Part 1: Photon Emission Rate

Given:

- Power of the light bulb, $P = 25\,W$
- Efficiency, $\eta = 20\% = 0.2$
- Wavelength of photons, $\lambda = 500\,nm = 500 \times 10^{-9}\,m$

First, we calculate the energy output per second due to the efficiency:

$$P_{\text{useful}} = \eta \times P = 0.2 \times 25 = 5\,W$$

Each photon has energy E_{photon} given by:

$$E_{\text{photon}} = \frac{hc}{\lambda}$$

Where:

- $h = 6.626 \times 10^{-34}\,J \cdot s$ (Planck's constant)
- $c = 3 \times 10^{8}\,m/s$ (speed of light)
- $\lambda = 500 \times 10^{-9}\,m$

$$E_{\text{photon}} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{500 \times 10^{-9}} = 3.9756 \times 10^{-19}\,J$$

The number of photons emitted per second N_{photon} is:

$$N_{\text{photon}} = \frac{P_{\text{useful}}}{E_{\text{photon}}} = \frac{5}{3.9756 \times 10^{-19}} \approx 1.26 \times 10^{19}\,\text{photons/s}$$

Part 2: Radiant Flux, Radiant Intensity, Radiant Exitance, and Energy

Given:

- Voltage $V = 2.4\,V$
- Current $I = 0.7\,A$
- Power $P = V \times I = 2.4 \times 0.7 = 1.68\,W$

Radiant Flux (Φ)

This is the total power emitted as radiation, assuming ideal conditions:

$$\Phi = 1.68\,W$$

Radiant Intensity (I)

Radiant intensity is the power emitted per unit solid angle, and for a sphere, we assume the light is emitted equally in all directions. The solid angle for a sphere is $4\pi\,sr$.

$$I = \frac{\Phi}{4\pi} = \frac{1.68}{4\pi} \approx 0.134\,W/sr$$

Radiant Exitance (M)

Radiant exitance is the power emitted per unit area. The surface area of the spherical light bulb is:

$$A = 4\pi r^2 = 4\pi \times (0.005\,m)^2 \approx 3.14 \times 10^{-4}\,m^2$$

$$M = \frac{\Phi}{A} = \frac{1.68}{3.14 \times 10^{-4}} \approx 5350\,W/m^2$$

Energy Emitted in 5 Minutes

Energy is power multiplied by time:

$$E = P \times t = 1.68\,W \times (5 \times 60)\,s = 504\,J$$

Part 3: Irradiance Received in the Eye

Given:

- Pupil diameter $d = 6\,mm = 0.006\,m$
- Distance from light bulb $r = 1\,m$

The area of the pupil is:

$$A_{\text{pupil}} = \pi \left(\frac{d}{2}\right)^2 = \pi \times \left(\frac{0.006}{2}\right)^2 \approx 2.83 \times 10^{-5}\,m^2$$

The irradiance E is given by:

$$E = \frac{I \times A_{\text{pupil}}}{r^2} = \frac{0.134\,W/sr \times 2.83 \times 10^{-5}}{1^2} \approx 3.79 \times 10^{-6}\,W/m^2$$

Part 4: Irradiance and Illuminance on the Table

Given:

- Power $P = 200\,W$
- Efficiency $\eta = 20\%$
- Wavelength $\lambda = 650\,nm$
- Distance from table $r = 2\,m$
- Luminous efficiency at 650 nm: $V(\lambda) = 0.1$

Radiant Flux (Φ)

Useful power output:

$$P_{\text{useful}} = 0.2 \times 200 = 40\,W$$

Irradiance (E)

The irradiance on the table is:

$$E = \frac{P_{\text{useful}}}{4\pi r^2} = \frac{40}{4\pi \times 2^2} \approx 0.796\,W/m^2$$

Illuminance (Photometric Quantity)

To find illuminance:

$$\text{Illuminance} = \text{Radiometric} \times 685 \times V(\lambda) = 0.796 \times 685 \times 0.1 \approx 54.55\,lm/m^2$$

Part 5: Luminous Intensity of the Unknown Source

Given:

- **Known light source:**
  - Luminous intensity $I_s = 40\,\mathrm{cd}$ (where cd stands for candela, the unit of luminous intensity equivalent to lumens per steradian, lm/sr)
  - Distance from the screen $r_s = 35\,\mathrm{cm} = 0.35\,\mathrm{m}$
- **Unknown light source:**
  - Distance from the screen $r_x = 65\,\mathrm{cm} = 0.65\,\mathrm{m}$

The **inverse square law** states that the intensity of light is inversely proportional to the square of the distance from the light source. This can be expressed mathematically as:

$$\frac{I_s}{I_x} = \left(\frac{r_x}{r_s}\right)^2$$

Where:

- I_s is the luminous intensity of the known source (40 cd),
- I_x is the luminous intensity of the unknown source (what we want to find),
- r_s is the distance from the screen to the known source (0.35 m),
- r_x is the distance from the screen to the unknown source (0.65 m).

