

```
In [144]: import numpy as np
import matplotlib.pyplot as plt
import sklearn
import scipy.spatial
import functools

%matplotlib inline
```

## 6.2.1

```
In [145]: ### Kernel function generators
def linear_kernel(X1, X2):
    """
    Computes the linear kernel between two sets of vectors.
    Args:
        X1 - an n1xd matrix with vectors x1_1,...,x1_n1 in the rows
        X2 - an n2xd matrix with vectors x2_1,...,x2_n2 in the rows
    Returns:
        matrix of size n1xn2, with x1_i^T x2_j in position i,j
    """
    return np.dot(X1,np.transpose(X2))

def RBF_kernel(X1,X2,sigma):
    """
    Computes the RBF kernel between two sets of vectors
    Args:
        X1 - an n1xd matrix with vectors x1_1,...,x1_n1 in the rows
        X2 - an n2xd matrix with vectors x2_1,...,x2_n2 in the rows
        sigma - the bandwidth (i.e. standard deviation) for the RBF/Gaussian kernel
    Returns:
        matrix of size n1xn2, with exp(-||x1_i-x2_j||^2/(2 sigma^2)) in position i,j
    """
    #TODO
    a = scipy.spatial.distance.cdist(X1, X2, 'sqeuclidean')
    K = np.exp(- a /(2*sigma**2))
    return K

def polynomial_kernel(X1, X2, offset, degree):
    """
    Computes the inhomogeneous polynomial kernel between two sets of vectors
    Args:
        X1 - an n1xd matrix with vectors x1_1,...,x1_n1 in the rows
        X2 - an n2xd matrix with vectors x2_1,...,x2_n2 in the rows
        offset, degree - two parameters for the kernel
    Returns:
        matrix of size n1xn2, with (offset + <x1_i,x2_j>)^degree in position i,j
    """
    #TODO
    return np.power(np.add(np.dot(X1,np.transpose(X2)), offset), degree)
```

## 6.2.2

```
In [146]: prototypes = np.array([-4,-1,0,2]).reshape(-1,1)
linear_kernel(prototypes, prototypes)
```

```
Out[146]: array([[16,  4,  0, -8],
 [ 4,  1,  0, -2],
 [ 0,  0,  0,  0],
 [-8, -2,  0,  4]])
```

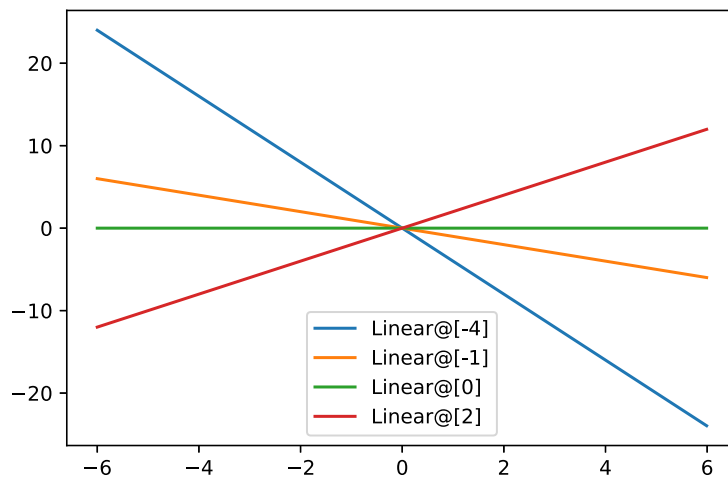
## 6.2.3

```
In [147]: xpts = np.arange(-5.0, 6, .01).reshape(-1,1)
xpts
```

