Proof of Concept for Modified Loss Function in Multi-Scale Noise Video Anomaly Detection

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Intuition

The original paper sets a broad interval for the noise scale, with $\sigma_{\text{low}} = 0.001$ and $\sigma_{\text{high}} = 1.0$. Empirical results from the paper show that the model performs best when using a noise scale centered around $\sigma = 0.33$, while performance significantly drops for values outside the range $[0.2, 0.5]$.

Our goal is to modify the weighting function $\lambda(\sigma)$ in the loss function to:

- Emphasize values near $\sigma = 0.33$, where the model achieves optimal performance.
- Minimize contributions from noise scales outside $[0.2, 0.5]$, which empirically show reduced performance.

By focusing on the most effective noise scales, we encourage the model to prioritize learning features that are particularly useful for anomaly detection, potentially enhancing detection performance.

Change in Loss Function

In the original loss function:

$$\mathcal{L} = \left[\lambda(\sigma) \left\| \nabla_{\tilde{\mathbf{x}}} f_\theta(\tilde{\mathbf{x}}, \sigma) - \frac{\tilde{\mathbf{x}} - \mathbf{x}}{\sigma^2} \right\|_2^2 + \beta f_\theta(\mathbf{x}, \sigma)^2\right], \tag{1}$$

the weighting function $\lambda(\sigma) = \sigma^2$ applies **uniformly** across all noise levels. Our modification redefines $\lambda(\sigma)$ to give higher priority to values near $\sigma = 0.33$ and reduce the impact of values outside the range $[0.2, 0.5]$.

Proposed Weighting Function $\lambda(\sigma)$

To emphasize the optimal noise scale at $\sigma = \sigma_0$ and decay contributions outside $[\sigma_0 - \sigma_{\text{spread}}, \sigma_0 + \sigma_{\text{spread}}]$, we define $\lambda(\sigma)$ as a Gaussian centered at $\sigma = \sigma_0$, with a standard deviation of σ_{spread}:

$$\lambda(\sigma) = \sigma^2 \cdot \exp\left(-\frac{(\sigma - \sigma_0)^2}{2 \times \sigma_{\text{spread}}^2}\right). \tag{2}$$

where $\sigma_0 = 0.33$ and $\sigma_{\text{spread}} = 0.075$.

This weighting function has the following properties:

- **Peaks near** $\sigma = 0.33$: Maximizes the contribution in the most effective noise scale region.
- **Diminishes outside** $[0.2, 0.5]$: Reduces the impact of suboptimal noise levels.

Final Formula for the Modified Loss Function

Substituting the new $\lambda(\sigma)$ into the loss function, we get:

$$\mathcal{L} = \left[\sigma^2 \cdot \exp\left(-\frac{(\sigma - \sigma_0)^2}{2 \times (\sigma_{\text{spread}})^2}\right) \cdot \left\| \nabla_{\tilde{\mathbf{x}}} f_\theta(\tilde{\mathbf{x}}, \sigma) - \frac{\tilde{\mathbf{x}} - \mathbf{x}}{\sigma^2} \right\|_2^2 + \beta f_\theta(\mathbf{x}, \sigma)^2\right]. \tag{3}$$

where $\sigma_0 = 0.33$ and $\sigma_{\text{spread}} = 0.075$

Why This Change Helps

- **Focus on Optimal Noise Scale**: By emphasizing learning at $\sigma = 0.33$, the model concentrates on the features most effective for anomaly detection, potentially improving accuracy.
- **Reduced Contribution of Suboptimal Noise Levels**: Values outside $[0.2, 0.5]$ contribute minimally due to the Gaussian weight, which reduces distracting influences from less effective noise levels.
- **Alignment with Empirical Results**: This weighting aligns directly with the empirical observations in the paper, thus making the training process more efficient and targeted.