Step-by-Step Solution

1. **Apply the Inverse Square Law:**

$$\frac{I_s}{I_x} = \left(\frac{r_x}{r_s}\right)^2$$

Substituting the given values:

$$\frac{40}{I_x} = \left(\frac{0.65}{0.35}\right)^2$$

2. **Simplify the Ratio:**

Calculate the ratio of the distances:

$$\frac{0.65}{0.35} = \frac{13}{7}$$

Now, square the ratio:

$$\left(\frac{13}{7}\right)^2 = \frac{169}{49} \approx 3.45$$

3. **Solve for I_x:**

Now, solve for the unknown luminous intensity I_x:

$$\frac{40}{I_x} = 3.45$$

$$I_x = \frac{40}{3.45} \approx 11.59\,\mathrm{cd}$$

Part 6: Radiant Exitance and Energy Emitted from a Diffuse Source

Given:

- Radiance $L = 5000\,W/(sr \cdot m^2)$
- Area of the source $A = 0.1 \times 0.1 = 0.01\,m^2$

Radiant Exitance (M)

For a diffuse emitter, radiant exitance M is related to radiance by:

$$M = \pi L = \pi \times 5000 \approx 15708\,W/m^2$$

Energy Emitted

The total power emitted is:

$$P = M \times A = 15708 \times 0.01 = 157.08\,W$$

Part 7: Radiant Exitance and Power for a Non-Diffuse Emitter

We are given:

- The radiance L of the non-diffuse emitter is described by the function $L = 6000 \cos\theta \, W/(sr \cdot m^2)$.
- The surface area of the light source is $A = 10 \times 10 \, cm = 0.1 \times 0.1 \, m^2 = 0.01 \, m^2$.

We need to calculate:

1. **Radiant Exitance** M (which is the power emitted per unit area).
2. The **total power** P emitted by the entire light source.

Step 1: Radiant Exitance (M)

Radiant exitance M is the total power emitted per unit area by the source into all directions. For a non-diffuse emitter, the radiant exitance is the integral of the radiance $L(\theta)$ over the hemisphere above the surface.

The general formula for radiant exitance M is:

$$M = \int_{\Omega} L(\theta) \cos\theta \, d\Omega$$

Where:

- Ω is the solid angle (over a hemisphere),
- θ is the angle relative to the surface normal,
- $L(\theta)$ is the radiance (directional power per unit area per solid angle),
- $\cos\theta$ is the cosine factor accounting for the projection of the radiance on the surface,
- $d\Omega = 2\pi \sin\theta \, d\theta$ is the differential solid angle in spherical coordinates.

Since we are given $L(\theta) = 6000 \cos\theta$, we substitute this into the formula for M:

$$M = \int_0^{\pi/2} 6000 \cos^2\theta \times 2\pi \sin\theta \, d\theta$$

Simplifying the integral:

$$M = 12000\pi \int_0^{\pi/2} \cos^2\theta \sin\theta \, d\theta$$

Now, let's solve the integral $\int_0^{\pi/2} \cos^2\theta \sin\theta \, d\theta$:

- We use the substitution $u = \cos\theta$, hence $du = -\sin\theta \, d\theta$.
- When $\theta = 0$, $u = 1$, and when $\theta = \frac{\pi}{2}$, $u = 0$.

The integral becomes:

$$\int_1^0 u^2(-du) = \int_0^1 u^2 \, du = \frac{u^3}{3}\Big|_0^1 = \frac{1}{3}$$

Thus, the radiant exitance M is:

$$M = 12000\pi \times \frac{1}{3} = 4000\pi \, W/m^2$$

$$M \approx 12566.37 \, W/m^2$$

Step 2: Total Power (P)

The total power P emitted by the light source is the radiant exitance M multiplied by the surface area A:

$$P = M \times A$$

Given that $A = 0.01 \, m^2$ and $M \approx 12566.37 \, W/m^2$, the total power is:

$$P = 12566.37 \times 0.01 = 125.66 \, W$$

Part 8: Radiance from the Sun

- The Sun is considered a diffuse emitter with:
  - Total power emitted by the Sun: $P_{\text{Sun}} = 3.91 \times 10^{26} \, W$
  - Surface area of the Sun: $A_{\text{Sun}} = 6.07 \times 10^{18} \, m^2$
- Distances:
  - Distance from the Sun to Earth: $d_{\text{Earth}} = 1.5 \times 10^{11} \, m$
  - Distance from the Sun to Mars: $d_{\text{Mars}} = 2.28 \times 10^{11} \, m$

Questions to Address:

1. Is the radiance from the Sun identical on Earth and Mars?
2. Why is it warmer on Earth?
3. Calculate the radiance from the Sun on Earth and Mars.
4. Find the solid angle the Sun subtends as seen from Earth and Mars.
5. How much energy is received on a $1 \, m^2$ surface on Earth and Mars?