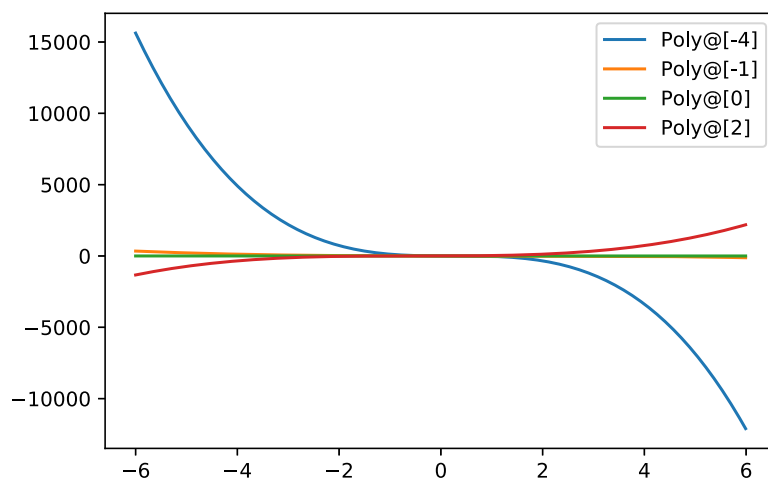
```
Out[147]: array([[ -5. ],
 [-4.99],
 [-4.98],
 ...,
 [ 5.97],
 [ 5.98],
 [ 5.99]])
```

```
In [148]: # Plot kernel machine functions
%config InlineBackend.figure_format = 'svg'
plot_step = .01
xpts = np.arange(-6.0, 6, plot_step).reshape(-1,1)
prototypes = np.array([-4,-1,0,2]).reshape(-1,1)

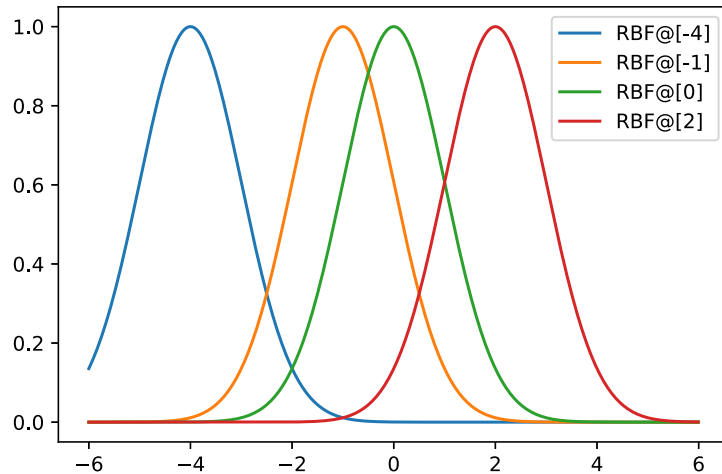
# Linear kernel
y = linear_kernel(prototypes, xpts)
for i in range(len(prototypes)):
    label = "Linear@" + str(prototypes[i,:])
    plt.plot(xpts, y[i,:], label=label)
plt.legend(loc = 'best')
plt.show()
```



```
In [149]: # Poly kernel
y = polynomial_kernel(prototypes, xpts, 1, 3)
for i in range(len(prototypes)):
    label = "Poly@" + str(prototypes[i,:])
    plt.plot(xpts, y[i,:], label=label)
plt.legend(loc = 'best')
plt.show()
```



```
In [150]: # RBF kernel
y = RBF_kernel(prototypes, xpts, 1)
for i in range(len(prototypes)):
    label = "RBF@"+str(prototypes[i,:])
    plt.plot(xpts, y[i,:], label=label)
plt.legend(loc = 'best')
plt.show()
```



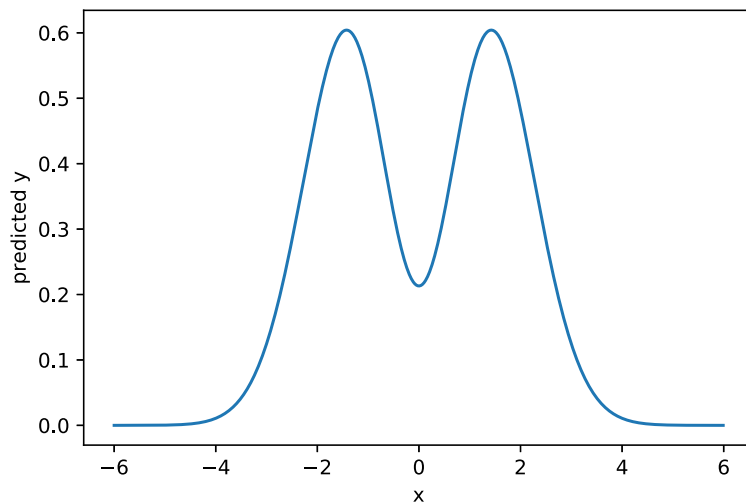
```
In [151]: class Kernel_Machine(object):
def __init__(self, kernel, prototype_points, weights):
    """
    Args:
        kernel(X1,X2) - a function return the cross-kernel matrix between rows of X1 and rows of X2 for kernel
        prototype_points - an Rxd matrix with rows mu_1,...,mu_R
        weights - a vector of length R with entries w_1,...,w_R
    """

    self.kernel = kernel
    self.prototype_points = prototype_points
    self.weights = weights

def predict(self, X):
    """
    Evaluates the kernel machine on the points given by the rows of X
    Args:
        X - an nxd matrix with inputs x_1,...,x_n in the rows
    Returns:
        Vector of kernel machine evaluations on the n points in X. Specifically, jth entry of return vector
        Sum_{i=1}^R w_i k(x_j, mu_i)
    """
    # TODO
    k = self.kernel(X,self.prototype_points)
    return np.dot(k,self.weights)
```

```
In [152]: prototypes = np.array([-1,0,1]).reshape(-1,1)
weights = np.array([1,-1,1]).reshape(-1,1)
k = functools.partial(RBF_kernel,sigma = 1)
km = Kernel_Machine(k,prototypes,weights)
```

```
In [153]: plot_step = .01
xpts = np.arange(-6.0, 6, plot_step).reshape(-1,1)
plt.plot(xpts, km.predict(xpts), label=label)
plt.xlabel('x')
plt.ylabel('predicted y')
plt.show()
```

