Adversarial Search

Chapter 5

Game Playing

Why do AI researchers study game playing?

1. It’s a good reasoning problem, formal and nontrivial.

2. Direct comparison with humans and other computer programs is easy.

What Kinds of Games?

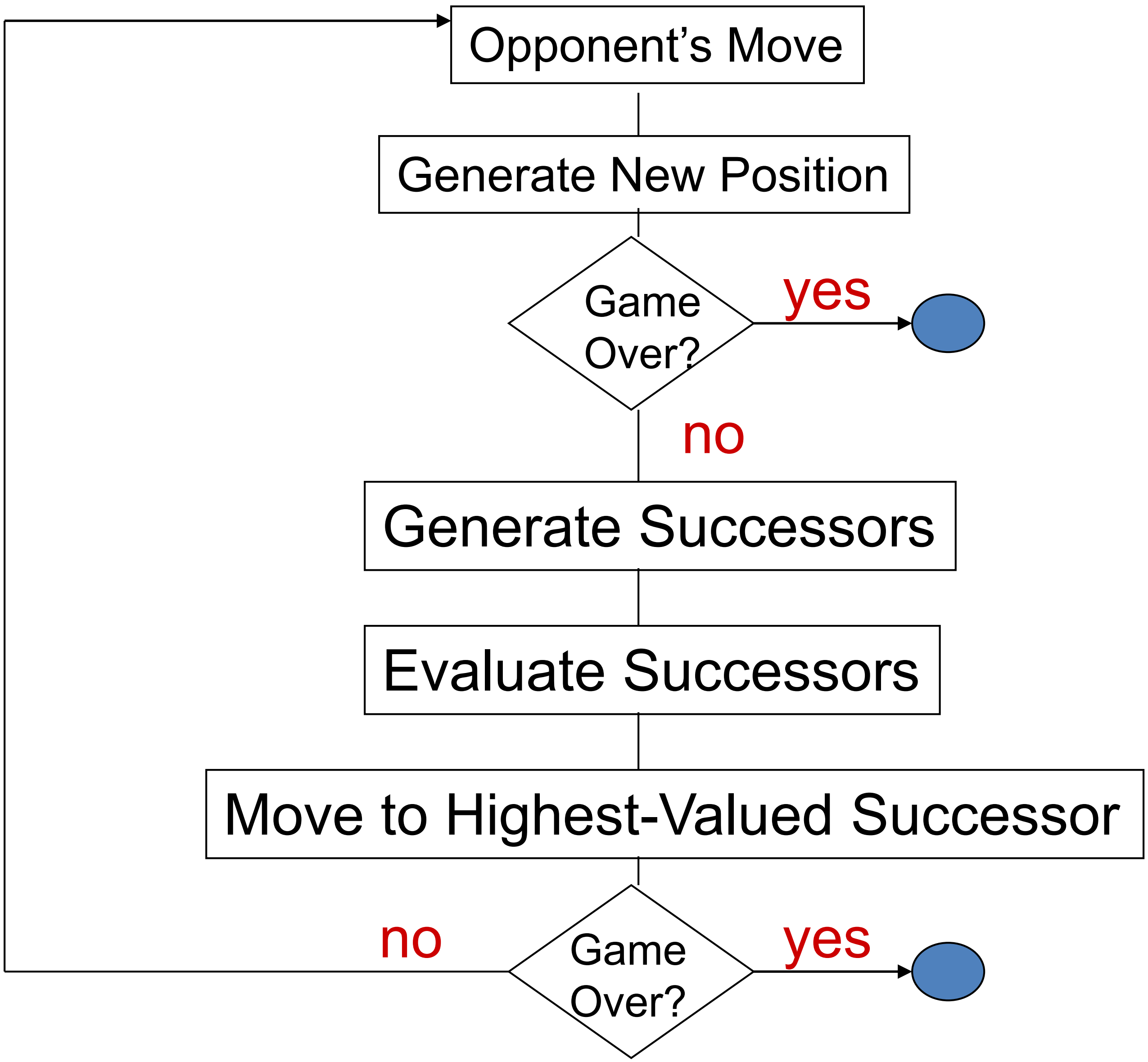
Mainly games of strategy with the following characteristics:

1. Sequence of moves to play
2. Rules that specify possible moves
3. Rules that specify a payment for each move
4. Objective is to maximize your payment

Games vs. Search Problems

- Unpredictable opponent → specifying a move for every possible opponent reply
- Time limits → unlikely to find goal, must approximate

Two-Player Game



Games as Adversarial Search

- States:
  - board configurations
- Initial state:
  - the board position and which player will move
- Successor function:
  - returns list of (move, state) pairs, each indicating a legal move and the resulting state
- Terminal test:
  - determines when the game is over
- Utility function:
  - gives a numeric value in terminal states

  (e.g., -1, 0, +1 for loss, tie, win)

Game Tree (2-player, Deterministic, Turns)

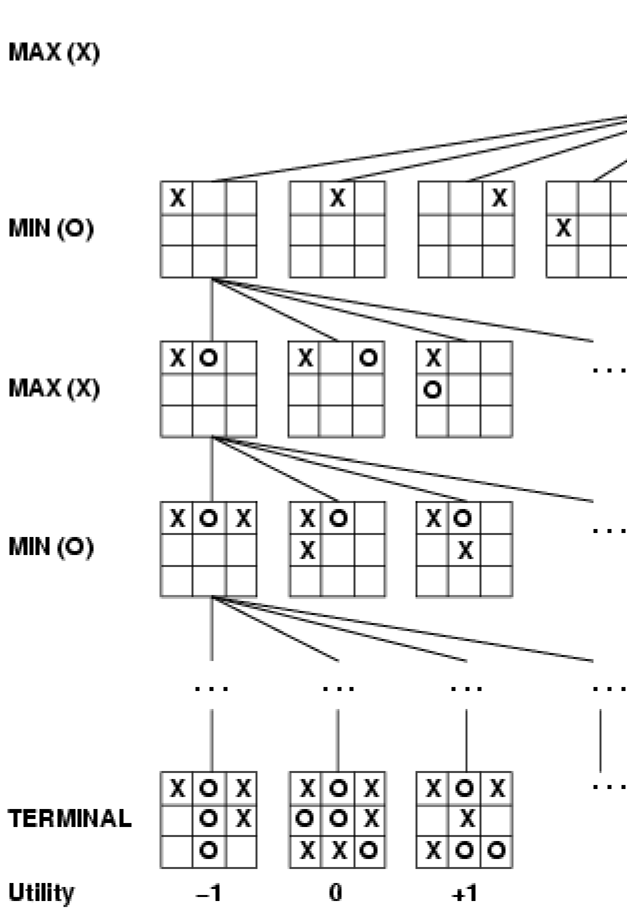
computer's turn

opponent's turn

computer's turn

opponent's turn

leaf nodes are evaluated



The computer is **Max**.
The opponent is **Min**.

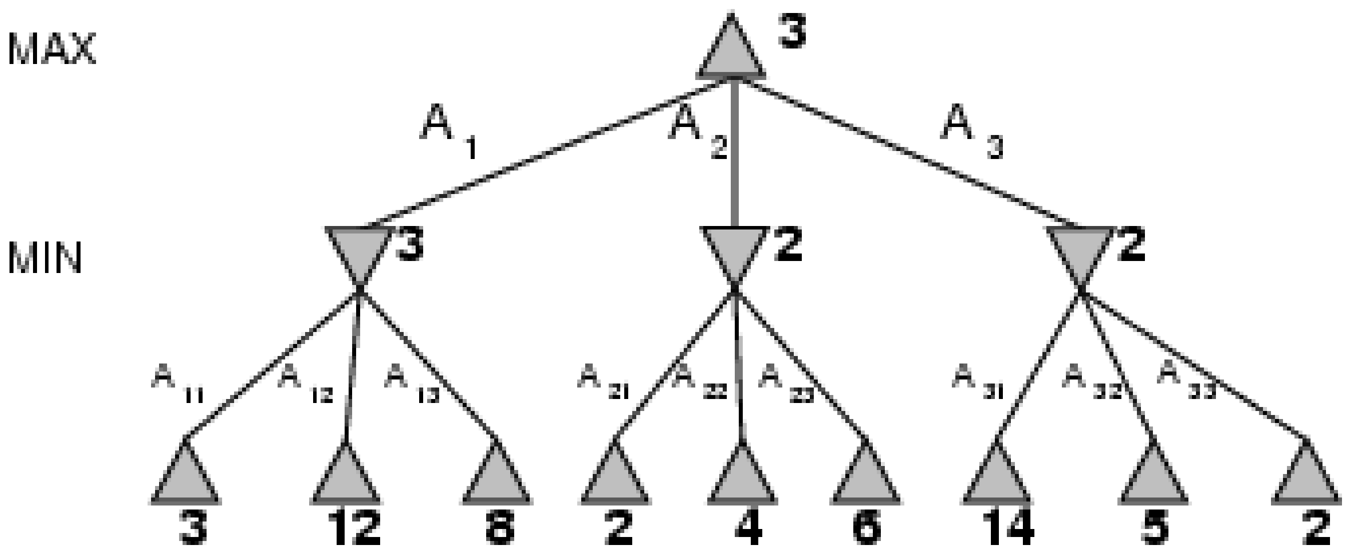
At the leaf nodes, the **utility function** is employed. Big value means good, small is bad.

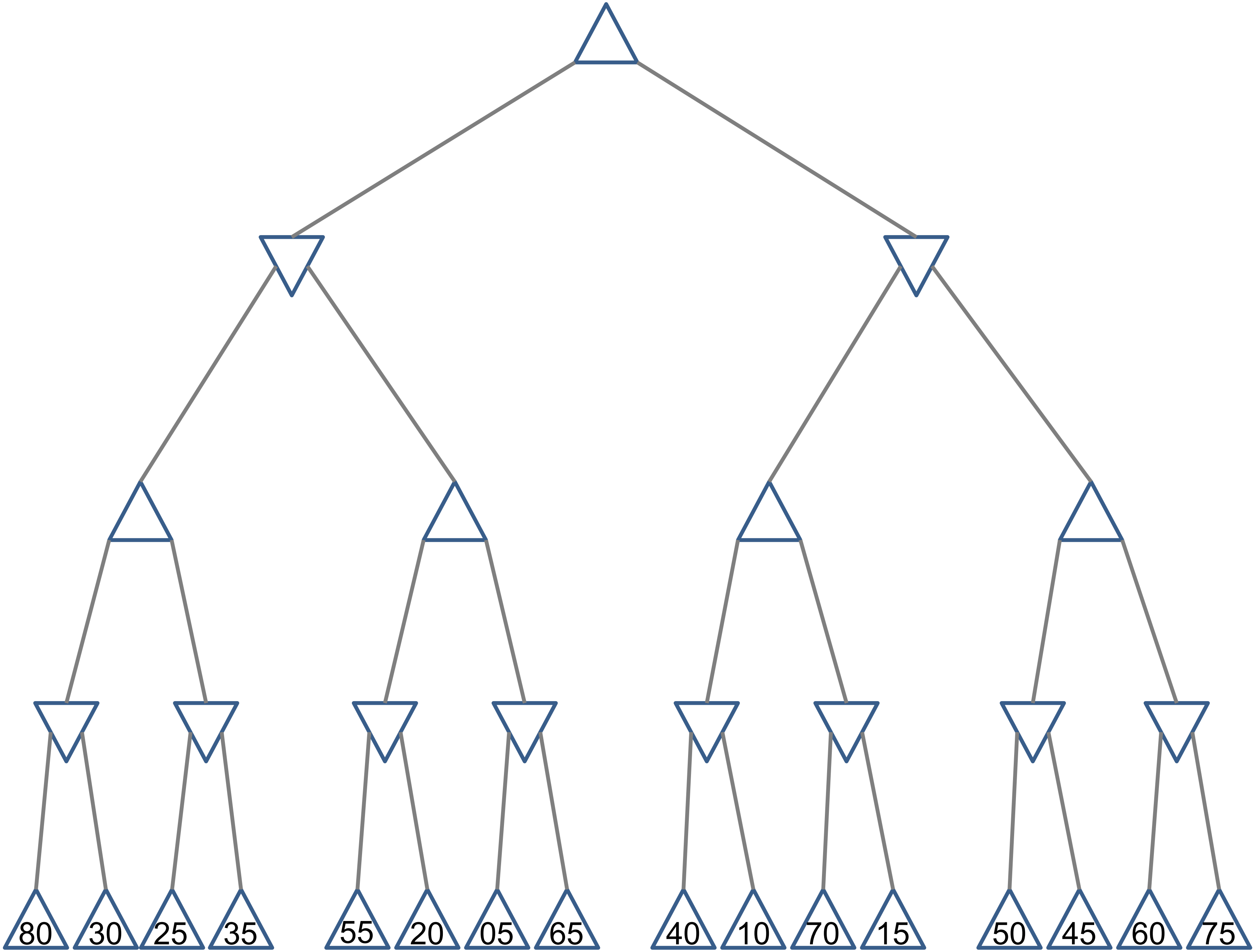
Mini-Max Terminology

- move: a move by both players
- ply: a half-move
- utility function: the function applied to leaf nodes
- backed-up value
  - of a max-position: the value of its largest successor
  - of a min-position: the value of its smallest successor
- minimax procedure: search down several levels; at the bottom level apply the utility function, back-up values all the way up to the root node, and that node selects the move.

Minimax

- Perfect play for deterministic games
- Idea: choose move to position with highest minimax value
  = best achievable payoff against best play
- E.g., 2-ply game:

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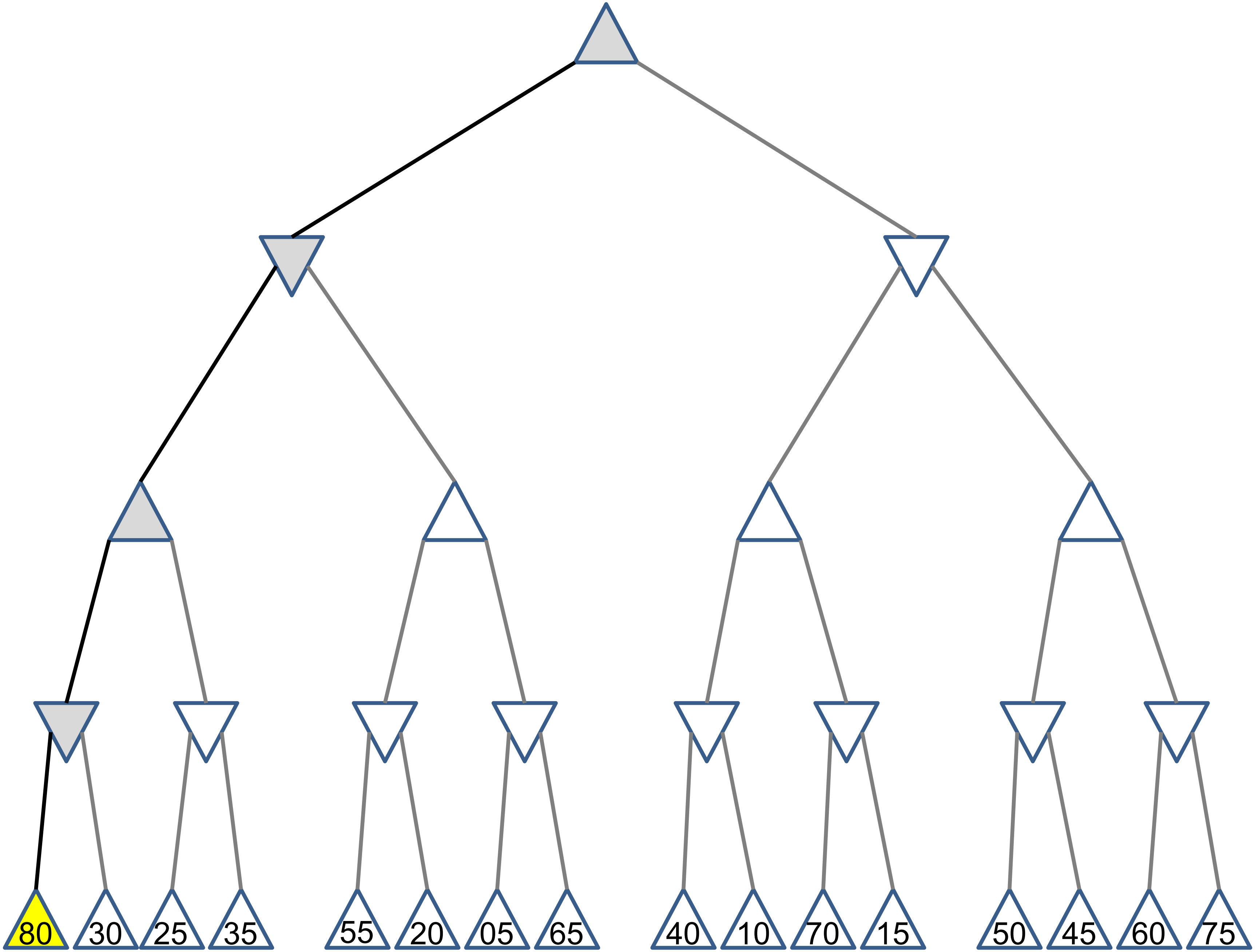
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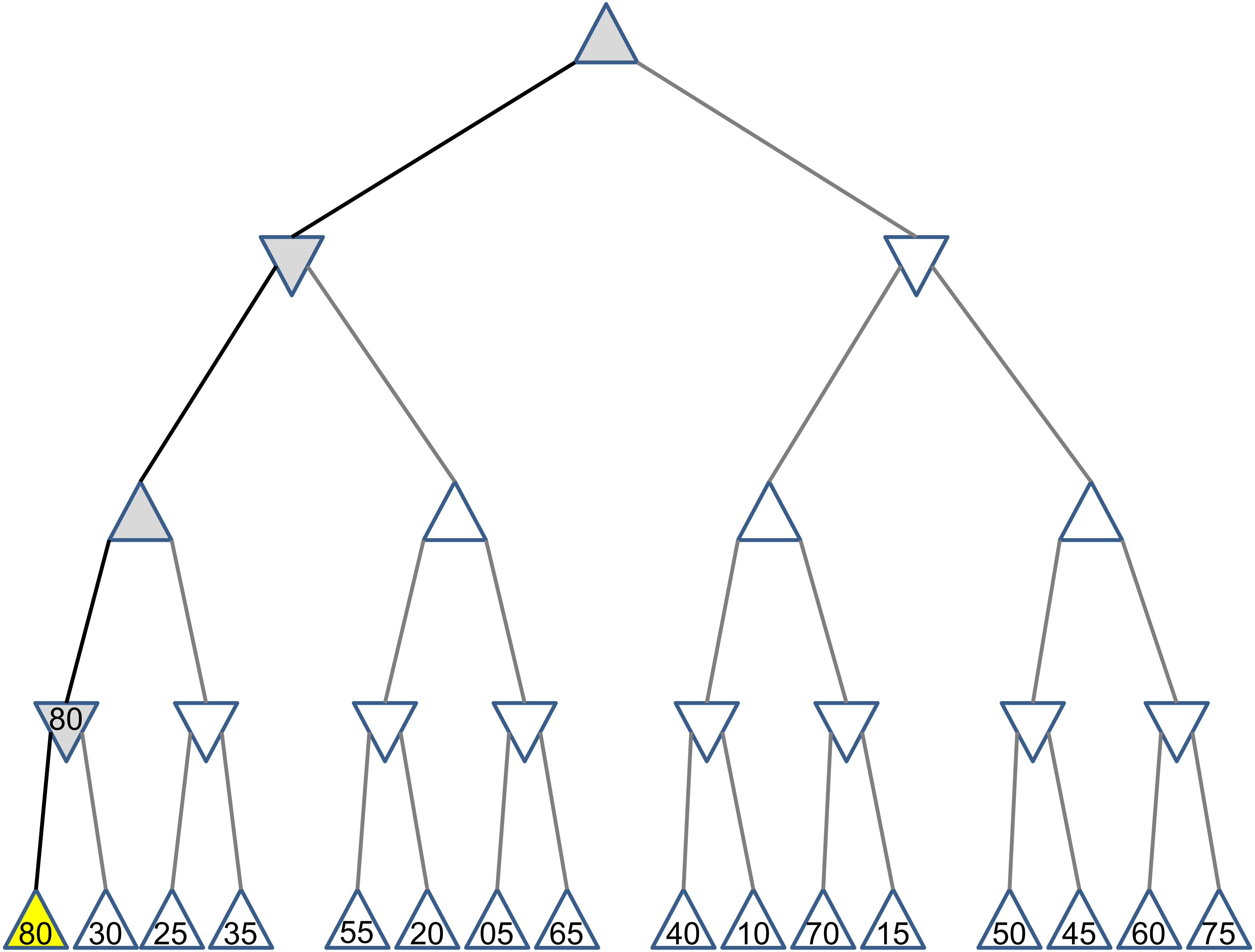
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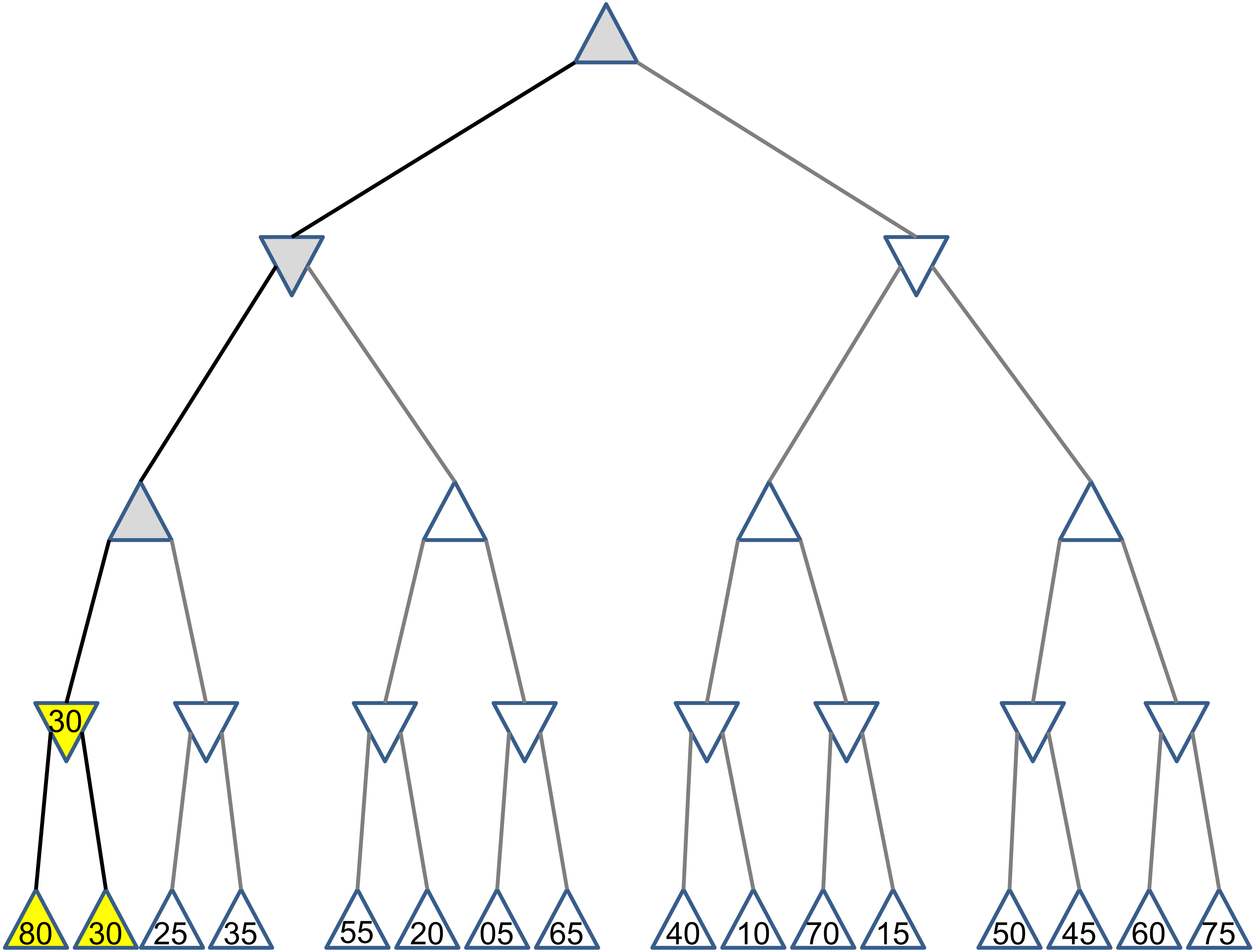
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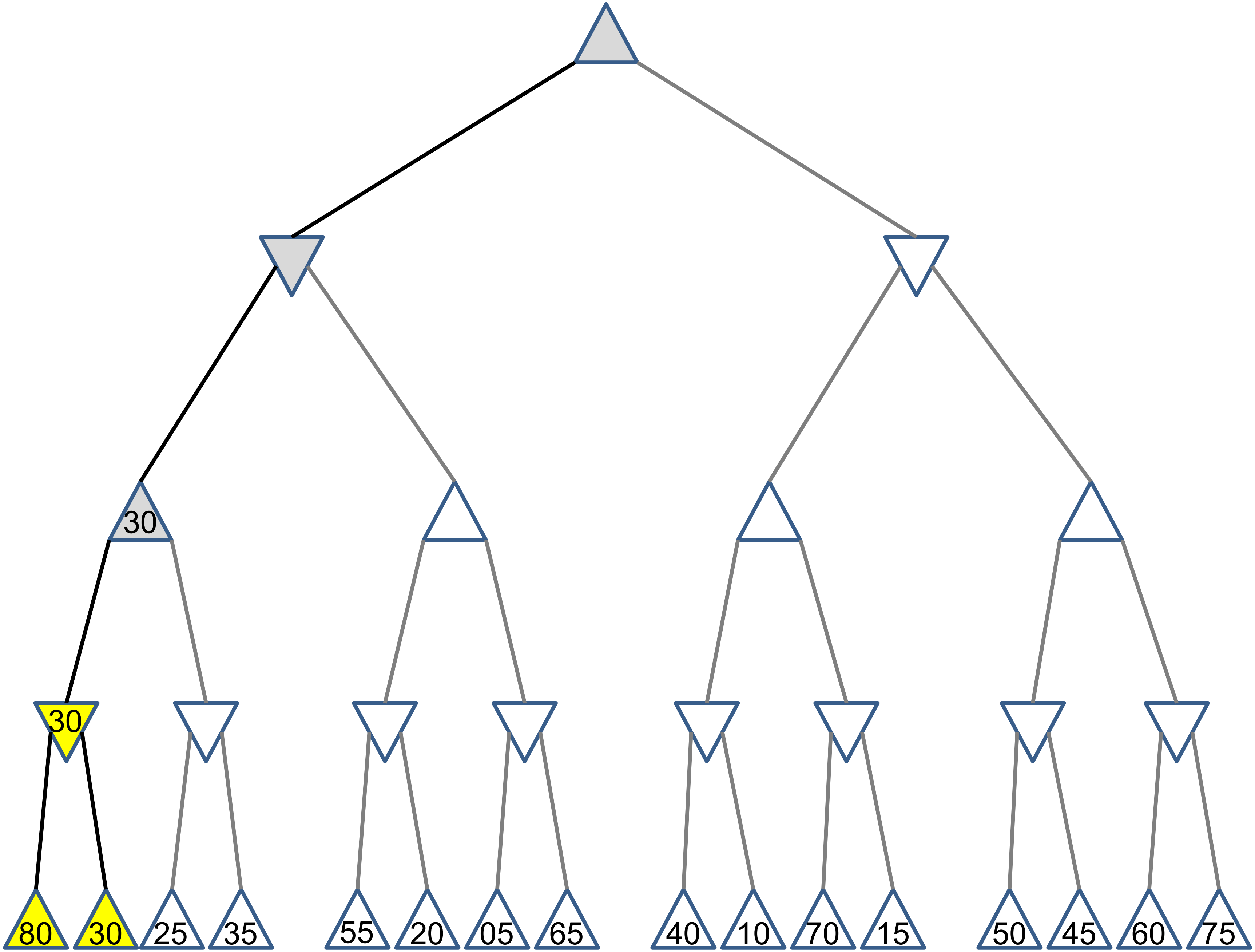
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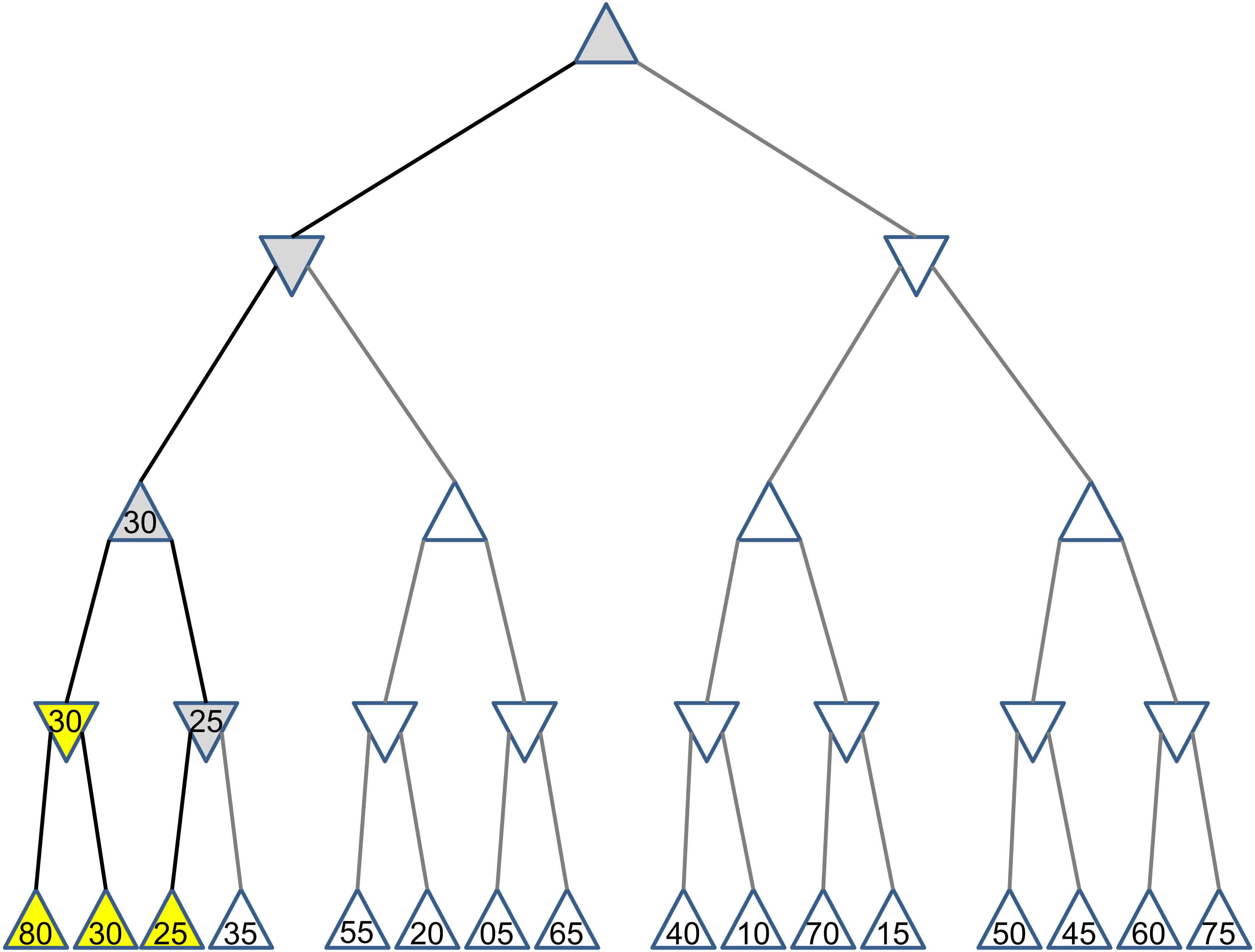
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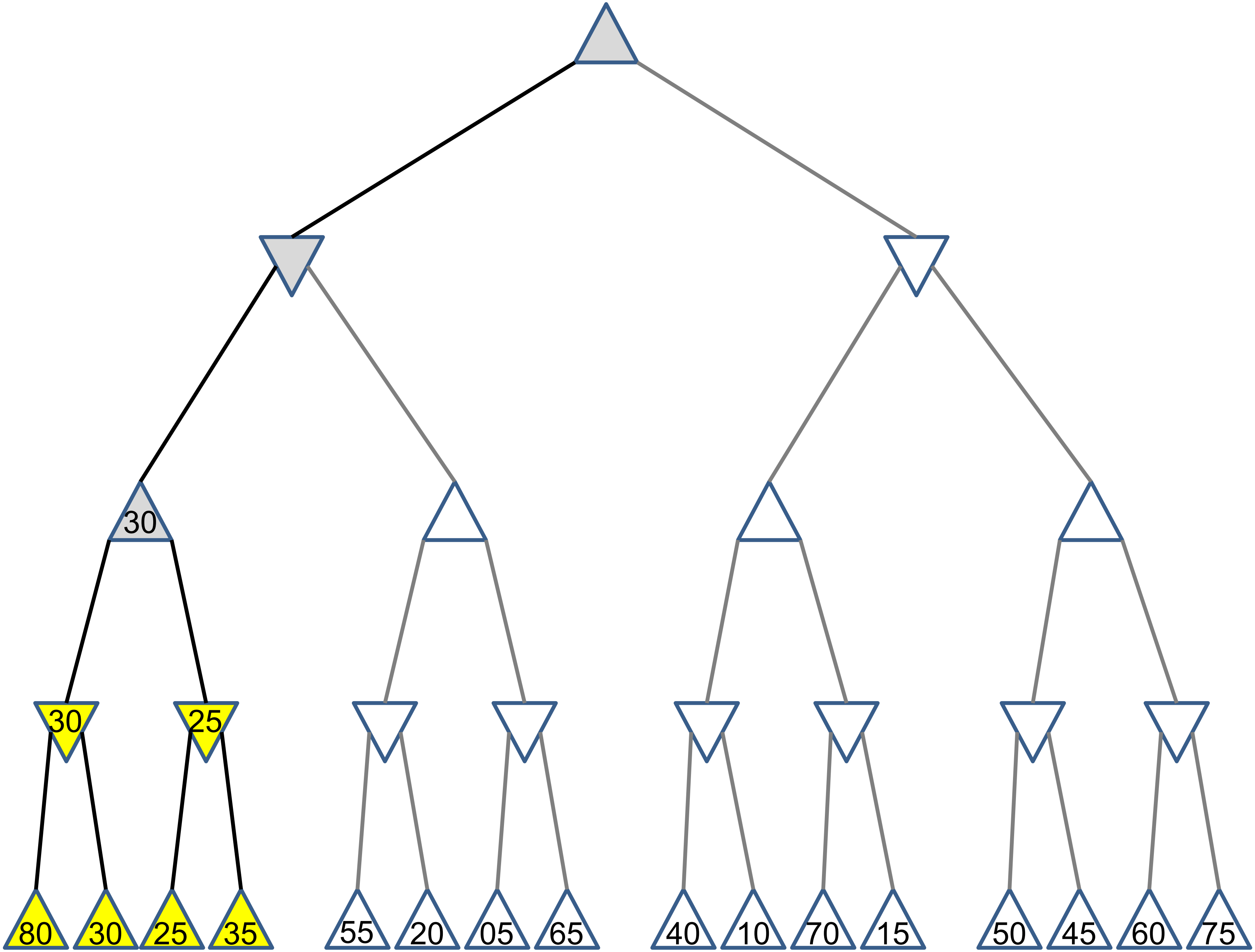
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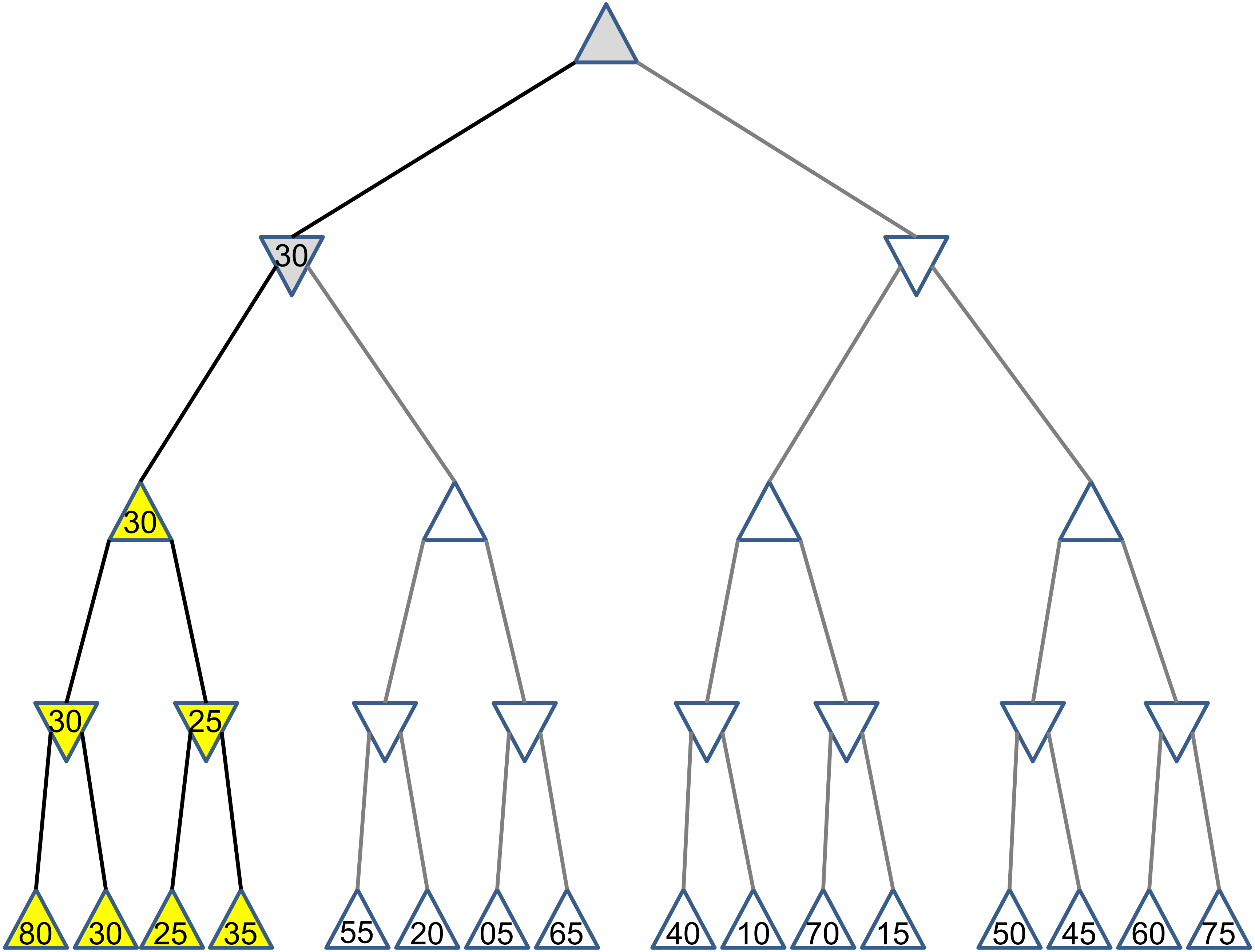
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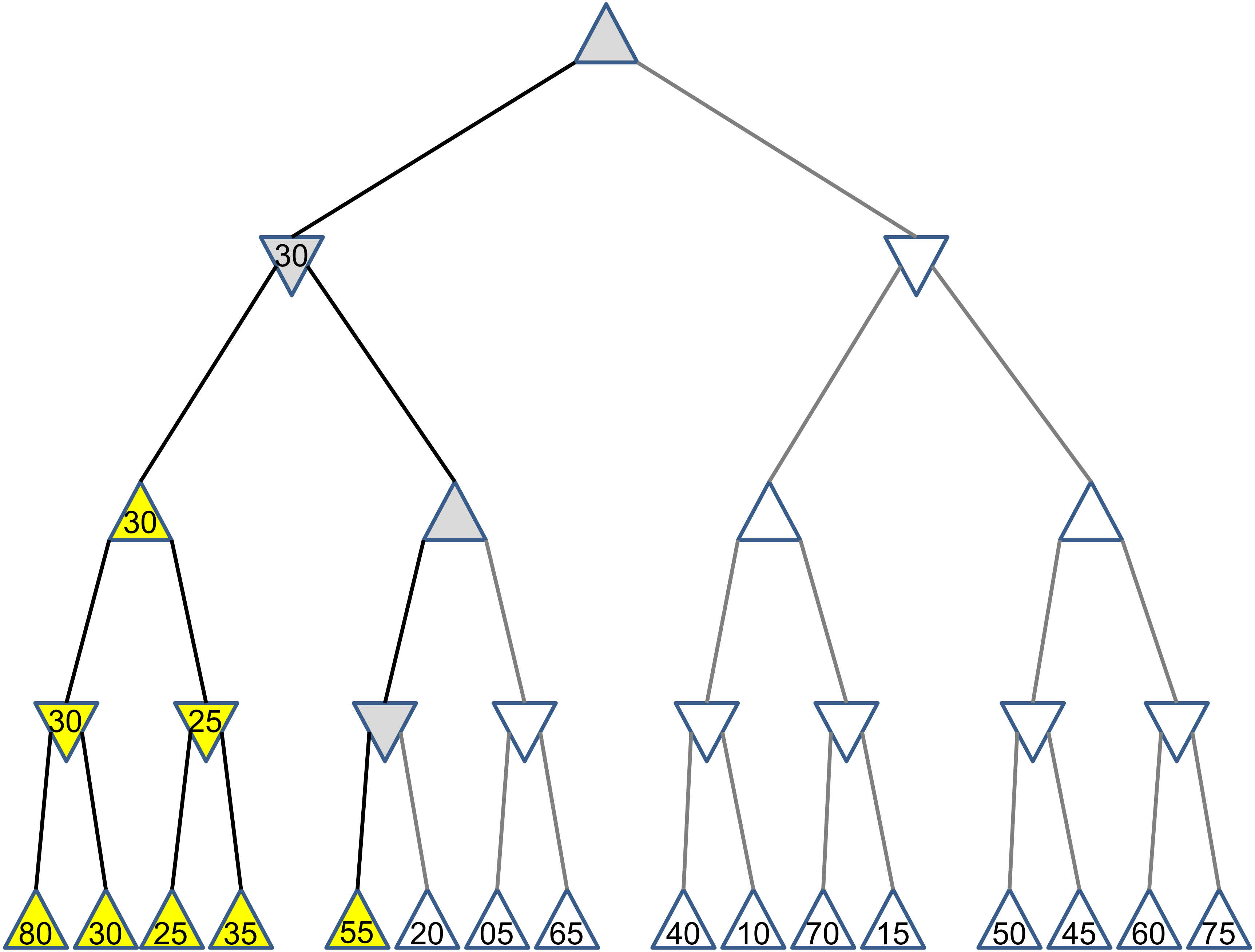
