Serie trigonométrica de Fourier

mayo 02, 2024

$\boxed{\text{Ejercicio A}}$

$$) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi t}{L} + b_n \operatorname{sen}\frac{n\pi}{L}t$$

Donde

T = periodo

$L = \frac{T}{2}$

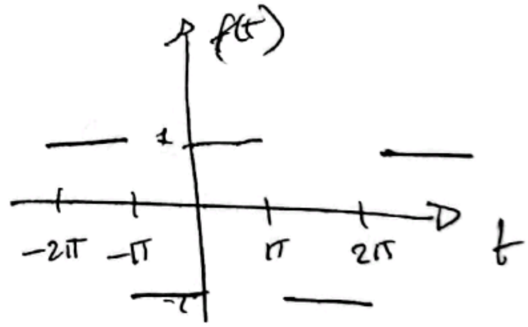
$$: \frac{1}{L}\int_{-L}^{L} f(t)\,dt$$

$$\frac{1}{L}\int_{-L}^{L} f(t) \cos\frac{n\pi}{L}t \, dt$$

$$\frac{1}{L}\int_{-L}^{L} f(t) \operatorname{sen}\frac{n\pi t}{L} \, dt$$

.o : Para una señal $f(t) = \begin{cases} 1 & 0 < t < \pi \\ -1 & \pi < t < 2\pi \text{ ó } -\pi < t < 0 \end{cases}$

l cálculo de los coeficientes:



$$\int_{-\pi}^{\pi} f(t)\,dt$$

$$\int_{-\pi}^{0} (1)dt + \int_{0}^{\pi} dt \Big]$$

$$-t\Big|_{-\pi}^{0} + t\Big|_{0}^{\pi} \Big] = \frac{1}{\pi}\Big[-(0 \cancel{+\pi}) + \cancel{(\pi - 0)}\Big] = \underline{0}$$

$$a_n = \frac{1}{\pi}\left[-\int_{-\pi}^{0} \cos\frac{n\pi t}{\pi}\,dt + \int_{0}^{\pi} \cos(nt)\,dt\right]$$

$$a_n = \frac{1}{\pi}\left[-\left(\frac{sen(nt)}{n}\right)\Bigg|_{-\pi}^{0} + \frac{sen(nt)}{n}\Bigg|_{0}^{\pi}\right]$$

$$a_n = \frac{1}{\pi}\left[(-1)\left(0 - \cancel{\frac{sen(-\pi n)}{n}}^{\,0}\right) + \cancel{\frac{sen(n\pi)}{n}}^{\,0} - 0\right] = 0$$

$$b_n = \frac{1}{\pi}\left[-\int_{-\pi}^{0} sen(nt)\,dt + \int_{0}^{\pi} sen(nt)\,dt\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{\cos(nt)}{n}\Bigg|_{-\pi}^{0} - \frac{\cos(nt)}{n}\Bigg|_{0}^{\pi}\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{\cancel{\cos(0)}^{\,1}}{n} - \underbrace{\frac{\cos(-n\pi)}{n}}_{(-1)^n} - \left(\underbrace{\frac{\cos(\pi n)}{n}}_{} - \frac{\cancel{\cos(0)}^{\,1}}{n}\right)\right]$$

$\cos(0) = 1 \longleftrightarrow \cos(2\pi) = 1$

$\cos(-\pi) = -1 \longleftrightarrow \cos(\pi) = -1$ $\boxed{\text{Impar} = -1}$

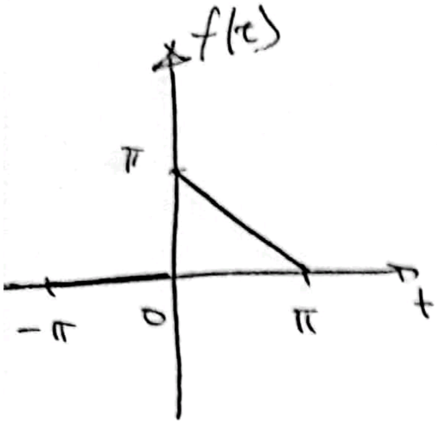
$(-1)^n$

$$b_n = \frac{1}{\pi}\left[\frac{2}{n} - 2\frac{(-1)^n}{n}\right] = \frac{2}{\pi n}\left[1 - (-1)^n\right]$$

∴ La expresión $f(t)$:

$$f(t) = \sum_{n=1}^{\infty}\left(\frac{2}{\pi n}\left[1 - (-1)^n\right]\right) sen(nt)$$

Video:
ercicio: "Series de Fourier #3." de FisicayMates. $\boxed{\text{Ejercicio B}}$



$$t = \begin{cases} 0 & -\pi < t < 0 \\ -t+\pi & 0 < t < \pi \end{cases}$$

$T = 2\pi$
$L = \pi$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi t}{L} + b_n \operatorname{sen}\frac{n\pi t}{L}$$

$$\frac{1}{L}\int_{-L}^{L} f(t)\,dt = \frac{1}{\pi}\int_{0}^{\pi}(-t+\pi)\,dt = \frac{1}{\pi}\left[-\frac{t^2}{2} + \pi t \Big|_0^{\pi}\right]$$

$$\frac{1}{\pi}\left[-\frac{\pi^2}{2} + \pi^2 - [0]\right] = \frac{1}{\cancel{\pi}}\left[\frac{\pi^{\cancel{2}}}{2}\right] = \frac{\pi}{2}$$

$$\frac{1}{L}\int_0^{\pi}(-t+\pi)\cos\frac{n\pi t}{\cancel{L}}\,dt = \frac{1}{\pi}\left[(-)\int_0^{\pi} t\cos(nt)\,dt + \pi\int_0^{\pi}\cos(nt)\,dt\right]$$