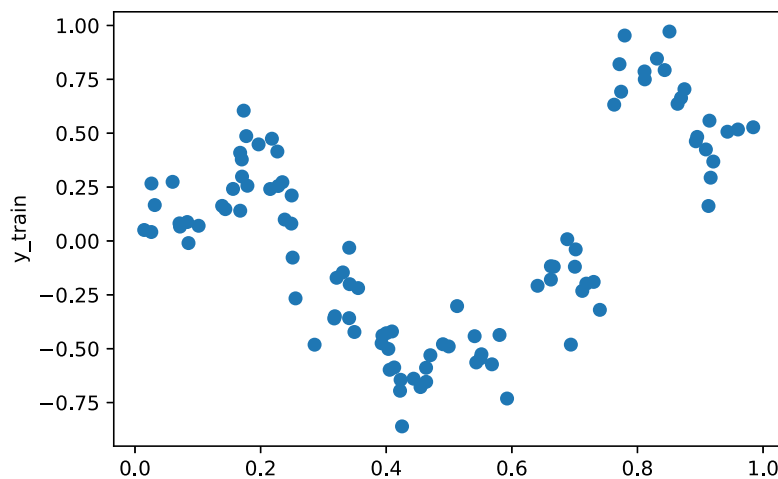


### 6.3.1

Load train & test data; Convert to column vectors so it generalizes well to data in higher dimensions.

```
In [154]: data_train, data_test = np.loadtxt("krr-train.txt"), np.loadtxt("krr-test.txt")
x_train, y_train = data_train[:,0].reshape(-1,1), data_train[:,1].reshape(-1,1)
x_test, y_test = data_test[:,0].reshape(-1,1), data_test[:,1].reshape(-1,1)
```

```
In [155]: plt.plot(x_train, y_train, 'o')
plt.xlabel('x_train')
plt.ylabel('y_train')
plt.show()
```



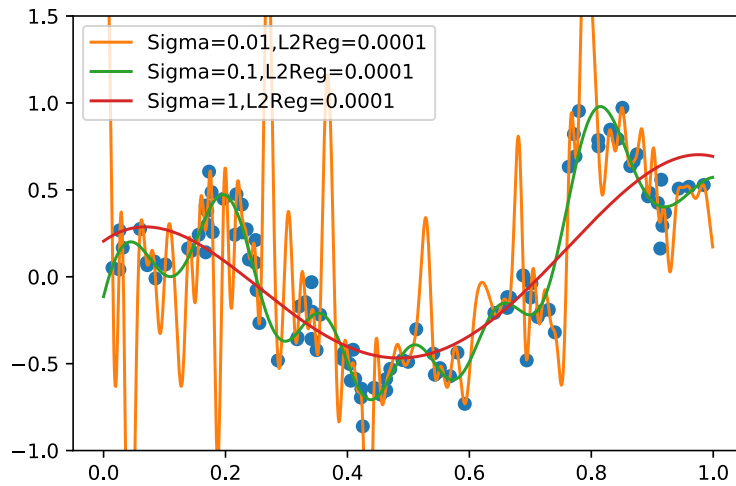
### 6.3.2

```
In [156]: def train_kernel_ridge_regression(X, y, kernel, l2reg):
# TODO
dim = X.shape[0]
alpha = np.dot(np.linalg.inv(l2reg * np.identity(dim) + kernel(X,X)), y)
return Kernel_Machine(kernel, X, alpha)
```

### 6.3.3

- From the graph below, we can see that:  $\sigma=0.01$  is more likely to over fit while  $\sigma=1$  is less.

