MAGNETISM

Chapter 27

Alessandro Bombelli

Operations & Environment

Faculty of Aerospace Engineering

Structure of the lecture

1. Magnets and Magnetic Fields
2. Electric Currents Produce Magnetic Fields
3. Force on Electric Current in Magnetic Field
4. Force on Electric Charge Moving in Magnetic Field
5. Torque on a Current Loop: Magnetic Dipole Moment
6. Applications
7. Mass Spectrometer

Learning objectives for today's lecture

After this lecture you should be able to:

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- Explain the concept of **magnetic dipole** and some cornerstone applications

Magnetism and Aerospace Engineering

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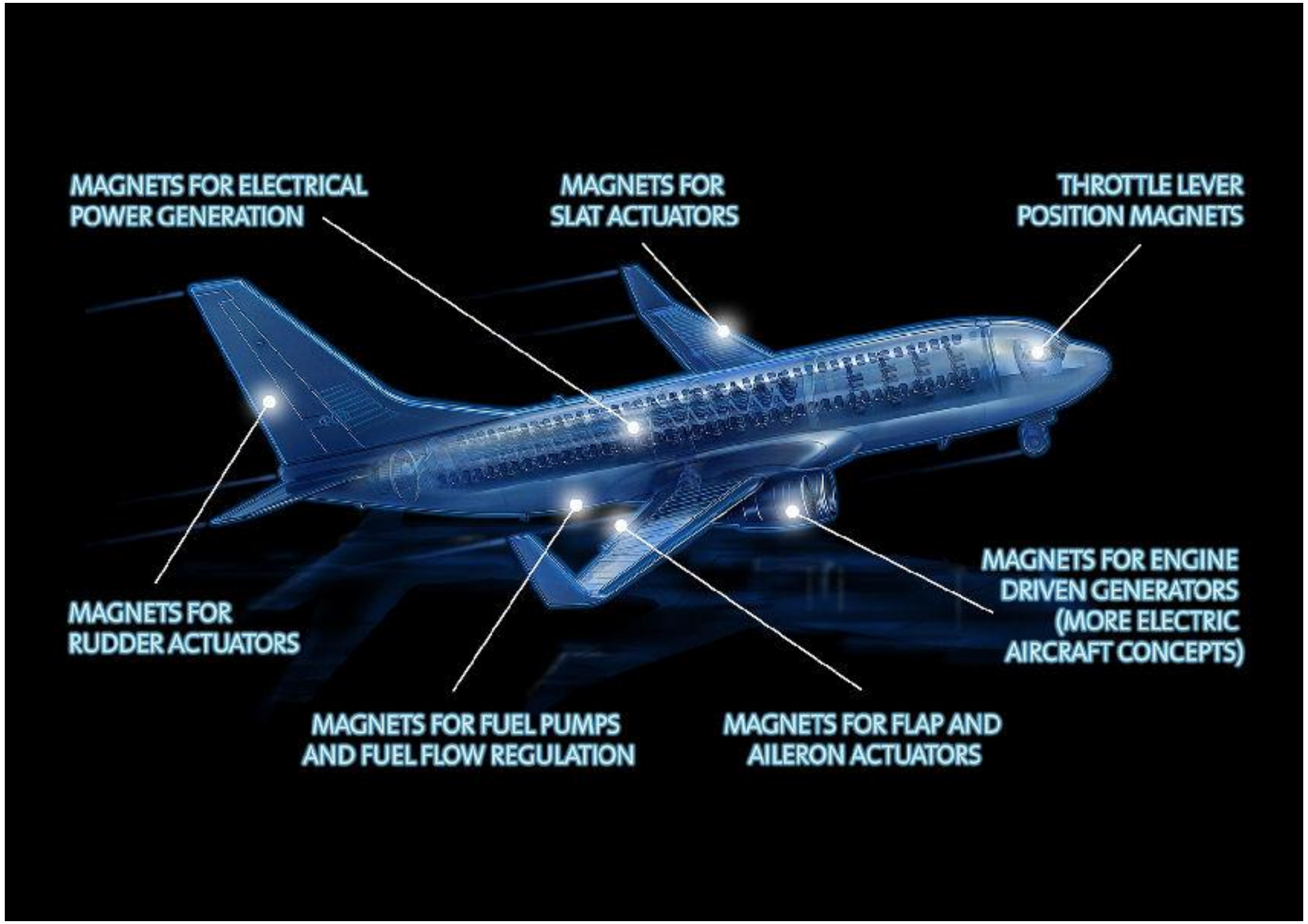

Credit: https://www.goudsmit.co.uk/magnets-in-the-aerospace-industry-goudsmit-uk/#:~:text=Many%20aircraft%20applications%20use%20powerful,Fuel%20pumps%20and%20flow%20regulation

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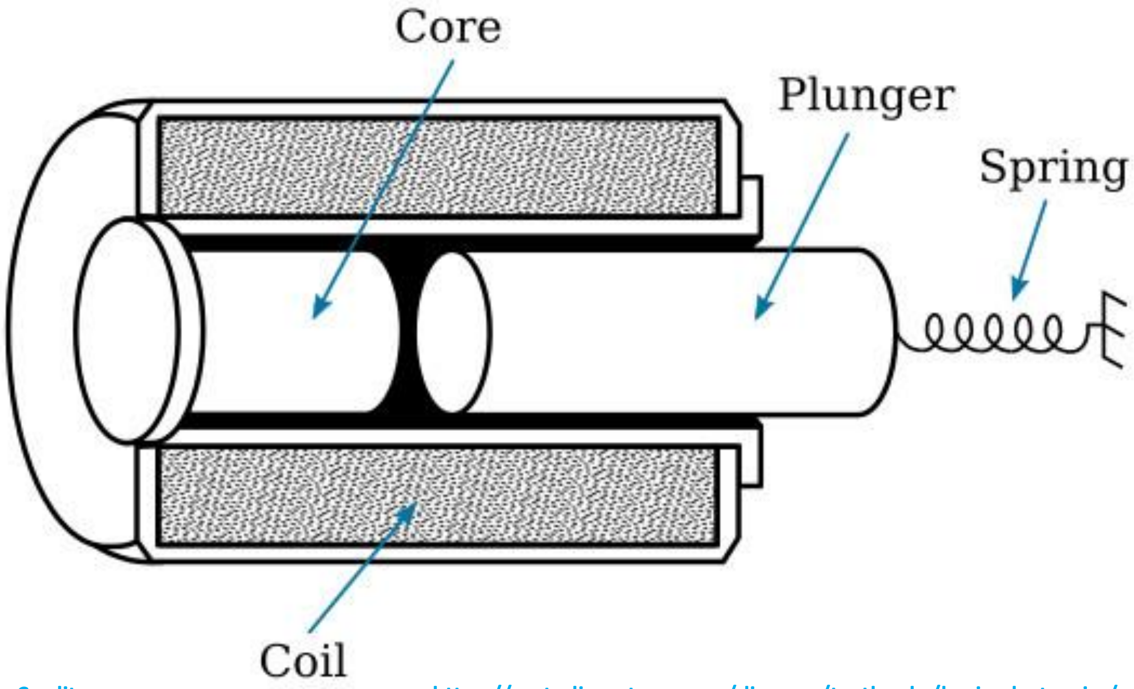
Electromagnetic rudder actuator

Credit: https://www.instructables.com/Micro-Magnetic-Actuator-for-RudderAileron-Control/

Video: https://www.youtube.com/watch?v=1c0BF4UsUx8

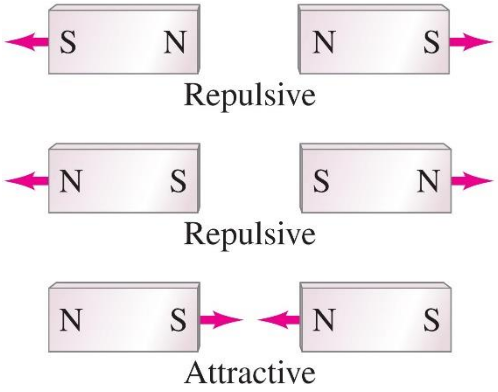


Credit: https://ecstudiosystems.com/discover/textbooks/basic-electronics/sensors-and-actuators/solenoids/#:~:text=A%20solenoid%20actuator%20is%20a,both%20at%20the%20same%20time.

27.1 – Magnets and Magnetic Fields

Magnets have two ends – poles – called north and south.

Like poles repel; unlike poles attract.



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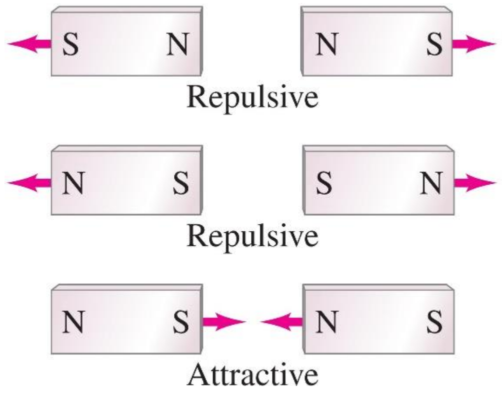
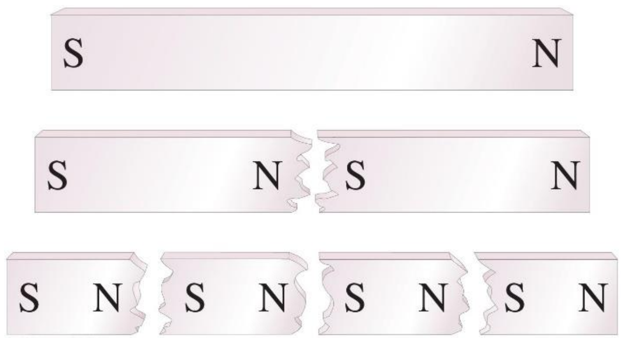


However, if you cut a magnet in half, you do not get a north pole and a south pole – you get two smaller magnets.



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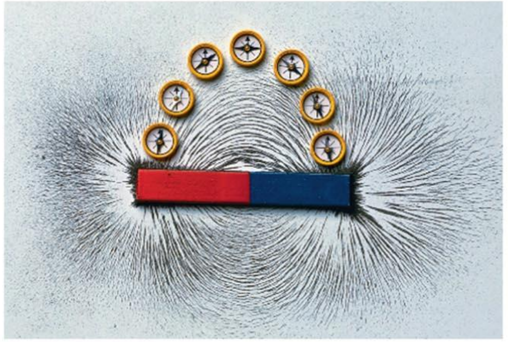
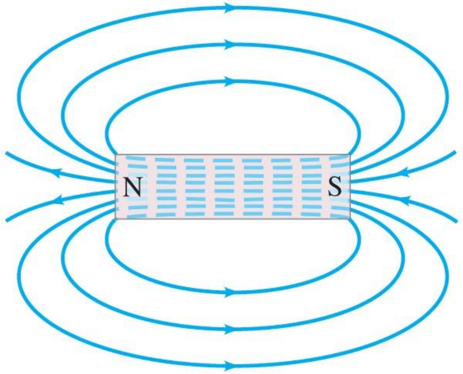
27.1 – Magnets and Magnetic Fields

Magnetic fields can be visualized using magnetic field lines, which are always closed loops.





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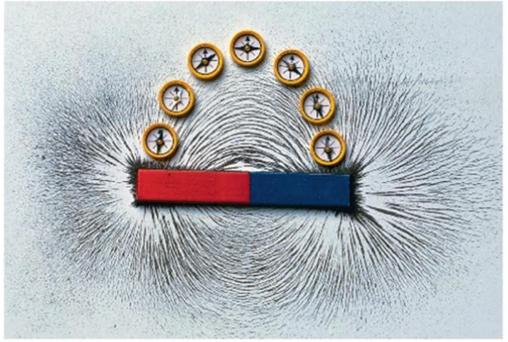

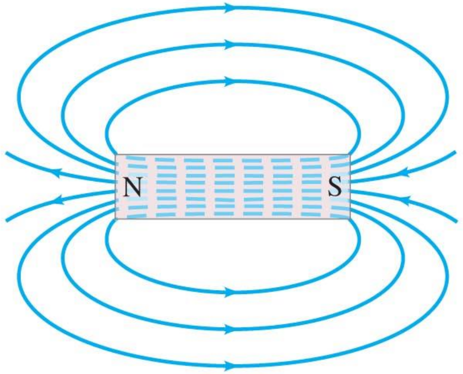

Notwithstanding, the quest for the discovery of the magnetic monopole continues.

Quest for the curious magnetic monopole continues

ATLAS experiment places some of the tightest limits yet on magnetic monopoles

15 SEPTEMBER, 2023 | By ATLAS collaboration

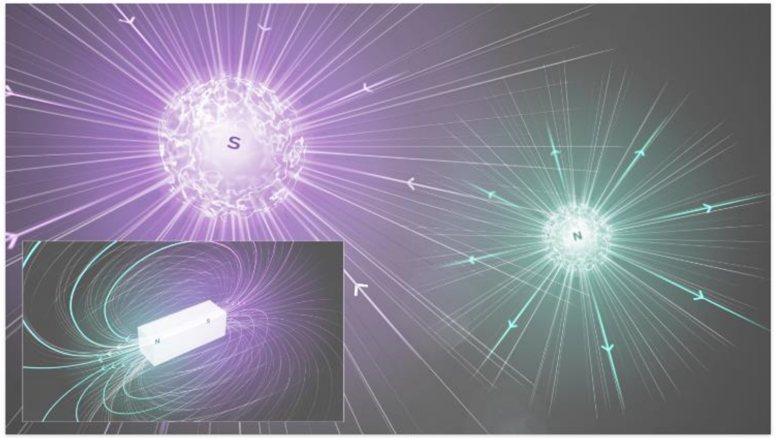

Illustration of magnetic monopoles (larger image) and a magnetic dipole (inset) (Image: CERN)

Magnets, those everyday objects we stick to our fridges, all share a unique characteristic: they always have both a north and a south pole. Even if you tried breaking a magnet in half, the poles would not separate – you would only get two smaller dipole magnets. But what if a particle could have a single pole with a magnetic charge? For over a century, physicists have been searching for such magnetic monopoles. A new study from the ATLAS collaboration at the Large Hadron Collider (LHC) places new limits on these hypothetical particles, adding new

Credit: https://www.home.cern/news/news/physics/quest-curious-magnetic-monopole-continues

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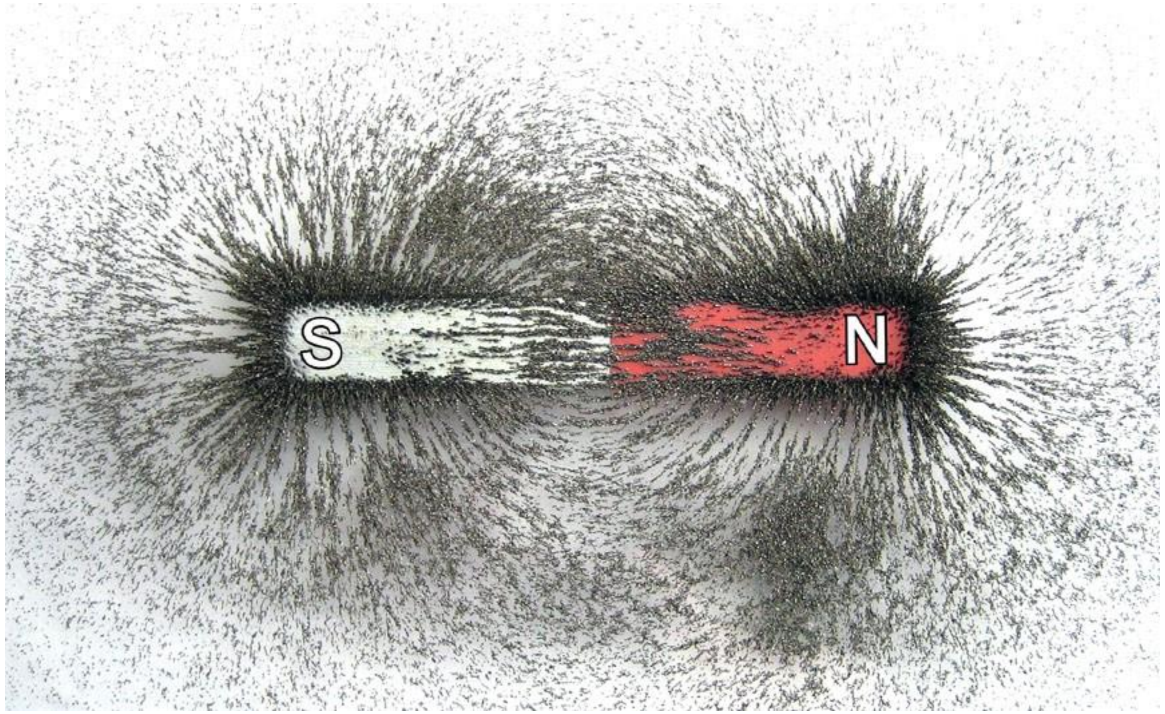
27.1 – Magnets and Magnetic Fields

A couple of properties of magnetic field lines:

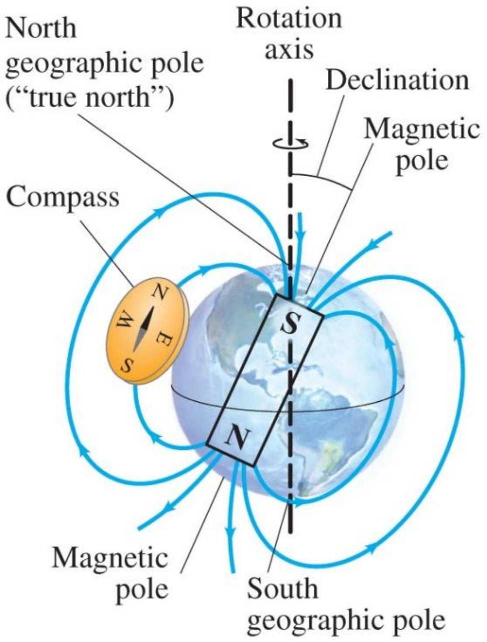
- The direction of the magnetic field is **tangent to a field line** at any point
- The **number** of lines per unit area is **proportional to the strength** of the magnetic field



27.1 – Magnets and Magnetic Fields

The Earth's magnetic field is similar to that of a bar magnet.

Note that the Earth's "North Pole" is really a south magnetic pole, as the north ends of magnets are attracted to it.



