

## Definitions

```
iter :: Int -> (a -> a) -> (a -> a)
iter n f
| n > 0 = f . iter (n - 1) f
| otherwise = id

reverse :: [a] -> [a]
reverse [] = []
reverse (x:xs) = reverse xs ++ [x]

elem :: a -> [a] -> a
elem y [] = False
elem y (x:xs) = if y == x then True else elem y xs

length :: [a] -> Int
length [] = 0
length (x:xs) = 1 + length xs

(++) :: [a] -> [a] -> [a]
(++) [] [] = []
(++) [] (y:ys) = y : ((++) [] ys)
(++) (x:xs) ys = x : ((++) xs ys)

map f [] = []
map f (x:xs) = f x : map f xs

abs x
| x < 0 = abs (-x)
| otherwise = x

signum x
| x < 0 = -1
| x == 0 = 0
| otherwise = 1
```

## Exercise 9.5

Prove for all finite lists  $xs$  and  $ys$ :  $\text{sum } (xs ++ ys) = \text{sum } xs + \text{sum } ys$  where

```
sum [] = 0
sum (x:xs) = x + sum xs

[] ++ ys = ys
(x:xs) ++ ys = x : (xs ++ ys)
```

## Exercise 9.6

Prove for all finite lists  $xs$ :  $xs ++ [] = xs$

Prove for all finite lists  $xs$ :  $(xs ++ ys) ++ zs = xs ++ (ys ++ zs)$

## Exercise 9.7

Prove for all finite lists  $xs$ :  $\text{sum } (\text{reverse } xs) = \text{sum } xs$

Prove for all finite lists  $xs$ :  $\text{length } (\text{reverse } xs) = \text{length } xs$

### Exercise 9.8

Prove for all finite lists `xs` and `ys`: `elem z (xs ++ ys) = elem z xs || elem z ys`

### Exercise 9.10

Prove for all finite lists `xs` and defined `n`: `take n xs ++ drop n xs = xs`, where

```
take 0 _ = []
take _ [] = []
take n (x:xs) = x : take (n-1) xs

drop 0 xs = xs
drop n [] = []
drop n (x:xs) = drop (n-1) xs
```

### Exercise 11.25

Prove for all `x`: `f.(g.h) x == (f.g).h x`

### Exercise 11.26

Prove for all `f`: `id.f = f`

### Exercise 11.29

Prove for all natural `n`: `iter n id = id`

### Exercise 11.30

Prove for all natural `x`: `abs.abs x = abs x`

Prove for all natural `x`: `signum.signum x = signum x`

### Exercise 11.31

Prove: `map f (ys ++ zs) = map f ys ++ map f zs`

### Exercise 11.34

Prove: `concat (map (map f) xs) = map f (concat xs)`

### Exercise 11.35

Prove: `(0<) . (+1) = (0<=)`