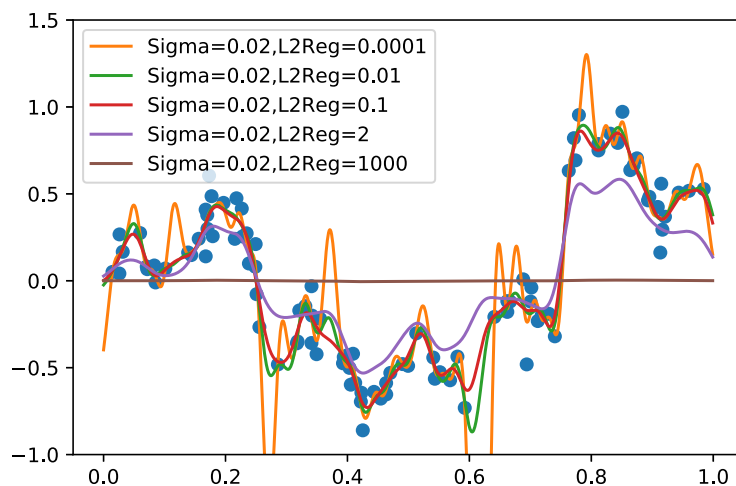
```
In [157]: plot_step = .001
xpts = np.arange(0 , 1, plot_step).reshape(-1,1)
plt.plot(x_train,y_train,'o')
l2reg = 0.0001
for sigma in [.01,.1,1]:
    k = functools.partial(RBF_kernel, sigma=sigma)
    f = train_kernel_ridge_regression(x_train, y_train, k, l2reg=l2reg)
    label = "Sigma="+str(sigma)+" , L2Reg="+str(l2reg)
    plt.plot(xpts, f.predict(xpts), label=label)
plt.legend(loc = 'best')
plt.ylim(-1,1.5)
plt.show()
```



### 6.3.4

- When  $\lambda \rightarrow \infty$ , the prediction function becomes a constant (looking like a straight line  $y=0$  from the figure)

```
In [158]: plot_step = .001
xpts = np.arange(0 , 1, plot_step).reshape(-1,1)
plt.plot(x_train,y_train,'o')
sigma = .02
for l2reg in [.0001,.01,.1,2,1000]:
    k = functools.partial(RBF_kernel, sigma=sigma)
    f = train_kernel_ridge_regression(x_train, y_train, k, l2reg=l2reg)
    label = "Sigma="+str(sigma)+" , L2Reg="+str(l2reg)
    plt.plot(xpts, f.predict(xpts), label=label)
plt.legend(loc = 'best')
plt.ylim(-1,1.5)
plt.show()
```