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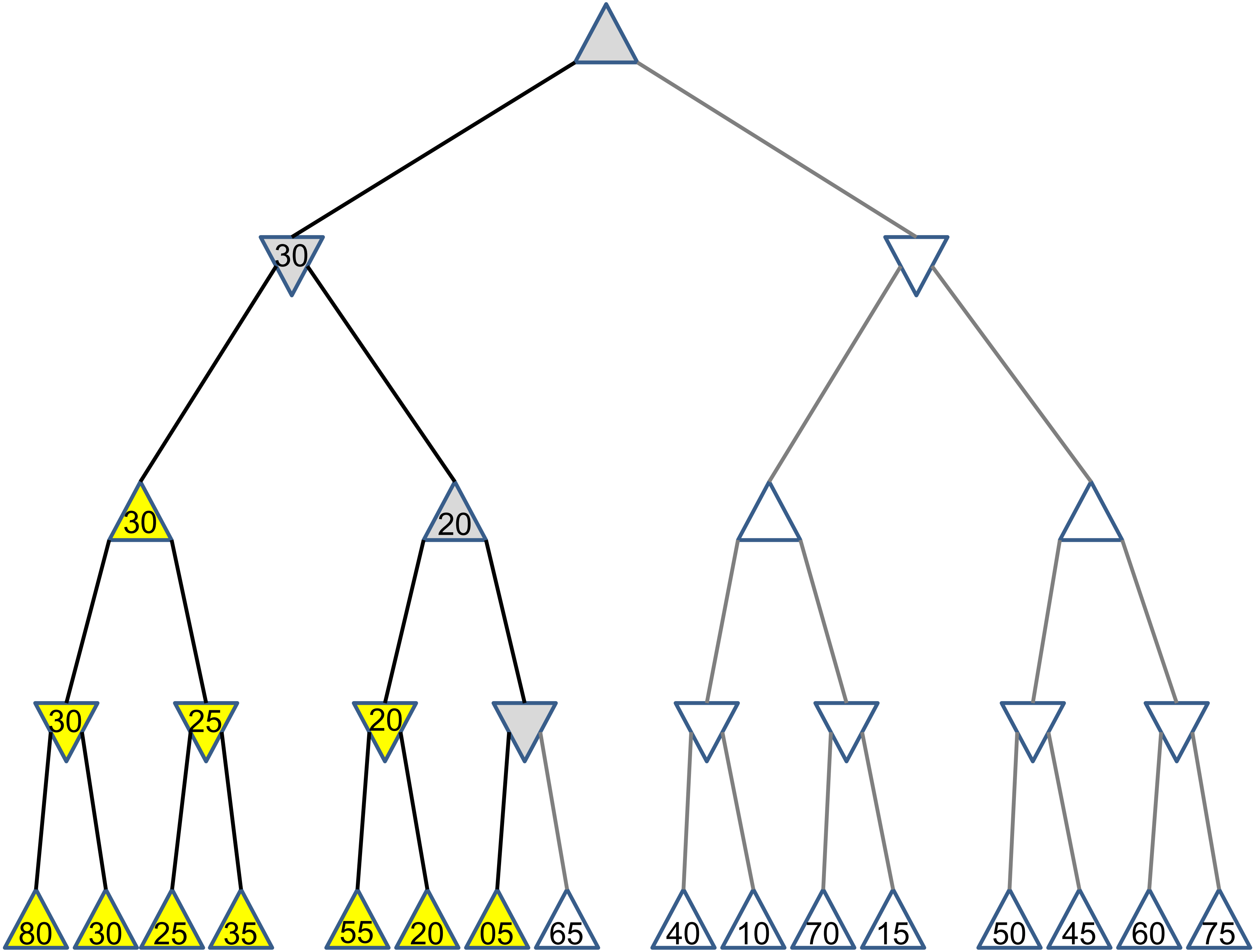
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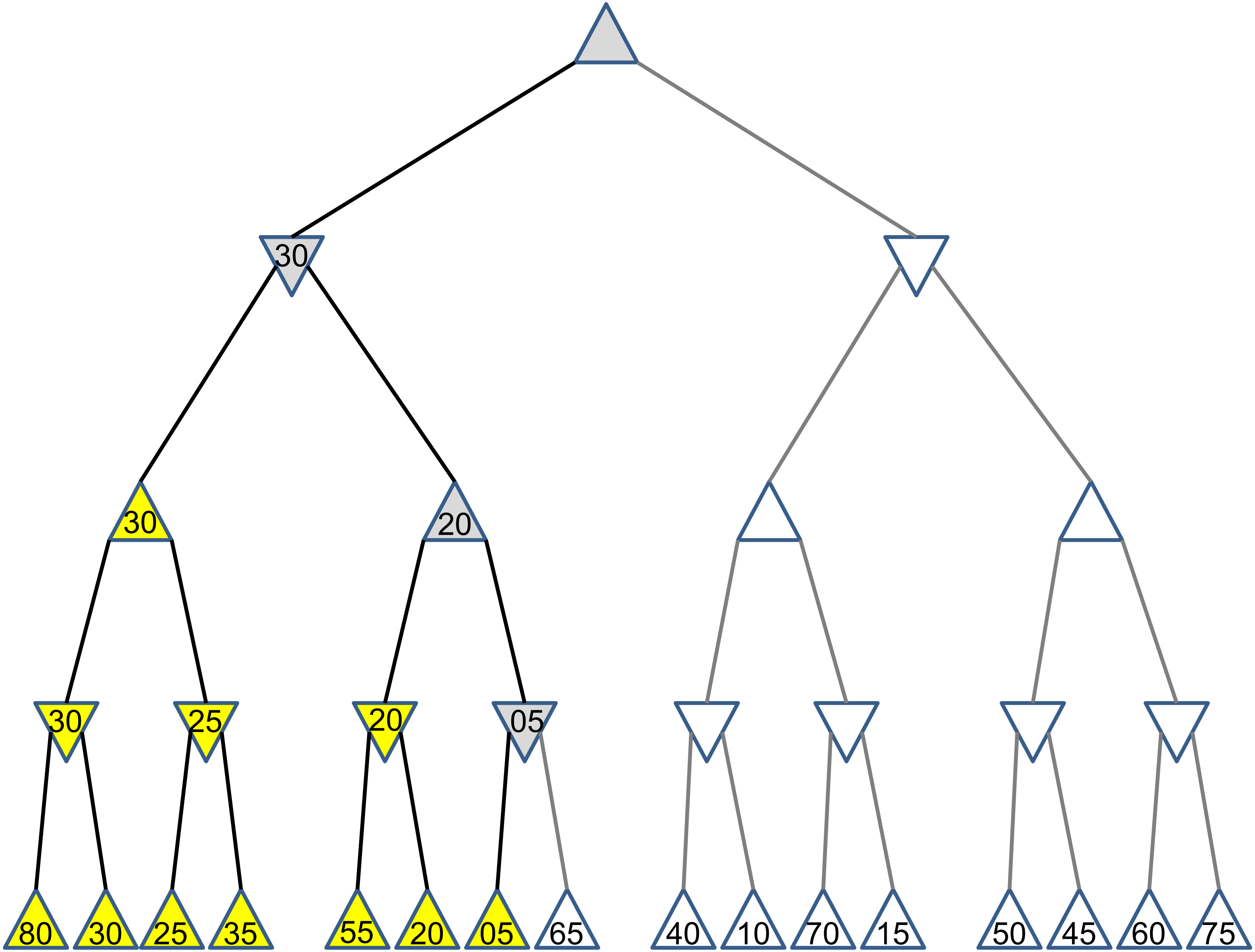
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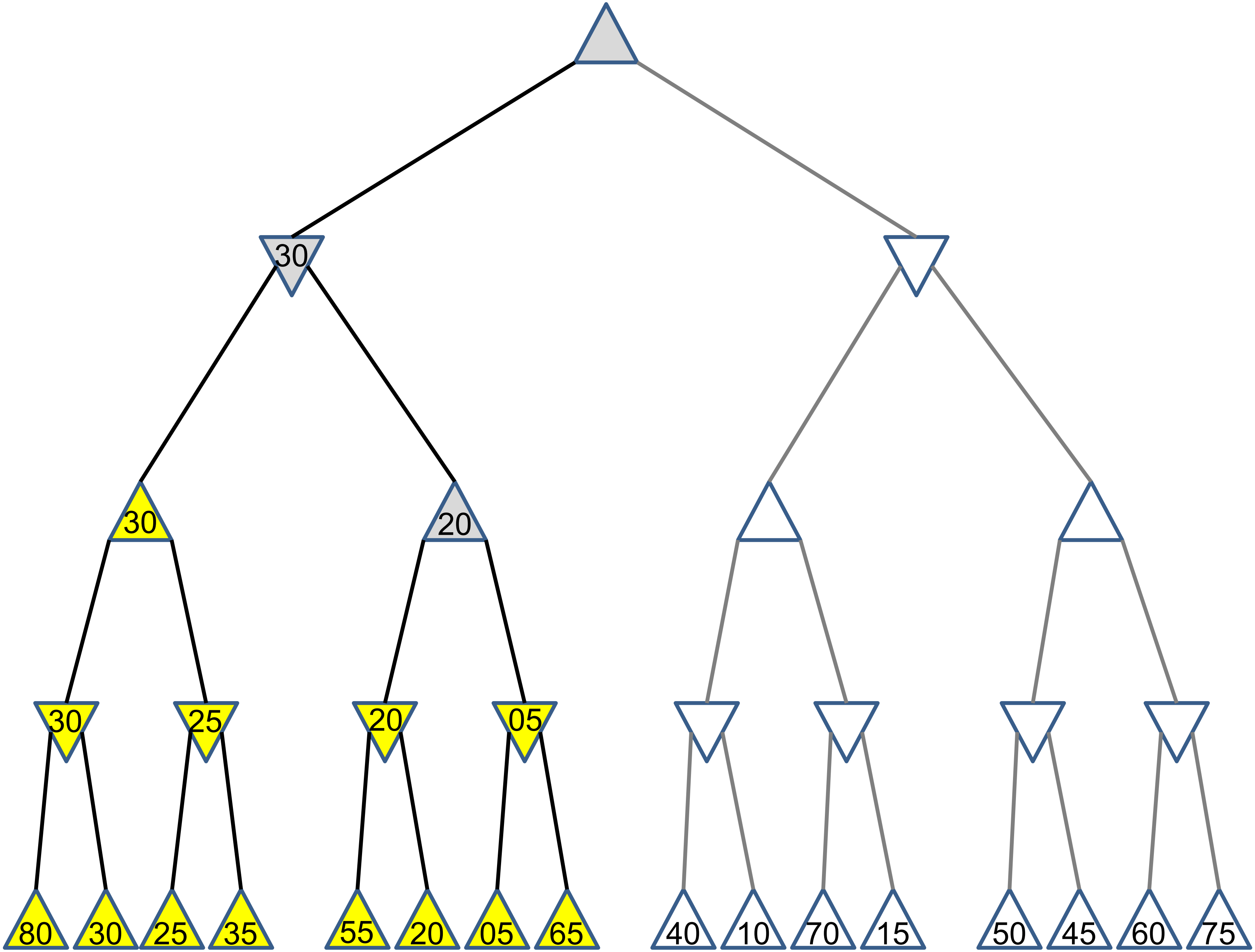
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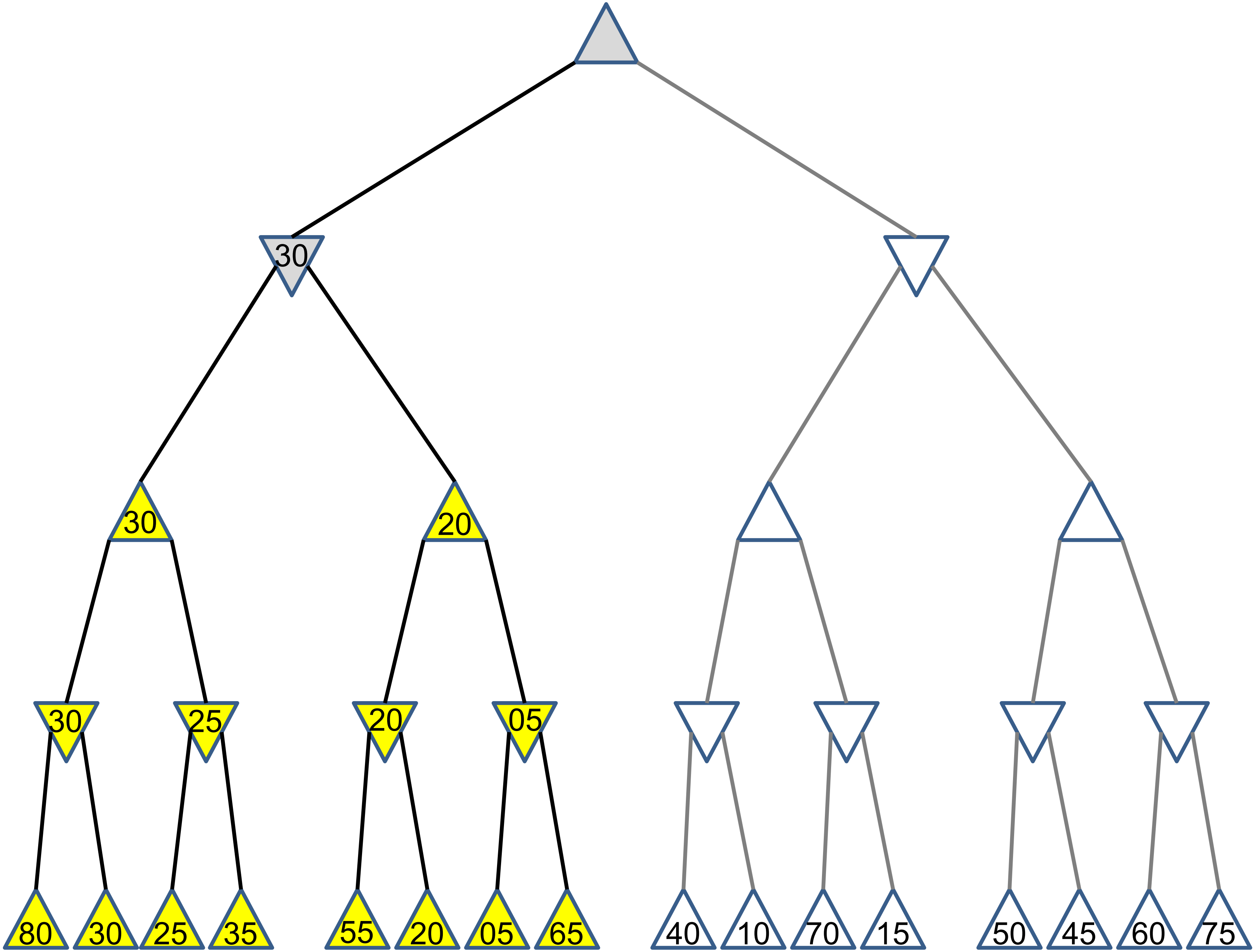
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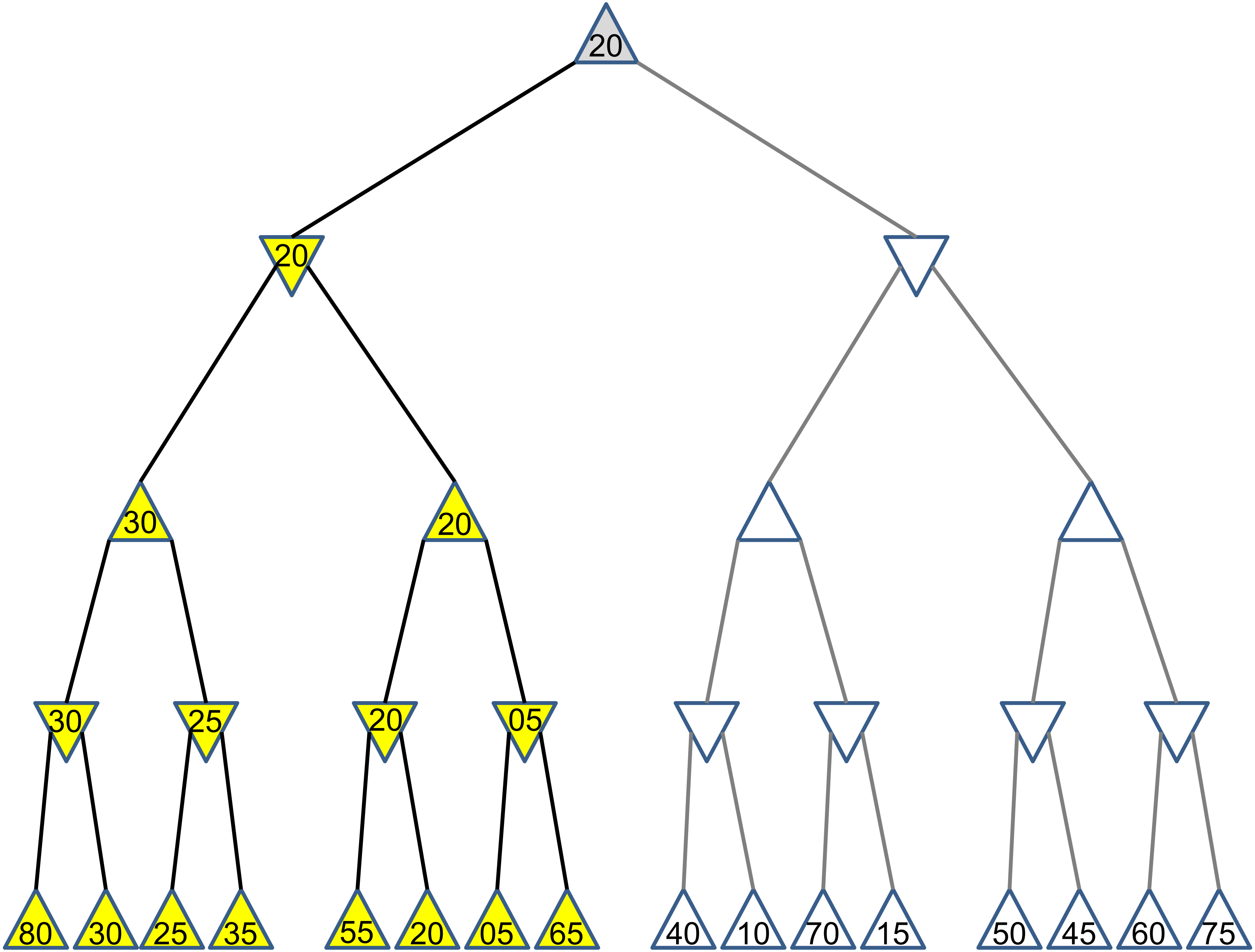
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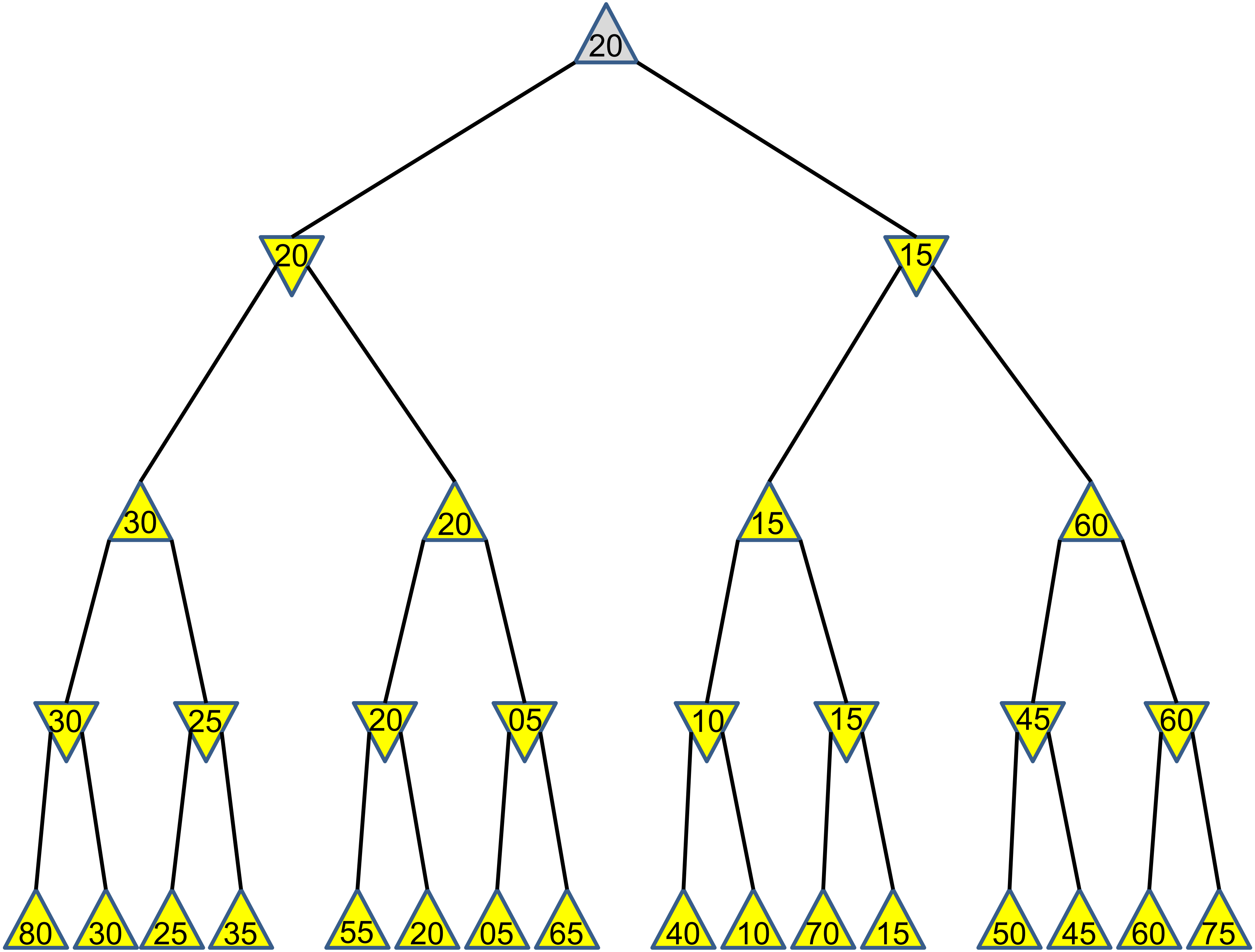
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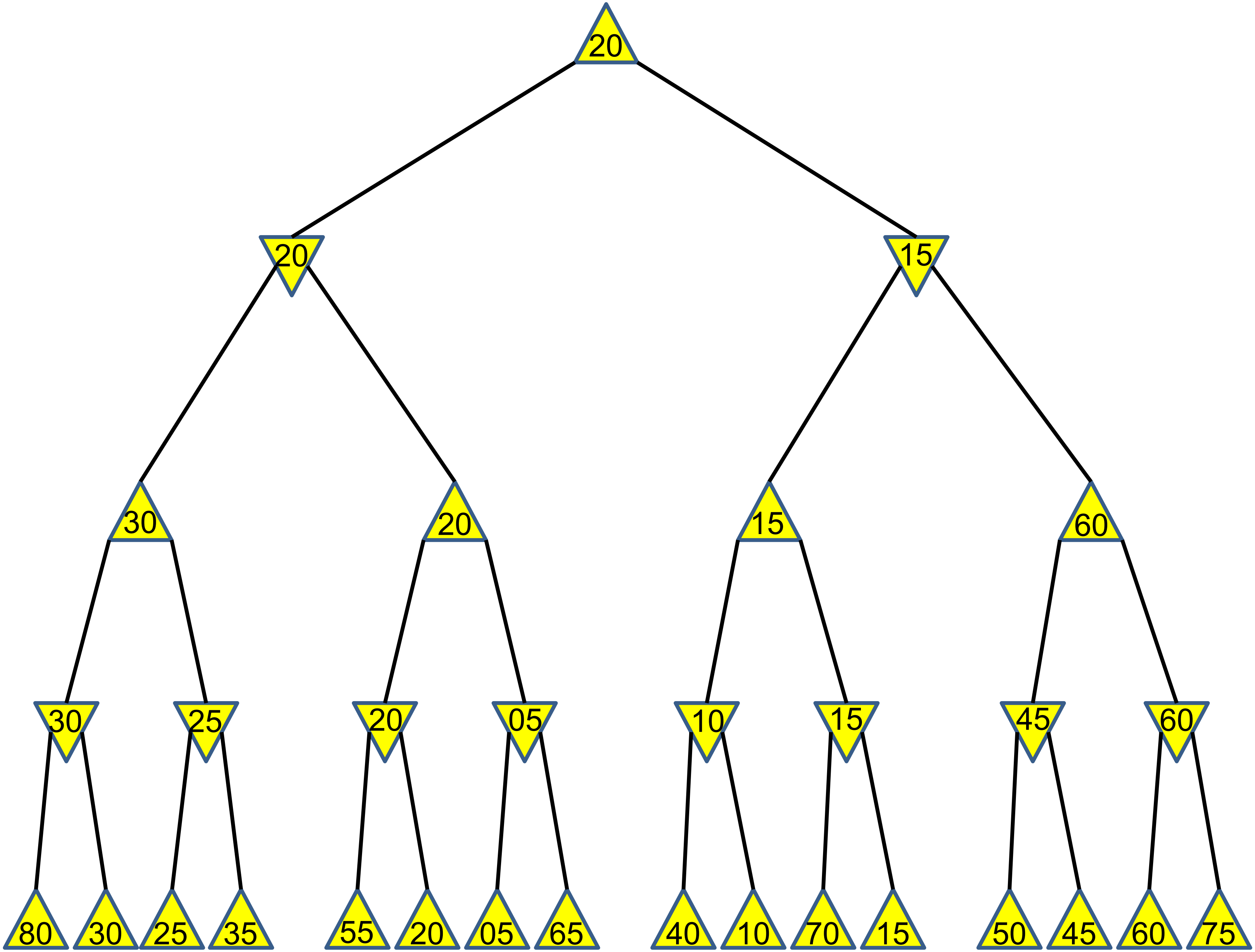
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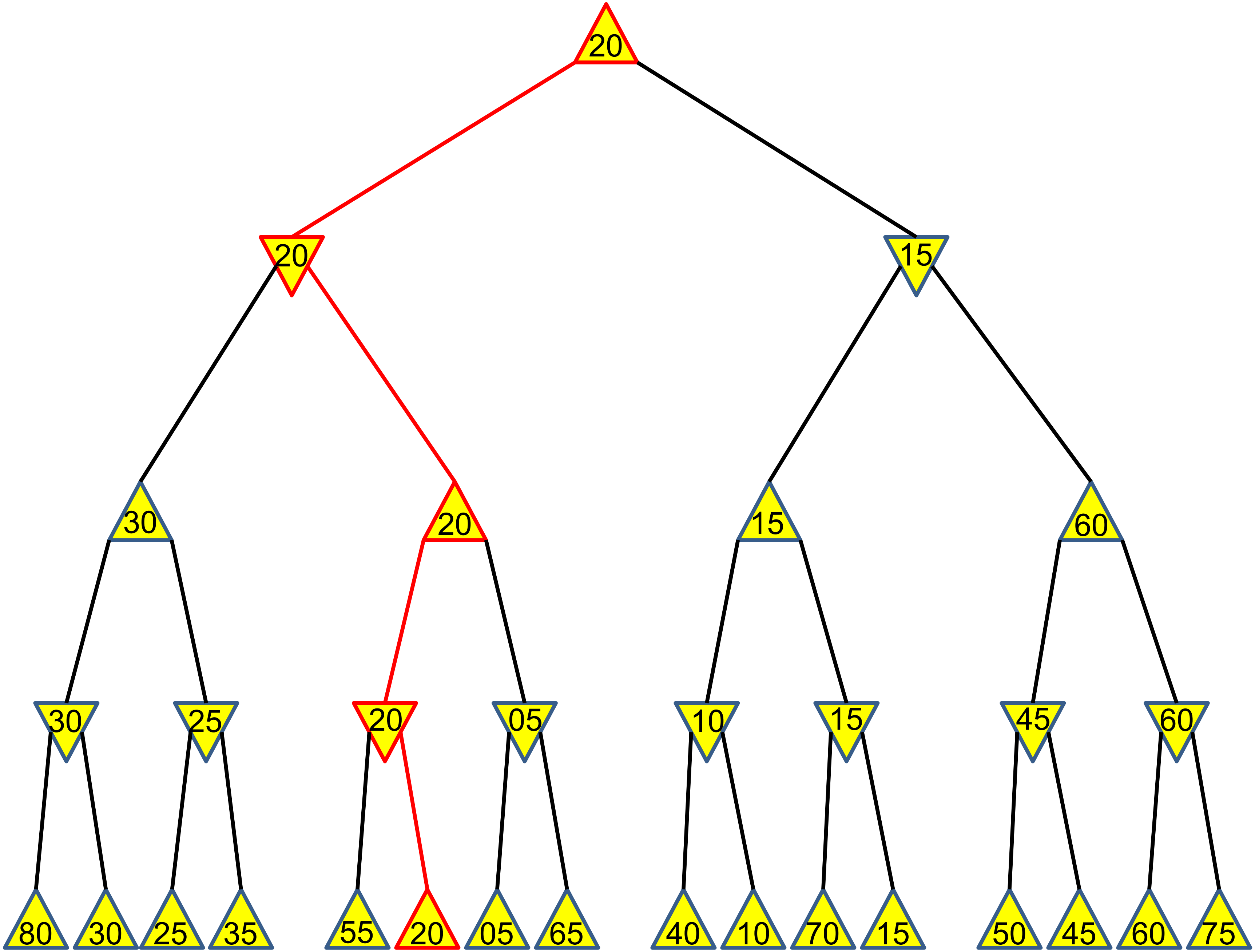
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Minimax Strategy

