are placed $80cm$ apart. The object is placed 60 cm in front of the first lens. Determine:

(a) The position of the image through the entire system

(b) The related magnification



(a)

Lens A : $\frac{1}{d_{iA}} = \frac{1}{f_A} - \frac{1}{d_{oA}} \Rightarrow\ d_{iA} = 0,3\ m$

Lens B: $\frac{1}{d_{iB}} = \frac{1}{f_B} - \frac{1}{d_{oB}} \Rightarrow d_{iB} = 0,5\ m$

$$\text{with } \mathrm{d_{oB}} = 0,8 - 0,3 = 0,5\ \mathrm{m}$$

(b)

$$m_A = -\frac{d_{iA}}{d_{0A}} = -\frac{0.3}{0.6} = -0.5$$

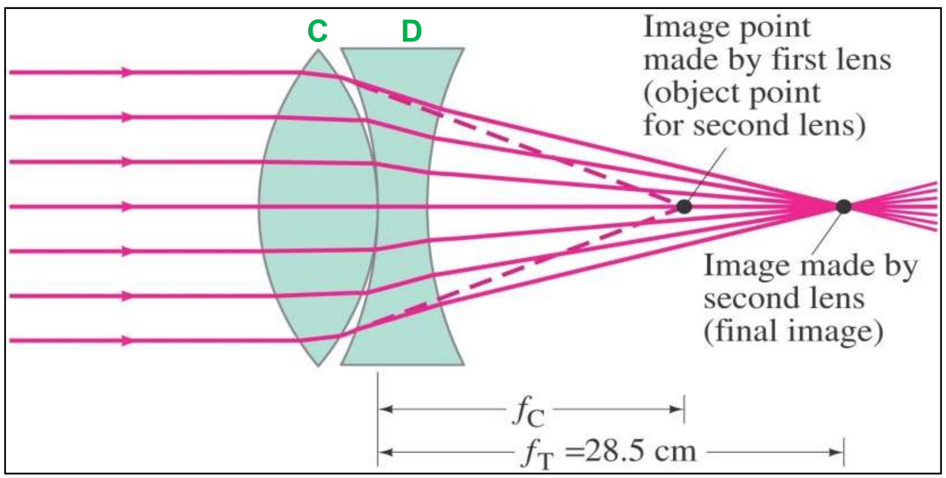
$$m_B = -\frac{d_{iB}}{d_{0B}} = -\frac{0.5}{0.5} = -1$$

$$m_{tot} = m_A m_B$$

33-3 Combinations of Lenses

Example 33-6: Measuring *f* for a diverging lens.

- To measure the **focal length** of a **diverging lens**, a **converging lens** is placed in contact with it. The Sun's rays are focused by this combination at a point 28.5 cm behind the lenses as shown. If the converging lens has a focal length f_C of 16.0 cm, what is the focal length f_D of the diverging lens? Assume both lenses are thin and the space between them is negligible.



Combination of lenses: $\frac{1}{d_o^T} + \frac{1}{d_i^T} = \frac{1}{f^T}$

Lens C: $\frac{1}{d_o^C} + \frac{1}{d_i^C} = \frac{1}{f^C}$

$$\Rightarrow \frac{1}{\infty} + \frac{1}{d_i^C} = \frac{1}{f^C} \Rightarrow d_i^C = f^C = 16\ cm$$

Lens D: $\frac{1}{d_o^D} + \frac{1}{d_i^D} = \frac{1}{f^D}$

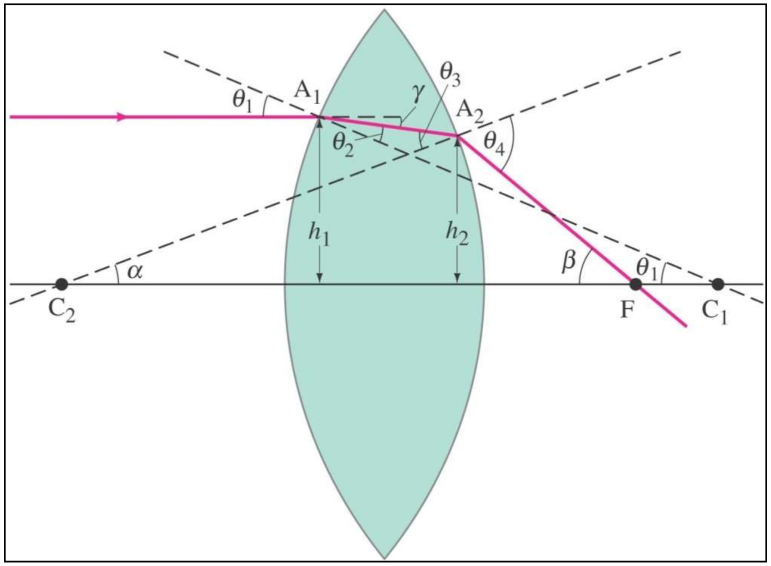
with $d_o^D = -d_i^C = -16\ cm$ And $d_i^D = d_i^T = 28{,}5\ cm$

Then $f^D = -36{,}48\ cm$

33-4 Lensmaker's Equation

- This useful equation relates the radii of curvature of the two lens surfaces, and the index of refraction, to the focal length:

$$\frac{1}{f} = (n_2 - n_1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (n_{lens} - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$



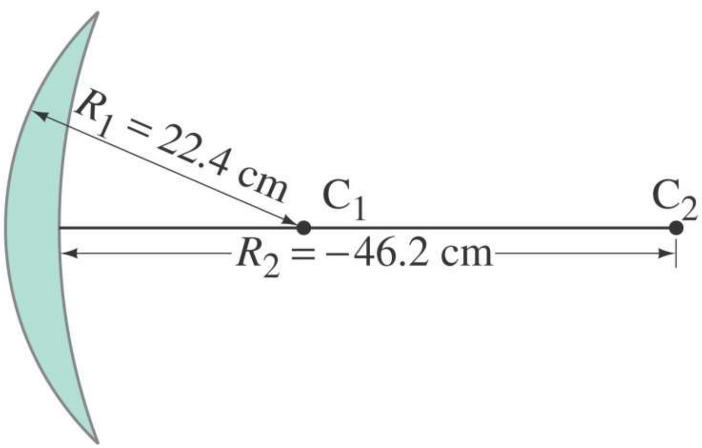
n_1: surrounding refractive index
n_2: lens refractive index

The equation is symmetrical in R_1 and R_2: if a lens is turned around, the focal length is the same, even if the two lens surfaces are different.