### 6.3.5

```
In [159]: from sklearn.base import BaseEstimator, RegressorMixin, ClassifierMixin

class KernelRidgeRegression(BaseEstimator, RegressorMixin):
    """sklearn wrapper for our kernel ridge regression"""

    def __init__(self, kernel="RBF", sigma=1, degree=2, offset=1, l2reg=1):
        self.kernel = kernel
        self.sigma = sigma
        self.degree = degree
        self.offset = offset
        self.l2reg = l2reg

    def fit(self, X, y=None):
        """
        This should fit classifier. All the "work" should be done here.
        """
        if (self.kernel == "linear"):
            self.k = linear_kernel
        elif (self.kernel == "RBF"):
            self.k = functools.partial(RBF_kernel, sigma=self.sigma)
        elif (self.kernel == "polynomial"):
            self.k = functools.partial(polynomial_kernel, offset=self.offset, degree=self.degree)
        else:
            raise ValueError('Unrecognized kernel type requested.')

        self.kernel_machine_ = train_kernel_ridge_regression(X, y, self.k, self.l2reg)

        return self

    def predict(self, X, y=None):
        try:
            getattr(self, "kernel_machine_")
        except AttributeError:
            raise RuntimeError("You must train classifier before predicting data!")

        return(self.kernel_machine_.predict(X))

    def score(self, X, y=None):
        # get the average square error
        return(((self.predict(X)-y)**2).mean())
```

```
In [160]: from sklearn.model_selection import GridSearchCV, PredefinedSplit
from sklearn.model_selection import ParameterGrid
from sklearn.metrics import mean_squared_error, make_scorer
import pandas as pd

test_fold = [-1]*len(x_train) + [0]*len(x_test) #0 corresponds to test, -1 to train
predefined_split = PredefinedSplit(test_fold=test_fold)
```

```
In [191]: param_grid = [{'kernel': ['RBF'], 'sigma': [0.02, 0.04, 0.05, 0.06, 0.07, 0.1, 0.5], 'l2reg': np.exp2(-np.arange(-1, 5, 0.5))},
                        {'kernel': ['polynomial'], 'offset': np.arange(-5, 5, 0.5), 'degree': np.arange(2, 10, 1), 'l2reg': np.exp2(-np.arange(-1, 5, 0.5))},
                        {'kernel': ['linear'], 'l2reg': [10, 5, 4, 3, 2, 1, 0.1, 0.01, 0.005]}]
kernel_ridge_regression_estimator = KernelRidgeRegression()
grid = GridSearchCV(kernel_ridge_regression_estimator,
                    param_grid,
                    cv = predefined_split,
                    scoring = make_scorer(mean_squared_error, greater_is_better = False)
                    # n_jobs = -1 #should allow parallelism, but crashes Python on my machine
                    )
grid.fit(np.vstack((x_train, x_test)), np.vstack((y_train, y_test)))
```

```
Out[191]: GridSearchCV(cv=PredefinedSplit(test_fold=array([-1, -1, ..., 0, 0])),
                        error_score='raise-deprecating',
                        estimator=KernelRidgeRegression(degree=2, kernel='RBF', l2reg=1, offset=1, sigma=1),
                        fit_params=None, iid='warn', n_jobs=None,
                        param_grid=[{'kernel': ['RBF'], 'sigma': [0.02, 0.04, 0.05, 0.06, 0.07, 0.1, 0.5], 'l2reg': array([2.
, 1.86607, 1.7411, 1.6245, 1.51572, 1.41421, 1.31951,
1.23114, 1.1487, 1.07177, 1.
, 0.93303, 0.87055, 0.81225,
0.75786, 0.70711, 0.65975, 0.61557, 0.57435, 0.53589, 0.5
...
0.03125, 0.0221 ])}, {'kernel': ['linear'], 'l2reg': [10, 5, 4, 3, 2, 1, 0.1, 0.01, 0.005]}],
                        pre_dispatch='2*n_jobs', refit=True, return_train_score='warn',
                        scoring=make_scorer(mean_squared_error, greater_is_better=False),
                        verbose=0)
```

```
In [192]: pd.set_option('display.max_rows', 20)
df = pd.DataFrame(grid.cv_results_)
# Flip sign of score back, because GridSearchCV likes to maximize,
# so it flips the sign of the score if "greater_is_better=False"
df['mean_test_score'] = -df['mean_test_score']
df['mean_train_score'] = -df['mean_train_score']
cols_to_keep = ["param_degree", "param_kernel", "param_l2reg", "param_offset", "param_sigma",
               "mean_test_score", "mean_train_score"]
df_toshow = df[cols_to_keep].fillna('-')
df_toshow.sort_values(by=["mean_test_score"])
```

```
/Users/jr/anaconda3/lib/python3.7/site-packages/sklearn/utils/deprecation.py:125: FutureWarning: You are accessing a training score ('split0_train_score'), which will not be available by default any more in 0.21. If you need training scores, please set return_train_score=True
  warnings.warn(*warn_args, **warn_kwargs)
/Users/jr/anaconda3/lib/python3.7/site-packages/sklearn/utils/deprecation.py:125: FutureWarning: You are accessing a training score ('mean_train_score'), which will not be available by default any more in 0.21. If you need training scores, please set return_train_score=True
  warnings.warn(*warn_args, **warn_kwargs)
/Users/jr/anaconda3/lib/python3.7/site-packages/sklearn/utils/deprecation.py:125: FutureWarning: You are accessing a training score ('std_train_score'), which will not be available by default any more in 0.21. If you need training scores, please set return_train_score=True
  warnings.warn(*warn_args, **warn_kwargs)
```

Out[192]:

	param_degree	param_kernel	param_l2reg	param_offset	param_sigma	mean_test_score	mean_train_score
248	-	RBF	0.176777	-	0.06	0.013805	0.014446
255	-	RBF	0.164938	-	0.06	0.013808	0.014337
241	-	RBF	0.189465	-	0.06	0.013810	0.014562
262	-	RBF	0.153893	-	0.06	0.013819	0.014233
234	-	RBF	0.203063	-	0.06	0.013824	0.014686
269	-	RBF	0.143587	-	0.06	0.013838	0.014136
361	-	RBF	0.058315	-	0.07	0.013843	0.014489
368	-	RBF	0.054409	-	0.07	0.013845	0.014409
227	-	RBF	0.217638	-	0.06	0.013847	0.014817
354	-	RBF	0.062500	-	0.07	0.013847	0.014575
...	...	...	...	...	...	...	...
2524	8	polynomial	0.176777	-3	-	26.118150	14.846931
1408	5	polynomial	2.828427	-1	-	42.627495	39.888049
1349	4	polynomial	0.031250	-0.5	-	83.919292	74.118846
745	3	polynomial	4.000000	-2.5	-	176.827125	155.535490
1687	5	polynomial	0.022097	-1.5	-	207.650239	160.865511
1665	5	polynomial	0.031250	-2.5	-	209.931191	163.009402
2043	7	polynomial	2.828427	-3.5	-	210.301447	163.146338
2922	9	polynomial	0.044194	-4	-	362.248725	208.787529
2647	8	polynomial	0.022097	-1.5	-	442.130500	252.449933
1642	5	polynomial	0.044194	-4	-	1487.962466	1139.643705