Step 1: Radiance from the Sun on Earth and Mars

Radiance of the Sun

Radiance L of a diffuse emitter is related to its total power P_{Sun} and surface area A_{Sun} by the equation:

$$L_{\text{Sun}} = \frac{P_{\text{Sun}}}{\pi A_{\text{Sun}}}$$

Substitute the given values:

$$L_{\text{Sun}} = \frac{3.91 \times 10^{26}}{\pi \times 6.07 \times 10^{18}} \approx 2.05 \times 10^{7}\, W/(m^2 \cdot sr)$$

This is the **intrinsic radiance** of the Sun, and it remains the same regardless of the distance from the Sun because radiance is a measure of the directional energy density from a surface. Therefore:

Answer to Question 1: Yes, the radiance of the Sun is the same on both Earth and Mars because radiance is intrinsic to the source and does not depend on distance.

Answer to Question 2: It is warmer on Earth because **irradiance** (the power received per unit area) decreases with distance, even though the intrinsic radiance is the same. Mars is farther from the Sun, so the energy per unit area received on Mars is less.

Step 2: Solid Angle Subtended by the Sun as Seen from Earth and Mars

The **solid angle** Ω subtended by an object (like the Sun) from a given point is calculated using the formula:

$$\Omega = \frac{A_{\text{Sun}}}{d^2}$$

Where:

- $A_{\text{Sun}} = 6.07 \times 10^{18}\, m^2$ (surface area of the Sun),
- d is the distance from the observer (either Earth or Mars).

Solid Angle from Earth:

Substitute the distance from the Sun to Earth $d_{\text{Earth}} = 1.5 \times 10^{11}\, m$:

$$\Omega_{\text{Earth}} = \frac{6.07 \times 10^{18}}{(1.5 \times 10^{11})^2} = \frac{6.07 \times 10^{18}}{2.25 \times 10^{22}} \approx 2.7 \times 10^{-4}\, sr$$

Solid Angle from Mars:

Similarly, for Mars at $d_{\text{Mars}} = 2.28 \times 10^{11}\, m$:

$$\Omega_{\text{Mars}} = \frac{6.07 \times 10^{18}}{(2.28 \times 10^{11})^2} = \frac{6.07 \times 10^{18}}{5.19 \times 10^{22}} \approx 1.17 \times 10^{-4}\, sr$$

Answer to Question 4: The solid angle subtended by the Sun is larger as seen from Earth ($2.7 \times 10^{-4}\, sr$) than from Mars ($1.17 \times 10^{-4}\, sr$), which explains why the Sun appears smaller from Mars.

Step 3: Energy Received on a 1 m² Surface on Earth and Mars

The **irradiance** E received on a surface at a distance d from the Sun is related to the Sun's total power P_{Sun} by the inverse square law:

$$E = \frac{P_{\text{Sun}}}{4\pi d^2}$$

Irradiance on Earth:

For Earth at $d_{\text{Earth}} = 1.5 \times 10^{11}\, m$:

$$E_{\text{Earth}} = \frac{3.91 \times 10^{26}}{4\pi(1.5 \times 10^{11})^2} = \frac{3.91 \times 10^{26}}{2.827 \times 10^{23}} \approx 1384\, W/m^2$$

Irradiance on Mars:

For Mars at $d_{\text{Mars}} = 2.28 \times 10^{11}\, m$:

$$E_{\text{Mars}} = \frac{3.91 \times 10^{26}}{4\pi(2.28 \times 10^{11})^2} = \frac{3.91 \times 10^{26}}{6.54 \times 10^{23}} \approx 598\, W/m^2$$

Thus, the irradiance (energy per second per square meter) is higher on Earth than on Mars, which explains why Earth is warmer.

Answer to Question 5:

- The irradiance on Earth is approximately **1384 W/m²**.
- The irradiance on Mars is approximately **598 W/m²**.