$\frac{d}{dt}$	$\int dt$
t	$\cos(nt)\,dt$
(1)	$\frac{\operatorname{sen}(nt)}{n}$
(0)	$-\frac{\cos(nt)}{n^2}$

$$\operatorname{sen}(nt) + \frac{\cos(nt)}{n^2} = \frac{nt\operatorname{sen}(nt) + \cos(nt)}{n^2}$$

$$\left[(-)\left[\frac{nt\,\cancel{\operatorname{sen}(nt)}^{\,0} + \cos(nt)}{n^2}\right|_0^{\pi} + \frac{\pi}{n}\,\cancel{\operatorname{sen}(nt)}^{\,0}\Big|_0^{\pi}\right]$$

$$)\left[\frac{\cos(n\pi)}{n^2} - \frac{1}{n^2}\right] = \frac{1}{\pi}\left[\frac{1}{n^2} - \frac{1}{n^2}(-1)^n\right]$$

$$a_n = \frac{1}{\pi n^2}\left[1 - (-1)^n\right]$$

$$b_n = \frac{1}{\pi}\left[-\int_0^{\pi} t\,\text{sen}(nt)\,dt + \int_0^{\pi} \pi\,\text{sen}(nt)\,dt \right]$$

$\frac{d}{dt}$	$\int dt$
t	$\text{sen}(nt)\,dt$
(1)	$-\frac{\cos(nt)}{n}$
(0)	$-\frac{\text{sen}(nt)}{n^2}$

$$b_n = \frac{1}{\pi}\left[(-)\left[-\frac{t\cos(nt)}{n} + \frac{\text{sen}(nt)}{n^2}\right]_0^{\pi} - \pi\left(\frac{\cos(nt)}{n}\right)\Bigg|_0^{\pi}\right]$$

$$b_n = \frac{1}{\pi}\left[(-)\left[-\frac{\pi\cos(n\pi)}{n} + \cancel{\frac{\text{sen}(n\pi)}{n^2}}^{\to 0} - \left(-\cancel{0\cos}^{\to 0} + \cancel{0}^{\to 0}\right)\right] - \pi\left[\frac{\cos(n\pi)}{n} - \frac{1}{n}\right]\right]$$

$$b_n = \frac{1}{\pi}\left[\cancel{\frac{\pi\cos(n\pi)}{n}} - \cancel{\frac{\pi\cos(n\pi)}{n}} + \frac{\pi}{n}\right] = \boxed{\frac{1}{n}} \checkmark$$

$$\therefore\; f(t) = \frac{\frac{\pi}{2}}{\frac{2}{1}} + \sum_{n=1}^{\infty}\left[\frac{1}{\pi n^2}\left[1-(-1)^n\right]\cos(nt) + \frac{1}{n}\,\text{sen}(nt)\right]$$

$$\frac{\pi}{4}$$

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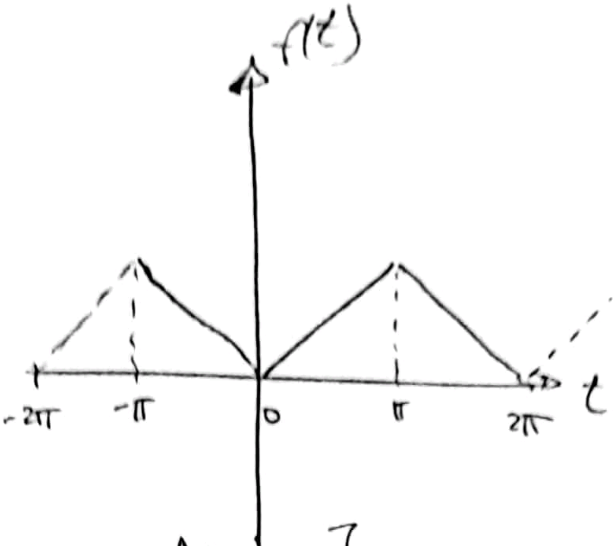
$\boxed{\text{Ejercicio C}}$

Para la siguiente función:

$$f(t)=\begin{cases} t & 0<t<\pi \\ -t & -\pi<t<0 \end{cases}$$



$$a_0=\frac{1}{L}\int_{-L}^{L} f(t)\,dt=\frac{1}{\pi}\left[\int_{-\pi}^{0}-t\,dt+\int_{0}^{\pi}t\,dt\right]$$

al final me queda $a_0=\pi$

$$a_n=\frac{1}{\pi}\left[-\int_{-\pi}^{0}(t)\cos(nt)\,dt+\int_{0}^{\pi}(t)\cos(nt)\,dt\right]$$

Usando la regla

$\frac{d}{dt}$		$\int dt$
$(-t)$		$\cos(nt)$
(-1)	$+$	$\frac{\text{sen}(nt)}{n}$
0	$-$	$-\frac{\cos(nt)}{n^2}$

llego a $a_n=\frac{2}{\pi n^2}\left[(-1)^n-1\right]$

y para b_n después de todo el desarrollo queda $b_n=0$

$$\therefore\ f(t)=\frac{\pi}{2}+\sum_{n=1}^{\infty}\left(\frac{2}{\pi n^2}\right)\left[(-1)^n-1\right]\cos(nt)$$