Math

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1 Prelude

Some math notes for myself. You can tell I'm not great at this, as the good math people know these off the top of their heads :P

2 General Notes

- Area of circumcircle of a triangle: $R = \frac{abc}{4A}$
- Sum of roots of polynomial: $-\frac{b}{a}$
- Product of roots of polynomial of degree n: $-1^n \frac{z}{a}$
- Sum of squares of root of a polynomial: $-\frac{b+2c}{a}$
- Binomial theorem: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$
- Sum of cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- Difference of cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- Sum of squares: $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
- Sum of cubes: $\sum_{k=1}^{n} k^3 = (\frac{n(n+1)}{2})^2$
- Sum of fourth power: $\sum_{k}^{n} k^4 = \frac{n(2n+1)(n+1)(3n^2+3n-1)}{30}$
- Sum of fifth power: $\sum_{k}^{n} k^5 = \frac{n^2(2n^2+2n-1)(n+1)^2}{12}$
- Inequality of arithmetic and geometric means: $\sqrt[n]{x_1 x_2 \cdots x_n} \leq \frac{x_1+x_2+\cdots+x_n}{n}$
  - Direct consequence: $\sqrt{ab} \leq \frac{a+b}{2} \leq \sqrt{\frac{a^2+b^2}{2}}$
- Convex functions inequality: $f(ty + (1-t)x) \leq tf(y) + (1-t)f(x)$
- Jensen's Inequality: $f(\sum_{k=1}^{n} t_k x_k) \leq \sum_{k=1}^{n} t_k f(x_k)$ where $\sum_{k=1}^{n} t_k = 1$
  - Direct consequence: $f(\frac{\sum_{k=1}^{n} x_k}{n}) \leq \frac{\sum k=1^n f(x_k)}{n}$
- Volume of partial sphere: $V = \frac{\pi h^2}{3}(3r - h) = \frac{\pi h}{6}(3a^2 + h^2)$
- Curved surface area of partial sphere: $A = 2\pi rh$
- Surface area of cone: $A = \pi rs + \pi r^2$
- Surface area of sphere: $A = 4\pi r^2$
- Newton's method: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
- Logarithm inequality: $1 - \frac{1}{x} \leq \ln(x) \leq x - 1$ for all $x > 0$, or $\frac{x}{1+x} \leq \ln(1+x) \leq x$
- Sum of geometric series: $S_n = \frac{a(1-r^n)}{1-r}$

Function	Expansion	Region of convergence
$\frac{1}{x}$	$\sum_{k=0}^{\infty}(-1)^k(x-1)^k$	$\lvert x-1\rvert < 1$
$\frac{1}{x+1}$	$\sum_{k=0}^{\infty}(-1)^k(x)^k$	$\lvert x\rvert < 1$
$\ln(x)$	$\sum_{k=0}^{\infty}(-1)^k\frac{(x-1)^k}{k}$	$0 < x \leq 2$
e^x	$\sum_{k=0}^{\infty}\frac{x^k}{k!}$	$\mathbb{R}$
$\sin(x)$	$\sum_{k=0}^{\infty}(-1)^k\frac{x^{2n+1}}{(2n+1)!}$	
$\cos(x)$	$\sum_{k=0}^{\infty}(-1)^k\frac{x^{2n}}{(2n)!}$	

Function	Inverse derivative	Integral
$sin(x)$	$\frac{1}{\sqrt{1-x^2}}$	$cos(x)$
$cos(x)$	$\frac{-1}{\sqrt{1-x^2}}$	$-sin(x)$
$tan(x)$	$\frac{1}{1+x^2}$	$-ln\lvert cos(x)\rvert$
$sec(x)$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$ln\lvert sec(x)+tan(x)\rvert$
$csc(x)$	$\frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$	$-ln\lvert csc(x)+cot(x)\rvert$
$cot(x)$	$\frac{-1}{1+x^2}$	$ln\lvert sin(x)\rvert$

3 Taylor Series Expansions

4 Trigonometric Derivatives and Integrals

5 Hyperbolic Derivatives and Integrals

Function	Inverse	Inverse derivative	Integral
$sinh(x)$	$ln(x+\sqrt{x^2+1})$	$\frac{1}{\sqrt{x^2+1}}$	$cosh(x)$
$cosh(x)$	$ln(x+\sqrt{x^2-1}); x \geq 1$	$\frac{1}{\sqrt{x^2-1}}$	$sinh(x)$
$tanh(x)$	$\frac{1}{2}ln(\frac{1+x}{1-x}); \lvert x\rvert < 1$	$\frac{1}{1-x^2}$	$ln(cosh(x))$
$sech(x)$	$cosh(\frac{1}{x}); 0 < x \leq 1$	$\frac{-1}{x\sqrt{1-x^2}}$	$atan(sinh(x))$
$csch(x)$	$sinh(\frac{1}{x}); x \neq 0$	$\frac{-1}{\lvert x\rvert\sqrt{1+x^2}}$	$ln(tanh(\frac{x}{2}))$
$coth(x)$	$tanh(\frac{1}{x}); \lvert x\rvert > 1$	$\frac{1}{1-x^2}$	$ln(sinh(x))$

6 Integrals of powers of trigonometric functions

7 Trigonometric substitution

8 Tan half angle substitution

Hmmmm hmmm hmmm