# Calibration 2 - Superposition & Entanglement

Congratulations, the first part of calibration has now been completed!

Let's move on to measurement and entanglement, the cornerstone phenomena of quantum computing, and the second part required before we can use our Quantum Kraken Device!

## Measurement and State Collapse

This is where statevectors will make more sense! Measurement is the process of observing a qubit's state, collapsing it into either $|0\rangle$ or $|1\rangle$ based on its probability distribution. The coefficients we saw in the first calibration step, or the statevector, determines these probabilities:

$P(|0\rangle) = |\alpha|^2$
$P(|1\rangle) = |\beta|^2$

For example:

- Initialize a qubit in $|0\rangle$.
- Apply an X gate to flip it to $|1\rangle$.
- Measure the qubit to observe $|1\rangle$ with 100% certainty.

Measurement outcomes are random when the qubit is in a superposition state - measuring $|+\rangle$ yields $|0\rangle$ or $|1\rangle$ with equal probability.

Remember how when a qubit is in the $|+\rangle$ state, both coefficients $\alpha$ and $\beta$ are the same value of $1/\sqrt{2}$? Well if we use the same probability distribution from above:

$P(|0\rangle) = |1/\sqrt{2}|^2 = 1/2 = 50\%$
$P(|1\rangle) = |1/\sqrt{2}|^2 = 1/2 = 50\%$

We can measure a qubit at any point on the Bloch sphere's surface. The result we get will be a probability distribution based on that location.

### Basis Measurement

When measuring a qubit, you're free to choose the **basis** in which to observe it—most commonly, the **Z (computational) basis** or the **X basis**.

```
nsec> quantum 1
Initialized 1-qubit state: |0>
Quantum interactive mode. Type 'exit' to quit, 'help' for commands, or 'list' to show applied gates.
q>: g X 0
Applying gate X to qubit 0...
q>: sv
Statevector:
|0> : (0.000000 + 0.000000i)
|1> : (1.000000 + 0.000000i)
q>: measure
Measured qubit state: |1>
Qubits have been reinitialized...
```

In the Z basis, the outcomes $|0\rangle$ or $|1\rangle$ reflect the "classical bit" perspective, matching how quantum computers typically store data. By contrast, measuring in the X basis (i.e., $|+\rangle$ or $|-\rangle$) probes a different aspect of the qubit's superposition. If measuring in the X basis, you are evaluating:

$P(|+\rangle) = |\alpha|^2$
$P(|-\rangle) = |\beta|^2$

Where: $|\psi\rangle = \alpha|+\rangle + \beta|-\rangle$

So now you can star to see how much information a "quantum bit" (qubit) can represent!

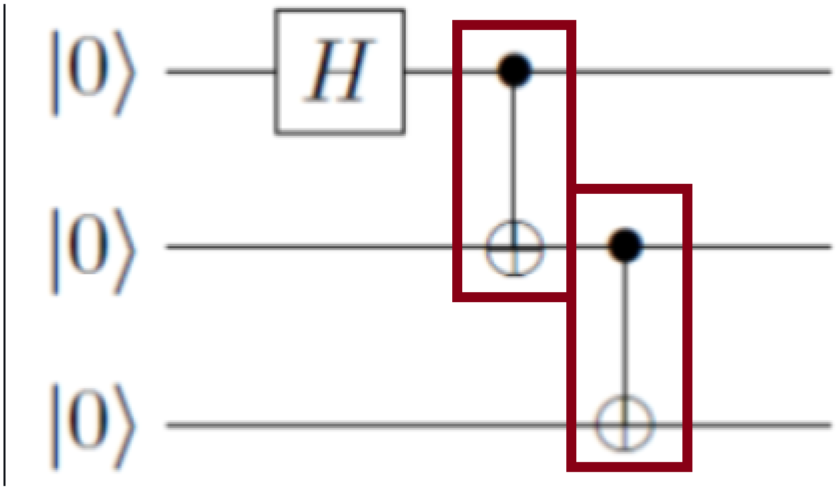
The measurement of the Quantum Kraken Device is always in the Computational Z basis (0 or 1), however knowledge of this conceptual measurement across an axis of the qubit is required as we move on past the calibration state, so keep it in mind!

