

Math

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1 Prelude

Some math notes for myself. You can tell I'm not great at this, as the good math people know these off the top of their heads :P

2 General Notes

- Area of circumcircle of a triangle: $R = \frac{abc}{4A}$
- Sum of roots of polynomial: $-\frac{b}{a}$
- Product of roots of polynomial of degree n : $-1^n \frac{z}{a}$
- Sum of squares of root of a polynomial: $-\frac{b+2c}{a}$
- Binomial theorem: $(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$
- Sum of cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- Difference of cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- Sum of squares: $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$
- Sum of cubes: $\sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2}\right)^2$
- Sum of fourth power: $\sum_{k=1}^n k^4 = \frac{n(2n+1)(n+1)(3n^2+3n-1)}{30}$
- Sum of fifth power: $\sum_{k=1}^n k^5 = \frac{n^2(2n^2+2n-1)(n+1)^2}{12}$
- Inequality of arithmetic and geometric means: $\sqrt[n]{x_1 x_2 \cdots x_n} \leq \frac{x_1 + x_2 + \cdots + x_n}{n}$
 - Direct consequence: $\sqrt{ab} \leq \frac{a+b}{2} \leq \sqrt{\frac{a^2+b^2}{2}}$
- Convex functions inequality: $f(tx + (1-t)y) \leq tf(y) + (1-t)f(x)$
- Jensen's Inequality: $f(\sum_{k=1}^n t_k x_k) \leq \sum_{k=1}^n t_k f(x_k)$ where $\sum_{k=1}^n t_k = 1$
 - Direct consequence: $f\left(\frac{\sum_{k=1}^n x_k}{n}\right) \leq \frac{\sum_{k=1}^n f(x_k)}{n}$

3 Taylor Series Expansions

Function	Expansion	Region of convergence
$\frac{1}{x-1}$	$\sum_{k=0}^{\infty} (-1)^k (x-1)^k$	$ x-1 < 1$
$\frac{1}{x+1}$	$\sum_{k=0}^{\infty} (-1)^k x^k$	$ x < 1$
$\ln(x)$	$\sum_{k=0}^{\infty} (-1)^k \frac{(x-1)^{k+1}}{k+1}$	$0 < x \leq 2$
e^x	$\sum_{k=0}^{\infty} \frac{x^k}{k!}$	\mathbb{R}
$\sin(x)$	$\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$	
$\cos(x)$	$\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$	