2989 rows × 7 columns

```
In [193]: a more convenient way to look at the table
grid
nbinstall(overwrite=True) # copies javascript dependencies to your /nbextensions folder
d_toshow = df[show].fillna(0)
d.show_grid(df_toshow,show_toolbar=True)
```

Add Row

Remove Row

✕

	param_degree	param_kernel	param_l2reg	param_offset	param_sigma	mean_test_score	mean_train_s
2961	9	polynomial	0.0221	-4.5	-	0.03036	0.04016
2940	9	polynomial	0.03125	-5.0	-	0.03042	0.0403
2960	9	polynomial	0.0221	-5.0	-	0.03049	0.03973
2962	9	polynomial	0.0221	-4.0	-	0.03109	0.0417
2941	9	polynomial	0.03125	-4.5	-	0.03136	0.04193
2920	9	polynomial	0.04419	-5.0	-	0.03201	0.04257
1974	6	polynomial	0.04419	2.0	-	0.03242	0.04862
2013	6	polynomial	0.0221	1.5	-	0.03244	0.04823
1955	6	polynomial	0.0625	2.5	-	0.03249	0.04756
2136	7	polynomial	0.70711	3.0	-	0.03255	0.04765
2155	7	polynomial	0.5	2.5	-	0.03257	0.0494
1936	6	polynomial	0.08839	3.0	-	0.03257	0.04727
2078	7	polynomial	2	4.0	-	0.03259	0.04889
2097	7	polynomial	1.41421	3.5	-	0.0326	0.04942
1917	6	polynomial	0.125	3.5	-	0.03261	0.04741
2059	7	polynomial	2.82843	4.5	-	0.03263	0.0488

- 1. Linear kernel

```
In [177]: q.get_changed_df()
```

Out[177]:

	param_degree	param_kernel	param_l2reg	param_offset	param_sigma	mean_test_score	mean_train_score
5822	-	linear	4.000	-	-	0.164510	0.206563
5823	-	linear	3.000	-	-	0.164512	0.206538
5821	-	linear	5.000	-	-	0.164513	0.206592
5824	-	linear	2.000	-	-	0.164522	0.206518
5825	-	linear	1.000	-	-	0.164540	0.206506
5826	-	linear	0.100	-	-	0.164565	0.206501
5827	-	linear	0.010	-	-	0.164569	0.206501
5828	-	linear	0.005	-	-	0.164569	0.206501
5820	-	linear	10.000	-	-	0.164591	0.206780

- 2. RBF kernel  
(the best RBF: l2reg = 0.176777, sigma = 0.06)



```
In [176]: q.get_changed_df()
```

Out[176]:

	param_degree	param_kernel	param_l2reg	param_offset	param_sigma	mean_test_score	mean_train_score
248	-	RBF	0.176777	-	0.06	0.013805	0.014446
255	-	RBF	0.164938	-	0.06	0.013808	0.014337
241	-	RBF	0.189465	-	0.06	0.013810	0.014562
262	-	RBF	0.153893	-	0.06	0.013819	0.014233
234	-	RBF	0.203063	-	0.06	0.013824	0.014686
269	-	RBF	0.143587	-	0.06	0.013838	0.014136
361	-	RBF	0.058315	-	0.07	0.013843	0.014489
368	-	RBF	0.054409	-	0.07	0.013845	0.014409
227	-	RBF	0.217638	-	0.06	0.013847	0.014817
354	-	RBF	0.062500	-	0.07	0.013847	0.014575
...	...	...	...	...	...	...	...
69	-	RBF	1.071773	-	0.5	0.058221	0.093002
62	-	RBF	1.148698	-	0.5	0.059118	0.094221
55	-	RBF	1.231144	-	0.5	0.060095	0.095519
48	-	RBF	1.319508	-	0.5	0.061154	0.096897
41	-	RBF	1.414214	-	0.5	0.062297	0.098356
34	-	RBF	1.515717	-	0.5	0.063527	0.099897
27	-	RBF	1.624505	-	0.5	0.064843	0.101521
20	-	RBF	1.741101	-	0.5	0.066248	0.103227
13	-	RBF	1.866066	-	0.5	0.067739	0.105014
6	-	RBF	2.000000	-	0.5	0.069317	0.106880

420 rows × 7 columns

- 3. Polynomial Kernel  
(the best Poly: degree =9, l2reg = 0.022097, offset = -4.5)

```
In [194]: q.get_changed_df()
```

```
Out[194]:
```