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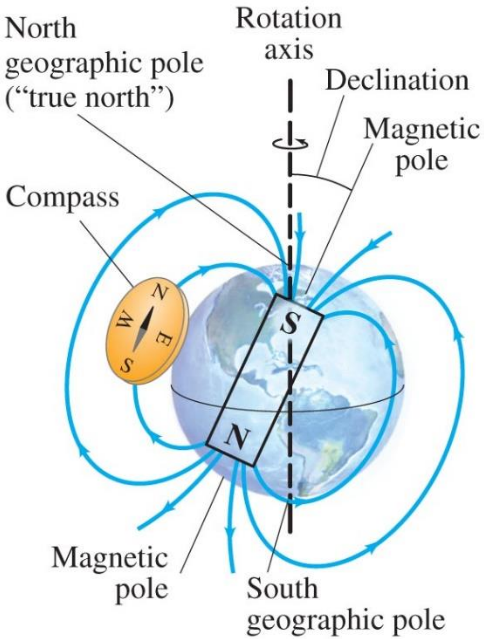

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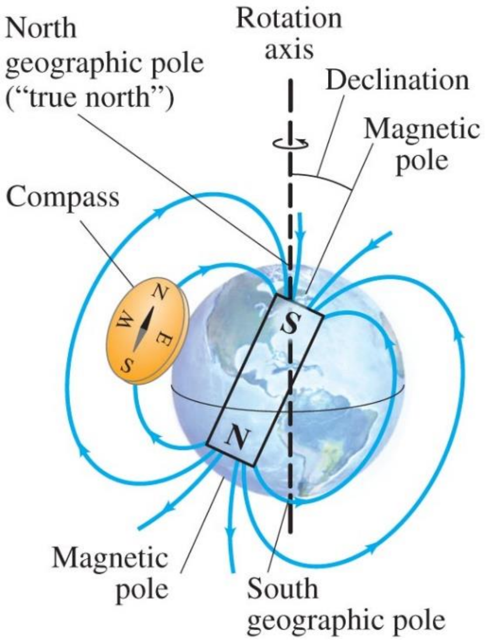
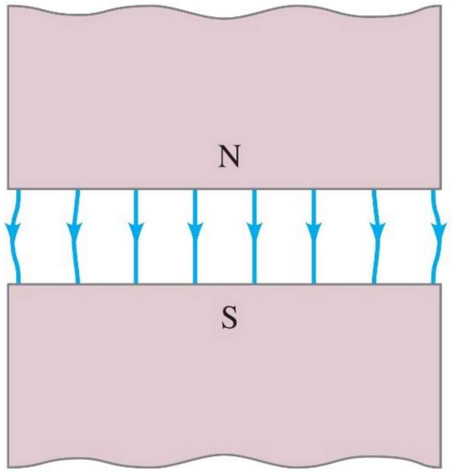


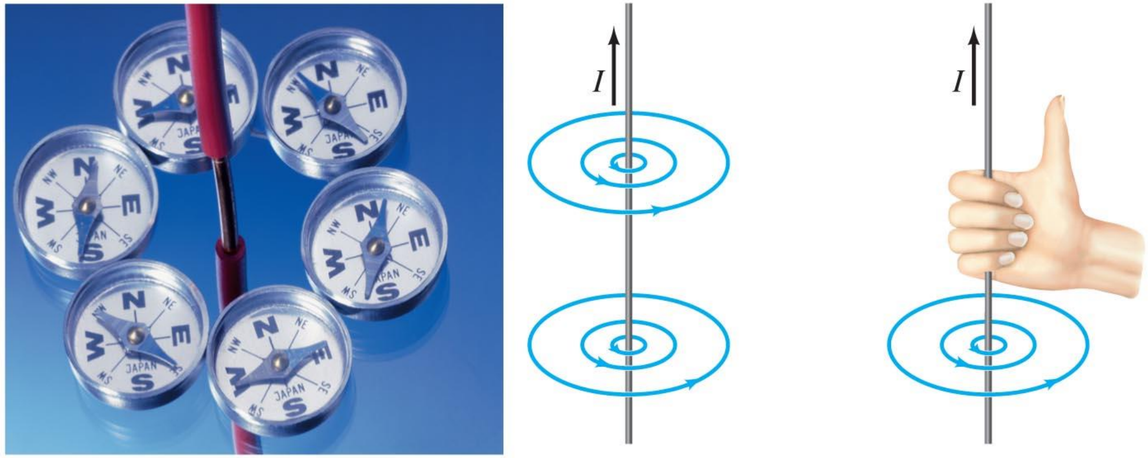
A uniform magnetic field is constant in magnitude and direction.

The field between these two wide poles is nearly uniform.



27.2 – Electric Currents produce Magnetic Fields

Experiment shows that an electric current produces a magnetic field. The direction of the field is given by a right-hand rule.



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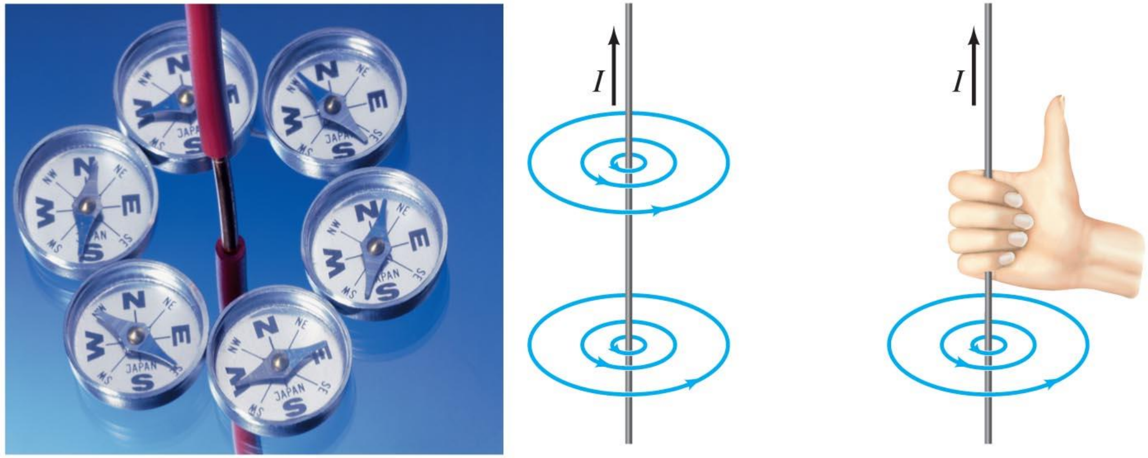
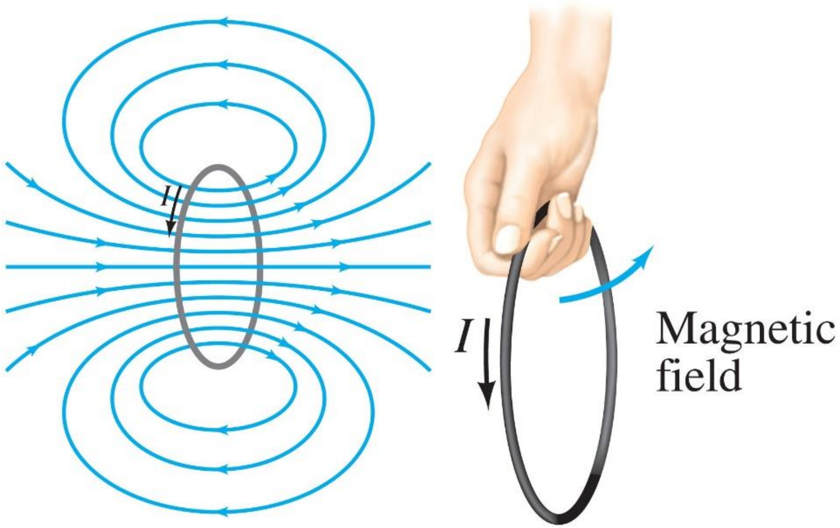


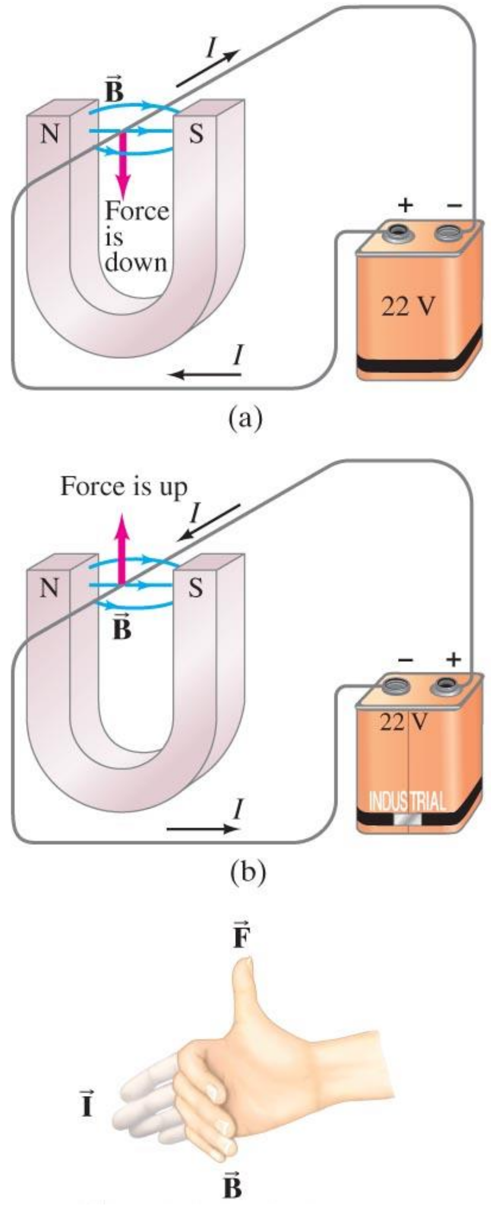
Here we see the field due to a current loop; the direction is again given by a right-hand rule.



27.3 – Force on an Electric Current in a Magnetic Field

A magnet exerts a force on a current-carrying wire. In vectorial form we have:

$$\vec{F} = I\,\vec{l} \times \vec{B}$$



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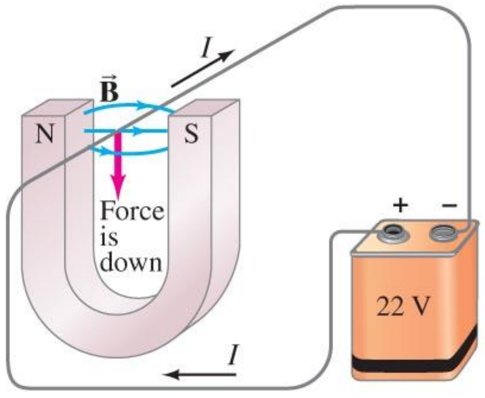
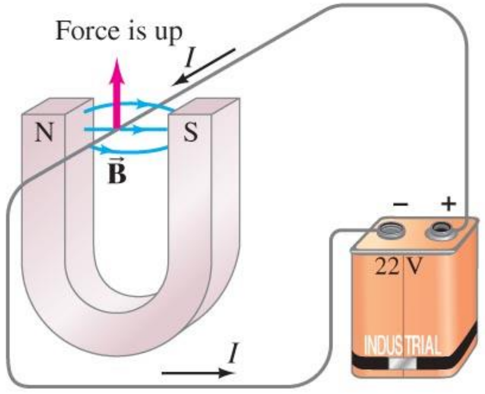


(a)



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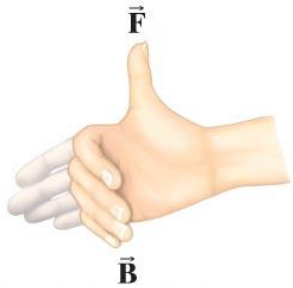

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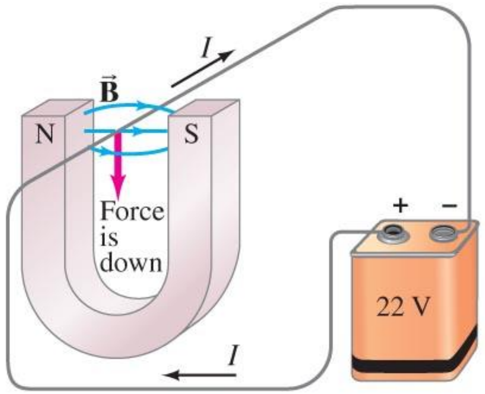
The units of $\vec{B}$ are $\frac{N}{A \cdot m}$ (Tesla)



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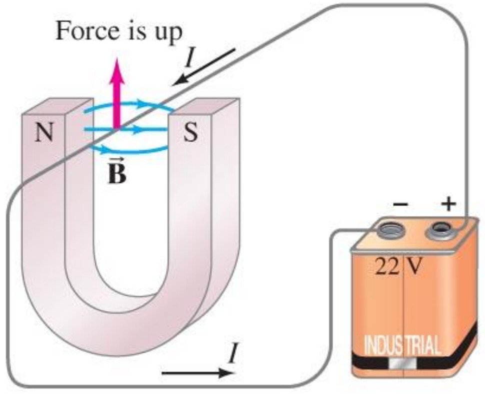

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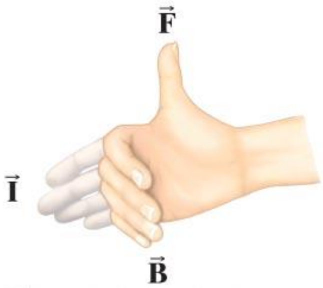

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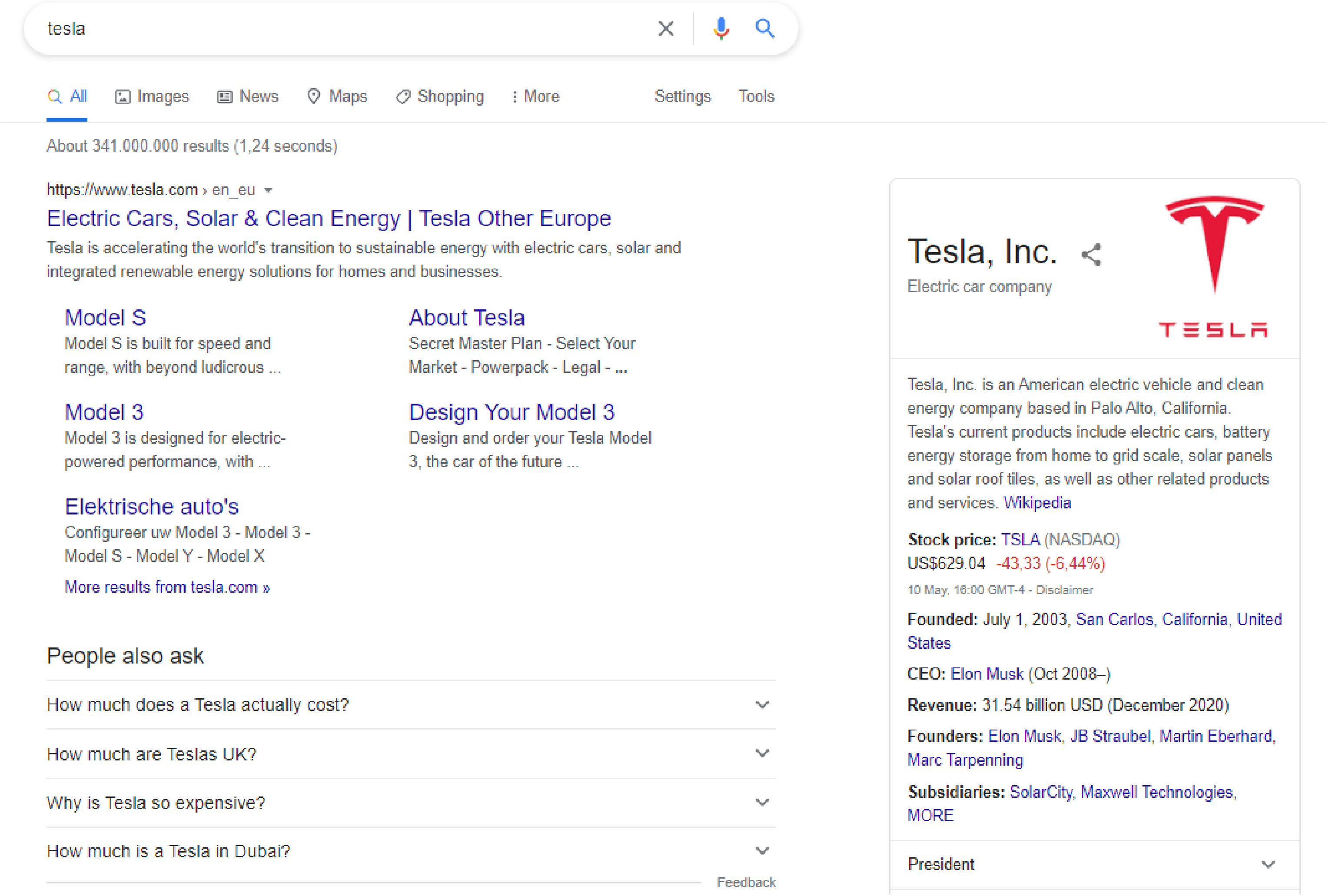

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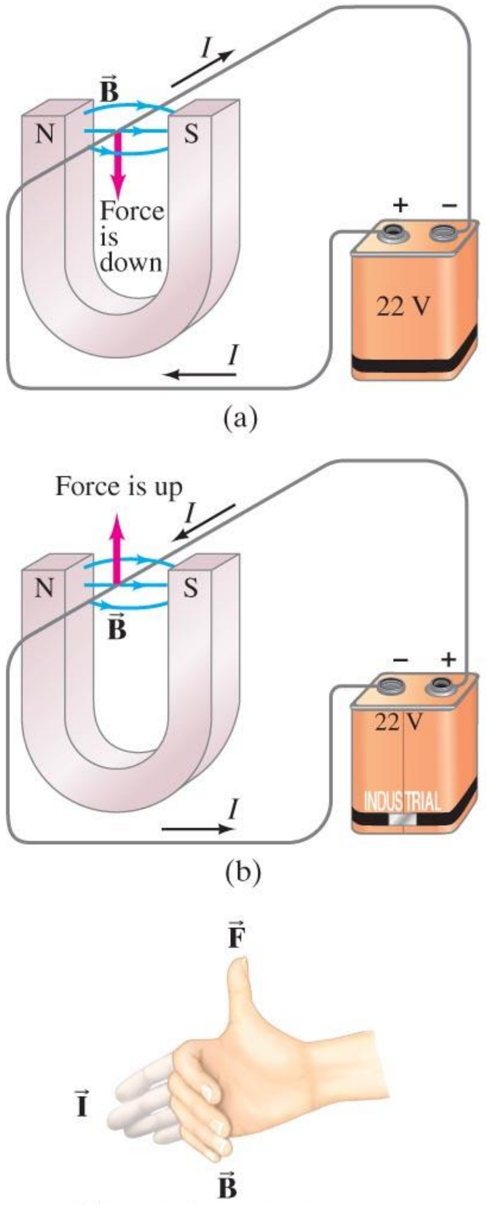
27.3 – Force on an Electric Current in a Magnetic Field

If the wire changes orientation with respect to the magnetic field, then we need to resort to the infinitesimal version of the cross product:



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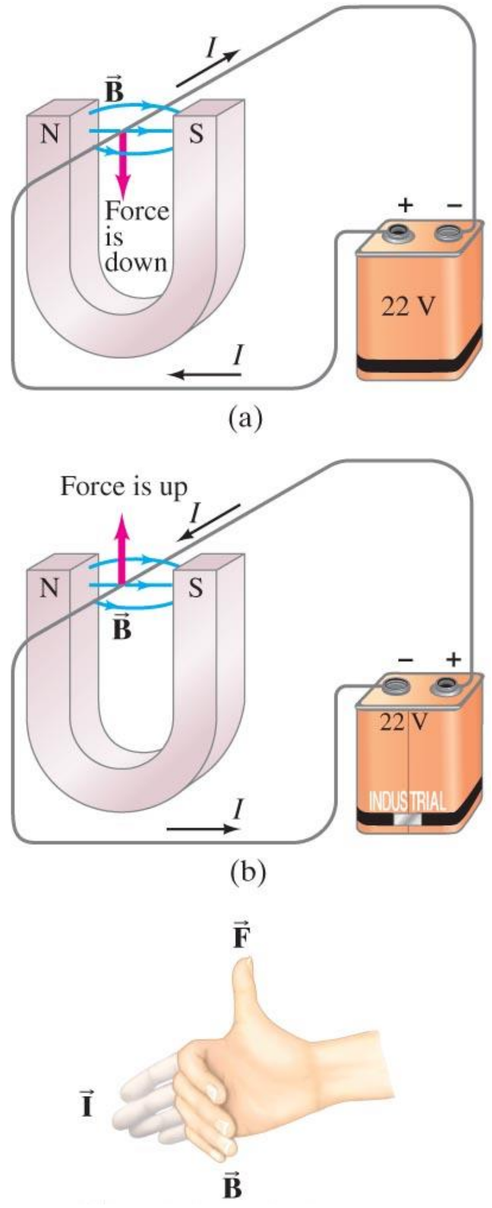
$$d\vec{F} = I\, d\vec{l} \times \vec{B} \; *$$

***: we assume the magnetic field is constant (vectorially)**



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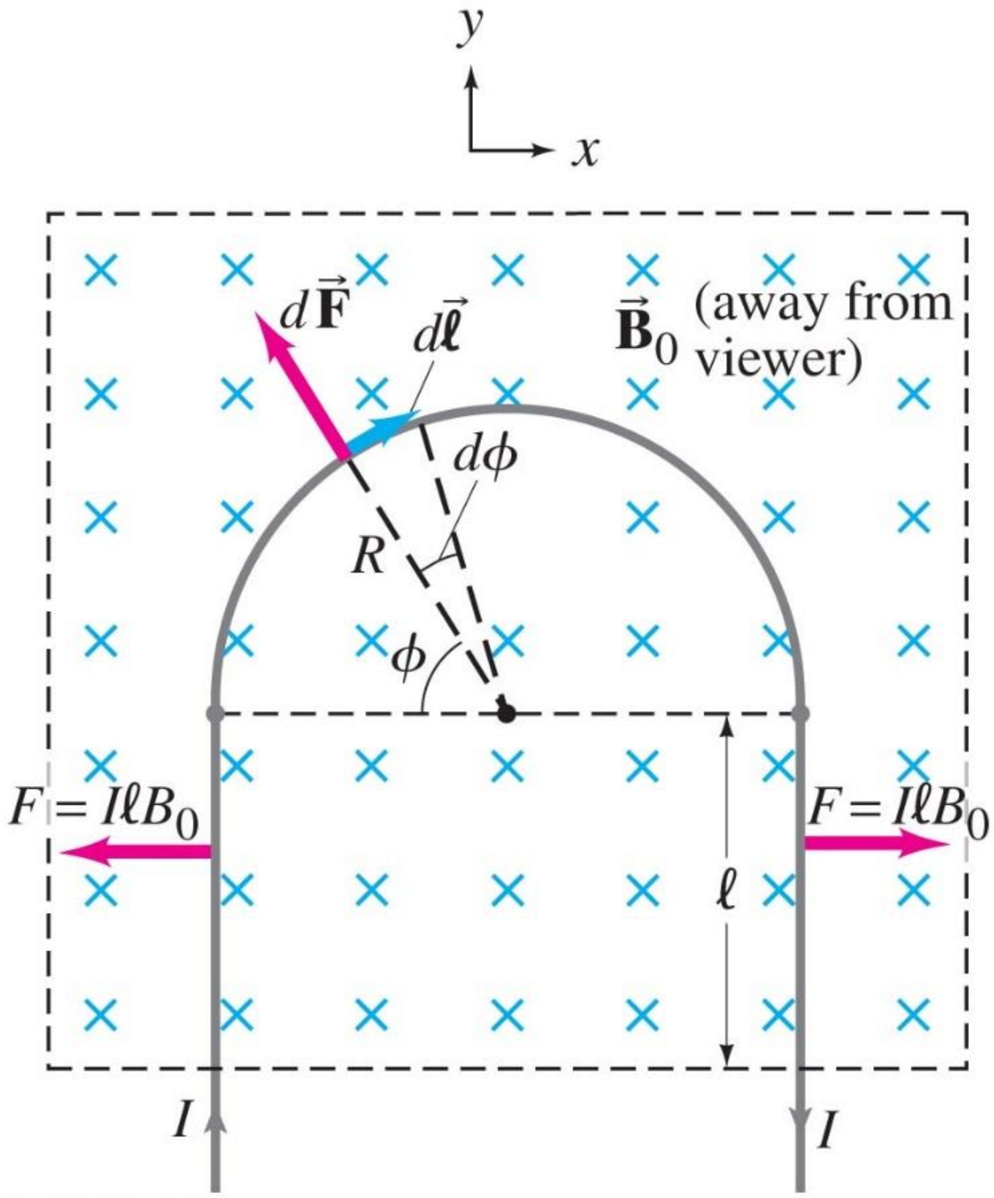
Magnetic force on a semicircular wire

Like what seen for electricity, symmetry can/should be exploited.