33-4 Lensmaker's Equation

Example 33-7: Calculating *f* for a converging lens.

- A convex meniscus lens is made from glass with n = 1.50. The radius of curvature of the convex surface is 22.4 cm and that of the concave surface is 46.2 cm.
  - What is the focal length?
  - Where will the image be for an object 2.00 m away?



Solution:

$$f = 87\ cm$$
$$d_i = 1{,}54\ m$$

Remark: attention to units

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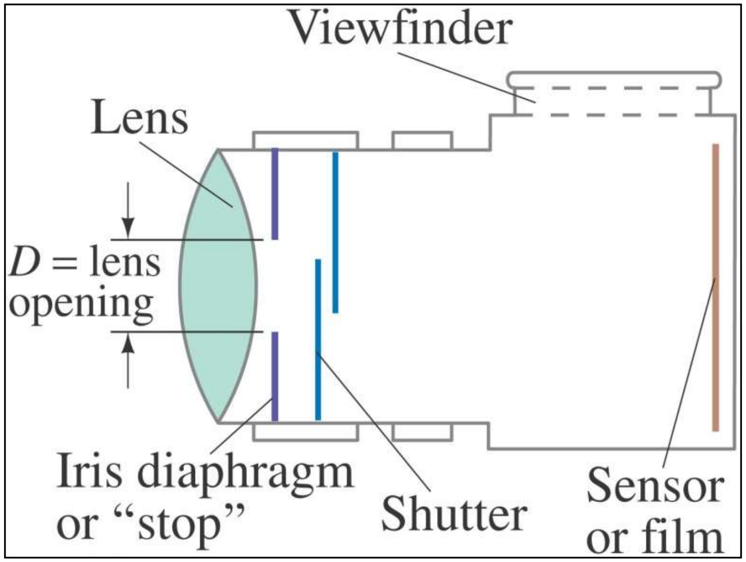
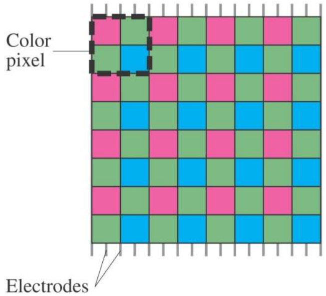
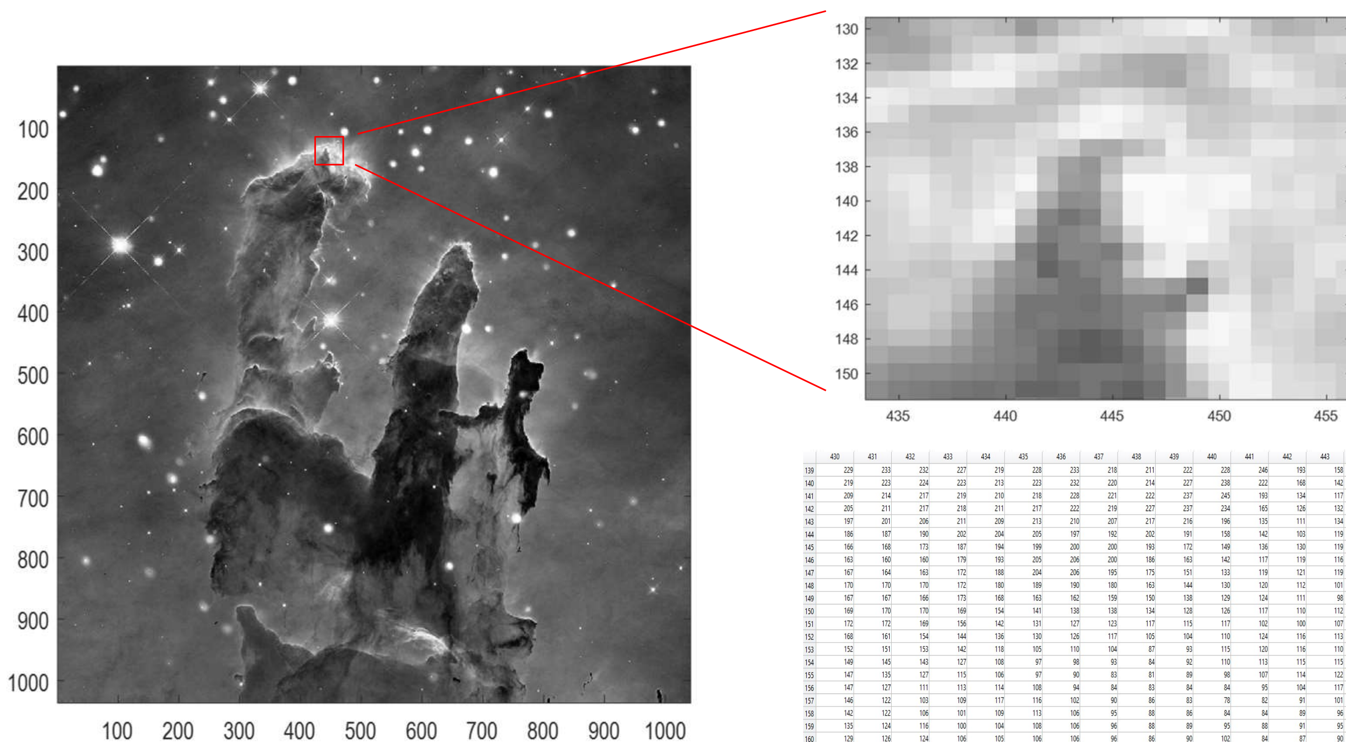
33-5 Cameras: Film and Digital