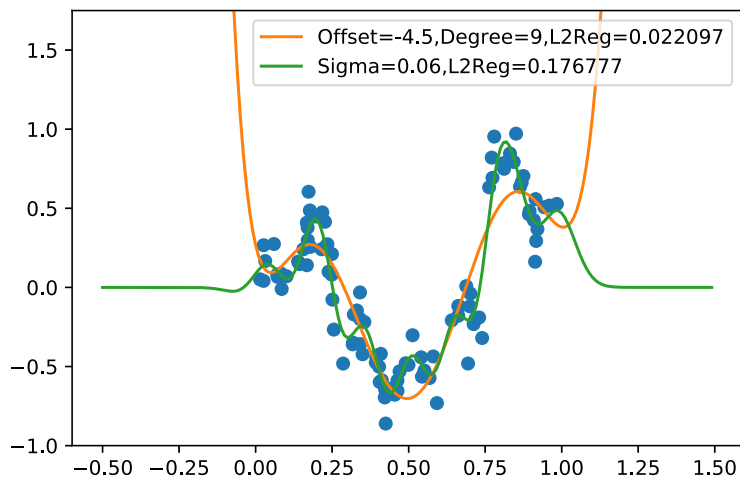
	param_degree	param_kernel	param_l2reg	param_offset	param_sigma	mean_test_score	mean_train_score
2961	9	polynomial	0.022097	-4.5	-	0.030362	0.040158
2940	9	polynomial	0.031250	-5	-	0.030418	0.040295
2960	9	polynomial	0.022097	-5	-	0.030488	0.039725
2962	9	polynomial	0.022097	-4	-	0.031091	0.041698
2941	9	polynomial	0.031250	-4.5	-	0.031362	0.041928
2920	9	polynomial	0.044194	-5	-	0.032005	0.042569
1974	6	polynomial	0.044194	2	-	0.032416	0.048621
2013	6	polynomial	0.022097	1.5	-	0.032440	0.048233
1955	6	polynomial	0.062500	2.5	-	0.032487	0.047561
2136	7	polynomial	0.707107	3	-	0.032550	0.047654
...	...	...	...	...	...	...	...
2524	8	polynomial	0.176777	-3	-	26.118150	14.846931
1408	5	polynomial	2.828427	-1	-	42.627495	39.888049
1349	4	polynomial	0.031250	-0.5	-	83.919292	74.118846
745	3	polynomial	4.000000	-2.5	-	176.827125	155.535490
1687	5	polynomial	0.022097	-1.5	-	207.650239	160.865511
1665	5	polynomial	0.031250	-2.5	-	209.931191	163.009402
2043	7	polynomial	2.828427	-3.5	-	210.301447	163.146338
2922	9	polynomial	0.044194	-4	-	362.248725	208.787529
2647	8	polynomial	0.022097	-1.5	-	442.130500	252.449933
1642	5	polynomial	0.044194	-4	-	1487.962466	1139.643705

2560 rows × 7 columns

## 6.3.6

**Answer:** it seems that the RBF kernel fits data better(it captures more twist in the curve), and the polynomial kernel seems to be more general.

```
In [196]: ## Plot the best polynomial and RBF fits you found
plot_step = .01
xpts = np.arange(-.5, 1.5, plot_step).reshape(-1,1)
plt.plot(x_train,y_train,'o')
#Plot best polynomial fit
offset= -4.5
degree = 9
l2reg = 0.022097
k = functools.partial(polynomial_kernel, offset=offset, degree=degree)
f = train_kernel_ridge_regression(x_train, y_train, k, l2reg=l2reg)
label = "Offset="+str(offset)+" ,Degree="+str(degree)+" ,L2Reg="+str(l2reg)
plt.plot(xpts, f.predict(xpts), label=label)
#Plot best RBF fit
sigma = 0.06
l2reg= 0.176777
k = functools.partial(RBF_kernel, sigma=sigma)
f = train_kernel_ridge_regression(x_train, y_train, k, l2reg=l2reg)
label = "Sigma="+str(sigma)+" ,L2Reg="+str(l2reg)
plt.plot(xpts, f.predict(xpts), label=label)
plt.legend(loc = 'best')
plt.ylim(-1,1.75)
plt.show()
```



### 6.3.7

- from hw1, we know that the bayes decision function for square loss is:  $E(y|x) = E(f(x) + \epsilon|x) = f(x)$
- from hw1, we know that the bayes risk for square loss is:  $Var(y) = Var(f(x) + \epsilon) = Var(f(x)) + Var(\epsilon) = 0.01$

