EES4404 : Renewable Energy and Smart Grid

Wind Energy

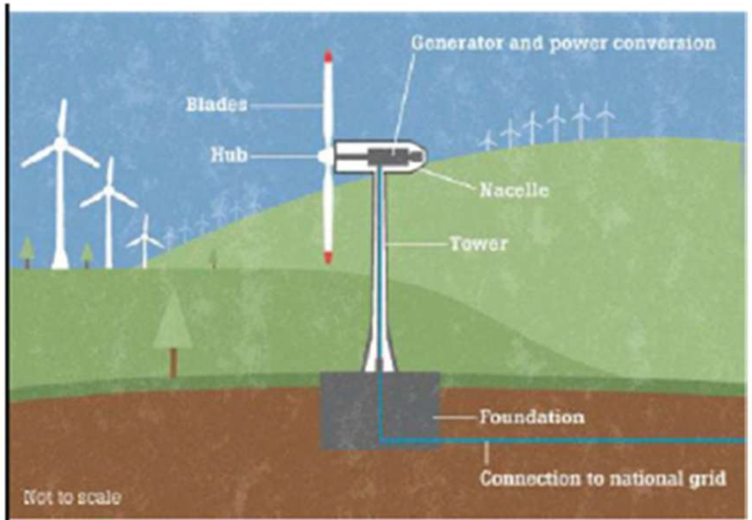
By Dr. Sahoo SK, NUS

World’s First Wind Turbine

- Vejen, Denmark.
- Built in 1891 by Dane Poul La Cour.
- He was an inventor and a high school teacher back then.
- This was the first wind turbine to generate electricity, which was used to electrolyze water, producing hydrogen for gas lights in the schoolhouse.

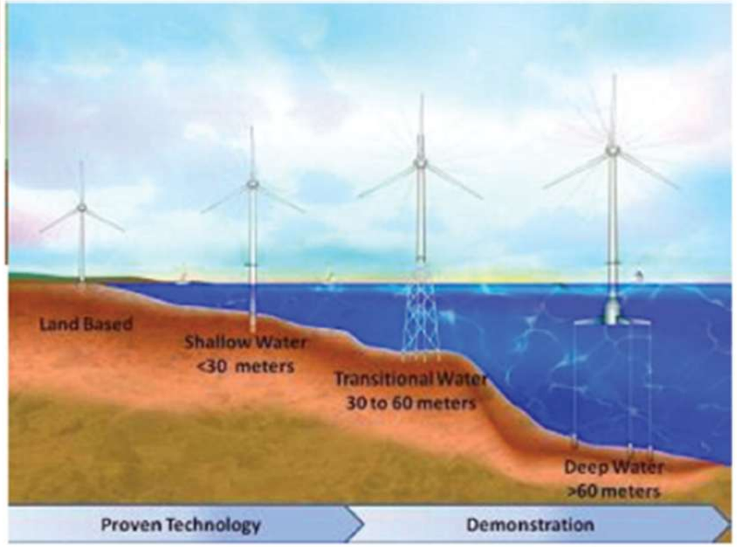
http://www.poullacour.dk/engelsk/menu.htm

Wind Power Plant Locations



source: http://www.ashden.org/wind

On-shore Wind Power Station



source: http://www.nrdc.org

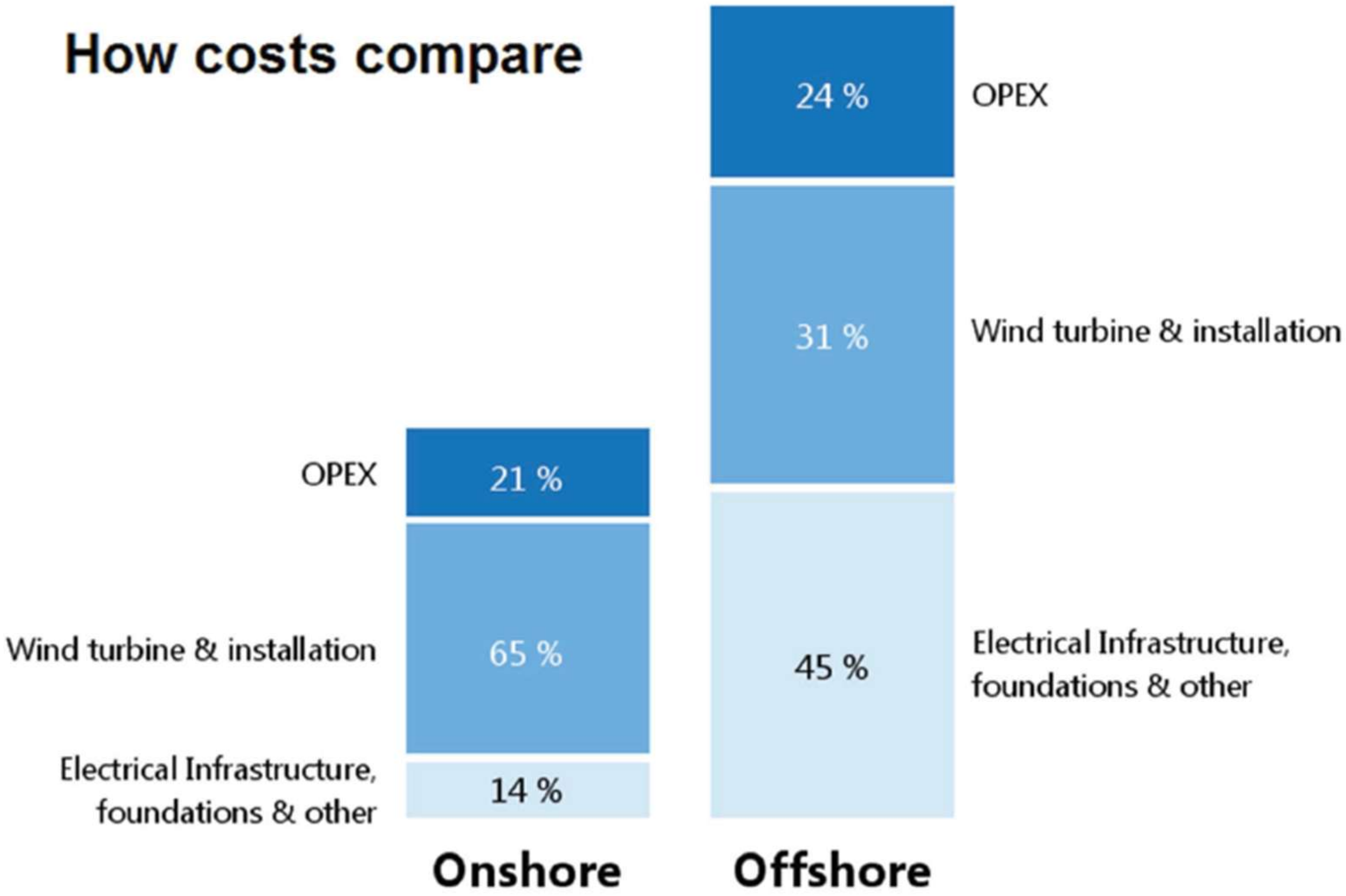
Off-shore Wind Power Station

Off Shore Wind Farms

Denmark Off-shore wind farms

- Higher wind speeds
- Less noise pollution
- Less visual impact
- Difficult to install and maintain
- Higher Energy losses due long distance transport

Onshore vs Offshore: Comparison of Costs



Largest On-Shore Wind Farm

Jiuquan wind power base is the world's biggest wind farm. Image courtesy of Popolon.

- The Gansu Wind Farm in China is the largest wind farm in the world
- Has more than 7,000 turbines arranged in rows that stretch along the sandy horizon
- Current capacity is 7900 MW, with a target capacity of 20,000 MW

Largest Off-Shore Wind Farm

The Hornsea offshore wind farm in the United Kingdom's North Sea.

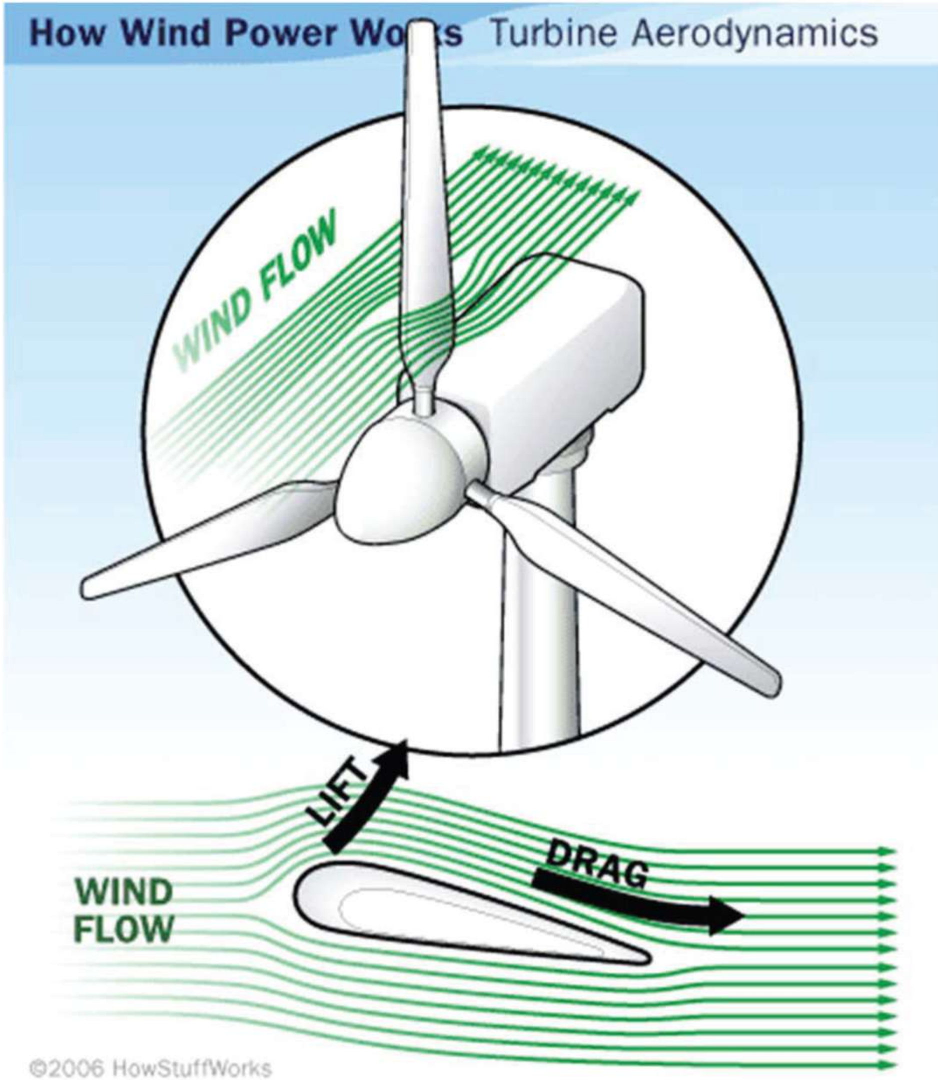
Wind Power Systems Terminology

- Wind-driven genearator
- Wind generator
- Wind turbine
- Wind-turbine generator (WTG)
- Wind energy conversion system (WECS)

Wind Energy

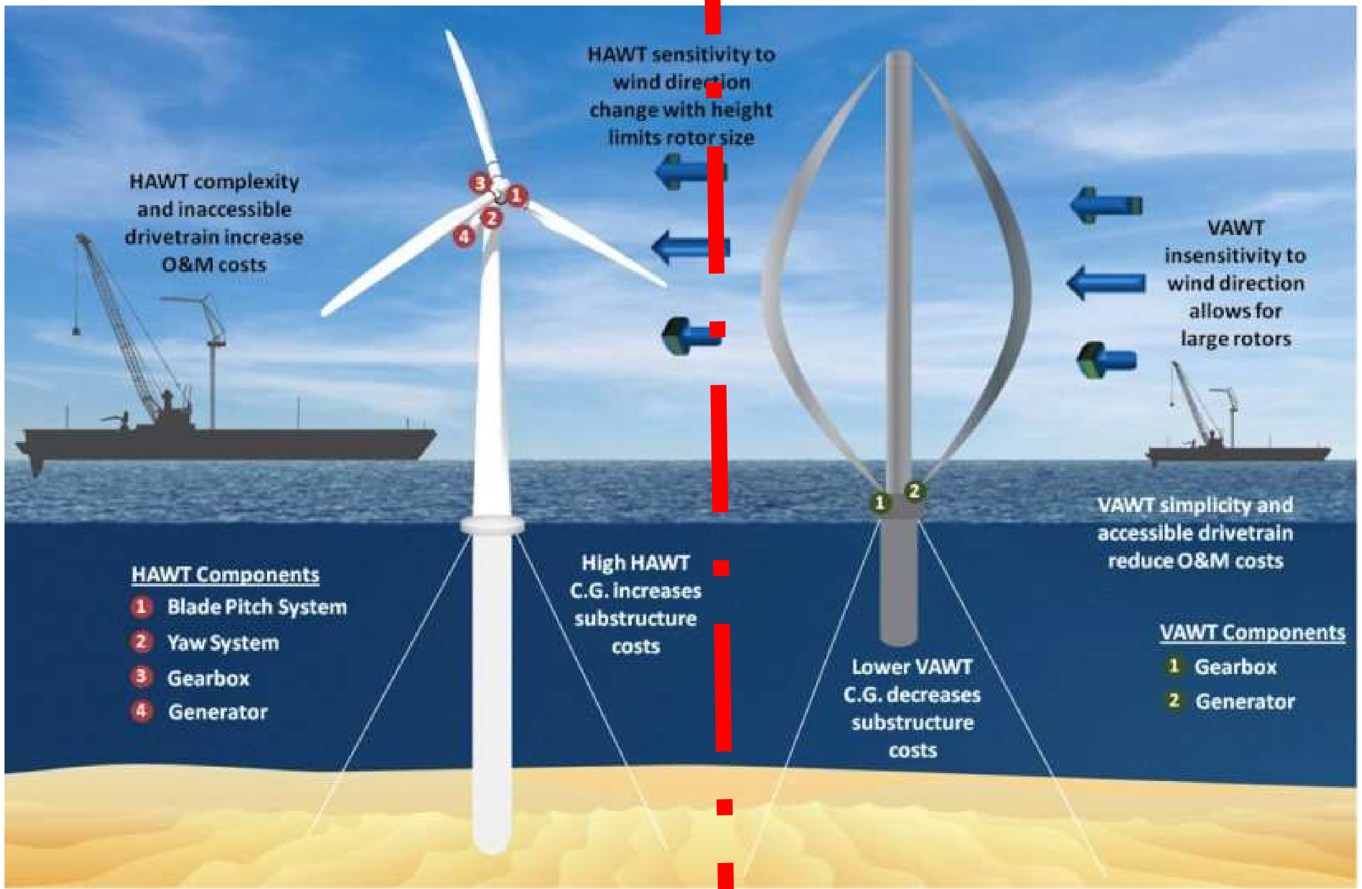
- A wind turbine extracts energy from moving air by slowing down the wind and transferring this energy into a spinning shaft, which usually turns a generator to produce electricity.

- The power in the wind that's available for harvest depends on both the wind speed and the area that's swept by the turbine blades.



Types of Wind Turbines

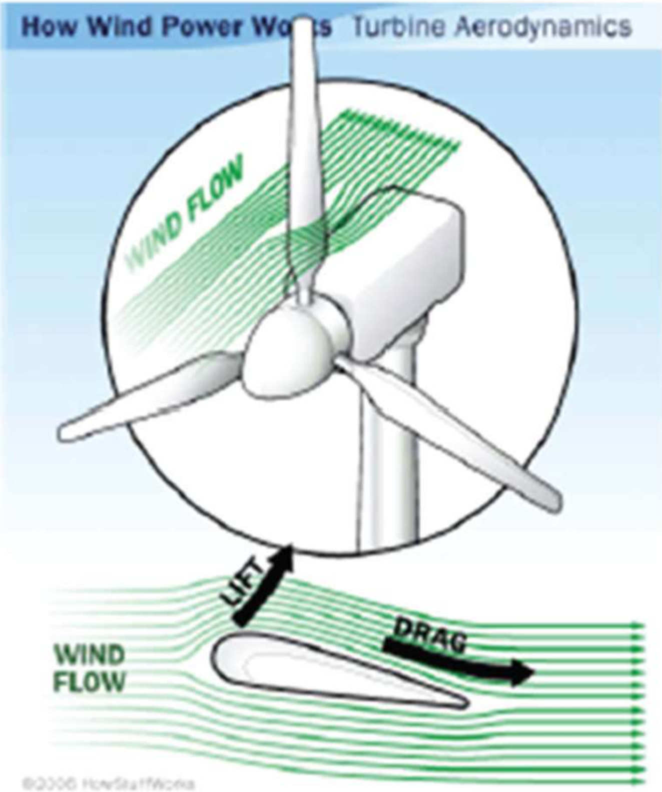
Horizontal axis wind turbines(HAWT) | Vertical axis wind turbines(VAWT)



HAWTs vs VAWTs

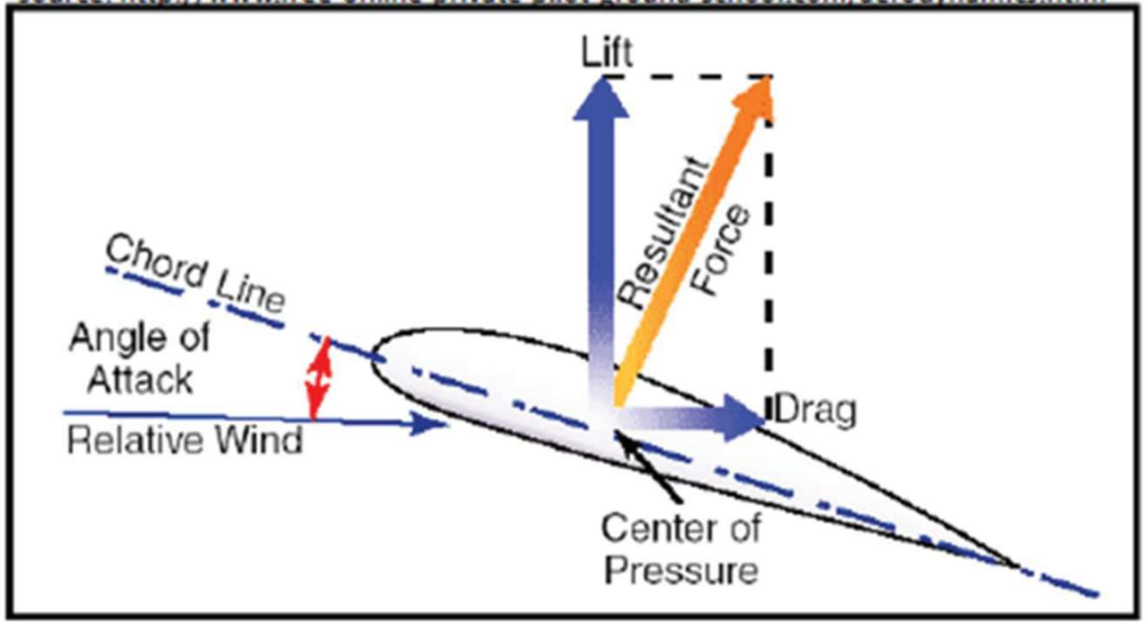
HAWT	VAWT
• The turbines need to be aligned with the wind direction. Needs yaw control.	• Capture wind energy from any direction because of the turbine's symmetry about its vertical axis. No need for yaw control.
• Capture wind energy at higher power	• Cannot capture wind energy at high altitude
• Power-train equipment located above ground, making maintenance difficult.	• Power-train equipment are located near the ground. Hence, maintenance is easier.

How Does the Rotor Turn?