- Basic parts of a camera:
  - Lens
  - Light-tight box
  - Shutter
  - Film or electronic sensor



A digital camera uses CCD sensors instead of film. The digitized image is sent to a processor for storage and later retrieval





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167	130	124	111	114	114	103	85	77	81	79	70	63	77	93	108	123	130	133

33-5 Cameras: Film and Digital

Camera adjustments:

- **Shutter speed:** controls the amount of time light enters the camera. A faster shutter speed makes a sharper picture (especially if the scene and/or the camera moves).
- **f-stop:** controls the maximum opening of the shutter. This allows the right amount of light to enter to properly expose the film and must be adjusted for external light conditions.
- **Focusing:** this adjusts the position of the lens so that the image is positioned on the film.

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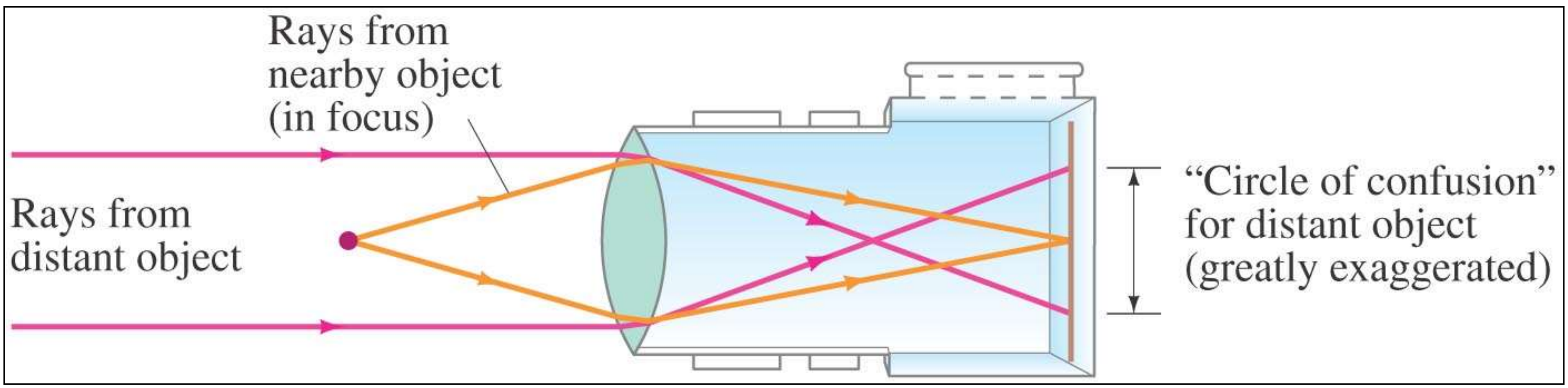
33-5 Cameras: Film and Digital

- There is a certain range of distances over which objects will be in focus; this is called the depth of field of the lens.
  - Objects closer or farther will be blurred.



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33-5 Cameras, Film and Digital

- There are different types of lenses available for cameras, besides the normal lens.
  - **Telephoto lens:** longer focal length, magnified image
  - **Wide-angle lens:** shorter focal length, wider field of view, smaller image
  - **Zoom lens:** adjustable focal length
  - **Digital zoom (in digital cameras):** enlarges pixels with loss of resolution

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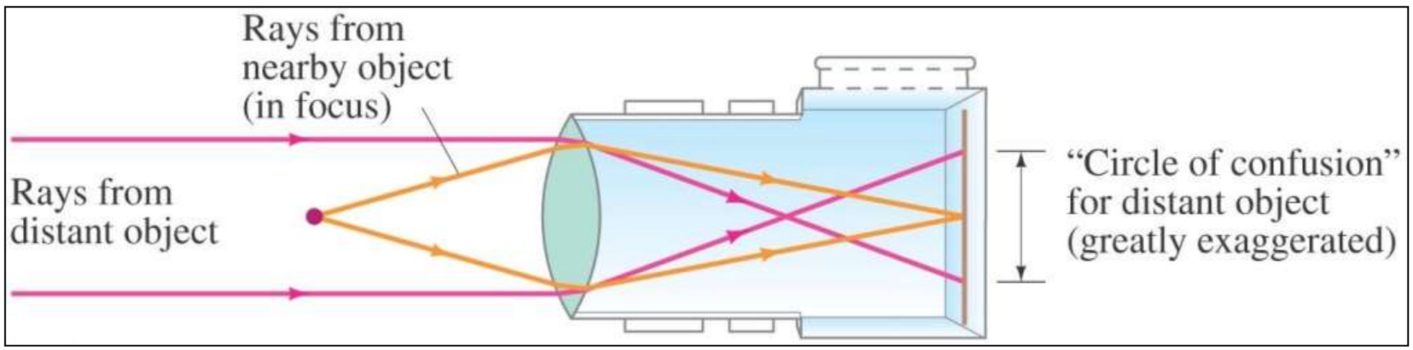
33-5 Cameras, Film and Digital

Example 33-8: Camera focus.

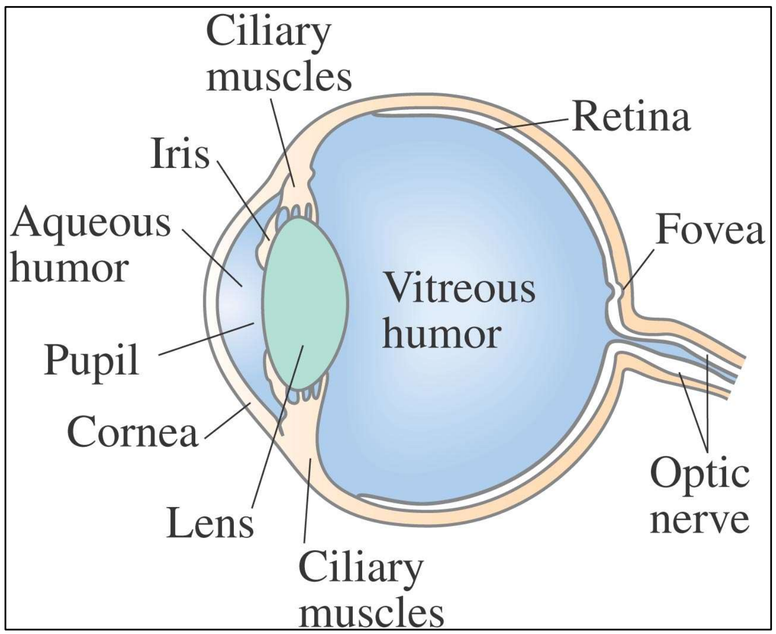
- How far must a 50.0-mm-focal-length camera lens be moved from its infinity setting to sharply focus an object 3.00 m away?



- **<u>Solution:</u>** the lens needs to move 0.8 mm away from the digital sensor, i.e. focusing by moving the lens (*f* is fixed).
- The eye focusses by changing ***f*** (this is called accommodation).

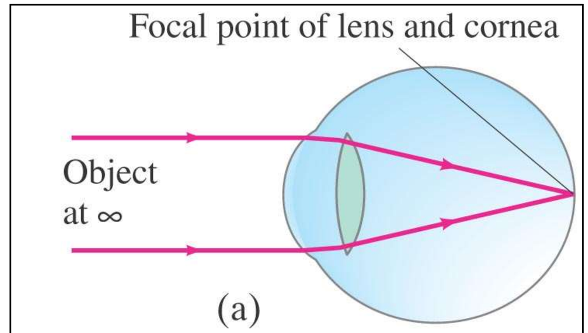
33-6 The Human Eye; Corrective Lenses

The human eye resembles a camera in its basic functioning, with an **adjustable lens**, the **iris**, and the **retina.**

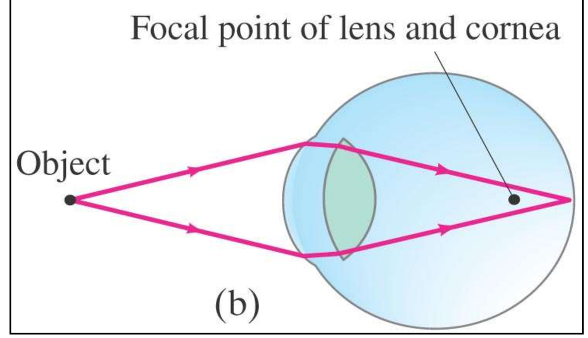


33-6 The Human Eye; Corrective Lenses

- Most of the refraction is done at the cornea's surface; the lens makes small adjustments to focus at different distances.
  - **Near point**: closest distance at which eye can focus clearly. Normal is about 25 cm. – *reading distance*
  - **Far point:** farthest distance at which object can be seen clearly. Normal is at infinity.



Relaxed eye, focused at infinity.



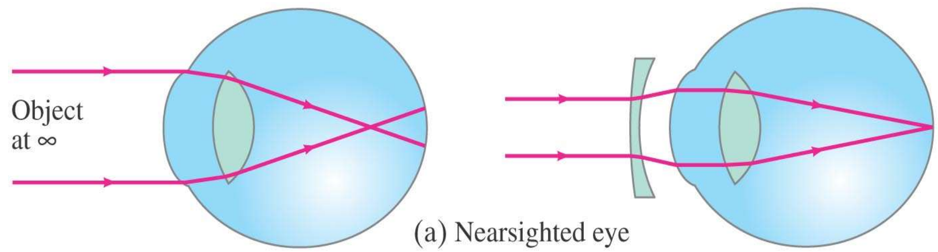
- **Remark:**
  - Nearsightedness (Myopia): far point is too close.
  - Farsightedness (Hyperopia): near point is too far away.

Image **blurred** on the **retina**

33-6 The Human Eye; Corrective Lenses

Nearsightedness eye (Myopia)

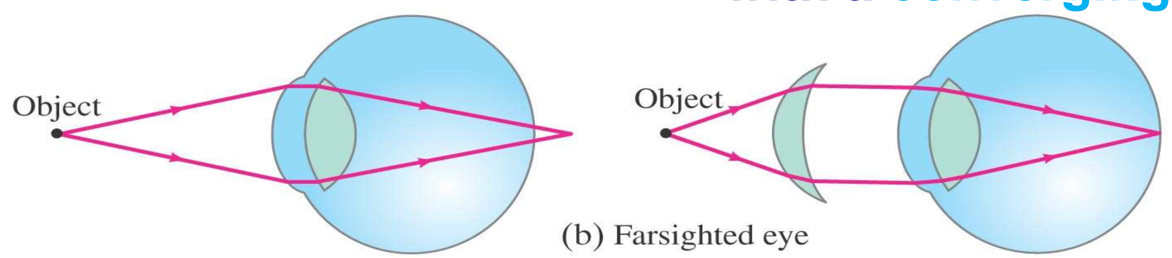
Nearsightedness can be corrected with a diverging lens.



(a) Nearsighted eye

Farsightedness eye (Hyperopia)

Farsightedness can be corrected with a converging lens.



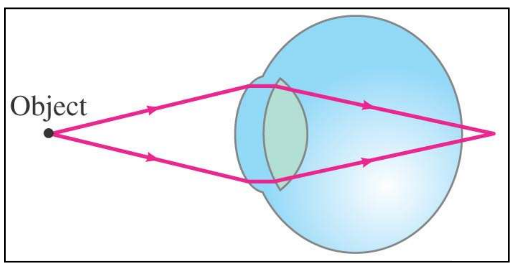
(b) Farsighted eye

33-6 The Human Eye; Corrective Lenses

Example 33-12: Farsighted eye.

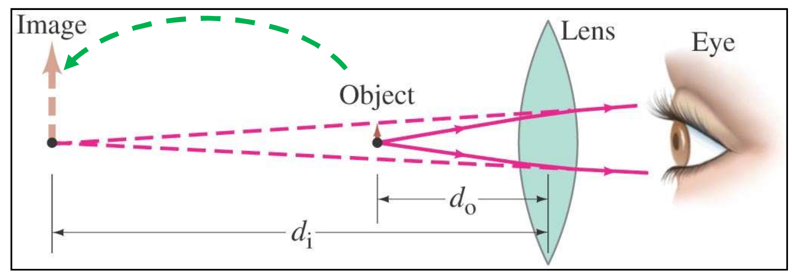
- Sue is farsighted with a near point of 100 cm. What lens power do reading glasses need for reading a newspaper at a distance of 25 cm when the lens is very close to the eye?

Initial situation



$\Rightarrow$

The glasses should change the position of the object to the left to have a better focus on the retina



$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{25\text{ cm}} - \frac{1}{-100\text{ cm}} = \frac{4-1}{100\text{ cm}} = \frac{1}{33\text{ cm}}$$

$$f = 33\text{ cm}$$

$$P = \frac{1}{f} = +3.0\text{ D}$$

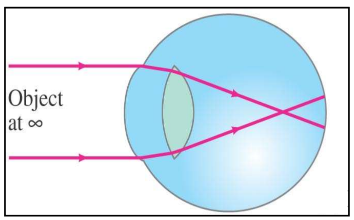
33-6 The Human Eye; Corrective Lenses

Example 33-13: Nearsighted eye.

- A nearsighted eye has near and far points of 12 cm and 17 cm, respectively.
  - What lens power is needed for this person to see distant objects clearly,
  - what then will be the near point? Assume that the lens is 2.0 cm from the eye (typical for eyeglasses).

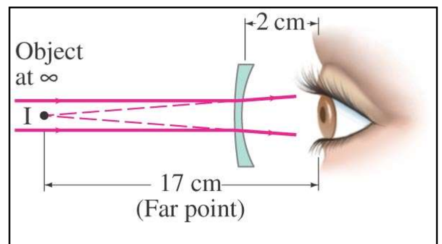
FAR Focus

Initial situation



⇒

The glasses should change the position of the object to the right to have a better focus on the retina (from ∞ to 17 cm)



(a)

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{\infty}-\frac{1}{-15\text{ cm}}=\frac{-1}{15\text{ cm}}$$

$$f=-15\text{ cm}$$

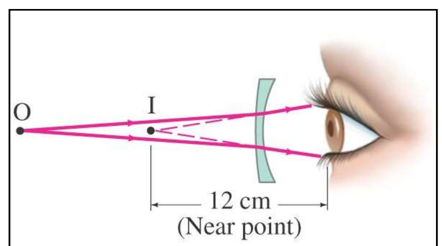
$$P=\frac{1}{f}=-6.7\text{ D}$$

NEAR focus

$$\frac{1}{d_o}=\frac{1}{f}-\frac{1}{d_i}=-\frac{1}{0.15\text{ m}}+\frac{1}{0.10\text{ m}}=\frac{-2+3}{0.30\text{ m}}=\frac{1}{0.30\text{ m}}$$

$$d_o=30\text{ cm}$$

What is the reading distance with the same glasses?

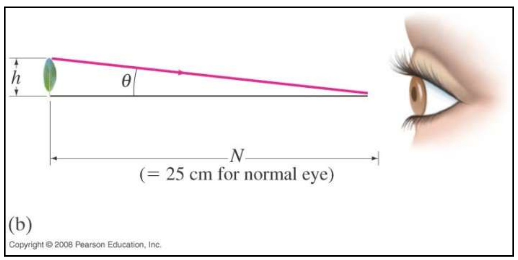


(b)

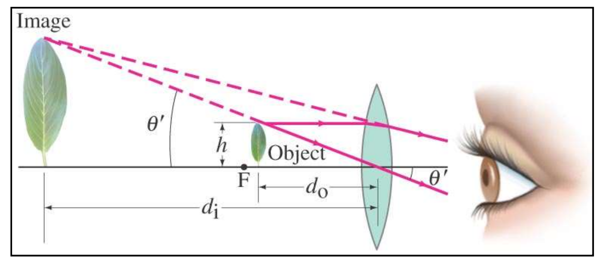
33-7 Magnifying Glass

- A magnifying glass (simple magnifier) is a converging lens. It allows us to focus on objects closer than the near point (25 cm for a normal eye), so that they make a larger, and therefore clearer, image on the retina.

Initial situation (without magnifying glass)



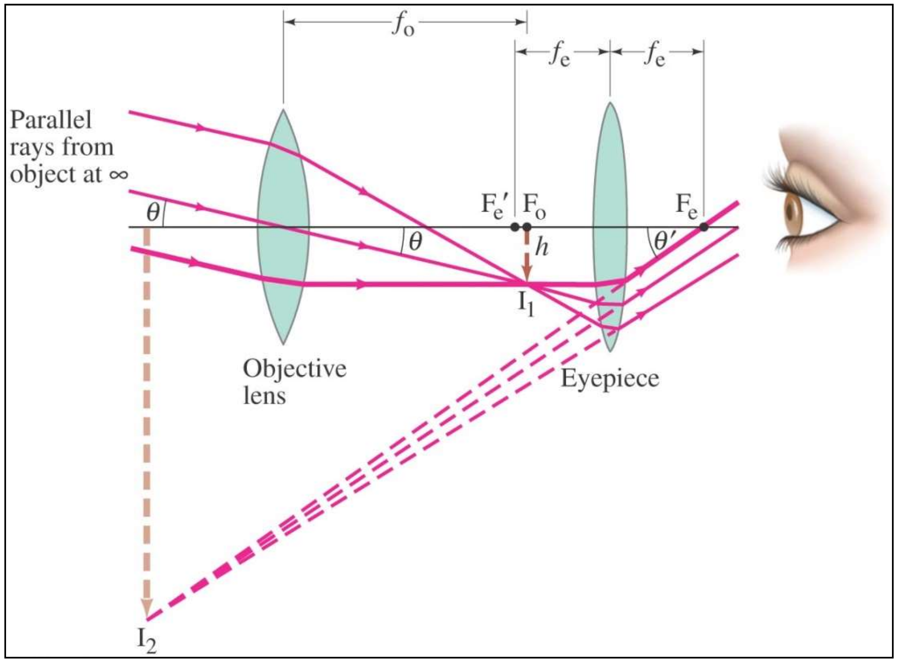
Final configuration (with magnifying glass)



- The power of a magnifying glass is described by its <u>angular</u> magnification: $m = \frac{\theta'}{\theta}$
  - $\tan(\theta) \approx \theta = \frac{h}{N}$ and $\tan(\theta') \approx \theta' = \frac{h}{d_o}\ with\ d_o \approx f\ \Rightarrow \boxed{m = \frac{N}{f}}$

33-8 Telescopes

- A refracting telescope consists of two lenses at opposite ends of a long tube. The objective lens is closest to the object, and the eyepiece is closest to the eye.
- The magnification is given by:



$$M = \frac{\theta'}{\theta} = \frac{(h/f_e)}{(h/f_o)} = -\frac{f_o}{f_e}. \qquad \text{[telescope]}$$

Remarks: virtual images are created for the eye.

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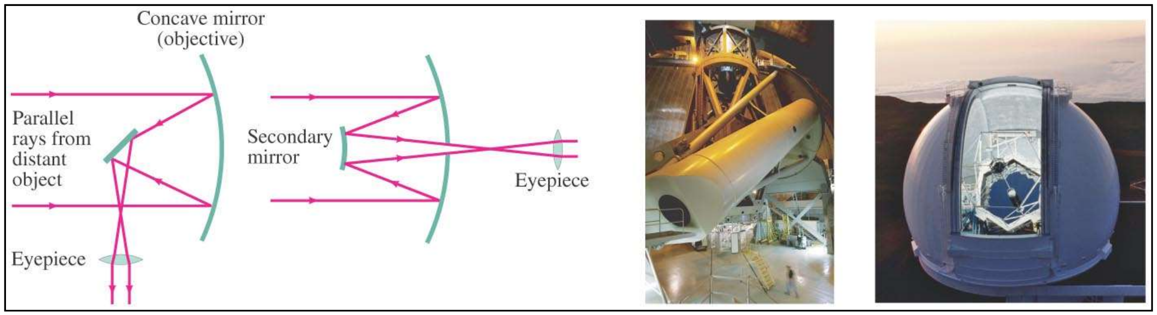
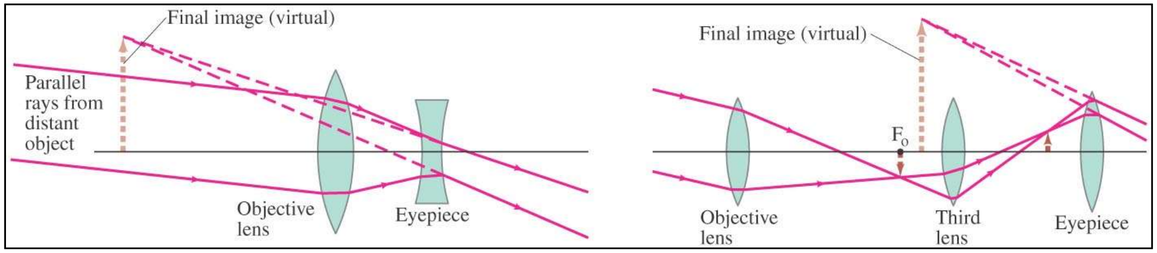
33-8 Telescopes

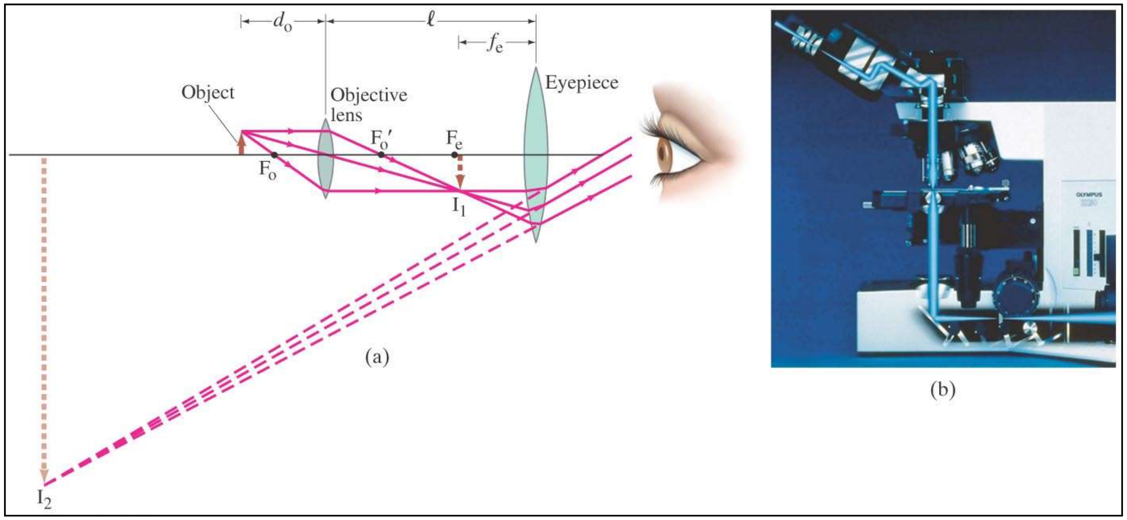
- Astronomical telescopes need to gather as much light as possible, meaning that the objective must be as large as possible. Hence, mirrors are used instead of lenses, as they can be made much larger and with more precision.



- A terrestrial telescope, used for viewing objects on Earth, should produce an upright image. Here are two models, a Galilean type and a spyglass:



33-9 Compound Microscope



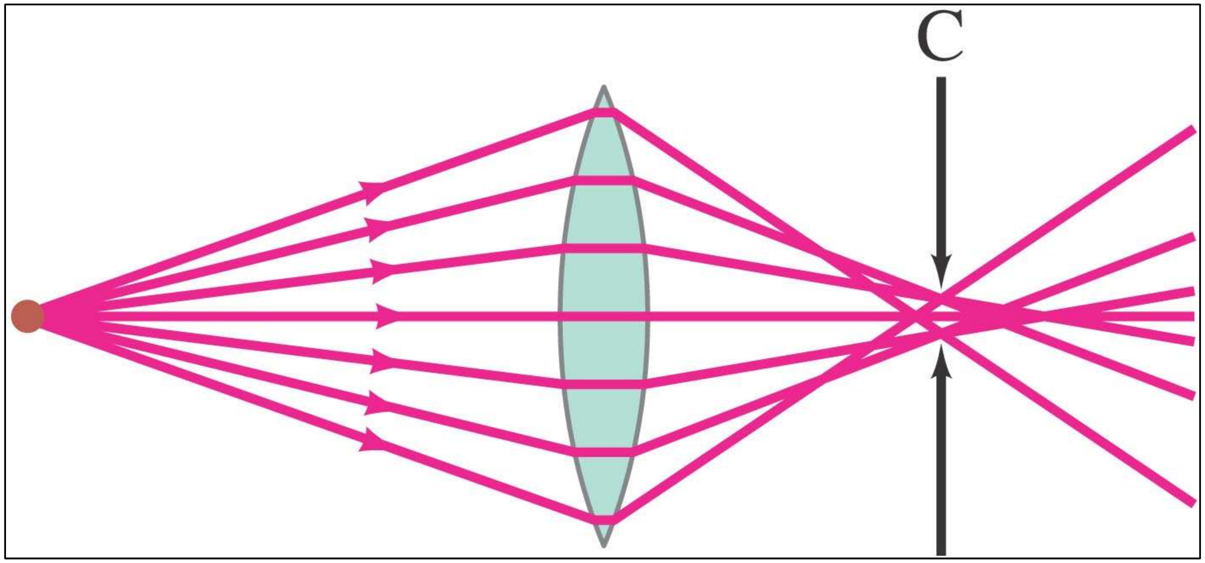
- The magnification is given by

$$M = M_e m_o = \left(\frac{N}{f_e}\right)\left(\frac{\ell - f_e}{d_o}\right) \qquad \text{[microscope]}$$

$$\approx \frac{N\ell}{f_e f_o}. \qquad [f_o \text{ and } f_e \ll \ell]$$

33-10 Aberrations of Lenses and Mirrors

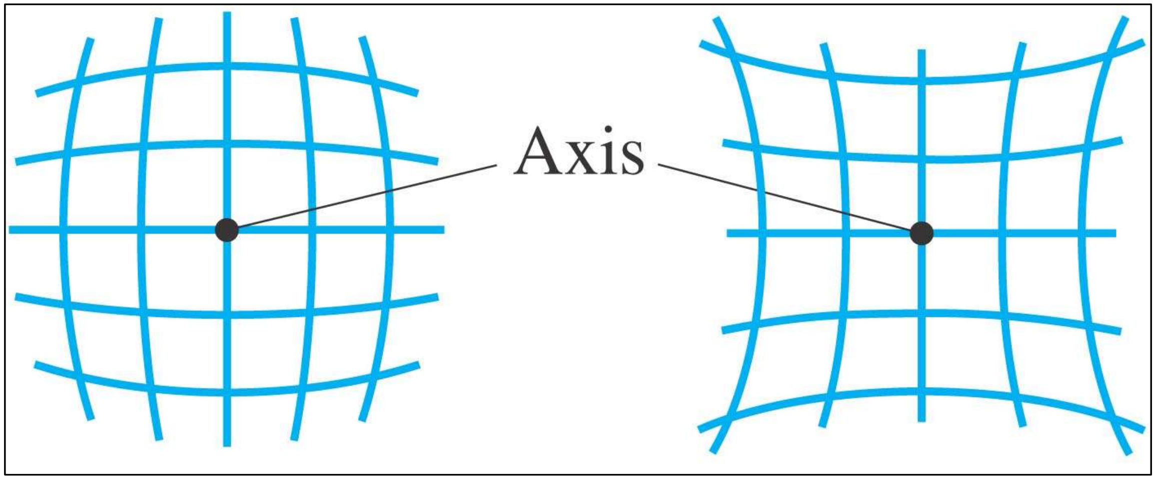
- Spherical aberration: rays far from the lens axis do not focus at the focal point



Solutions: compound-lens systems; use only central part of lens.

33-10 Aberrations of Lenses and Mirrors

- Distortion: caused by variation in magnification with distance from the lens.
  - Barrel and pincushion distortion:



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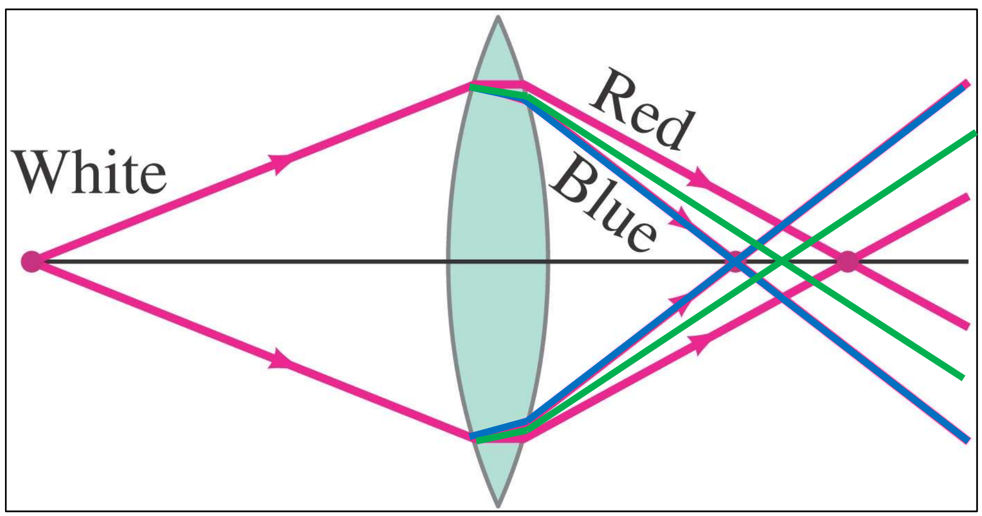
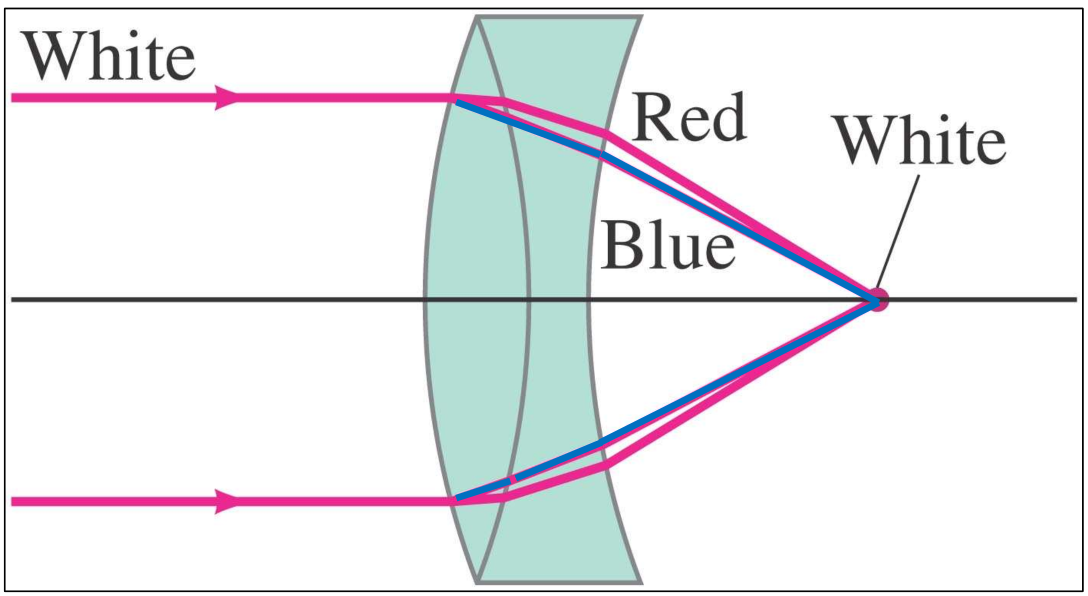
33-10 Aberrations of Lenses and Mirrors

- Chromatic aberration: light of different wavelengths has different indices of refraction and focuses at different points.



33-10 Aberrations of Lenses and Mirrors

- Solution: Achromatic doublet, made of lenses of two different materials



Summary of Chapter 33 (1 of 4)

- Lens uses refraction to form real or virtual image.
- Converging lens: rays converge at focal point.
- Diverging lens: rays appear to diverge from focal point.
- Power is given in diopters $\left(m^{-1}\right)$:

$$P = \frac{1}{f}.$$

Summary of Chapter 33 (2 of 4)

- **Thin lens equation:**

$$\frac{1}{d_\mathrm{o}} + \frac{1}{d_\mathrm{i}} = \frac{1}{f}.$$

- **Magnification:**

$$m = \frac{h_\mathrm{i}}{h_\mathrm{o}} = -\frac{d_\mathrm{i}}{d_\mathrm{o}}.$$

Summary of Chapter 33 (3 of 4)

- Camera focuses image on film or electronic sensor; lens can be moved and size of opening adjusted (f-stop).
- Human eye also makes adjustments, by changing shape of lens and size of pupil.
- Nearsighted eye is corrected by diverging lens.
- Farsighted eye is corrected by converging lens.

Summary of Chapter 33 (4 of 4)

- **Magnification of simple magnifier:**

$$M = \frac{\theta'}{\theta} = \frac{h/f}{h/N} = \frac{N}{f}. \qquad \left[\begin{array}{c}\text{eye focused at } \infty; \\ N = 25\text{ cm for normal eye}\end{array}\right]$$

- **Telescope: objective lens or mirror plus eyepiece lens. Magnification:**

$$M = \frac{\theta'}{\theta} = \frac{(h/f_e)}{(h/f_o)} = -\frac{f_o}{f_e}. \qquad \text{[telescope]}$$