- Why do we take the min value every other level of the tree?
- These nodes represent the opponent's choice of move.
- The computer assumes that the human will choose that move that is of least value to the computer.

Minimax algorithm
Adversarial analogue of DFS

function MINIMAX-DECISION(*state*) **returns** *an action*

  $v \leftarrow$ MAX-VALUE(*state*)
  return the *action* in SUCCESSORS(*state*) with value v

---

function MAX-VALUE(*state*) **returns** *a utility value*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s))
  return v

---

function MIN-VALUE(*state*) **returns** *a utility value*

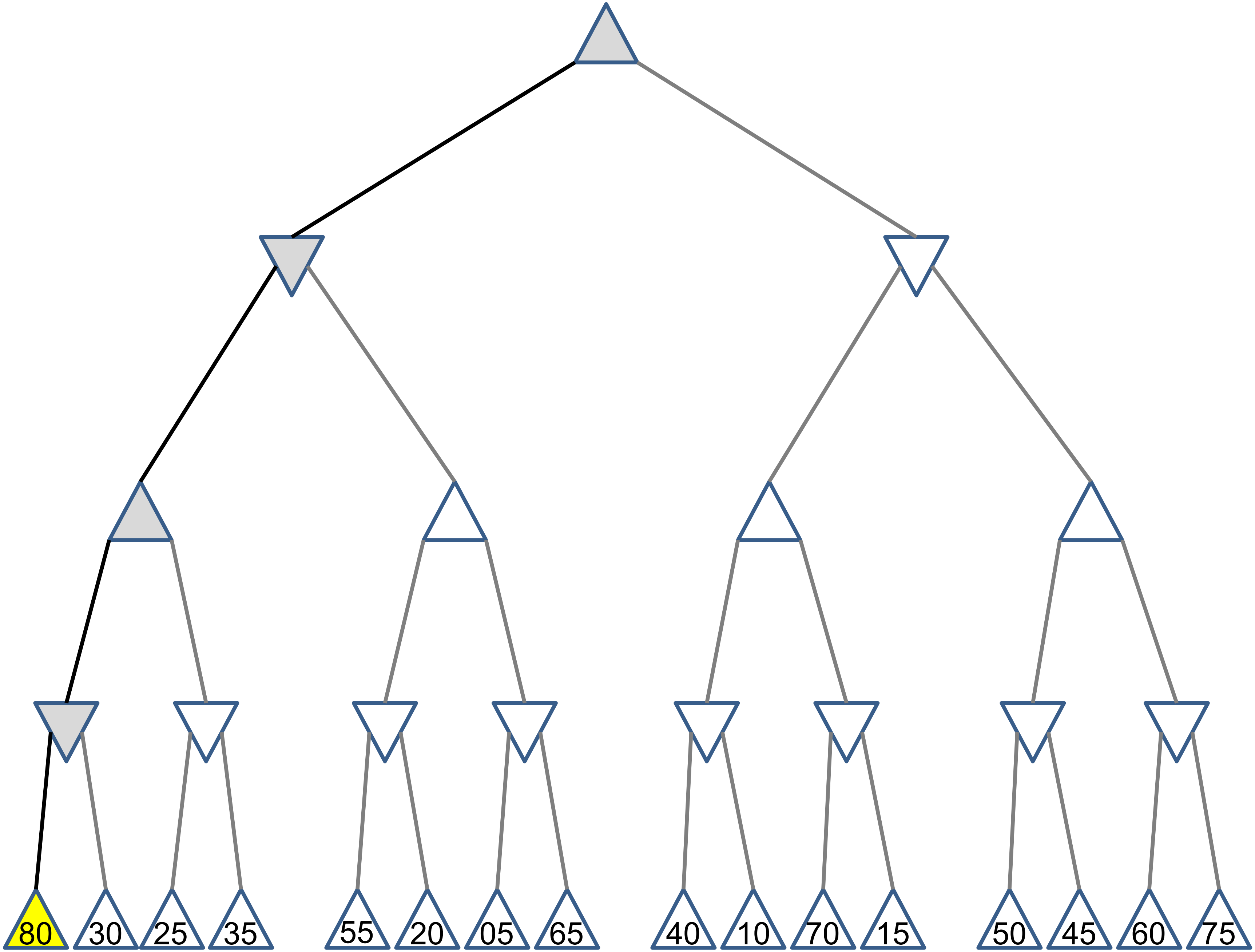
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow \infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(s))
  return v

Properties of Minimax

- Complete?
  - Yes (if tree is finite)
- Optimal?
  - Yes (against an optimal opponent)
  - No (does not exploit opponent weakness against suboptimal opponent)
- Time complexity?
  - $O(b^m)$
- Space complexity?
  - O(bm) (depth-first exploration)

Good Enough?

- Chess:
  - branching factor b≈35
  - game length m≈100
  - search space $b^m \approx 35^{100} \approx 10^{154}$
- The Universe:
  - number of atoms $\approx 10^{78}$
  - age $\approx 10^{18}$ seconds
  - 10^8 moves/sec x 10^{78} x 10^{18} = 10^{104}
- Exact solution completely infeasible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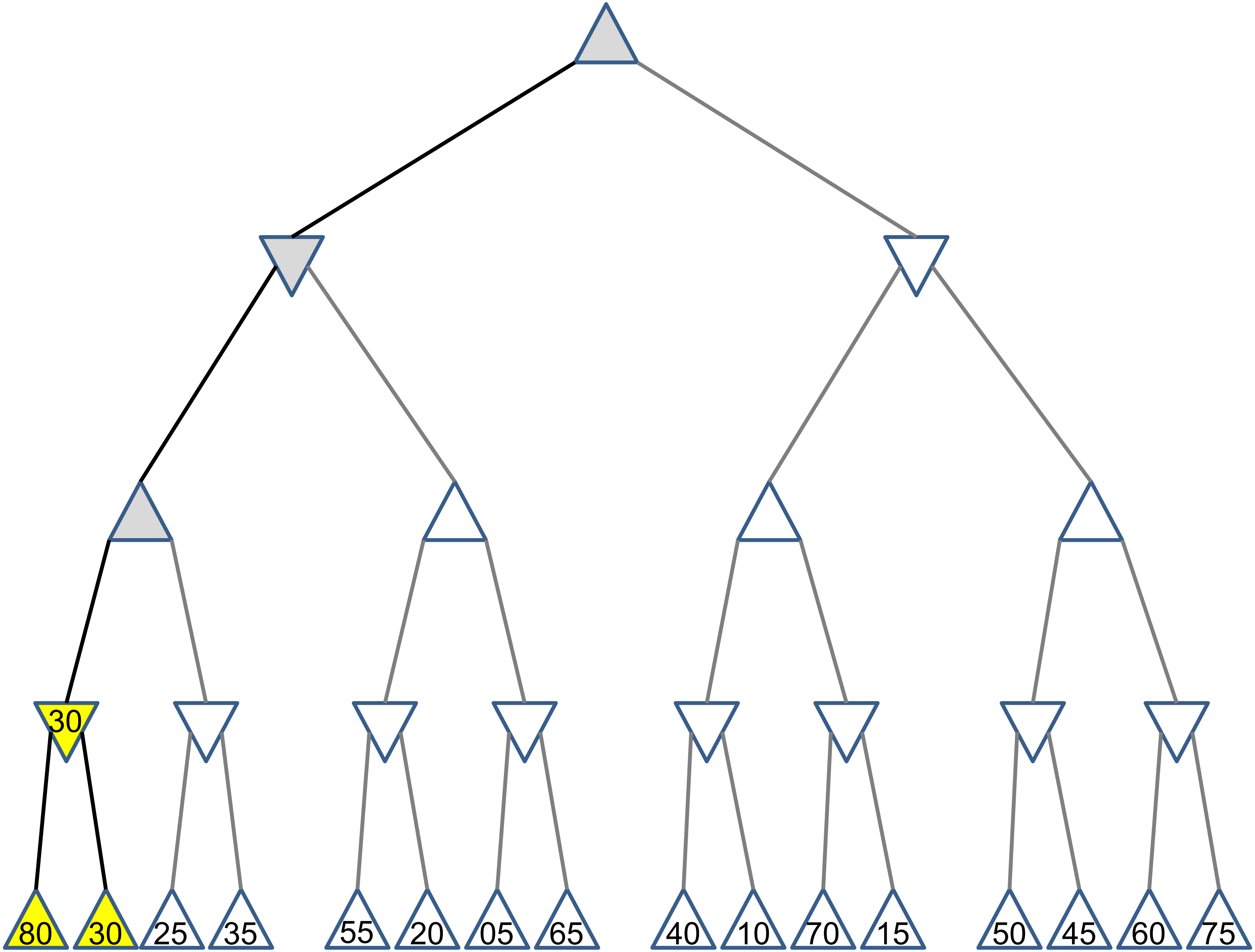

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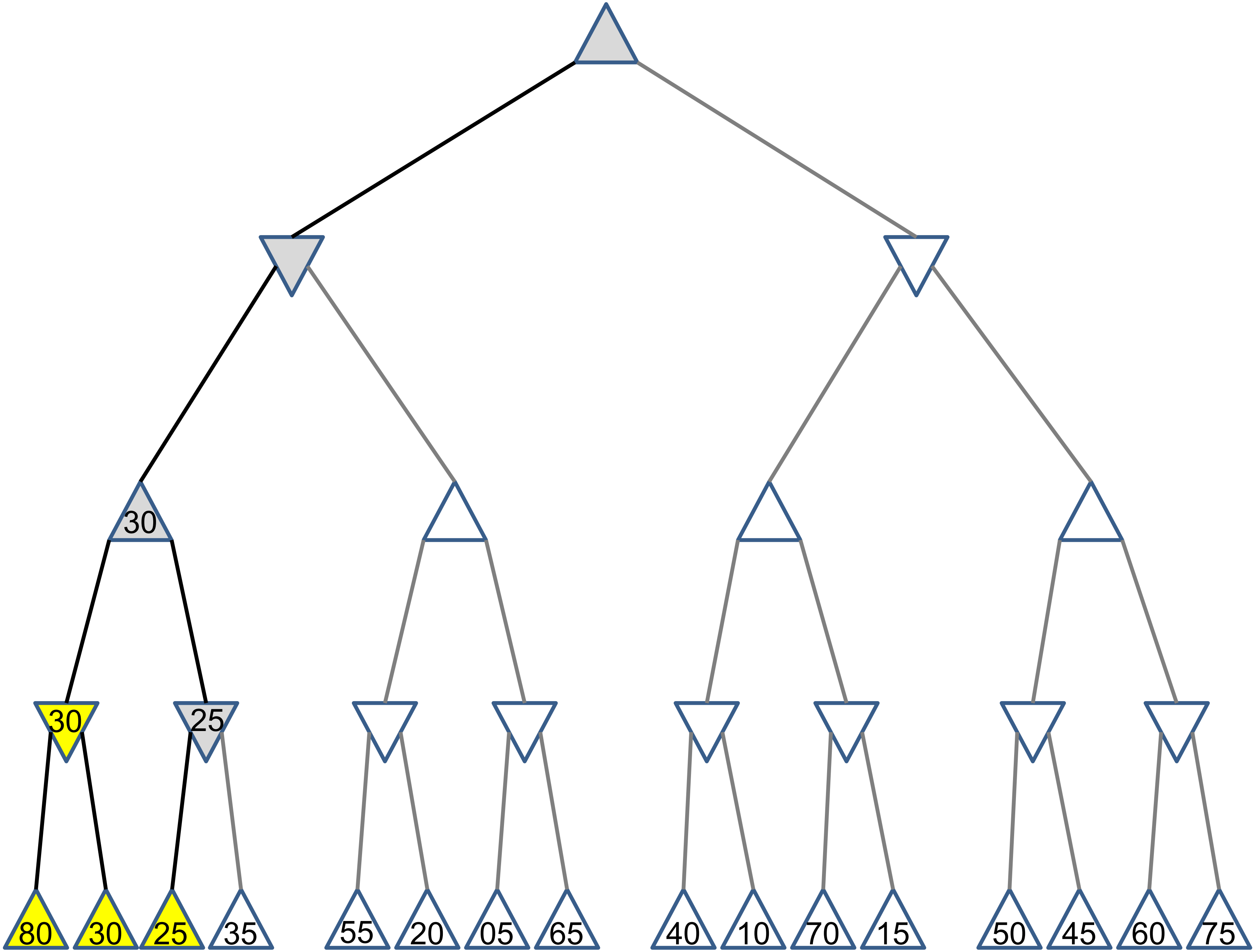
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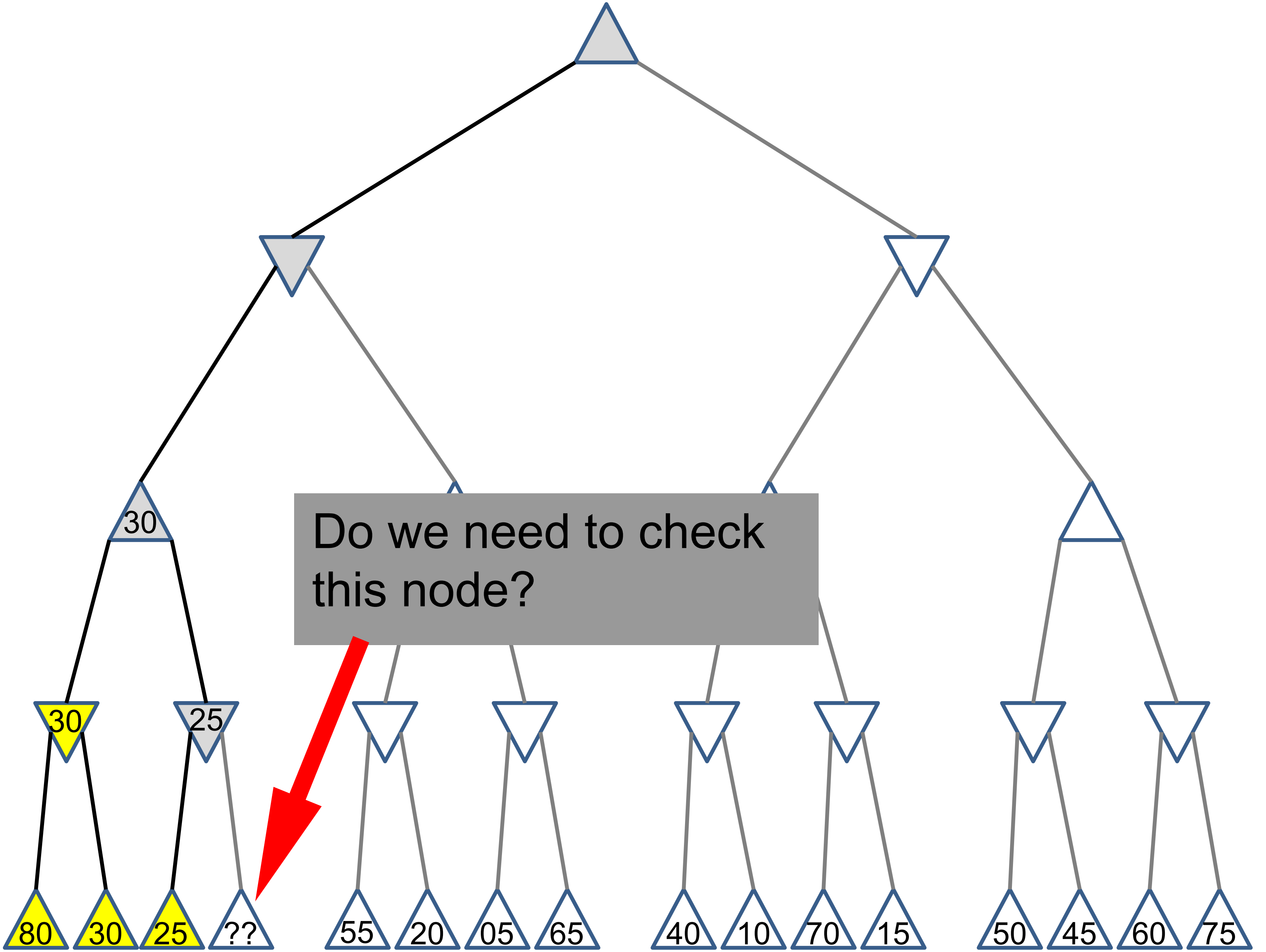
Do we need to check this node?
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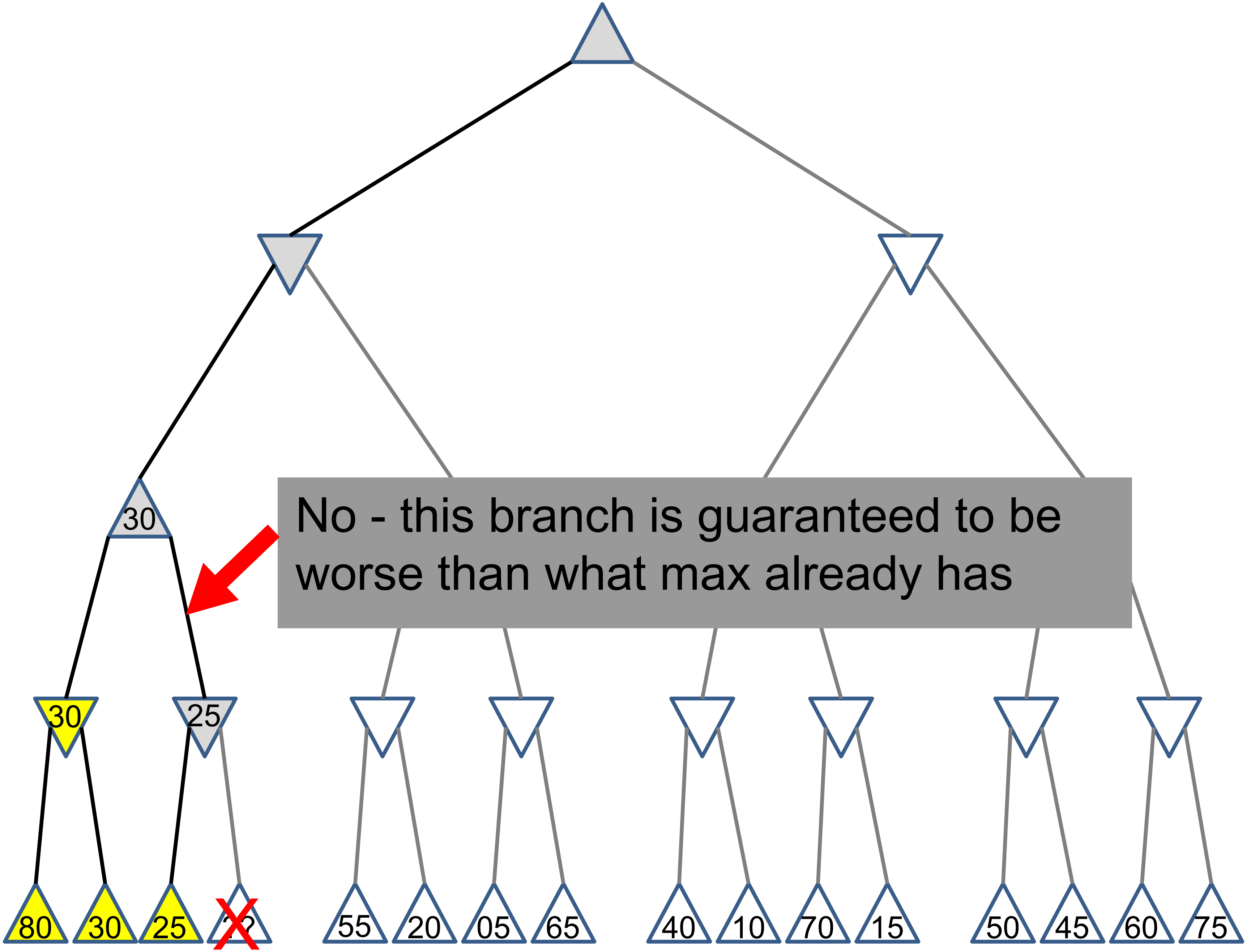
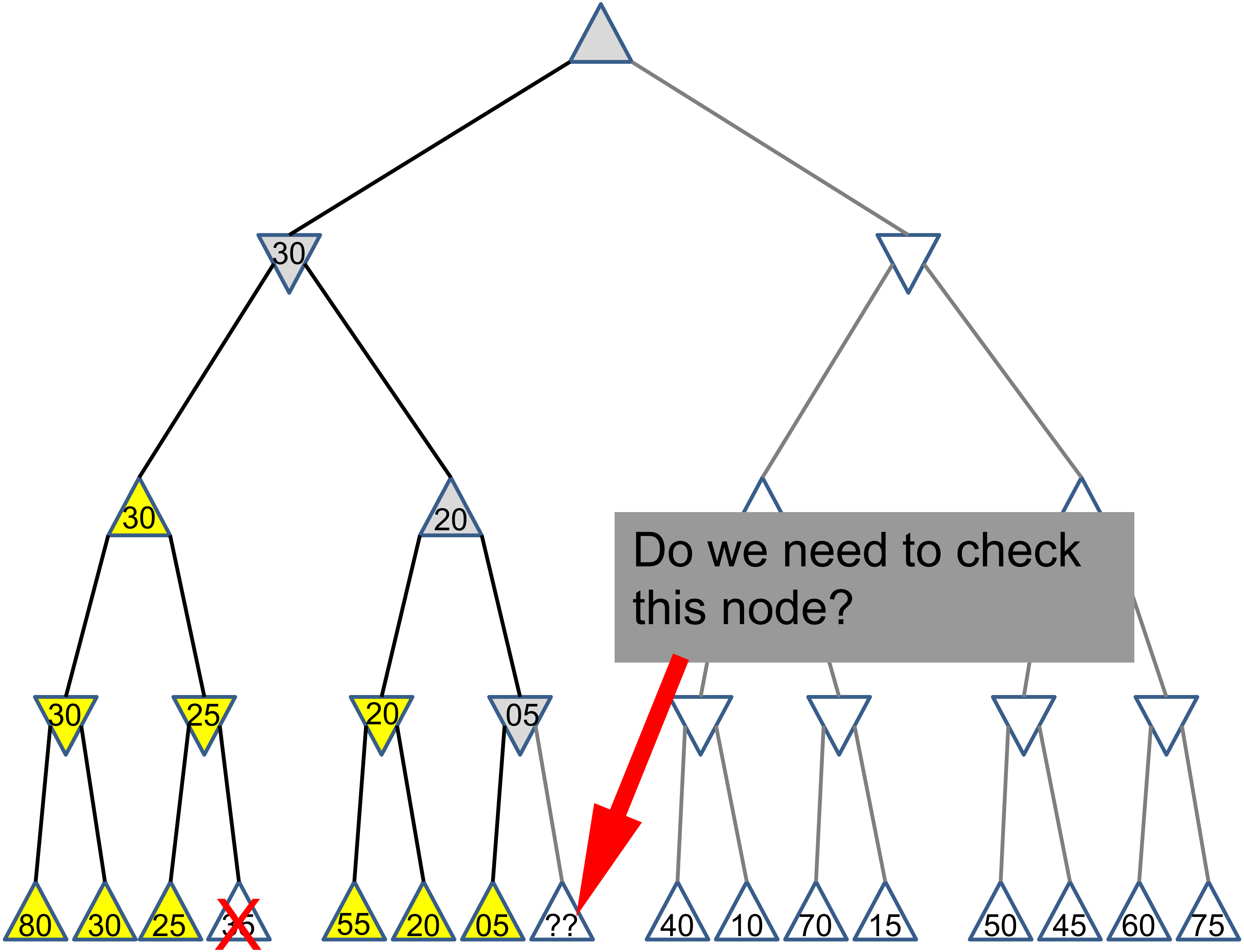


No - this branch is guaranteed to be worse than what max already has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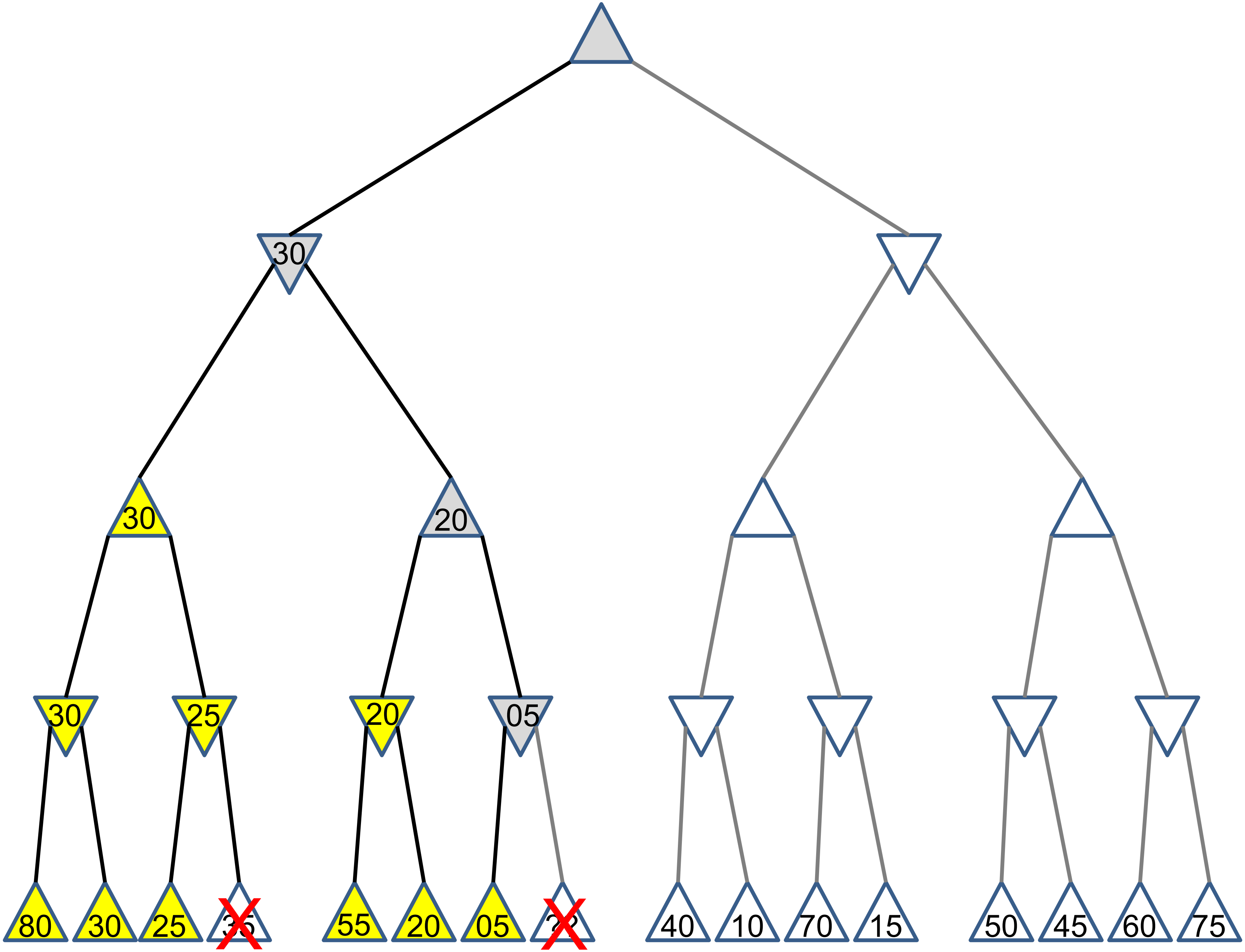

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Alpha-Beta

- The alpha-beta procedure can speed up a depth-first minimax search.
- Alpha: a lower bound on the value that a max node may ultimately be assigned

$$v \geq \alpha$$

- Beta: an upper bound on the value that a minimizing node may ultimately be assigned

$$v \leq \beta$$

Alpha-Beta

```
MinVal(state, alpha, beta){
  if (terminal(state))
          return utility(state);
  for (s in children(state)){
     child = MaxVal(s,alpha,beta);
     beta = min(beta,child);
     if (alpha>=beta) return child;
  }
  return best child (min); }
```

alpha = the **highest** value for **MAX** along the path

beta = the **lowest** value for **MIN** along the path

Alpha-Beta

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MaxVal(state, alpha, beta){
  if (terminal(state))
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     alpha = max(alpha,child);
     if (alpha>=beta) return child;
  }
  return best child (max); }
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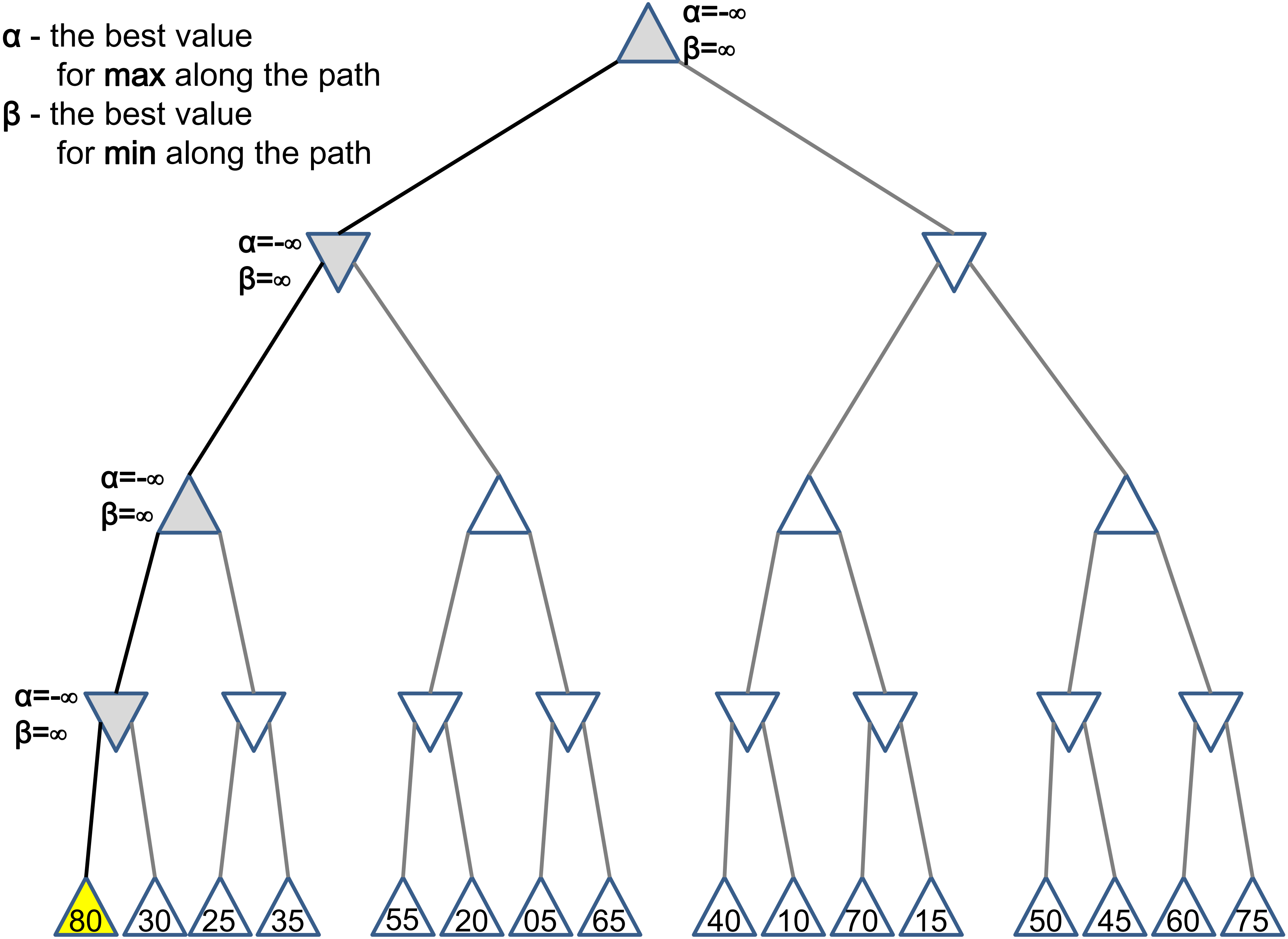

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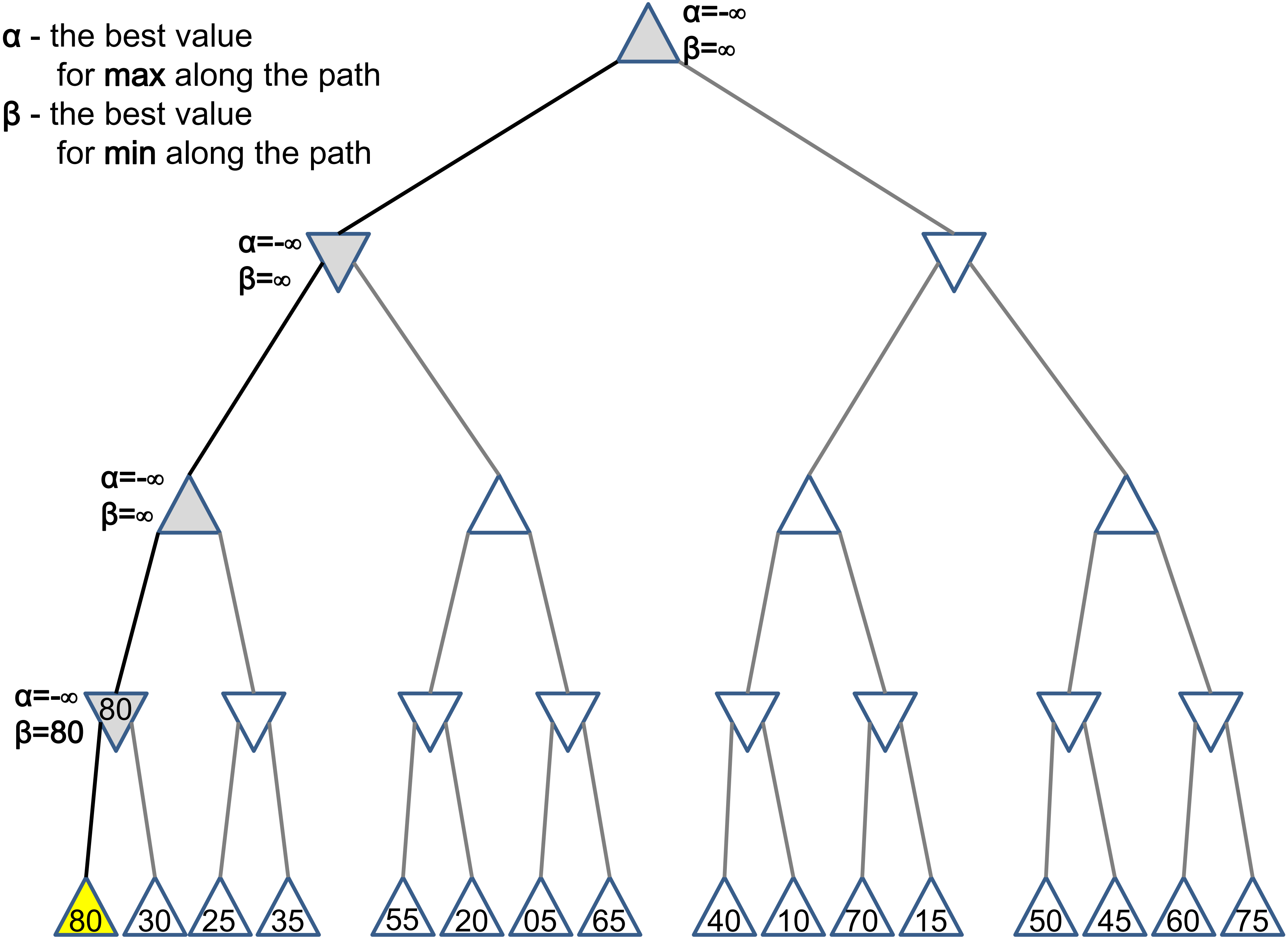

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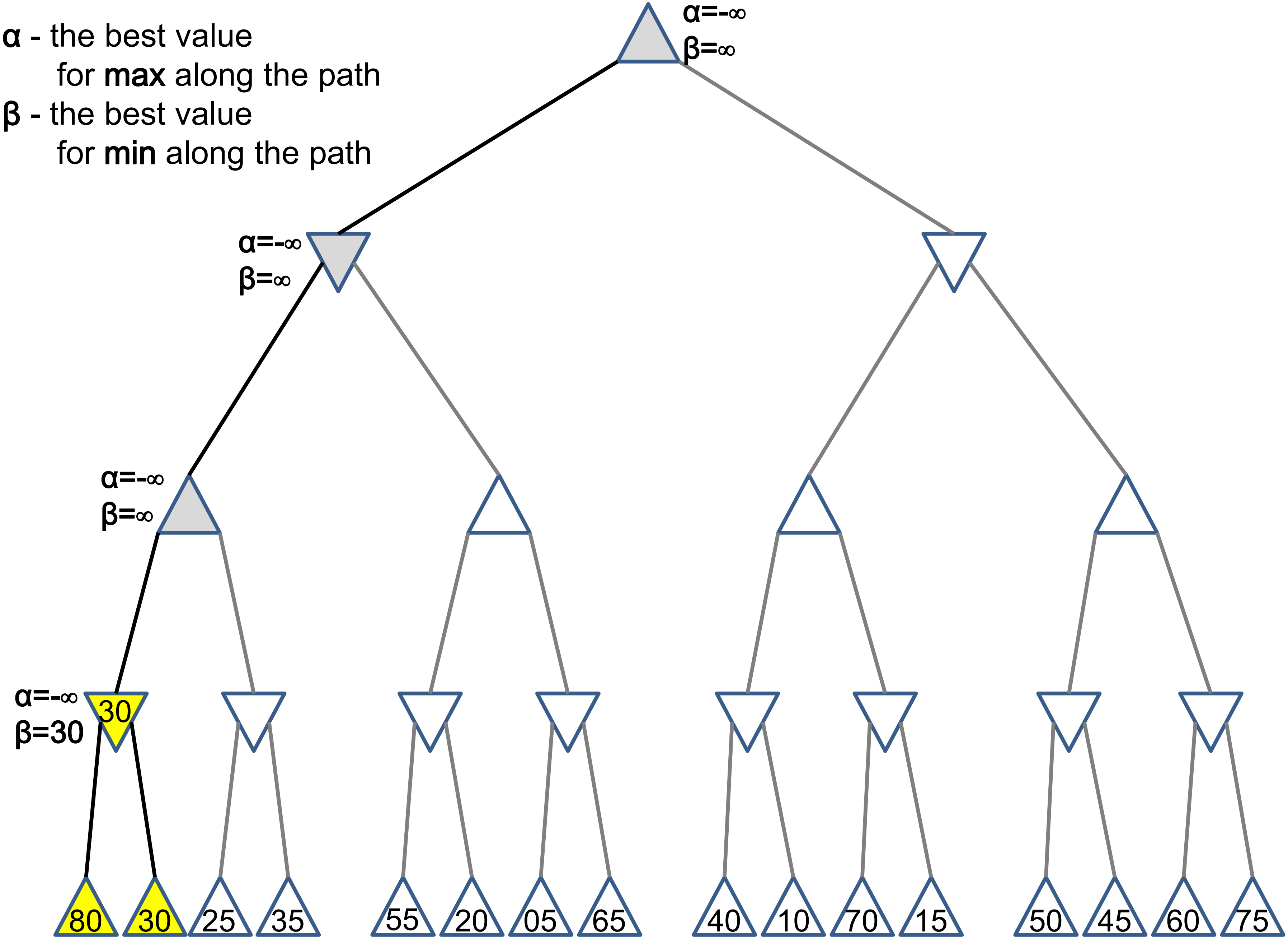

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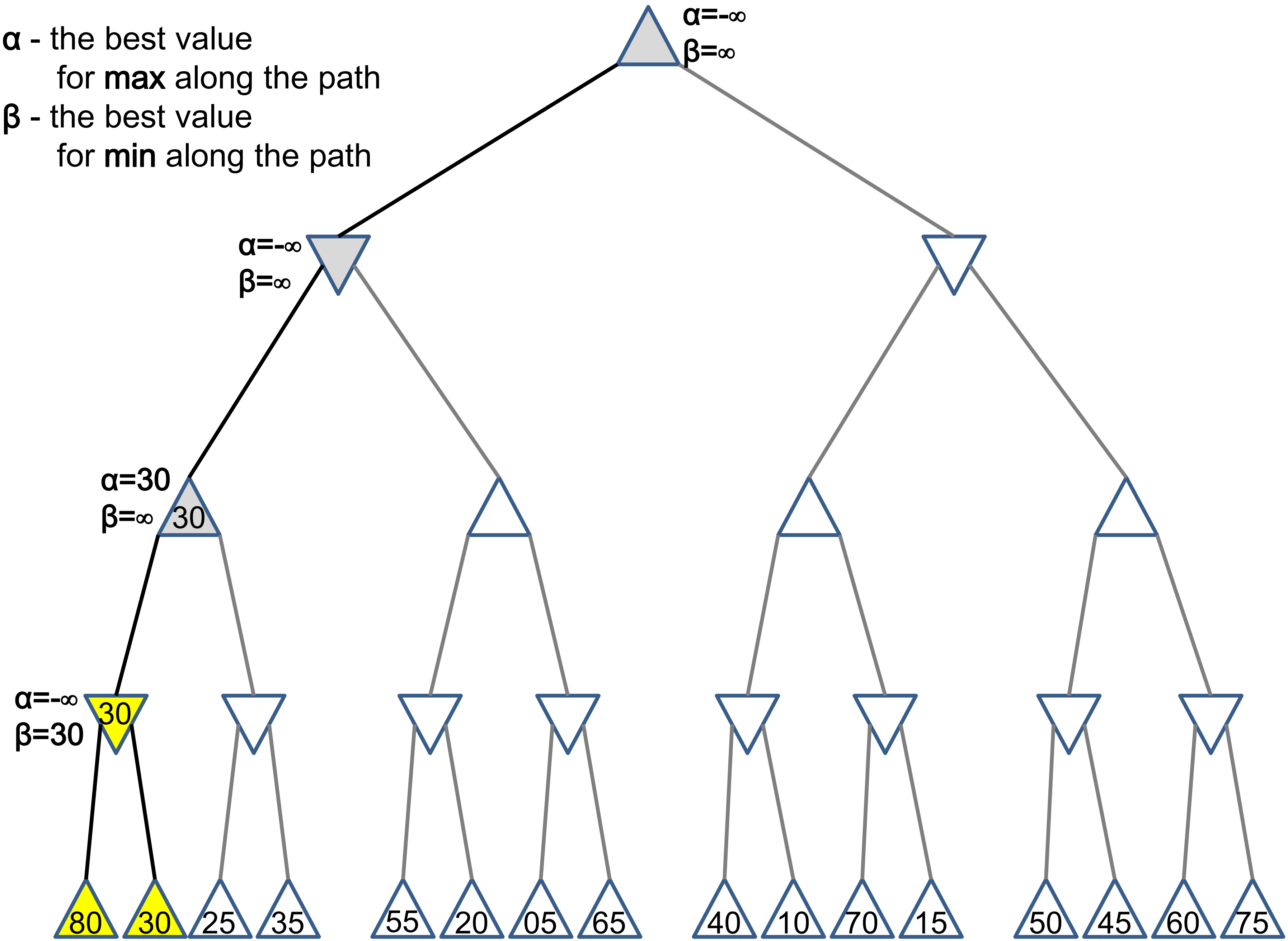

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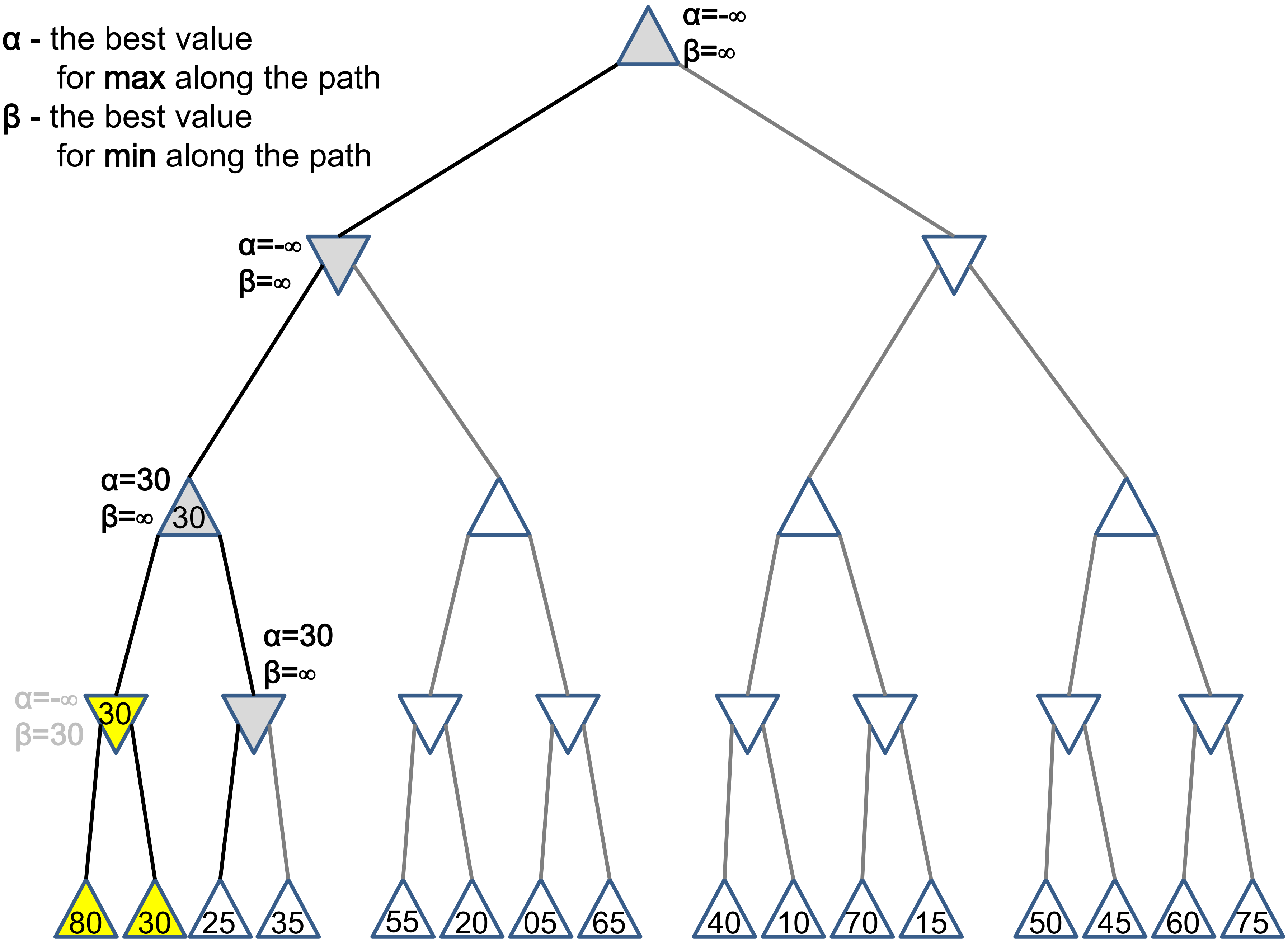

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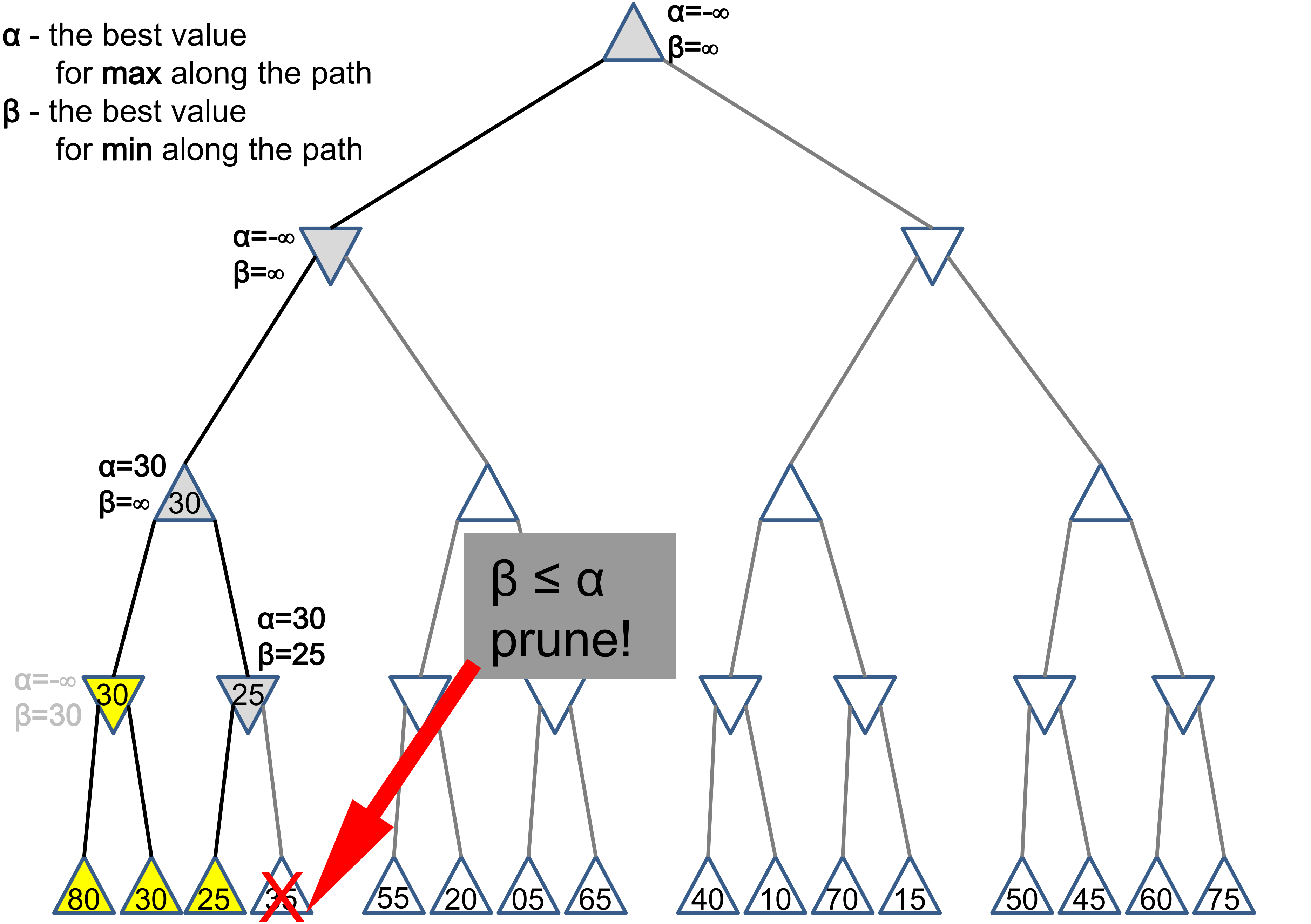

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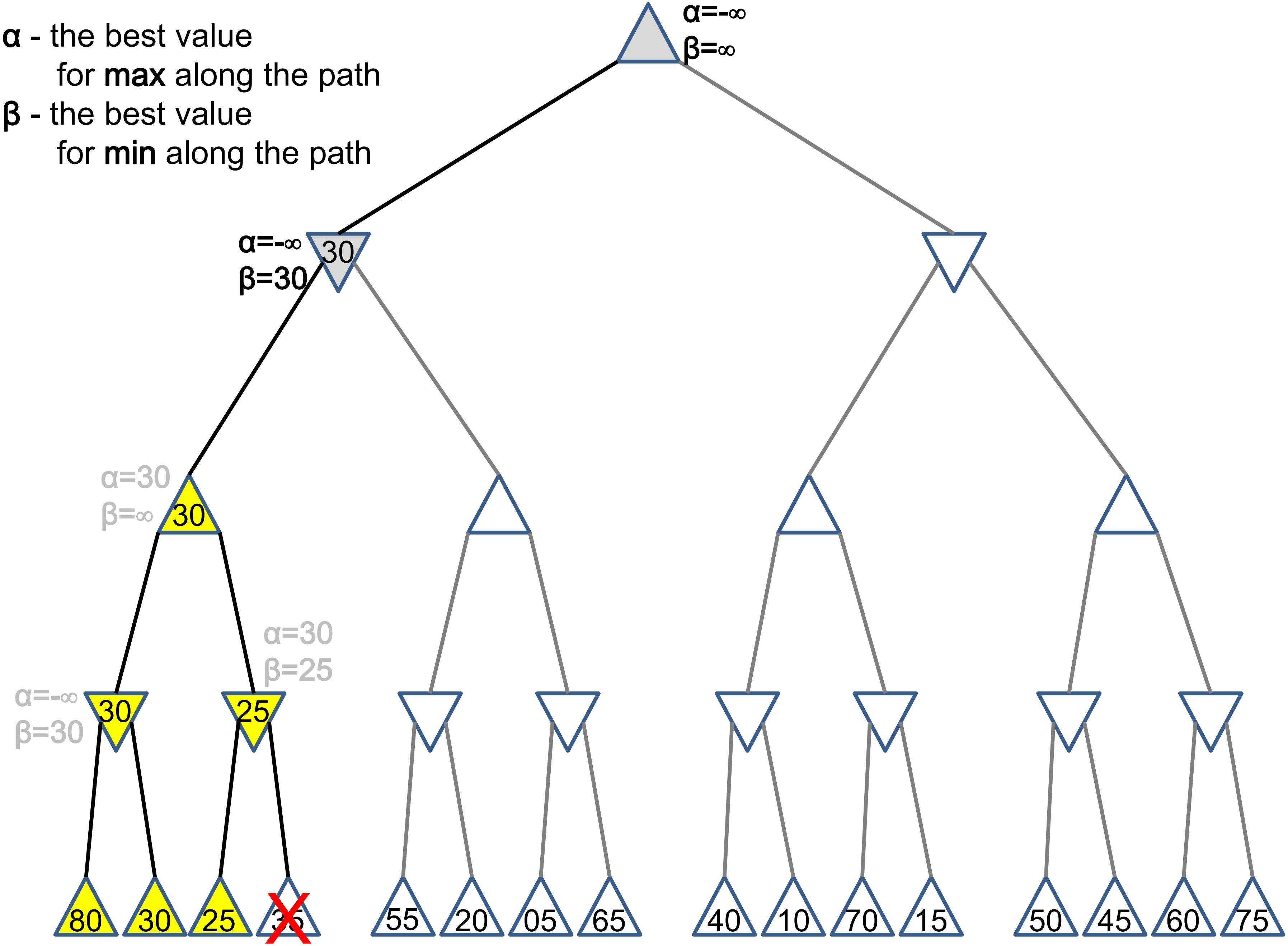

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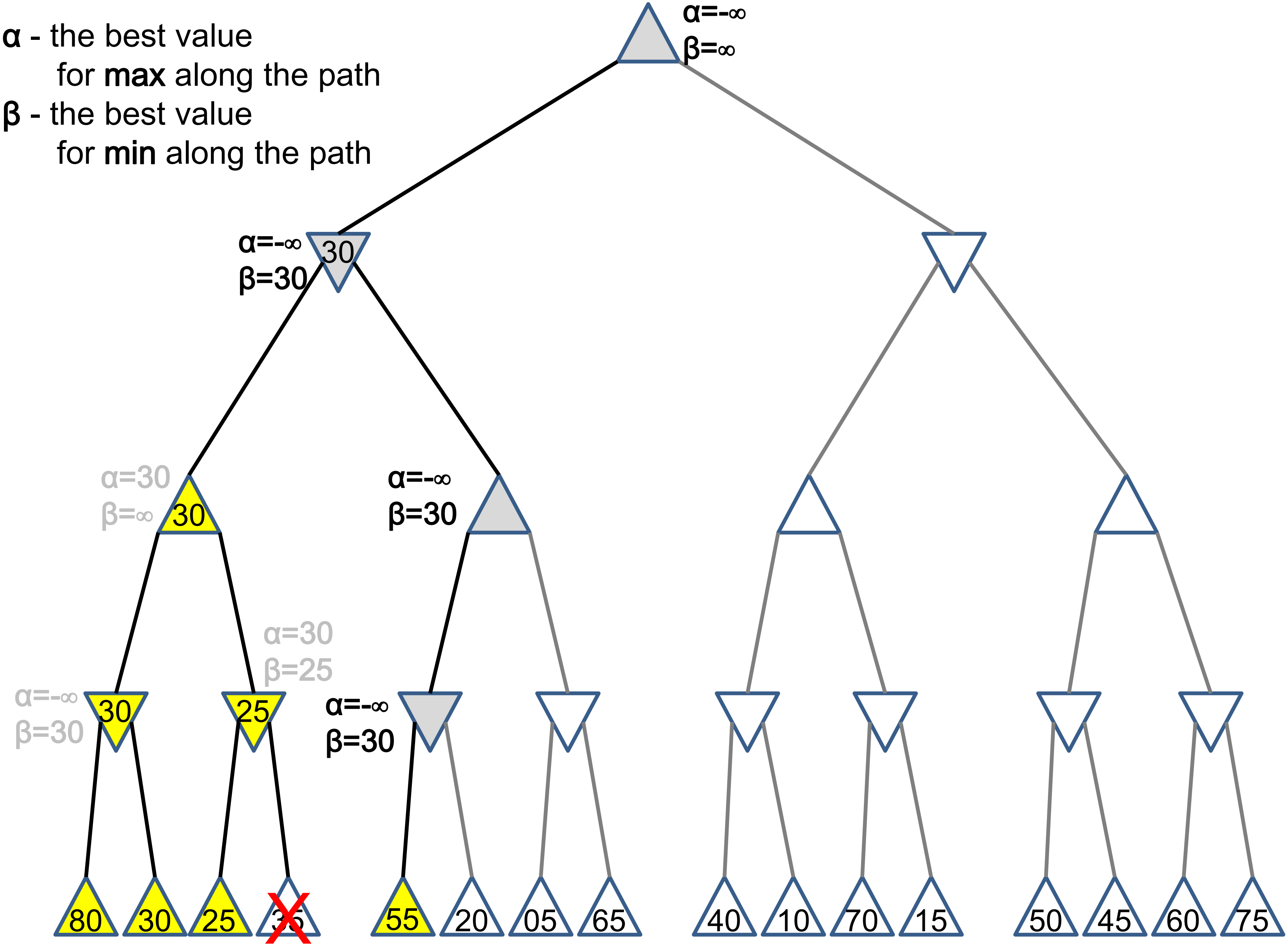

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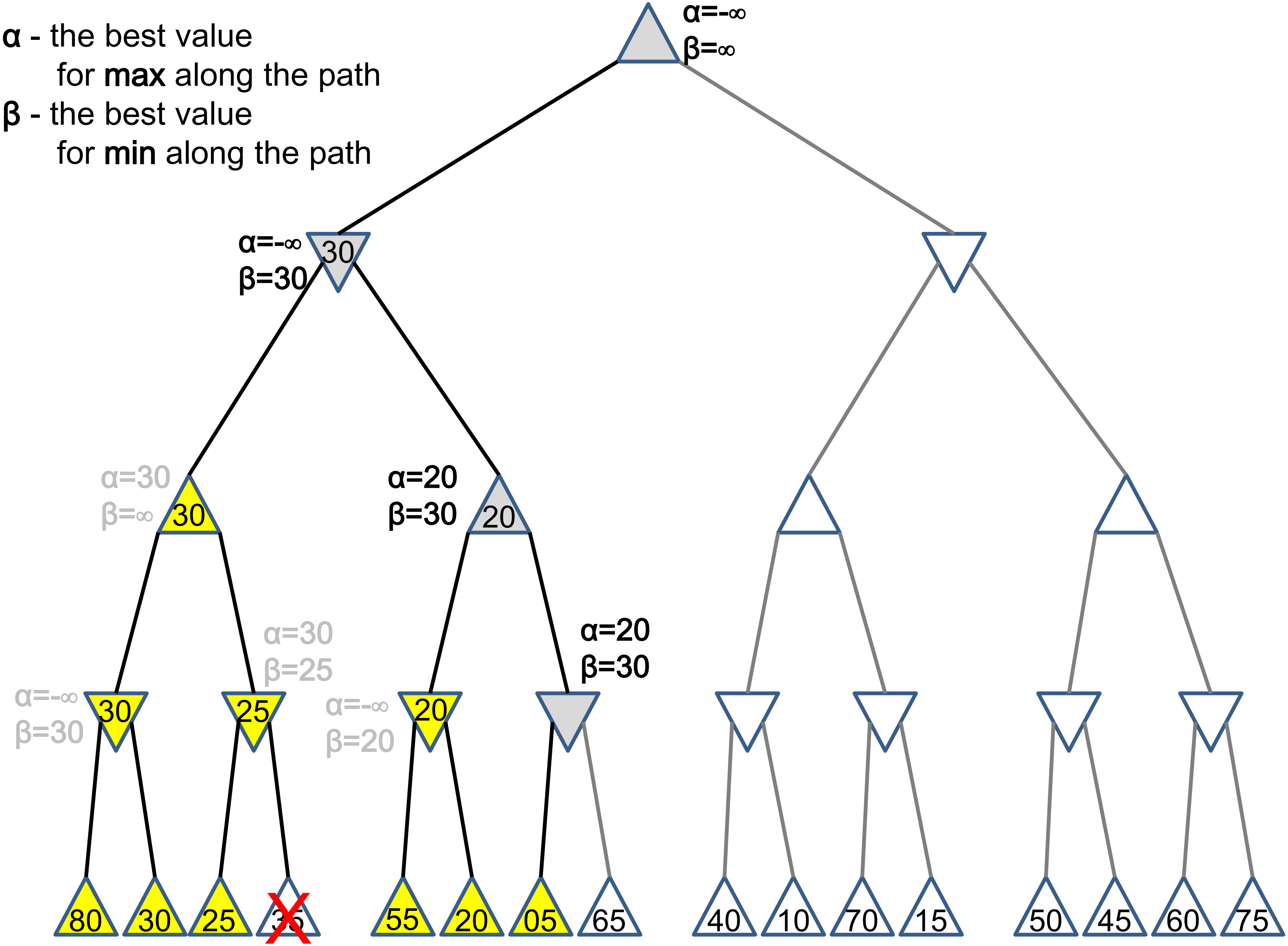

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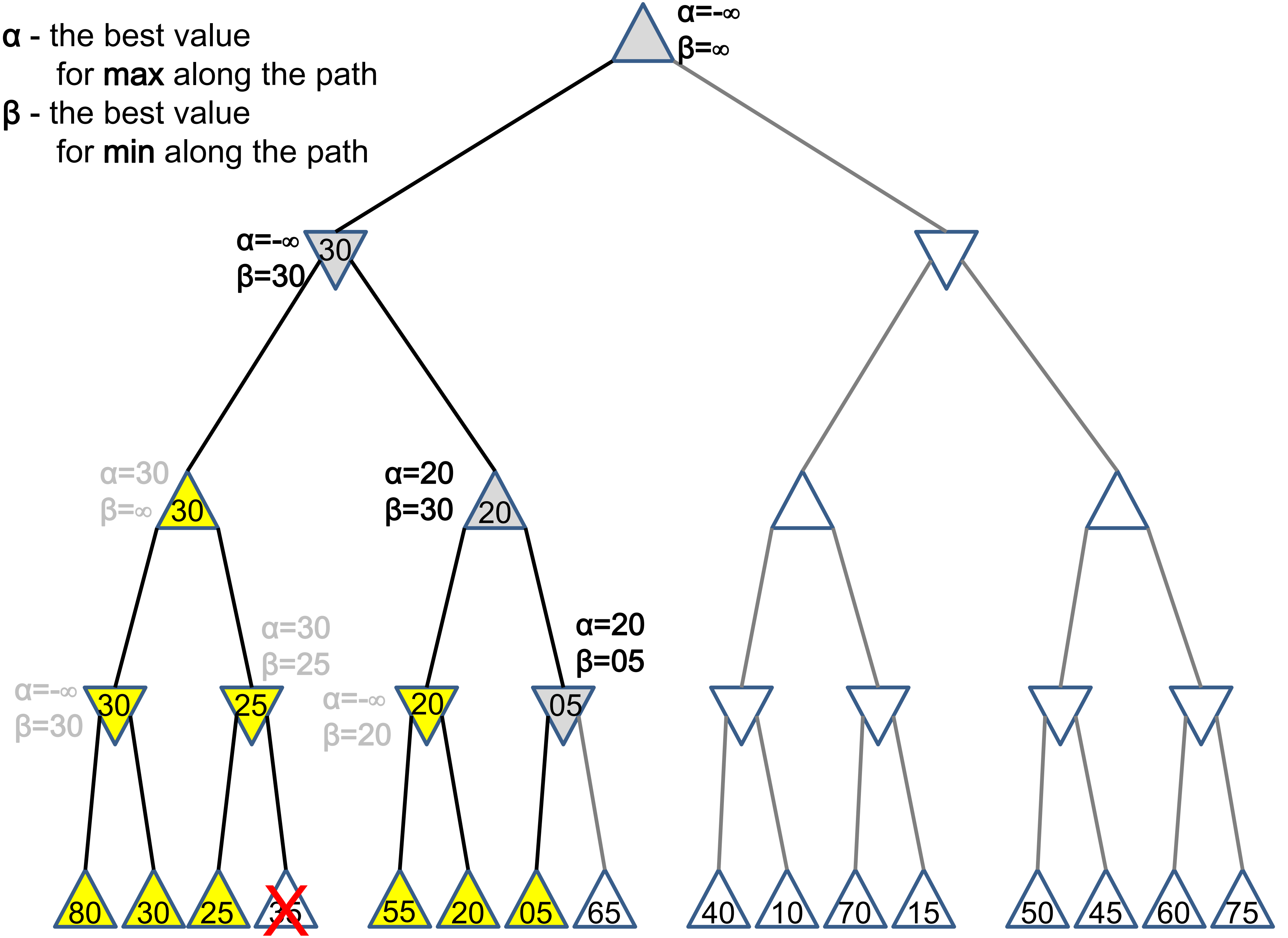

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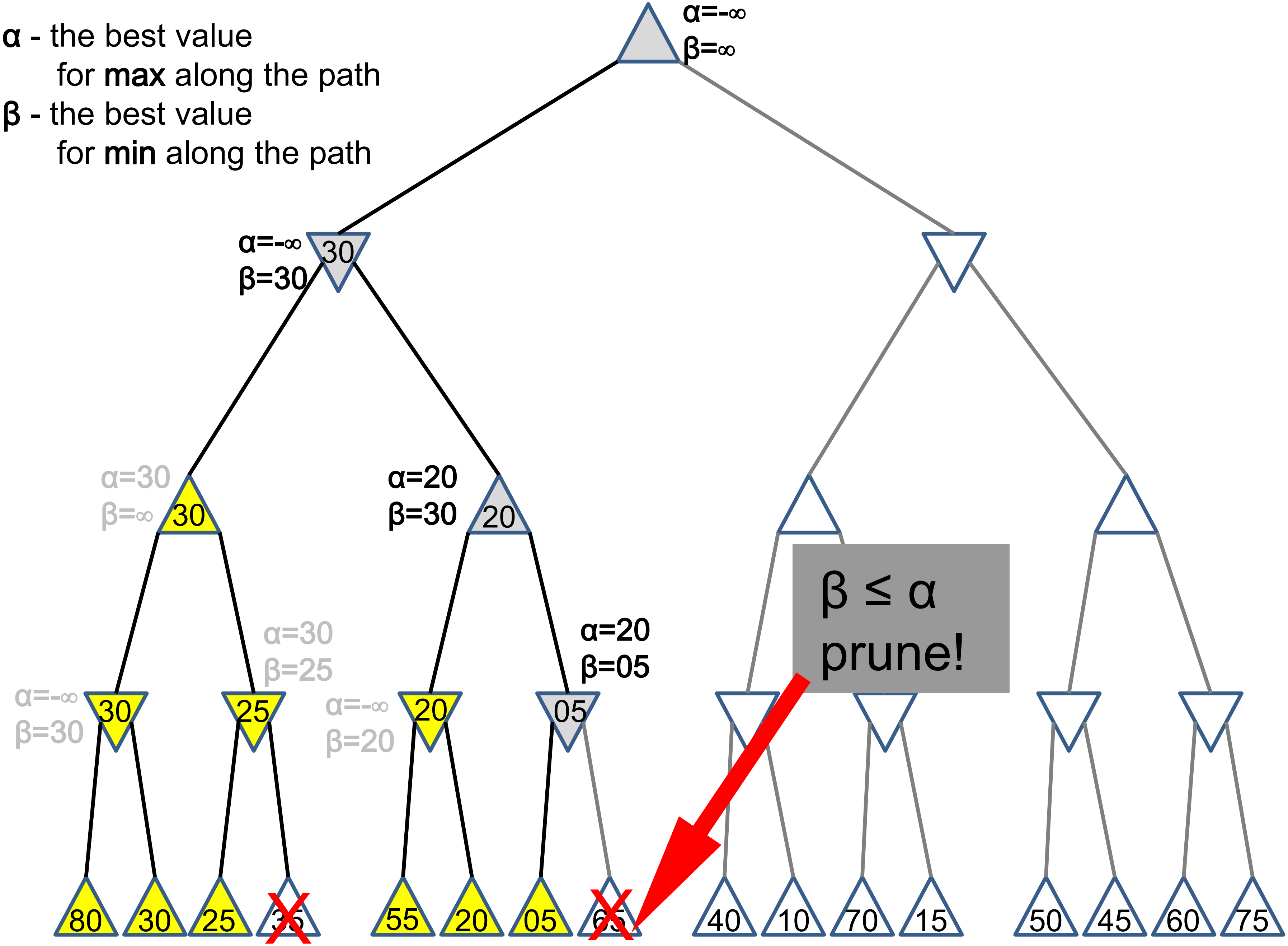

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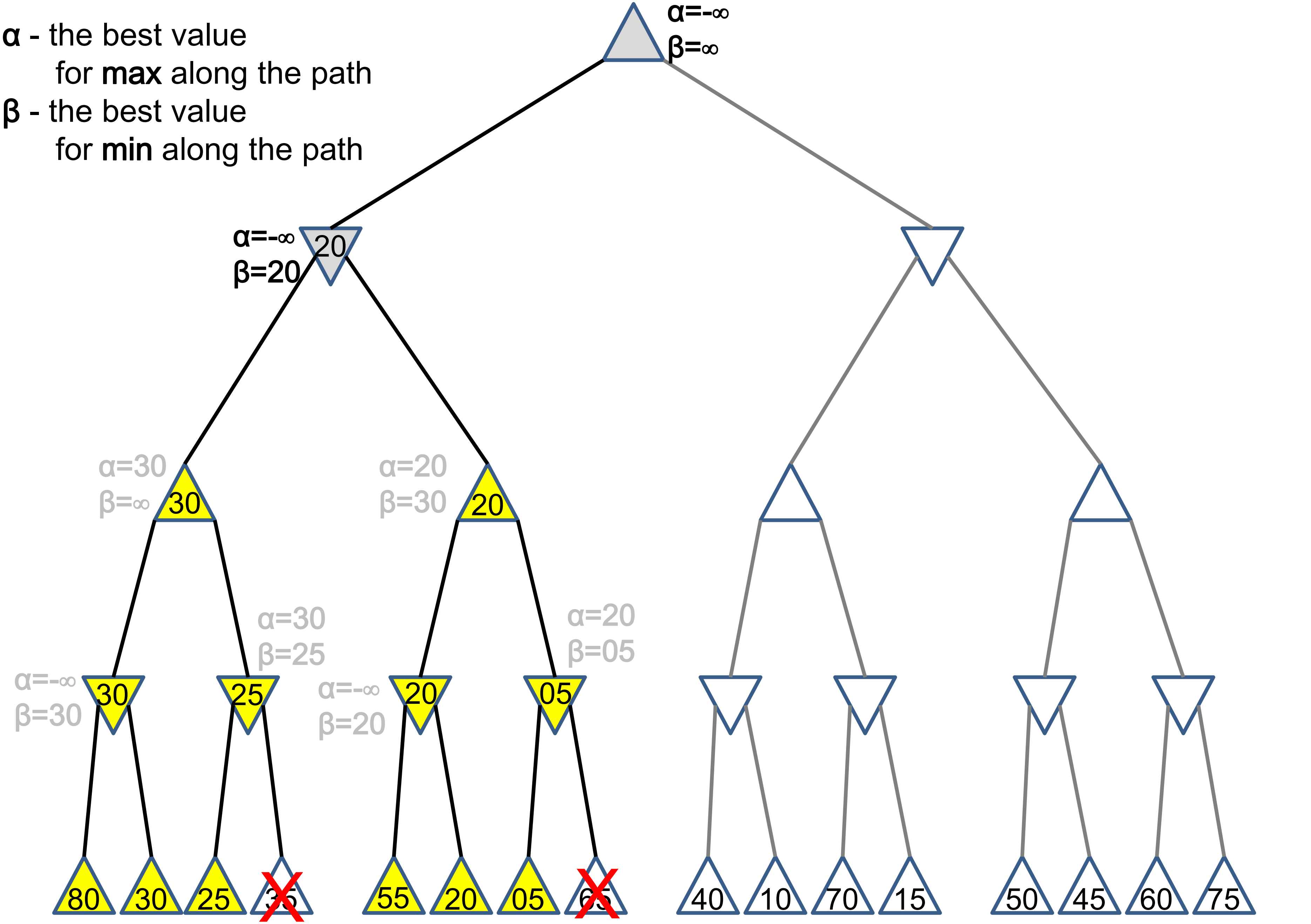

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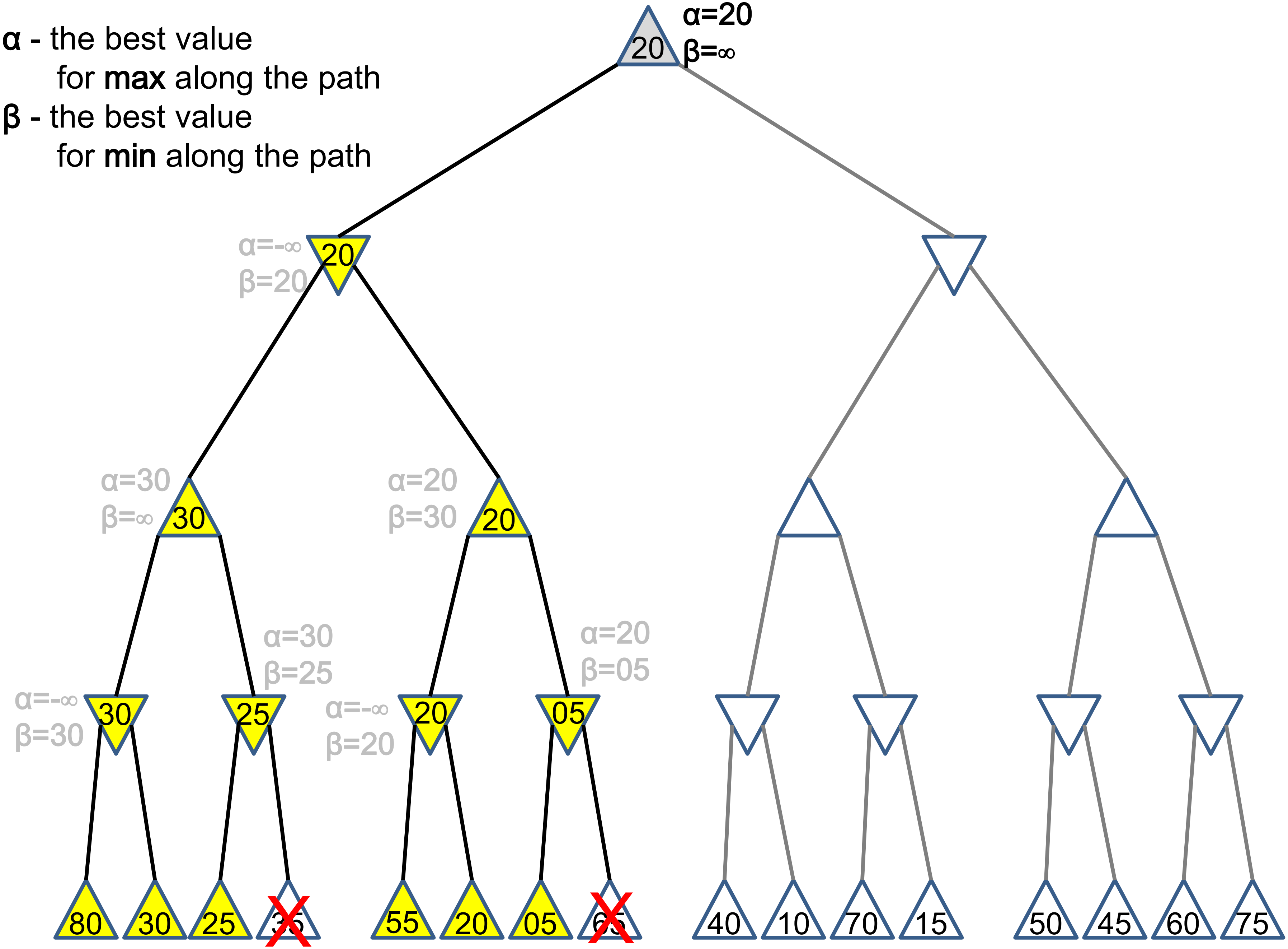

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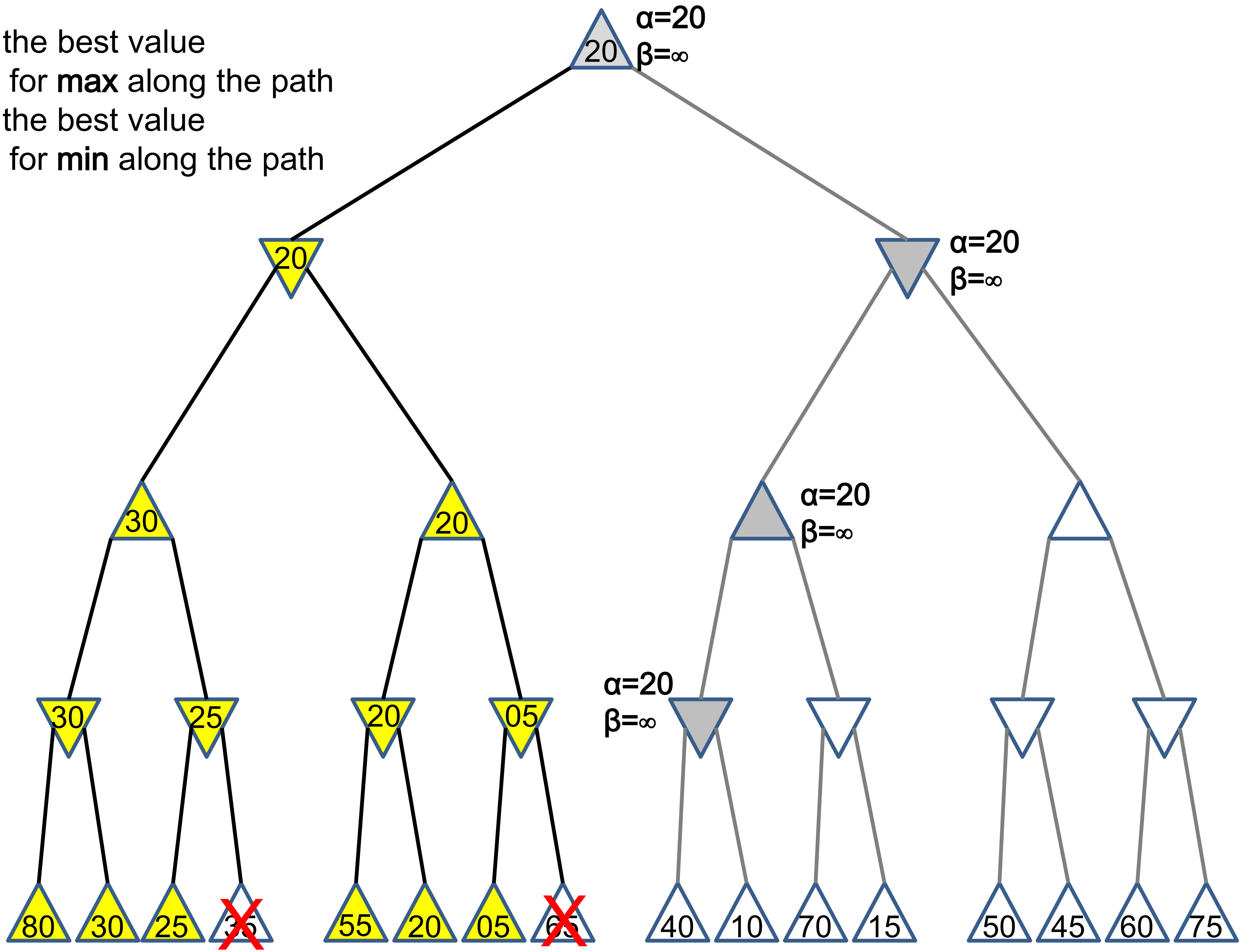

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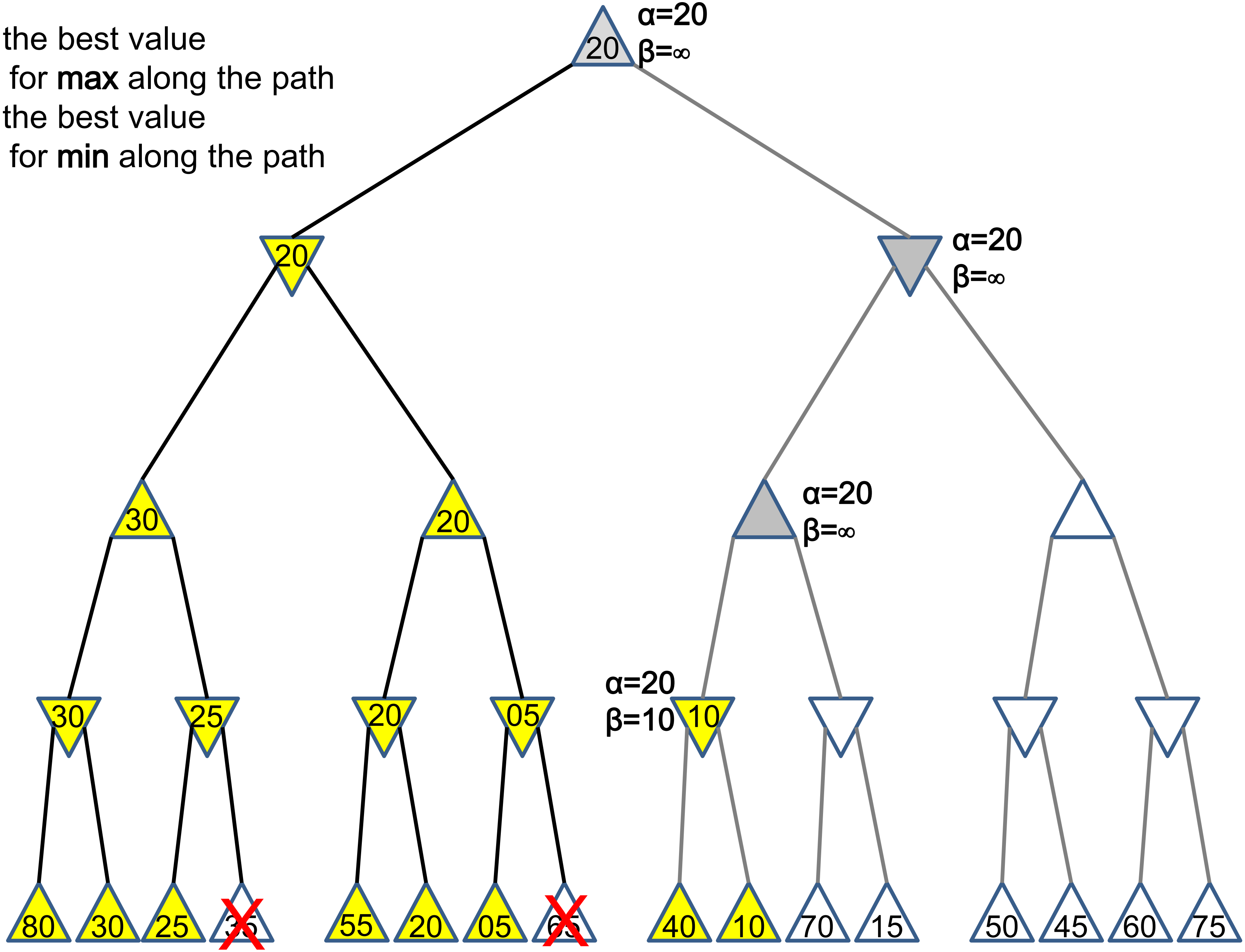

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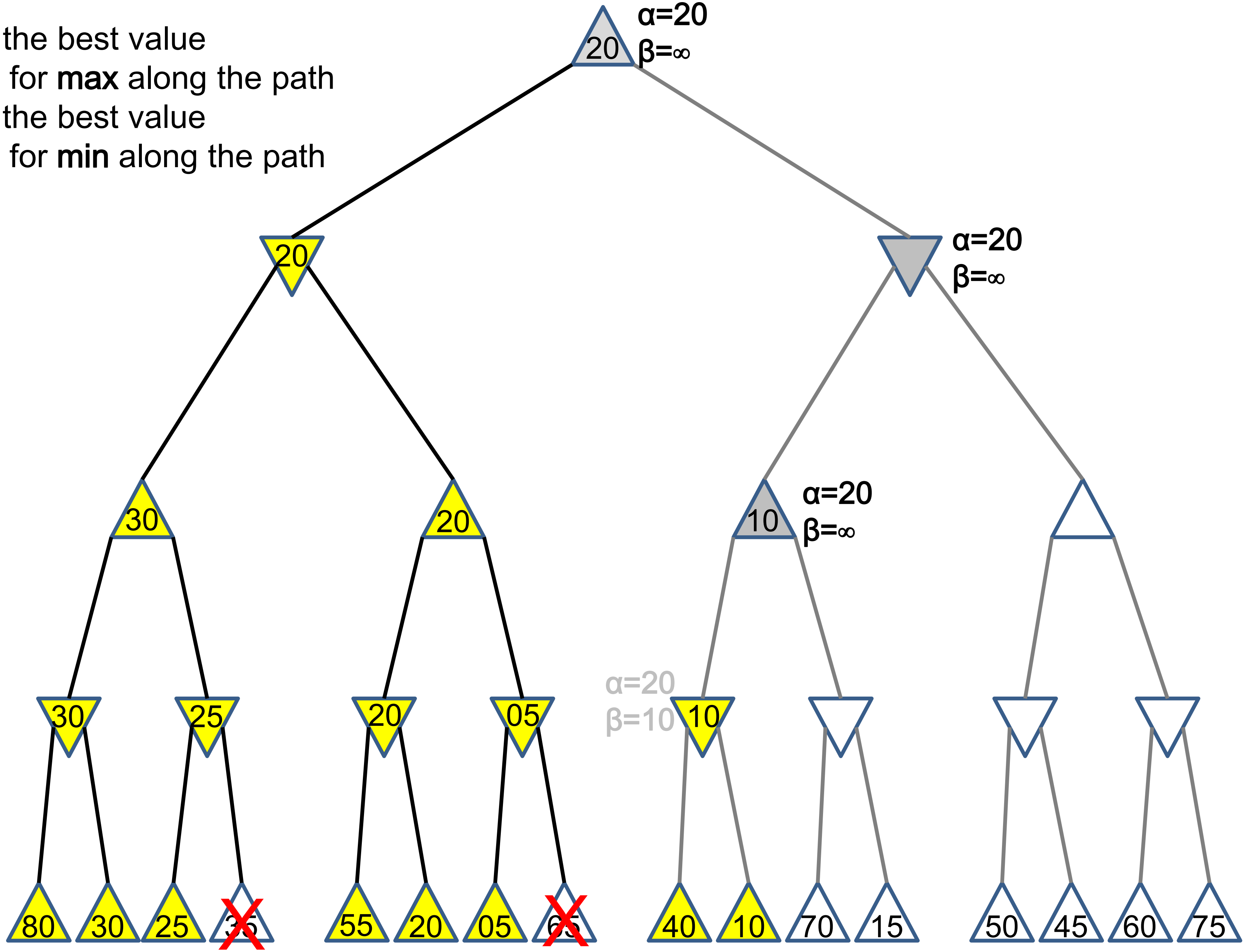

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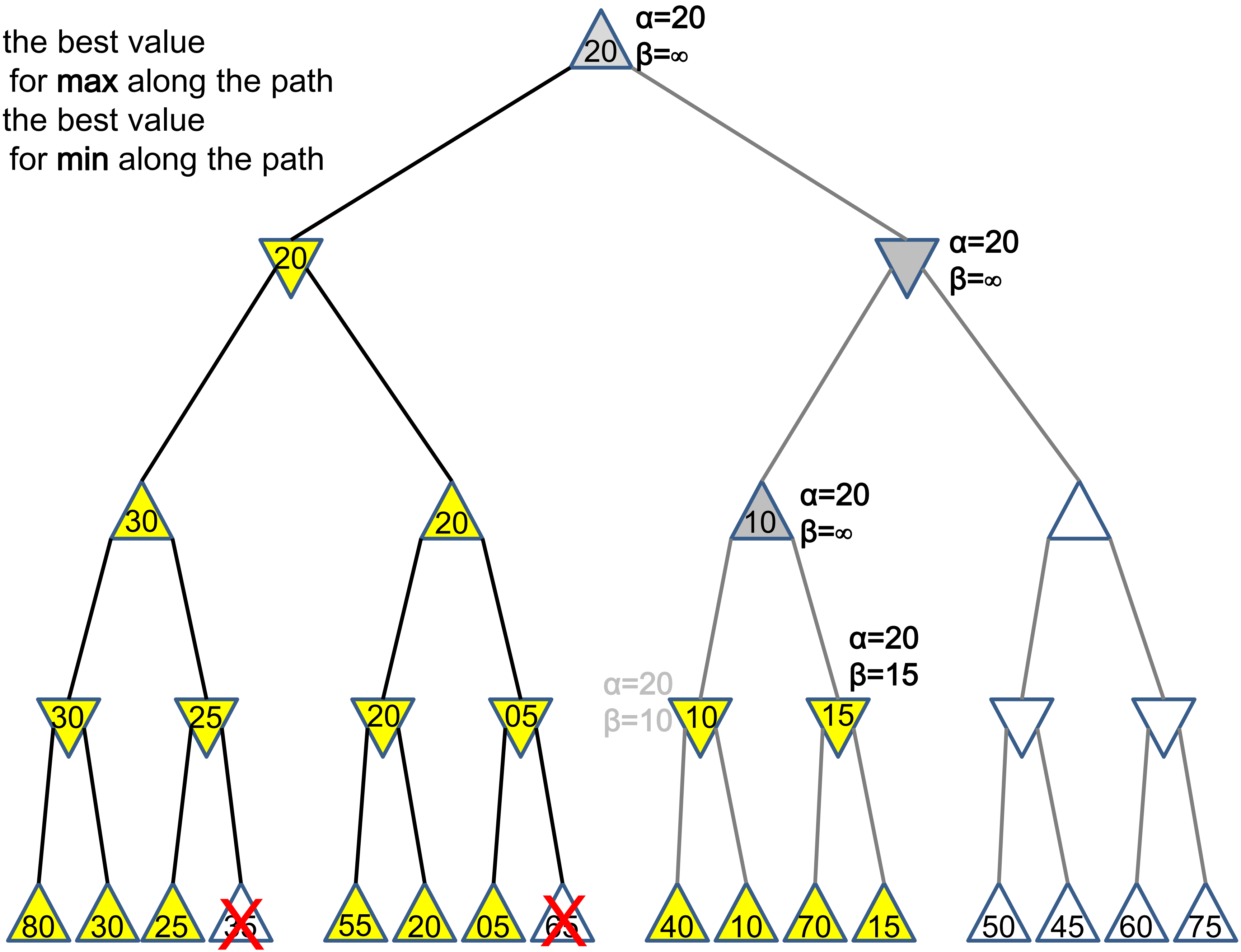

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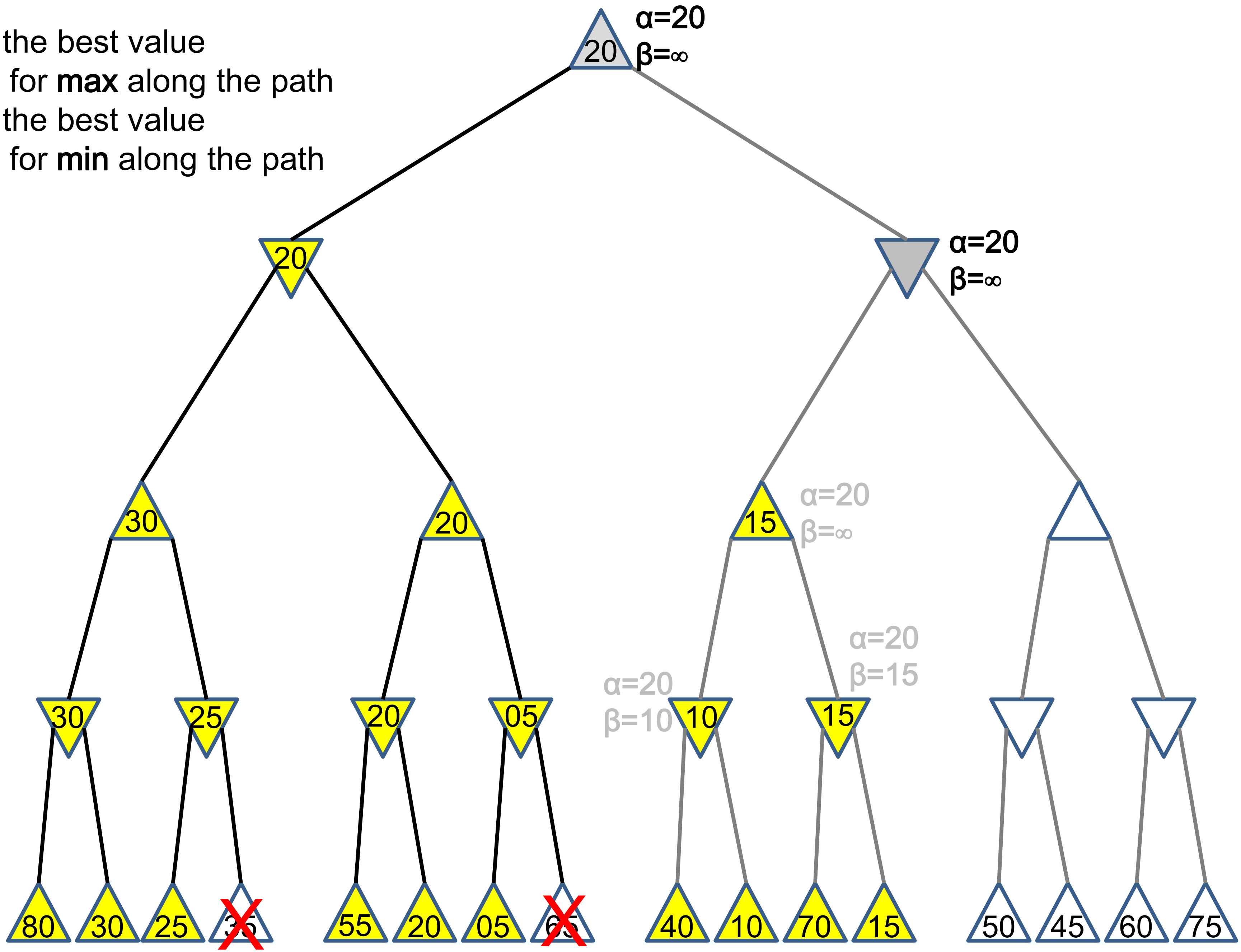

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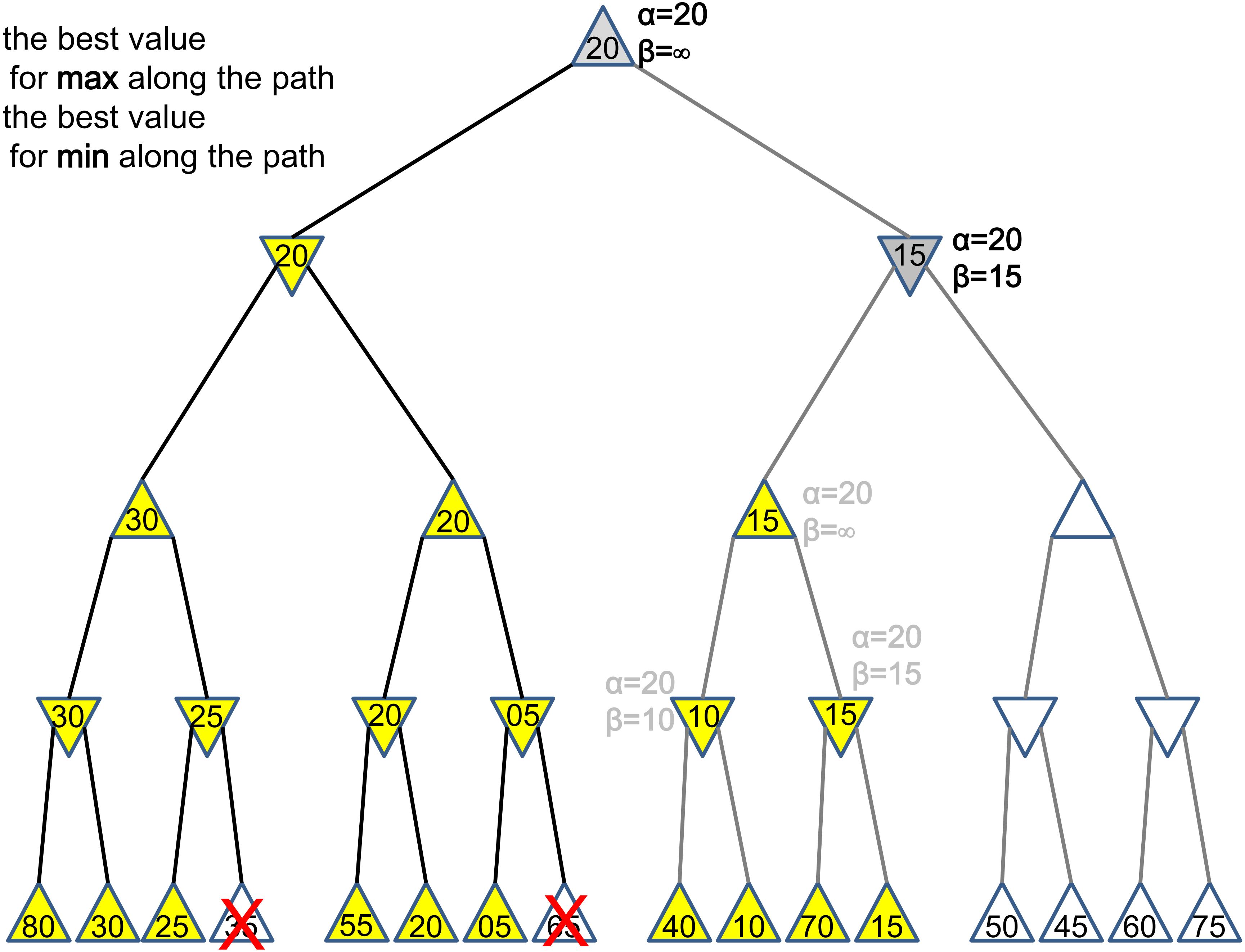
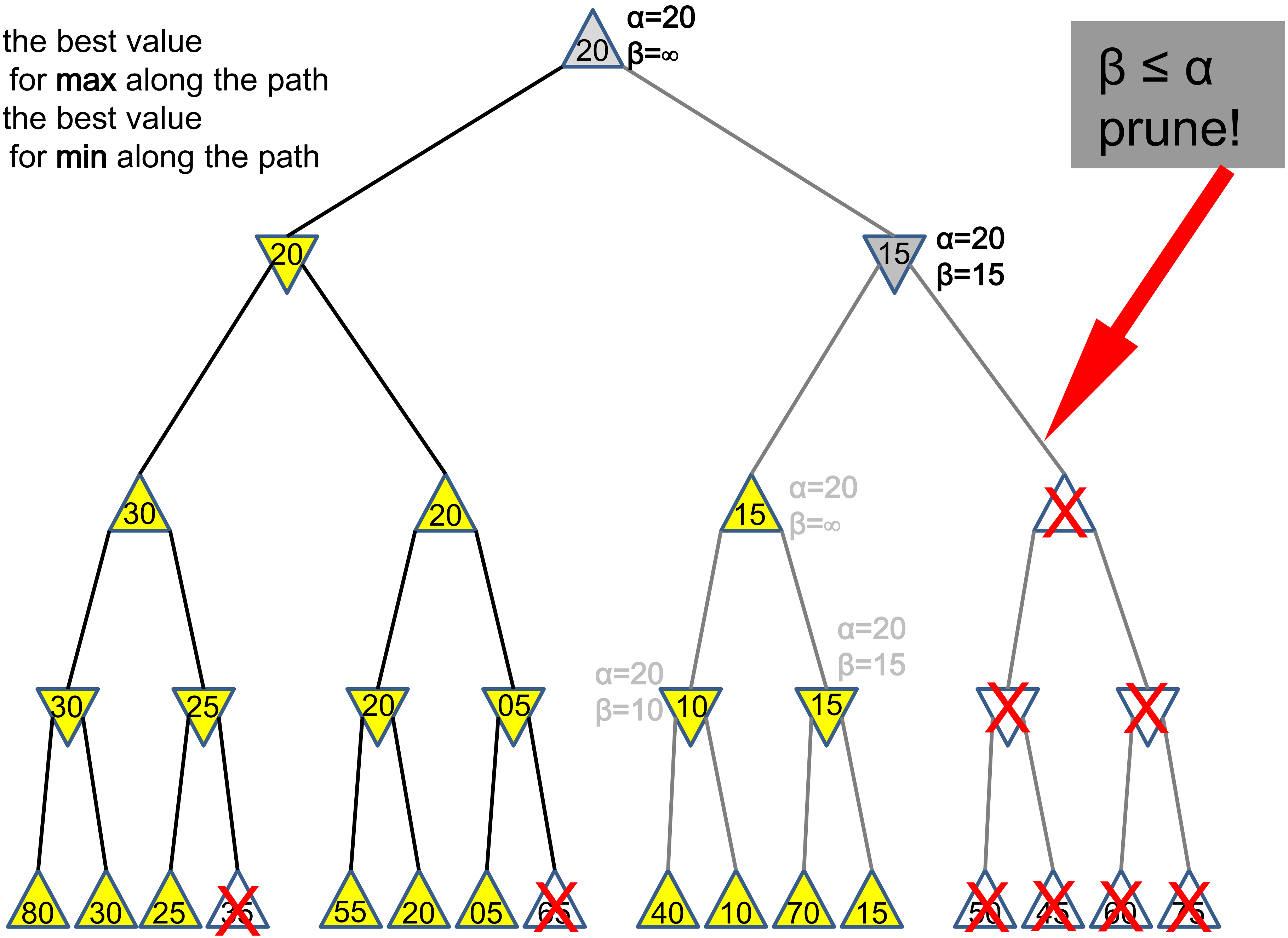


α - the best value
for **max** along the path

β - the best value
for **min** along the path



Bad and Good Cases for Alpha-Beta Pruning

- Bad: Worst moves encountered first

```
                  4                    MAX
         +--------------+--------------+
         2              3              4         MIN
     +----+----+    +----+----+    +----+----+
     6    4    2    7    5    3    8    6    4       MAX
   +--+ +--+ +--+ +-+-+ +--+ +--+ +--+ +--+ +--+--+
   6  5 4  3 2  1 1 3 7  4 5 2 3 8  2 1 6 1  2  4
```

- Good: Good moves ordered first

```
                  4                    MAX
         +--------------+--------------+
         4              3              2         MIN
     +----+----+    +----+----+    +----+----+
     4    6    8    3    x    x    2    x    x       MAX
   +--+ +--+ +--+ +--+            +-+-+
   4  2 6  x 8  x 3  2            1 2 1
```

- If we can order moves, we can get more benefit from alpha-beta pruning

Properties of α-β

- Pruning does not affect final result. This means that it gets the exact same result as does full minimax.
- Good move ordering improves effectiveness of pruning
- With "perfect ordering," time complexity = $O(b^{m/2})$
  → doubles depth of search
- A simple example of reasoning about ‘which computations are relevant’ (a form of metareasoning)

Good Enough?

- Chess:
  - branching factor $b \approx 35$
  - game length $m \approx 100$
  - search space $b^{m/2} \approx 35^{50} \approx 10^{77}$

The universe can play chess - can we?

- The Universe:
  - number of atoms $\approx 10^{78}$
  - age $\approx 10^{18}$ seconds
  - 10^{8} moves/sec x 10^{78} x 10^{18} = 10^{104}