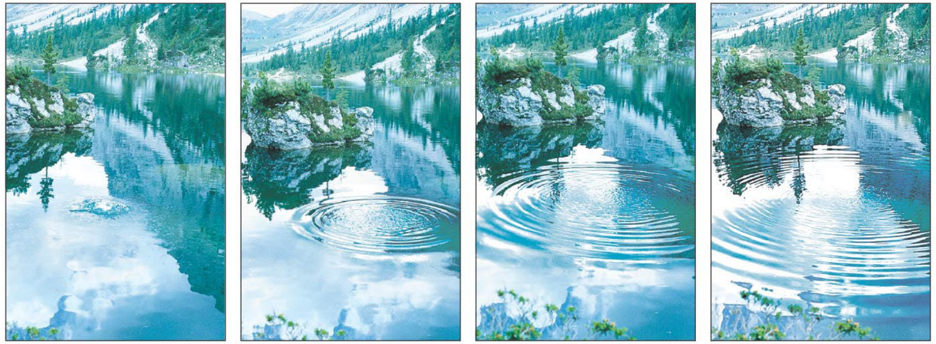
WAVE MOTION

Chapter 15



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Operations & Environment section

Faculty of Aerospace Engineering

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Position in the syllabus

14. ~~Oscillations~~
15. **Waves**
16. Sound
17. Temperature and the ideal gas law
18. Thermodynamics
19. Electricity and circuits
20. Electromagnetism
21. Optics

STEPPING STONE FOR FURTHER LECTURES

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Structure of the lecture

Learning objectives for today's lecture

After this lecture you should be able to:

- Describe different **types of waves** and their main characteristics.
- Analyze the **energy** transported by waves.
- Calculate the mathematical representation of a traveling wave using the **wave equation.**

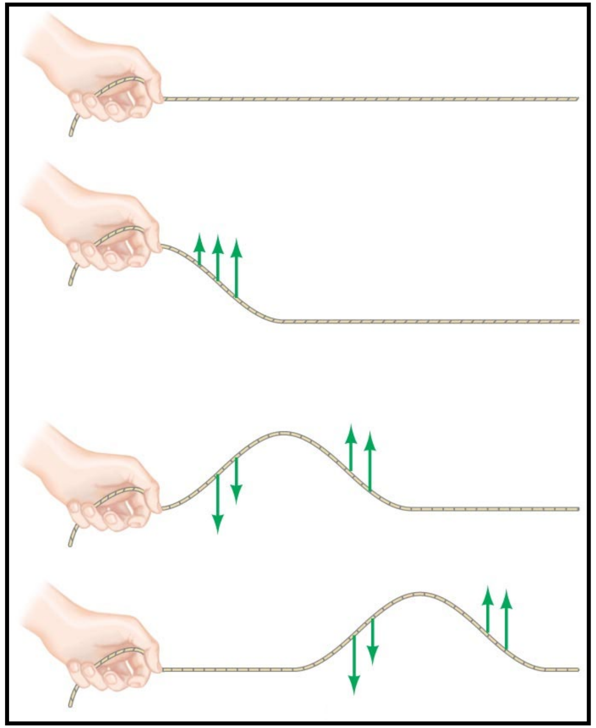
Assumed prior knowledge

- Basic trigonometry (cosine, etc.)
- Basic mechanics and kinematics (Newton’s laws, etc.)
- Differential equations
- Concepts learned in **Chapter 14 - Oscillations**

15.1 – Characteristics of wave motion

A wave consists of **oscillations that move** **without carrying matter** with them.

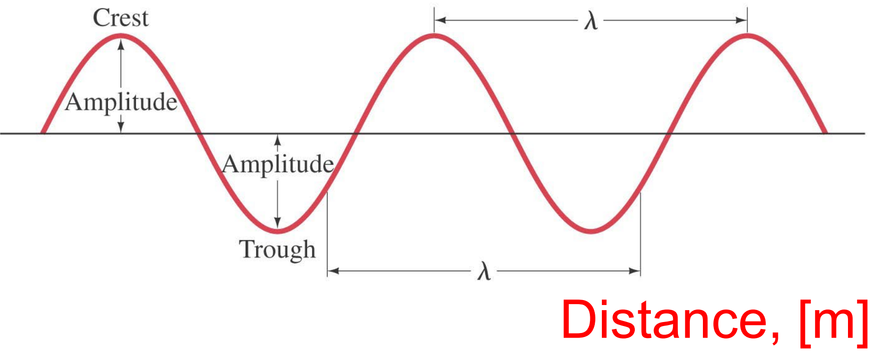
All types of traveling waves transport **energy**.



A single wave pulse begins with a **vibration** that is transmitted through internal forces in the medium.

Continuous waves start with vibrations too. If the vibration follows a Simple Harmonic Motion (**SHM**), then the wave will be **sinusoidal**.

15.1 – Characteristics of wave motion - Definitions



- **Amplitude** (A) is the maximum displacement, [m].
- **Period** (T) is the time required to complete one cycle, [s].
- **Frequency** (f) is the number of cycles completed per second, [Hz = 1/s].
- **Wavelength** (λ) is the distance between two points of equal phase (e.g. two crests), [m].
- **Wave velocity** (v), [m/s]: $\boxed{v = \frac{\lambda}{T} = \lambda f}$

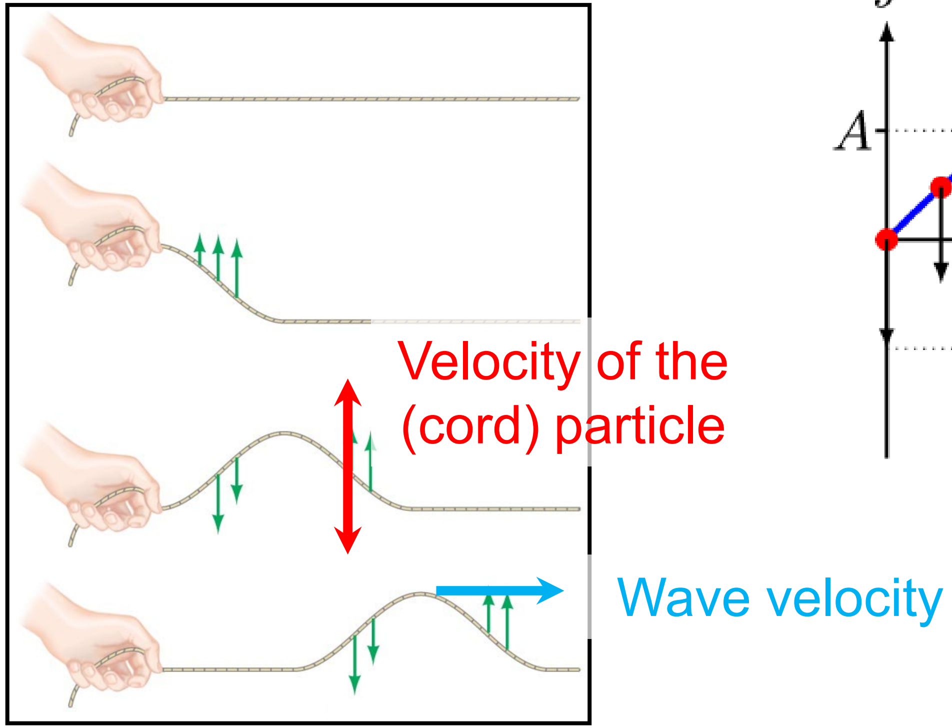
For a constant wave velocity v

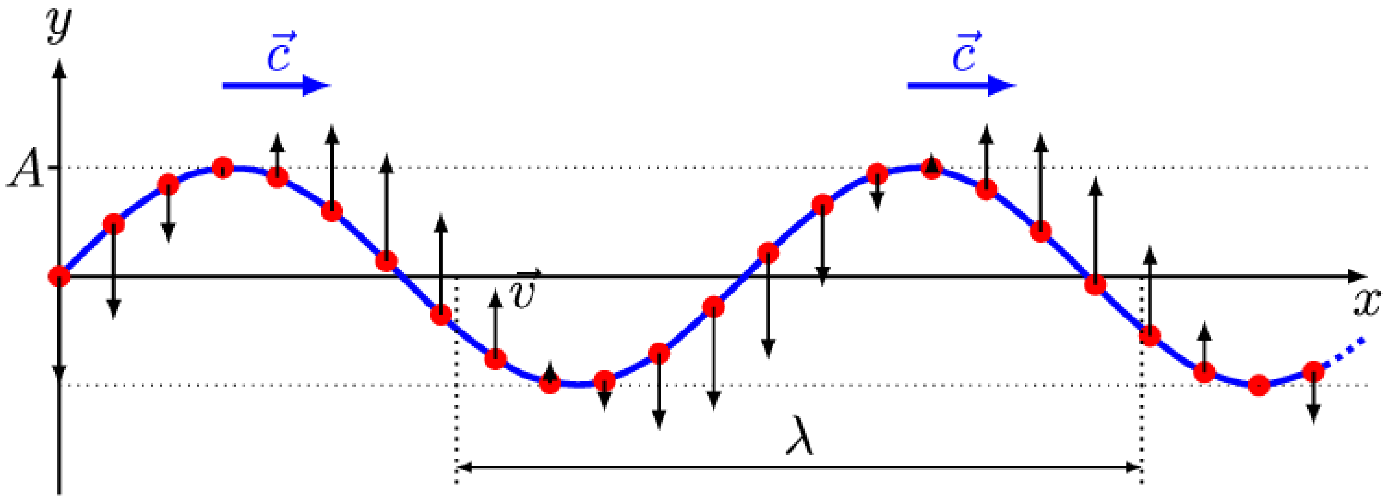
$\lambda \uparrow \rightarrow f \downarrow$

$\lambda \downarrow \rightarrow f \uparrow$

15.1 – Characteristics of wave motion: wave velocity

The velocity of a **particle** (on the cord) is not equal to the **wave velocity**!

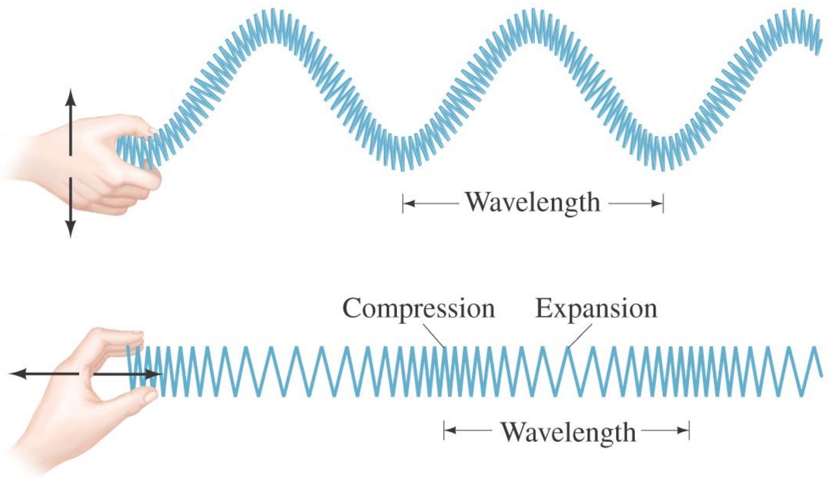


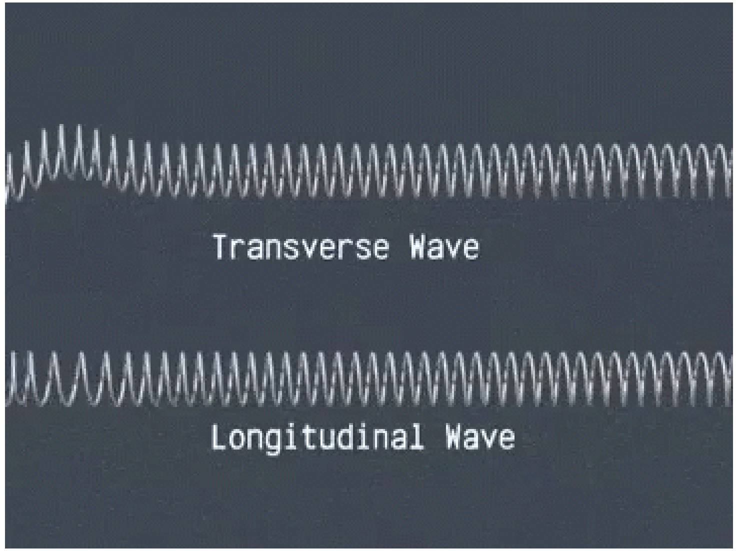


15.2 – Types of waves: Transverse and longitudinal

Transverse wave

Longitudinal wave





The motion of particles in a wave can be either perpendicular to the wave direction (**transverse**) or parallel to it (**longitudinal**).

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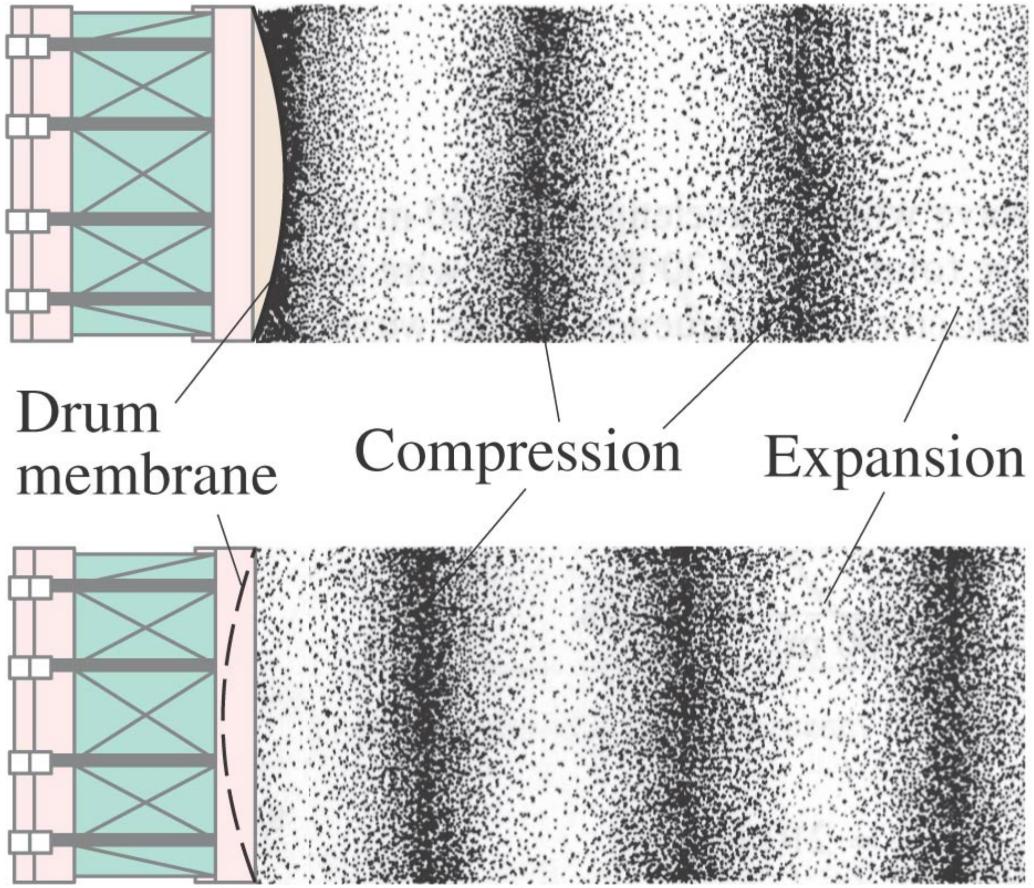
15.2 – Types of waves: Example of longitudinal wave

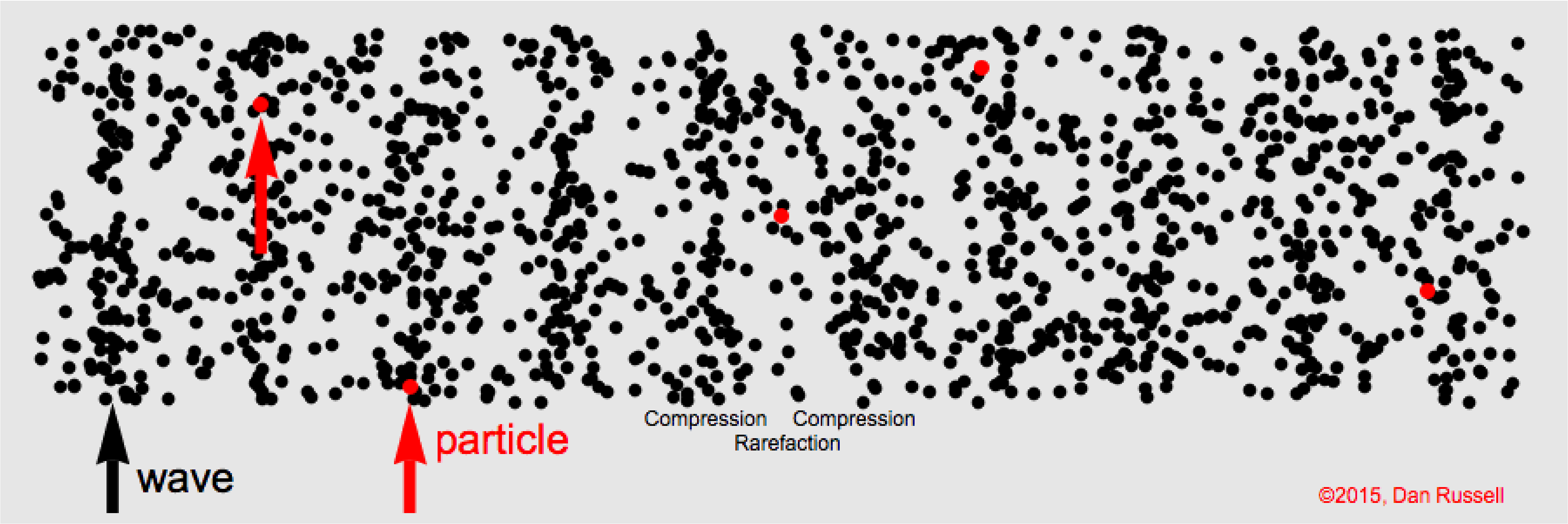


Sound is an example of **longitudinal** wave.

This figure represents a sound wave (black dots represent air particles) separated half a period ($T/2$) apart

15.2 – Types of waves: Example of longitudinal wave

The (oscillation) velocity of a **particle** is not equal to the **wave velocity**!



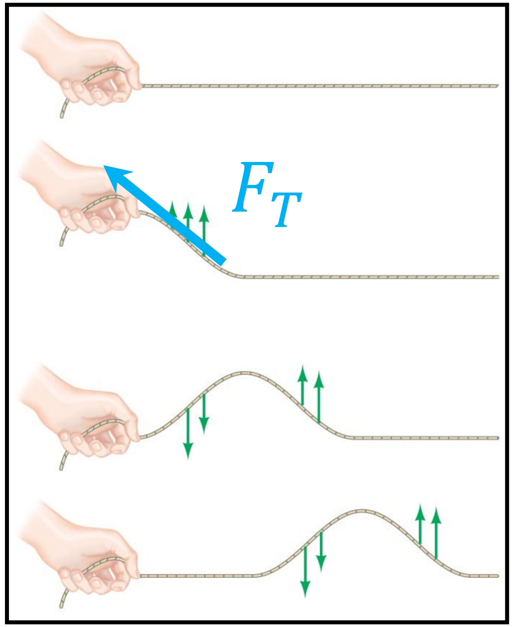
15.2 – Propagation of transverse waves

The wave propagation velocity ($v = \lambda f$) **depends on the medium** in which the wave propagates.