Source: http://green-energy-center.blogspot.com/2008/07/wind-turbine-blade-design-designing.html

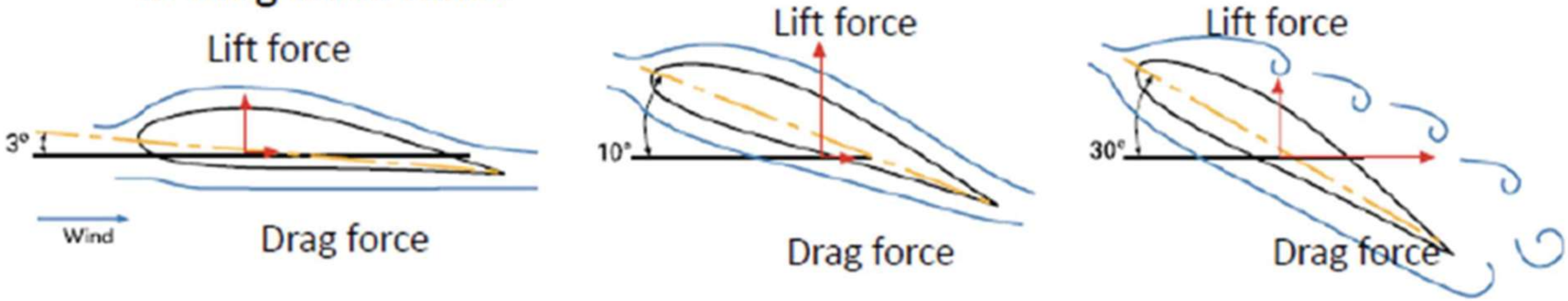
source: http://www.free-online-private-pilot-ground-school.com/aerodynamics.html



Air moving over top of airfoil has more distance to travel →Air pressure on top is lower than under airfoil
→ "Lift" is created

Angle of Attack

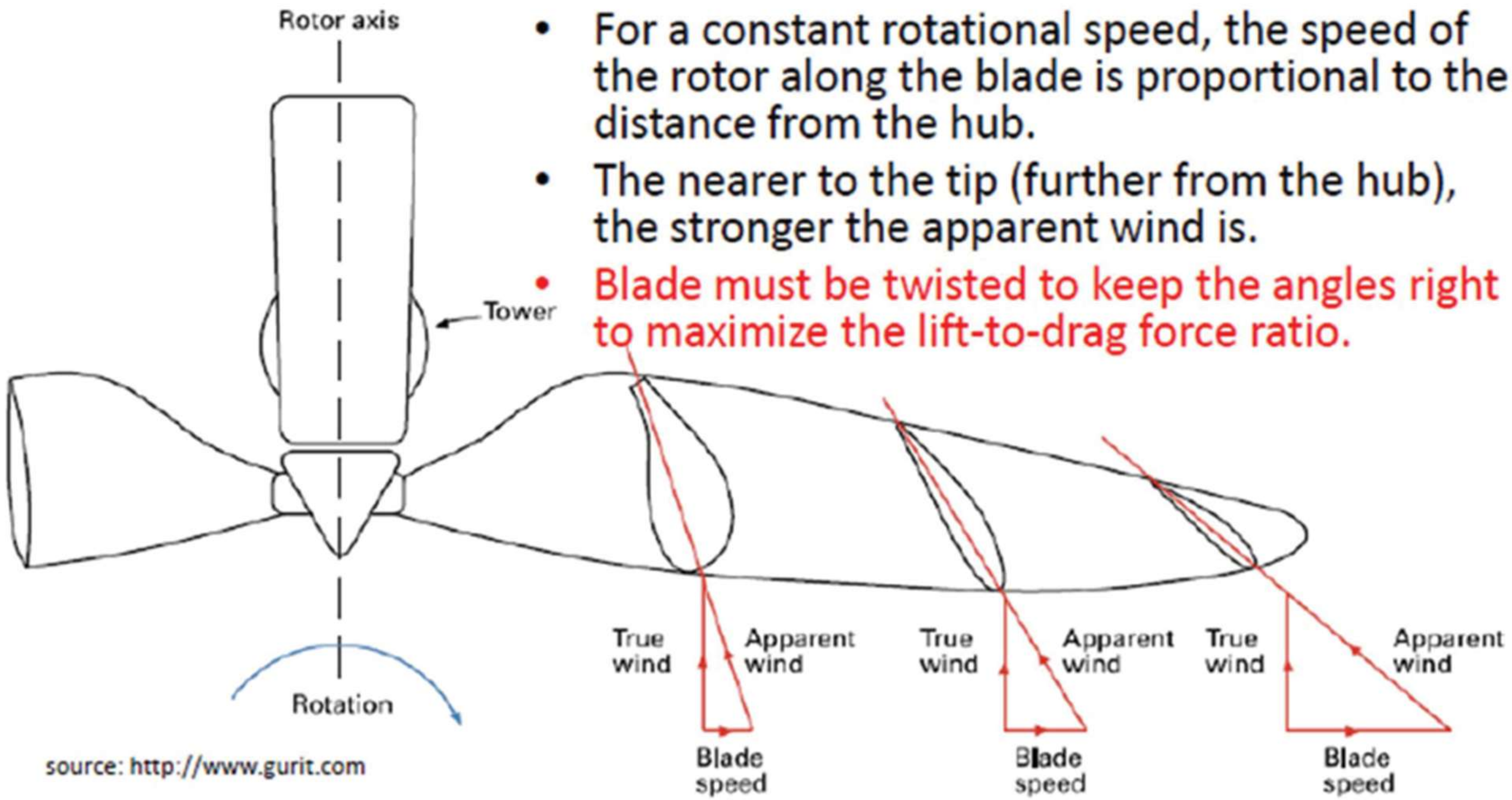
- Angle of attack improves lift. It should be set at the maximum lift-to-drag force ratio.



- Too high angle of attack can cause 'stall'.
  - Wind on top of the airfoil no longer attach to the surface
  - Drag force also reduce the effect of lift force and slow down the rotor.

- This means that we can control the speed of wind turbine by controlling the 'angle of attack'.
  - Decrease angle of attack → decrease lift-to-drag ratio (pitch control)
  - Increase angle of attack → decrease lift-to-drag ratio (stalling)

Rotor Speed Along the Blade



source: http://www.gurit.com

- For a constant rotational speed, the speed of the rotor along the blade is proportional to the distance from the hub.
- The nearer to the tip (further from the hub), the stronger the apparent wind is.
- Blade must be twisted to keep the angles right to maximize the lift-to-drag force ratio.

Number of Blades

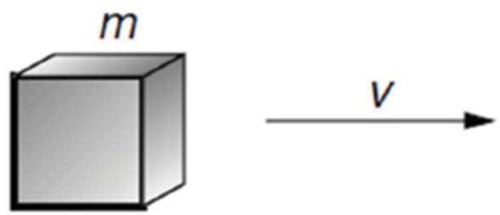
- Multi-blade wind mills need high starting torque and low speed for water pumping action.
- Fewer blades allow the turbine to spin faster → **smaller generator**
- **Two or three blades are most commonly used in modern wind turbines**

Source:
http://www.wind-energy-the-facts.org
http://www.climatechangeconnection.org
http://www.sti.nasa.gov

How to calculate wind power output?

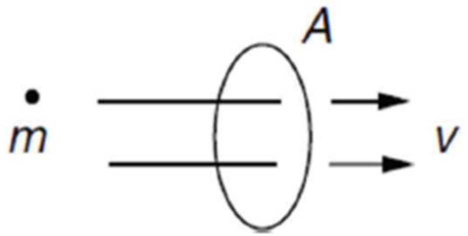
Power in the Wind

Consider a “packet” of air with mass *m moving at a speed v. Its kinetic energy* K.E., is given by the familiar relationship:



$$\text{K.E.} = \frac{1}{2} m v^2$$

Since power is energy per unit time, the power represented by a mass of air moving at velocity *v through area A will be*



$$\text{Power through area } A = \frac{\text{Energy}}{\text{Time}} = \frac{1}{2}\left(\frac{\text{Mass}}{\text{Time}}\right) v^2$$

Power in the Wind

The mass flow rate $\dot{m}$, *through area A, is the product of air density* ρ, *speed* v, and cross-sectional area A:

$$\left(\frac{\text{Mass passing through A}}{\text{Time}}\right) = \dot{m} = \rho A v$$

Substituting in equation for power: $P_w = \dfrac{1}{2}\dot{m}v$

$$\boxed{P_w = \frac{1}{2}\rho A v^3}$$

P_w is the power in the wind (watts)
ρ is the air density (kg/m^3)
A is the cross-sectional area
v = windspeed normal to A (m/s)

Power density (specific power) = power per square meter (Watts/m²)

Observations from the Equation for power

- Power in the wind depends on,
  - Air density,
  - Area that wind flow through (i.e. swept area of the turbine rotor), and
  - Wind speed.

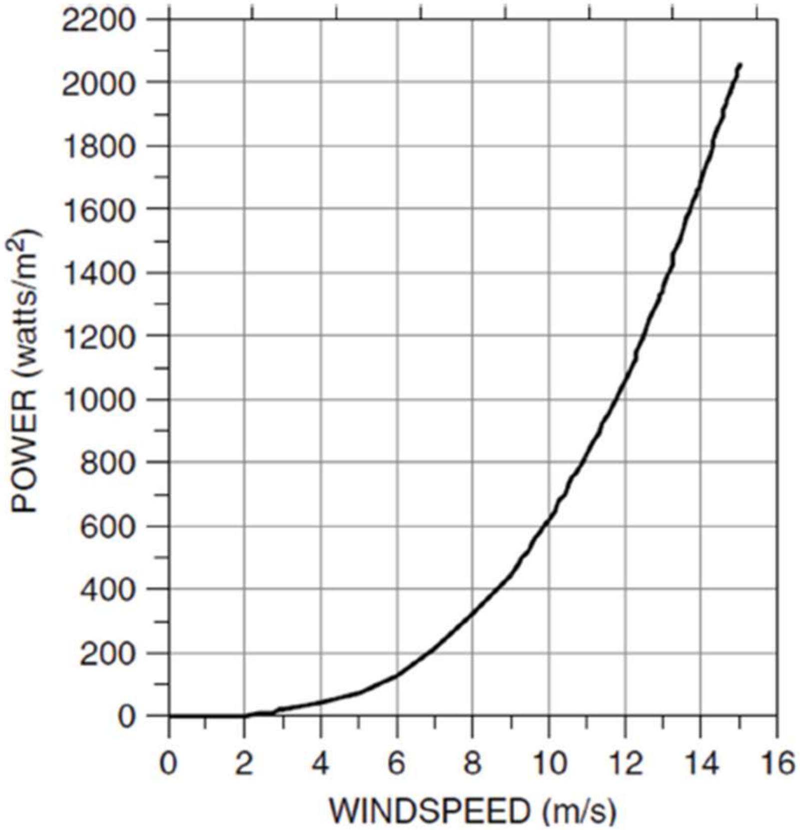
$$\boxed{P_w = \frac{1}{2}\rho A v^3}$$

- Power increases as the cube of wind speed.

- Can we calculate power using *average* wind speed?

Power in the wind, per square meter of cross section, at 15^{o}C atm pressure

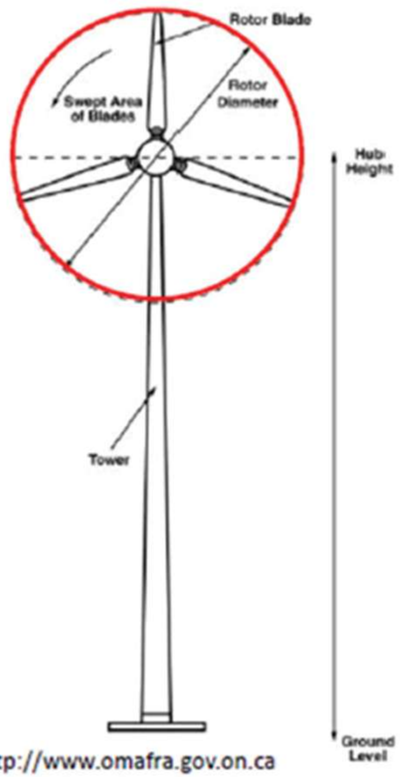
Windspeed (m/s)	Windspeed (mph)	Power (W/m^2)
0	0	0
1	2.24	1
2	4.47	5
3	6.71	17
4	8.95	39
5	11.19	77
6	13.42	132
7	15.66	210
8	17.90	314
9	20.13	447
10	22.37	613
11	24.61	815
12	26.84	1058
13	29.08	1346
14	31.32	1681
15	33.56	2067



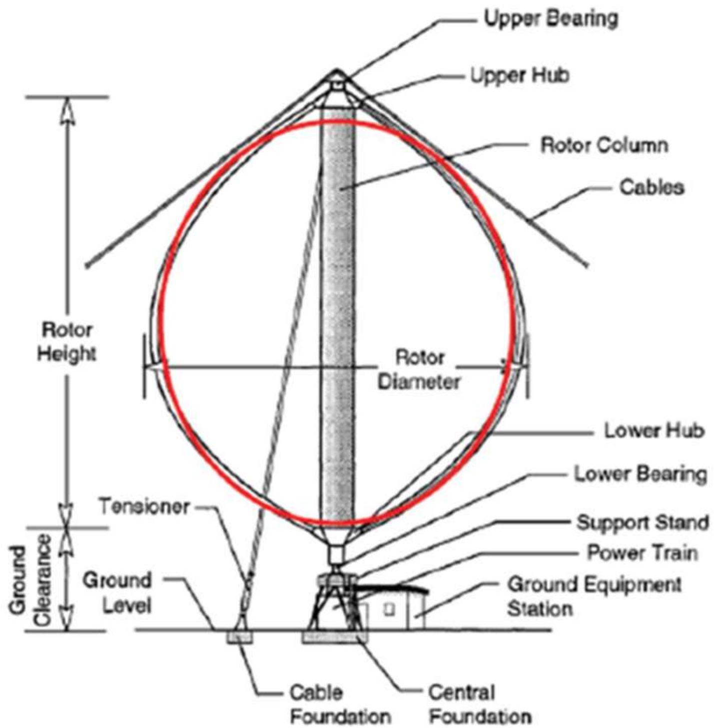
Even a small increase in wind speed results in a large increase in power

Rotor Swept Area

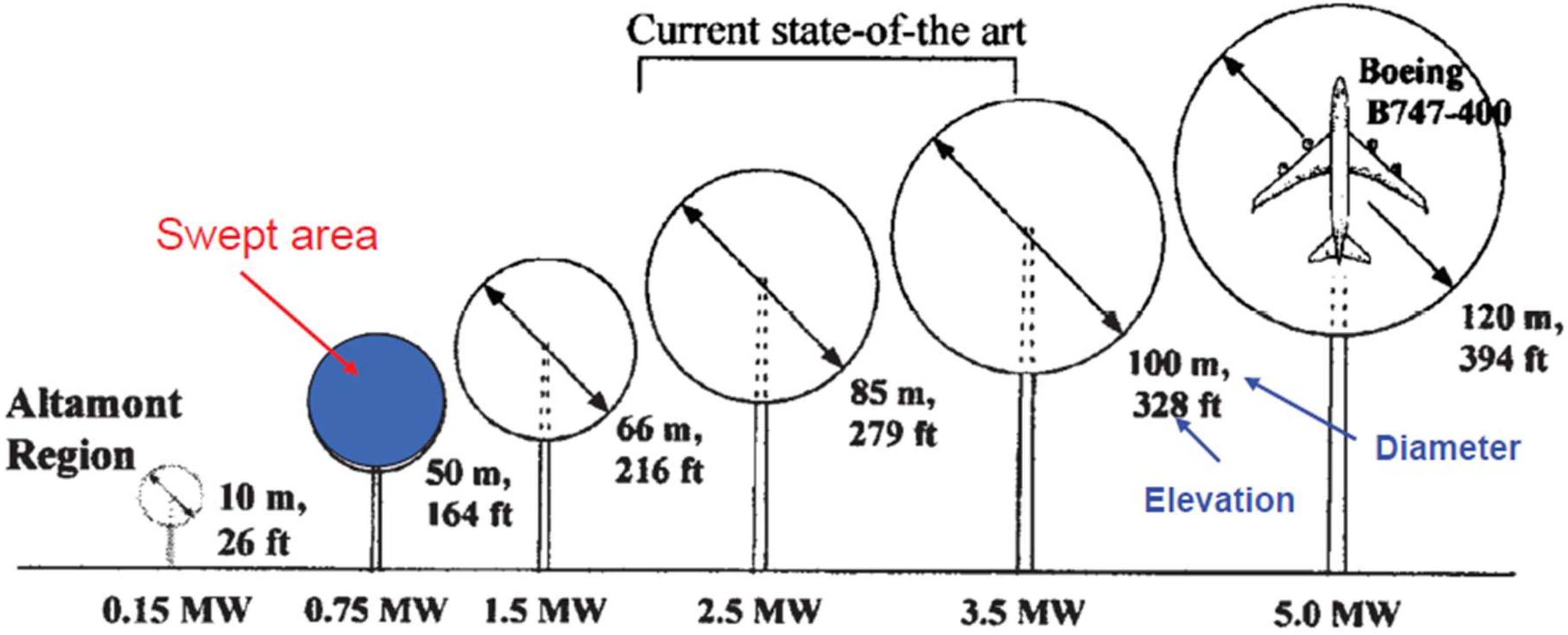
The rotor swept area, A, is important because the rotor is the part of the turbine that captures the wind energy.



source: http://www.omafra.gov.on.ca



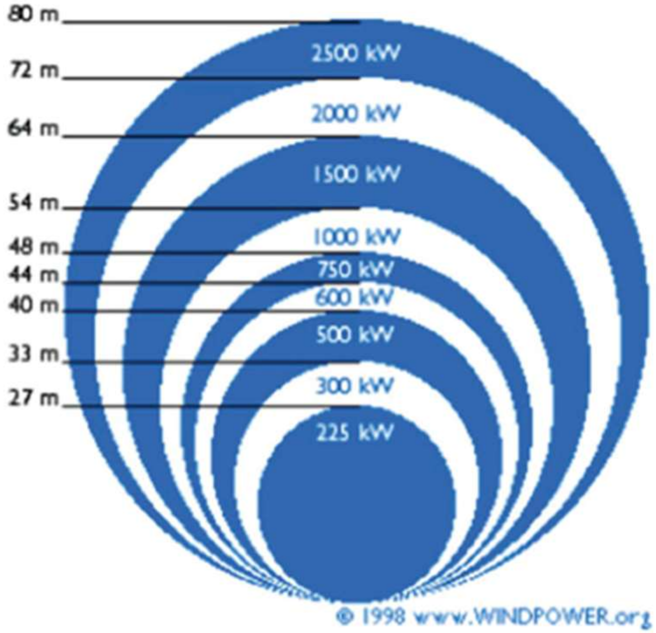
Power vs Swept Area



$$P_w = \frac{1}{2}\rho A v^3$$

The larger the rotor, the more energy it can capture

Power vs Swept Area



- Power increases as proportional to swept area of the rotor.

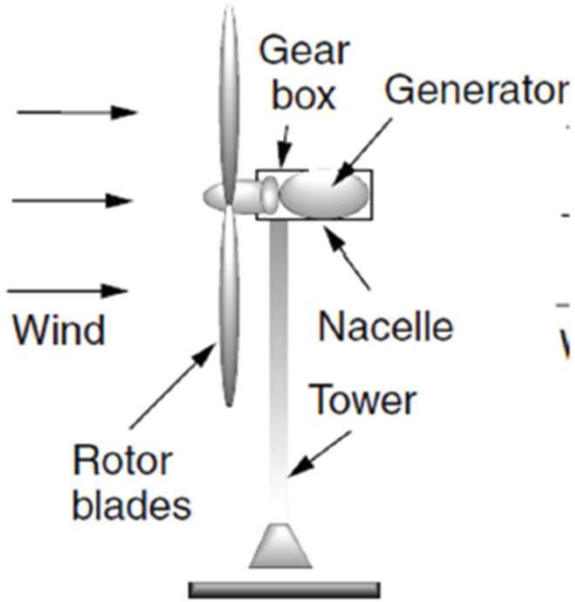
$$A = (\pi/4)D^2$$

- This implies that power is proportional to square of the diameter; the bigger, the better.
- This explains **economies of scale** of wind turbines.

The larger the diameter of its blades, the more power it is capable of extracting from the wind.

Power in the Wind – Effect of Turbine Diameter

HAWT: $A = (\pi/4)D^2$

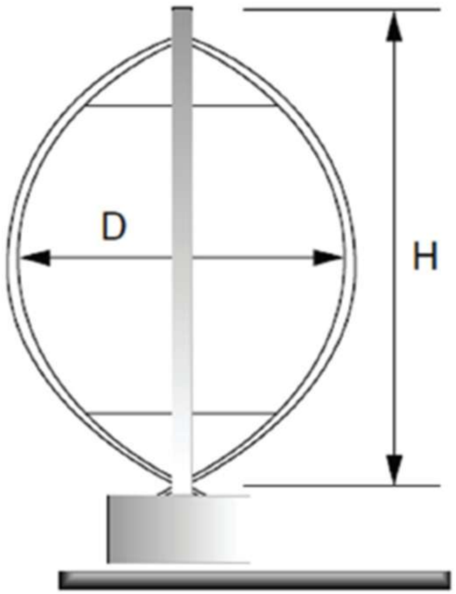


Doubling the diameter increases the power available by a factor of 4.

The cost of a turbine increases roughly in proportion to blade diameter, but power is proportional to diameter squared, so bigger machines have proven to be more cost effective.

Power in the Wind – Effect of Turbine Diameter

VAWT: $A \cong \frac{2}{3} D \cdot H$



Increase in Diameter causes only a linear increase in the power available

Power vs Air Density

Power in the wind depends on,

– Air density,

– Area that wind flows through (i.e. swept area of the turbine rotor), and

– Wind speed.

- At 15°C and 1 atmosphere, $\rho = 1.225\ kg/m^3$.

- Density = weight/volume

Power vs Air Density

The density of dry air can be calculated using the **ideal gas law**, expressed as a function of **temperature** and pressure:

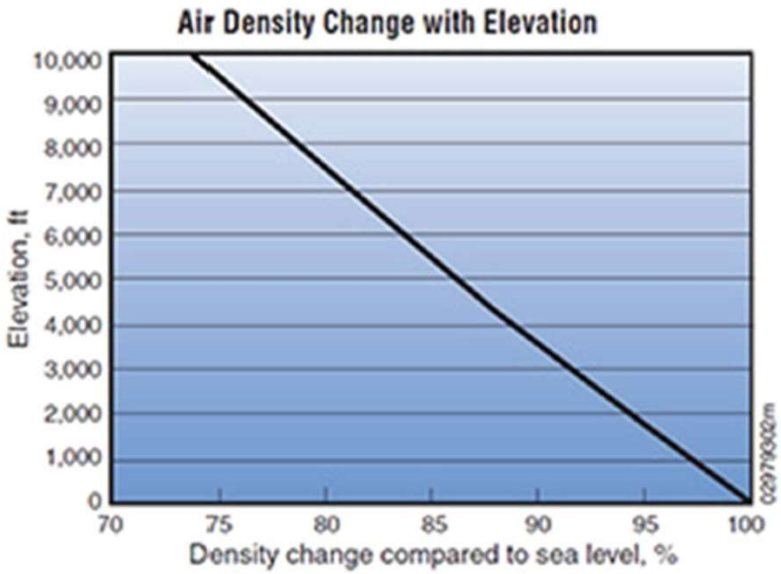
$$\rho = \frac{P}{RT}$$

where ρ is the air density, P is absolute **pressure**, R is the **specific gas constant** for dry air, and T is absolute temperature.

This means that air density depends on atmospheric pressure (P) and temperature (T).

Power vs Air Density

- **Air pressure itself is a function of altitude**
- The air is most dense at sea level and thins with increased altitude.
- During the winter, the turbine should produce more power than in the summer at the same average wind speed.



Temperature and Altitude Corrections

Temperature and altitude corrections for air density can be made using these correction factors

$$\boxed{\rho = 1.225 K_T K_A}$$

TABLE 6.1 Density of Dry Air at a Pressure of 1 Atmosphere[a]

Temperature (°C)	Temperature (°F)	Density (kg/m^3)	Density Ratio (K_T)
−15	5.0	1.368	1.12
−10	14.0	1.342	1.10
−5	23.0	1.317	1.07
0	32.0	1.293	1.05
5	41.0	1.269	1.04
10	50.0	1.247	1.02
15	**59.0**	**1.225**	1.00
20	68.0	1.204	0.98
25	77.0	1.184	0.97
30	86.0	1.165	0.95
35	95.0	1.146	0.94
40	104.0	1.127	0.92

[a] The density ratio K_T is the ratio of density at T to the density at the standard (boldfaced) 15°C.

TABLE 6.2 Air Pressure at 15°C as a Function of Altitude

Altitude (meters)	Altitude (feet)	Pressure (atm)	Pressure Ratio (K_A)
0	0	1	1
200	656	0.977	0.977
400	1312	0.954	0.954
600	1968	0.931	0.931
800	2625	0.910	0.910
1000	3281	0.888	0.888
1200	3937	0.868	0.868
1400	4593	0.847	0.847
1600	5249	0.827	0.827
1800	5905	0.808	0.808
2000	6562	0.789	0.789
2200	7218	0.771	0.771

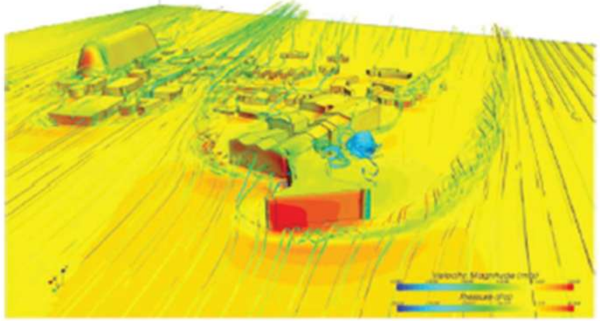
Impact of Tower Height

Wind speed near the ground is greatly affected by the friction that air experiences.

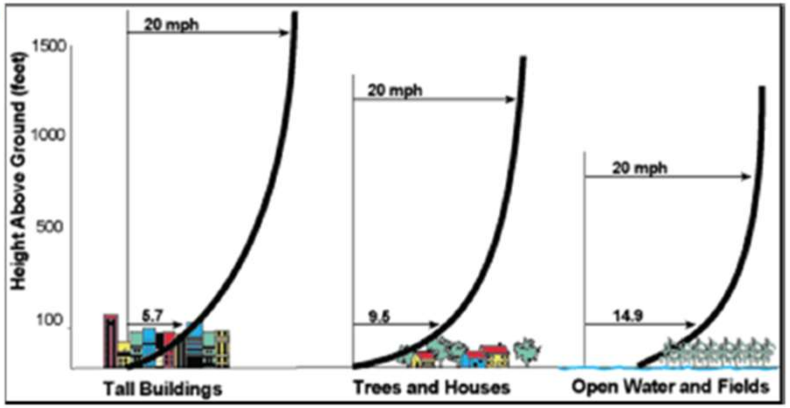
- Smooth surface, such as sea --> less friction.
- Rough surface, such as city with tall buildings --> more friction.
- Wind speed as a function of,

– Height,

– Earth's surface.



source: www.nasa.gov



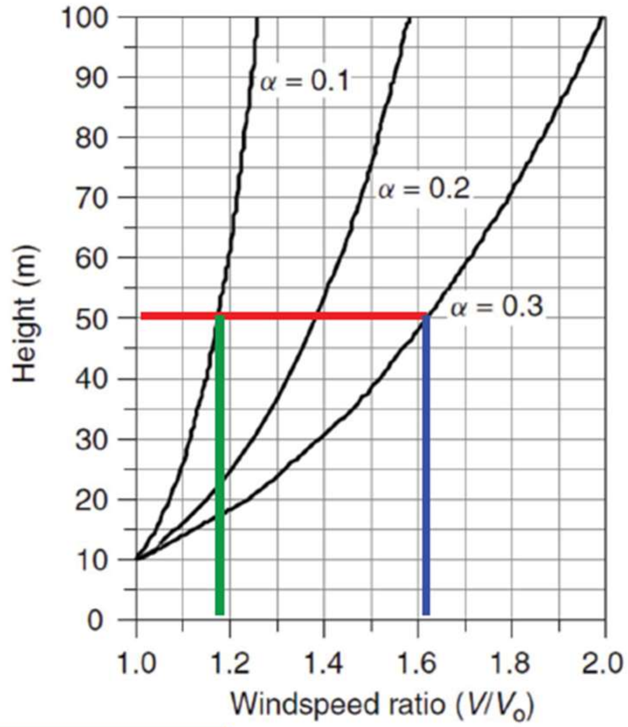
One way to get more power output from a wind system is to increase the height to which the blades are exposed.

Power in the Wind - Impact of Tower Height

$$\left(\frac{v}{v_0}\right) = \left(\frac{H}{H_0}\right)^{\alpha}$$

v is the windspeed at height H
v_0 is the windspeed at height H_0
α is the friction coefficient

Since power in the wind varies as the cube of windspeed, we can rewrite above eqn to indicate the relative power of the wind at height *H v/s* the power at the reference height of H_0 *as:*



$$\boxed{\left(\frac{P}{P_0}\right) = \left(\frac{1/2\rho A v^3}{1/2\rho A v_0^3}\right) = \left(\frac{v}{v_0}\right)^3 = \left(\frac{H}{H_0}\right)^{3\alpha}}$$

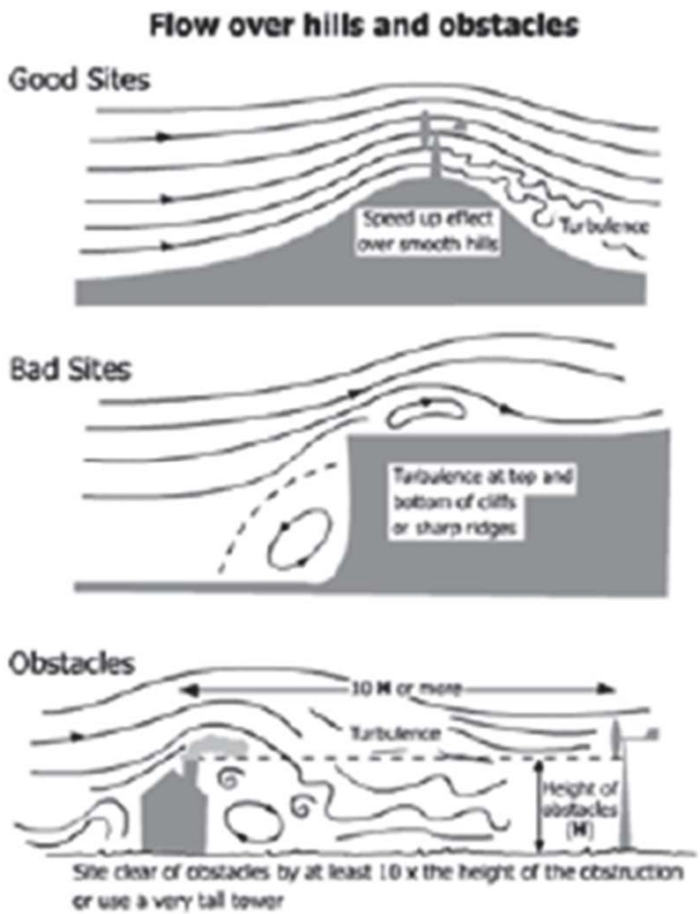
These are just approximation, nothing beats actual site measurements!!

Friction coefficient and Roughness Class

Terrain Characteristics	Friction Coefficient α
Smooth hard ground, calm water	0.10
Tall grass on level ground	0.15
High crops, hedges and shrubs	0.20
Wooded countryside, many trees	0.25
Small town with trees and shrubs	0.30
Large city with tall buildings	0.40

$$\left(\frac{v}{v_0}\right) = \left(\frac{H}{H_0}\right)^{\alpha}$$

The friction coefficient, α, depends on the terrain over which the wind is blowing. For open terrain, a value of 1/7 is often used.

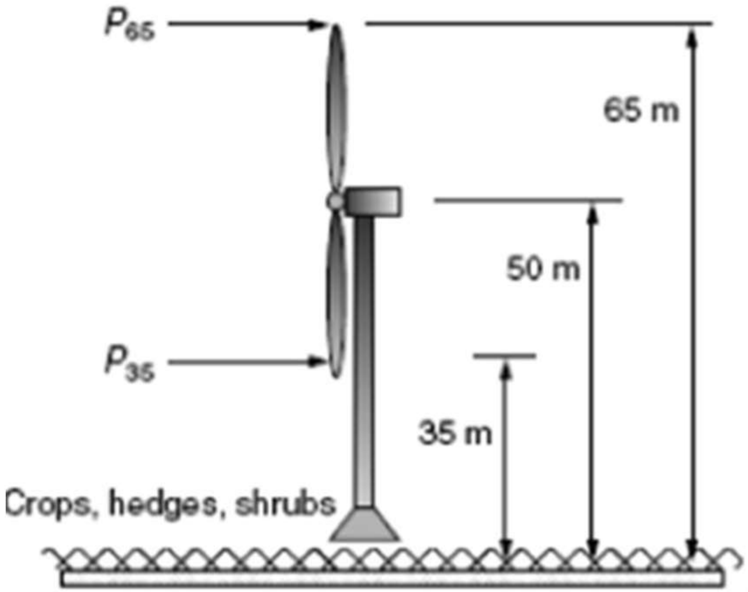


Example

A wind turbine with a 30-m rotor diameter is mounted with its hub at 50 m above a ground surface that is characterized by shrubs and hedges.

Estimate the ratio of specific power in the wind at the highest point that a rotor blade tip reaches to the lowest point that it falls to.

Solution



The ratio of velocities:

$$\left(\frac{v}{v_0}\right) = \left(\frac{H}{H_0}\right)^{\alpha}$$

$$\left(\frac{v_{65}}{v_{35}}\right) = \left(\frac{65}{35}\right)^{0.2} = 1.13$$

Since power density varies as cube of the velocity,

$$\left(\frac{P_{65}}{P_{35}}\right) = \left(\left(\frac{65}{35}\right)^{0.2}\right)^{3} = \left(\frac{65}{35}\right)^{0.2\times3} = 1.45$$

Rotor Stress

As seen in the previous example, the blade at the top of its rotation can experience much higher wind speeds than at the bottom of its rotation. This results in flexing of the blade.

It can also:

- Increase noise.
- Contribute to blade fatigue, which can lead to blade failure.

Power extracted from the Wind

Albert Betz's Formulation

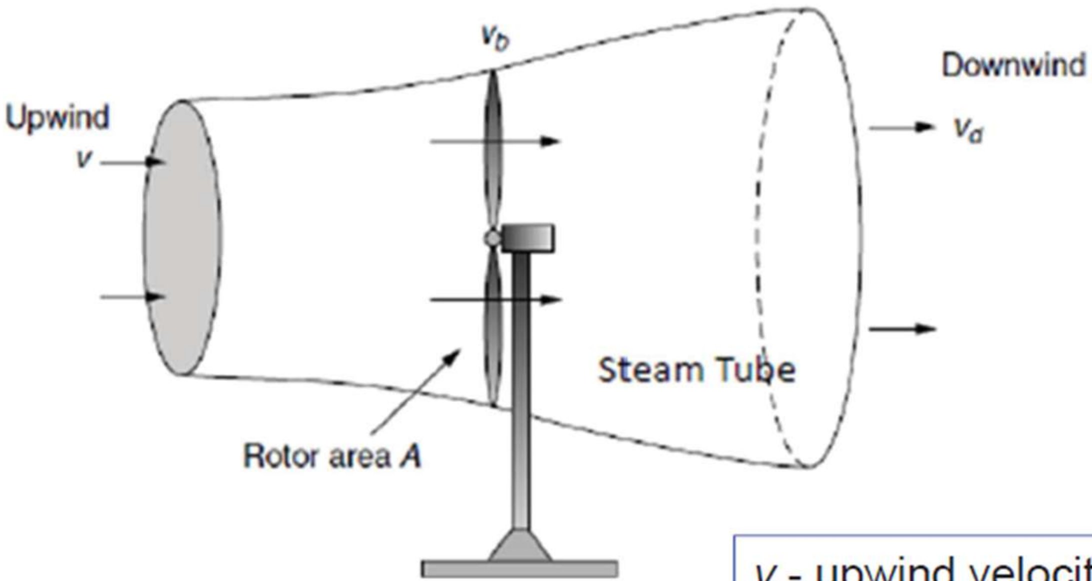
Betz derivation explains the constraint that limits the ability of a wind turbine to convert kinetic energy in the wind to mechanical power.

Albert Betz,
German physicist, 1885-1968

- Turbine blades remove energy from wind, hence wind is slowed down as a portion of its kinetic energy is extracted.

- The wind leaving the turbine is of lower pressure than the incident wind, and therefore its volume expands.

Albert Betz's Formulation..



$$P_b = \frac{1}{2}\dot{m}(v^2 - v_d^2)$$

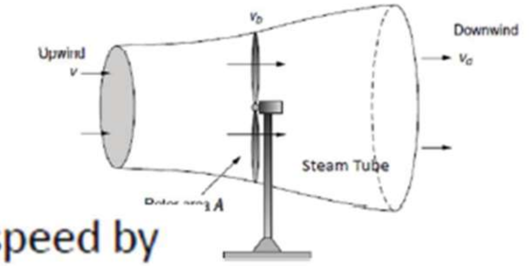
$$\dot{m} = \rho A v_b$$

v - upwind velocity of the undisturbed wind
v_b - velocity of the wind through the plane of the rotor blades
v_d - downwind velocity
$\dot{m}$ - mass flow rate of air within the stream tube (It's the same everywhere)
P_b - The power extracted by the blades

Power Extracted

Assume that the velocity of wind v_b is just the average of the upwind and downwind speed,

$$P_b = \frac{1}{2}\rho A \left(\frac{v + v_d}{2}\right)(v^2 - v_d^2)$$



Denote the ratio between upwind and downwind speed by

$$\lambda = \left(\frac{v_d}{v}\right)$$

Substitute v_d, then we have,

$$P_b = \frac{1}{2}\rho A \left(\frac{v + \lambda v}{2}\right)(v^2 - \lambda^2 v^2)$$

$$= \underbrace{\frac{1}{2}\rho A v^3}_{\text{Power in the wind}} \cdot \boxed{\underbrace{\left[\frac{1}{2}(1+\lambda)(1-\lambda^2)\right]}_{\text{Fraction extracted}}}$$

Rotor Efficiency

Define Rotor efficiency as,

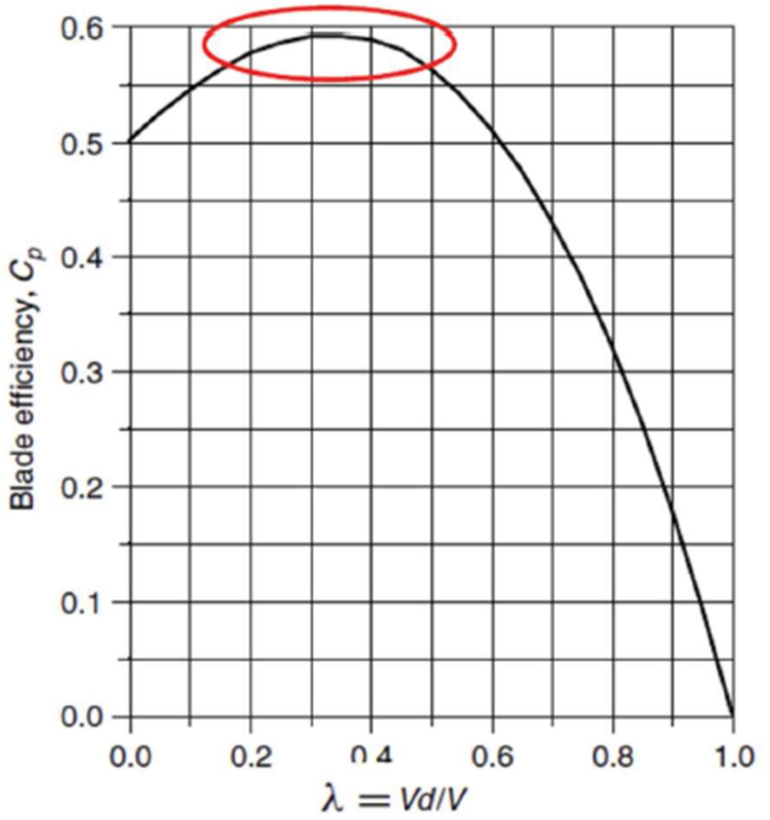
$$C_P = \frac{1}{2}(1+\lambda)(1-\lambda^2)$$

Fundamental relationship for power delivered by the rotor,

$$\boxed{P_b = \frac{1}{2}\rho A v^3 \cdot C_p}$$

How should we design λ so that we can have better rotor efficiency?

This is maximum rotor efficiency in theory.



Maximum Rotor Efficiency

$$\frac{dC_p}{d\lambda} = \frac{1}{2}[(1+\lambda)(-2\lambda) + (1-\lambda^2)] = 0$$

$$= \frac{1}{2}[(1+\lambda)(-2\lambda) + (1+\lambda)(1-\lambda)]$$

$$= \frac{1}{2}(1+\lambda)(1-3\lambda) = 0$$

The blade efficiency will be maximum if it slows the wind to one-third of the upwind speed. $\lambda = \frac{v_d}{v} = \frac{1}{3}$

Maximum Rotor Efficiency…

We can now find the maximum rotor efficiency,

Substituting λ=1/3
in Efficiency $= \frac{1}{2}(1+\lambda)(1-\lambda^2)$

$$\text{Maximum rotor efficiency} = \frac{1}{2}\left(1+\frac{1}{3}\right)\left(1-\frac{1}{3^2}\right)$$

$$= \frac{16}{27} = 0.593 = 59.3\%$$

The max. efficiency of the rotor occurs when the downstream velocity is slowed to ⅓ of its upwind velocity.

Betz's Law

- Maximum theoretical efficiency of a rotor is 59.3%.
- This is known as the Betz efficiency or Betz's law.

- How close are modern wind turbine to this Betz limit?
  - –Around 80% of the limit, 45-50%

Tip Speed Ratio (TSR)

- For a given wind speed, the rotor efficiency depends on the speed of rotation of the blades.
- TSR is the speed at rotor tip divided by the wind speed.

$$\frac{\text{Rotor tip speed}}{\text{Wind speed}} = \frac{\text{rpm} \times \pi D}{60\ v}$$

D: diameter (m)
v: wind speed (m/s)

– If TSR is high, it means that the blade spins too fast, and that the blade will experience turbulent wind.

– If TSR is low, it means that the blade spins too slowly, and it can not efficiently capture wind energy.

The optimal TSR gives the maximum efficiency that a turbine can extract wind energy.

Rotor Efficiency vs TSR

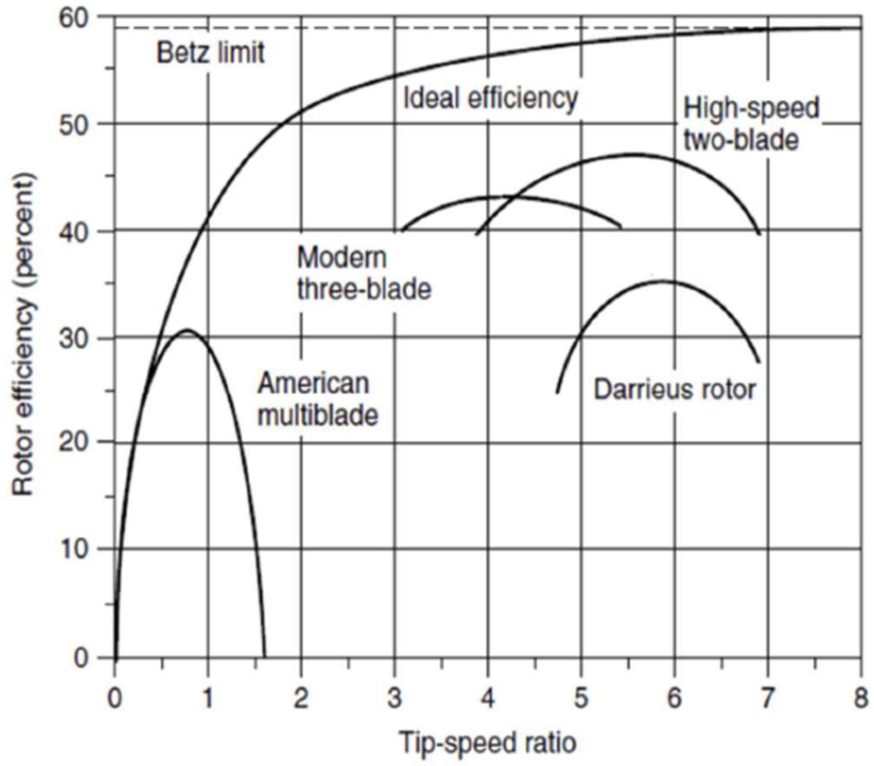
- For a given wind speed, rotor efficiency is a function of the rate at which a rotor turns.

  - If Rotor turns too slow letting too much wind to pass

    => efficiency drops.

  - If Rotor turns too fast causing turbulence

    =>efficiency drops.



Example

A 40-m, three-bladed wind turbine produces 600 kW at a wind speed of 14 m/s. Air density is the standard 1.225 kg/m3. Under these conditions,

a. At what rpm does the rotor turn when it operates with a TSR of 4.0?

b. What is the tip speed of the rotor?

c. If the generator needs to turn at 1800 rpm, what gear ratio is needed to match the rotor speed to the generator speed?

d. What is the efficiency of the complete wind turbine (blades, gear box, generator) under these conditions?

Solution

(a) At what rpm does the rotor turn when it operates with a TSR of 4.0?

$$\text{Tip-Speed-Ratio (TSR)} = \frac{\text{Rotor tip speed}}{\text{Wind speed}} = \frac{\text{rpm} \times \pi D}{60\ v}$$

$$\text{rpm} = \frac{\text{TSR} \times 60\ v}{\pi D}$$

$$= \frac{4 \times 60\ \text{s/min} \times 14\ \text{m/s}}{40\pi\,\text{m/rev}} = 26.7\ \text{rev/min}$$

That's about 2.2 seconds per revolution . . . pretty slow!

Solution..

(b) What is the tip speed of the rotor?

The tip of each blade is moving at

$$\text{Tip speed} = \frac{26.7 \text{ rev/min} \times \pi 40 \text{ m/rev}}{60 \text{ s/ min}}$$

$$= 55.9 \text{ m/s}$$

Notice that even though 2.2 s/rev sounds slow; the tip of the blade is moving at a rapid 55.9 m/s, or 125 mph.

Solution..

(c) If the generator needs to turn at 1800 rpm, what gear ratio is needed to match the rotor speed to the generator speed?

$$\text{Gear ratio} = \frac{\text{Generator rpm}}{\text{Rotor rpm}}$$

$$= \frac{1800}{26.7} = 67.4$$

Solution…

(d) What is the efficiency of the complete wind turbine (blades, gear box, generator) under these conditions?

The power in the wind is:

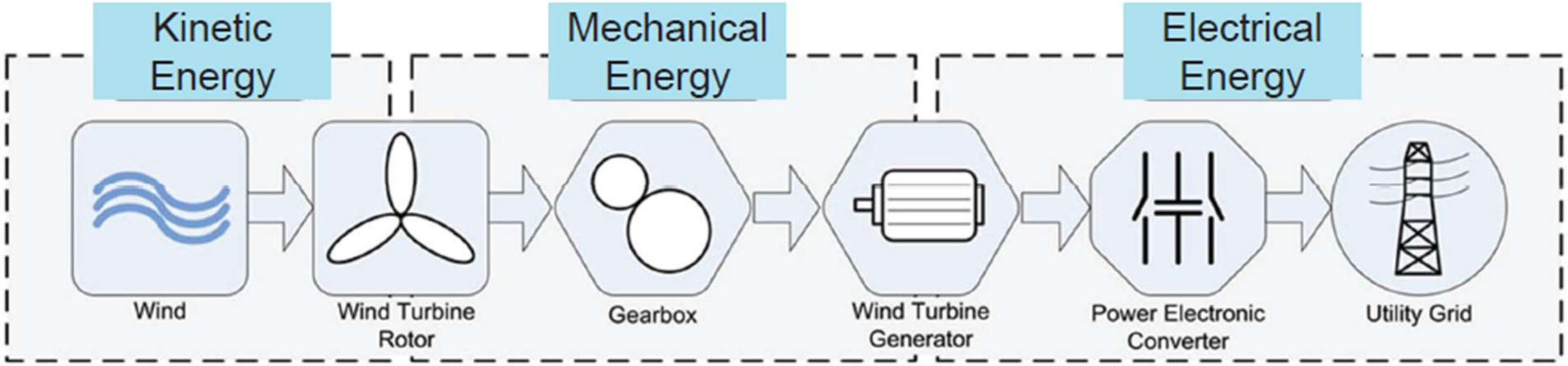
$$P_w = \frac{1}{2}\rho A v_w^3 = \frac{1}{2} \times 1.225 \times \frac{\pi}{4} \times 40^2 \times 14^3 = 2112\ kW$$

Out of the generator is given as $600\ kW$.

Overall efficiency = $\frac{600\ kw}{2112\ kW} = 0.284 = 28.4\%$

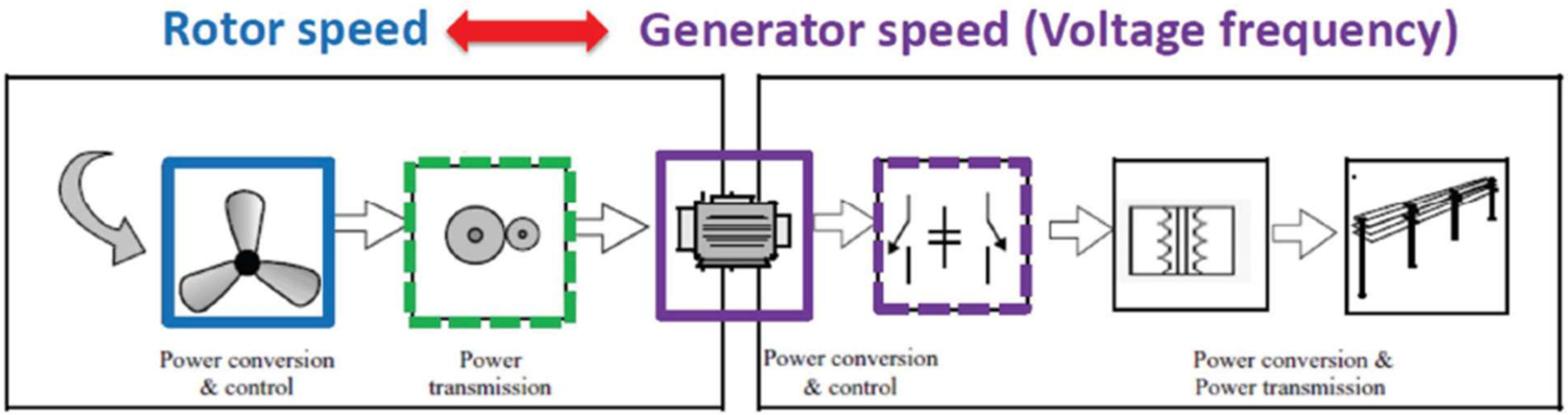
Wind Energy Conversion Systems (WECS)

Wind Energy Conversion Systems (WECS)



- The blades convert the kinetic energy from the wind into rotating shaft power (mechanical energy).
- This rotational energy is converted to electrical energy using a generator.
- In a generator, conductors move through a magnetic field to generate voltage and current.

Energy Conversion and Control

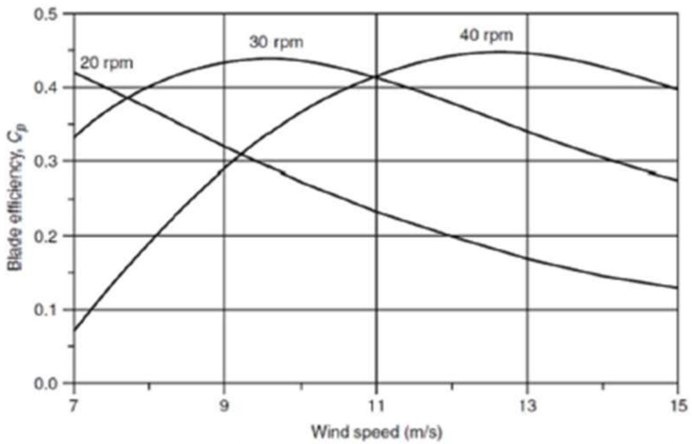


Kinetic energy
to
Mechanical power

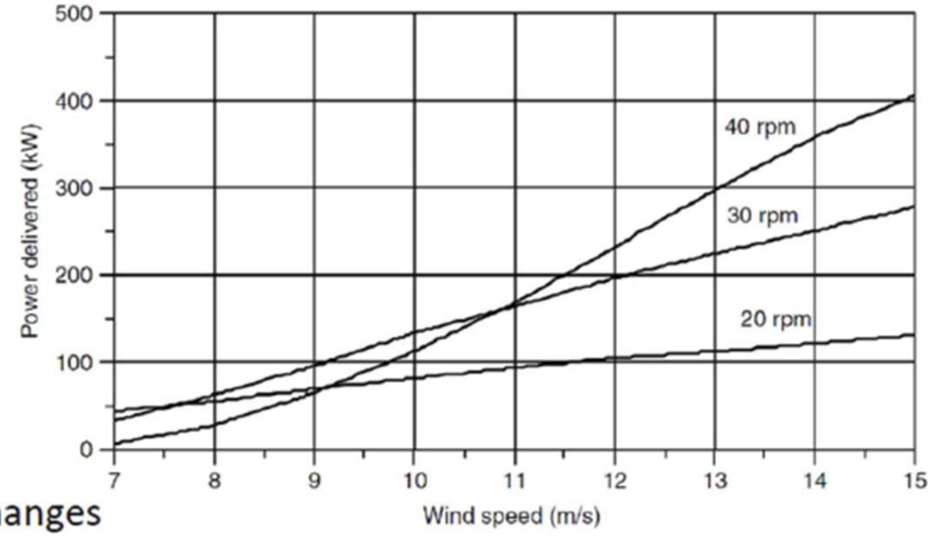
Mechanical power
to
Electrical power

Optimal Rotor speed (TSR)

- **For rotor maximum efficiency, turbine blades should change their speed as wind speed changes.**



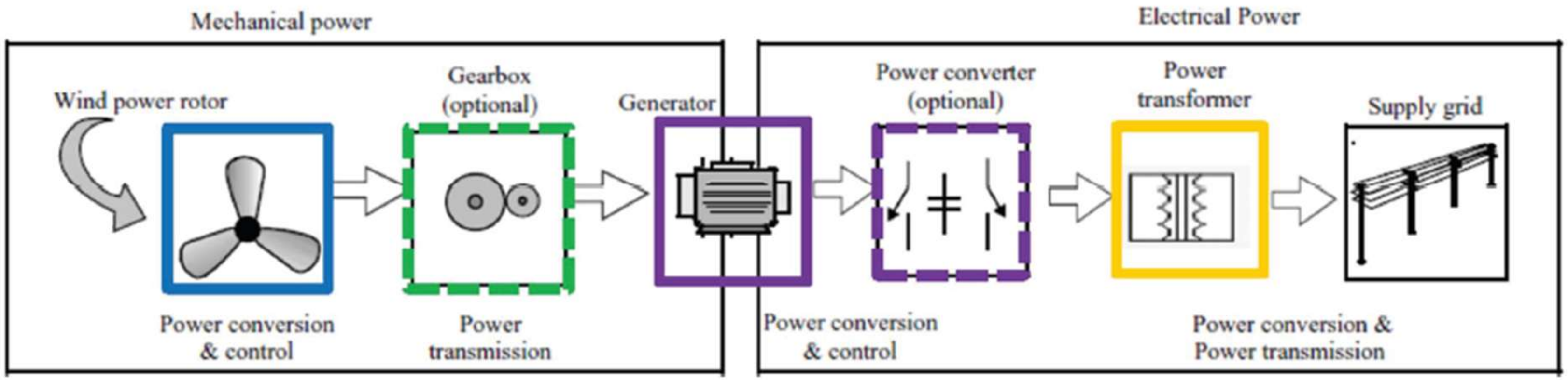
Rotation speed changes as wind speed changes



Three-step rotational speed adjustment

The **challenge** is to design a machine that can accommodate **variable rotor speeds** at a **fixed generator speed**.

Main Components of Wind Energy Conversion Systems (WECS)



Power extracted by the rotor at low rotational speed, typically at 10-15 rpm.

Gearbox is used to increase rotational speed to a higher rpm to generate 50/60Hz electricity.

Generators and power converters are the main components to make sure that WECS produce high quality voltage supplies.

Transformers are used to step-up the voltage to connect the generator to the grid.

When the wind speed varies, how can we optimally extract wind energy while maintaining a constant voltage frequency?

Energy Conversion and Control

Turbine speed control	Generator speed control
Goal: • To achieve highest rotor efficiency i.e. to operate at optimal TSR • To protect the turbine from strong winds	Goal: To maintain constant frequency of the voltage output
How: • Adjust angle of attack at the turbine blades • Stall or pitch control	How: • Multiple gearboxes design for different rotor speed to generator speed ratios. • Different generator designs and power converters are used to adjust the voltage frequency to be the same at the grid frequency.

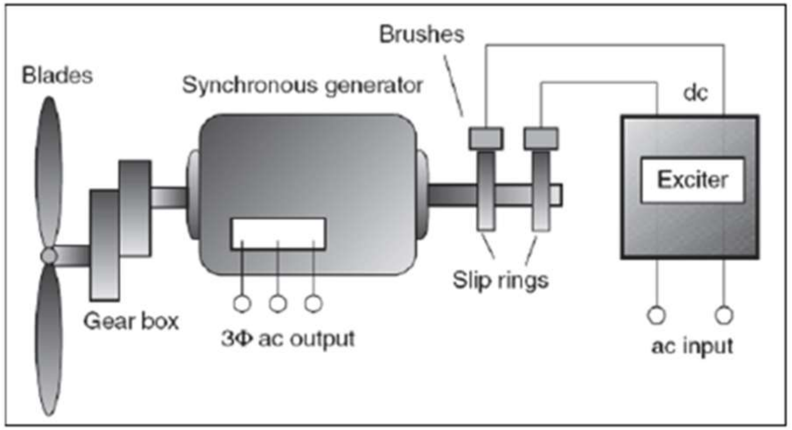
Types of Generators

Synchronous Generators:

Operate at a constant rotational speed to generate constant voltage frequency.

Require separate DC source for magnetic field.

Fixed rotor speed. Need gear box to adjust rotor speed.

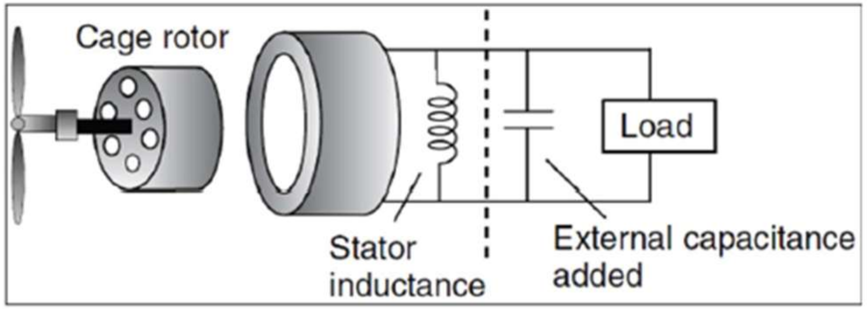


Asynchronous generators:

Induction generator operating at a rotational speed higher than voltage frequency.

Absence of separate DC source, easy to maintain but require reactive power support.

Flexible rotor speed. Can design rotor circuit to adjust rotor speed.

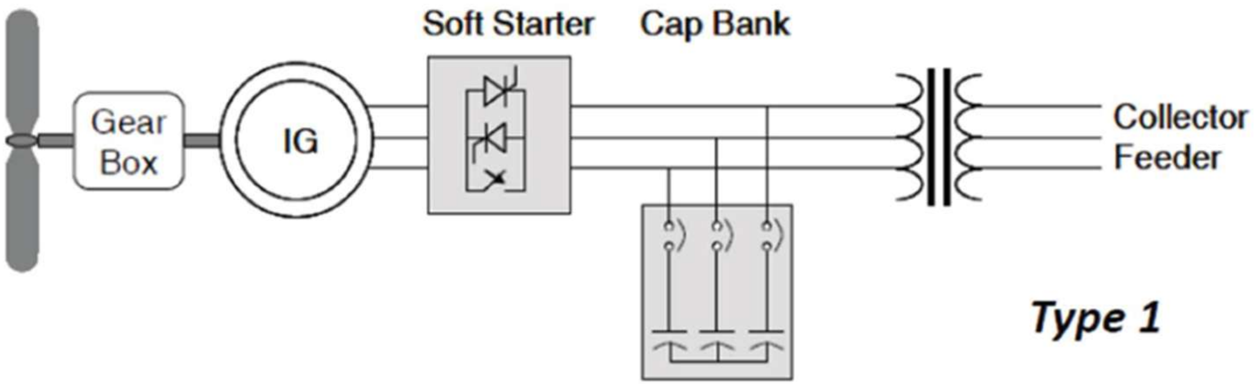


WTGs Classification by Speed Control

Wind turbine generators can be divided into 5 types.

1. Type 1: Fixed speed (1-2% variation)
2. Type 2: Limited variable speed (10% variation)
3. Type 3: Variable speed with partial power electronic conversion (30% variation)
4. Type 4: Variable speed with full power electronic conversion (full variation)
5. Type 5: Variable speed with mechanical torque converter to control synchronous speed (full variation)

Type 1: Fixed Speed Systems

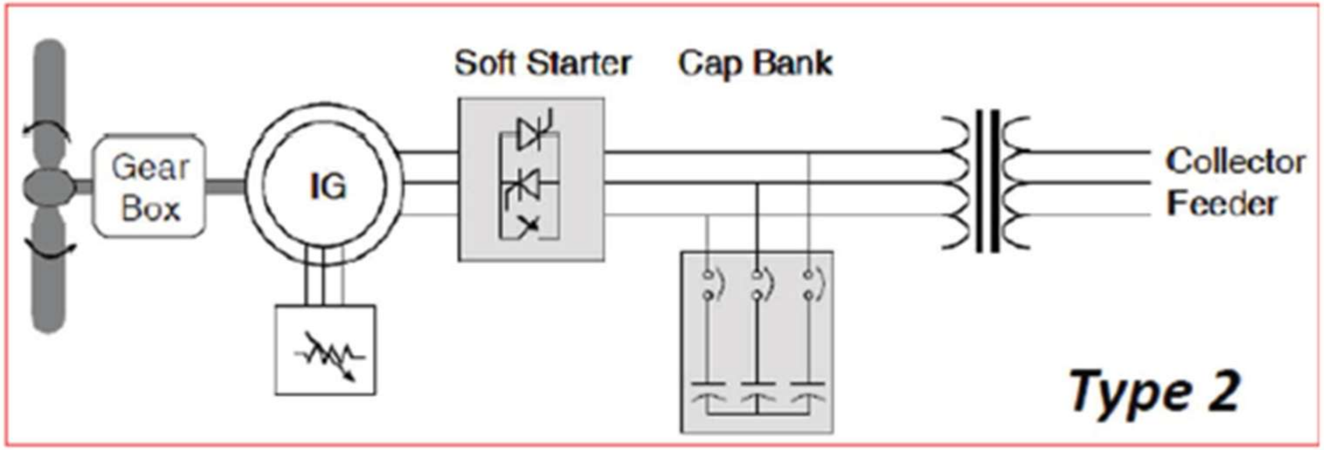


- Turbine speed is fixed (or nearly fixed with 1-2% variations) to electrical grid's frequency. This implies that the turbine may not operate at optimal TSR.
- Use aerodynamic to control turbine blades by stall or pitch control.
- Simple and reliable construction of electrical parts while cause higher stress in mechanical parts.
- Use either synchronous or induction generators and connect them directly to the grid. Induction generators are preferred due to their low maintenance and cost.

Type 2-4: Variable Speed Systems

- Decouple the electrical grid frequency and mechanical rotor frequency using power electronics systems to interface the grid, allowing variable speed operation to achieve optimal TSR.
- Use synchronous or asynchronous (induction) generators with power electronics.

Type 2: Variable Speed Systems



- Variable speed induction generators (VSIG) are often used in wind energy conversion systems to harness energy efficiently from varying wind speeds.
- The rotor resistance control method involves adjusting rotor circuit's resistance to control the generator's speed

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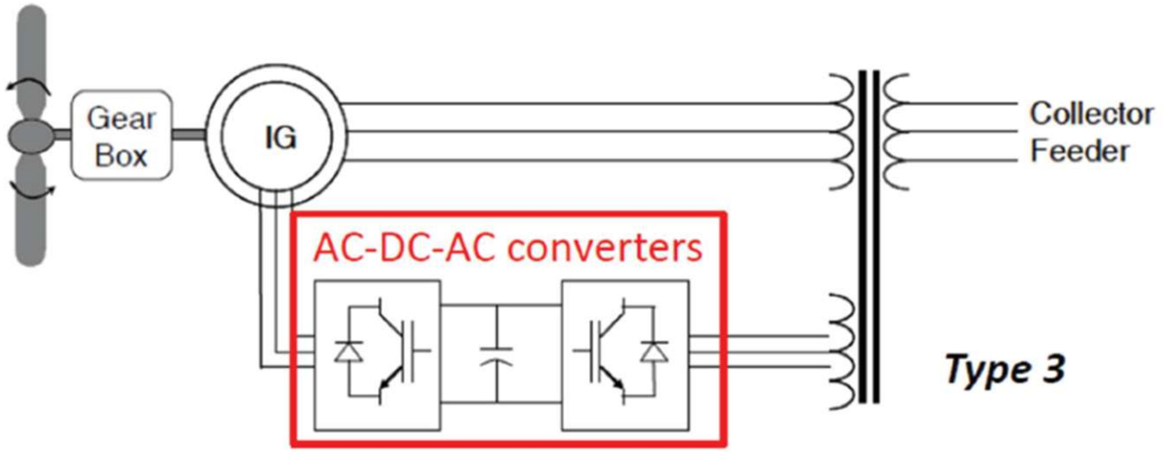
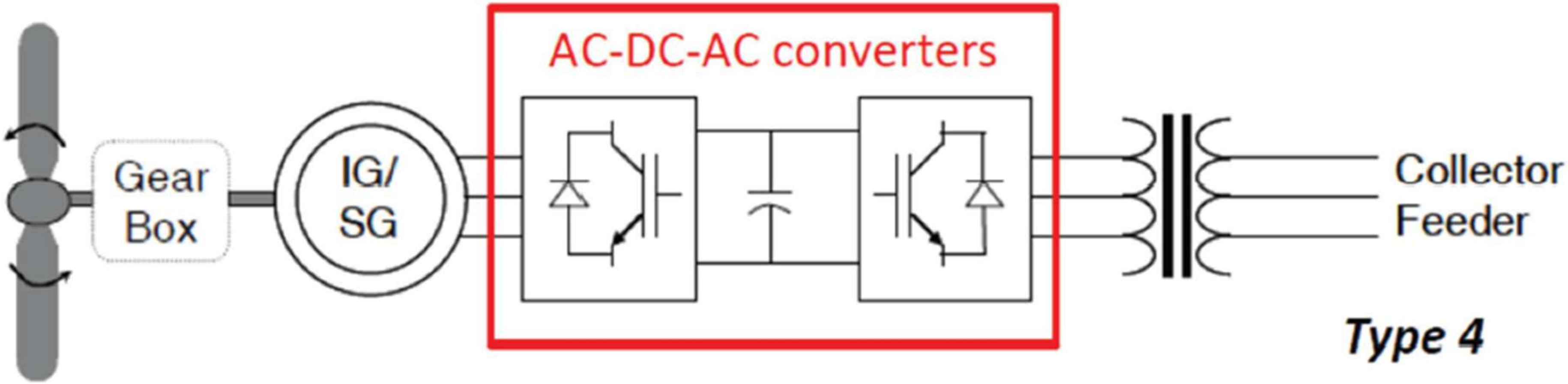
Type 3: Doubly Fed Induction Generator(DFIG)



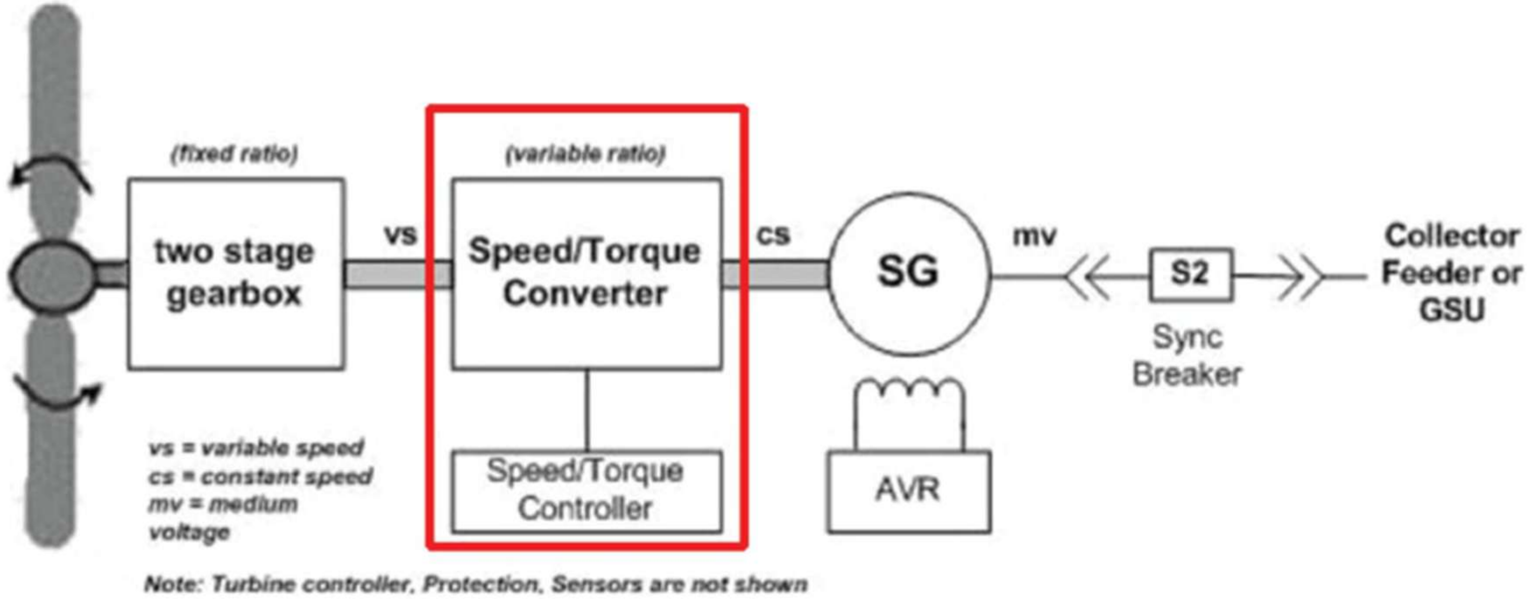
- Instead of variable resistors in Type 2, this Type 3 design adds AC-DC-AC converters to the rotor circuit.
- Rotor frequency is decoupled from grid frequency.
- The machine can still be synchronized with the grid while the wind speed varies.

Type 4: Indirect Grid Connection



- Allows the turbine to rotate at its optimal speed.
- AC output from generator frequency is different and decoupled from grid frequency.
- AC-DC-AC converter is used to connect the AC output to the grid.
- Full control and flexibility in the design and operation of wind turbine.
- The ratings of power electronics are higher than Type 3.

Type 5: Mechanical control



Speed/torque converter:

- To achieve maximum power, $P = T \times \omega$
- To adjust the rotor speed according to grid frequency.
- Operate as typical synchronous generators

Speed Control for Wind Turbines

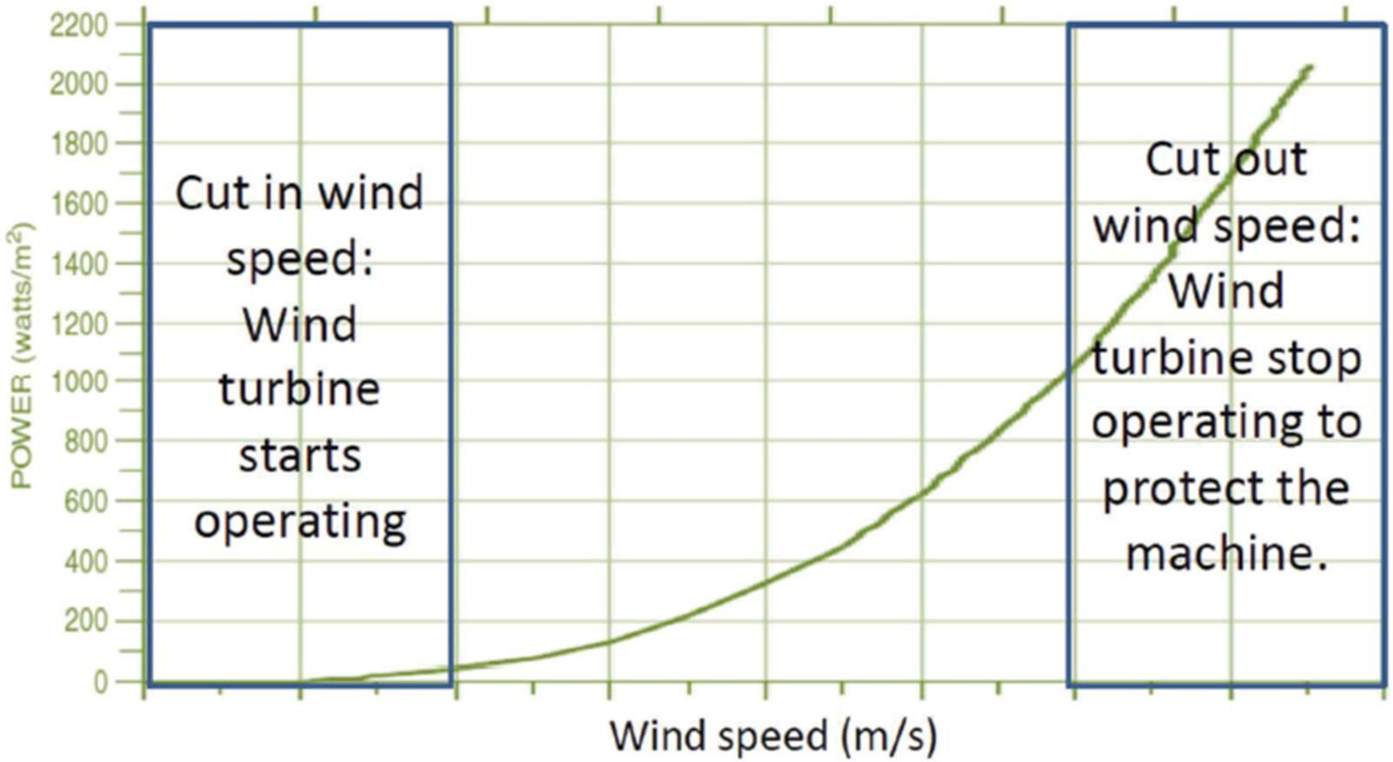
Speed can be controlled for 2 purposes.

1. To achieve high energy conversion efficiency while producing constant voltage frequency during normal operation.
2. To protect wind turbine during turbulent weather.

Turbine speed can be controlled through:

1. Electrical parts: Generator, Power converters
2. Mechanical parts: Gearbox, Yaw control, Turbine blade (aerodynamic design)

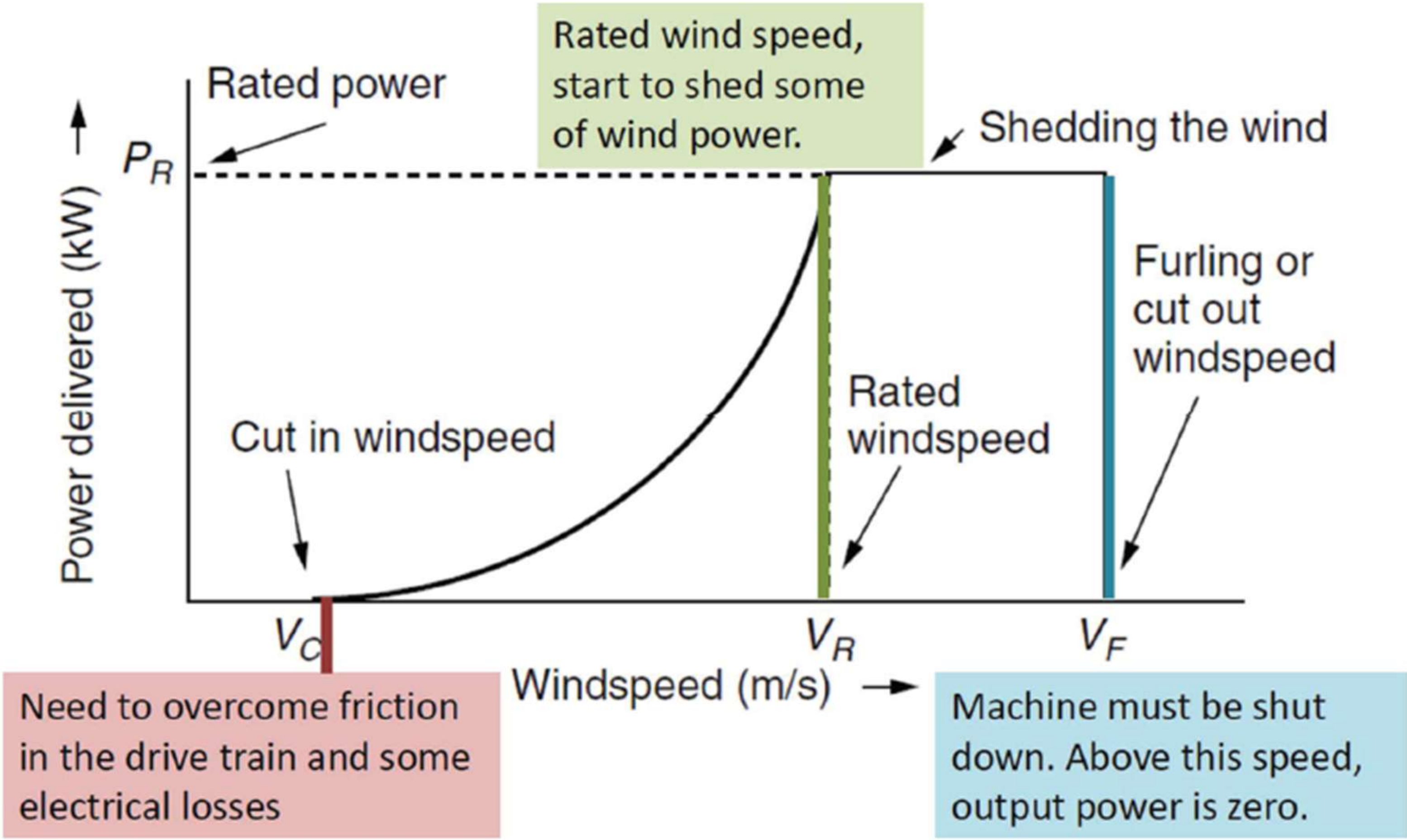
Extracted Power from Wind



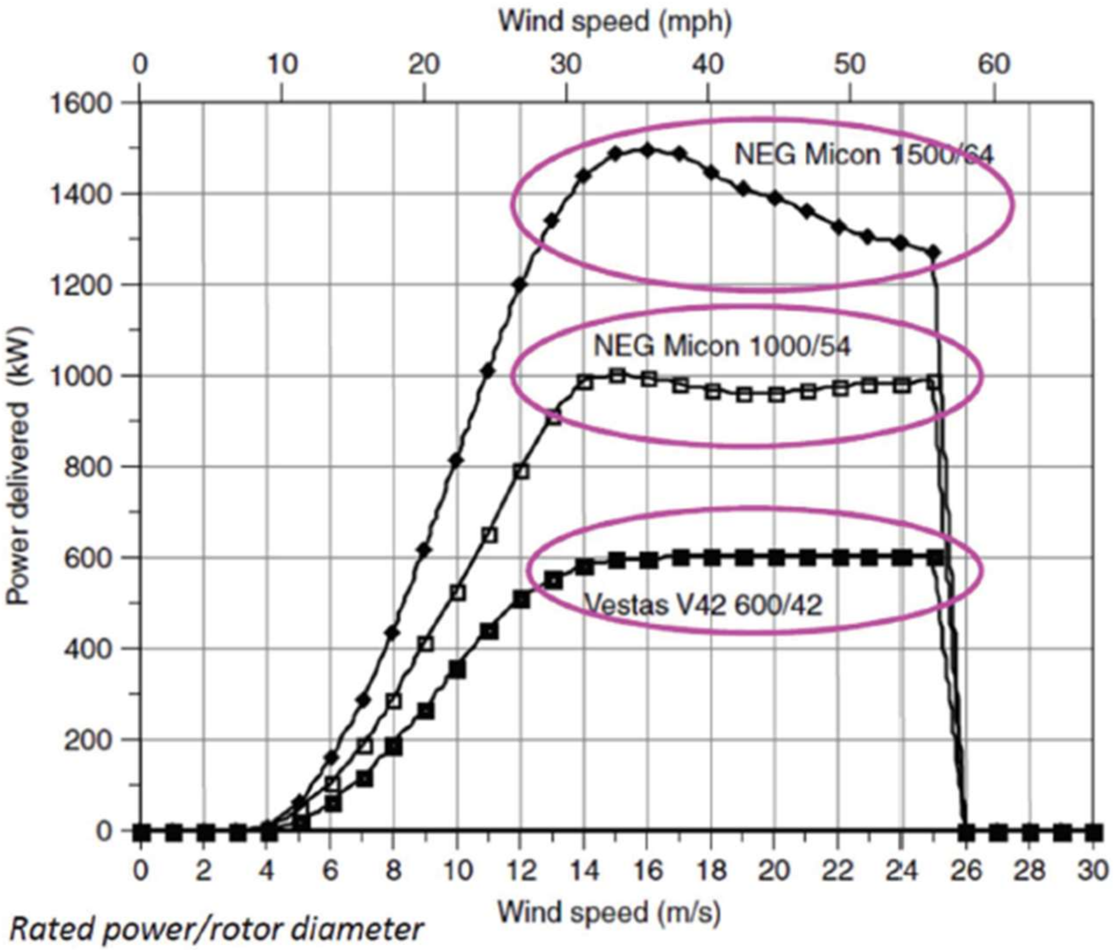
What Strong Wind Can Do

This 300ft wind turbine in Ayrshire, UK exploded into flames when it was buffeted by high winds

Ideal Power Curve



Real Power Curve



Rated power/rotor diameter

It is difficult to find rated wind speed for a large wind turbine.

How to Shed Wind Power

Pitch-controlled turbines

– Active control by reducing ‘angle of attack’

Stall-controlled turbines.

– Passive control using pure aerodynamic design.

Active stall control.

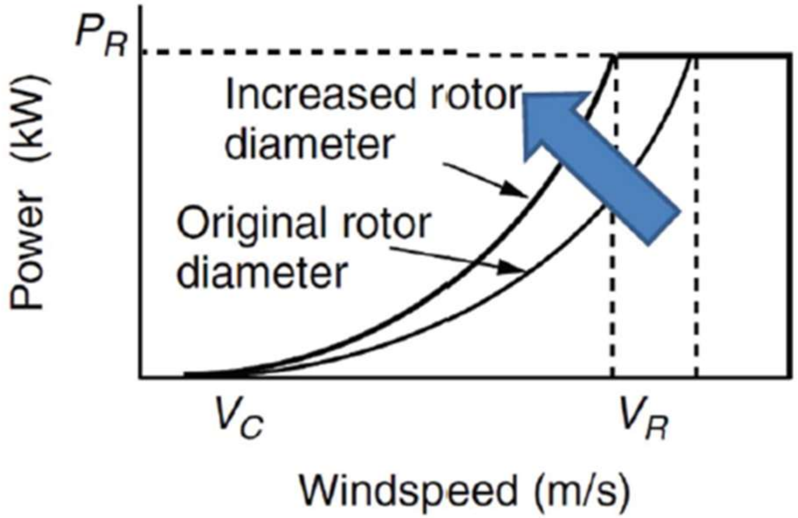
– Induce stall for large wind turbine by increasing ‘angle of attack’.

Passive yaw control

– Small kW size turbine, causing axis of turbine to move off the wind.

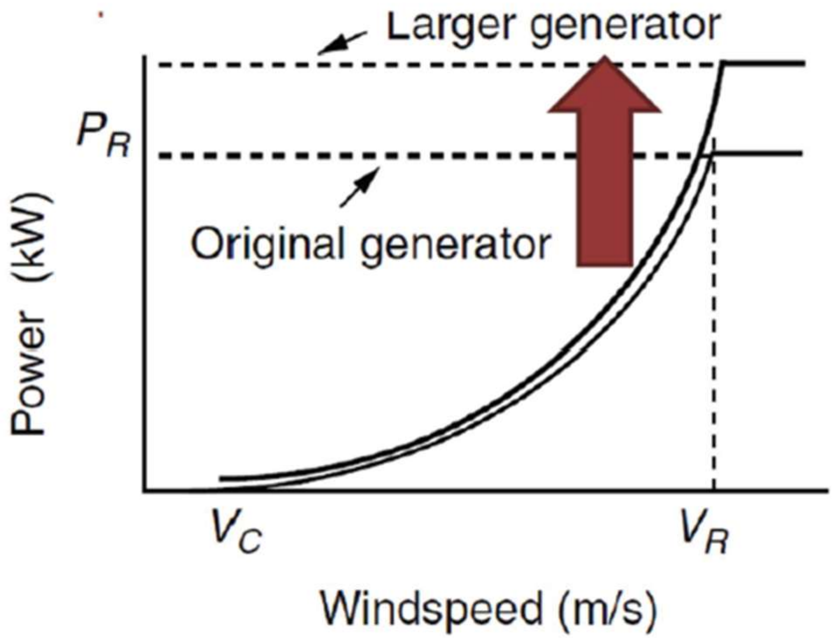
Rotor Diameter vs Gen. Rated Power

With the same generator, rated power is reached at lower wind speed.



This strategy increases output power for lower-speed winds.

With the same rotor diameter, rated power is increased to new rated power



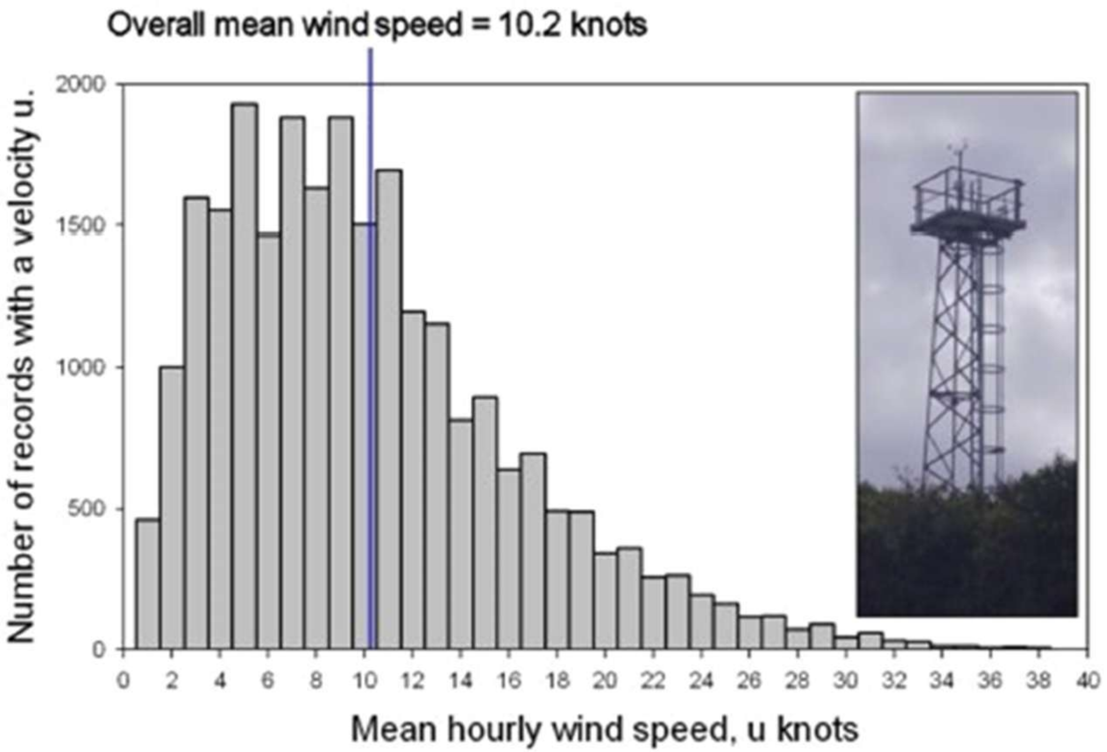
This strategy increases output power for higher-speed winds.

Wind Speed Statistics
and
Average Power in the Wind

“Average power” and “power at average speed”

“Average power” is not the same as “power at average speed” as wind power is proportional to cube of wind speed.

Wind Speed Statistics and "average" wind speed



Histogram of hourly wind speeds at Plymouth, Mountbatten.
(Years 2005 to 2007 - 25,203 valid data points)

(adapted from Crown copyright data supplied by the Met Office)

1 knot = 0.5144 m/s

Average Wind Speed calculation

$$v_{\text{avg}} = \frac{\text{Miles of wind}}{\text{Total hours}} = \frac{3\ \text{h} \cdot 0\ \text{mile/hr} + 3\ \text{h} \cdot 5\ \text{mile/h} + 4\ \text{h} \cdot 10\ \text{mile/h}}{3 + 3 + 4\ \text{h}}$$

$$v_{\text{avg}} = \left(\frac{3\ \text{h}}{10\ \text{h}}\right) \times 0\ \text{mph} + \left(\frac{3\ \text{h}}{10\ \text{h}}\right) \times 5\ \text{mph} + \left(\frac{4\ \text{h}}{10\ \text{h}}\right) \times 10\ \text{mph} = 5.5\ \text{mph}$$

A more general expression for the above two equations would be:

Probability that the speed is v_i!!

$$v_{\text{avg}} = \frac{\sum_i [v_i \cdot (\text{hours @ } v_i)]}{\sum \text{hours}} = \sum_i [v_i \cdot \boxed{(\text{fraction of hours @ } v_i)}]$$

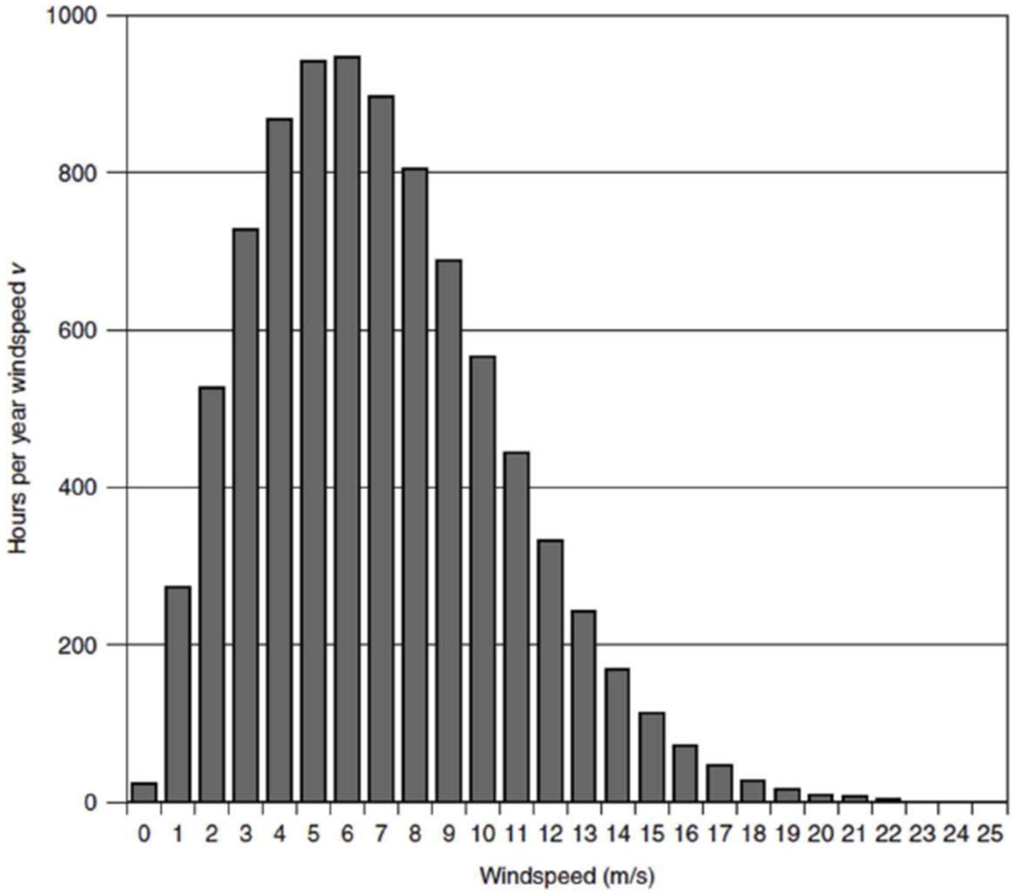
$$(v^3)_{\text{avg}} = \frac{\sum_i [{v_i}^3 \cdot (\text{hours @ } v_i)]}{\sum \text{hours}} = \sum_i [{v_i}^3 \cdot (\text{fraction of hours @ } v_i)]$$

$$(v^3)_{\text{avg}} = \sum_i [{v_i}^3 \cdot \text{probability}(v = v_i)]$$

Example

Using the data given below, find the average windspeed and the average power in the wind (W/m^2). Assume air density = 1.225 kg/m^3.

v (m/s)	Hrs/yr
0	24
1	276
2	527
3	729
4	869
5	941
6	946
7	896
8	805
9	690
10	565
11	444
12	335
13	243
14	170
15	114
16	74
17	46
18	28
19	16
20	9
21	5
22	3
23	1
24	1
25	0
Total hrs	8,760



Solution

The average windspeed is

$$v_{\text{avg}} = \sum_i [v_i \cdot (\text{Fraction of hours @ } v_i)] = 7.0 \text{ m/s}$$

The average value of v^3 is

$$(v^3)_{\text{avg}} = \sum_i [v_i{}^3 \cdot (\text{Fraction of hours @ } v_i)] = 653.24$$

The average power in the wind is

$$P_{\text{avg}} = \tfrac{1}{2}\rho(v^3)_{\text{avg}} = 0.5 \times 1.225 \times 653.24 = 400 \text{ W/m}^2$$

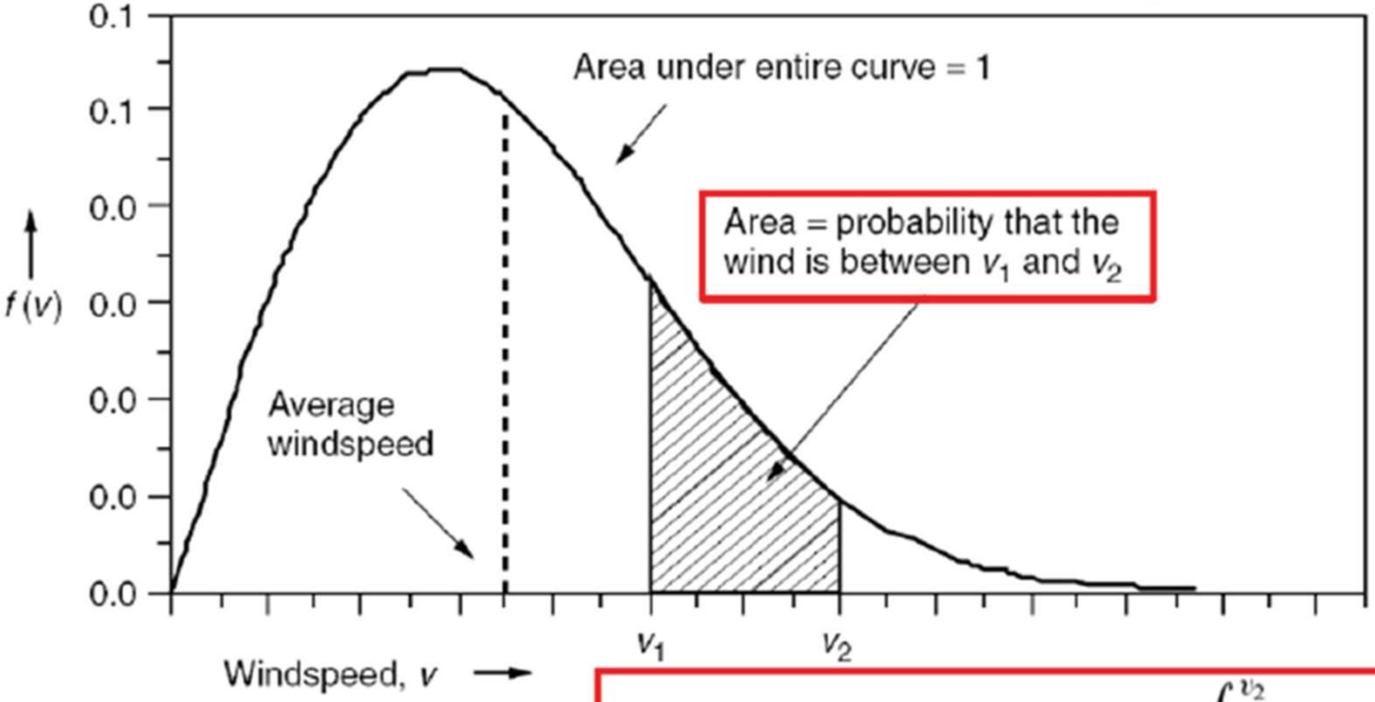
If we had miscalculated average power in the wind using the 7 m/s *average* windspeed, we would have found:

$$\boxed{\begin{aligned} P_{\text{average}}(\text{WRONG}) &= \tfrac{1}{2}\rho(v_{\text{avg}})^3 \\ &= 0.5 \times 1.225 \times 7.0^3 = 210 \text{ W/m}^2 \end{aligned}}$$

Wind Speed Distribution Function

Instead of discrete histogram, the information can be better presented using a continuous function, called a probability distribution function (PDF)

Normalized from total hours in one year (8760 hours).



$$\text{hours/yr } (v_1 \le v \le v_2) = 8760 \int_{v_1}^{v_2} f(v)\, dv$$

Weibull Distribution

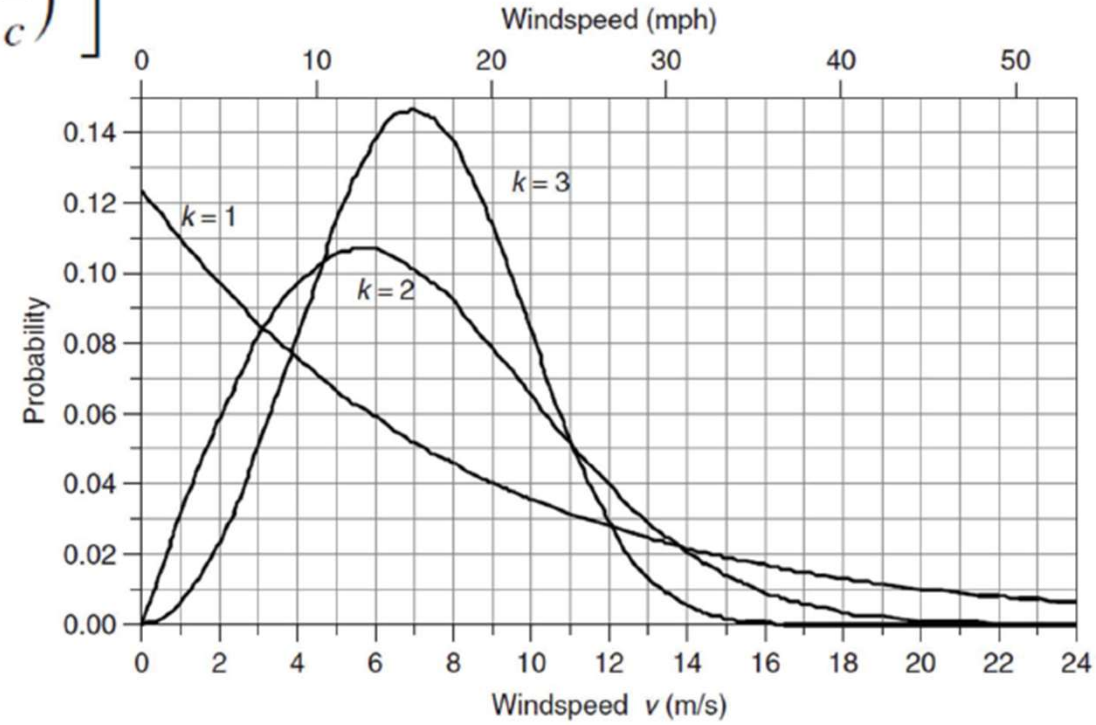
A very general expression that is often used as the starting point for characterizing the statistics of windspeeds is called the *Weibull probability density function*:

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^{k}\right]$$

k = shape parameter
c = scale parameter

k=1, too much of low speed
k=3, Too symmetric.
k=2, Just about right!

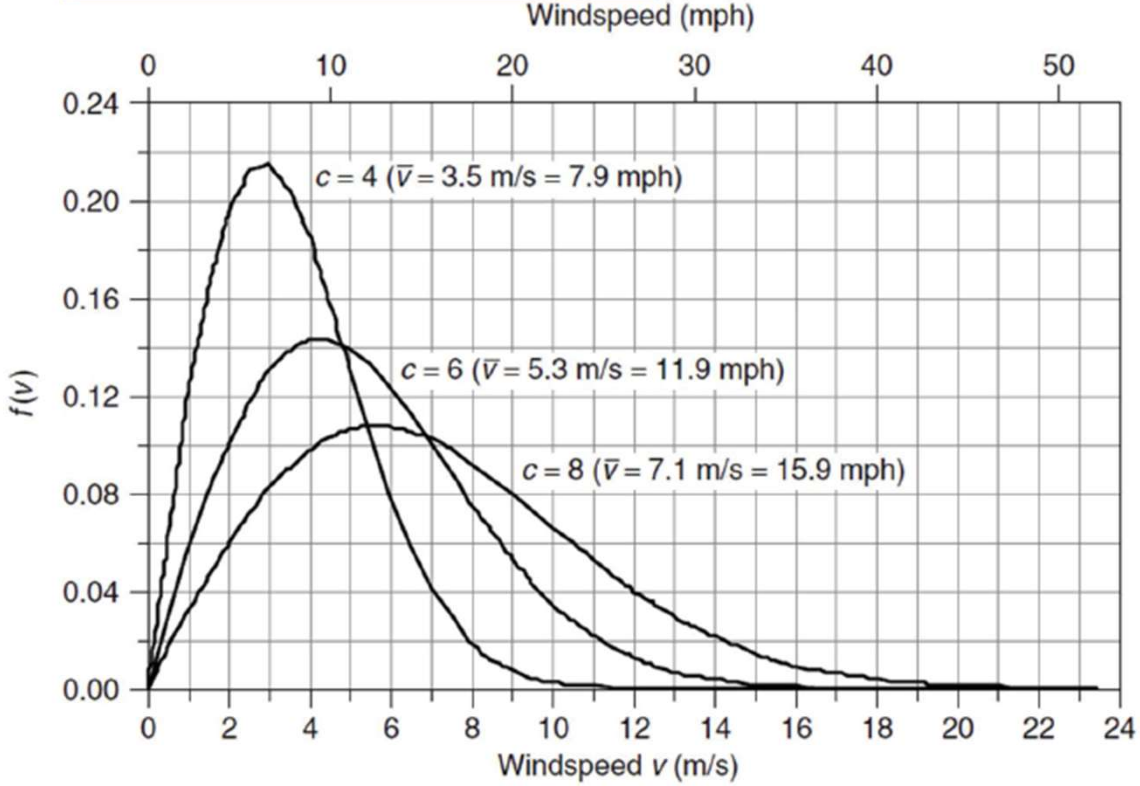
A particular form of the Weibull distribution is referred to as the Rayleigh distribution and occurs when k=2.



Rayleigh Distribution

$$f(v) = \frac{2v}{c^2} \exp\left[-\left(\frac{v}{c}\right)^2\right]$$

k = shape parameter = 2
c = scale parameter



The average wind speed is higher as the scale parameter increases.

Would there be any relationship between average wind speed and scale parameter, c?

Rayleigh PDF: Average Wind Speed

From $$f(v) = \frac{2v}{c^2} \exp\left[-\left(\frac{v}{c}\right)^2\right]$$

$$\bar{v} = \int_0^\infty v \cdot f(v)\ dv = \int_0^\infty \frac{2v^2}{c^2} \exp\left[-\left(\frac{v}{c}\right)^2\right]$$

Substituting in equation for standard integrals

$$= \frac{\sqrt{\pi}}{2} c \cong 0.886c$$

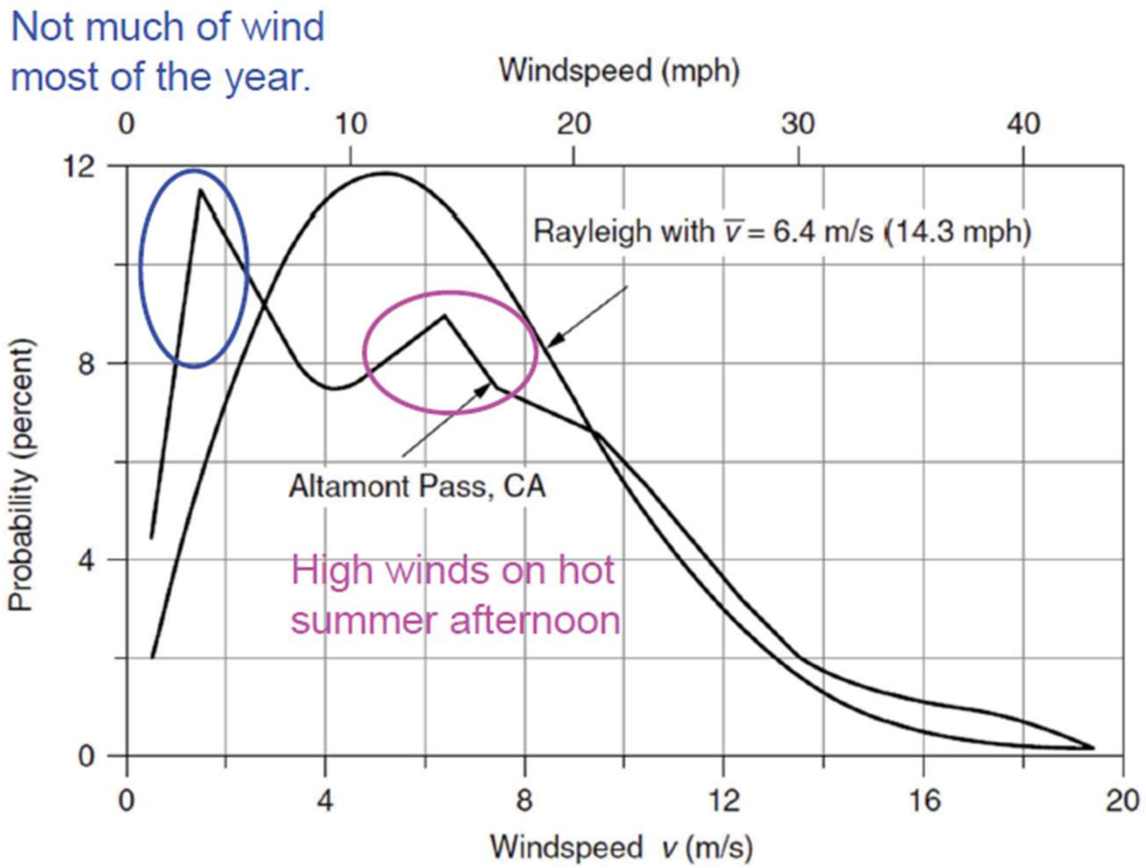
Or, we can write: $$c = \frac{2}{\sqrt{\pi}} \bar{v} \cong 1.128\ \bar{v}$$

The Rayleigh probability density function can be written as follows in terms of average wind speed.

$$\boxed{f(v) = \frac{\pi v}{2\bar{v}^2} \exp\left[-\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2\right]}$$

The starting point for wind prospecting is to gather enough site data to be able to estimate average windspeed.

Wind Speed PDF at Altamont Pass, CA



It is important to gather real wind data rather than relying on Rayleigh distribution

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Rayleigh PDF: Average Power

Coupling average windspeed with the assumption that the wind speed distribution follows Rayleigh statistics enables us to find the average power in the wind.

$$P_{avg} = \frac{1}{2}\rho A (v^3)_{avg}$$

$$(v^3)_{avg} = \int_0^\infty v^3 \cdot f(v)dv$$

$$= \int_0^\infty v^3 \cdot \frac{2v}{c^2} \exp\left[-\left(\frac{v}{c}\right)^2\right] dv \quad = \frac{3}{4} c^3 \sqrt{\pi}$$

Substitute $c = \frac{2}{\sqrt{\pi}}\bar{v}$ ➡ $(v^3)_{avg} = \frac{3}{4}\sqrt{\pi}\left(\frac{2\bar{v}}{\sqrt{\pi}}\right)^3$

$$= \frac{6}{\pi}\bar{v}^3 = 1.91\ \bar{v}^3$$

$$\boxed{\bar{P} = \frac{6}{\pi} \cdot \frac{1}{2}\rho A \bar{v}^3}$$

This means that the average power in the wind can be found in terms of average wind speed. The average power is equal to **1.91 times** the average power found at the average wind speed.

Example

Estimate the average power in the wind at a height of 50 m when the wind speed at 10 m averages 6 m/s.

Assume Rayleigh statistics;

Standard friction coefficient $\alpha = \frac{1}{7}$;

Standard air density $\rho = 1.225 \frac{kg}{m^3}$.

Solution

We first adjust the winds at 10 m to those expected at 50 m

$$\overline{v}_{50} = \overline{v}_{10}\left(\frac{H_{50}}{H_{10}}\right)^{\alpha} = 6 \cdot \left(\frac{50}{10}\right)^{1/7} = 7.55 \text{ m/s}$$

the average wind power density would be

$$\begin{aligned}\overline{P}_{50} &= \frac{6}{\pi} \cdot \frac{1}{2}\rho\overline{v}^3 \\ &= \frac{6}{\pi} \cdot \frac{1}{2} \cdot 1.225 \cdot (7.55)^3 = 504 \text{ W/m}^2\end{aligned}$$

Wind Energy Potential

Wind Energy Potential

It is important to translate wind speed to power density for the following purposes.

- Energy planning
- Optimal land use
- Avoid potentially damaging the environment
- Proximity to transmission capability
- Economic viability

Requires wind speed and elevation assumptions.

Standard Wind Power Classification

Wind Power Class	Avg Windspeed at 10 m (m/s)	Avg Windspeed at 10 m (mph)	Wind Power Density at 10 m (W/m^2)	Wind Power Density at 50 m (W/m^2)
1	0–4.4	0–9.8	0–100	0–200
2	4.4–5.1	9.8–11.4	100–150	200–300
3	5.1–5.6	11.4–12.5	150–200	300–400
4	5.6–6.0	12.5–13.4	200–250	400–500
5	6.0–6.4	13.4–14.3	250–300	500–600
6	6.4–7.0	14.3–15.7	300–400	600–800
7	7.0–9.5	15.7–21.5	400–1000	800–2000

[a] Assumptions include Rayleigh statistics, ground friction coefficient $\alpha = 1/7$, sea-level 0°C air density 1.225 kg/m^3, 10-m anemometer height, 50-m hub height.

Measure wind speed at 10-m height above the ground.

Estimate wind speed and power density at 50 m.

Overall Efficiency

Average power in the wind (W/m²) → Power extracted from the wind (W/m²), depending turbine blade (rotor) efficiency → Electrical power output (W/m²), depending on the gearbox and generator efficiency

$$P_{wind} = \frac{1}{2}\rho A\omega^3 \qquad P_{blade} = \frac{1}{2}\rho A\omega^3 \cdot C_p$$

(Assume $C_p = 0.45$)

(Assume gearbox and generator efficiency has combined efficiency of 2/3)

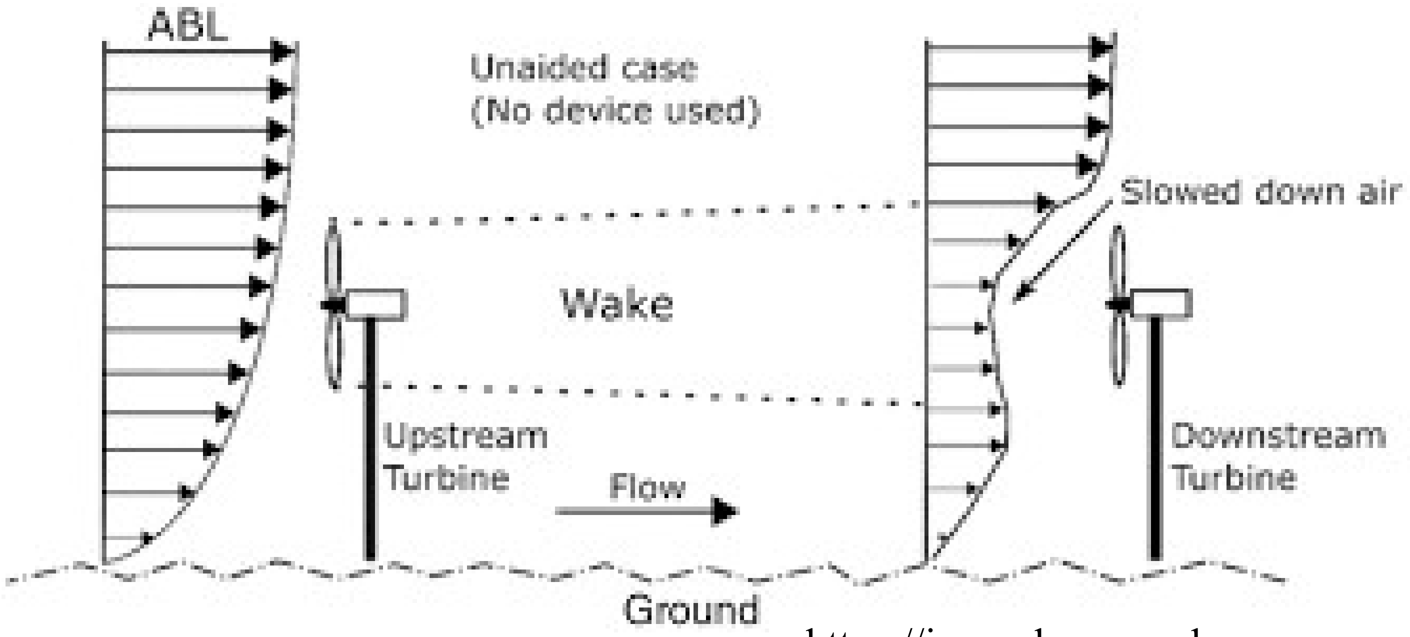
$$\text{Overall efficiency} = \frac{P_{electrical}}{P_{wind}} = 0.45 \times \frac{2}{3} = 0.3 = 30\%$$

Energy Estimate of Wind Farms

- Large number of wind turbines.
- Clustered together at the same windy site.
- Centralized operation and maintenance, reducing cost.

- Spacing between wind turbines needs to be properly designed.
  - To optimally capture wind energy.
  - To reduce ‘wake effect’.

Wake effect



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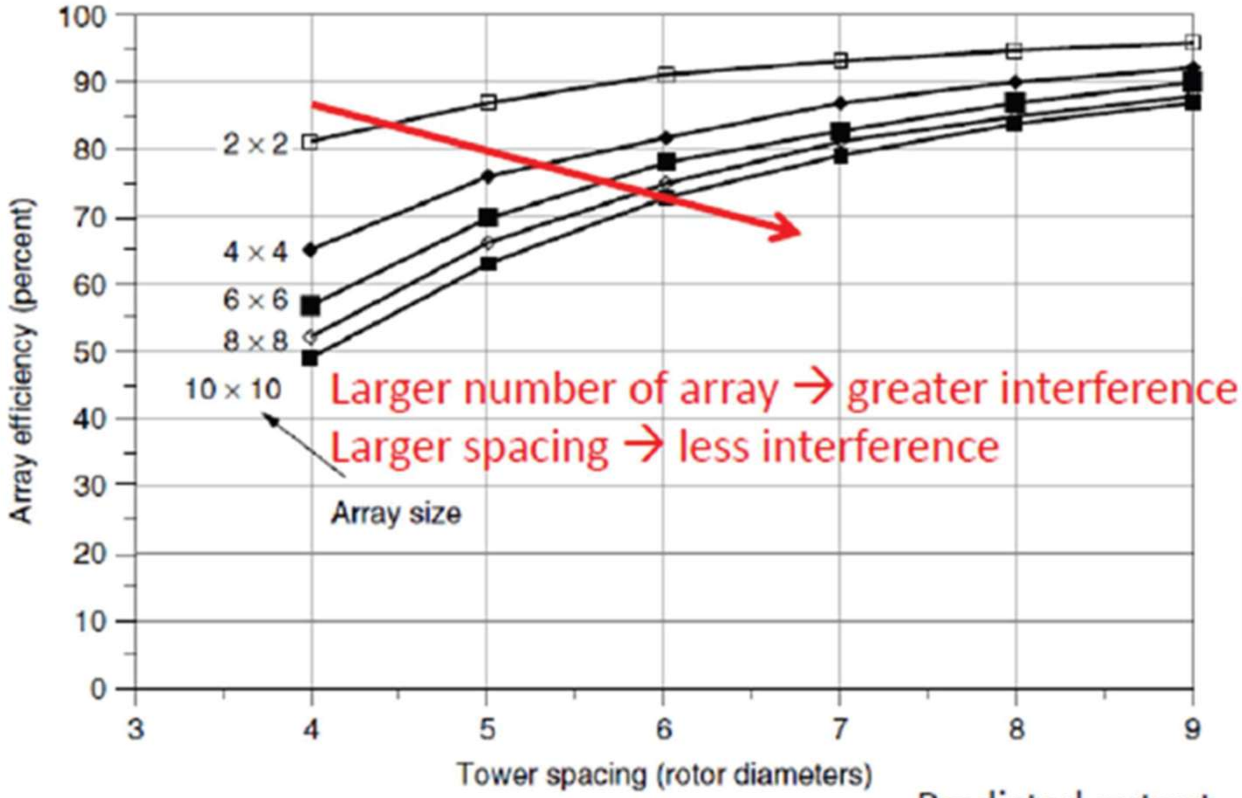
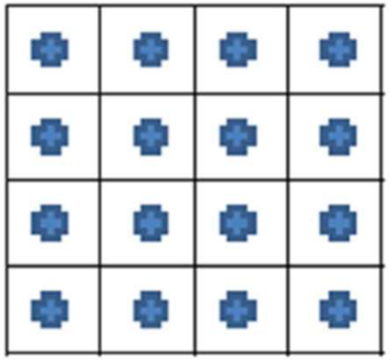
- The 'wake effect' is the trail left by each turbine where wind speeds are reduced.
- The wind regime generates additional turbulence to that already produced by the terrain, affecting nearby wind turbines and even neighboring wind farms.

Tower Spacing Study



2 x 2 square array



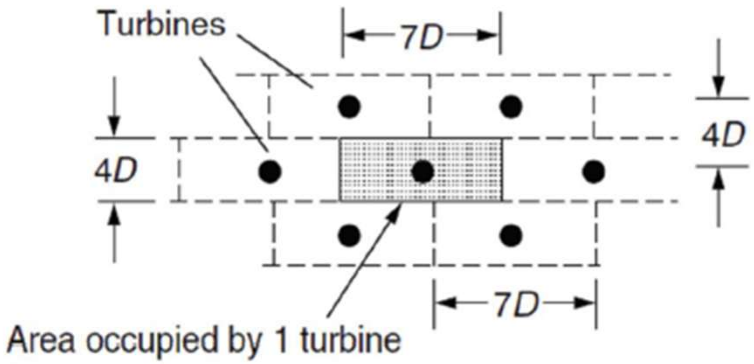
4 x 4 square array

$$\text{Array efficiency} = \frac{\text{Predicted output}}{\text{Output with no interference}}$$

Example

Suppose that a wind farm has 4-rotor-diameter tower spacing along its rows, with 7-diameter spacing between rows (4D × 7D).

Assume 30% wind turbine efficiency and an array efficiency of 80%.



Find the annual energy production per unit of land area in an area with 400-W/m2 winds at hub height (the edge of 50 m, Class 4 winds).

$$\frac{Energy}{Land\ area} = \frac{Power\ density\ of\ wind\ turbine \times swept\ area\ of\ rotor \times turbine\ efficiency \times array\ efficiency}{land\ area\ for\ one\ turbine} \times hours$$

Solution

The land area occupied by one turbine is $4D \times 7D = 28D^2$.

The rotor area $\frac{\pi}{4}D^2$.

The yearly energy produced by one turbine =

$$= Wind\ power\ density \times rotor\ area \times turbine\ efficiency \times array\ efficiency \times hours\ per\ year$$

$$= 400 \times \frac{\pi}{4}D^2 \times\ 0.3 \times 0.8 \times 8760 = 660153.6D^2$$

$$\frac{Energy}{Land\ Area} = \frac{660153.6D^2}{28D^2} = 23.58\ kWh/m^2$$

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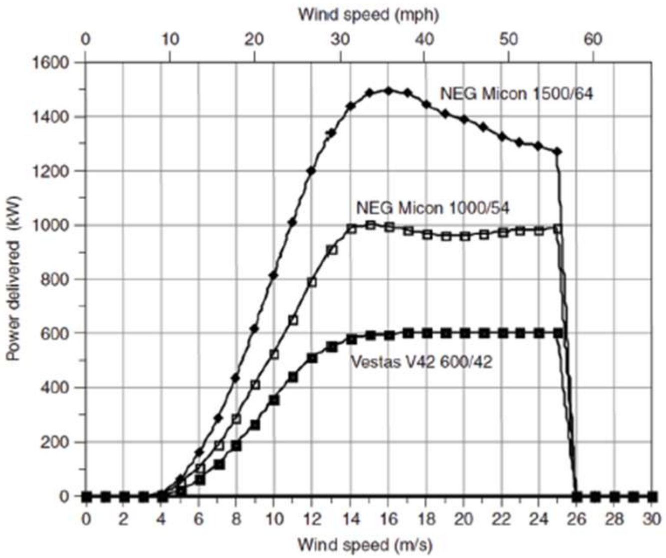
From Power to Energy

- **Energy = power x time.**
- Note that in the previous example, it is assumed that overall wind turbine efficiency is 30%.
  - **This assumption includes the efficiency of all mechanical and electrical parts.**
  - **This efficiency is the average value over all wind speeds.**
- A more accurate way to compute wind **energy** estimate is to use ‘power curve’
  - Recall that power curve gives a relationship between output electrical power as a function of wind speed.
- Wind speed is also a function of time, represented by Weibull or Rayleigh statistics.

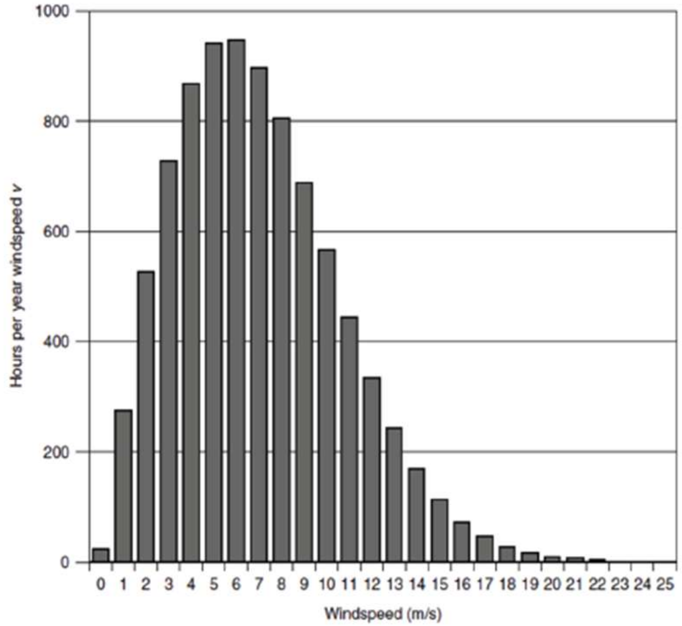
Annual Energy Production: Spreadsheet Method



Electrical power output at any given speed

v (m/s)	Hrs/yr
0	24
1	276
2	527
3	729
4	869
5	941
6	946
7	896
8	805
9	690
10	565
11	444
12	335
13	243
14	170
15	114
16	74
17	46
18	28
19	16
20	9
21	5
22	3
23	1
24	1
25	0
Total hrs	8,760

Wind speed varies according to Weibull and Rayleigh statistics



We need to combine **power produced at any wind speed** with the **hours of that wind speed** to find total energy produced in a year

If only the average wind speed is known, then Rayleigh PDF is used.

Example

Suppose that a NEG Micon 60-m diameter wind turbine having a rated power of 1000 kW is installed at a site having Rayleigh wind statistics with an average wind speed of 7m/s at the hub height. The generator output for various wind speeds are given on the right.

a) Find the annual energy generated.
b) From the result, find the overall average efficiency of this turbine in these winds.
c) Find the productivity in terms of kWh/yr delivered per m² of swept area.

	Manufacturer:	NEG Micon
	Rated Power (kW):	1000
	Diameter (m):	60
Avg. Windspeed		
v (m/s)	v(mph)	kW
0	0	0
1	2.2	0
2	4.5	0
3	6.7	0
4	8.9	33
5	11.2	86
6	13.4	150
7	15.7	248
8	17.9	385
9	20.1	535
10	22.4	670
11	24.6	780
12	26.8	864
13	29.1	924
14	31.3	964
15	33.6	989
16	35.8	1000
17	38.0	998
18	40.3	987
19	42.5	968
20	44.7	944
21	47.0	917
22	49.2	889
23	51.5	863
24	53.7	840
25	55.9	822
26	58.2	0

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Solution

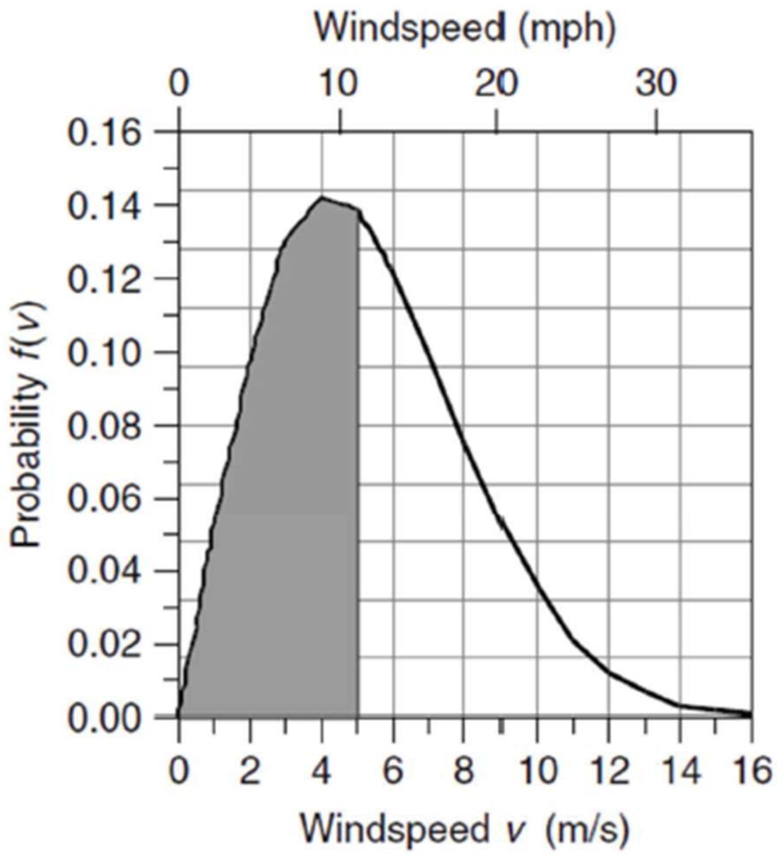
Assumption: Rayleigh wind statistics with average wind speed of 7 m/s at hub height.

Step 1: Find the probability of each wind speed. How???

Step 2: Find the energy produced at each wind speed.

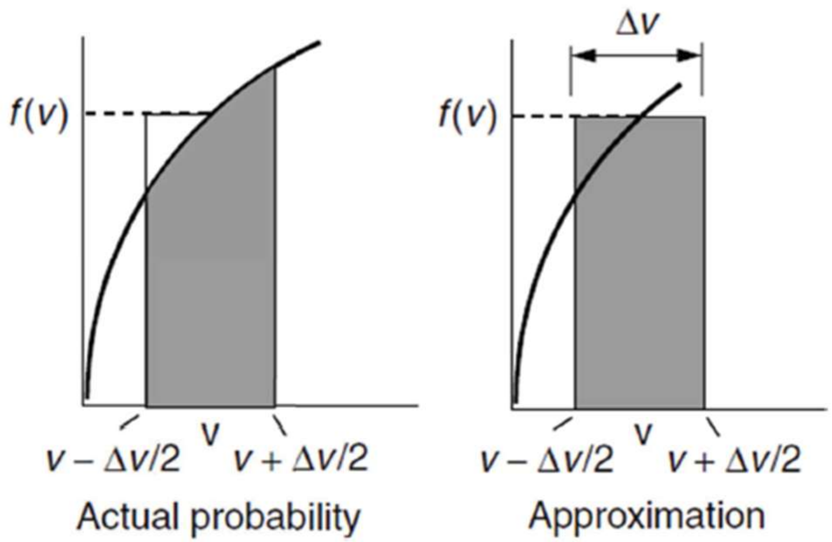
Step 3: Annual energy generated = summation of energy produced at each wind speed

Probability Approximation



$$f(v) = \frac{\pi v}{2\bar{v}^2} \exp\left[-\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2\right]$$

We can discretize a continuous PDF and say that the probability that the wind blows at v is just f(v).



$$\text{Area} = \int_{v-\Delta v/2}^{v+\Delta v/2} f(v)\, dv \approx f(v)\Delta v$$

The energy produced at any wind speed e.g. 6 m/s.

		NEG
	Manufacturer:	Micon
	Rated Power (kW):	1000
	Diameter (m):	60
Avg. Windspeed		
v (m/s)	v(mph)	kW
0	0	0
1	2.2	0
2	4.5	0
3	6.7	0
4	8.9	33
5	11.2	86
6	13.4	150
7	15.7	248
8	17.9	385
9	20.1	535
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20	44.7	944
21	47.0	917
22	49.2	889
23	51.5	863
24	53.7	840
25	55.9	822
26	58.2	0

at 6 m/s the NEG Micon 1000/60 generates 150 kW

the Rayleigh p.d.f. at 6 m/s in a regime with 7-m/s average windspeed is

$$f(v) = \frac{\pi v}{2\bar{v}^2} \exp\left[-\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2\right]$$

$$= \frac{\pi \cdot 6}{2 \cdot 7^2} \exp\left[-\frac{\pi}{4}\left(\frac{6}{7}\right)^2\right] = 0.10801$$

In a year with 8760 h, our estimate of the hours the wind blows at 6 m/s is

$$\text{Hours @6 m/s} = 8760 \text{ h/yr} \times 0.10801 = 946 \text{ h/yr}$$

So the energy delivered by 6-m/s winds is

$$\text{Energy (@6 m/s)} = 150 \text{ kW} \times 946 \text{ h/yr}$$

$$= 141,929 \text{ kWh/yr}$$

Do this for all speeds and add them up!

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The overall average efficiency of this turbine

Windspeed (m/s)	Power (kW)	Probability $f(v)$	Hrs/yr at v	Energy (kWh/yr)
0	0	0.000	0	0
1	0	0.032	276	0
2	0	0.060	527	0
3	0	0.083	729	0
4	33	0.099	869	28,683
5	86	0.107	941	80,885
6	150	0.108	946	141,929
7	248	0.102	896	222,271
8	385	0.092	805	310,076
9	535	0.079	690	369,126
10	670	0.065	565	378,785
11	780	0.051	444	346,435
12	864	0.038	335	289,551
13	924	0.028	243	224,707
14	964	0.019	170	163,779
15	989	0.013	114	113,101
16	1000	0.008	74	74,218
17	998	0.005	46	46,371
18	987	0.003	28	27,709
19	968	0.002	16	15,853
20	944	0.001	9	8,709
21	917	0.001	5	4,604
22	889	0.000	3	2,347
23	863	0.000	1	1,158
24	840	0.000	1	554
25	822	0.000	0	257
26	0	0.000	0	0
			Total:	$\boxed{2,851,109}$

average power in the wind for a 60-m rotor diameter

$$\overline{P} = \frac{6}{\pi} \cdot \frac{1}{2}\rho A\overline{v}^3$$

$$= \frac{6}{\pi} \times 0.5 \times 1.225 \times \frac{\pi}{4}(60)^2 \times (7)^3$$

$$= 1.134 \times 10^6 \text{ W} = 1134 \text{ kW}$$

In a year with 8760 h, the energy in the wind is

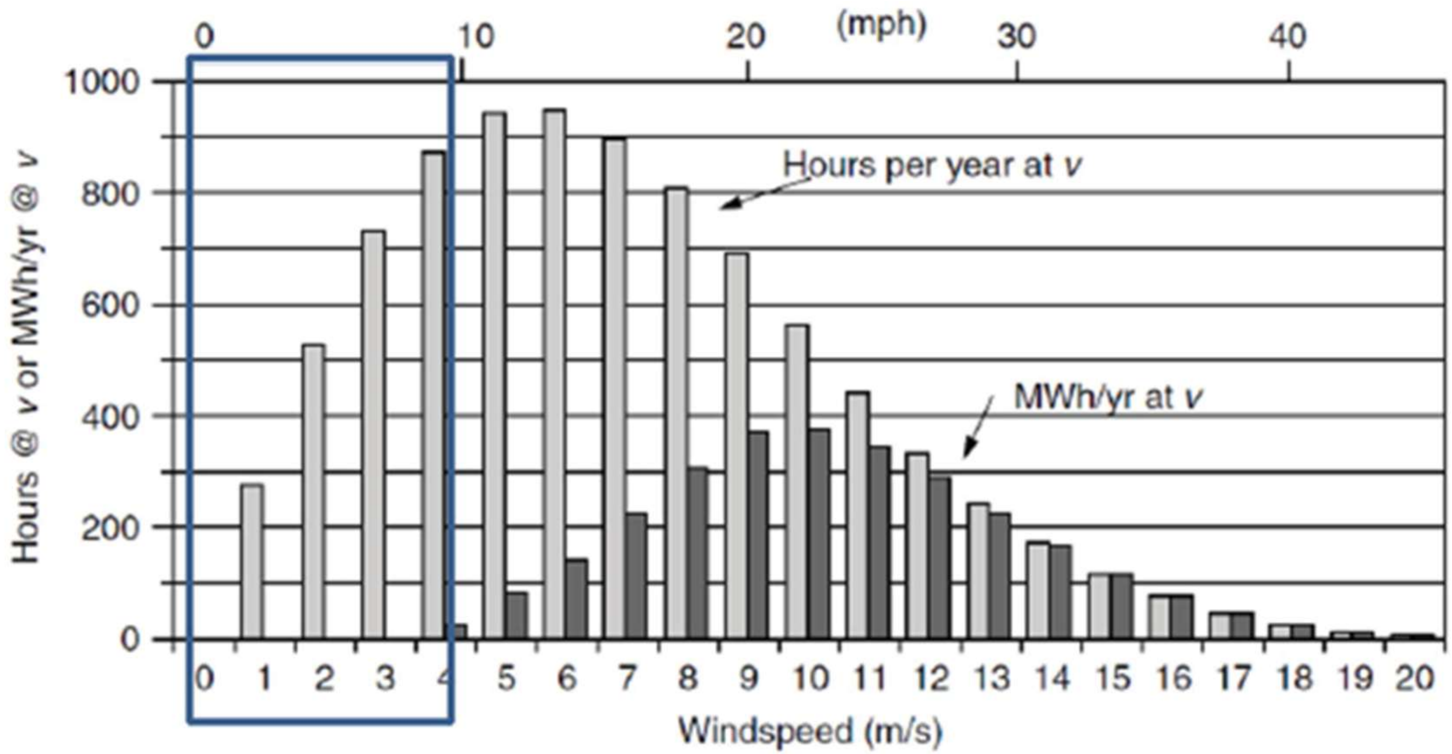
$$\text{Energy in wind} = 8760 \text{ h/yr} \times 1134 \text{ kW}$$

$$= 9.938 \times 10^6 \text{ kWh}$$

$$\text{Average efficiency} = \frac{2.85 \times 10^6 \text{ kWh/yr}}{9.938 \times 10^6 \text{ kWh/yr}}$$

$$= 0.29 = 29\%$$

Hours and MWh per Year over the speed range



Little or no energy produced during low speed wind.

Annual Energy Production: Capacity Factor Method

Capacity factor is a measure of the fraction of actual energy delivered to the rated energy output in one year.

$$\text{CF} = \frac{\text{Actual energy delivered}}{\text{Rated power} \times 8760}$$

$$\text{Annual energy (kWh/yr)} = P_R \text{ (kW)} \times 8760 \text{ (h/yr)} \times \text{CF}$$

where P_R is the rated power (kW) and CF is the capacity factor

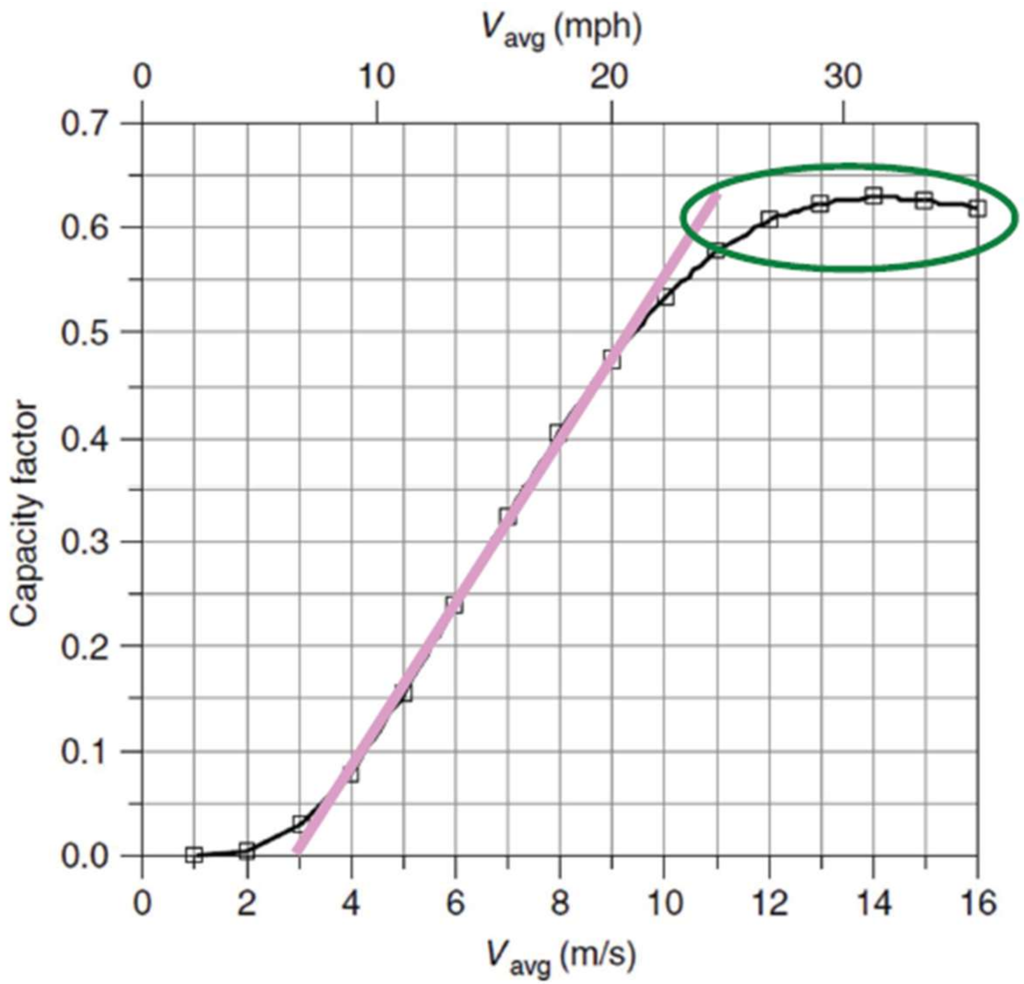
$$\text{CF} = \frac{\text{Actual energy delivered}}{P_R \times 8760}$$

another way to express it is

$$\text{CF} = \frac{\text{Actual energy delivered/8760 h/yr}}{P_R} = \frac{\text{Average power}}{\text{Rated power}}$$

- Dimensionless quantity between 0 and 1.
- CF is meant for calculating 'actual energy delivered' given that we know the rated power.

CF for NEG Micon 1000/60



In this region, more and more winds are above the cut out wind speed and CF level drops.

- Assume Rayleigh wind statistics and vary average wind speed.
- CF varies quite linearly with average wind speed.

Capacity Factor Curve

- Assumption: Wind statistics (Weibull or Rayleigh) with average wind speed of X m/s at hub height.
- Step 1: Find the probability of each wind speed.

This step depends on wind site.

- Step 2: Find the energy produced at each wind speed.

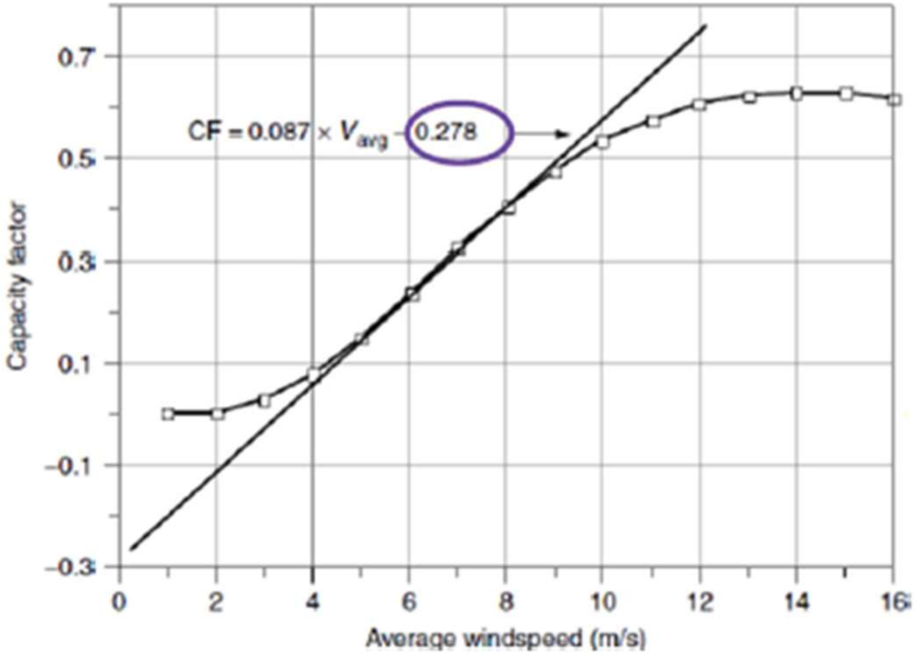
This step depends on machine power curve.

- Step 3: Annual energy generated = summation of energy produced at each wind speed

$$\text{CF} = \frac{\text{Actual energy delivered}}{\text{Rated power} \times 8760}$$

Linear Approximation of CF Curve

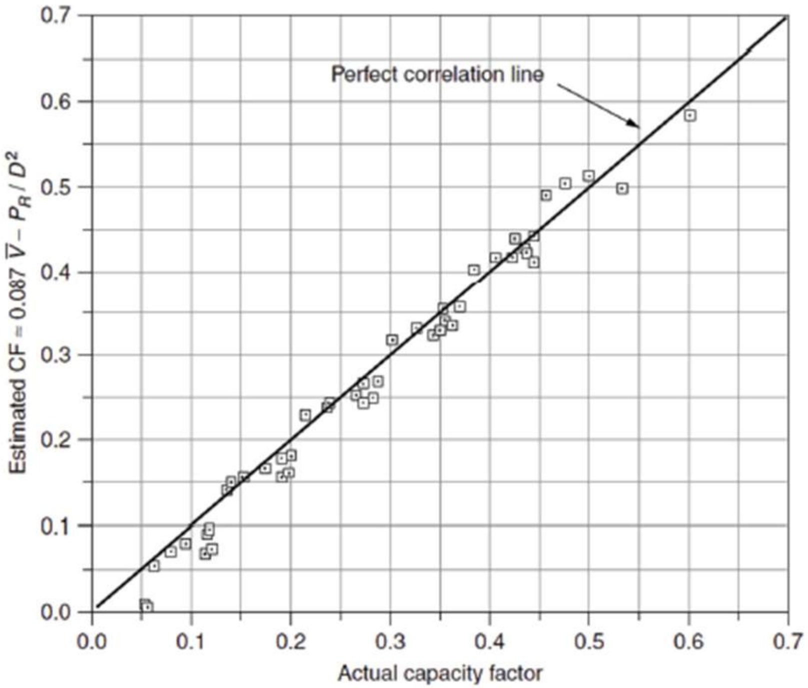
$$\mathrm{CF} = m\overline{V} + b$$



- For NEG Micon 1000/60,
  - Rated power = 1000 kW.
  - Rotor diameter = 60 m.
  - $\dfrac{P_R}{D^2} = \dfrac{1000 \text{ kW}}{(60 \text{ m})^2} = 0.278$
- Nice coincidence!

Estimated Capacity Factor

$$\mathrm{CF} = 0.087\overline{\mathrm{V}} - \frac{P_R}{D^2}$$



- Only the following information is needed.
  - Average wind speed
  - Rated power
  - Rotor diameter
- Quite useful relationship when little data of wind speed and turbine are known.
- If the data is available, this is NOT a suitable method to replace the spreadsheet method!!

Estimate of energy delivered from a turbine of diameter D:

$$\text{Annual energy (kWh/yr)} = 8760 \cdot P_R(\text{kW}) \left\{ 0.087\overline{V}(\text{m/s}) - \frac{P_R(\text{kW})}{[D(\text{m})]^2} \right\}$$

Example

The Whisper H900 wind turbine has a 900-W generator with 2.13-m blades.

In an area with 6-m/s average wind speed, estimate the approximated energy delivered.

Solution

$$\mathrm{CF} = 0.087\overline{V} - \frac{P_R}{D^2}$$

$$= 0.087 \times 6 - \frac{0.90}{2.13^2} = 0.324$$

The energy delivered in a year's time

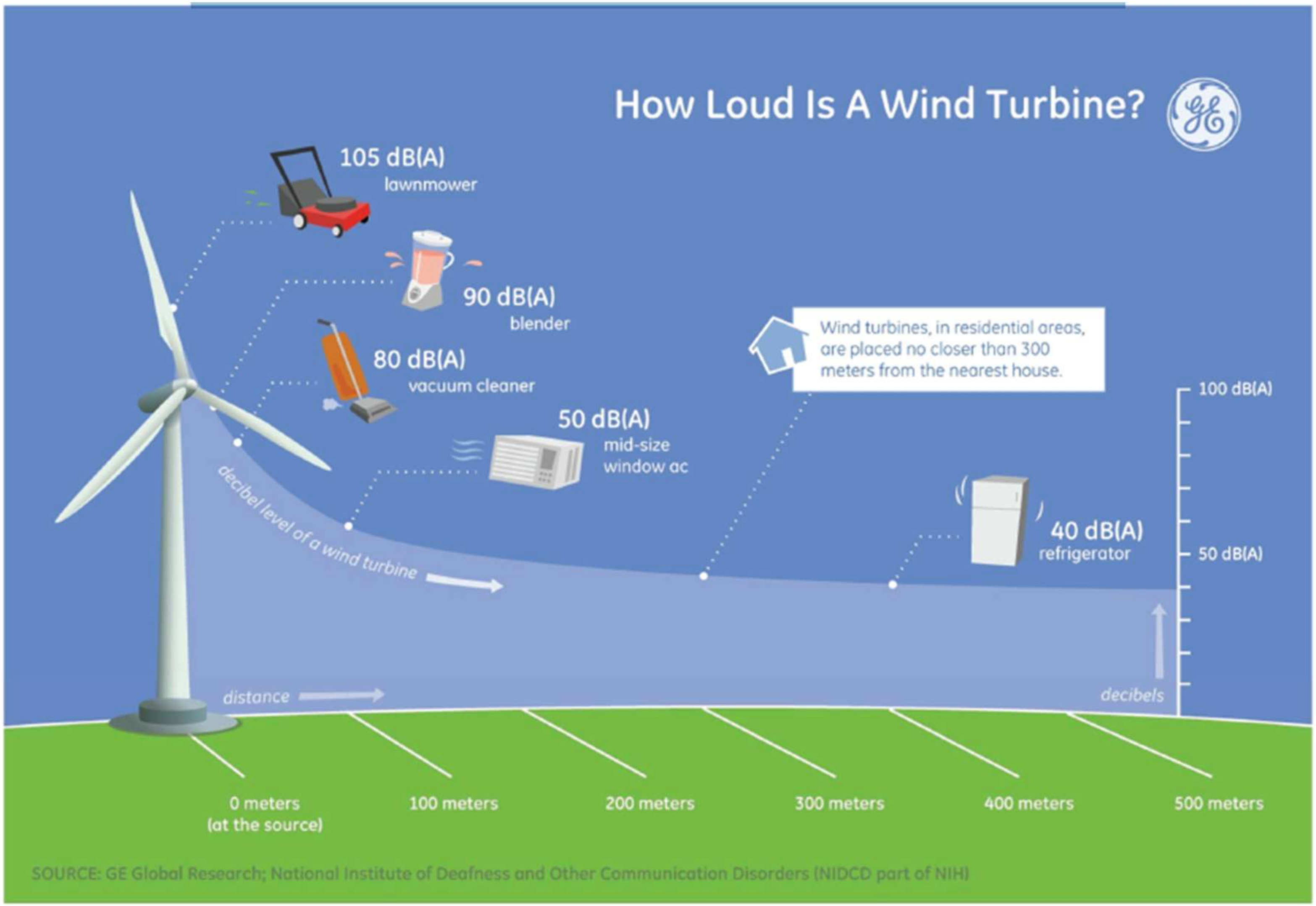
$$\text{Energy} = 8760\ \text{h/yr} \times 0.90\ \text{kW} \times 0.324$$

$$= 2551\ \text{kWh/yr}$$

Environmental Impact Of Wind Energy

- Noise
- Impact on wildlife such as birds and bats in case of on-shore wind turbine.
- Impact on marine life in case of offshore wind turbines.
- Visual impact

Noise Generated by Wind Turbine



Summary

- **Power in the wind**
  - As a function of swept area, air density, and cube of wind speed.
  - Air density is a function of temperature and altitude.
  - Impact of tower height and ground surface on wind speed and power.

- **Power *extracted* from the wind**
  - Rotor efficiency
  - Tip-speed ratio

- **Wind Energy Conversion Systems (WECS)**
  - Different types of wind turbine generators

- **Speed control for maximum power output and to protect wind turbine during turbulent weather.**

Summary

- Average wind speed and power in the wind with Rayleigh statistics
- Overall wind energy efficiency
- Wind farm: spacing of wind turbines
- Annual electrical energy output estimate
  - Exact method using spreadsheet
  - Approximate method using capacity factor
- Environmental impacts of wind energy systems