Define a function total

```
total :: (Integer → Integer) → (Integer → Integer) so that total f is the function which at value n gives the total:
f 0 + f 1 + ... + f n
```

Given a function f of the type $a \rightarrow b \rightarrow c$, write down a lambda abstraction that describes the function of type $b \rightarrow a \rightarrow c$ which behaves like f but which takes it arguments in the other order

Using this expression, give a definition of the function:

flip ::
$$(a \rightarrow b \rightarrow c) \rightarrow (b \rightarrow a \rightarrow c)$$

which reverses the order in which its functions arguments takes its arguments

```
uncurry :: (a -> b -> c) -> ((a, b) -> c)

($) :: (a -> b) -> a -> b

(:) :: a -> [a] -> [a]

(.) :: (b -> c) -> (a -> b) -> a -> c
```

- What is the effect of uncurry (\$) ?
- What is its type?
- Answer a similar question for uncurry (:) and uncurry (.)

What are the effects and types of

- 1. uncurry uncurry
- 2. curry uncurry

Can you define functions:

```
curry3 :: ((a, b, c) \rightarrow d) \rightarrow (a \rightarrow b \rightarrow c \rightarrow d))
uncurry3 :: (a \rightarrow b \rightarrow c \rightarrow d) \rightarrow ((a, b, c) \rightarrow d)
```

which perform the analogue of curry and uncurry but for three arguments rather than two? Can you use curry and uncurry in these definitions?

```
Give calculations of
iter 3 double 1
(comp2 succ (*)) 3 4
comp2 sq add 3 4
iter :: Integer -> (a -> a) -> (a -> a)
iter n f
  | n > 0 = f \cdot iter (n-1) f
  | otherwise = id
double :: Num a => a -> a
double x = 2*x
add :: Num a => a -> a -> a
add x y = x + y
sq :: Num a => a -> a
sq x = x*x
succ :: Integer → Integer
succ n = n+1
comp2 :: (a -> b) -> (b -> b -> c) -> (a -> a-> c)
comp2 f g = (\x y -> g (f x) (f y))
```

What is the type and effect of the function:

```
\n → iter n succ
```

Give an alternative 'constructive' definition of iter which creates the list of n copies of f

```
[f,f...,f]
```

and then composes these functions by folding the operator '.' to give

f . f f

Define the function

```
splits :: [a] \rightarrow ([a], [a])
```

which defines the list of all the ways that a list can be split in two

e.g.

```
splits "Spy" = [("", "Spy"), ("S", "py"), ("Sp", "y"), ("Spy", "")]
```

Using the list comprehension notation, define the functions:

```
sublists, subsequences :: [a] → [[a]]
```

which return all the sublists and subsequences of a list. A sublist is obtained omitting some of the elements of a list; a subsequence is a continous block from a list. E.g. [2,4] and [3,4] are sublists of [2,3,4] but only [3,4] is a subsequence.

Define the infinite lists of factorial and Fibonacci numbers,

```
factorial = [1,1,2,6,24,120,720,...]
fibonacci = [0,1,1,2,3,5,8,13,21]
```

Give a definition of the function

```
factors :: Integer -> [Integer]
```

which returns a list containing the factors of a positive integer.

```
factors 12 = [1,2,3,4,6,12]
```

Using this function or otherwise, define the list of numbers whose only prime factors are 2, 3 and 5. So called Hamming Numbers

```
hamming = [2,3,4,5,6,8,9,10,12,15]
```

Define the function

```
runningSums :: [Int] → [Int]
which calculates the running sums
[0,a0, a0+a1,a1+a2+a3,...]
of a list
[a0, a1, a2,...]
```

- How would you merge two infinite lists, assuming that they are sorted?
- How would you remove duplicates from the list which results?
- As an example, how would you merge the lists of powers of 2 and 3?