For a **transverse wave on a cord**, it is given by:

$$\boxed{v = \sqrt{\frac{F_T}{\mu}}}$$

- F_T is the **tension force**, [N]
- $\mu = \frac{m}{l}$ is the **mass per unit length** of the rope, [kg/m]



15.2 – Propagation of longitudinal waves

Generalizing this idea to **longitudinal** waves, the propagation velocity depends on the (square root of the) ratio between the **elastic restoring force** of the medium and the **mass density** (~inertia).

$$\boxed{v = \sqrt{\frac{\text{elastic force}}{\text{inertia}}}} \quad \left\{ \begin{array}{l} \boxed{v = \sqrt{\dfrac{\mathrm{E}}{\rho}}} \\ \\ \boxed{v = \sqrt{\dfrac{\mathrm{B}}{\rho}}} \end{array} \right.$$

For a wave propagating along a **solid**:

- E is the elastic or Young modulus, $[\mathrm{N/m^2}]$.
- ρ is the volumetric density, $[\mathrm{kg/m^3}]$.

For a wave propagating through a **fluid**:

- B is the bulk modulus, $[\mathrm{N/m^2}]$.

15.2 – Propagation of waves (general idea)

$$\boxed{v = \sqrt{\frac{\text{elastic force}}{\text{inertia}}}}$$

$$Force \uparrow, Inertia \downarrow, \rightarrow v \uparrow$$

$$v = \sqrt{\frac{F_T}{\mu}}$$ Transverse wave in a **rope**

$$v = \sqrt{\frac{\mathrm{E}}{\rho}}$$ Longitudinal wave in **solid**

$$v = \sqrt{\frac{\mathrm{B}}{\rho}}$$ Longitudinal wave in **fluid**

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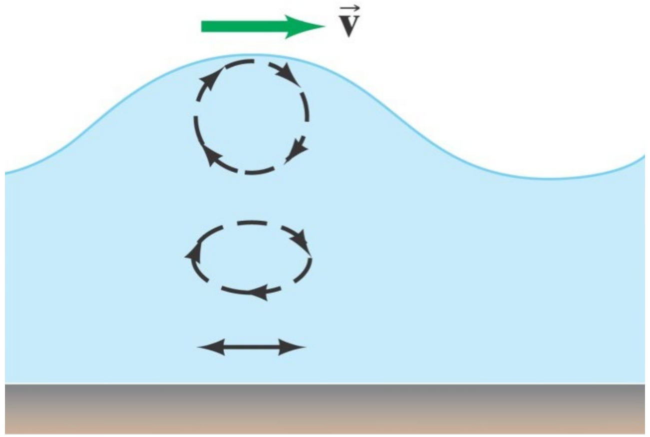
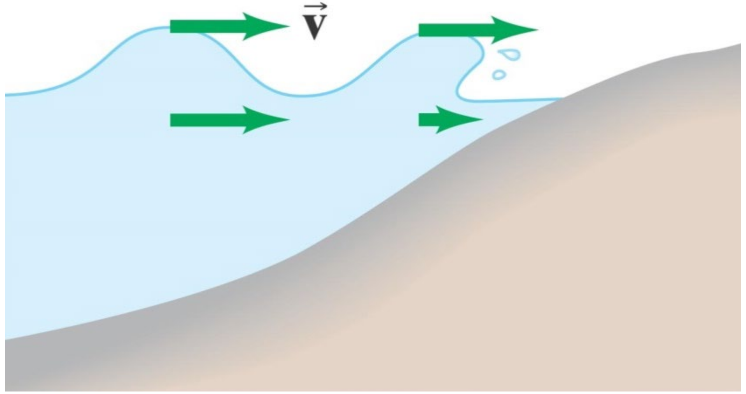
15.2 – Propagation of waves (general idea)

In a **solid**, **both longitudinal and transverse** waves can propagate (e.g. earthquakes).

In **fluids**, however, **only longitudinal waves** can propagate, since in the transverse direction, a fluid has no restoring force (fluids are deformable).

Surface waves (e.g. a sea wave) travel along the boundary between two media (in this case water and air) and can be a combination of both (transverse and longitudinal):





15.3 – Energy transported by waves

If we consider **sinusoidal waves**, the particles move in **simple harmonic motion (SHM)** (see Chapter 14).

By looking at the energy of a particle in the medium the wave is propagating, we find:

$$E = \frac{1}{2} k A^2$$

From Chapter 14, we know that:

$$\omega = \sqrt{\frac{k}{m}} = 2\pi f \quad \rightarrow \quad k = 4\pi^2 m f^2$$

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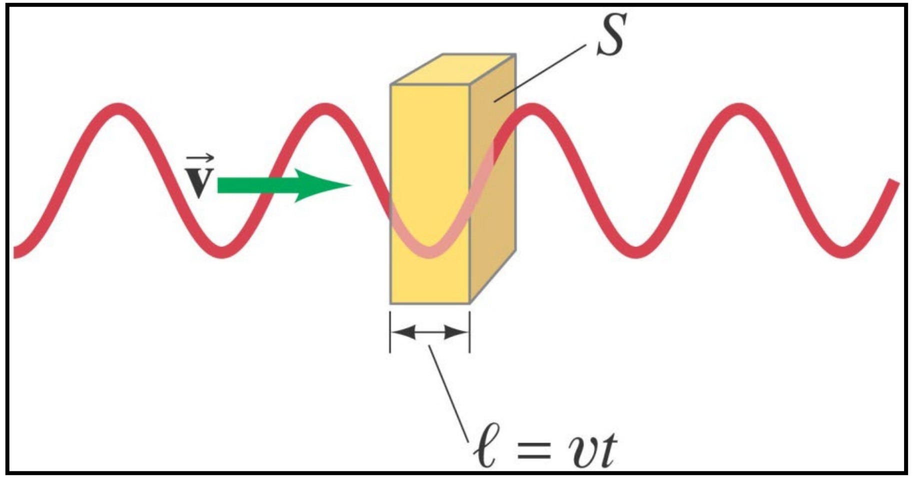
15.3 – Energy transported by waves

$$E = \frac{1}{2}kA^2 \qquad\qquad k = 4\pi^2 m f^2$$

If we consider a *slice* of the medium, we can express the mass of the particle as:



$$\rho = \frac{m}{V} = \frac{m}{Sl} = \frac{m}{Svt}$$

$$m = \rho S v t$$

$$E = \frac{1}{2}4\pi^2 m f^2 A^2 = 2\pi^2 \rho S v t f^2 A^2$$

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15.3 – Energy transported by waves

$$E = \frac{1}{2}4\pi^2 m f^2 A^2 = 2\pi^2 \rho S v t f^2 A^2, \qquad [\mathrm{J}]$$

The **power** is defined as:

$$P = \frac{E}{t} = 2\pi^2 \rho S v f^2 A^2, \qquad [\mathrm{W}]$$

Assuming that the **medium has the same density everywhere**, the **intensity** of the wave (i.e. power/surface) is:

$$I = \frac{P}{S} = 2\pi^2 \rho v f^2 A^2, \qquad [\mathrm{W/m^2}]$$

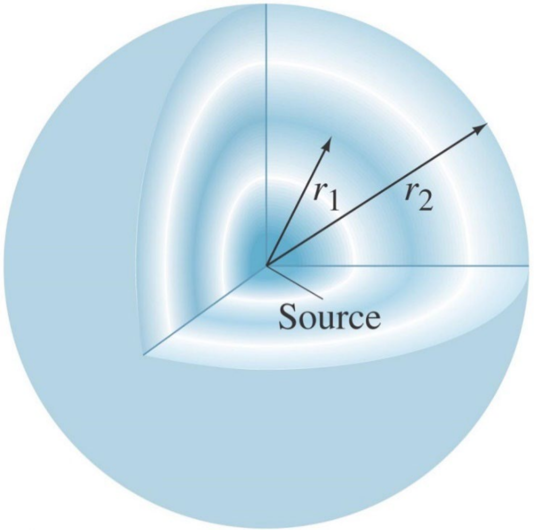
15.3 – Energy transported by waves

$$I = \frac{P}{S} = 2\pi^2 \rho v f^2 A^2, \qquad [\mathrm{W/m^2}]$$

$$I \propto A^2 f^2$$

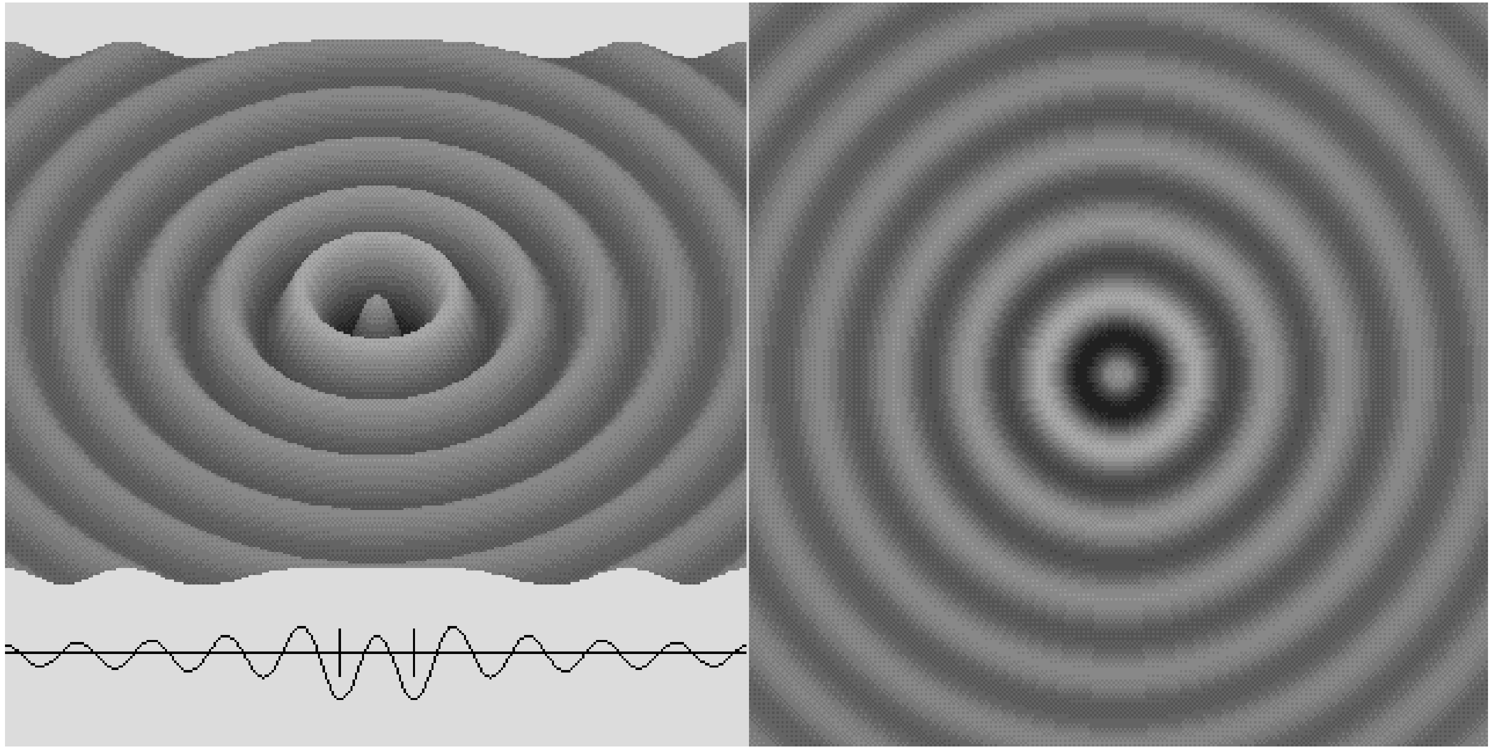
The intensity of the wave is proportional to the **square of the frequency** and to the **square of the amplitude**.

If a wave is able to spread out **three-dimensionally** from its source, and the **medium is uniform**, then the wave is spherical. If the **power output is constant**, then we have:



$$4\pi r_1^2 I_1 = 4\pi r_2^2 I_2 \qquad I \propto \frac{1}{r^2}$$

15.3 – Energy transported by waves



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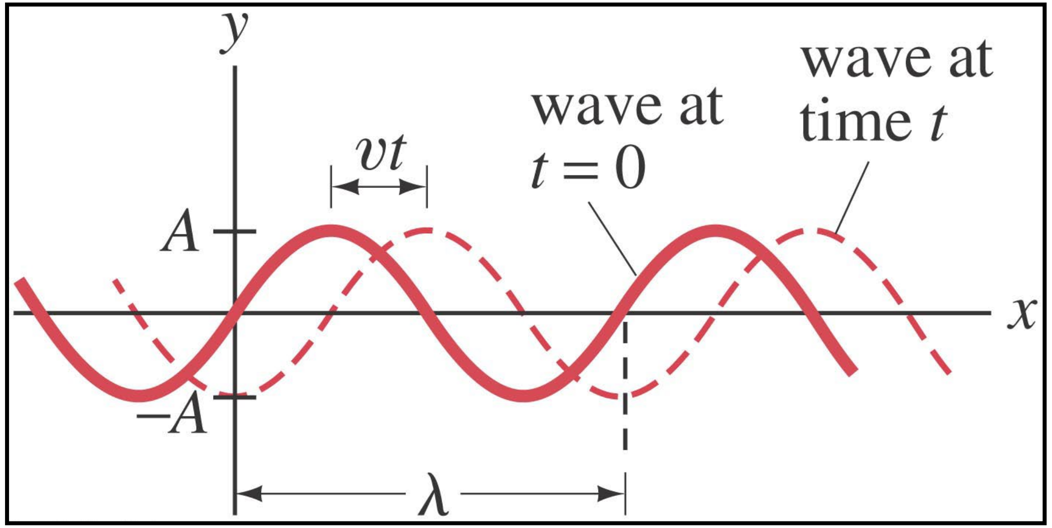
For 3D (spherical) waves we then have:

$$\boxed{I \propto \frac{A^2 f^2}{r^2}}$$

For 1D waves, *I* and *A* do not decrease with distance *r* (if we ignore friction)

15.4 – Mathematical representation of a traveling wave

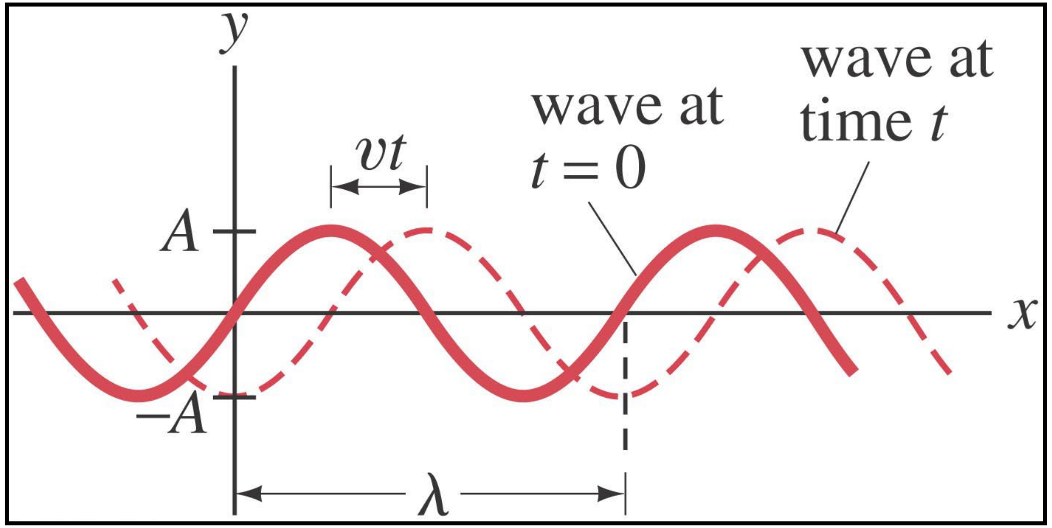
The **particle displacement** (D) generated by a sinusoidal wave at a given instant (**frozen time**, here $t = 0$) is given by:



$$\boxed{D(x) = A \sin \frac{2\pi}{\lambda} x}$$

15.4 – Mathematical representation of a traveling wave

After a certain time t, the wave has traveled a distance vt so we can write:



$$\boxed{D(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]}$$

This minus sign indicates that the wave is traveling in the **positive** x direction. For a wave traveling in the **negative** x direction, we would have a plus + sign.

In general, any wave traveling along the x-axis has the form: $D(x \pm vt)$

15.4 – Mathematical representation of a traveling wave

$$D(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

We can rewrite this equation as:

$$D(x,t) = A\sin(kx - \omega t)$$

k here is the **wave number**, [rad/m] **(Don't confuse** with the spring stiffness constant!)

Here v is the **wave propagation speed**.

$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f = \frac{2\pi v}{\lambda} \qquad v = \frac{\omega}{k}$$

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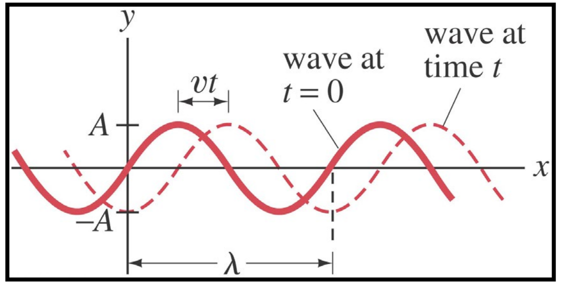
15.4 – Mathematical representation of a traveling wave

$$D(x,t) = A\sin(kx - \omega t)$$



For $t = 0$ (initial wave shape): $D(x,0) = A\sin(kx)$

For $x = 0$ (motion in x = 0 over time): $D(0,t) = A\sin(-\omega t)$

Simple harmonic motion once again!

In a general case, we can also have an **initial phase shift** ϕ

$$\boxed{D(x,t) = A\sin(kx - \omega t + \phi)}$$

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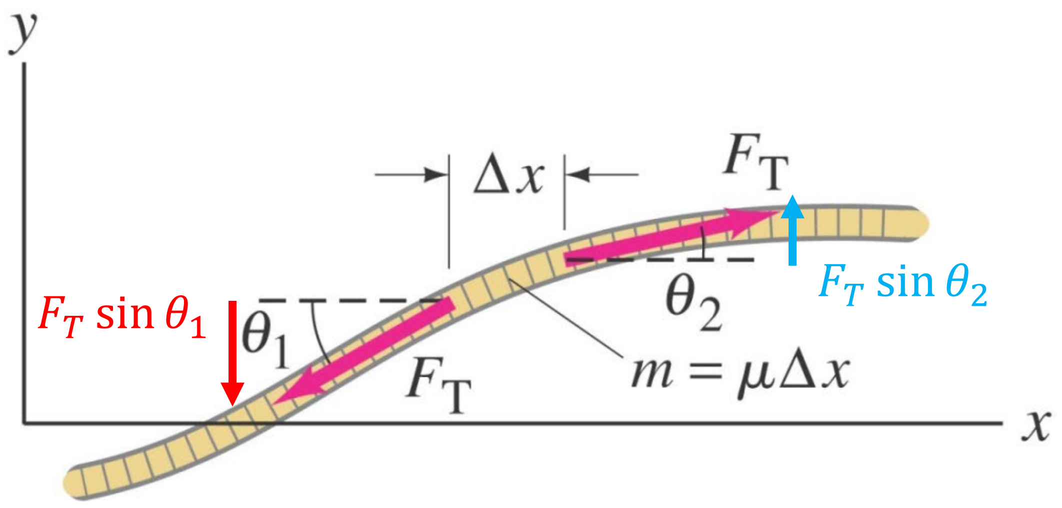
15.5 – The wave equation

If we now focus on a segment (Δx) of a string under tension, we have:

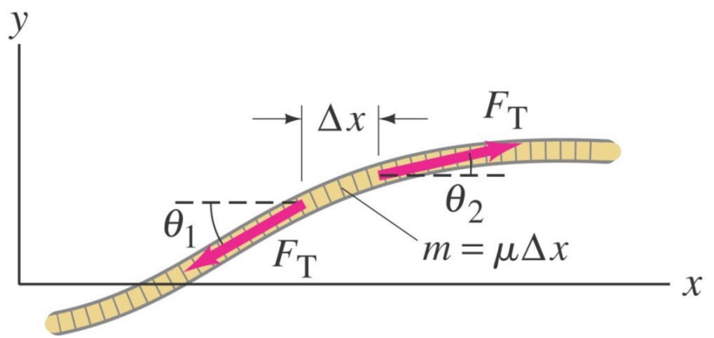


Newton's second law applied on the y-axis gives us:

$$\sum F_y = m a_y$$

$$F_T \sin\theta_2 - F_T \sin\theta_1 = \underbrace{(\mu\Delta x)}_{m} \underbrace{\frac{\partial^2 D}{\partial t^2}}_{a_y}$$

15.5 – The wave equation



$$F_T \sin\theta_2 - F_T \sin\theta_1 = (\mu\Delta x)\frac{\partial^2 D}{\partial t^2}$$

For small angles, we have that: $$\sin\theta \approx \tan\theta = \frac{\partial D}{\partial x} = s$$

where s is the slope of the string

$$F_T s_2 - F_T s_1 = (\mu\Delta x)\frac{\partial^2 D}{\partial t^2} \qquad F_T\frac{\Delta s}{\Delta x} = \mu\frac{\partial^2 D}{\partial t^2}$$

If we take $\Delta x \to 0$:

$$F_T\frac{\Delta s}{\Delta x} = F_T\frac{\partial}{\partial x}\left(\frac{\partial D}{\partial x}\right) = F_T\frac{\partial^2 D}{\partial x^2} \qquad F_T\frac{\partial^2 D}{\partial x^2} = \mu\frac{\partial^2 D}{\partial t^2}$$

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15.5 – The wave equation

$$F_T \frac{\partial^2 D}{\partial x^2} = \mu \frac{\partial^2 D}{\partial t^2}$$

$$\frac{\partial^2 D}{\partial x^2} = \frac{\mu}{F_T} \frac{\partial^2 D}{\partial t^2}$$

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$\boxed{\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 D}{\partial t^2}}$$

This is the **one-dimensional wave equation**. It is a second-order partial differential equation in x and t.

This is only valid for small amplitudes w.r.t. λ

General **sinusoidal waves** are solutions to this equation

$$D(x,t) = A \sin(kx - \omega t)$$

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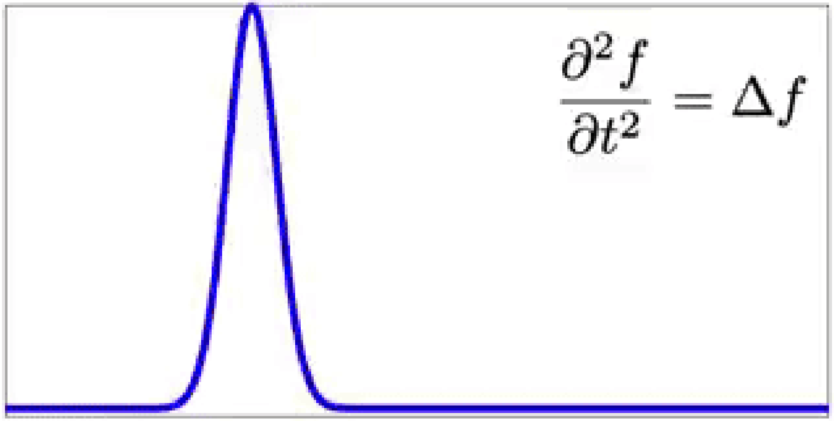
15.5 – Other second-order differential equations

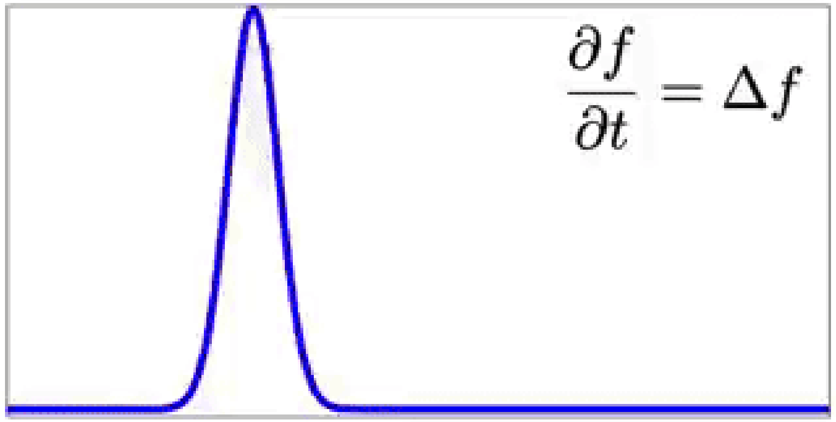
Wave equation:

$$\boxed{\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}}$$



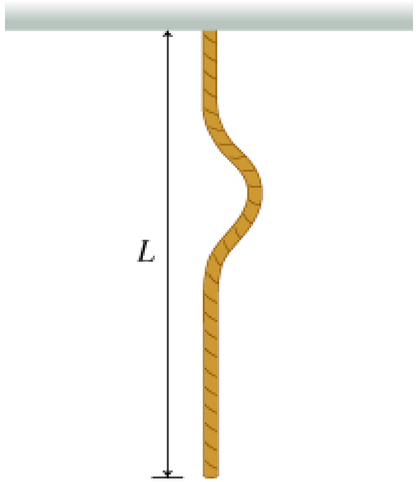
Heat equation:

$$\frac{\partial^2 u}{\partial x^2} = k\frac{\partial u}{\partial t}$$



15.5 – Brain teaser for next lecture

A uniform rope of length L, mass per unit length μ, and negligible stiffness hangs from a solid fixture in the ceiling. The free lower end of the rope is struck sharply at time t=0.



What is the time t it takes the resulting wave on the rope to travel to the ceiling, be reflected, and return to the lower end of the rope?

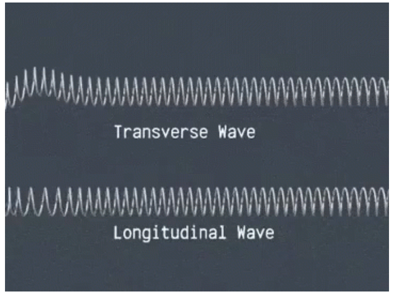
WRAP-UP

Wrap-up: revisit learning objectives

After this lecture you should be able to:

- Describe different **types of waves** and their main characteristics.

$$v = \sqrt{\frac{\text{elastic force}}{\text{inertia}}}$$



- Analyze the **energy** transported by waves.

$$I = \frac{P}{S} = 2\pi^2 \rho v f^2 A^2$$

- Calculate the mathematical representation of a traveling wave using the **wave equation.**

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$$

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For next lecture

- 15.6 – The principle of superposition
- 15.7 – Reflection and transmission
- 15.8 – Interference
- 15.9 – Standing waves and resonance
- 15.10 – Refraction
- 15.11 – Diffraction

- 16.1 – Characteristics of sound
- 16.2 – Mathematical representation of longitudinal waves

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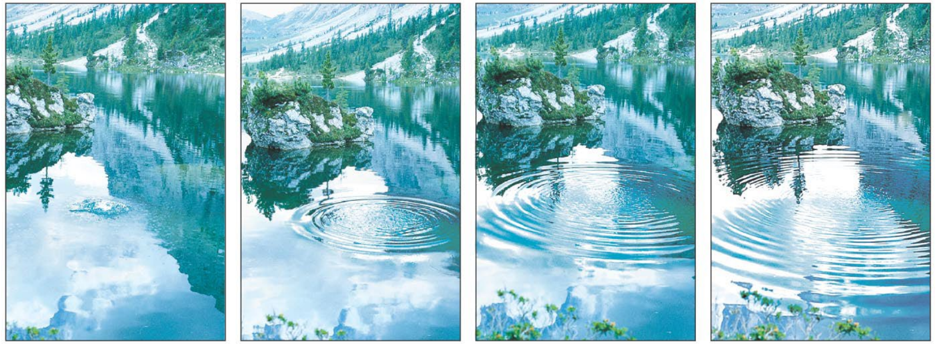
WAVE MOTION

Chapter 15



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~~14. Oscillations~~

15. Waves

16. Sound

17. Temperature and the ideal gas law

18. Thermodynamics

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STEPPING STONE FOR FURTHER LECTURES

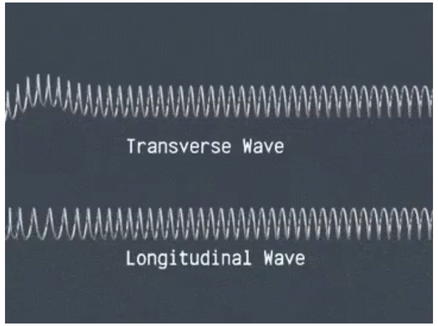
Structure of the lecture

1. Characteristics of wave motion
2. Types of waves: Transverse and longitudinal
3. Energy transported by waves
4. Mathematical representation of a travelling wave
5. The wave equation
6. The principle of superposition
7. Reflection and transmission
8. Interference
9. Standing waves and resonance
10. Refraction
11. Diffraction

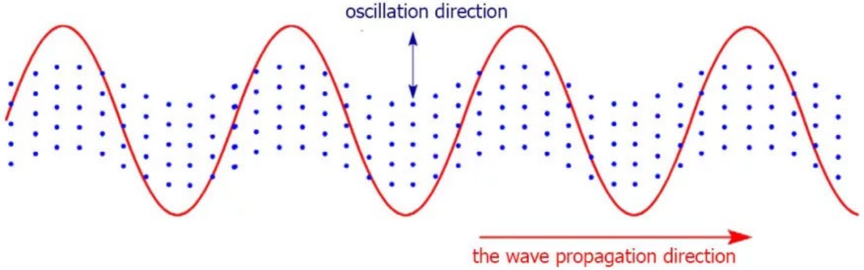
Quick reminder or last lecture

$$v = \sqrt{\frac{\text{elastic force}}{\text{inertia}}}$$

Types of waves and propagation velocity



Rope: $v = \sqrt{\frac{F_T}{\mu}}$ Solid: $v = \sqrt{\frac{\mathrm{E}}{\rho}}$ Fluid: $v = \sqrt{\frac{\mathrm{B}}{\rho}}$



Energy:

$$I = \frac{P}{S} = 2\pi^2 \rho v f^2 A^2$$

$$I \propto A^2 f^2$$

Wave equation (1D):

$$\boxed{\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}}$$

$$\boxed{D(x,t) = A\sin(kx - \omega t + \phi)}$$

Learning objectives for today's lecture

After this lecture you should be able to:

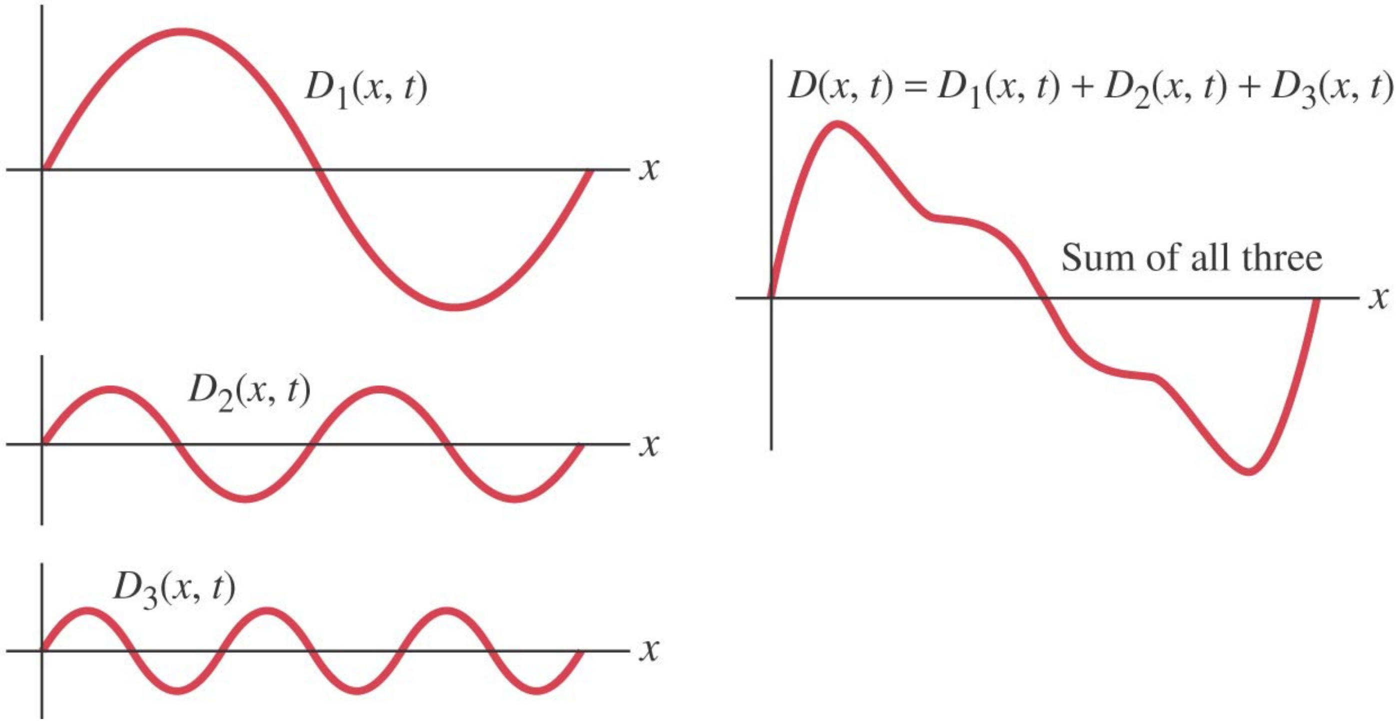
- Describe the principle of **superposition** and wave **interference** phenomena.
- Explain the phenomena of wave **reflection**, **refraction**, **diffraction**, and **transmission**.
- Describe **standing waves** and calculate their main characteristics.

15.6 – The principle of superposition

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$$

If $D_1(x,t)$ and $D_2(x,t)$ are solutions to the wave equation, then $aD_1(x,t) + bD_2(x,t)$ is also a solution. This is called the **superposition principle**.

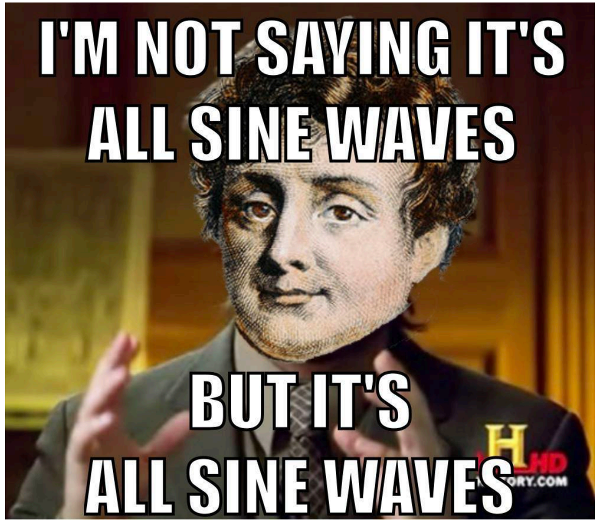
Another way to see this is that the displacement $D(x,t)$ at any point x is the sum of the displacements of all waves passing throught that point at that instant t.



15.6 – Fourier's theorem

Any* periodic wave can be expressed as the sum of sinusoidal waves of different amplitudes, frequencies, and phases.

$$\boxed{f(t) = \sum_{i=0}^{\infty} A_i \sin(\omega_i t + \phi_i)}$$

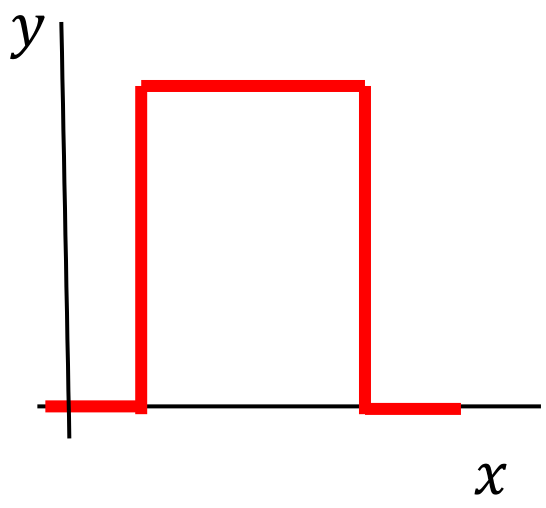


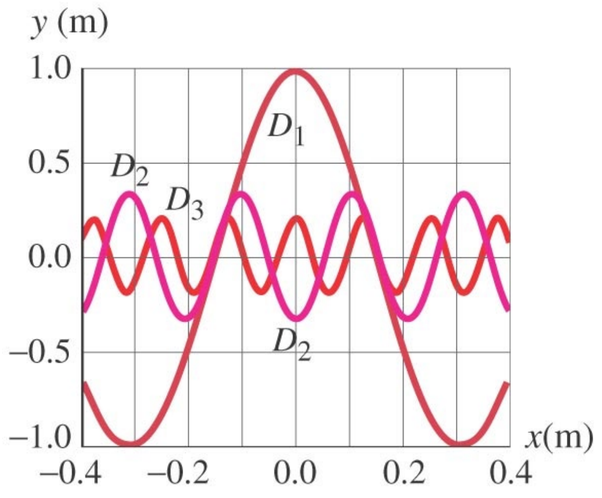
Joseph Fourier
(1768-1830)

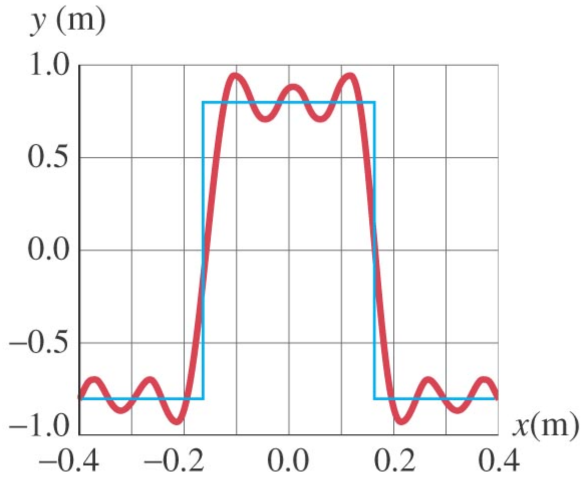
* Some conditions do apply (out of scope)

15.6 – Fourier's theorem example

How to make a square wave





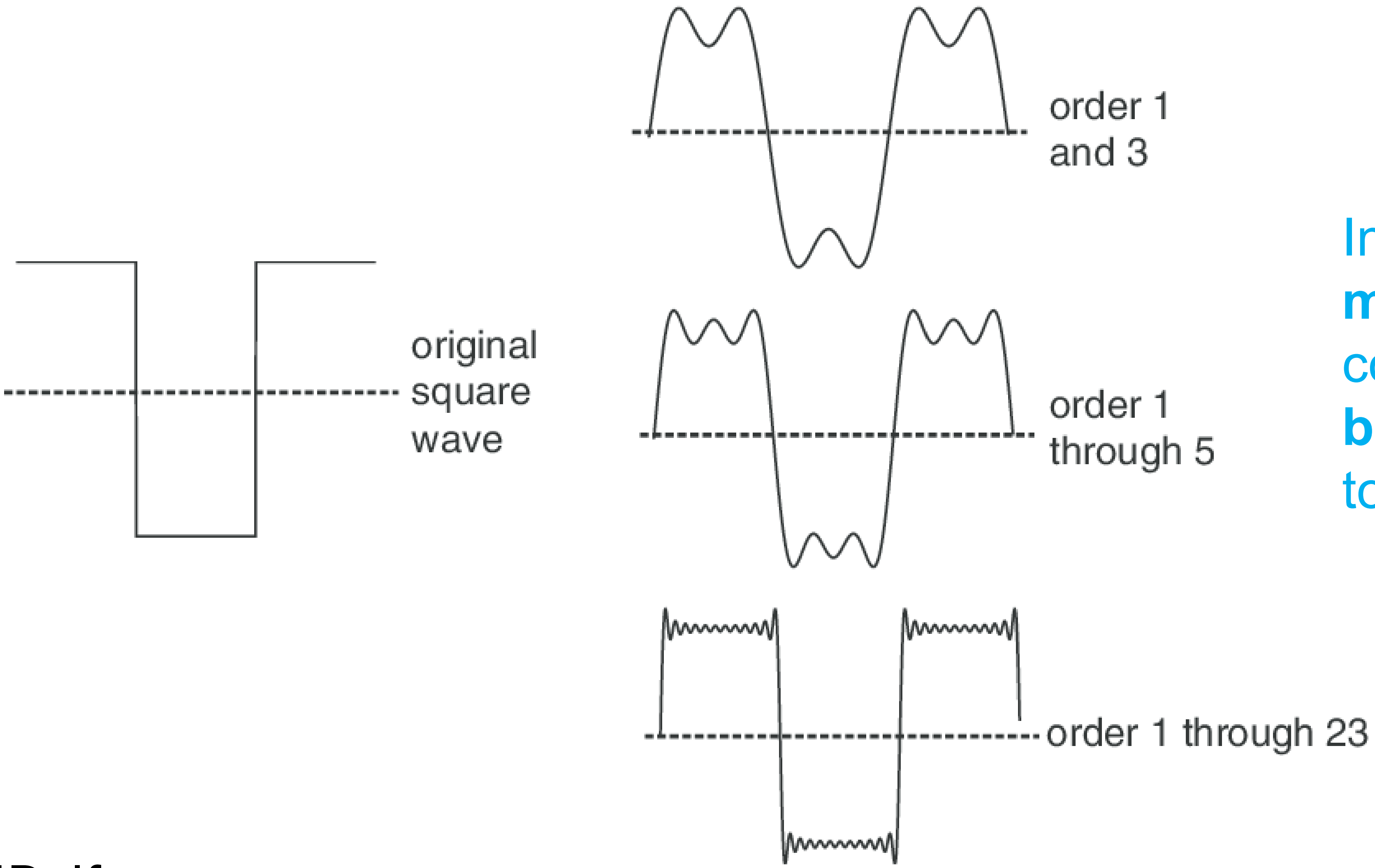


$$D_1 = A\cos kx$$

$$D_3 = \frac{-A}{3}\cos 3kx$$

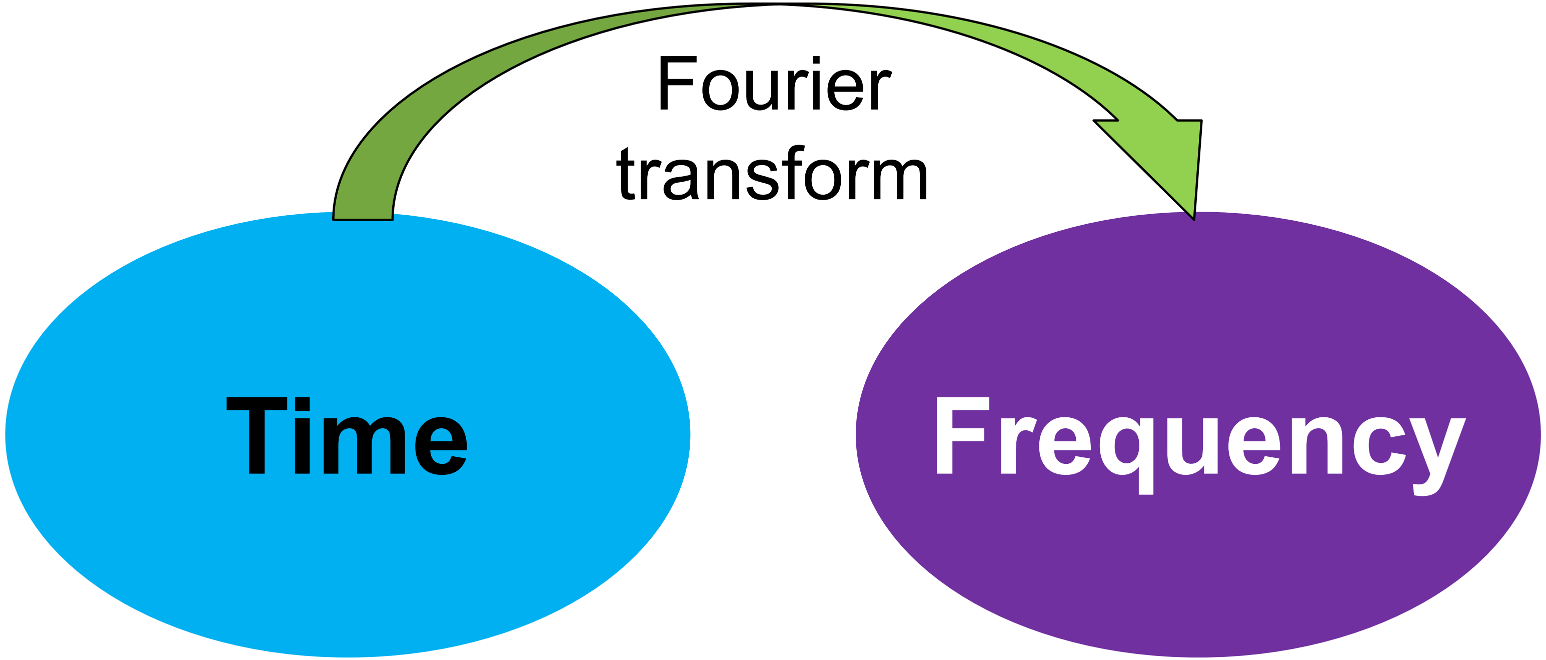
$$D_5 = \frac{A}{5}\cos 5kx$$

15.6 – Fourier's theorem example



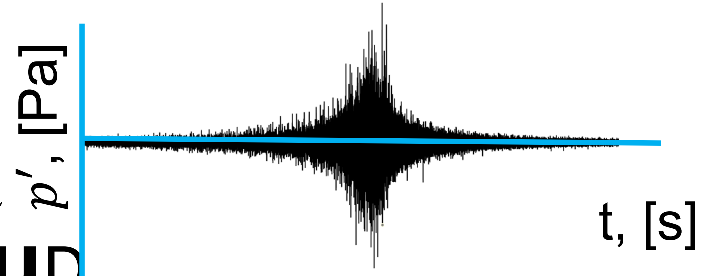
In general, **the more terms** we consider, **the better the match** to the original wave

15.6 – Fourier transform

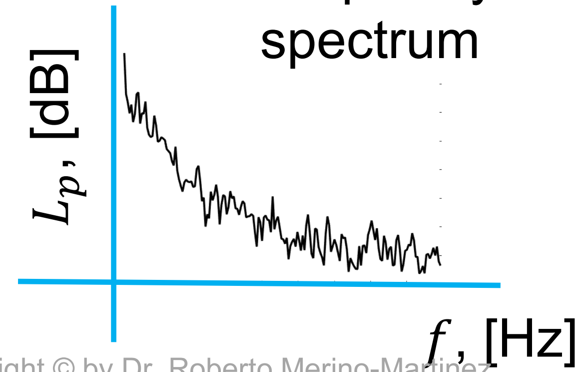


This concept is the base for the **Fourier transform**, a very important mathematical tool which allows us to analyze which frequencies are present in complex waves.

Measured signal (sound wave)



Frequency spectrum



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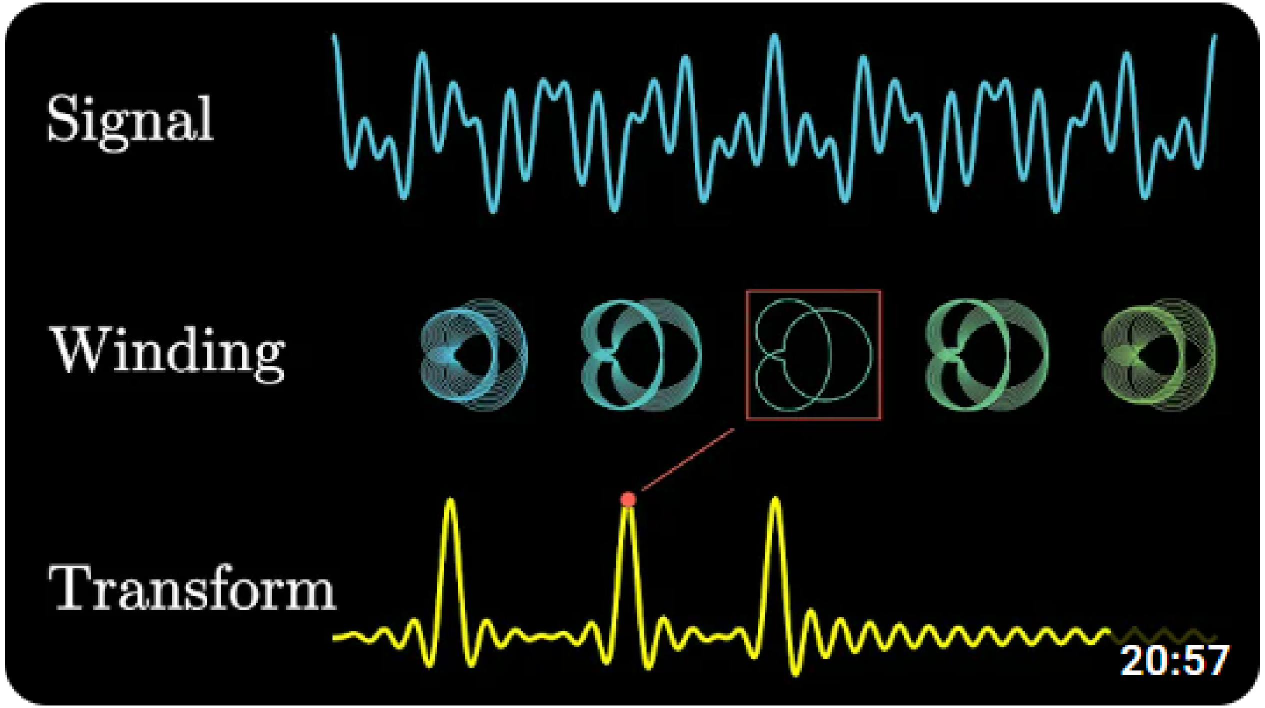
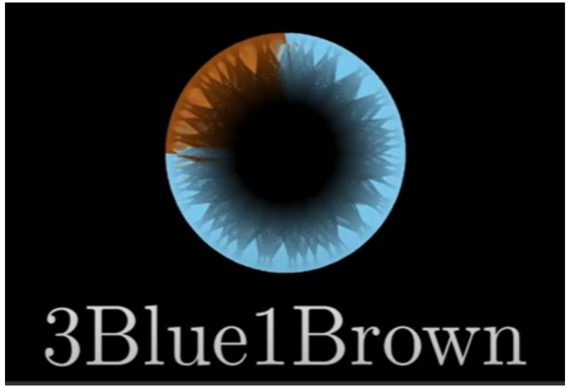
15.6 – Fourier transform

In this animation the complex wave f is decomposed into a sum of sine and cosine waves of different amplitudes, frequencies, and phases. Analyzing these gives us the **frequency spectrum** $\hat{f}$ of this wave.

© Wikimedia commons

15.6 – Fourier transform





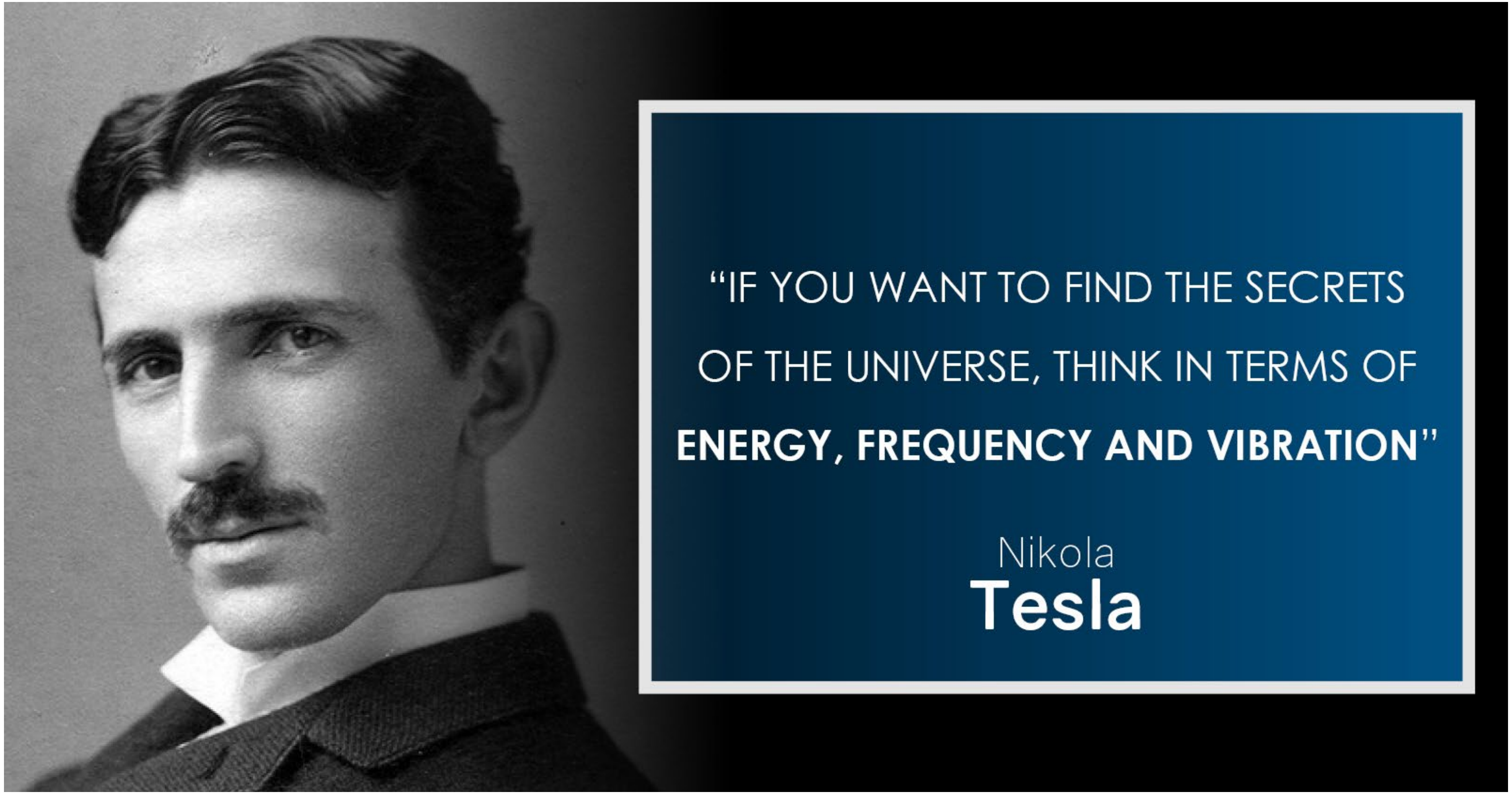
Another very illustrative YouTube video is in the channel 3Blue1Brown is called:

"***But what is the Fourier Transform? A visual introduction***."

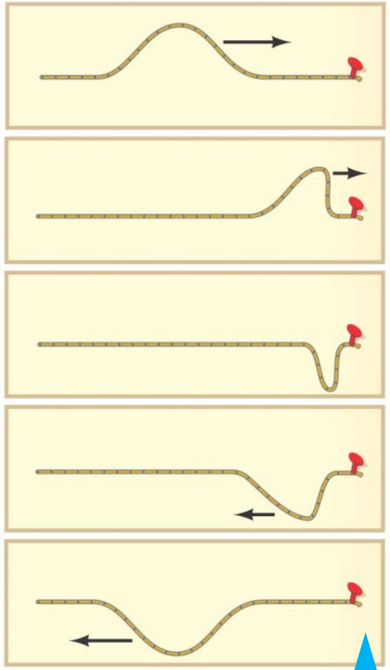
Very recommended if you want to learn more about this topic!

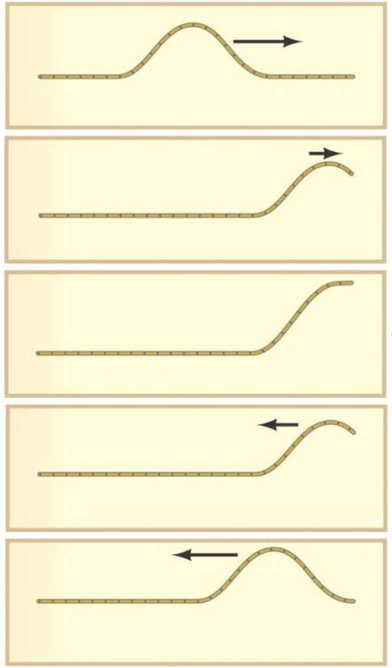
Link to the video

15.6 – Energy, Frequency, and Vibration

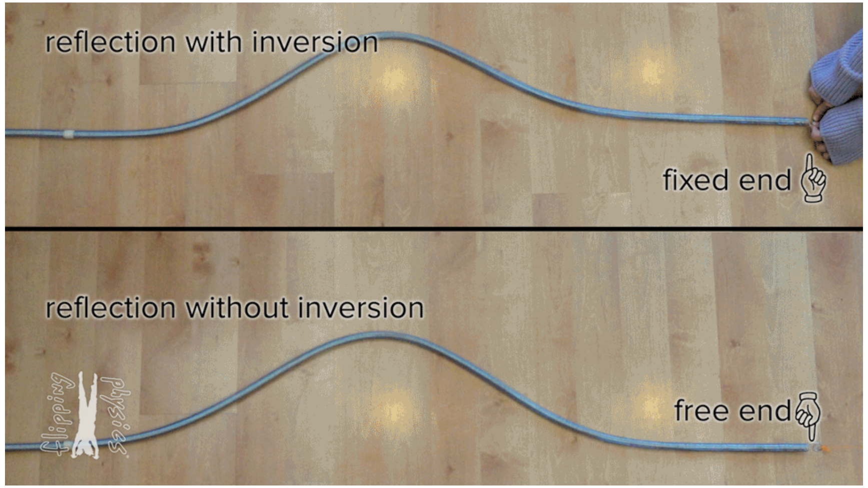


15.7 – Reflection and transmission





A wave reaching the end of its medium, but where the medium is still free to move, will be reflected, and its reflection will be **upright**.