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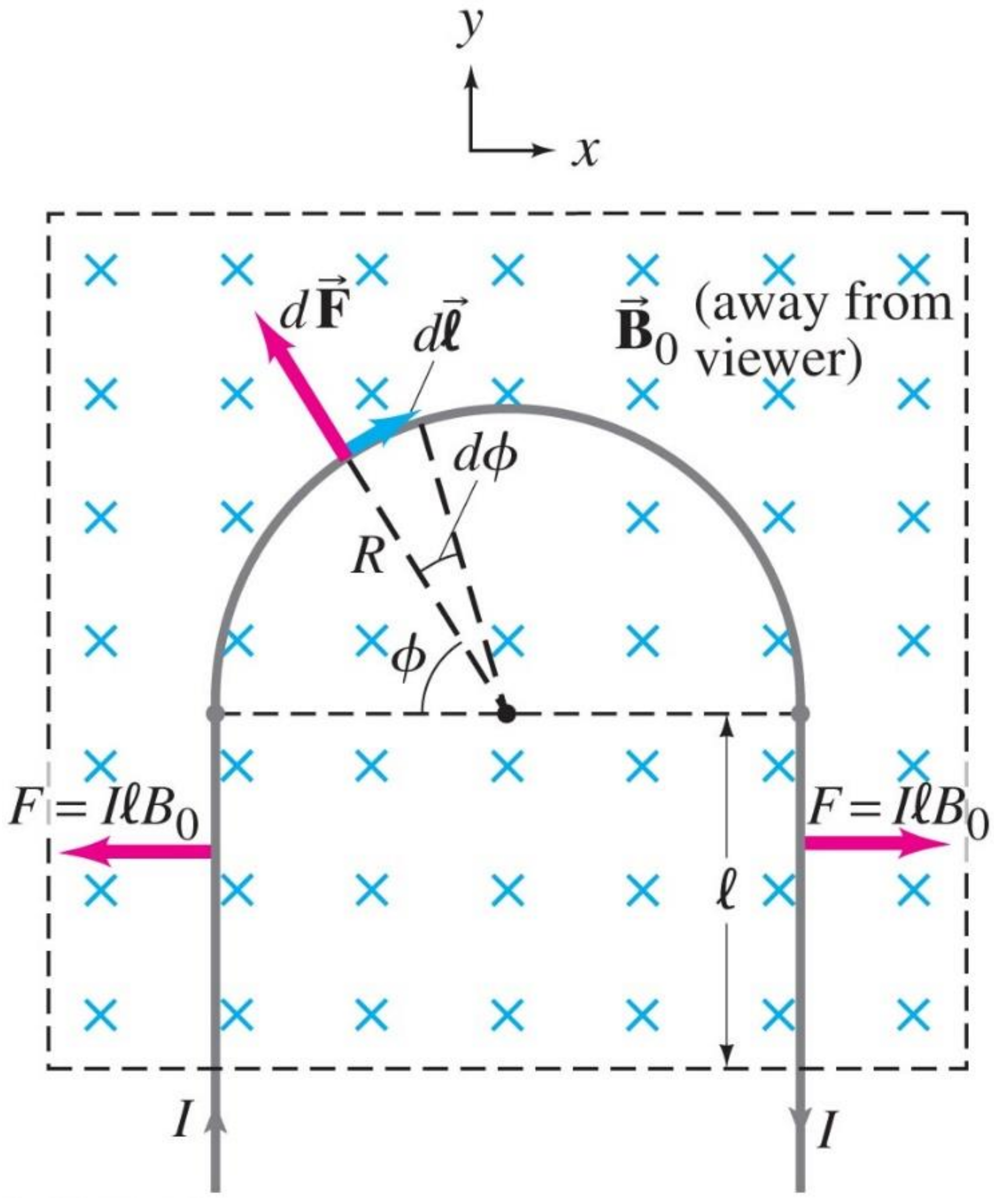
The contributions of the two vertical wires cancel each other (as a matter of fact, the exercise states that we deal just with a semicircular wire ☺).



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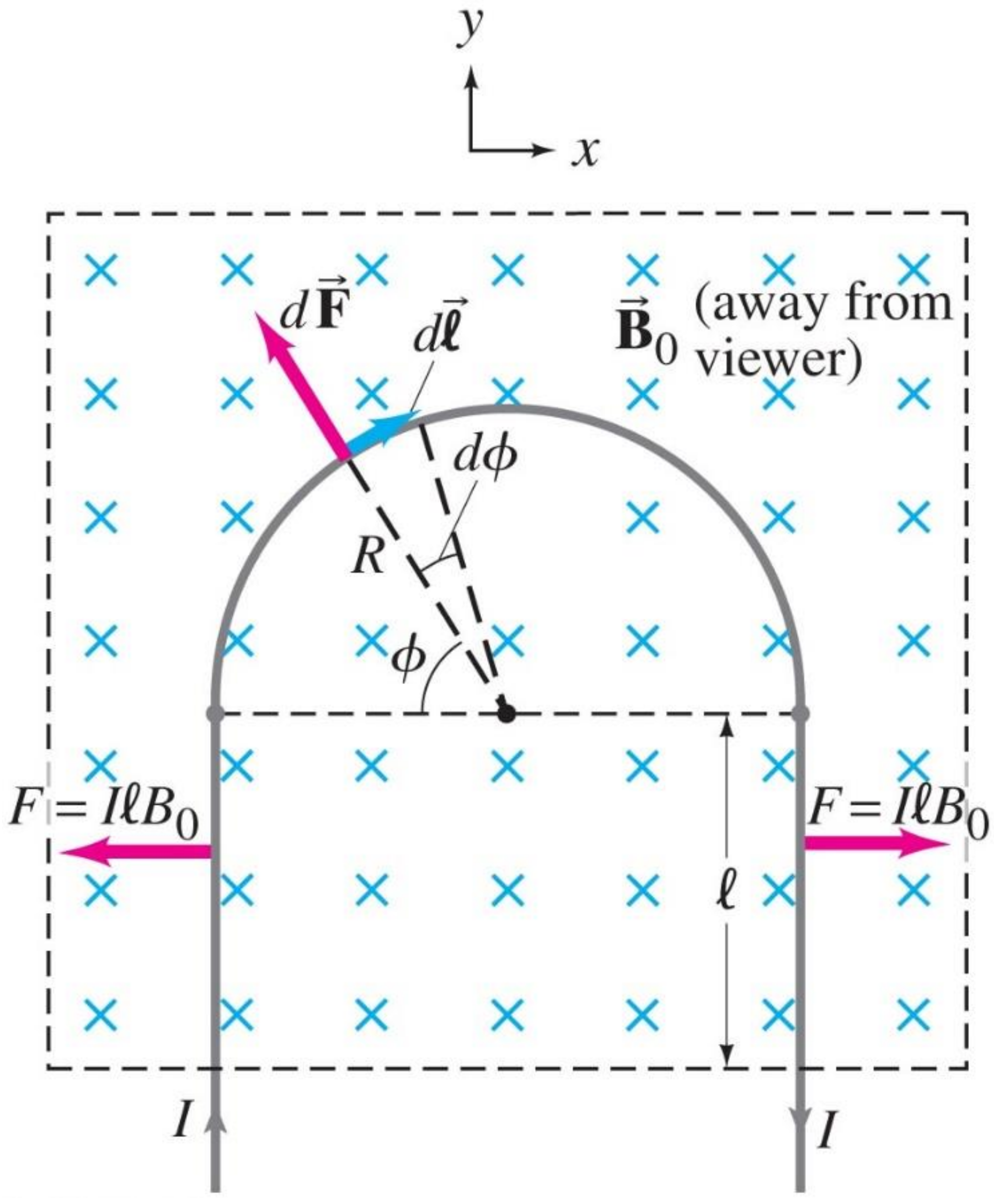

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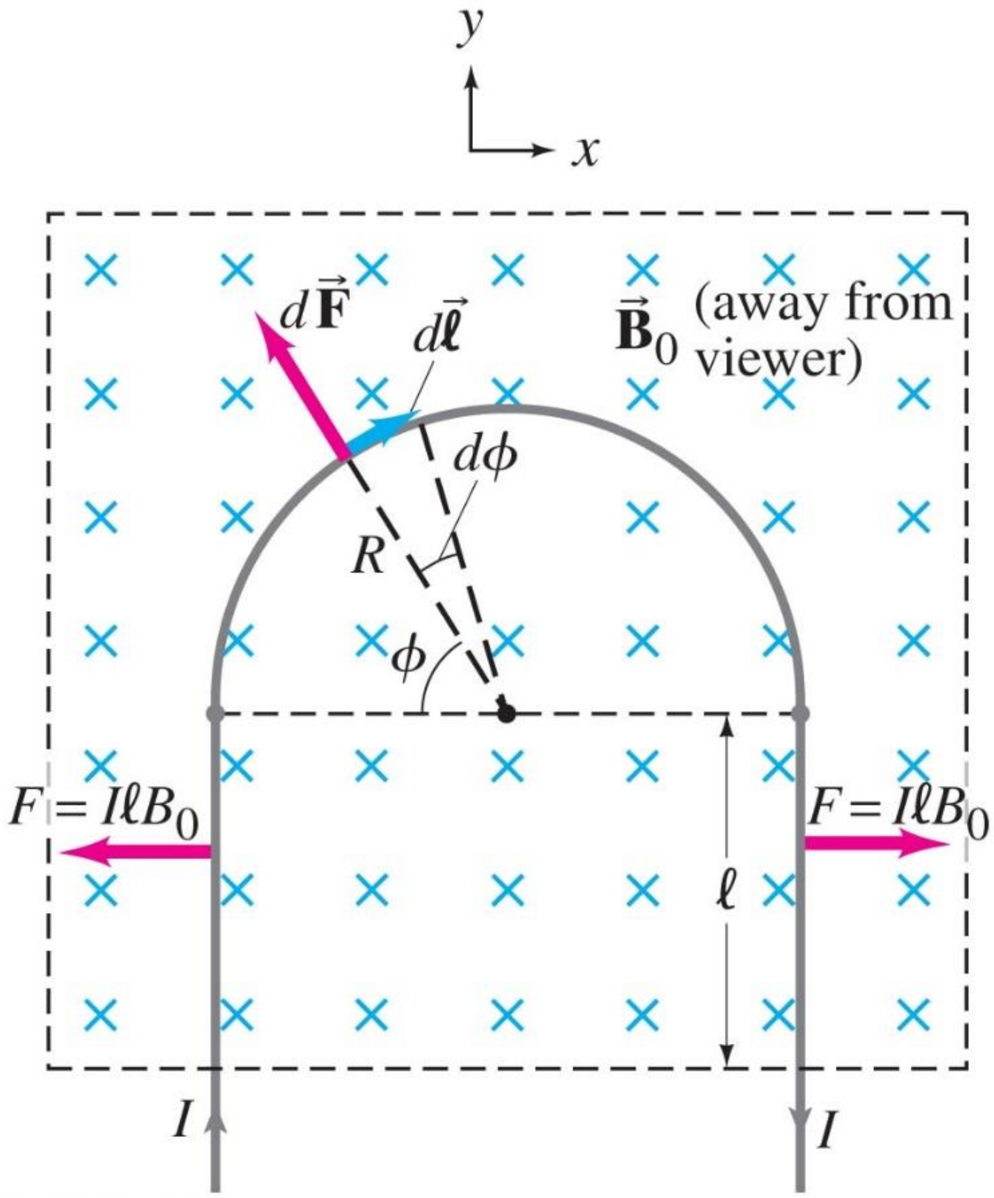
$$d\vec{F} = I\, d\vec{l} \times \vec{B}_0$$
$$|dl| = R\, d\phi$$
$$|dF_x| = -|dF| * cos(\phi)$$
$$|dF_y| = |dF| * sin(\phi)$$



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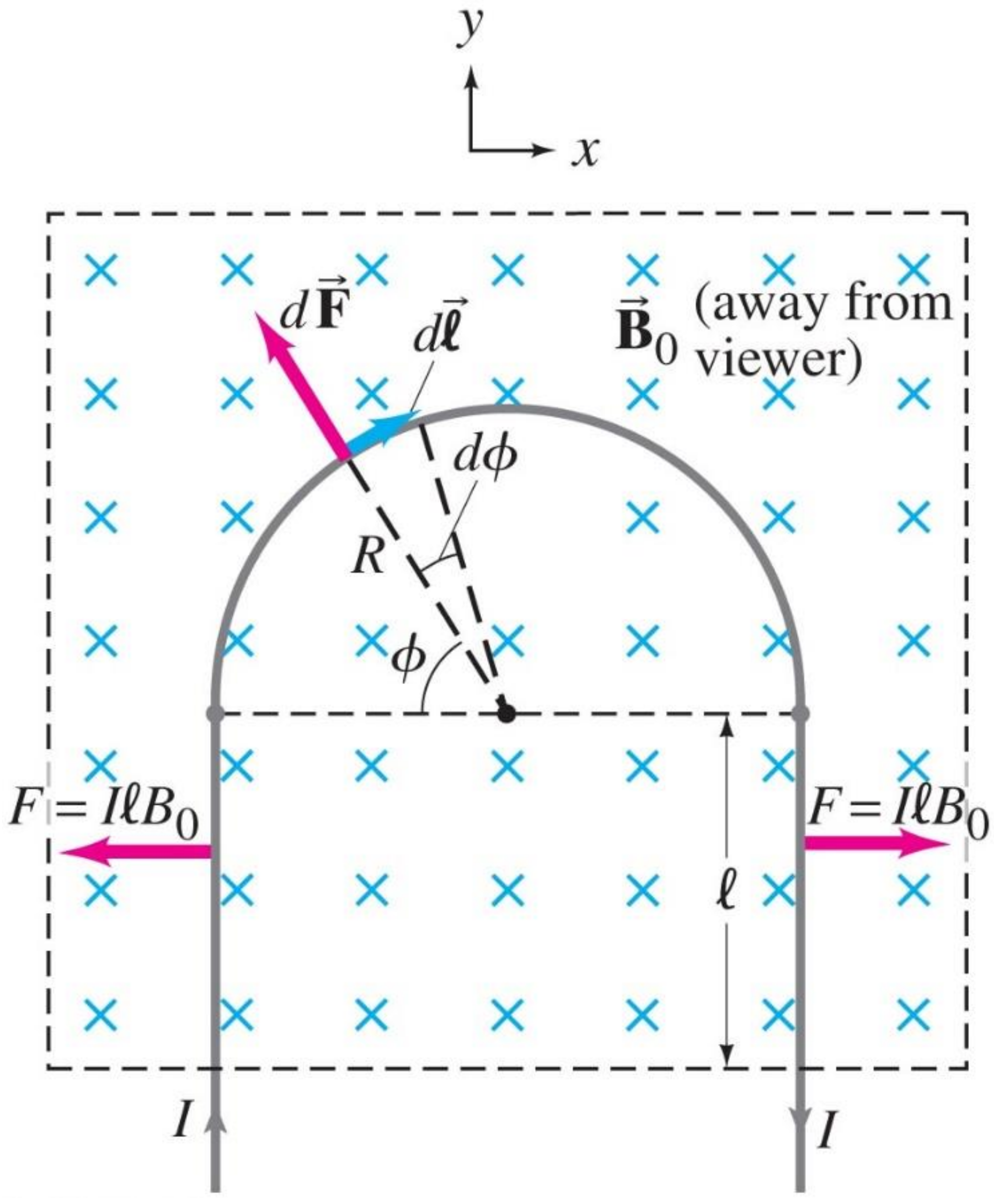
Because we need to consider the whole semicircular wire from $\phi = 0$ to $\phi = \pi$, the dF_x force contributions cancel out.