### 6.4.1

-**Answer:** not linearly separable, not quadratically separable. we can use some RBF kernel to separate the data properly.

```

In [197]: # Load and plot the SVM data
#load the training and test sets
data_train,data_test = np.loadtxt("svm-train.txt"),np.loadtxt("svm-test.txt")
x_train, y_train = data_train[:,0:2], data_train[:,2].reshape(-1,1)
x_test, y_test = data_test[:,0:2], data_test[:,2].reshape(-1,1)

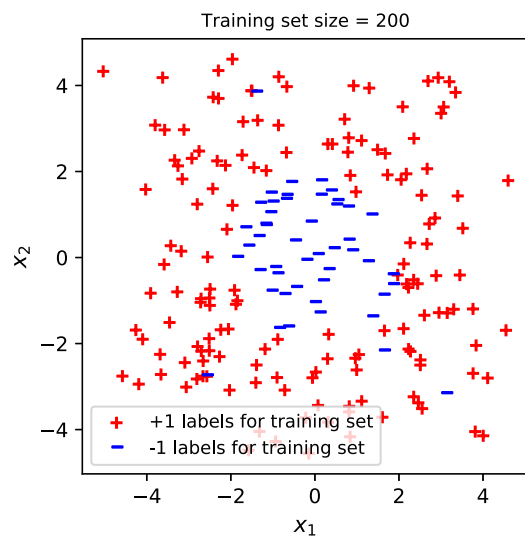
#determine predictions for the training set
yplus = np.ma.masked_where(y_train[:,0]<=0, y_train[:,0])
xplus = x_train[~np.array(yplus.mask)]
yminus = np.ma.masked_where(y_train[:,0]>0, y_train[:,0])
xminus = x_train[~np.array(yminus.mask)]

#plot the predictions for the training set
figsize = plt.figaspect(1)
f, (ax) = plt.subplots(1, 1, figsize=figsize)

pluses = ax.scatter (xplus[:,0], xplus[:,1], marker='+', c='r', label = '+1 labels for training set')
minuses = ax.scatter (xminus[:,0], xminus[:,1], marker=r'$-$', c='b', label = '-1 labels for training set')

ax.set_ylabel(r"$x_2$", fontsize=11)
ax.set_xlabel(r"$x_1$", fontsize=11)
ax.set_title('Training set size = %s'% len(data_train), fontsize=9)
ax.axis('tight')
ax.legend(handles=[pluses, minuses], fontsize=9)
plt.show()

```



```

In [ ]: def train_soft_svm(x_train, y_train, k, ...):

```

```

In [23]: # Code to help plot the decision regions
# (Note: This code isn't necessarily entirely appropriate for the questions asked. So think about what you are doing)

sigma=1
k = functools.partial(RBF_kernel, sigma=sigma)
f = train_soft_svm(x_train, y_train, k, ...)

#determine the decision regions for the predictions
x1_min = min(x_test[:,0])
x1_max = max(x_test[:,0])
x2_min = min(x_test[:,1])
x2_max = max(x_test[:,1])
h=0.1
xx, yy = np.meshgrid(np.arange(x1_min, x1_max, h),
                     np.arange(x2_min, x2_max, h))

Z = f.predict(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)

#determine the predictions for the test set
y_bar = f.predict(x_test)
yplus = np.ma.masked_where(y_bar<=0, y_bar)
xplus = x_test[~np.array(yplus.mask)]
yminus = np.ma.masked_where(y_bar>0, y_bar)
xminus = x_test[~np.array(yminus.mask)]

#plot the learned boundary and the predictions for the test set
figsize = plt.figure(figsize=figsize)
f, (ax) = plt.subplots(1, 1, figsize=figsize)
decision = ax.contourf(xx, yy, Z, cmap=plt.cm.coolwarm, alpha=0.8)
pluses = ax.scatter(xplus[:,0], xplus[:,1], marker='+', c='b', label = '+1 prediction for test set')
minuses = ax.scatter(xminus[:,0], xminus[:,1], marker='-$-$', c='b', label = '-1 prediction for test set')
ax.set_ylabel(r"$x_2$", fontsize=11)
ax.set_xlabel(r"$x_1$", fontsize=11)
ax.set_title('SVM with RBF Kernel: training set size = %s' % len(data_train), fontsize=9)
ax.axis('tight')
ax.legend(handles=[pluses, minuses], fontsize=9)
plt.show()

```

```

-----
NameError                                Traceback (most recent call last)
<ipython-input-23-2d02eela610> in <module>
      4 sigma=1
      5 k = functools.partial(RBF_kernel, sigma=sigma)
----> 6 f = train_soft_svm(x_train, y_train, k, ...)
      7
      8 #determine the decision regions for the predictions

NameError: name 'train_soft_svm' is not defined

```

In [ ]: