#### CS3642-01 Programming Assignment #2 (Fall 2023)

**Due: October 13, 2023 (11:30PM)** 

To implement Breadth-First Search (BFS), Uniform-Cost Search (UCS), Best-First Search (BFS), and A\* Algorithm to solve the following 8-puzzle problem (i.e. find the goal):

## 8-puzzle Problem:

The 8-puzzle consists of eight numbered, movable tiles set in a 3x3 frame. One cell of the frame is always empty thus making it possible to move an adjacent numbered tile into the empty cell. Start with a random state (cannot be fixed). The goal state is listed below.

1	2	3
8		4
7	6	5

The program is to change the initial configuration into the goal configuration. A solution to the problem is an appropriate sequence of moves. You must write your own codes for the algorithms. Make sure your submission meets all of the requirements and free of plagiarism.

Your program should be able to address any initial configuration and provide a table of statistics below in your PDF file.

Algorithm	Average number of nodes visited (you need repeat each algorithm several times with different initial configuration)	Average run time in your program	Your comment on these algorithms
<b>Breadth First</b>			
Search			
<b>Uniform Cost</b>			
Search			
Best First			
Search			
A*using			
Nilsson's			
sequence score			

You may write your code in a contemporary language of your choice; typical languages would include C/C++, Python, Java, Ada, Pascal, Smalltalk, Lisp, and Prolog. A GUI interface is preferred.

- 1. Submit a PDF file of your well-commented source program, your design and your printed outputs (screen shots). Please include your codes in your PDF file. It is plagiarism to take any codes from the website or others. Try to understand the algorithm and implement the algorithm by your own. You must have all the information required in your PDF file.
- 2. Provide a video presentation of your programming assignment in MP3, YouTube, or any media.
- 3. Please upload items 1) and 2) above separately to D2L.

## 4. Restriction: No zipped files.

Adding the following two sections (I and II) at the beginning of your PDF including your code and outputs.

# I. Your Information:

// Course: Artificial

Intelligence

// Student name: Raehyeong Lee

// Student ID: 000996758 // Assignment #: Assignment 2

// Assignment #. Assignment 2 // Due Date: October 13, 2023

// Signature: Rashyeong Lee (Your signature assures that everything is your own work. Required.)

// Score: (Note: Score will be posted on D2L)

# II. The statistics table:

Algorithm	Average number of nodes visited (you need repeat each algorithm several times with different initial configuration)	Average run time in your experiments	Your comment on these algorithms
Breadth First Search	20+23+3+25+13+14=16.3	6	This must be the largest but the results was not working in right way.
Uniform Cost Search	20+23+3+25+13+14=16.3	6	It seems UCS and A* algorithms are not operating properly. The result of the algorithms is always same with BFS above.
Best First Search	60+63+3+75+95+16=52	6	This algorithm must be shortest average but, in my program, it has the biggest average number of nodes moving.
A*using Nilsson's sequence score	20+23+3+25+13+14=16.3	6	It seems UCS and A* algorithms are not operating properly. The result of the algorithms is always same with BFS above.

#### 4th I/O 1st I/O **Initial State: Initial State:** 063 123 154 8 4 5 287 706 Breadth-First Search Result: **Breadth-First Search Result:** Number of moves: 20 Number of moves: 3 **Uniform-Cost Search Result: Uniform-Cost Search Result:** Number of moves: 3 Number of moves: 20 **Best-First Search Result:** Best-First Search Result: Number of moves: 60 Number of moves: 3 A\* Algorithm Result: A\* Algorithm Result: Number of moves: 20 Number of moves: 3 2<sup>nd</sup> I/O 5th I/O **Initial State: Initial State:** 167 136 053 845 284 702 Breadth-First Search Result: Breadth-First Search Result: Number of moves: 23 Number of moves: 13 **Uniform-Cost Search Result: Uniform-Cost Search Result:** Number of moves: 23 Number of moves: 13 **Best-First Search Result:** Best-First Search Result: Number of moves: 63 Number of moves: 95 A\* Algorithm Result: A\* Algorithm Result: Number of moves: 13 Number of moves: 23 3rd I/O 6th I/O **Initial State: Initial State:** 253 8 1 7 3 5 0 164 246 087 Breadth-First Search Result: Breadth-First Search Result:

Breadth-First Search Result:

Number of moves: 25

Uniform-Cost Search Result:

Number of moves: 24

Uniform-Cost Search Result:

Number of moves: 25

Number of moves: 14

Best-First Search Result:

Number of moves: 75

Number of moves: 16

A\* Algorithm Result:

A\* Algorithm Result:

Number of moves: 14

Number of moves: 25

#### **Program Codes**

```
goal\_state = [[1, 2, 3], [8, 0, 4], [7, 6, 5]]
movements = [(-1, 0), (1, 0), (0, -1), (0, 1)]
def find_empty_tile(state):
    for j in range(3):
       if state[i][j] == 0:
          return i, j
def is_valid(state):
 return all(0 \le \text{state}[i][j] \le 9 \text{ for } i \text{ in range}(3) \text{ for } j \text{ in range}(3))
def print_state(state):
 for row in state:
    print(" ".join(map(str, row)))
  print()
def bfs(initial_state):
  queue = [(initial_state, [])]
  visited = set()
  while queue:
    state, path = queue.pop(0)
    if state == goal_state:
       return path
    empty_i, empty_j = find_empty_tile(state)
    for move_i, move_j in movements:
       new_i, new_j = empty_i + move_i, empty_j + move_j
       if 0 \le \text{new}_i \le 3 and 0 \le \text{new}_j \le 3:
          new_state = copy.deepcopy(state)
          new_state[empty_i][empty_j], new_state[new_i][new_j] = new_state[new_i][new_j], new_state[empty_i][empty_j]
          if tuple(map(tuple, new_state)) not in visited and is_valid(new_state):
            queue.append((new_state, path + [(new_i, new_j)]))
            visited.add(tuple(map(tuple, new_state)))
def ucs(initial_state):
  priority_queue = [(0, initial_state, [])]
  visited = set()
  while priority_queue:
    cost, state, path = heapq.heappop(priority_queue)
    if state == goal_state:
       return path
    empty_i, empty_j = find_empty_tile(state)
    for move_i, move_j in movements:
       new_i, new_j = empty_i + move_i, empty_j + move_j
       if 0 \le \text{new}_i \le 3 and 0 \le \text{new}_j \le 3:
          new_state = copy.deepcopy(state)
          new\_state[empty\_i][empty\_j], new\_state[new\_i][new\_j] = new\_state[new\_i][new\_j], new\_state[empty\_i][empty\_j]
          if tuple(map(tuple, new_state)) not in visited and is_valid(new_state):
            new_cost = cost + 1
             heapq.heappush(priority_queue, (new_cost, new_state, path + [(new_i, new_j)]))
            visited.add(tuple(map(tuple, new_state)))
def heuristic(state):
  total\_distance = 0
  for i in range(3):
    for j in range(3):
       if state[i][j] != 0:
```

```
target_i, target_j = divmod(state[i][j] - 1, 3)
          total_distance += abs(target_i - i) + abs(target_j - j)
  return total_distance
def best_first_search(initial_state, heuristic):
  priority_queue = [(heuristic(initial_state), initial_state, [])]
  visited = set()
  while priority_queue:
     _, state, path = heapq.heappop(priority_queue)
     if state == goal_state:
       return path
     empty_i, empty_j = find_empty_tile(state)
     for move_i, move_j in movements:
       new\_i,\,new\_j = empty\_i + move\_i,\,empty\_j + move\_j
       if 0 \le \text{new}_i < 3 and 0 \le \text{new}_j < 3:
          new_state = copy.deepcopy(state)
          new_state[empty_i][empty_j], new_state[new_i][new_j] = new_state[new_i][new_j], new_state[empty_i][empty_j]
          if tuple(map(tuple, new_state)) not in visited and is_valid(new_state):
             heapq.heappush(priority\_queue, (heuristic(new\_state), new\_state, path + [(new\_i, new\_j)]))
             visited.add(tuple(map(tuple, new_state)))
def nilsson_sequence_score(state):
  score = 0
     for j in range(3):
       if state[i][j] != 0:
          target_i, target_j = divmod(state[i][j] - 1, 3)
          if (i, j) != (target_i, target_j):
             score += 2
            if (i, j) != (0, 0) and state[i][j] == goal_state[i][j]:
            elif(i, j) != (0, 1) and state[i][j] == goal\_state[i][j]:
  return score
def a_star_nilsson(initial_state):
  priority_queue = [(nilsson_sequence_score(initial_state) + 0, initial_state, [])]
  visited = set()
  while priority_queue:
     _, state, path = heapq.heappop(priority_queue)
     if state == goal_state:
       return path
     empty_i, empty_j = find_empty_tile(state)
     for move_i, move_j in movements:
       new_i, new_j = empty_i + move_i, empty_j + move_j
       if 0 \le \text{new_i} < 3 and 0 \le \text{new_j} < 3:
          new_state = copy.deepcopy(state)
          new\_state[empty\_i][empty\_j], new\_state[new\_i][new\_j] = new\_state[new\_i][new\_j], new\_state[empty\_i][empty\_j]
          if tuple(map(tuple, new_state)) not in visited and is_valid(new_state):
             new_cost = len(path) + 1
            heapq.heappush(priority\_queue, (new\_cost + nilsson\_sequence\_score(new\_state), new\_state, path + [(new\_i, new\_j)])) \\
            visited.add(tuple(map(tuple, new_state)))
initial\_state = [[2, 5, 3], [1, 6, 4], [0, 8, 7]]
print("Initial State:")
print_state(initial_state)
print("Breadth-First Search Result:")
bfs_result = bfs(initial_state)
print("Number of moves:", len(bfs_result))
for move in bfs_result:
 print(move)
print("Uniform-Cost Search Result:")
ucs_result = ucs(initial_state)
print("Number of moves:", len(ucs_result))
for move in ucs_result:
```

```
print(move)
print("Best-First Search Result:")
best_first_search_result = best_first_search(initial_state, heuristic)
print("Number of moves:", len(best_first_search_result))
for move in best_first_search_result:
    print(move)
print("A* Algorithm Result:")
a_star_result = a_star_nilsson(initial_state)
print("Number of moves:", len(a_star_result))
for move in a_star_result:
    print(move)
```

## **Screenshots of Output examples**

```
Initial State:
2 8 3
1 6 4
7 0 5
0 6 3
1 5 4
2 8 7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    Number of moves: 5
(1, 1)
(0, 1)
(0, 0)
(1, 0)
(1, 1)
   Breadth-First Search (BFS) Result:
(1, 0)
(2, 0)
(2, 1)
(1, 1)
(0, 1)
(0, 2)
(1, 2)
(1, 1)
(2, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        Number of moves: 5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    (1, 1)
(0, 1)
(0, 0)
(1, 0)
(1, 1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    (2, 2)
(1, 2)
(1, 1)
(0, 1)
(0, 0)
(1, 0)
(1, 1)
(2, 1)
(2, 2)
(1, 2)
(1, 1)
(2, 1)
(2, 2)
(1, 1)
(2, 1)
(2, 2)
(1, 1)
   (1, 1)
   Uniform-Cost Search (UCS) Result:
   Number of moves: 20
   (1, 0)
(2, 0)
(2, 1)
(1, 1)
(0, 1)
(0, 2)
(1, 2)
(1, 1)
(2, 1)
(2, 2)
(1, 1)
(2, 0)
(2, 0)
(2, 1)
(2, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1, 2)
(1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        A* Algorithm Result:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        Number of moves: 5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    Number of moves: 5
(1, 1)
(0, 1)
(0, 0)
(1, 0)
(1, 1)
PS C:\Users\lrh14>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              A* Algorithm Result:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           Number of moves: 20
   Best-First Search Result:
```

```
Initial State:

2 8 3
1 6 4
7 0 5

Breadth-First Search (BFS) Result:
Number of moves: 5
(1, 1)
(0, 0)
(1, 0)
(1, 1)
Uniform-Cost Search (UCS) Result:
Number of moves: 5
(1, 1)
(0, 0)
(1, 0)
(1, 1)
Best-First Search Result:
Number of moves: 15
(2, 2)
(1, 1)
(0, 0)
(1, 1)
(0, 1)
(0, 0)
(1, 1)
(2, 1)
(2, 2)
(1, 1)
(2, 1)
(2, 2)
(1, 1)
(2, 1)
(2, 2)
(1, 1)
(2, 1)
(2, 2)
(1, 1)
(2, 1)
(2, 2)
(1, 1)
(2, 1)
(2, 1)
(2, 2)
(1, 1)
(2, 1)
(2, 2)
(1, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(3, 1)
(4, 1)
(5, 1)
(6, 0)
(1, 0)
(1, 1)
(9, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 1)
(2, 1)
(2, 1)
(2, 2)
(1, 2)
(1, 1)
(2, 1)
(2, 2)
(1, 2)
(1, 1)
(2, 1)
(2, 2)
(1, 2)
(1, 1)
(2, 1)
(2, 1)
(2, 2)
(1, 2)
(1, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 2)
(1, 2)
(1, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 2)
(1, 2)
(1, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 2)
(1, 2)
(1, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(2, 1)
(3, 1)
(4, 1)
(5, 1)
(6, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
(1, 0)
```

```
A* Algorithm Result:
Number of moves: 20
(0, 1)
(0, 2)
(1, 1)
(1, 0)
(2, 0)
(2, 1)
(2, 2)
(1, 1)
(1, 2)
(0, 1)
(1, 1)
(1, 0)
(0, 0)
(0, 1)
(1, 1)
(1, 0)
(2, 0)
(2, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
```

```
Initial State:
2 5 3
1 6 4
8 8 7
Breadth-First Search (BFS) Result:
Number of moves: 14
(2, 1)
(2, 2)
(3, 1)
(4, 0)
(2, 0)
(2, 1)
(4, 1)
(4, 1)
(5, 2)
(2, 1)
(4, 1)
(5, 2)
(2, 1)
(5, 1)
(6, 0)
(7, 0)
(8, 0)
(8, 0)
(9, 0)
(1, 1)
(1, 1)
(1, 1)
(1, 1)
(2, 2)
(2, 1)
(3, 1)
(4, 1)
(5, 0)
(7, 0)
(8, 0)
(8, 0)
(9, 0)
(1, 0)
(1, 0)
(2, 0)
(2, 1)
(3, 1)
(4, 1)
(5, 1)
(6, 1)
(7, 1)
(8, 0)
(8, 0)
(9, 0)
(1, 0)
(1, 0)
(2, 0)
(2, 1)
(3, 1)
(4, 1)
(5, 2)
(5, 1)
(6, 1)
(7, 2)
(8, 1)
(9, 0)
(9, 0)
(1, 0)
(1, 0)
(2, 0)
(2, 0)
(2, 1)
(3, 1)
(4, 1)
(5, 2)
(5, 1)
(6, 1)
(7, 2)
(8, 1)
(9, 1)
(9, 0)
(9, 0)
(1, 0)
(2, 0)
(2, 1)
(3, 1)
(4, 1)
(5, 2)
(5, 1)
(6, 1)
(7, 2)
(8, 1)
(9, 0)
(9, 0)
(1, 0)
(1, 0)
(2, 0)
(2, 1)
(3, 1)
(4, 1)
(5, 2)
(5, 1)
(6, 1)
(7, 2)
(7, 2)
(8, 1)
(9, 0)
(9, 0)
(9, 0)
(9, 0)
(1, 0)
(9, 0)
(1, 0)
(1, 0)
(1, 0)
(2, 1)
(3, 1)
(4, 1)
(5, 1)
(5, 1)
(6, 1)
(7, 1)
(8, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1)
(9, 1
```