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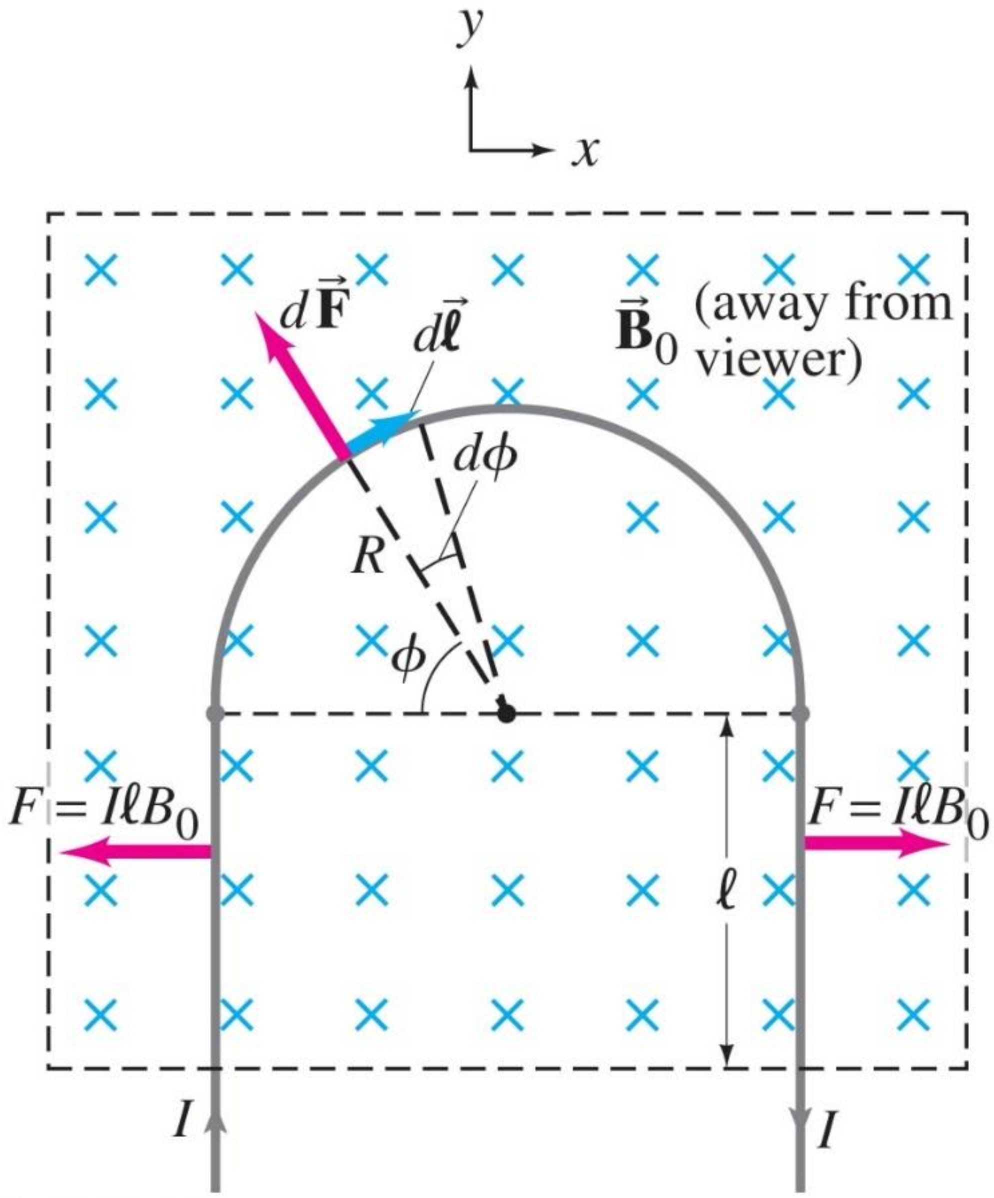
Plugging the expressions of $|dl|$ and $|dF_y|$ into the infinitesimal expression of $d\vec{F}$ and integrating:



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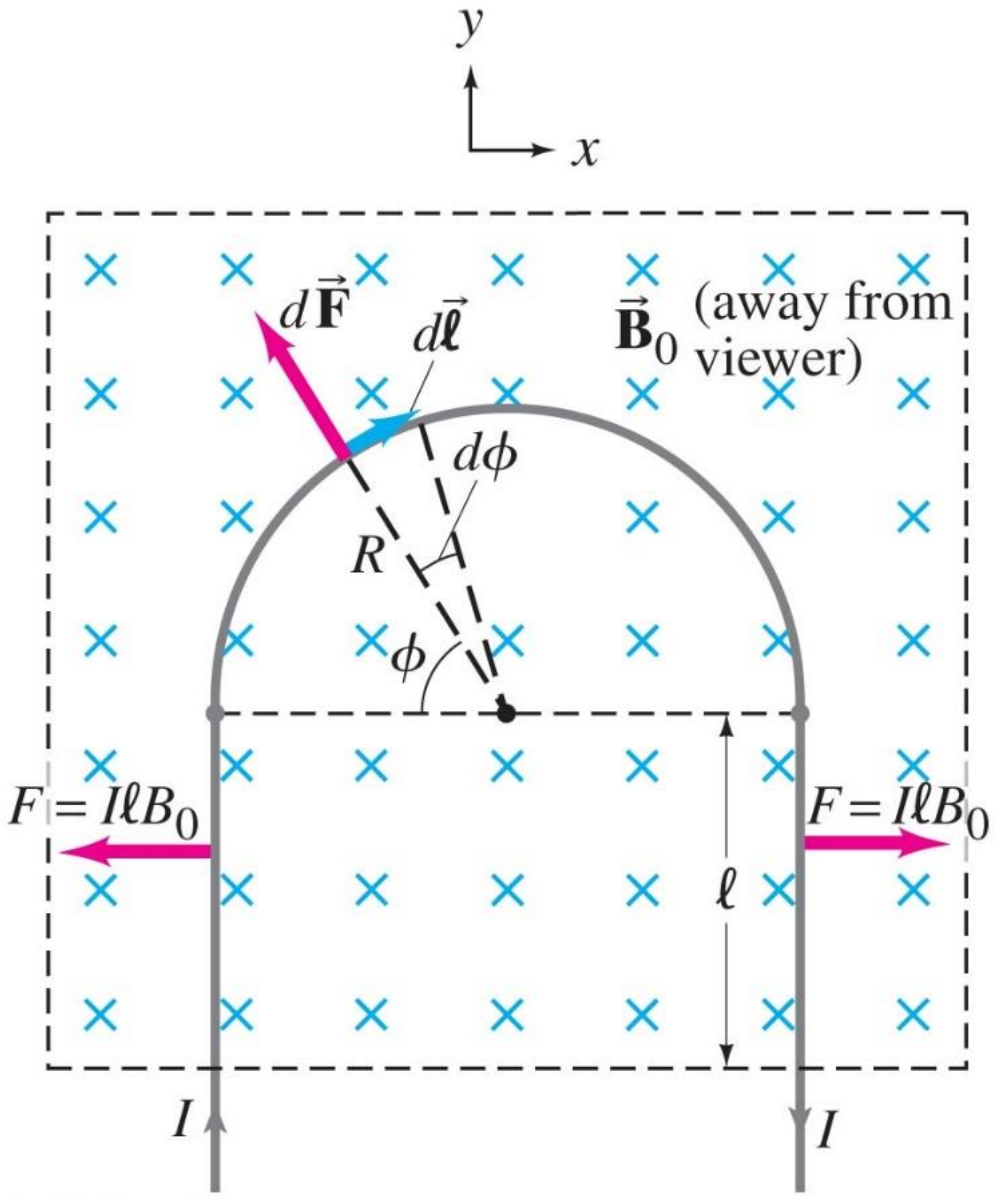
$$F = F_y = \int_0^{\pi} IRB_0 sin\phi d\phi$$

$$F = F_y = 2\int_0^{\frac{\pi}{2}} IRB_0 sin\phi d\phi = 2IRB_0$$



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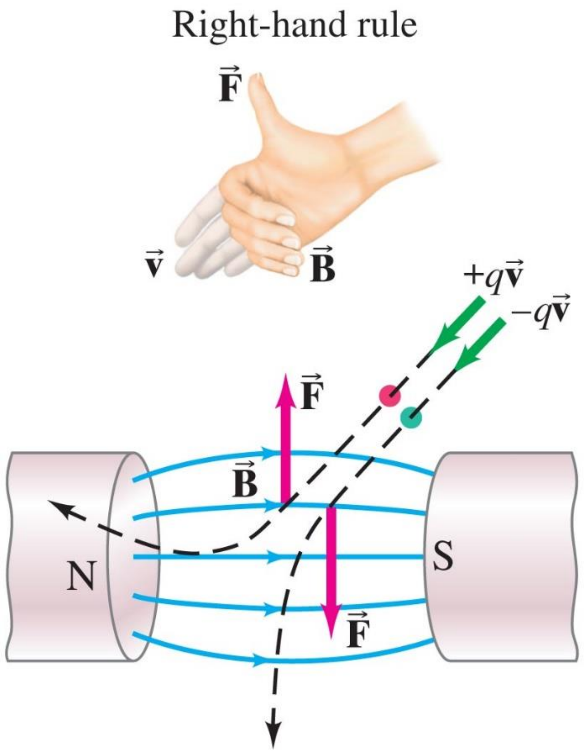
27.4 – Force on an Electric Charge Moving in a Magnetic Field

The force on a moving charge in a magnetic field is:

$$\vec{F} = q\,\vec{v} \times \vec{B}$$



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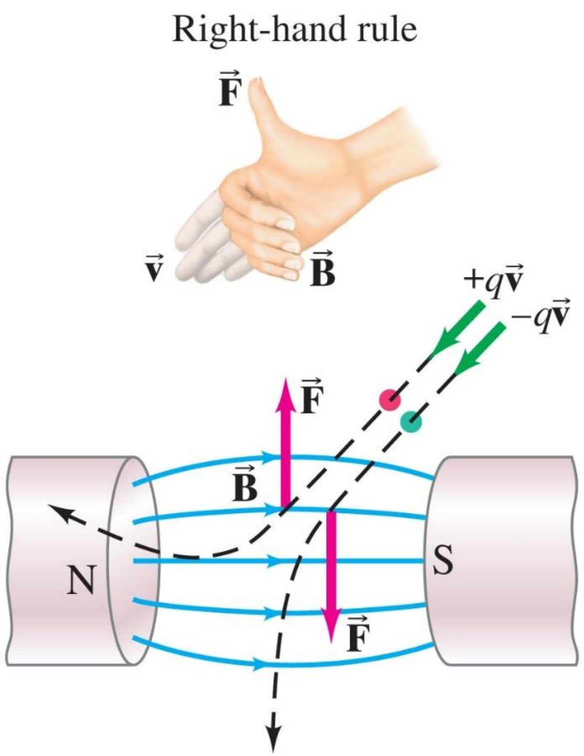
where q must appear with the proper sign (see figure to the right)! Once again, the direction is given by a right-hand rule.



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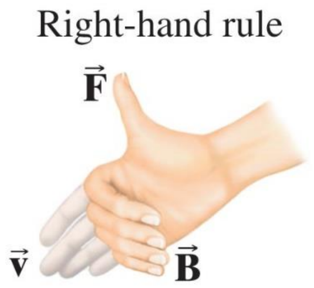
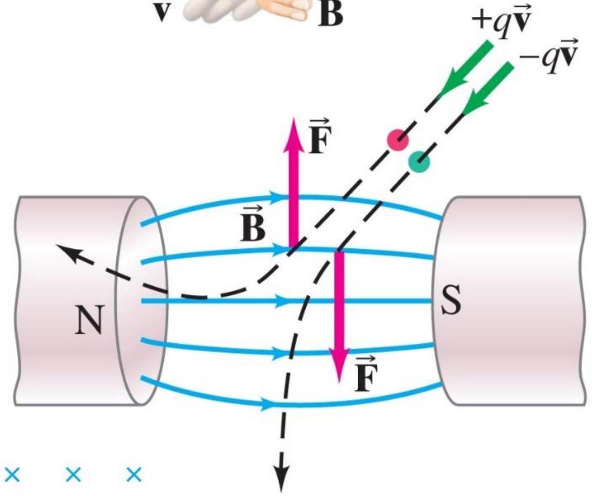
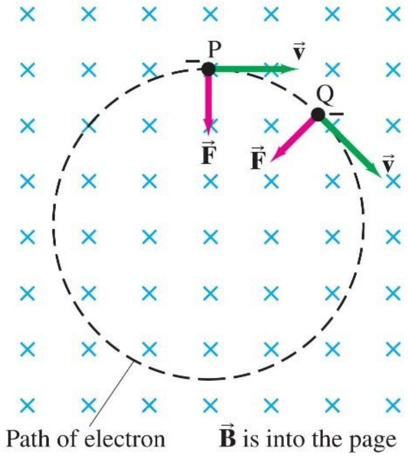
If a charged particle is moving perpendicular to a uniform magnetic field, its path will be a circle.



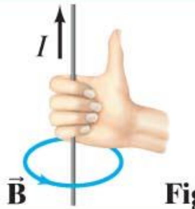
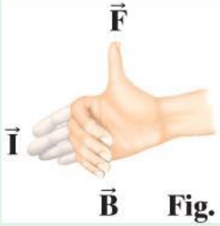
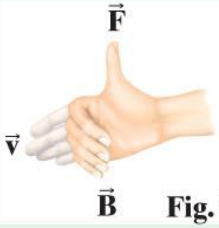




27.4 – Force on an Electric Charge Moving in a Magnetic Field

A summary of the vectorial relationships analyzed so far:

TABLE 27–1 Summary of Right-hand Rules (= RHR)

Physical Situation	Example	How to Orient Right Hand	Result
1. Magnetic field produced by current (RHR-1)	 **Fig. 27–8c**	Wrap fingers around wire with thumb pointing in direction of current I	Fingers point in direction of $\vec{\mathbf{B}}$
2. Force on electric current I due to magnetic field (RHR-2)	 **Fig. 27–11c**	Fingers point straight along current I, then bend along magnetic field $\vec{\mathbf{B}}$	Thumb points in direction of the force $\vec{\mathbf{F}}$
3. Force on electric charge $+q$ due to magnetic field (RHR-3)	 **Fig. 27–15**	Fingers point along particle's velocity $\vec{\mathbf{v}}$, then along $\vec{\mathbf{B}}$	Thumb points in direction of the force $\vec{\mathbf{F}}$

Electron's path in a uniform magnetic field

An electron travels at $2.0 \times 10^7 \frac{m}{s}$ in a plane perpendicular to a uniform $0.010\,T$ magnetic field. Describe its path quantitatively.

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The force due to magnetic field is counterbalanced by the centripetal force:

$$\boldsymbol{qvB = \frac{mv^2}{r} \rightarrow r = \frac{mv}{qB}}$$

The resulting radius is 1.1 cm.

Stopping a charged particle

Can a magnetic field be used to stop a single charged particle, as an electric field can?

Stopping a charged particle

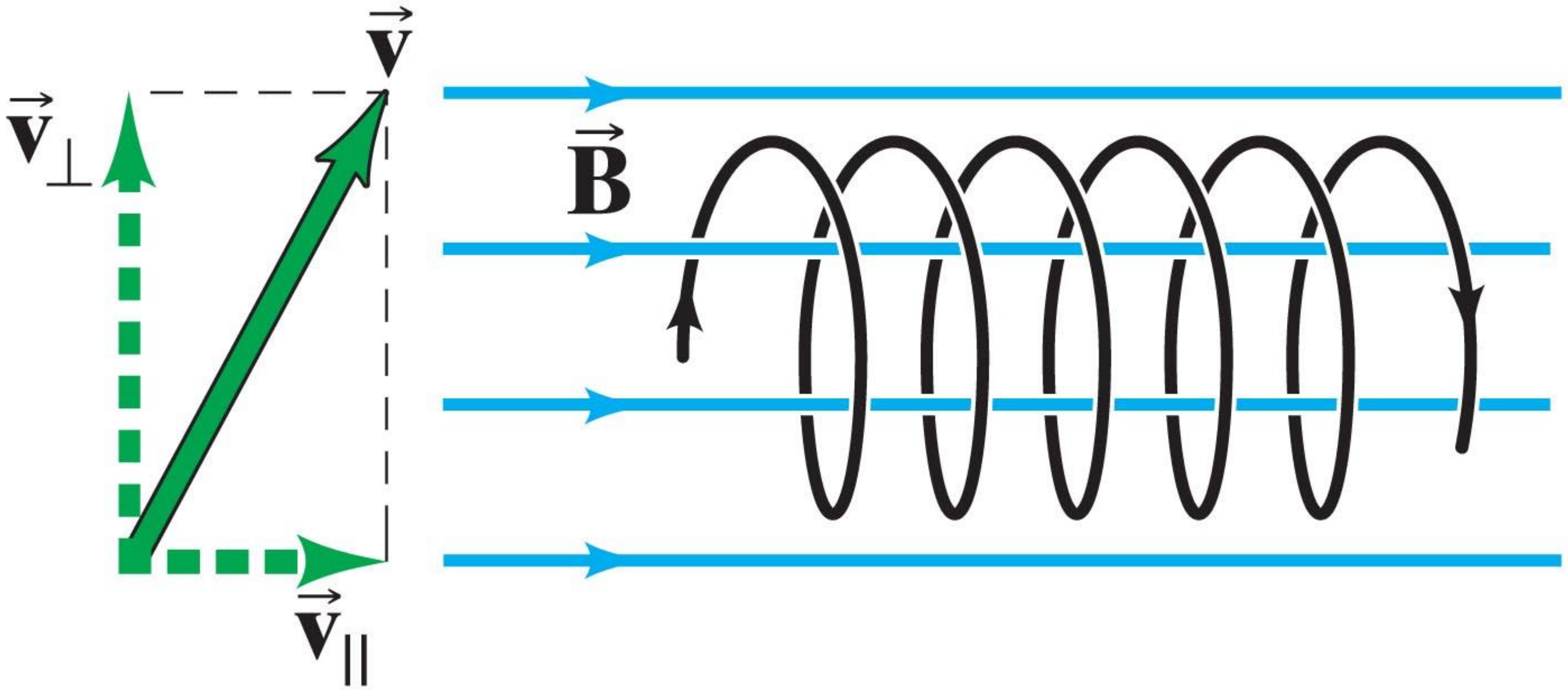
Can a magnetic field be used to stop a single charged particle, as an electric field can?

NO! Since the force generated is always perpendicular to the velocity, the magnetic field can only affect the velocity vector in terms of direction, not in terms of magnitude

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

The force is perpendicular to both the velocity and the magnetic field.

Charged particle moving (not perpendicularly) in magnetic field



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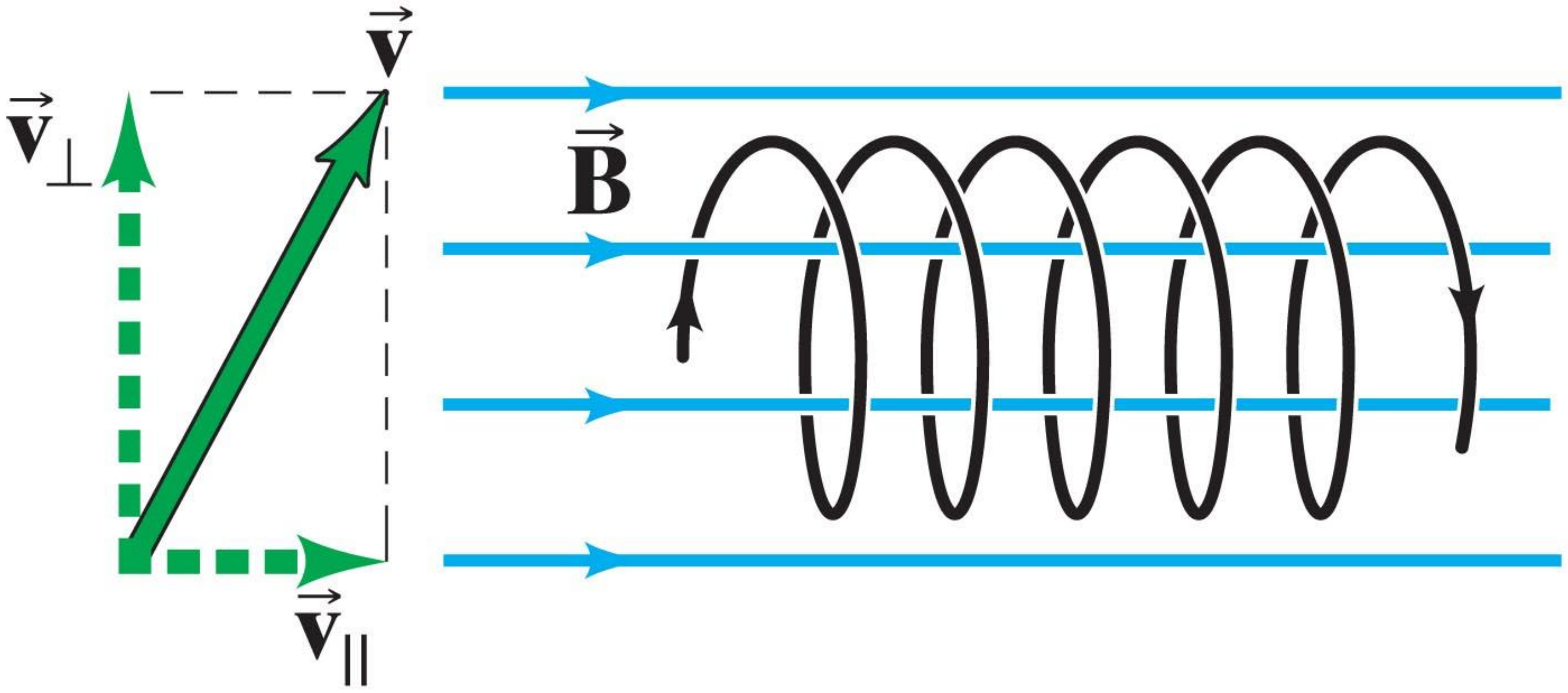


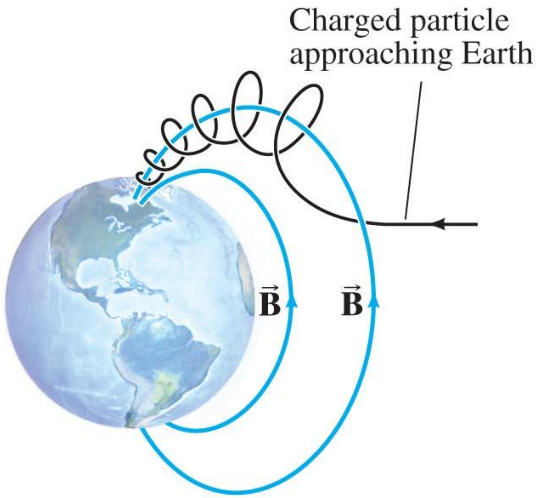
$$\vec{v}_{||} \times \vec{B} = 0$$

$$\left|\vec{v}_\perp \times \vec{B}\right| = v_\perp B$$

$$r = \frac{m v_\perp}{qB}$$

Given r, determine period T. The distance between “consecutive” helixes is $p = T v_{||}$

Charged particle moving (not perpendicularly) in magnetic field



Credit: https://en.wikipedia.org/wiki/File:Church_of_light.jpg

The aurora borealis (northern lights) is caused by charged particles from the solar wind spiraling along the Earth's magnetic field, and colliding with air molecules (see here: https://en.wikipedia.org/wiki/Aurora).

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27.4 – Lorentz equation

What if we now add an extra player to the equation, namely an electric field E? A charged particle q moving inside a region where both an electric and magnetic field exist follows the Lorentz equation:

$$\vec{\boldsymbol{F}} = \boldsymbol{q}(\vec{\boldsymbol{E}} + \vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}})$$

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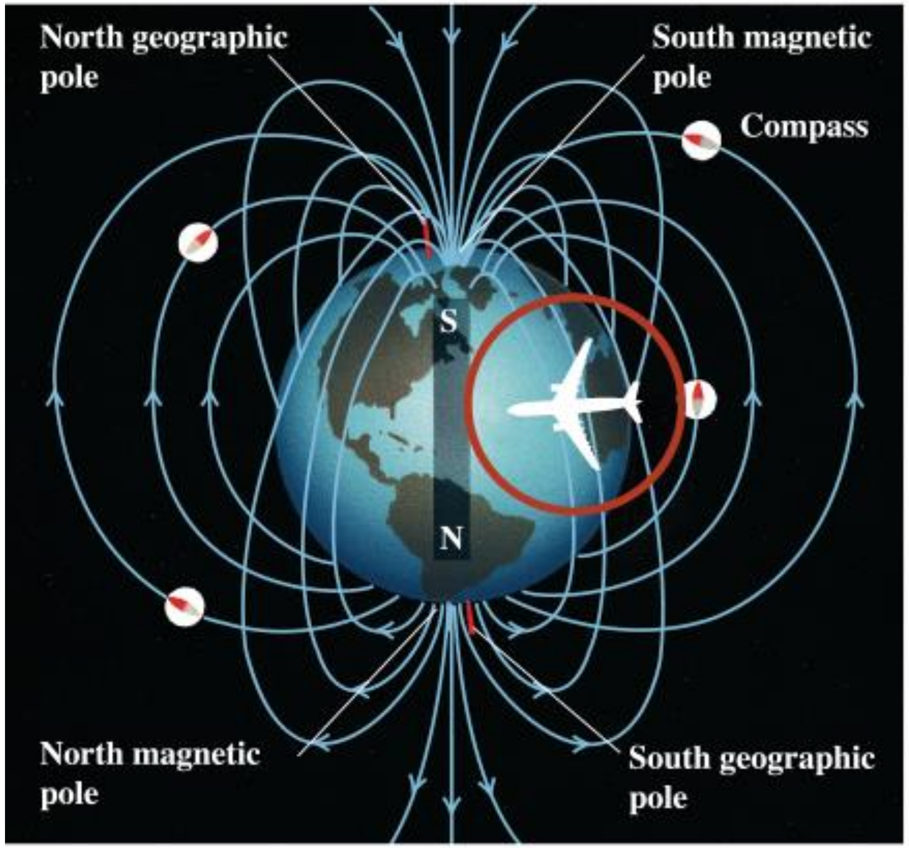
- The component of the force due to the electric field is parallel to the electric field itself (as seen previously in this course)
- The component of the force due to the magnetic field is perpendicular to both the velocity and the magnetic field itself (as seen previously in this lecture)

27.4 – Lorentz equation

How about the effect of the magnetic field of Earth on aircraft?

The magnitudes involved are such that not much will happen to the fuselage (and to passengers). See here: https://physics.stackexchange.com/questions/112217/what-is-the-effect-of-earth-s-magnetic-field-on-airplanes



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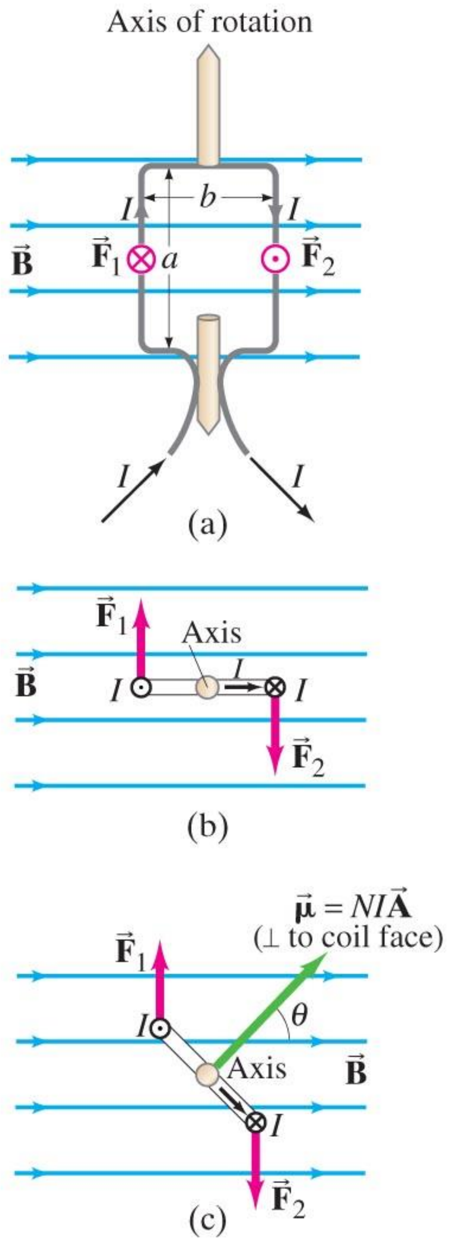
27.5 – Torque on a Current Loop; Magnetic Dipole Moment

The forces on opposite sides of a current loop will be equal and opposite (if the field is uniform and the loop is symmetric), but there may be a torque.



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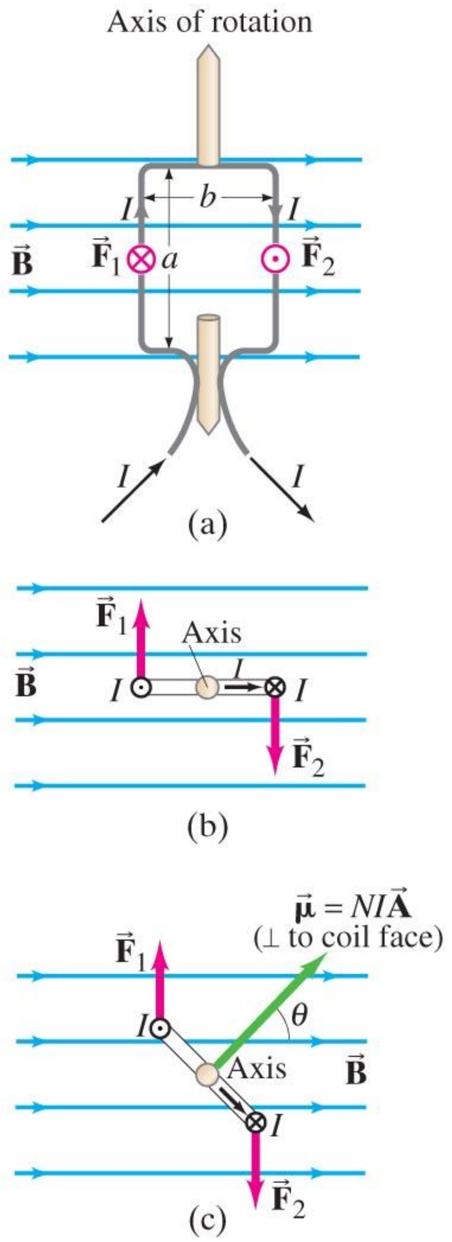
The torques produced by F_1 and F_2 act in the same direction (and opposite orientations), so the total torque is the sum of the two torques in Fig (b):

$$\tau = Ia\frac{b}{2}B + Ia\frac{b}{2}B = IabB$$



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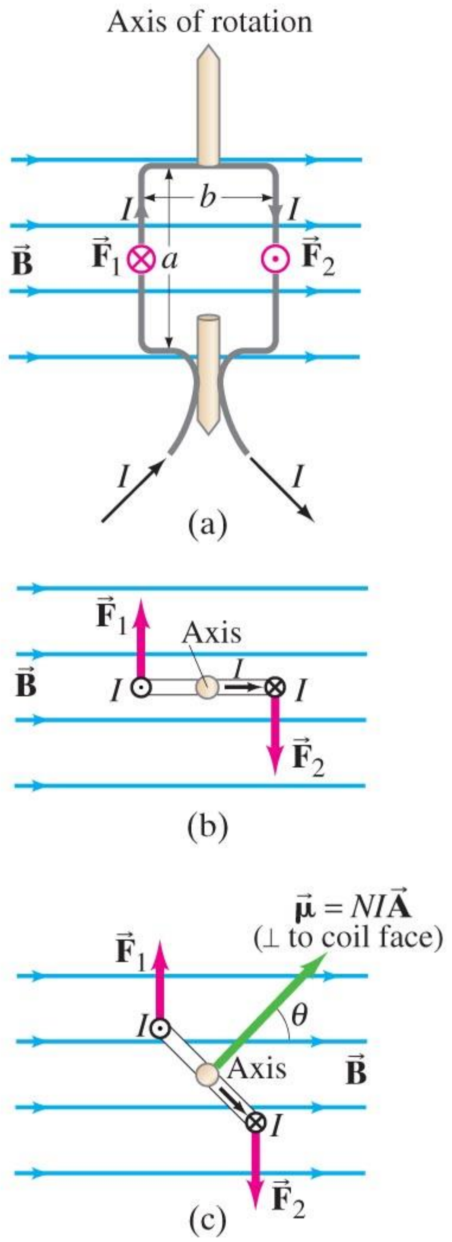
If we define $\boldsymbol{A = ab}$ we can re-write:

$$\boldsymbol{\tau = IAB}$$

If there are multiple ($\boldsymbol{N}$) coils, the expression becomes:

$$\boldsymbol{\tau = NIAB = \mu B}$$



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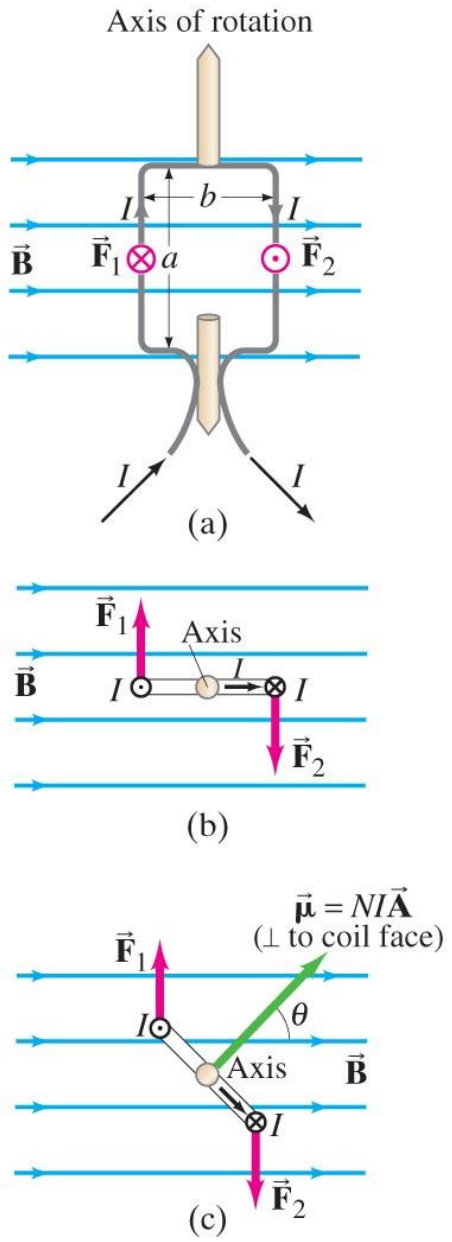
where $\boldsymbol{\vec{\mu} = NI\vec{A}}$ is called magnetic dipole moment (always perpendicular to coil face).



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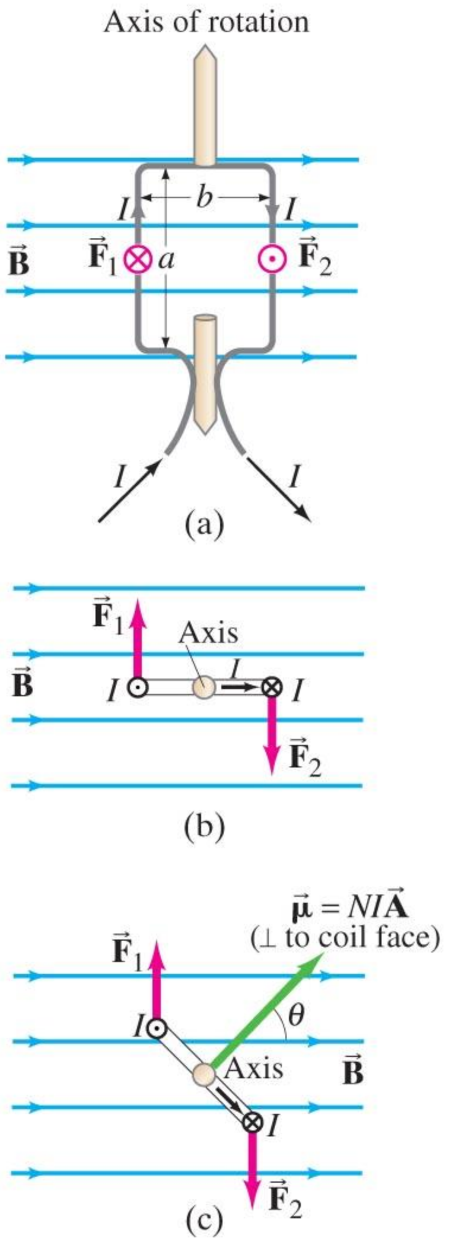
Note that the two forces F_1 and F_2 remain perpendicular to both the electric current and the magnetic field.



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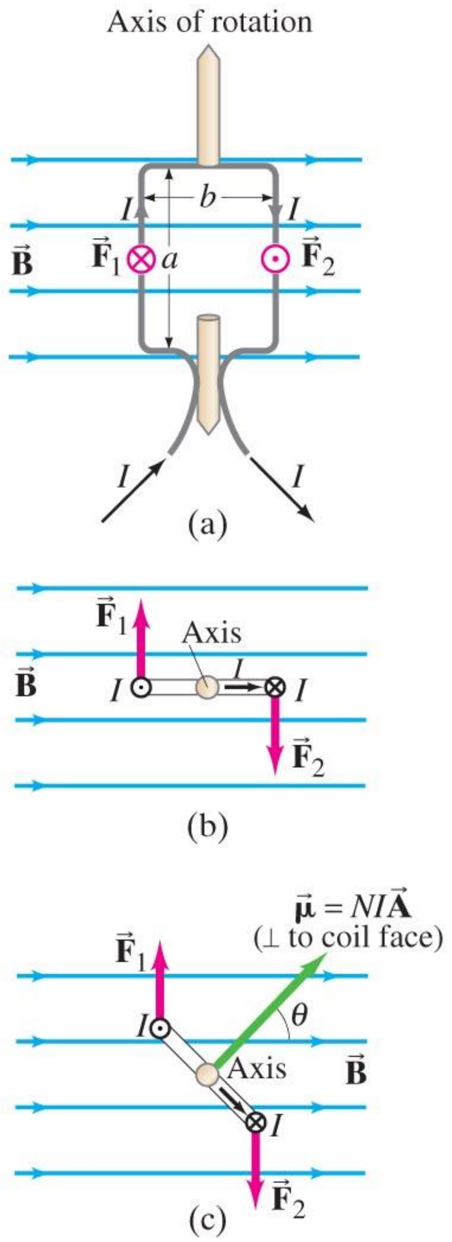
Note that the two forces F_1 and F_2 remain perpendicular to both the electric current and the magnetic field.

Hence, the torque is not constant over time and we can use the vectorial form of the magnetic dipole moment

$$\vec{\mu} = NI\vec{A}$$

to define the torque in vectorial form as:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



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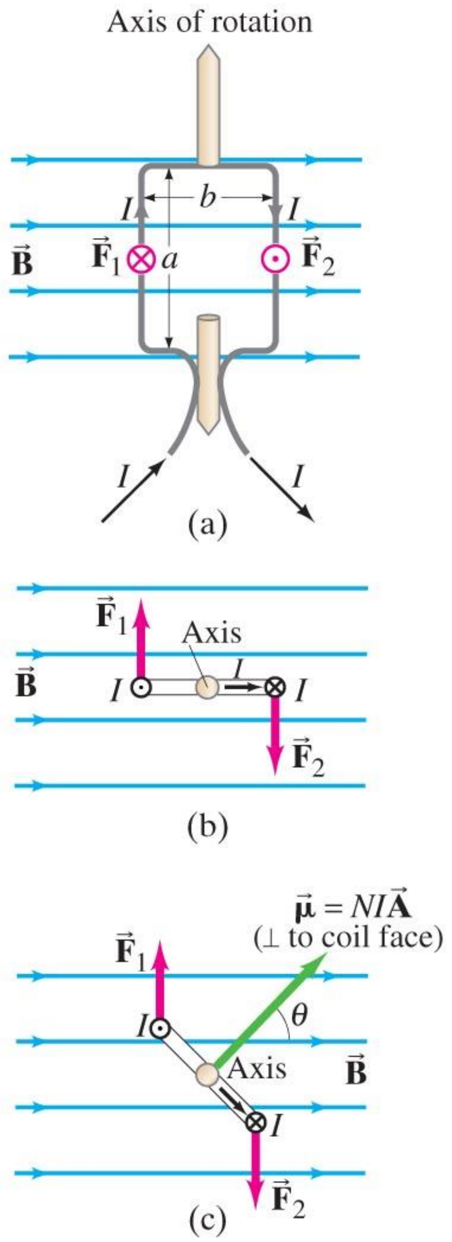
- Torque is largest when coil is parallel to magnetic field ($\theta = \frac{\pi}{2}$)
- Torque is zero when coil is perpendicular to magnetic field ($\theta = 0$)



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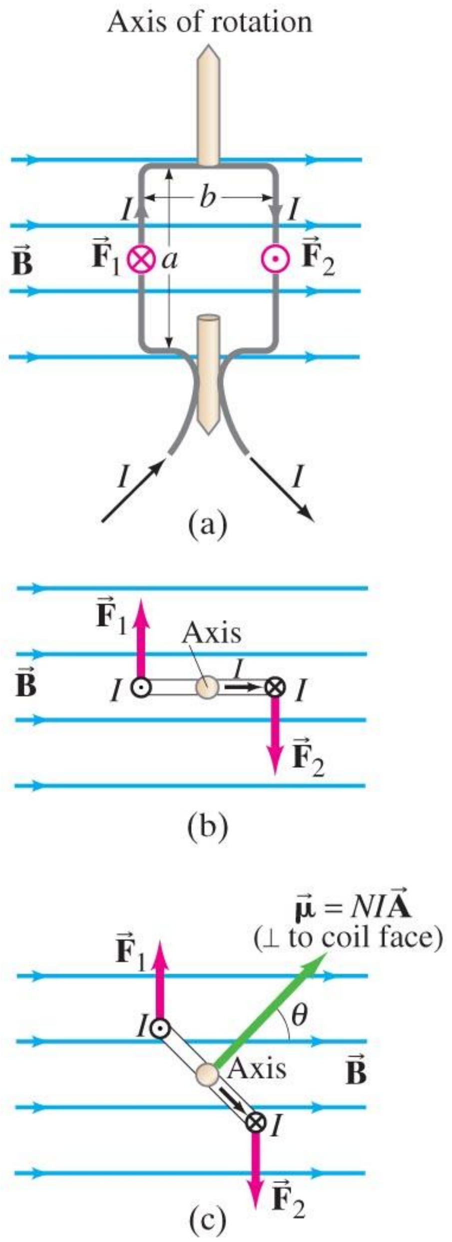
27.5 – Torque on a Current Loop; Magnetic Dipole Moment

Conversely, the potential energy of the system is defined as:

$$U = -\mu B \cos\theta$$

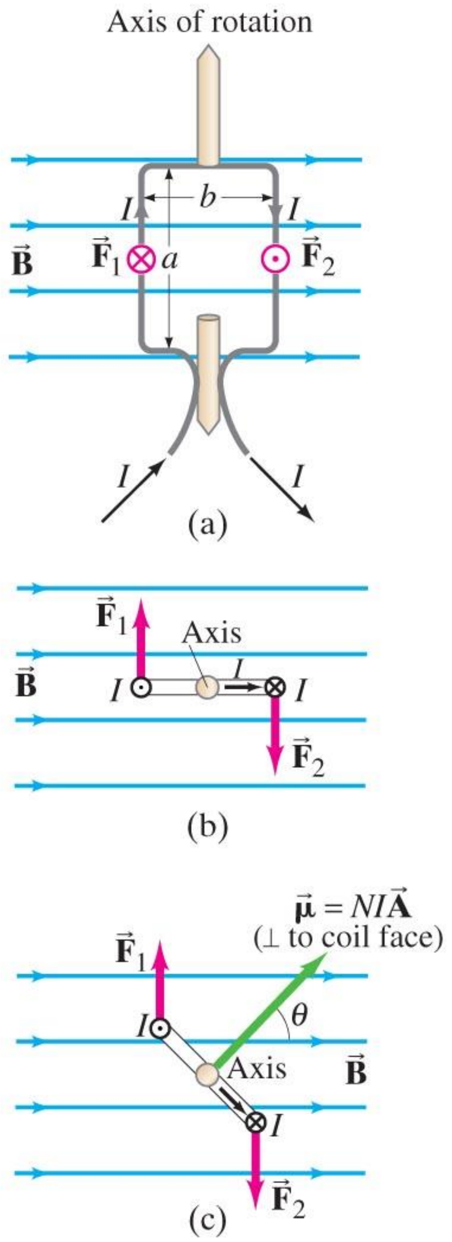


27.5 – Torque on a Current Loop; Magnetic Dipole Moment

Conversely, the potential energy of the system is defined as:

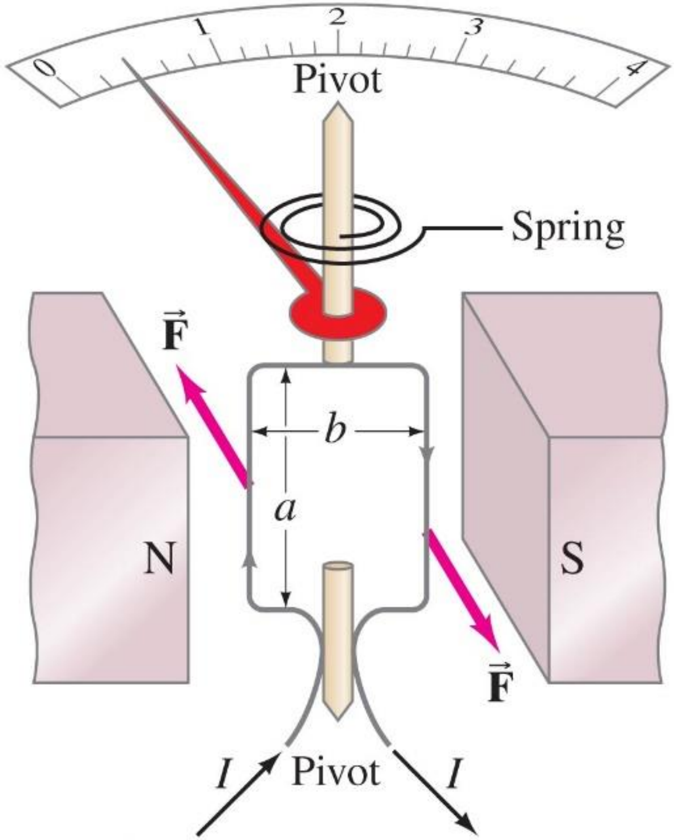
$$U = -\mu B \cos \theta$$

- It is a scalar.
- It is maximum (in absolute value) when the torque is at its minimum and vice versa. Recall the analogy with the spring-mass system.



27.6 – Applications: Galvanometer

A galvanometer is used both in ammeters and voltmeters (recall Chapter 26) to measure an unknown current in a circuit.



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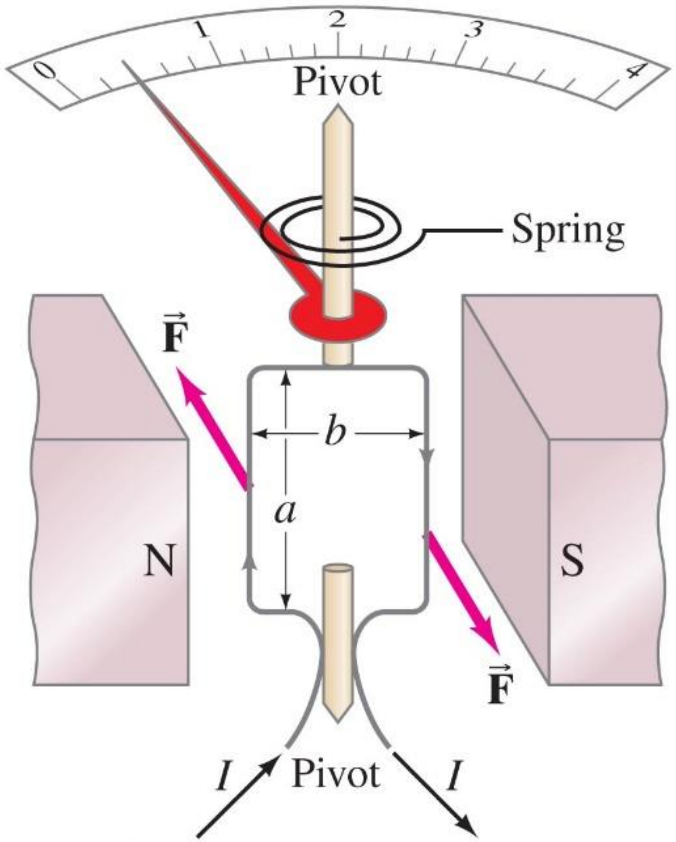
A galvanometer is used both in ammeters and voltmeters (recall Chapter 26) to measure an unknown current in a circuit.

Two torques acting on the coil: the one caused by the magnetic field and the one caused by a rotational spring: the torque wants to rotate the loop around its axis of rotation while a spring counteracts this tendency:

$$\begin{cases} \tau = NIAB\sin\theta \\ \tau = k\phi \end{cases}$$



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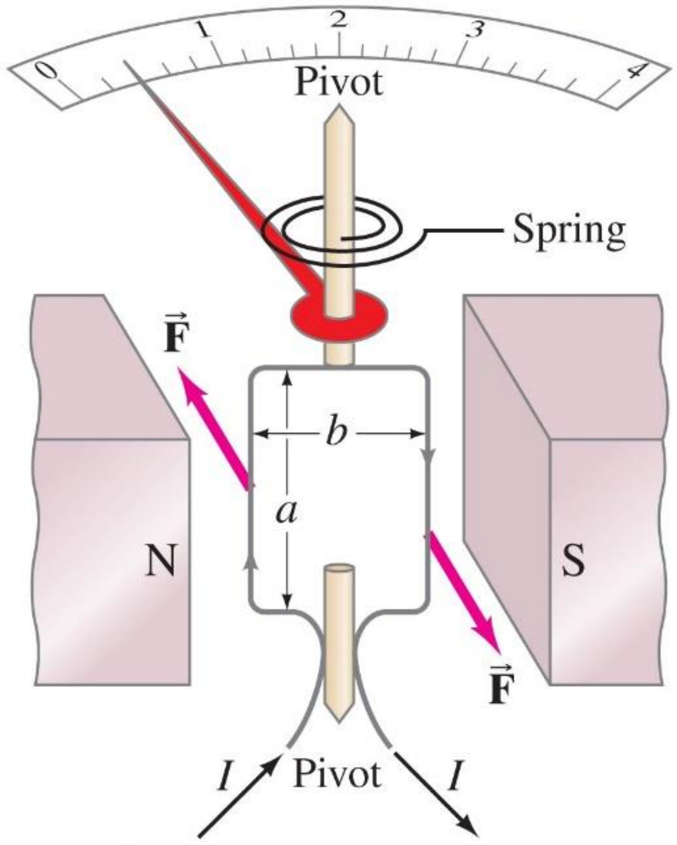

Our goal is to map the rotation of the pointer (ϕ) into the unknown current (I).

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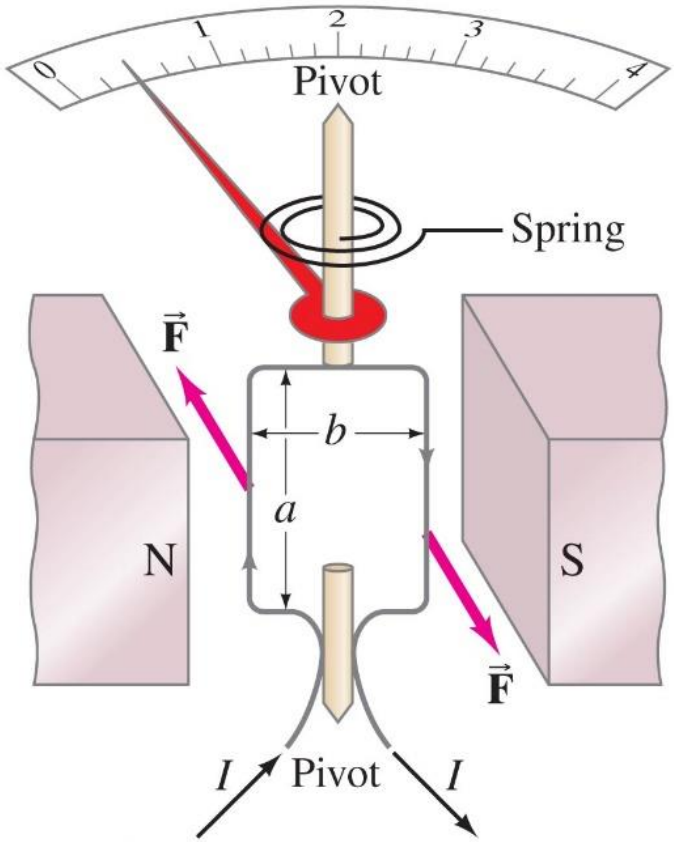
27.6 – Applications: Galvanometer

Equating the two terms from the previous slide:

$$\boldsymbol{\phi = \frac{NIAB\sin\theta}{k}}$$

which comes with the complication that the deflection does not linearly depend on the current because of the sine operator (we do not know the rotation of the coil, as it is not generally visible/measurable).



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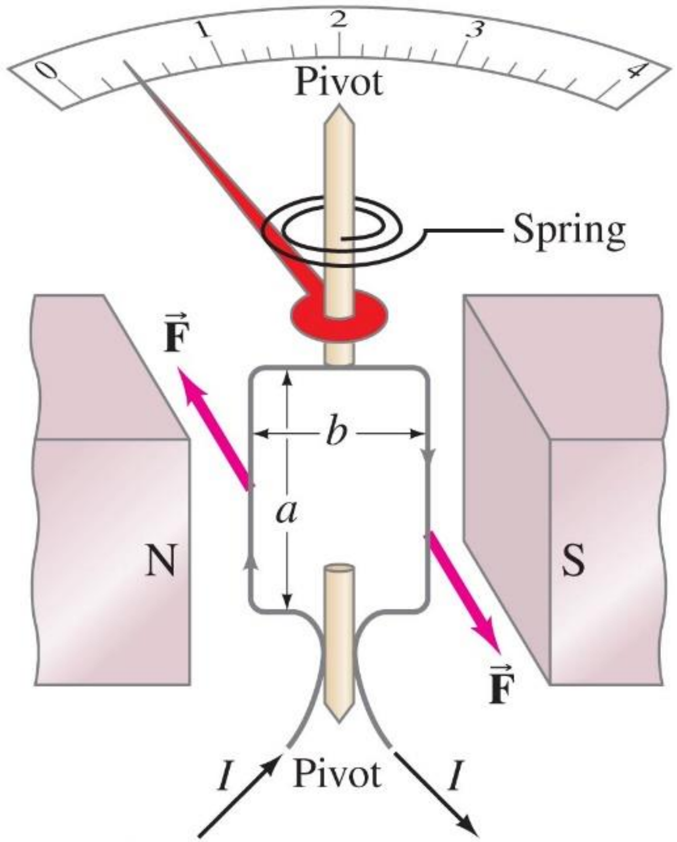
Hence, a cylindrical iron core is placed around the coil. The effect is that it concentrates field lines so that they are always perpendicular to the force ($\theta = \frac{\pi}{2}$).



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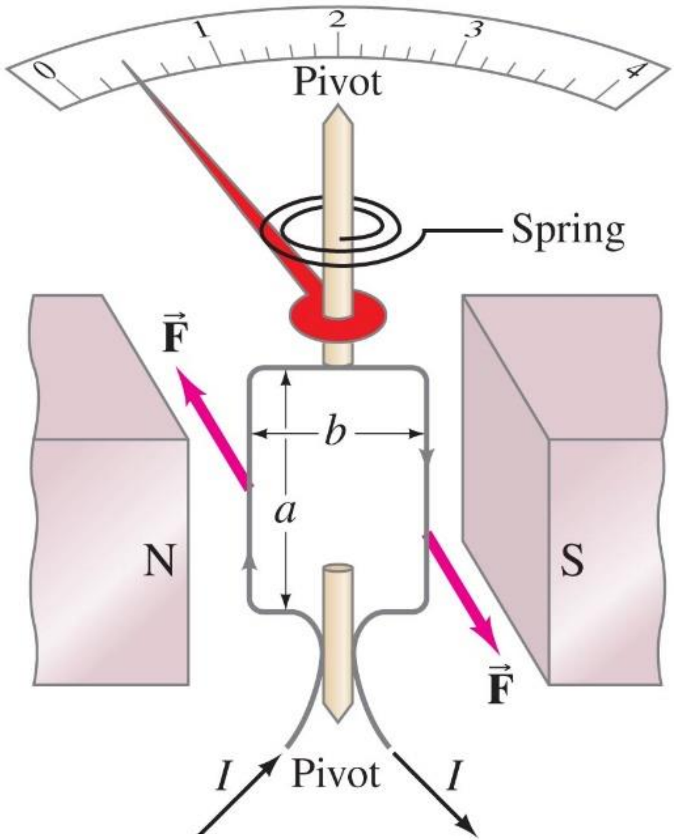
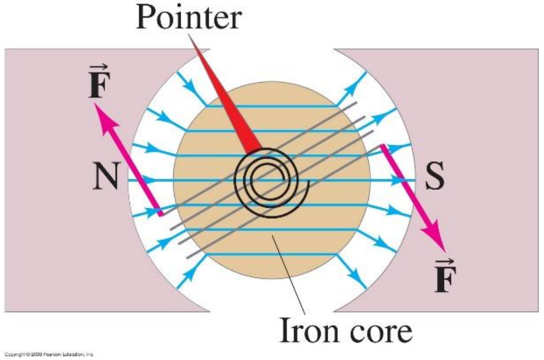
$$I = \frac{\phi k}{NAB} = f(\phi)$$





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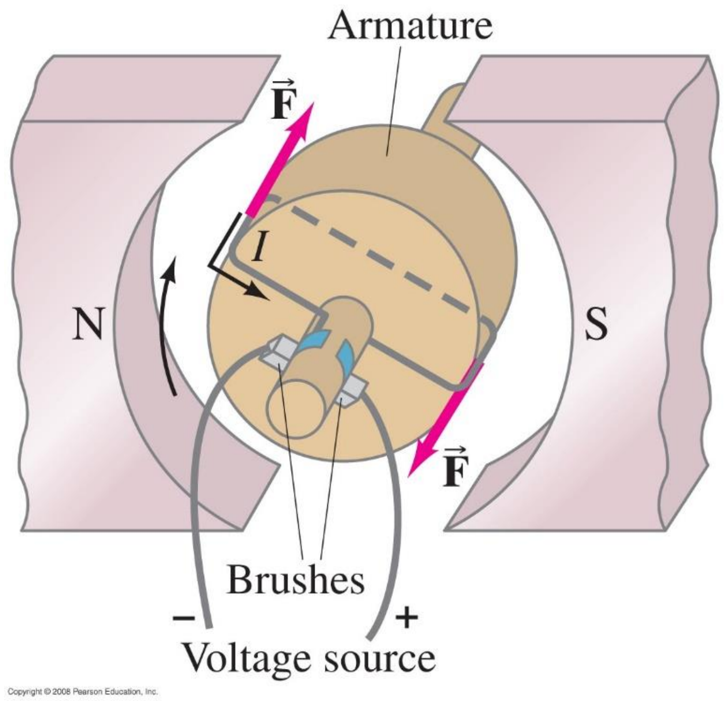
27.6 – Electric motor

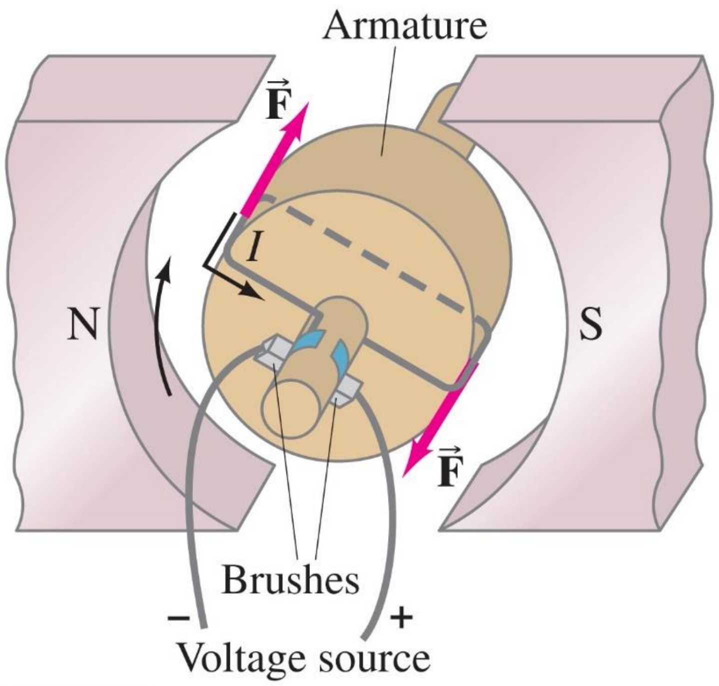
An electric motor **relies on the same working principle as the galvanometer**.

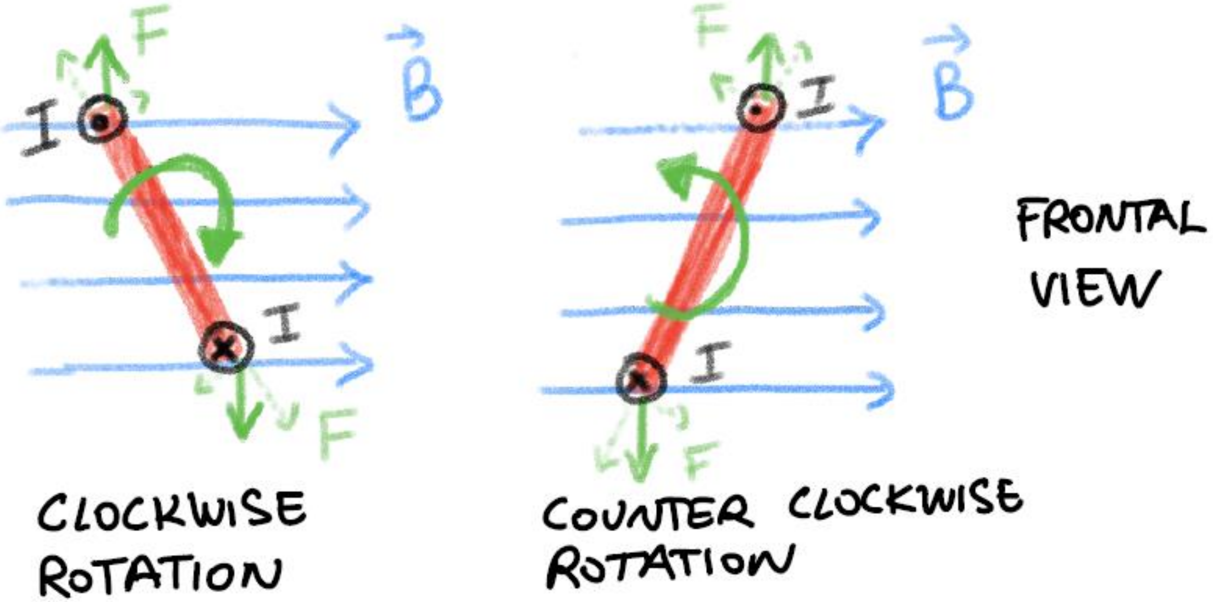


27.6 – Electric motor

An electric motor **relies on the same working principle as the galvanometer**.

It translates electric current into rotational energy. What is an issue given that we discussed so far?



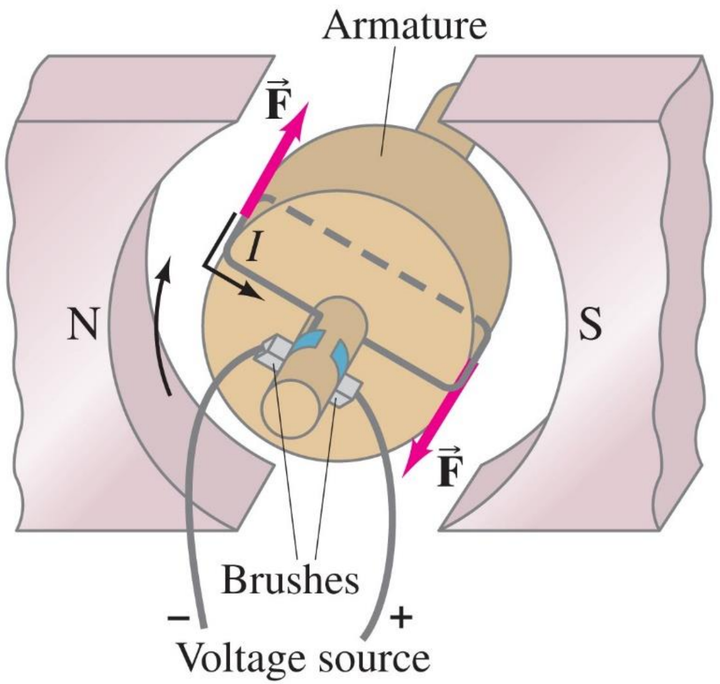


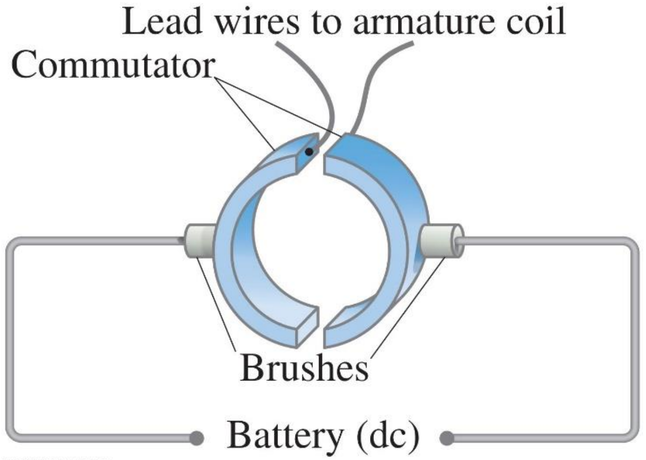
27.6 – Electric motor

If we want the rotor to constantly rotate in the same direction (as it is usually the case with a motor), we need to reverse the direction of the current every half rotation.

This can be achieved with a combination of commutators (that rotate with the shaft) and brushes (that are fixed).

Explanation video: https://www.youtube.com/watch?v=CWulQ1ZSE3c





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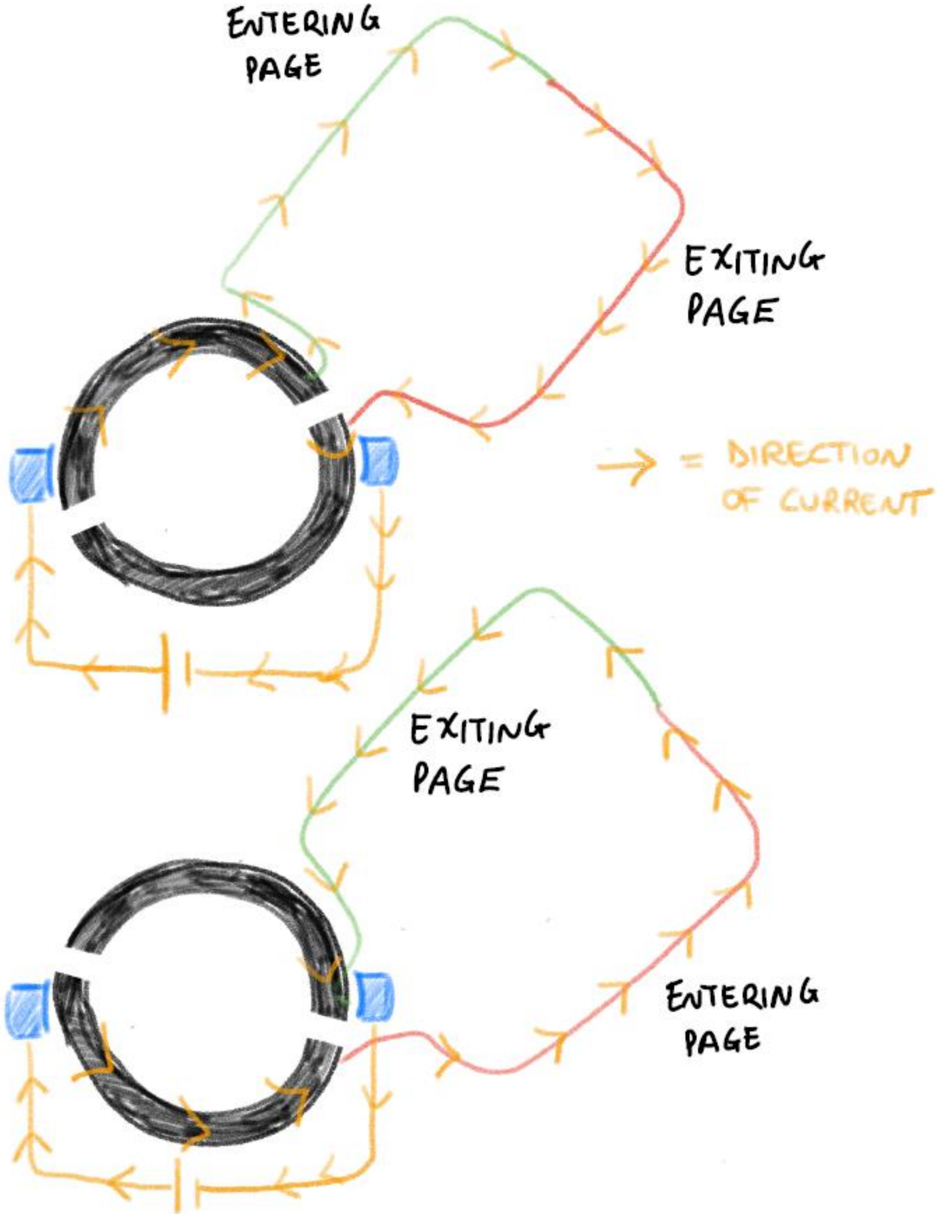
27.6 – Electric motor

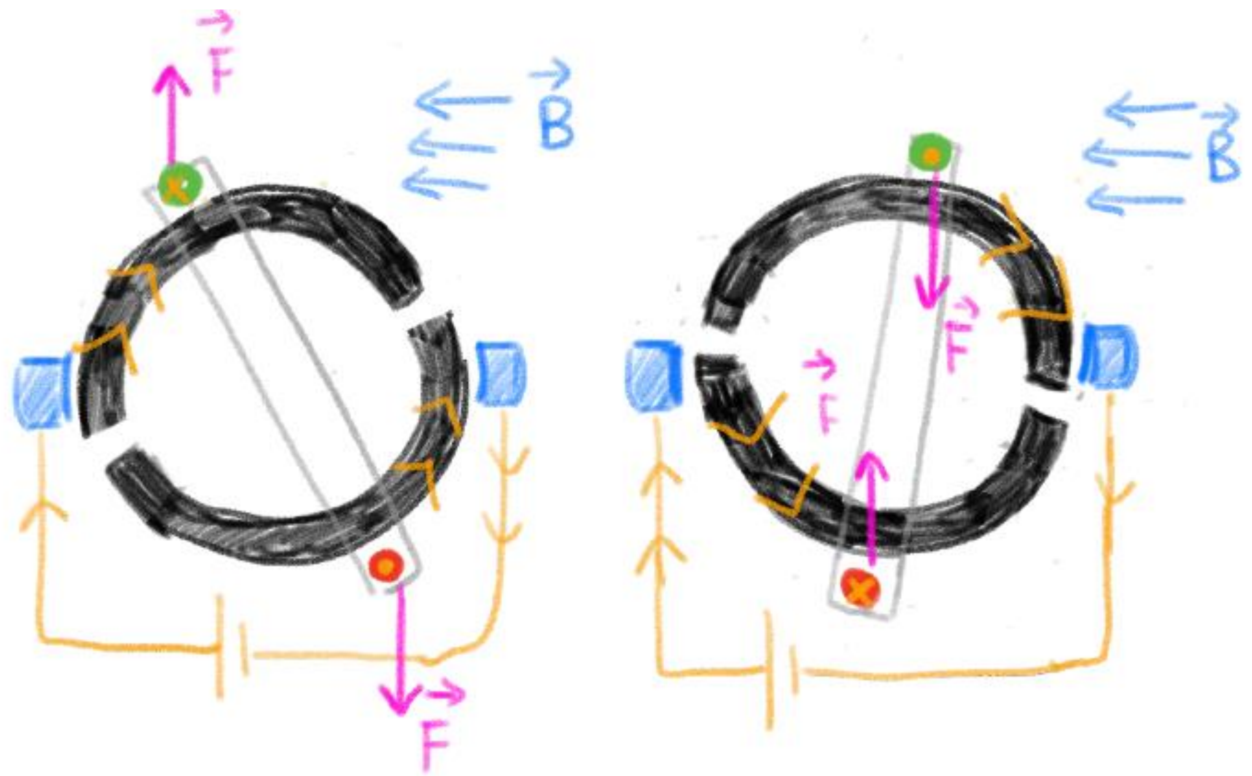
As long as the commutator of the green part of the coil is connected with the left brush (top figure), then the current points into the page in the green part and out of the page in the red part.

Once the commutator of the green part of the coil connects to the right brush (bottom figure), the orientations of the current are reversed.



27.6 – Electric motor

This is the frontal view of the same two cases as the previous slide.



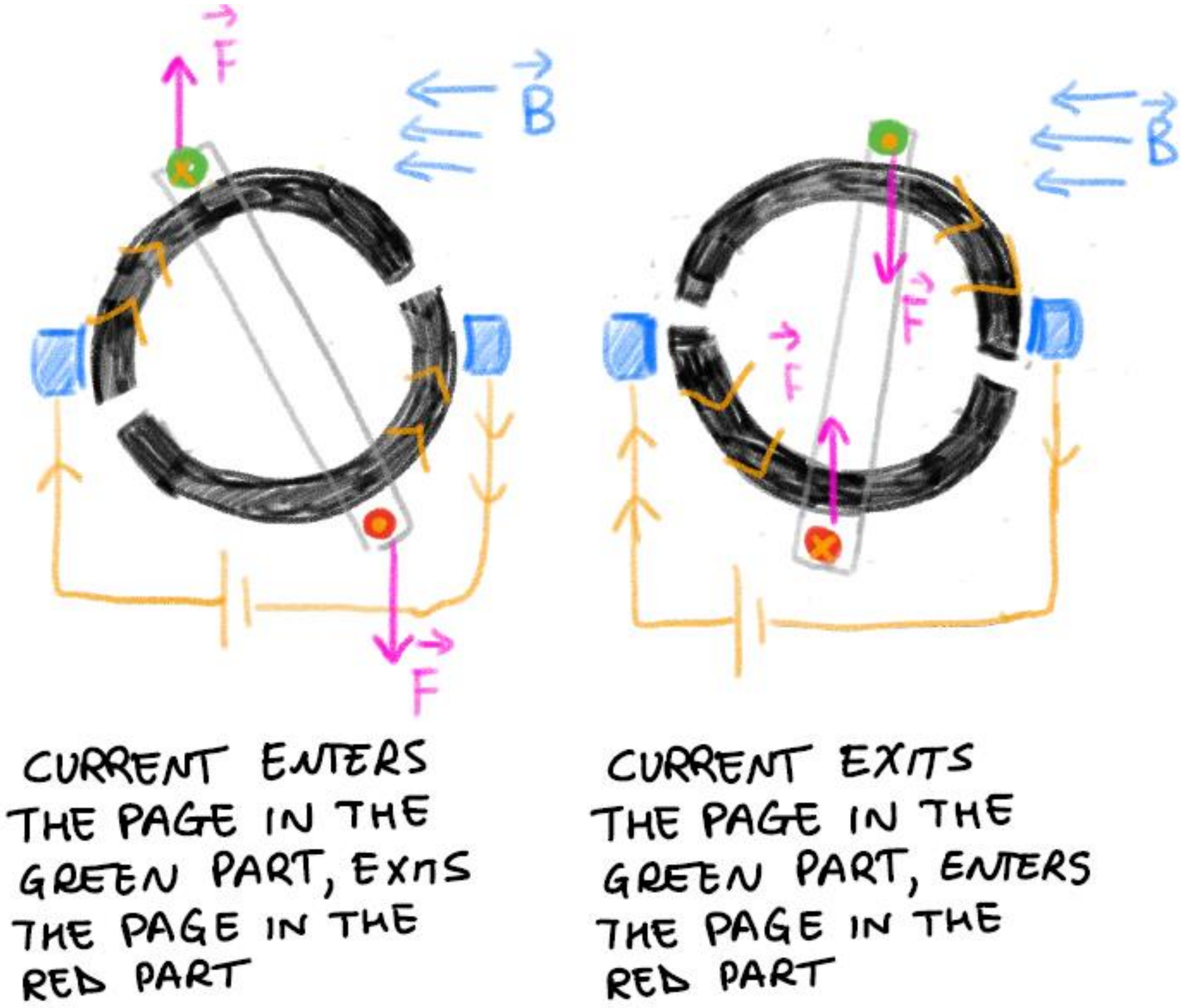
CURRENT ENTERS THE PAGE IN THE GREEN PART, EXITS THE PAGE IN THE RED PART

CURRENT EXITS THE PAGE IN THE GREEN PART, ENTERS THE PAGE IN THE RED PART

27.6 – Electric motor

This is the frontal view of the same two cases as the previous slide.

On the left, given the orientations of the currents in the green and red parts of the coil, the two components of the forces perpendicular to the coil result in a clockwise torque.



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27.6 – Electric motor

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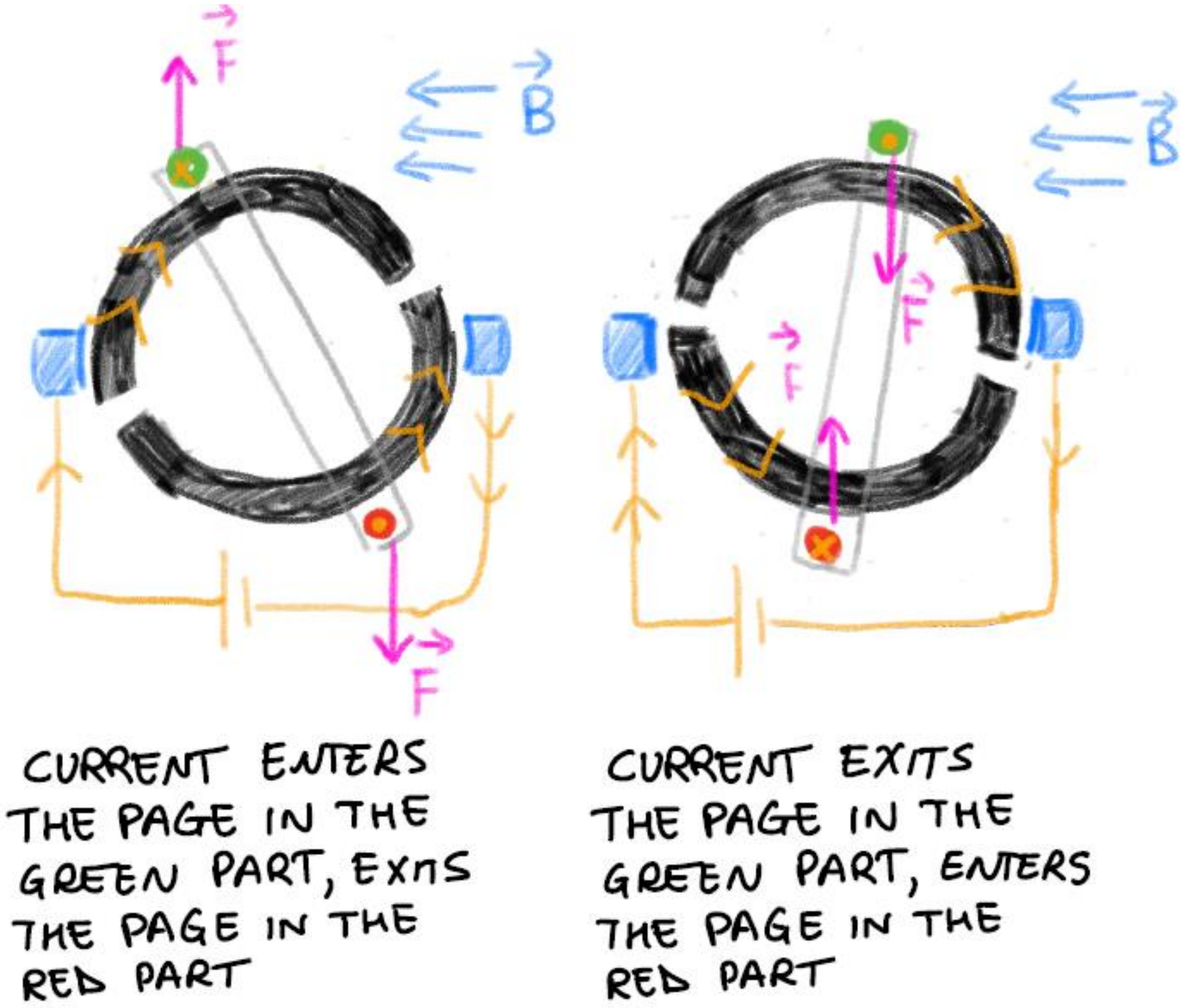
On the left, given the orientations of the currents in the green and red parts of the coil, the two components of the forces perpendicular to the coil result in a clockwise torque.

On the right, the orientations of the currents in the green and red parts of the coil are reversed and the coil has surpassed the vertical axis of symmetry. The two components of the forces perpendicular to the coil still result in a clockwise torque.



27.6 – Electric motor

Having fixed the rotational problem, it might be argued that, with the current setting, the maximum torque is achieved only twice per complete revolution.

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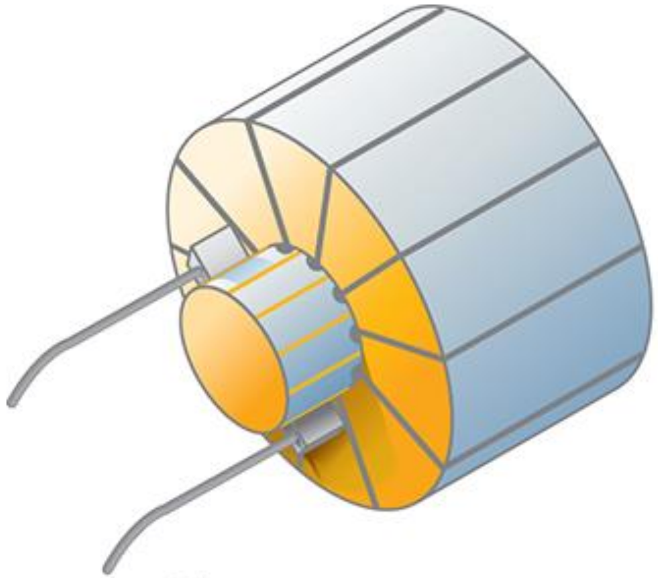
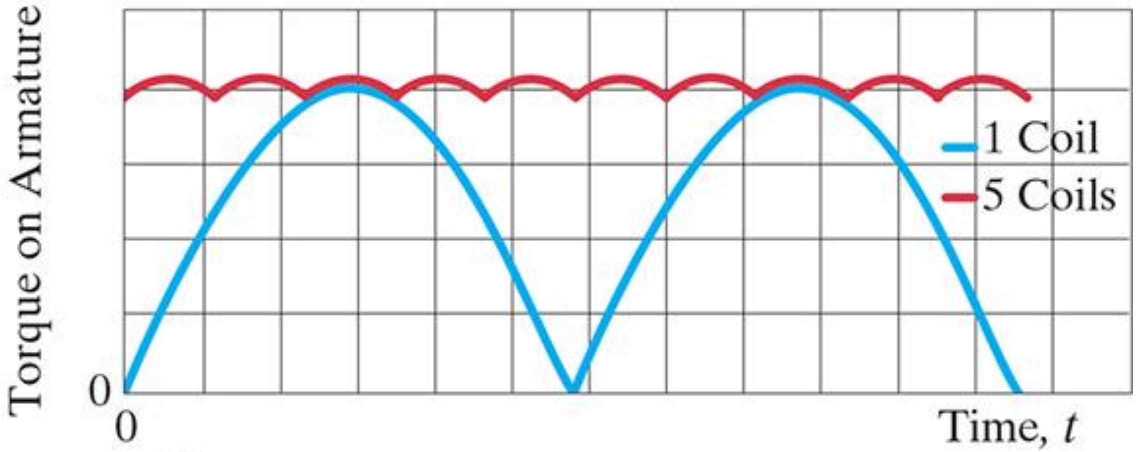
Having fixed the rotational problem, it might be argued that, with the current setting, the maximum torque is achieved only twice per complete revolution.

If we equip the motor with multiple displaced coils (windings), each connected to a different portion of the armature and with current flowing there for a small portion of the revolution, we obtain a much steadier torque.



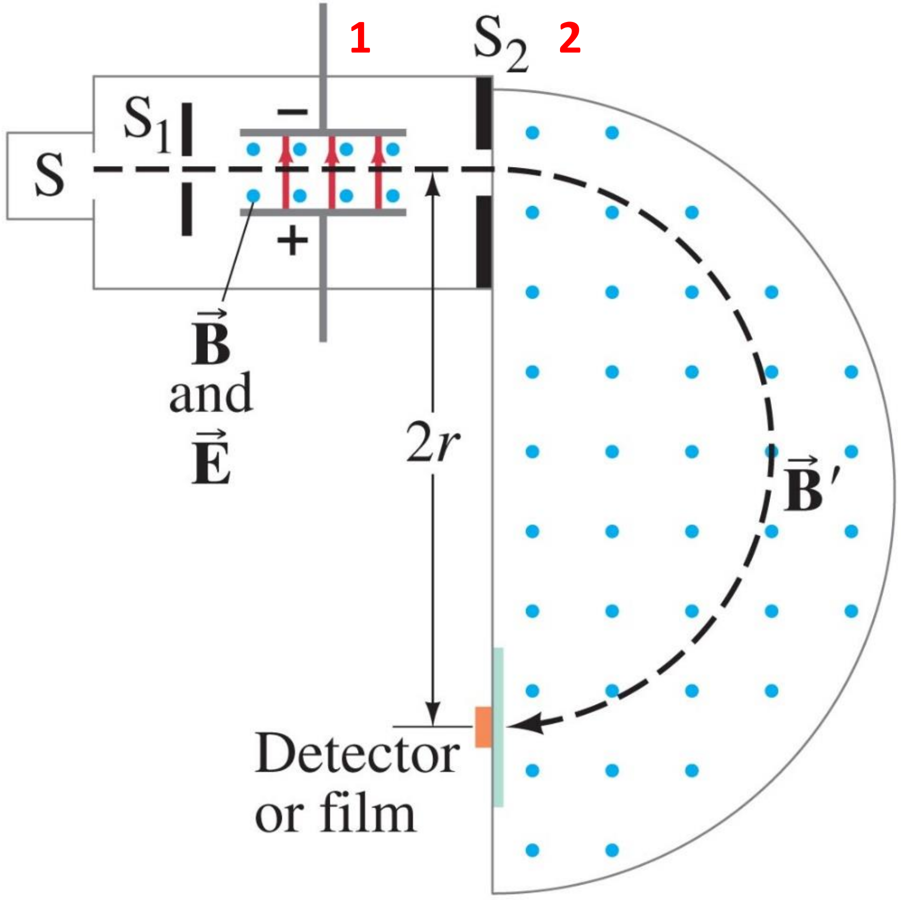
(a)



(b)

27.9 – Mass spectrometer

Goal: using a velocity selector and a magnetic field to determine the mass of an unknown element.
We can heat the analyzed sample to produce ions that are accelerated into region 1.



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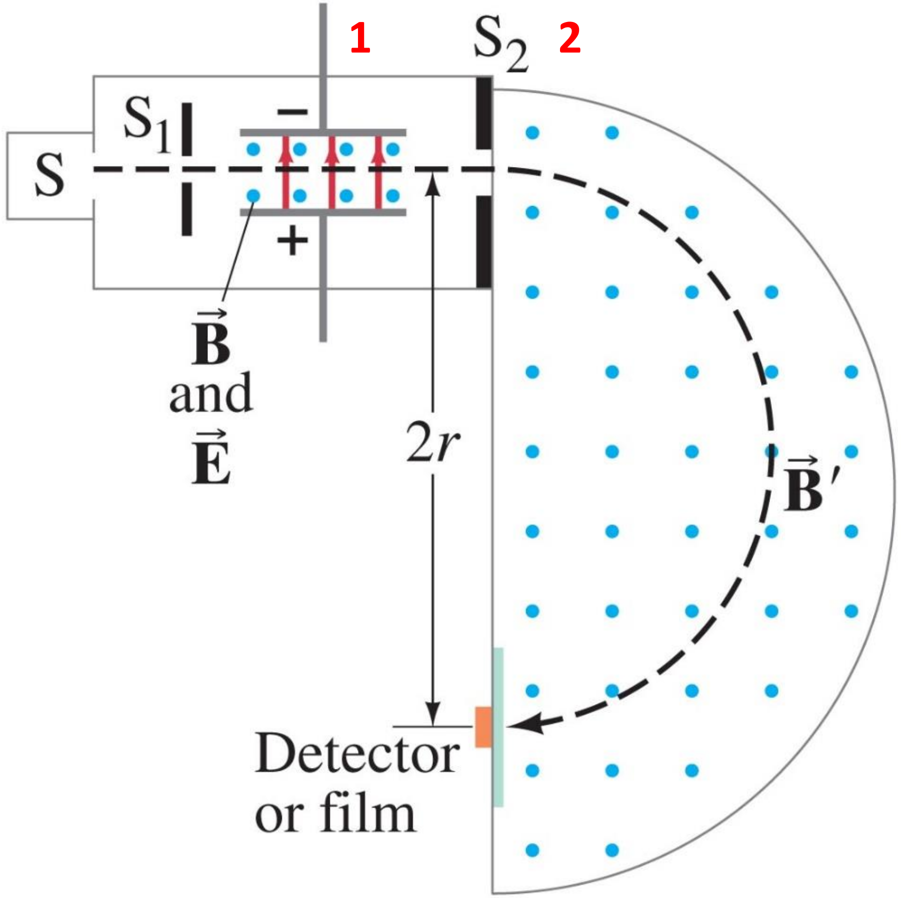

In region 1, we do have a velocity selector with known (and perpendicular) electric field E and magnetic field B. Hence, because of the velocity selector properties, the only ions that exit the selector via slit S_2 are the ones that entered it via S_1 with velocity:

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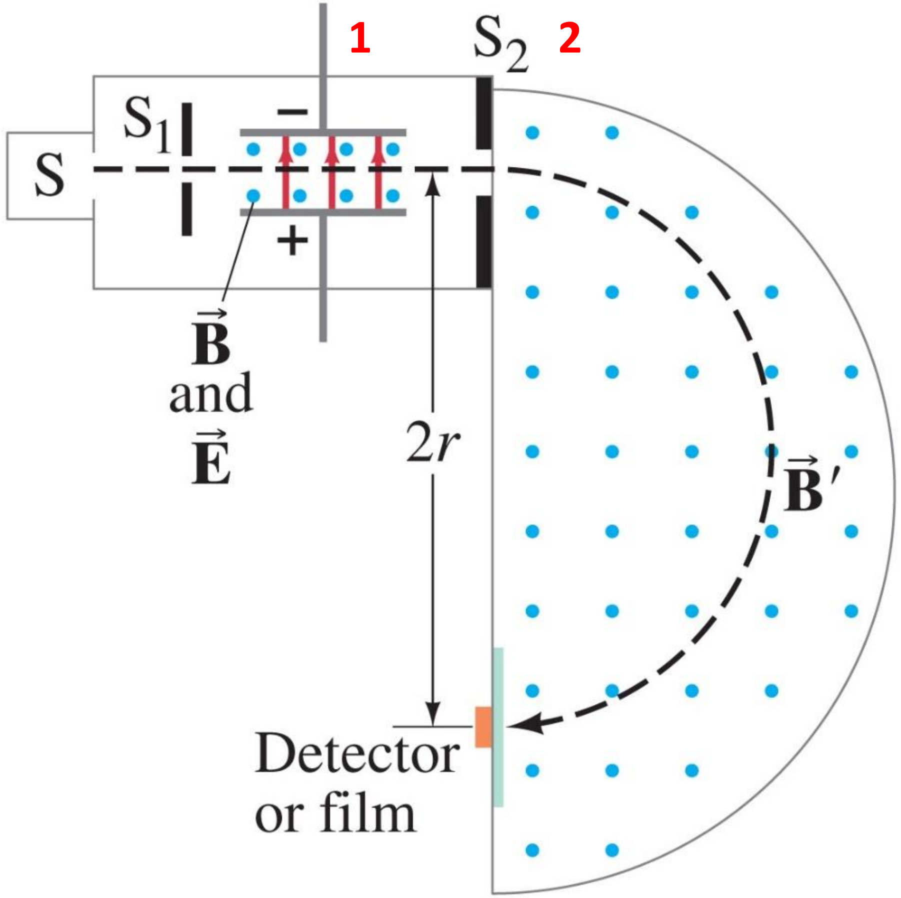

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$$qE = qvB \rightarrow v = \frac{E}{B}$$

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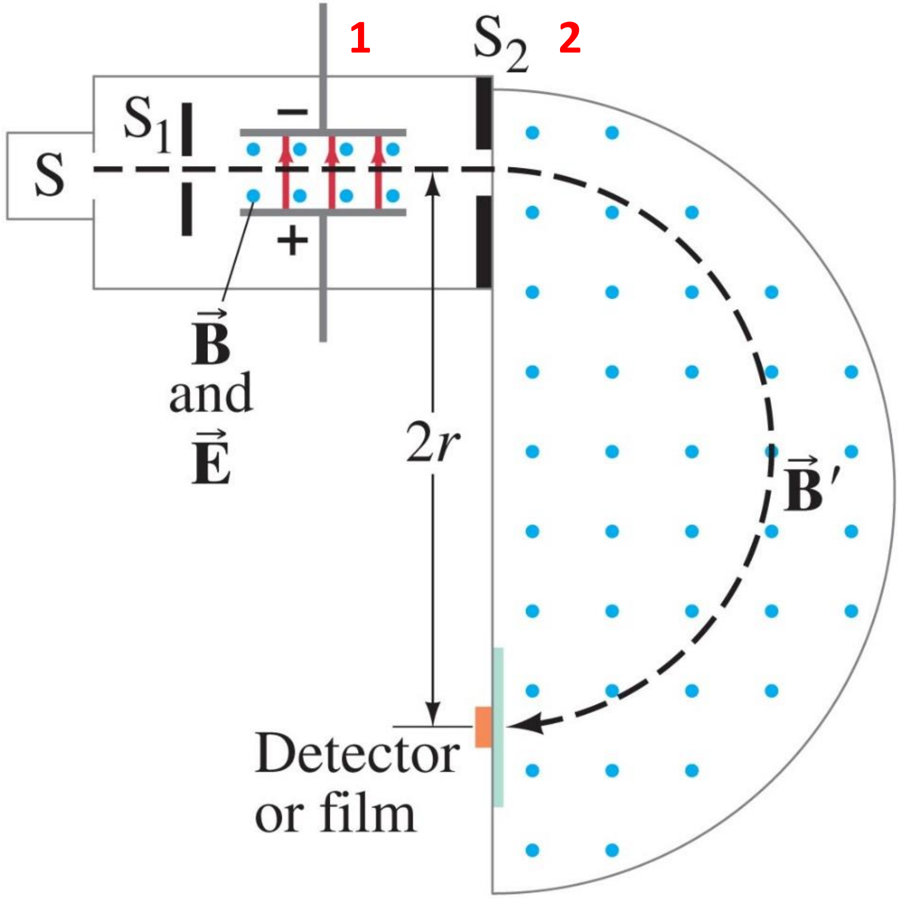

In region 2, we only have a magnetic field B'. By determining where the ion impacts the lower portion of the spectrometer (e.g., via a detector or film), we can determine the mass combining the two equations:

$$\frac{mv^2}{r} = qvB'$$

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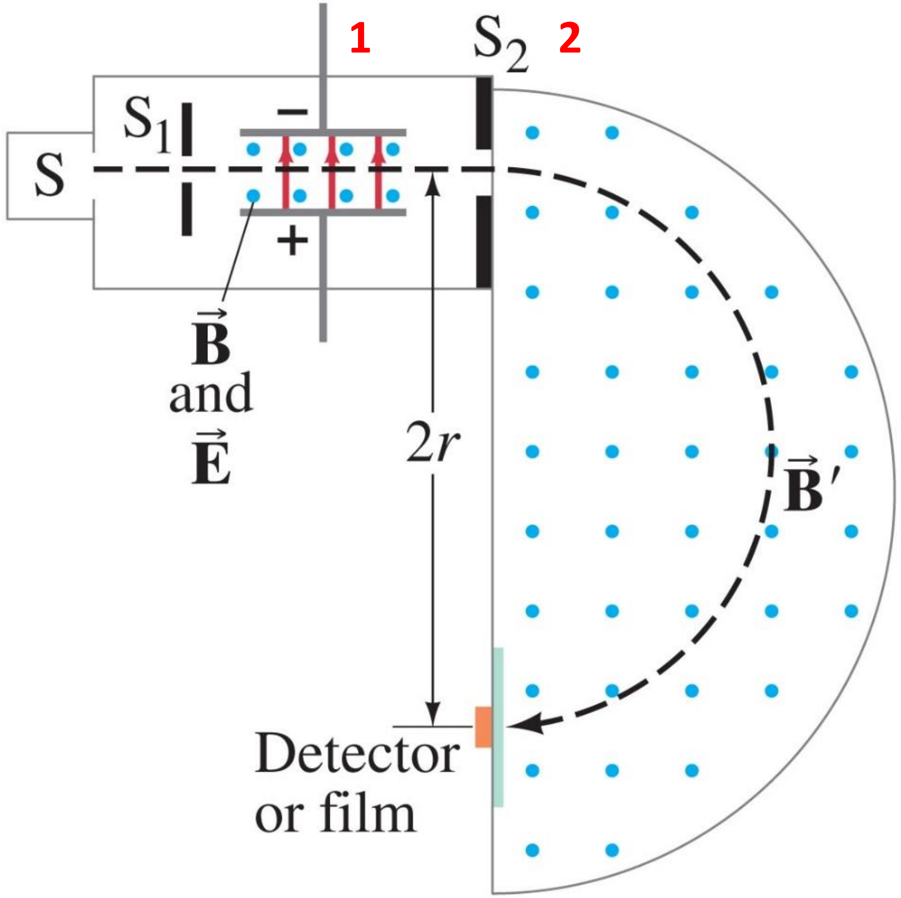

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that result into:

$$m = \frac{qB'r}{v} = \frac{qB'Br}{E}$$

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We indeed detect a second element on our detection film with radius 26.2 cm. Which element might that be?

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Carbon (C): $\boldsymbol{r_C = \frac{m_C E}{qB'B}}$

Unknown element (X): $\boldsymbol{r_X = \frac{m_X E}{qB'B}}$

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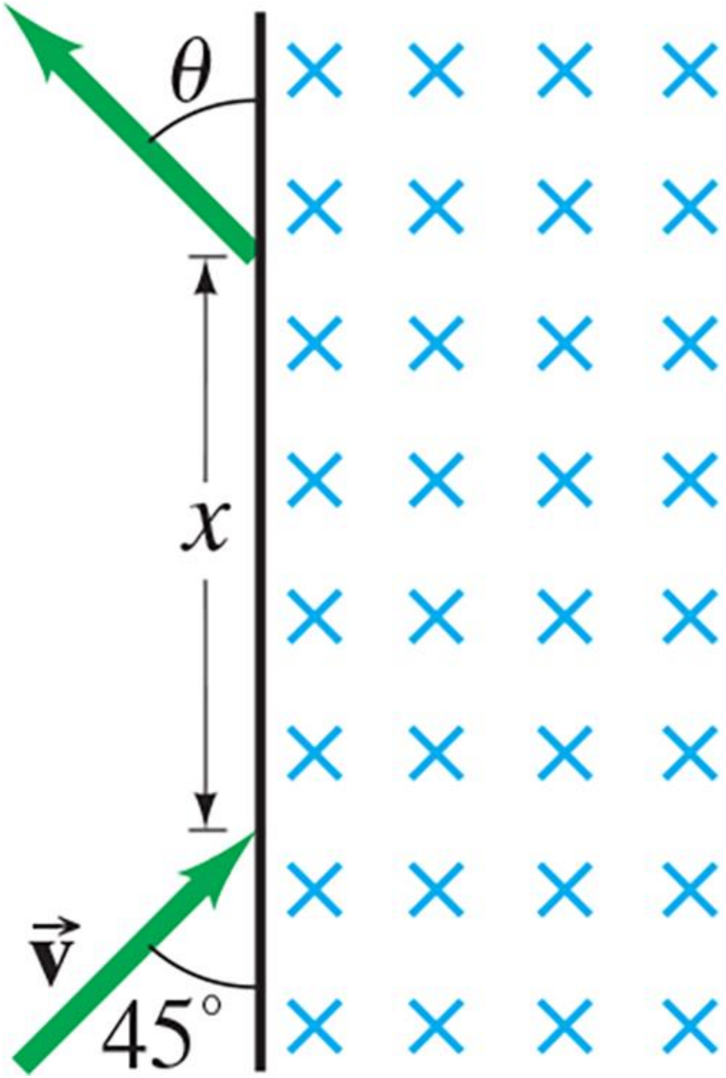
Unknown element (X): $\boldsymbol{r_X = \frac{m_X E}{qB'B}}$

It follows that $\boldsymbol{\frac{r_C}{m_C} = \frac{r_X}{m_X} \rightarrow m_X = m_C \frac{r_X}{r_C}} = 14u$ (nitrogen or isotope)

Homework

$$v = 1.8 \times 10^5 \frac{m}{s}$$
$$B = 0.850\ T$$

Determine the values of θ and x.



Wrap-up: revisiting Learning objectives

After today's lecture you should be able to:

- Explain the **relationship between electric current and a magnetic field** in terms of resulting force and some cornerstone applications
- Explain and apply the concept of **Lorentz equation**
- Explain the concept of **magnetic dipole** and some cornerstone applications:

$$\boldsymbol{d\vec{F} = I\, d\vec{l} \times \vec{B}}$$

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- Explain the concept of **magnetic dipole** and some cornerstone applications:

$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Wrap-up: revisiting Learning objectives

<u>After today's lecture you should be able to:</u>

- Explain the **relationship between electric current and a magnetic field** in terms of resulting force and some cornerstone applications
- Explain and apply the concept of **Lorentz equation**
- Explain the concept of **magnetic dipole** and some cornerstone applications:

$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{\mu} = NI\,\vec{A}$$

TU Delft

MAGNETISM

Chapter 27

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