A wave hitting an **obstacle** will be reflected, and its reflection will be **inverted**.

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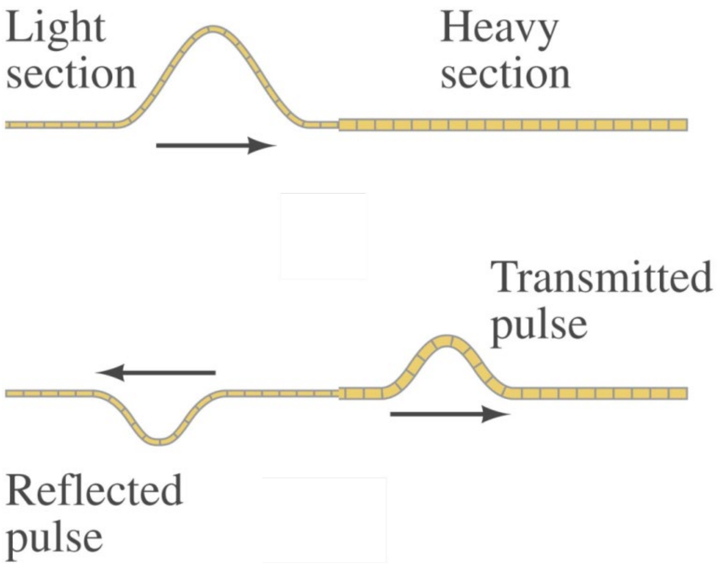
15.7 – Reflection and transmission

A wave encountering a **denser medium** will be partly **reflected** and partly **transmitted**.

As the wave speed is less in the denser medium, the wavelength will be shorter **(frequency does not change!).**



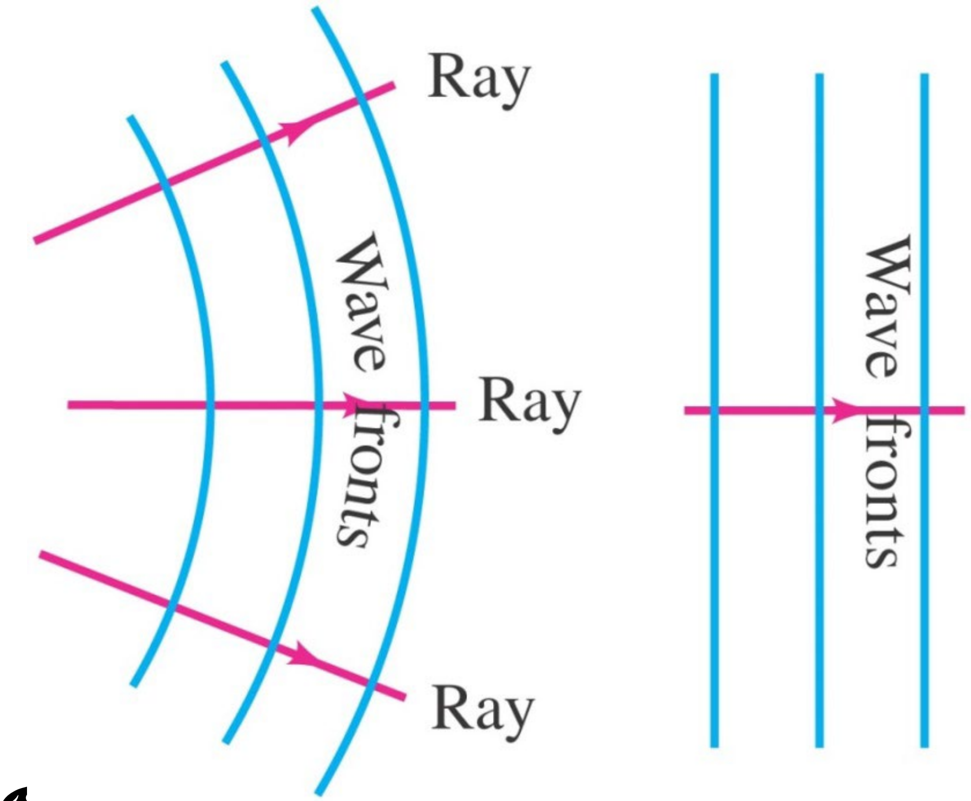
$$v = \sqrt{\frac{F_T}{\mu}} \qquad v = \frac{\lambda}{T} = \lambda f$$

$$\mu \uparrow \rightarrow \ v \downarrow \ \rightarrow \ \lambda \downarrow$$

$$\boldsymbol{f = \text{constant}}$$

15.7 – Reflection and transmission

Two- or three-dimensional waves can be represented by **wave fronts**, which are curves of surfaces where **all the waves have the same phase**.



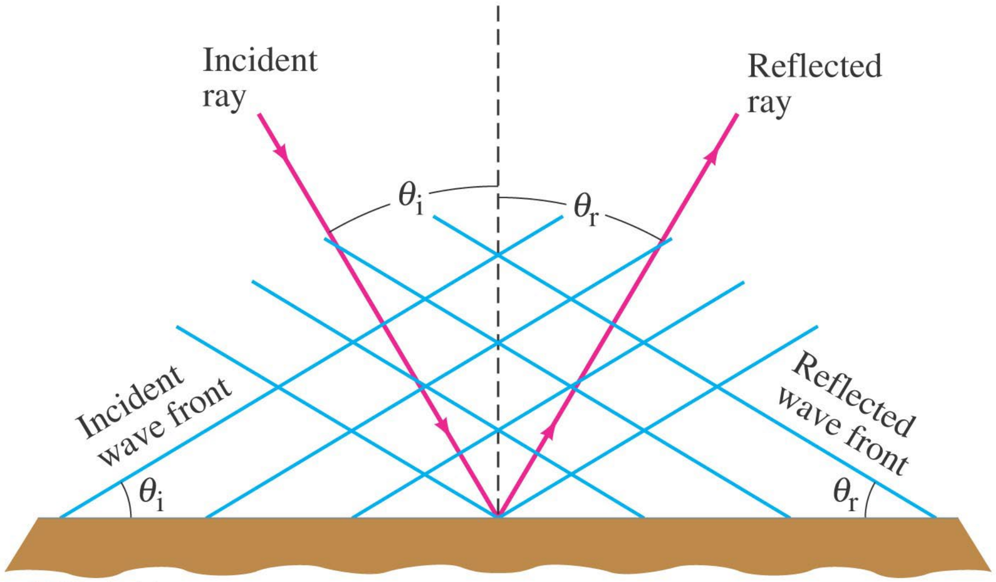
Lines **perpendicular** to the wave fronts are called **rays**; they point in the **direction of propagation** of the wave.

Wave fronts **(very) far from the source** have lost their curvature: **plane waves** (e.g. ocean waves, sun rays).

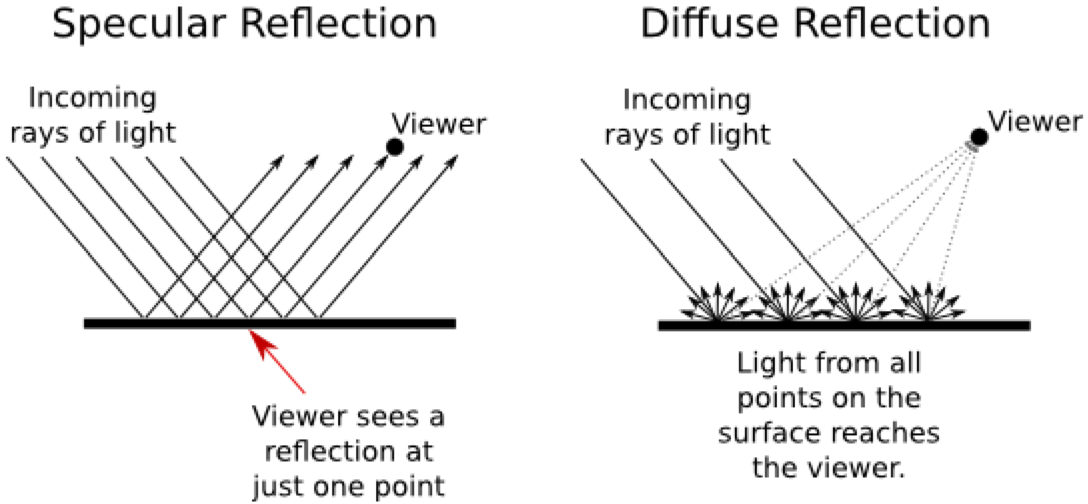
15.7 – Reflection and transmission

For **specular reflection**, (e.g. plane mirror) the angle of incidence θ_i equals the angle of reflection θ_r:

$$\theta_i = \theta_r$$



For **diffuse reflection**, the wave is reflected in many angles



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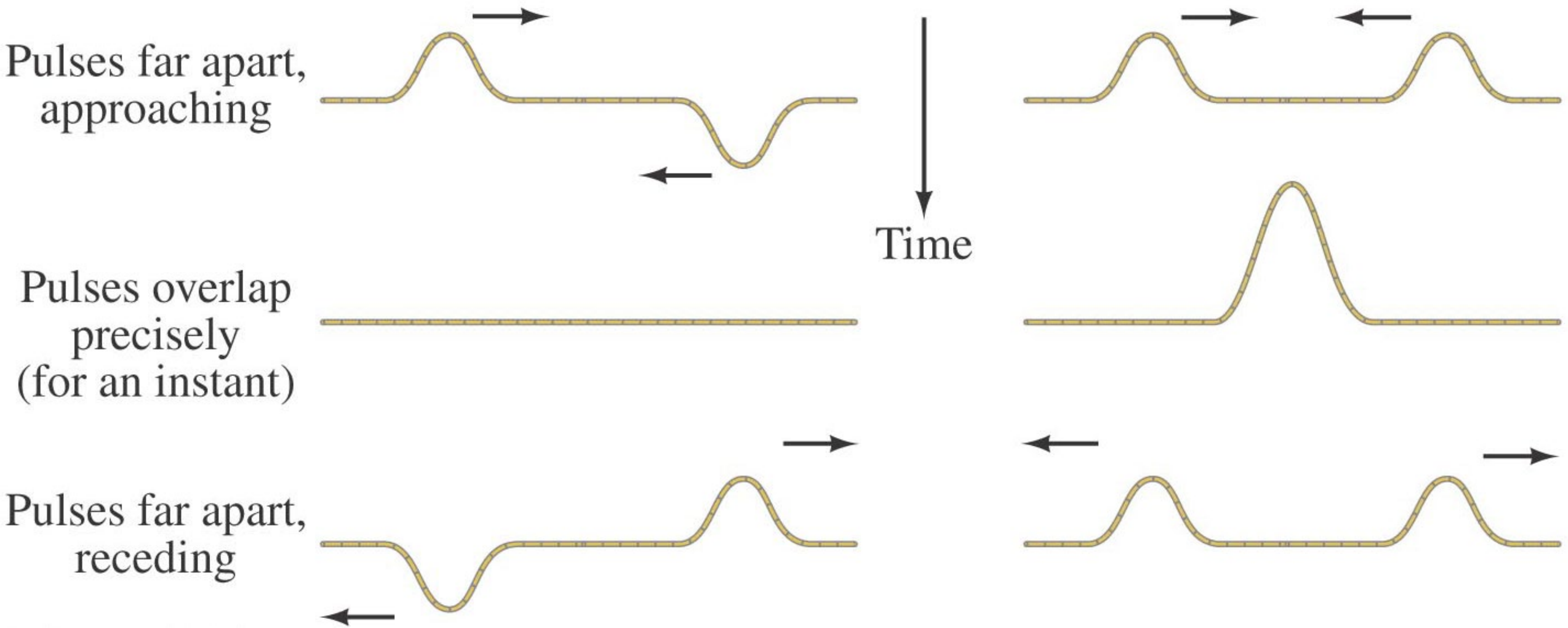
15.8 – Interference

Following the superposition principle, when two waves pass through the same point, the displacement is the **sum of the individual displacements**.

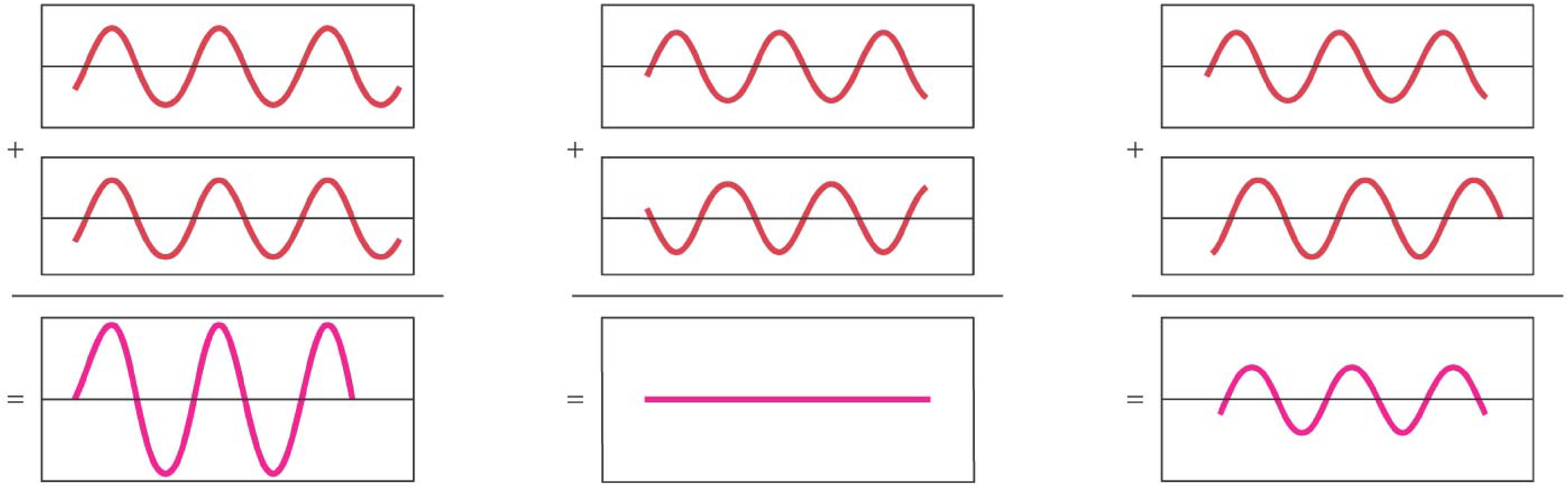


Destructive interference

Constructive interference

15.8 – Interference

This concept is closely related to the phase of the wave, so we can talk about waves "in-phase, $\Delta\phi = 0$" or "out of phase, $\Delta\phi = \pi$".

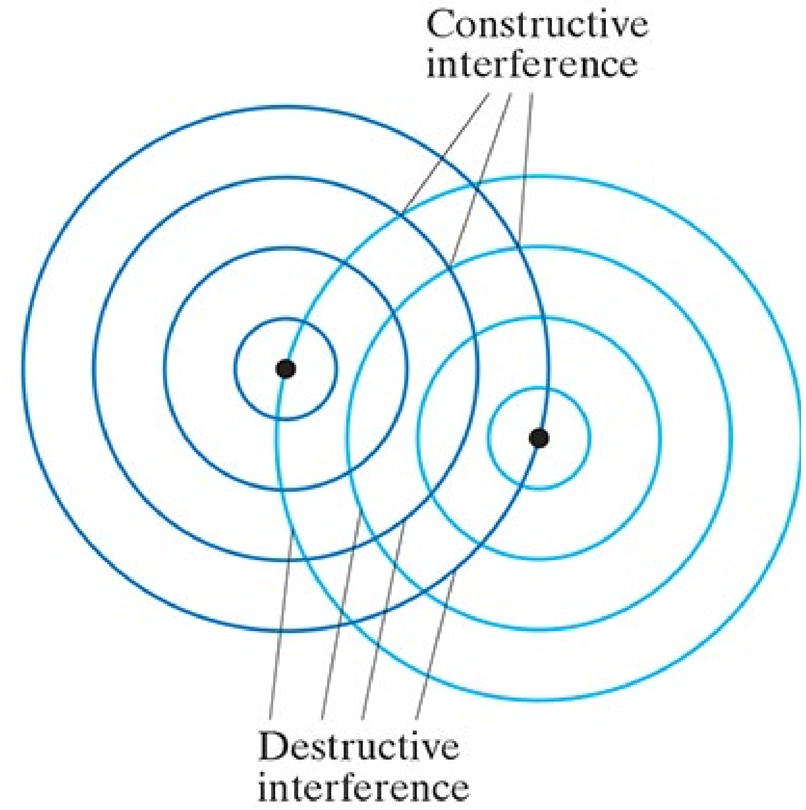


(a) Waves in phase

(b) Waves out of phase

(c) Partially out of phase

15.8 – Interference



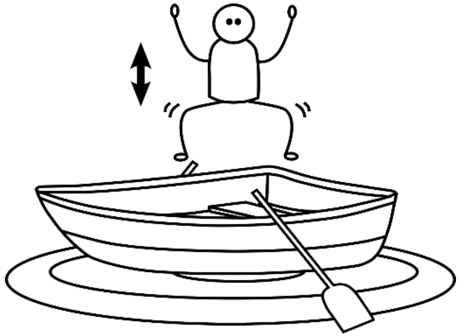
Wave front interaction

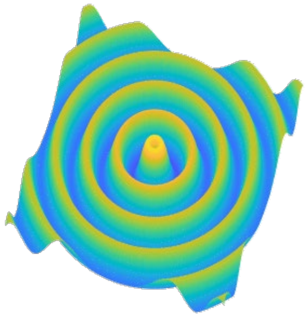
15.8 – Interference

Link to the video

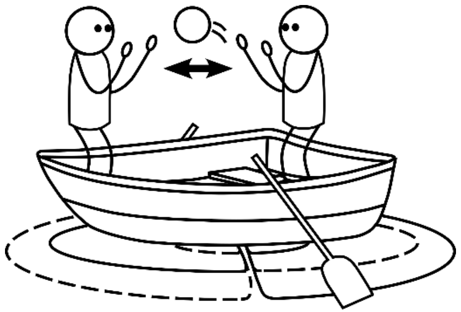
15.8 – Interference – Practical example

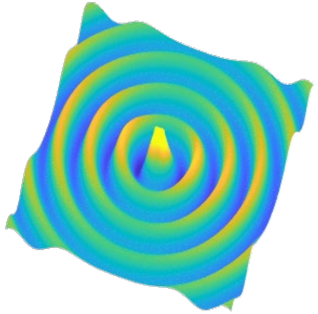
Monopole



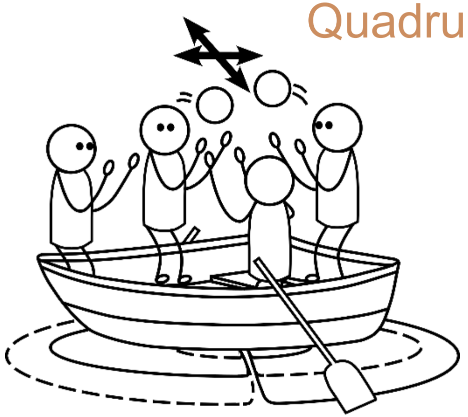


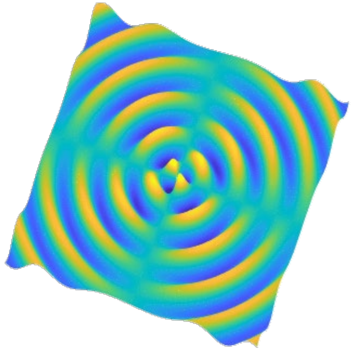
Dipole





Quadrupole





Animations kindly borrowed from:

Dr. Lourenco Tercio Lima Pereira

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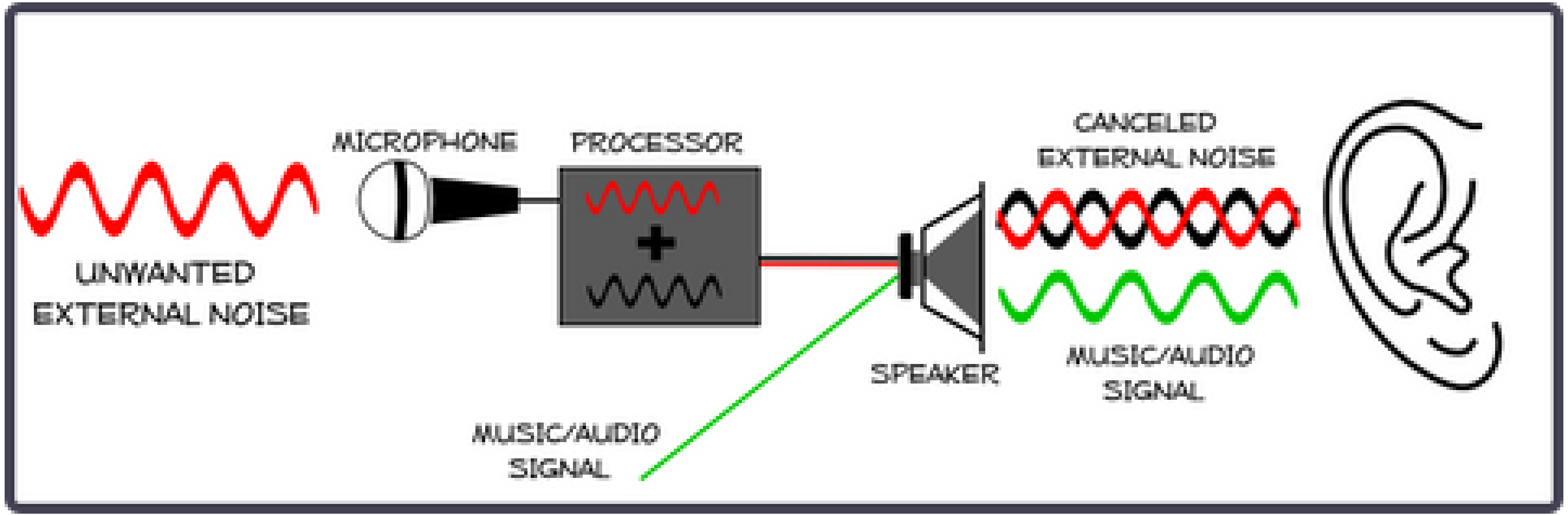
15.8 – Interference – Noise cancelling

Main concept behind active noise-cancelling headphones.



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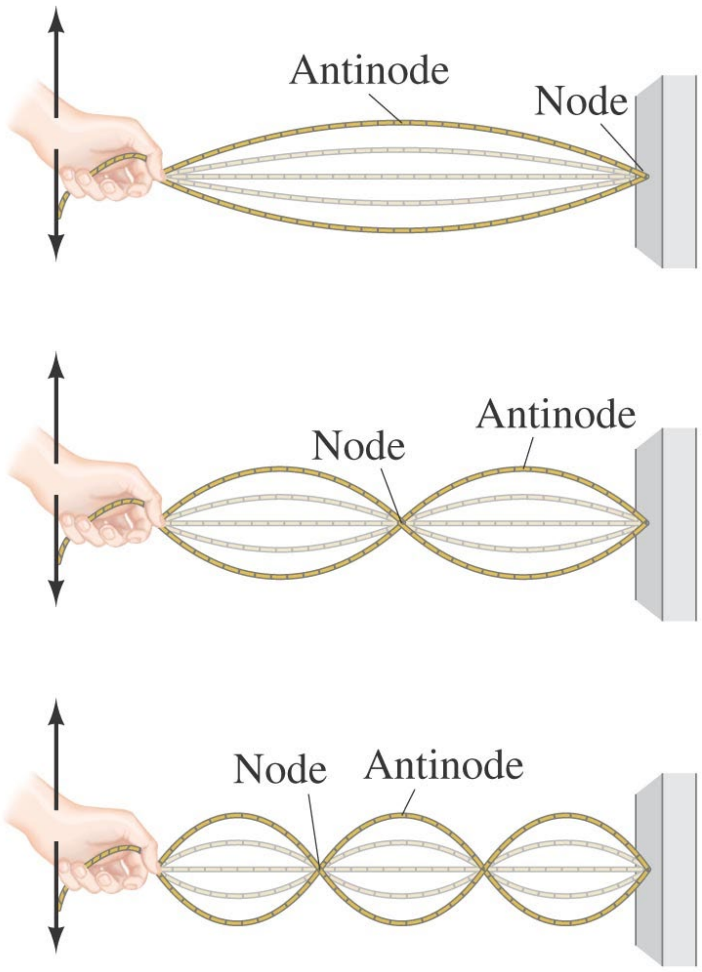
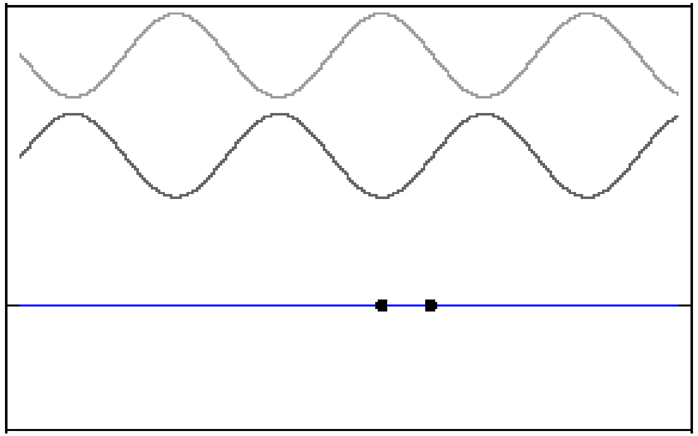
15.9 – Standing waves and resonance

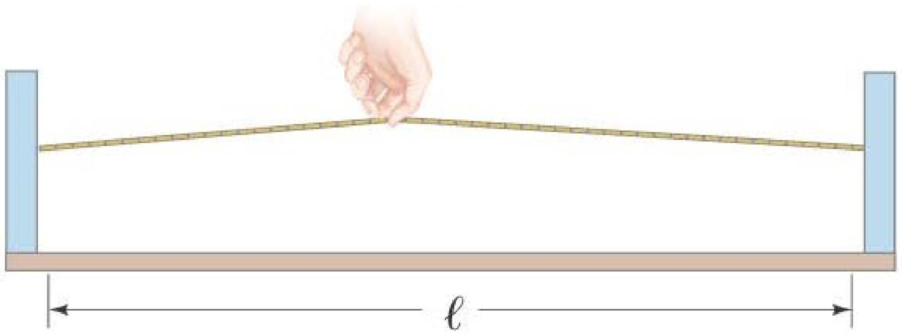


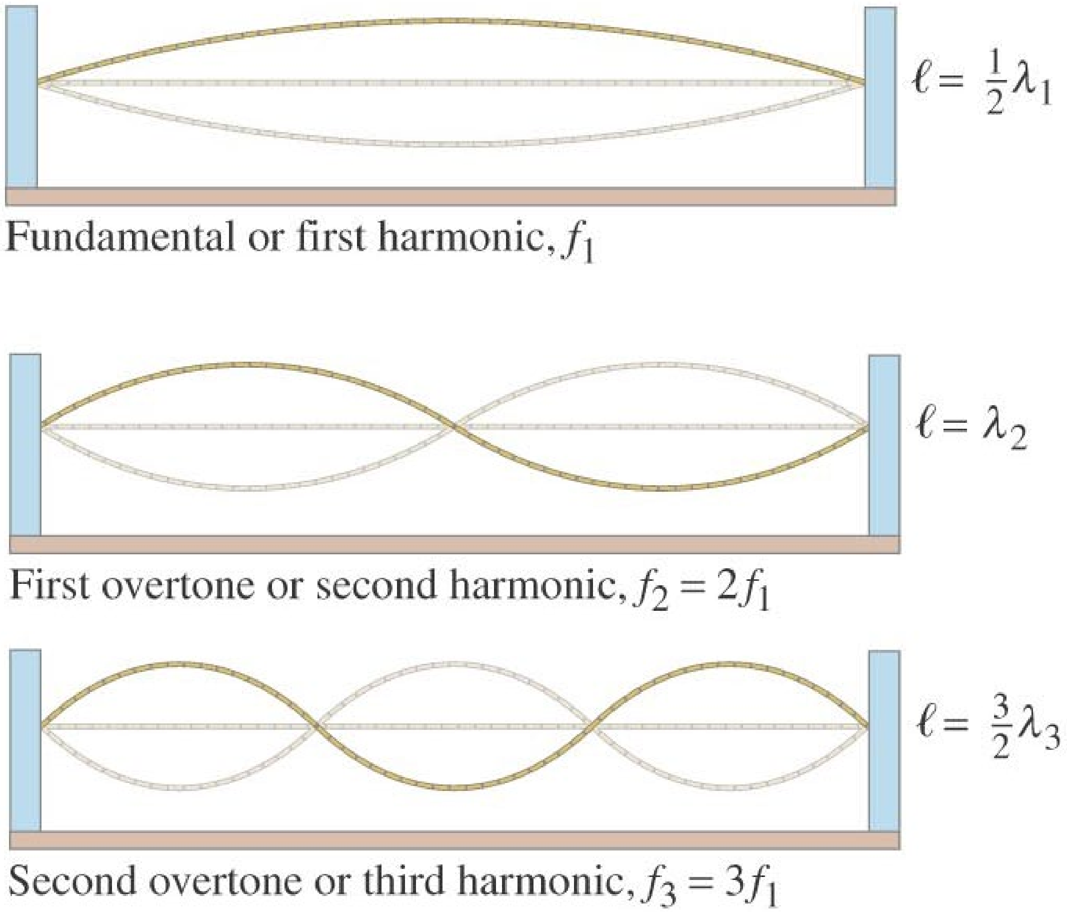
Standing waves occur when both ends of a string are fixed. In that case, only waves which are **motionless at the ends of the string can persist**.

There are **nodes**, where the amplitude is always zero, and **antinodes**, where the amplitude varies from zero to the maximum value.



15.9 – Standing waves and resonance





The frequencies of the standing waves on a particular string are called **resonant frequencies** (they persist longer).

They are also referred to as the **fundamental** and **higher harmonics**.

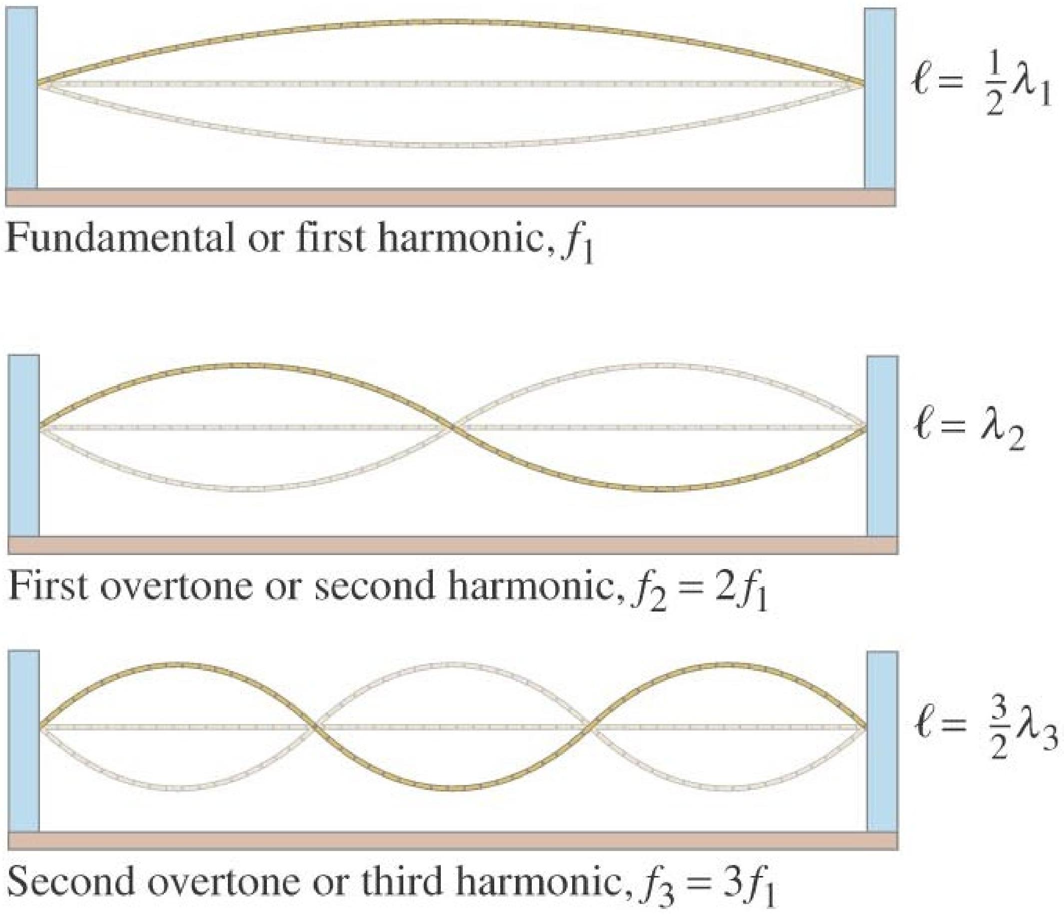
15.9 – Standing waves and resonance

The wavelength of the n^{th} harmonic for a string of length l is:

$$\lambda_n = \frac{2l}{n} \qquad n = 1, 2, 3 \ldots$$

The frequencies associated to these wavelengths are:

$$f_n = \frac{v}{\lambda_n} = \frac{vn}{2l} = nf_1$$



15.9 – Standing waves and resonance

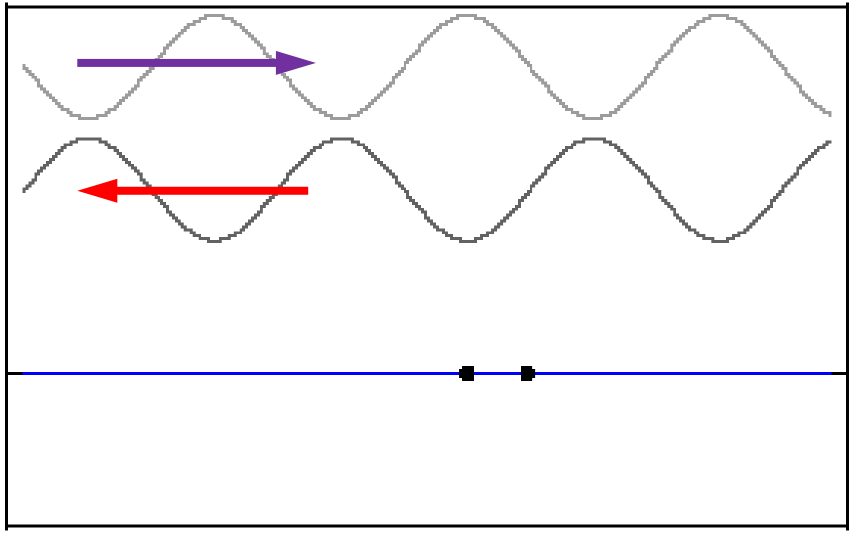
$$D_1(x,t) = A\sin(kx - \omega t)$$

$$D_2(x,t) = A\sin(kx + \omega t)$$

$D_1(x,t)$

$D_2(x,t)$

$D_1 + D_2$

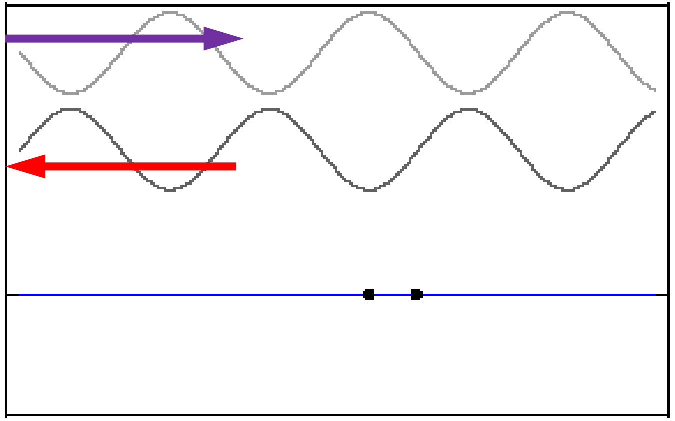


Using the superposition principle:

$$D_1 + D_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

15.9 – Standing waves and resonance

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$



$$D_1 + D_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

$$D_1 + D_2 = A[\sin kx \cos \omega t - \cancel{\cos kx \sin \omega t}] + A[\sin kx \cos \omega t + \cancel{\cos kx \sin \omega t}]$$

$$\boxed{D_1 + D_2 = 2A \sin kx \cos \omega t}$$

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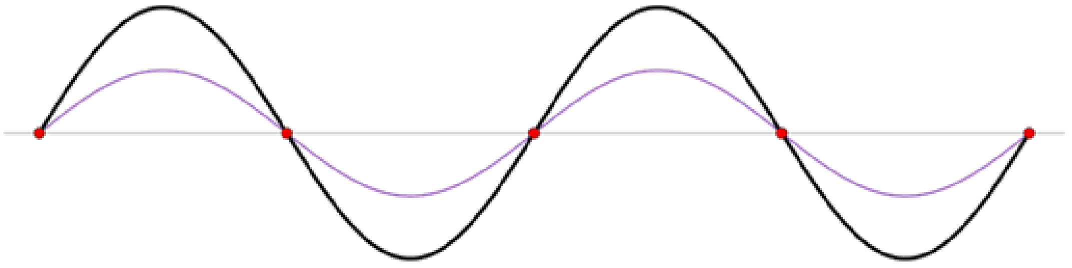
15.9 – Standing waves and resonance

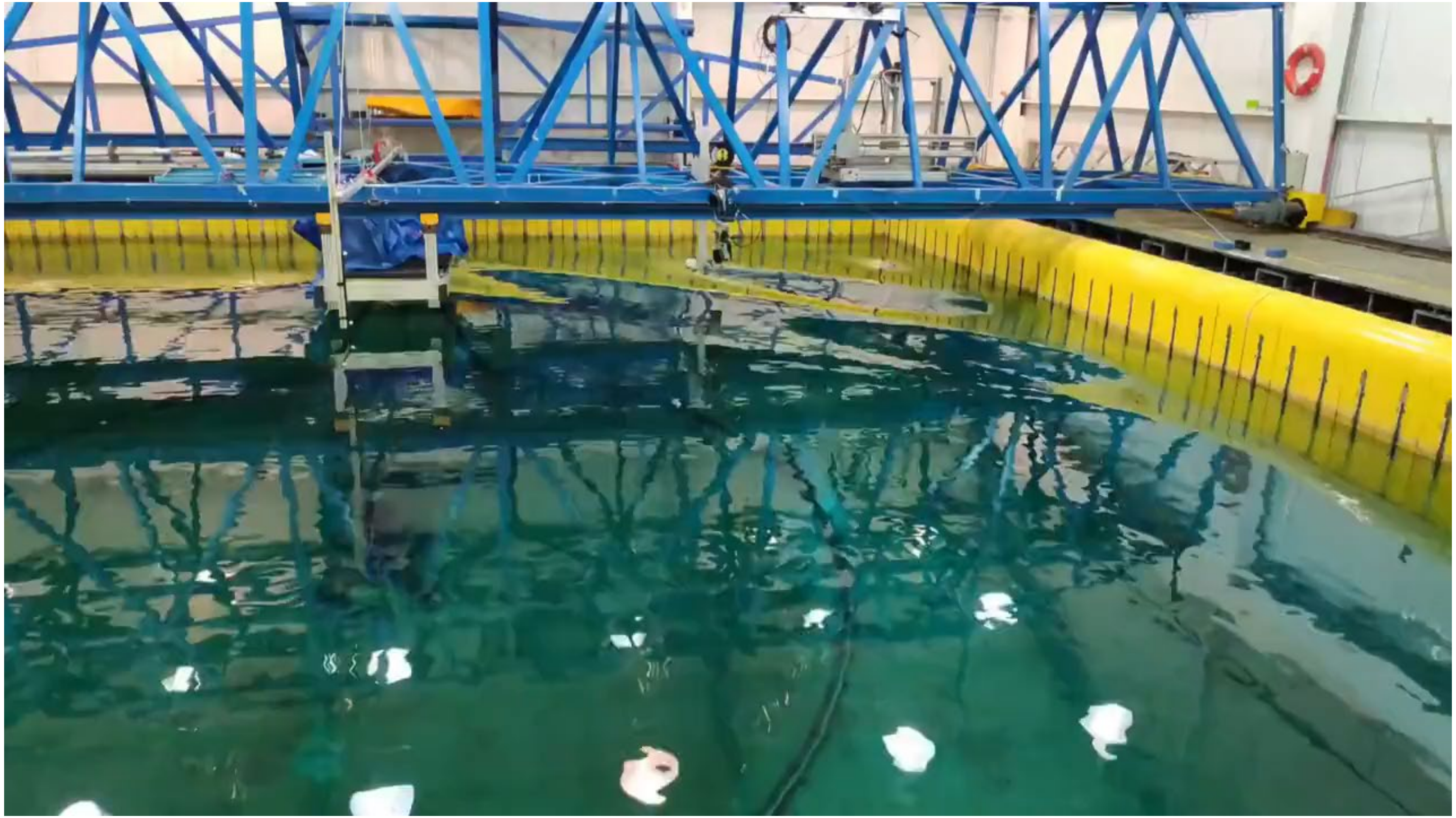
$$\boxed{D_1 + D_2 = 2A \underbrace{\sin kx}_{\text{Waveform in space “frozen in time”}} \underbrace{\cos \omega t}_{\text{Time-varying amplitude}}}$$



MORE ON THIS NEXT CHAPTER ☺

A particle at any position x vibrates in **simple harmonic motion** with the **same frequency**, but the **amplitude depends on x**.

15.9 – Standing waves and resonance (2D case)



Source: Twitter @Jousefm2

15.10 – Refraction

If the wave enters a medium where the **wave speed is different**, it will be **refracted**—its wave fronts and rays will **change direction**.

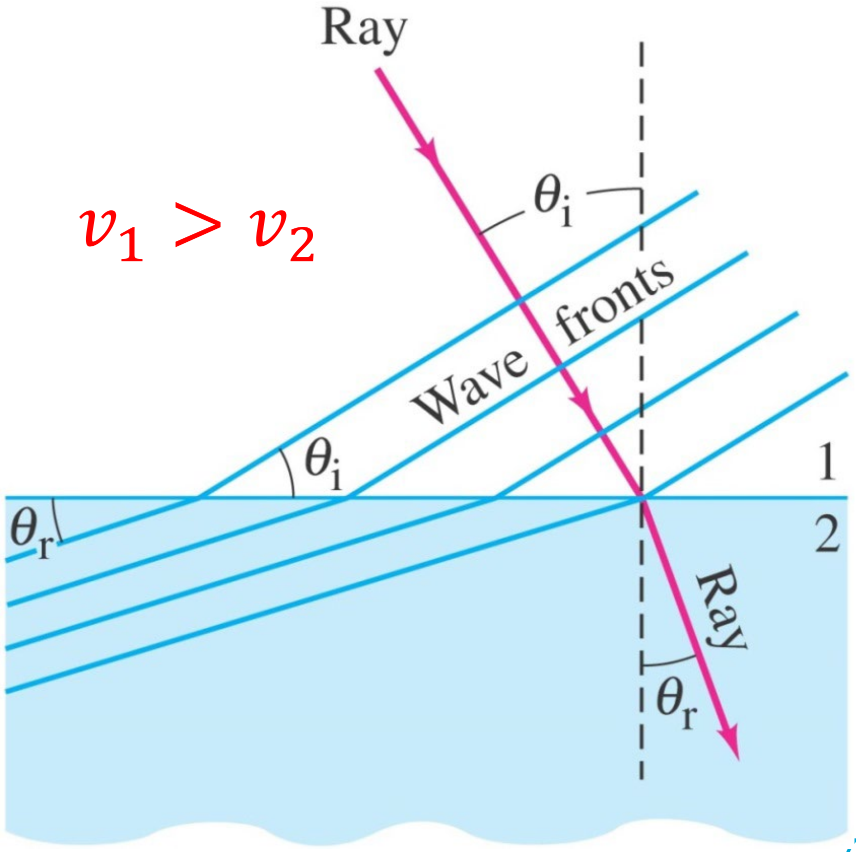
We can calculate the **angle of refraction**, using the famous law of **refraction** or **Snell's law:**

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

$$v = \lambda f$$

$$v \downarrow \rightarrow \lambda \downarrow$$

$v_1 > v_2$

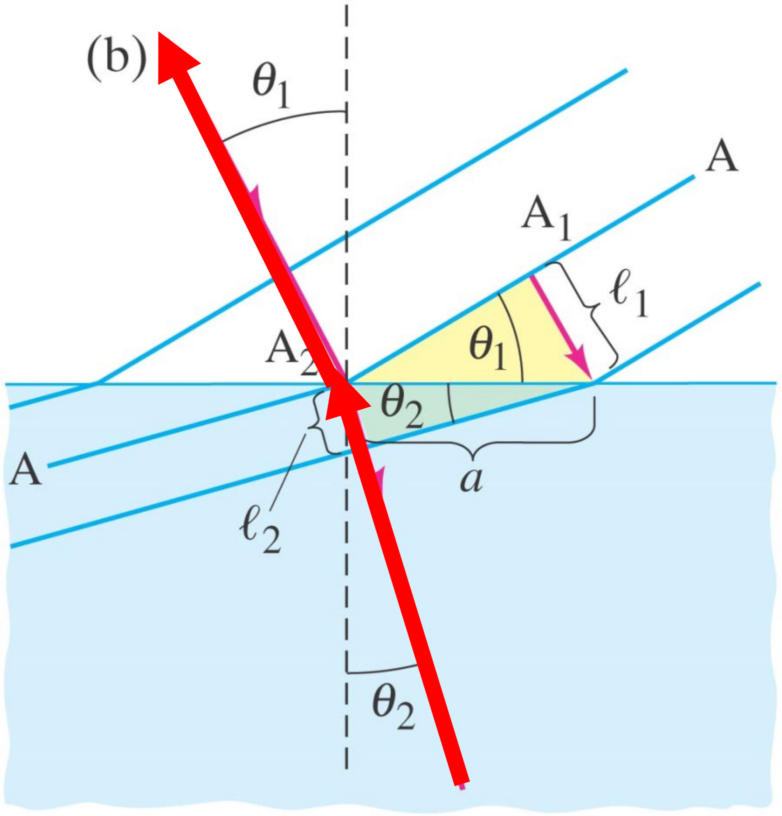


Since the **frequency remains constant**, the wavelength λ **needs to adjust accordingly**.

15.10 – Refraction

The law of refraction works both ways - a wave going from a slower medium to a faster one would follow the red line in the **other direction**.

$v_1 > v_2$



$$\sin\theta_1 = \frac{l_1}{a} = \frac{v_1 t}{a}$$

$$\sin\theta_2 = \frac{l_2}{a} = \frac{v_2 t}{a}$$

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

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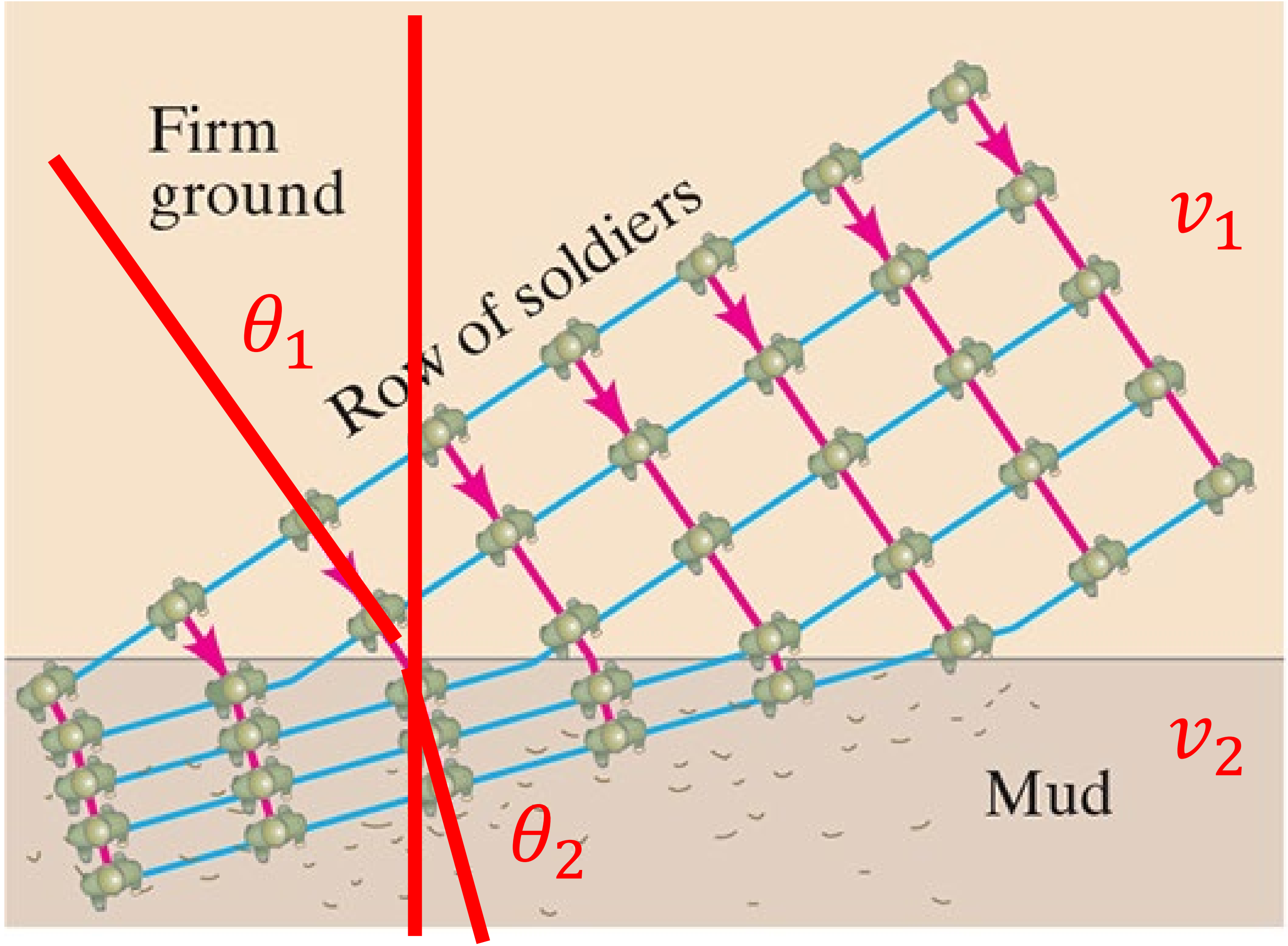
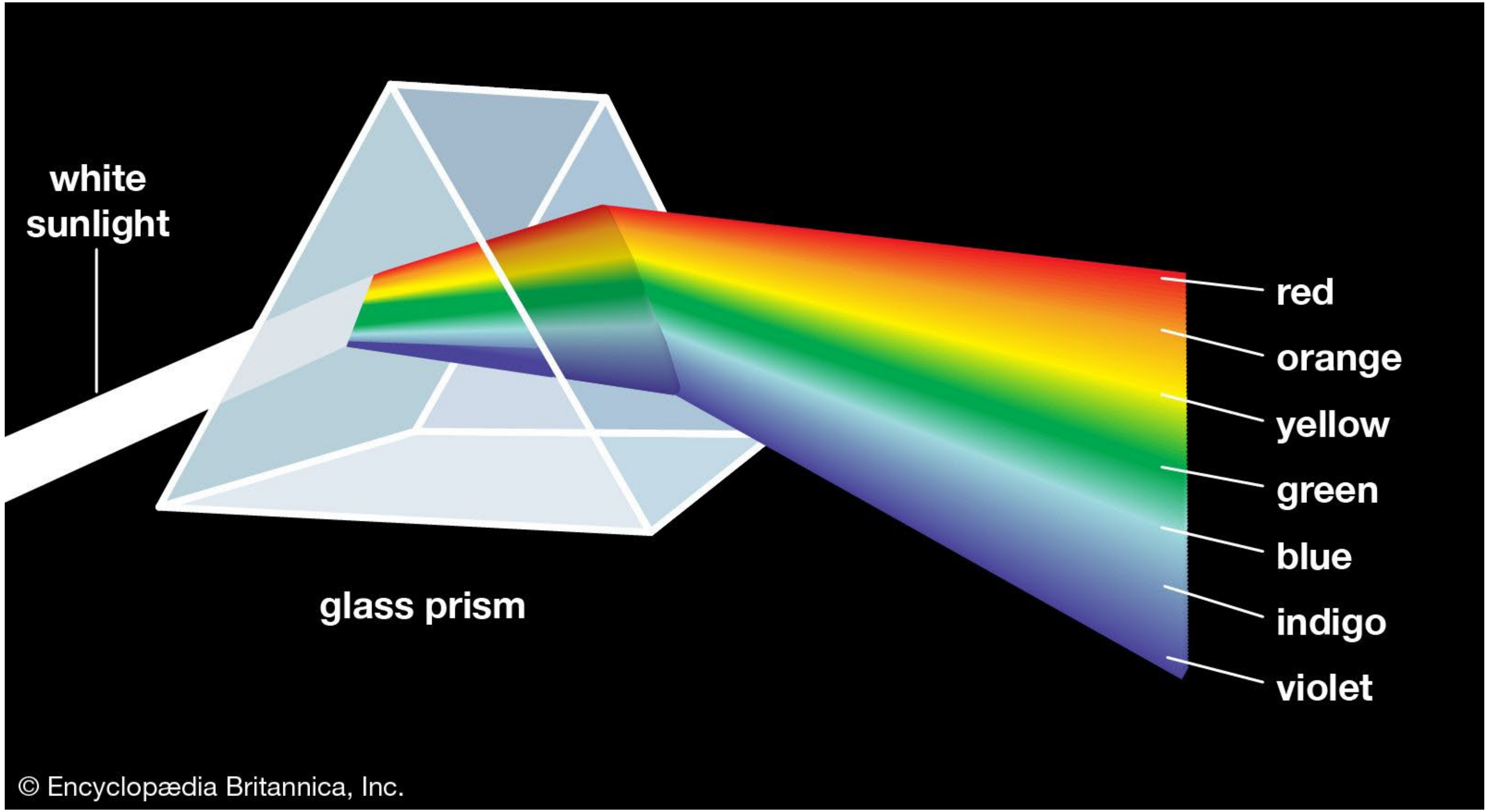
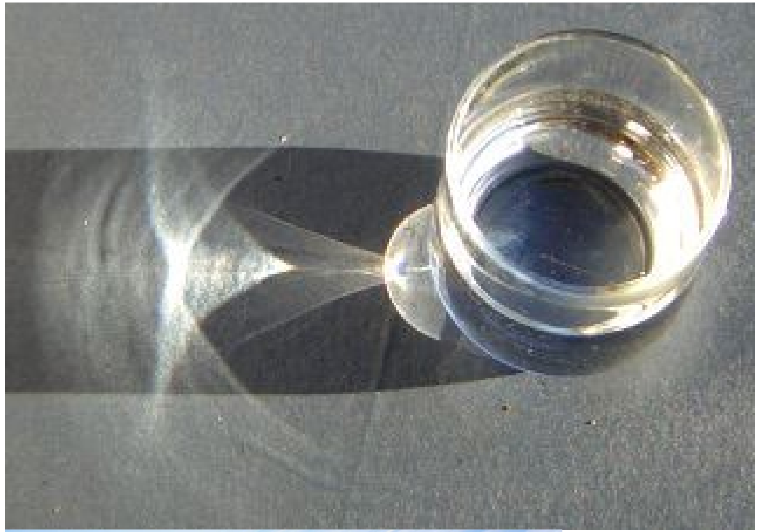
15.10 – Refraction analogy



$v_1 > v_2$

15.10 – Refraction example (optics)