### Quantum State Collapse

The last point to highlight about quantum measurement, is that when measuring a qubit, it collapses the state into a defined basis state. Once collapsed, the quantum properties of the qubit go away. You need to reinitialize the qubit start over. Let's see how that works with multiple qubits as well.

## Quantum Circuit Description

We've seen how a qubit can be represented and how gates operate on the qubit itself, but how do we show multiple qubits operating together? This can be done in a way that's similar to classical circuits! Reading left-to-right, where the initial qubit state is mentioned on the left of each "qubit line", and sequential gate operations are added in order that they are executed. When multi-qubit gates are applied, you can see the gate crossing between qubit lines, such as below.




These particular lines (highlighted read) represents the CNOT gate, so let's see what that does.

## Entanglement and the CNOT Gate

Entanglement links the states of two or more qubits such that the state of one qubit directly affects the others, no matter the distance between them. The CNOT (Controlled-NOT) gate is a fundamental operation to create entanglement.

A CNOT gate works by using the first qubit as a control (represented as a black dot in the circuit) and the second qubit as the target (represented as the circled cross in the diagram). When the first qubit is evaluated as |1⟩ the gate performs a NOT on the second qubit. Otherwise when the first control qubit is |0⟩ the target qubit is left unchanged.

- Start with two qubits,. both in |0⟩.
- Apply an H gate to the first qubit to create superposition.
- Use a CNOT gate with the first qubit as control and the second as target.

The resulting state is called an entangled Bell state:

|ψ⟩ = (|00⟩ + |11⟩) / √2

```
Statevector:
|00> : (0.707107 + 0.000000i)
|01> : (0.000000 + 0.000000i)
|10> : (0.000000 + 0.000000i)
|11> : (0.707107 + 0.000000i)
```

If either qubit is measured, the other collapses into the same state, maintaining their correlation. So even if you measure just one, if you measure a 0, the other qubit has collapsed to a 0, even without having measured it directly.

$$\mathrm{CNOT}(0 \to 1) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

## Action on |00⟩

In vector form, |00⟩ corresponds to $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$:

$$\mathrm{CNOT}(0 \to 1) \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 \\ 0 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 0 \\ 0 \cdot 1 + 0 \cdot 0 + 1 \cdot 0 + 0 \cdot 0 \\ 0 \cdot 1 + 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = |00\rangle.$$

## Action on |01⟩

Similarly, |01⟩ is $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$. Multiply:

$$\mathrm{CNOT}(0 \to 1) \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \cdot 0 + 0 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 \\ 0 \cdot 0 + 0 \cdot 1 + 0 \cdot 0 + 1 \cdot 0 \\ 0 \cdot 0 + 0 \cdot 1 + 1 \cdot 0 + 0 \cdot 0 \\ 0 \cdot 0 + 1 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = |11\rangle.$$

## Concept: Turning CNOT into CZ

A **Controlled-Z** gate applies a Z gate (phase flip) to the target qubit **only if** the control qubit is |1⟩.

1. **CNOT vs. CZ**

- **CNOT** flips |x⟩ on the target qubit **if** the control is |1⟩.
- **CZ** flips the phase (Z) on the target qubit **if** the control is |1⟩.

Using these transformations, we can **convert** a CNOT into a CZ with a few extra steps. To apply **CZ gate (control, target)** using **H gate, CNOT gate, H gate** on the **target** qubit. Because the Hadamard gate transforms Z into X and vice versa on a single qubit, sandwiching a **CNOT** gate with two **H** gates on the target effectively changes the "bit-flip if control=1" action into a "phase-flip if control=1."

## Calibration Challenge 2

- Calibration 2a: Using two qubits, create the positive **Bell Pair** below. Once done print out the statevector hash and submit to compare the calibration.

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

- Calibration 2b: Using three qubits, create the **GHZ (Greenberger–Horne–Zeilinger) state** below. Once done print out the statevector hash and submit to compare the calibration.

$$|\psi\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$

- Calibration 2c: Using three qubits, create a **Cluster State** that has the below statevector, **where every qubit is entangled**. Once done print out the statevector hash and submit to compare the calibration.

$$\frac{1}{2}\left(|000\rangle + |110\rangle + |001\rangle + |111\rangle\right)$$