Transformation Matrices Used:

1. Translation Matrix: $T(tx, ty, tz)$

$$\begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Scaling Matrix: $S(sx, sy, sz)$

$$\begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Rotation Matrix around X-axis: $R_x(\theta)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Rotation Matrix around Y-axis: $R_y(\theta)$

$$\begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. Orthographic Projection Matrix: $O(l, r, b, t, n, f)$

$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6. Perspective Projection Matrix: $P(fov, aspect, near, far)$

$$\begin{bmatrix} \frac{1}{\text{aspect}\cdot\tan(\frac{fov}{2})} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\frac{fov}{2})} & 0 & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & -\frac{2\cdot far\cdot near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Current Transformation Matrix (CTM) Formulas:

Isometric View (Part 1):

$$CTM_{\text{iso}} = O \cdot R_y(45^\circ) \cdot R_x(35.264^\circ)$$

Where:

- O is the orthographic projection matrix.
- $R_y(45^\circ)$ rotates the cube 45° around the Y-axis.
- $R_x(35.264^\circ)$ rotates the cube by $\arctan(1/\sqrt{2}) \approx 35.264^\circ$ around the X-axis.

One-Point Perspective (Part 2):

$$CTM_{\text{one}} = P \cdot T(0,0,-5) \cdot S(0.5,0.5,0.5)$$

Where:

- P is the perspective projection matrix.
- $T(0,0,-5)$ translates the cube 5 units away from the camera.
- $S(0.5,0.5,0.5)$ scales the cube to half its original size.

Two-Point Perspective (Part 2):

$$CTM_{\text{two}} = P \cdot T(-1, 0, -5) \cdot R_y(45^\circ) \cdot S(0.5, 0.5, 0.5)$$

Where:

- $T(-1, 0, -5)$ translates the cube left and away from the camera.
- $R_y(45^\circ)$ rotates the cube 45° around the Y-axis.
- $S(0.5, 0.5, 0.5)$ scales the cube to half its original size.

Three-Point Perspective (Part 2):

$$CTM_{\text{three}} = P \cdot T(1, 0, -5) \cdot R_y(45^\circ) \cdot R_x(30^\circ) \cdot S(0.5, 0.5, 0.5)$$

Where:

- $T(1, 0, -5)$ translates the cube right and away from the camera.
- $R_y(45^\circ)$ rotates the cube 45° around the Y-axis.
- $R_x(30^\circ)$ rotates the cube 30° around the X-axis.
- $S(0.5, 0.5, 0.5)$ scales the cube to half its original size.

In each case, the Current Transformation Matrix (CTM) is applied to the vertex positions in the vertex shader to transform the cube from its original position in model space to its final position in clip space. The order of matrix multiplication is crucial: transformations are applied from right to left.

For example, in the three-point perspective, the scaling is applied first, followed by the X-rotation, then the Y-rotation, the translation, and finally the perspective projection. This approach enables complex transformations by combining simpler, intuitive transformations, which is a powerful technique in computer graphics for positioning and projecting 3D objects onto a 2D screen.