15.10 – Refraction example

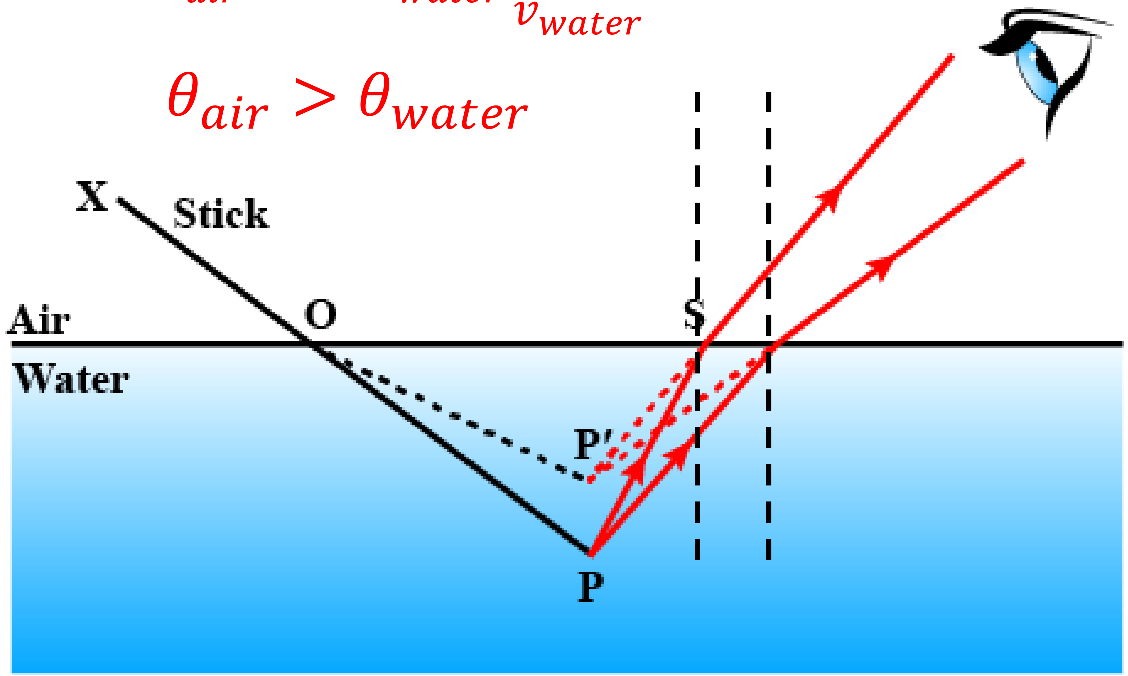
For light:

$$v_{air} = 299{,}702 \text{ km/s}$$

$$v_{water} = 225{,}000 \text{ km/s}$$

$$\sin\theta_{air} = \sin\theta_{water}\frac{v_{air}}{v_{water}}$$

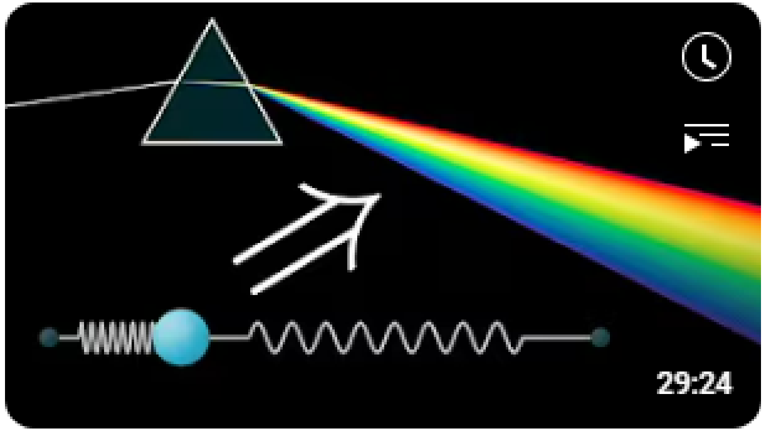
$$\theta_{air} > \theta_{water}$$



15.10 – Refraction – More information

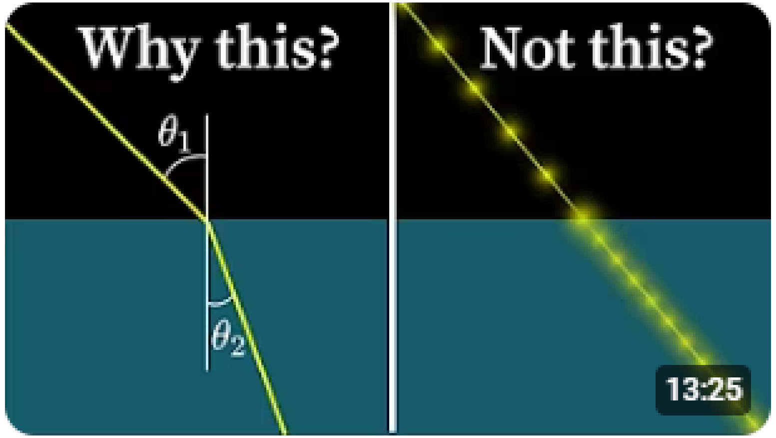
Why no two people see the same rainbow

Link to the video



But why would light "slow down"? | Visualizing Feynman's lecture on the...

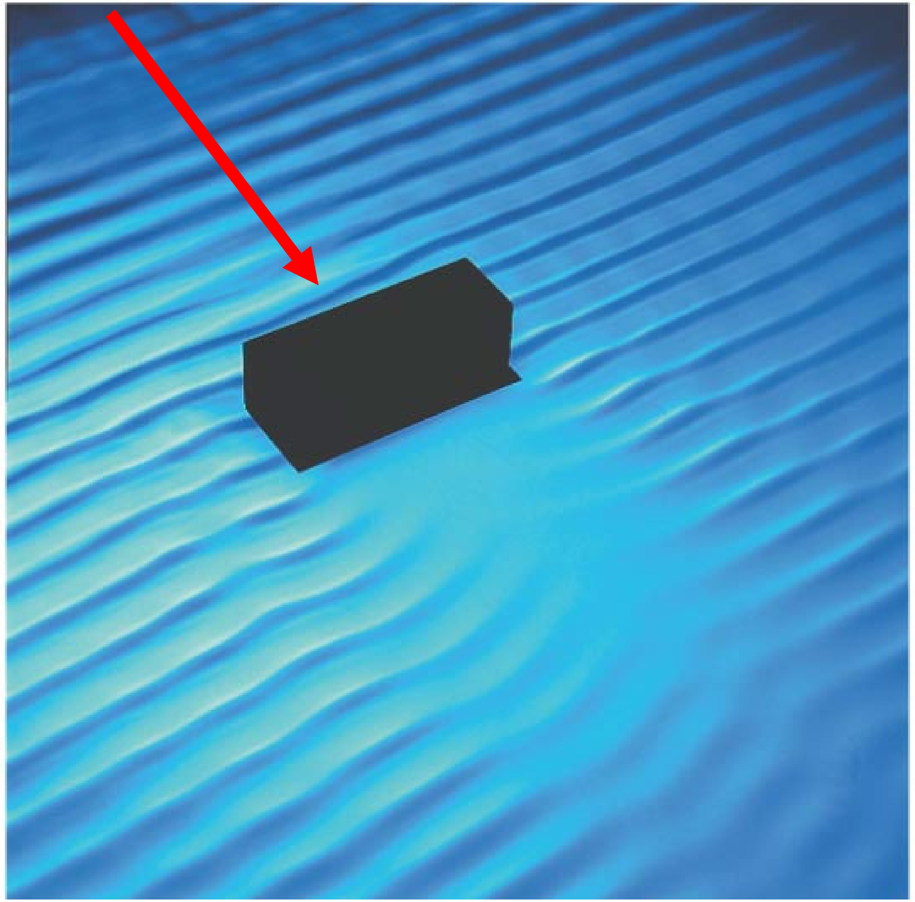
Link to the video



Answering viewer questions about refraction

Link to the video

15.11 – Diffraction



When waves encounter an **obstacle**, they “bend” around it, creating a sort of **shadow region**.

This phenomenon is called **diffraction**.

In essence:

More diffraction = more waves behind the obstacle

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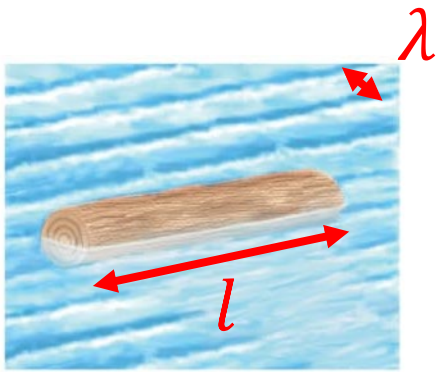
15.11 – Diffraction

Like most wave phenomena, diffraction depends on the ratio of the **wavelength λ** and the **size** of the **obstacle l**.

$$\frac{\lambda}{l}$$

Only if the wavelength is **smaller than the size of the object** will there be a significant shadow region, i.e. there is **hardly diffraction**.

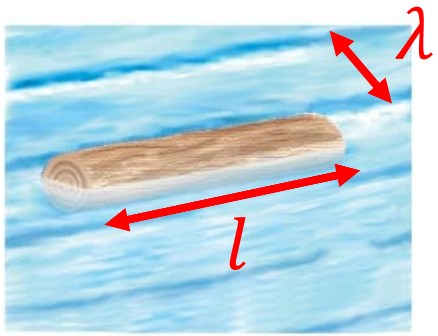
$$\frac{\lambda}{l} \ll 1$$



Short-wavelength waves passing log

Clear shadow region

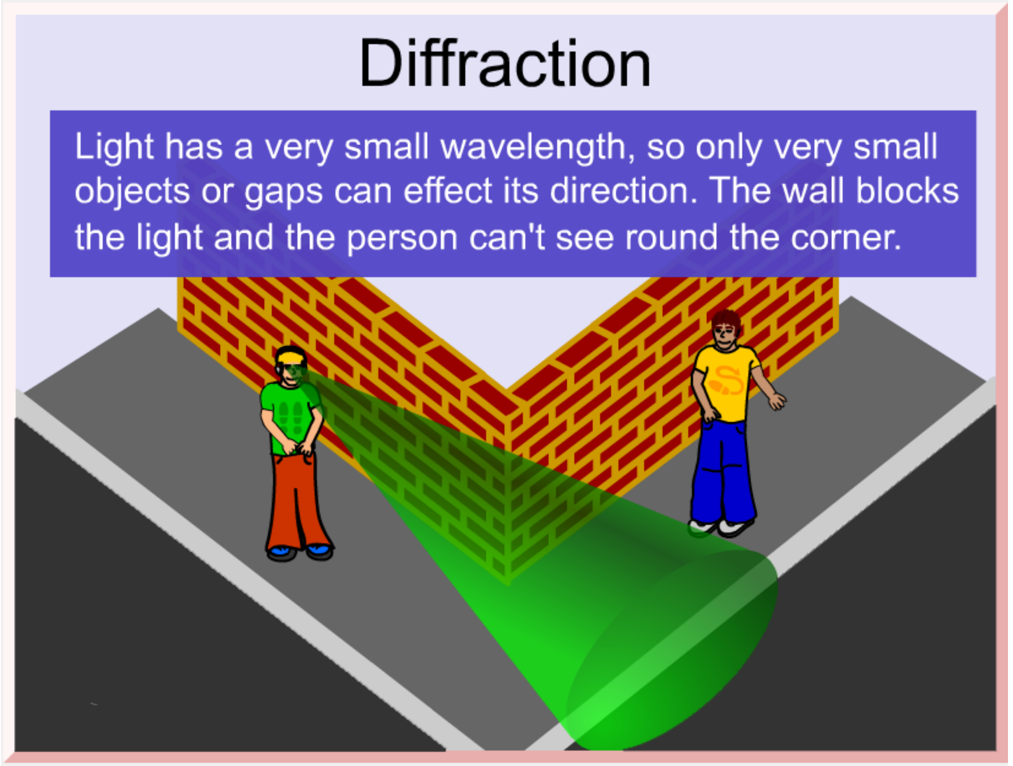
$$\frac{\lambda}{l} \approx 1$$

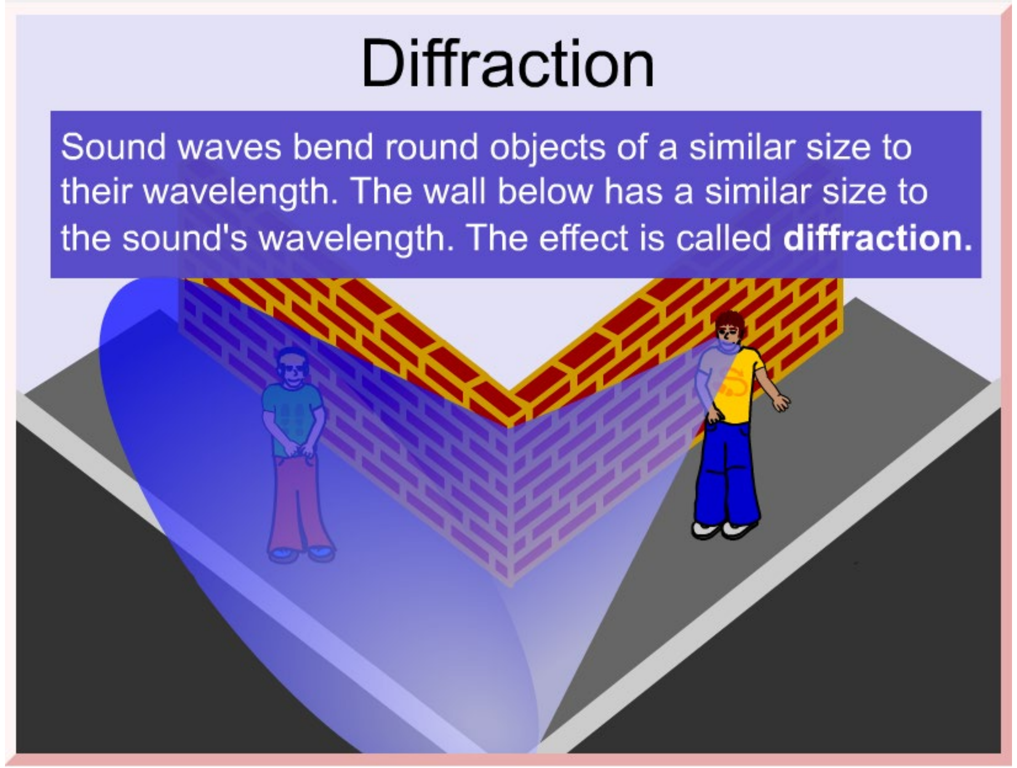


Long-wavelength waves passing log

More diffraction in shadow region

15.11 – Diffraction





$$\frac{\lambda}{l} \ll 1$$

$$\frac{\lambda}{l} \approx 1$$

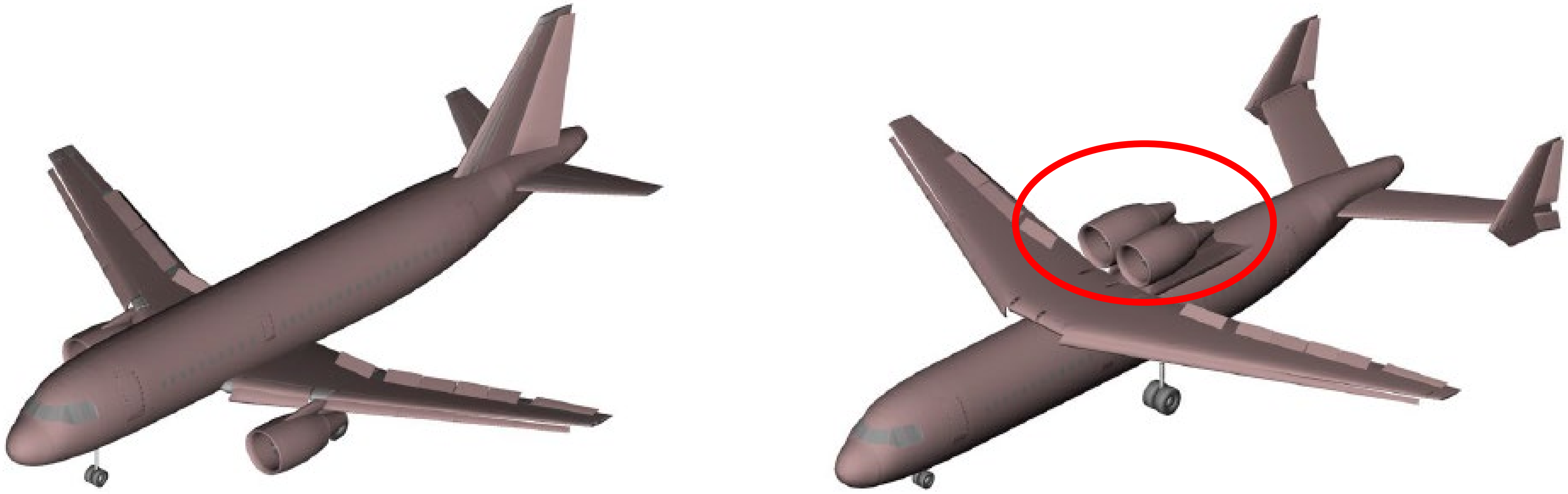
15.11 – Diffraction example – noise barriers

© https://www.yente.com/

Noise barriers along highways are **more useful for $\lambda < l$**, where l is the barrier dimension.

Hence, they <u>do not work well for very low frequencies</u>

15.11 – Diffraction example – engine noise shielding



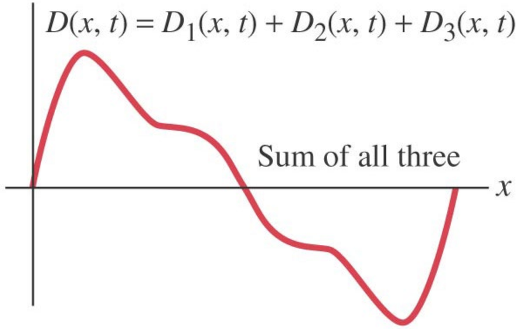
Pictures kindly borrowed from:

Dr. Lothar Bertsch (DLR)

Wrap-up: revisit learning objectives

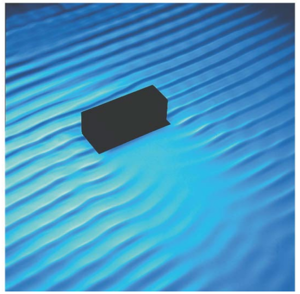
After this lecture you should be able to:

- Describe the principle of **superposition** and wave **interference** phenomena



- Explain the phenomena of wave **reflection**, **refraction**, **diffraction**, and **transmission**.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$



- Describe **standing waves** and calculate their main characteristics.

$$\lambda_n = \frac{2l}{n}$$

$$f_n = \frac{v}{\lambda_n} = \frac{vn}{2l} = nf_1$$

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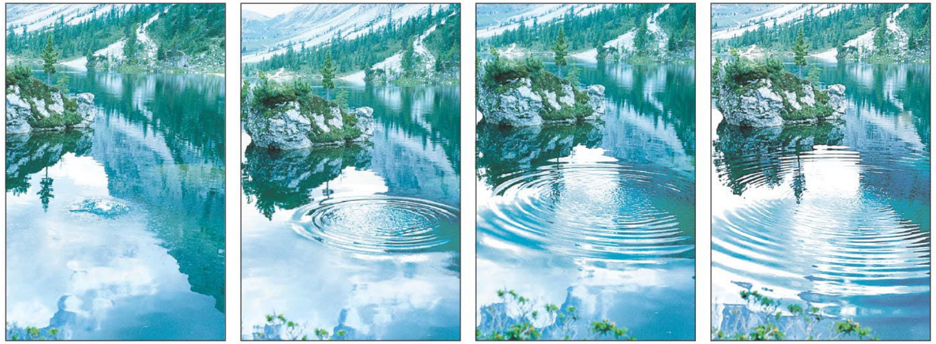
WAVE MOTION

Chapter 15



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