Summary of Answers

1. **Is the radiance identical on Earth and Mars?**
   - Yes, the radiance is the same because it is an intrinsic property of the Sun and does not depend on distance.
2. **Why is it warmer on Earth?**
   - Although the radiance is the same, the irradiance (energy received per unit area) is greater on Earth because Earth is closer to the Sun.
3. **Radiance on Earth and Mars:**
   - The radiance of the Sun, L_{Sun}, is approximately $2.05 \times 10^7\, W/(m^2 \cdot sr)$, and it is the same on both planets.
4. **Solid Angle:**
   - The solid angle subtended by the Sun as seen from Earth is $2.7 \times 10^{-4}\, sr$, while from Mars it is $1.17 \times 10^{-4}\, sr$.
5. **Energy received:**
   - On a $1\, m^2$ surface, Earth receives $1384\, W/m^2$ and Mars receives $598\, W/m^2$.

Part 9: Irradiance of horizontal plate

We are given:

- **Radiance of the sky**: $L_{\text{sky}} = 1000\ W/(m^2 \cdot sr)$.
- **Condition 1**: The entire sky is visible from the plate.
- **Condition 2**: Only a conical section of the sky with a half-angle of 30° is visible from the plate.

We are to calculate the **irradiance** E, which is the total power received per unit area at the center of the plate, under both conditions.

Step-by-Step Solution

Step 1: Irradiance from a Diffuse Radiance Source

Irradiance E from a diffuse source (such as the sky) is related to the radiance L and the solid angle Ω subtended by the source. In general, the irradiance from a source emitting uniformly in all directions above a horizontal surface is:

$$E = \int_{\Omega} L \cos\theta \, d\Omega$$

Where:

- L is the radiance of the source,
- θ is the angle between the direction of the incoming light and the normal to the surface,
- $d\Omega$ is the differential solid angle.

For the sky, $L_{\text{sky}} = 1000\ W/(m^2 \cdot sr)$, and $d\Omega = 2\pi \sin\theta \, d\theta$ in spherical coordinates.

Step 2: Irradiance for the Entire Sky (Condition 1)

When the entire sky is visible, the irradiance E is integrated over a hemisphere above the plate, from $\theta = 0$ (directly overhead) to $\theta = \pi/2$ (the horizon).

Thus, the irradiance is:

$$E = \int_0^{\pi/2} L_{\text{sky}} \cos\theta \times 2\pi \sin\theta \, d\theta$$

Substitute the given value $L_{\text{sky}} = 1000\ W/(m^2 \cdot sr)$:

$$E = 2\pi \times 1000 \times \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta$$

The integral $\int_0^{\pi/2} \cos\theta \sin\theta \, d\theta$ can be simplified:

$$\int_0^{\pi/2} \cos\theta \sin\theta \, d\theta = \frac{1}{2}$$

Thus, the irradiance for the entire sky is:

$$E_{\text{full sky}} = 2\pi \times 1000 \times \frac{1}{2} = 1000\pi\ W/m^2$$

This gives:

$$E_{\text{full sky}} \approx 3141.59\ W/m^2$$

Step 3: Irradiance for a Conical Section of the Sky (Condition 2)

In this case, only a conical section of the sky with a half-angle of 30° (or $\pi/6$ radians) is visible. The irradiance is now integrated over this conical section, from $\theta = 0$ (directly overhead) to $\theta = \pi/6$.

The irradiance for this conical section is:

$$E_{\text{cone}} = 2\pi \times 1000 \times \int_0^{\pi/6} \cos\theta \sin\theta \, d\theta$$

We solve the integral $\int_0^{\pi/6} \cos\theta \sin\theta \, d\theta$:

$$\int_0^{\pi/6} \cos\theta \sin\theta \, d\theta = \frac{1}{2} \left(\sin^2\theta\right) \Big|_0^{\pi/6}$$

Substitute the limits of integration:

$$= \frac{1}{2} \left(\sin^2\left(\frac{\pi}{6}\right) - \sin^2(0)\right)$$

Since $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$, we have:

$$= \frac{1}{2} \left(\left(\frac{1}{2}\right)^2 - 0\right) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

Thus, the irradiance for the conical section is:

$$E_{\text{cone}} = 2\pi \times 1000 \times \frac{1}{8} = 250\pi \, W/m^2$$

This gives:

$$E_{\text{cone}} \approx 785.4 \, W/m^2$$

Final Answers:

1. **Irradiance with the entire sky visible: $E_{\text{full sky}} \approx 3141.59 \, W/m^2$.**
2. **Irradiance with only a conical section of the sky visible (half-angle 30°): $E_{\text{cone}} \approx 785.4 \, W/m^2$.**