# Proportional-Derivative Control of a One Degree of Freedom Helicopter System: Dynamics and Implementation

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#### **Abstract**

This study delves into the dynamics and control strategies of a One Degree of Freedom (One-DOF) helicopter system, serving as a simplified model for understanding rotary-wing aircraft behavior. The research focuses on addressing challenges related to stability and precision in helicopter flight, with a specific emphasis on the application of a Proportional-Derivative (PD) controller. The mechanical structure incorporates essential components such as a coreless motor, MPU 6050 IMU sensor, L298 motor driver, and an Arduino UNO microprocessor. Through the LabVIEW LINX Toolkit, a serial communication interface is established with the Arduino UNO to acquire gyroscope data from the MPU6050. Experimental results highlight the empirical determination of PD controller coefficients (Kp and Td) through systematic testing, showcasing the PD controller's effectiveness in regulating the system's motion.

Keywords: Rotary-Wing Aircraft Dynamics, PD Controller Tuning, One-DOF Helicopter Model, Arduino UNO Interface

## 1. Introduction

The one-degree-of-freedom helicopter, commonly known as the single-rotor helicopter, serves as a pivotal and streamlined model essential for comprehending the intricate dynamics and control principles inherent in rotary-wing aircraft. Within this system, the helicopter's motion is intentionally confined to a singular rotational degree of freedom along its axis, strategically providing an optimal platform for the in-depth study of fundamental control strategies. This deliberate simplification distills the inherent complexities associated with full-scale helicopters, allowing researchers and engineers to focus with precision on critical aspects of control theory and dynamics.

Within the realm of helicopter operations, a paramount challenge resides in the maintenance of stability and precision during flight. To address this challenge, sophisticated control algorithms are employed, with the Proportional-Derivative (PD) controller emerging as a preeminent choice. The PD controller stands as a universally embraced feedback control mechanism, skillfully adjusting the system's behavior based on both the prevailing error and its rate of change. This strategic implementation empowers engineers to masterfully stabilize and regulate the helicopter's motion, ensuring a responsive and meticulously controlled flight.

## 2. Materials

The essential components employed in this study are meticulously cataloged in Table 1, presenting a detailed inventory accompanied by visual representations, including Arduino UNO, Sensor Module, Coreless Motor, Propeller, L298N Motor Driver, and filament for the 3D printing of the structure.



Table 1: Table of Components

## 3. Mathematical Model of the System

The equation of motion of the system is given by:

$$J\ddot{\theta} = f_{th}L_m + m_m g L_m \theta + m_s g L_s \theta$$

The Laplace-transformed equation is given by:

$$F_{th}(s) = \frac{J}{L_m} s^2 \Theta(s) + m_m g s \Theta(s) - \frac{L_s}{L_m} m_s g s \Theta(s)$$

$$F_{th}(s) = \left(\frac{J}{L_m}s^2 + m_m g s - \frac{L_s}{L_m} m_s g s\right) \Theta(s)$$

$$G(s) = \frac{1}{\frac{J}{J_{m}}s^{2} + m_{m}gs - \frac{L_{s}}{J_{m}}m_{s}gs}$$

Given  $m_m = 4.1674$  grams,  $m_s = 1.688$  grams,  $L_m = 6.2$  cm,  $L_s = 6.2$  cm,  $m_{rod} = 3.7146$  grams, and  $L_{rod} = 16.1$  cm, the open loop transfer function is given by:

$$G(s) = \frac{1931.464}{s^2 + 57.944}$$

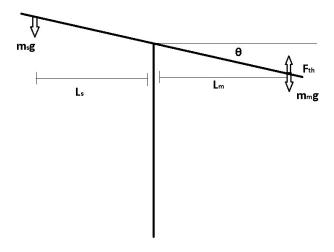


Figure 1: A Schematic of the System

The block diagram of the system us presendted in figure 3.

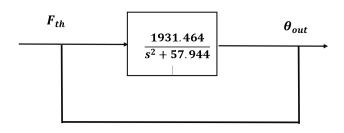


Figure 2: Block Diagram of the System

## 4. System Analysis and Controller Design

In the course of our investigation utilizing LabVIEW, we conducted a comprehensive analysis of the system, the results of which are presented in Figure 3 showcasing the system's step response and Figure 4 illustrating the root locus. Notably, our findings indicate that the system exhibits instability, with dominant poles positioned along the imaginary axis. This significant observation provides valuable insights into the dynamic behavior of the system, specifically highlighting the presence of critical stability issues warranting further investigation and potential remedial measures.

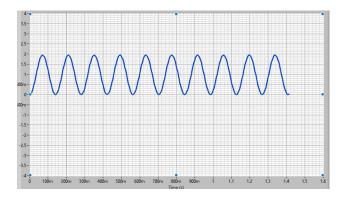


Figure 3: Step Response of the Uncompensated System

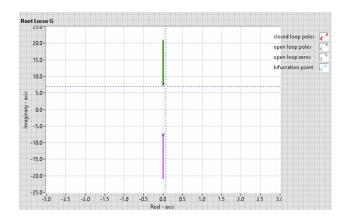


Figure 4: Root locus of the Uncompensated System

Commencing the design of the Proportional-Derivative (PD) controller involves the determination of critical parameters, specifically the critical gain ( $K_{cr}$ ) and critical time ( $P_{cr}$ ), essential for configuring the system's stability and performance characteristics. And they were obtained as follows:

The characteristic equation is given by:

$$\delta = s^2 + 1989.408$$

So, The critical gain is obtained from the gain of the system as:  $K_{crb} = 1931.464$ .

By substituting  $s=j\omega$  into the characteristic equation, we obtain:

$$\Delta = (i\omega)^2 + 1989.408$$

Simplifying further:

$$\omega^2 = 1989.408$$

Hence, 
$$\omega = 44.6$$

Therefore, the corresponding Critical periodic time  $(P_{cr})$  is given by:

$$P_{cr} = \frac{\omega}{2\pi} = 7.1 \, s$$

So, the values of the Proportional Gain  $(K_p)$  and Derivative Time  $(T_d)$  obtained using LabVIEW are  $K_p = 4.5$  and  $T_d = 0.1$ , respectively.

The Root Locus and Step Response of the compensated system are illustrated in Figures 5 and 6, respectively. The analysis reveals a notable enhancement in system stability, achieving a settling time of less than 100 ms.

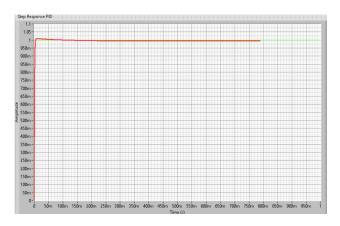


Figure 5: Step Response of the Compensated System

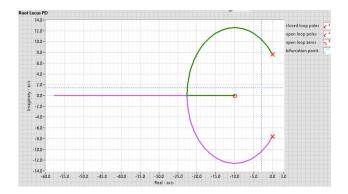


Figure 6: Root locus of the Compensated System

## 5. Real world Implementation

## 5.1. Hardware:

The project commenced with the acquisition of a 3D CAD model, meticulously tailored to meet our design specifications. After the design refinement, the model underwent fabrication through precision 3D printing technology. The mechanical

configuration of the One-Degree-of-Freedom (One-DOF) helicopter system features a foundational base and two vertical plates, strategically separated by a support plate. Notably, a cylindrical structure, intricately connected to the main rod through precision bearings, constitutes a pivotal component of the system. Figure 7 encapsulates the comprehensive Solid Works drawing, providing an insightful visual representation of the intricacies inherent in the One-DOF helicopter's mechanical structure.

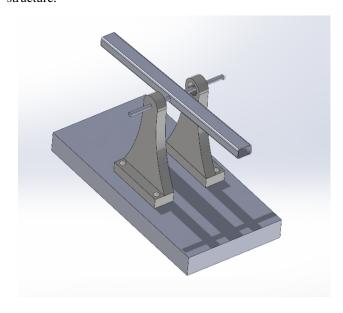


Figure 7: CAD Model of One-DOF Helicopter

The interconnection of the various components of the system, including the Coreless Motor, IMU sensor (MPU 6050), Motor Driver (L298), and Arduino UNO microprocessor, is illustrated in Figure 8.

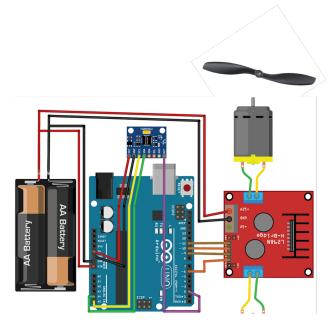


Figure 8: Interconnection of Components

## 5.2. Software:

The LabVIEW LINX Toolkit facilitated the interfacing with the Arduino UNO through serial communication. The interface was tailored to acquire gyroscope readings from the MPU6050. The implementation involves a while loop to continuously retrieve angle readings from the IMU, calculate motor power based on these readings, and apply the power to the plant. Figures (9) and (10) depict the front panel and block diagram of the LINX VI, respectively.

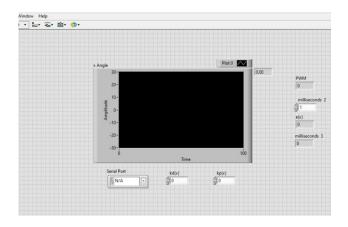


Figure 9: Front Panel illustrating real-time gyroscope readings

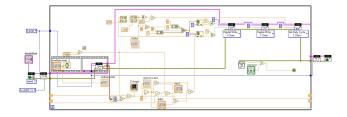


Figure 10: Block Diagram depicting the dynamic motor power application for plant control

## 5.3. Tuning of PD Controller Parameters:

The PD controller parameter coefficients,  $K_p$  and  $T_d$ , were empirically determined in the experimentation process. Figure (11-a, b) showcases the system's response under different  $K_p$  and  $T_d$  values. Through analysis, it was observed that  $K_p = 4$  and  $T_d = 0.01$  yielded the optimal response, demonstrating the effectiveness of these tuned parameters.

## **Future Recommendations**

\*\*State Feedback Theories Implementation:\*\* Explore
the application of advanced state feedback theories for
control system design. Investigate techniques such as Linear Quadratic Regulator (LQR) or State Feedback with
Observer for improved stability and performance.

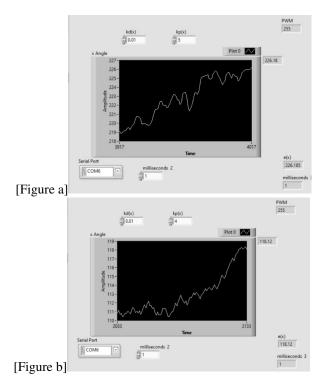


Figure 11: Various Instances from the Tuning Trials of PD Controller Parameters

- \*\*Sensor Reading Filtering with Kalman Filter:\*\* Implement Kalman filtering techniques for sensor data fusion to enhance the accuracy and reliability of the system's measurements. This can lead to improved control precision and robustness against noise and disturbances.
- 3. \*\*Extension to Three Degrees of Freedom (3-DOF):\*\*
  Consider expanding the project to a Three Degrees of Freedom (3-DOF) helicopter system. This extension would involve additional design considerations, such as incorporating lateral motion, and providing a more comprehensive platform for studying and implementing control strategies.

## Conclusion

In conclusion, this project has laid the groundwork for the exploration and implementation of sophisticated control strategies and sensor technologies in the design of a One-Degree-of-Freedom helicopter system. The successful application of a Proportional-Derivative controller showcased the system's stability and precision during vertical motion. Future endeavors could further advance the project by incorporating state feedback theories for enhanced control, implementing Kalman filters to refine sensor readings, and expanding the system to Three Degrees of Freedom (3-DOF) for a more comprehensive study. These recommendations open avenues for deeper research, innovation, and the potential application of the developed insights in diverse technological domains.

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