Media_Mix_Model

June 16, 2025

0.1 Importing Required Libraries

These libraries are necessary for data manipulation, visualization, and statistical modeling.

[1]:
```
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import numpy as np
import statsmodels.api as sm
from statsmodels.tsa.seasonal import seasonal_decompose
from statsmodels.stats.diagnostic import acorr_ljungbox, het_breuschpagan
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
```

0.2 Exploratory Data Analysis (EDA)

This section explores the dataset's structure, content, and variable distributions.

0.3 Data Preparation Summary

This step involves loading the dataset, checking for missing or duplicate values, and understanding the basic structure.

0.3.1 1. Loading Data and data description

[2]:
```
data = pd.read_csv("../data/data.csv")
```

[3]:
```
data.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 6240 entries, 0 to 6239
Data columns (total 16 columns):
 #   Column                   Non-Null Count  Dtype
---  ------                   --------------  -----
 0   geo                      6240 non-null   object
 1   date                     6240 non-null   object
 2   tv_impression            6240 non-null   int64
 3   radio_impression         6240 non-null   int64
 4   print_impression         6240 non-null   int64
 5   search_impression        6240 non-null   int64
 6   social_impression        6240 non-null   int64
```

```
 7   competitor_sales_control  6240 non-null   float64
 8   sentiment_score_control   6240 non-null   float64
 9   tv_spend                  6240 non-null   float64
 10  radio_spend               6240 non-null   float64
 11  print_spend               6240 non-null   float64
 12  search_spend              6240 non-null   float64
 13  social_spend              6240 non-null   float64
 14  population                6240 non-null   float64
 15  sales                     6240 non-null   float64
dtypes: float64(9), int64(5), object(2)
memory usage: 780.1+ KB
```

[4]:
```
data.describe()
```

[4]:
```
       tv_impression  radio_impression  print_impression  search_impression  \
count   6.240000e+03      6.240000e+03      6.240000e+03       6.240000e+03
mean    8.851635e+05      5.206055e+05      2.605050e+05       1.806810e+06
std     8.193451e+05      6.206231e+05      5.551842e+05       1.304818e+06
min     0.000000e+00      0.000000e+00      0.000000e+00       0.000000e+00
25%     2.536458e+05      0.000000e+00      0.000000e+00       8.083565e+05
50%     6.796705e+05      3.106525e+05      0.000000e+00       1.505379e+06
75%     1.324174e+06      8.075525e+05      2.547340e+05       2.566118e+06
max     5.192032e+06      4.397540e+06      5.610156e+06       7.635147e+06

       social_impression  competitor_sales_control  sentiment_score_control  \
count       6.240000e+03               6240.000000              6240.000000
mean        9.816780e+05                 -0.044412                -0.033675
std         9.141768e+05                  1.210164                 1.175572
min         0.000000e+00                 -4.825634                -4.230392
25%         2.675945e+05                 -0.872736                -0.808085
50%         7.462475e+05                 -0.050346                -0.036433
75%         1.476708e+06                  0.743633                 0.739391
max         6.975542e+06                  3.924682                 4.320259

           tv_spend   radio_spend   print_spend  search_spend  social_spend  \
count   6240.000000   6240.000000   6240.000000   6240.000000   6240.000000
mean    6490.659147   5019.278558   1935.793164  14080.185617   7649.147993
std     6008.031135   5983.571424   4125.531614  10168.239254   7123.184932
min        0.000000      0.000000      0.000000      0.000000      0.000000
25%     1859.914025      0.000000      0.000000   6299.393250   2085.072625
50%     4983.835500   2995.072650      0.000000  11731.178500   5814.694600
75%     9709.803000   7785.800675   1892.909150  19997.351500  11506.376500
max    38071.730000  42397.700000  41688.645000  59499.484000  54352.810000

          population          sales
count    6240.000000    6240.000000
mean   542418.832750  211330.038098
```

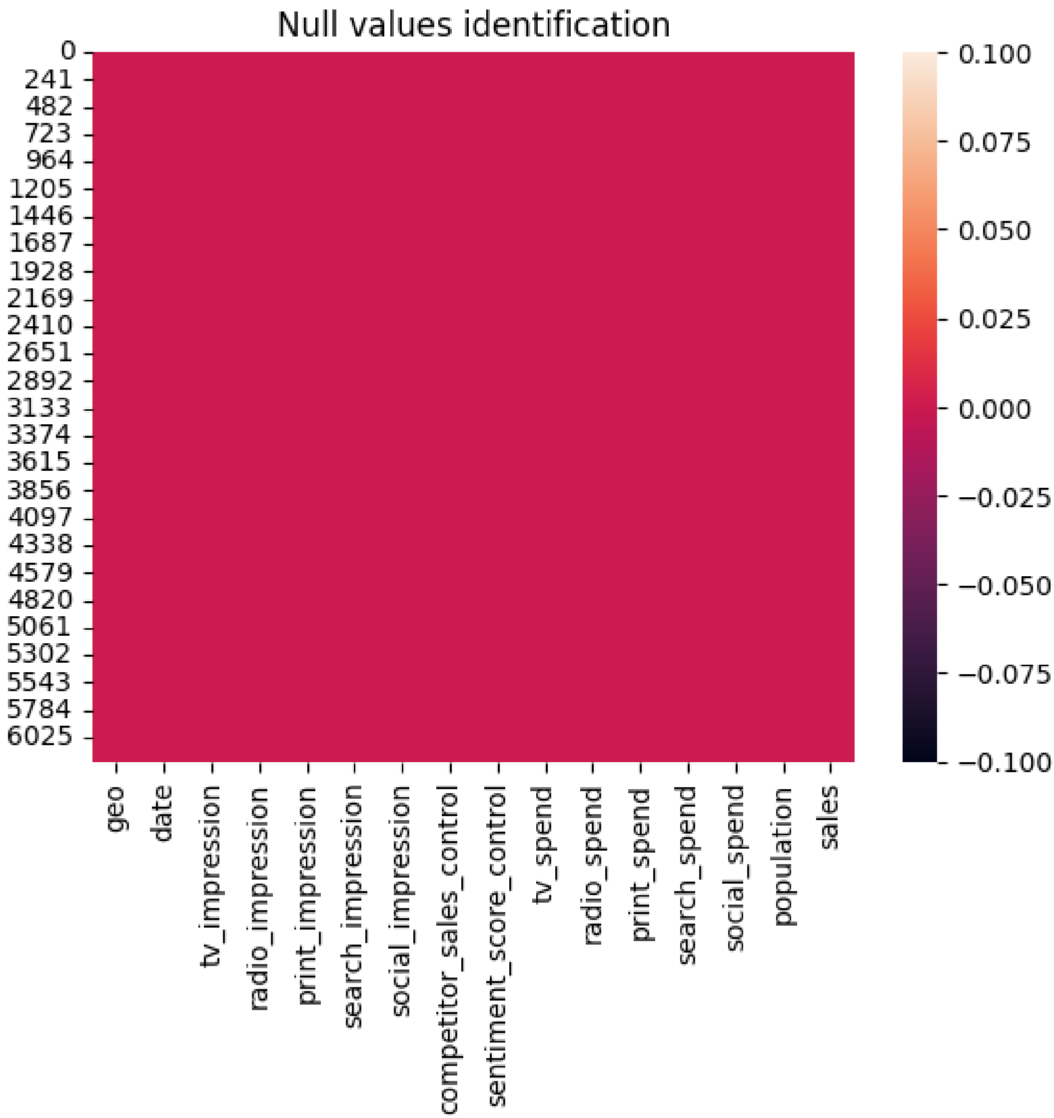
```
std    242839.719887  121605.908396
min    136670.940000    9742.282233
25%    335176.345000  114323.174425
50%    560478.825000  193995.582650
75%    736033.810000  287918.213625
max    994048.940000  739823.338900
```

[5]:
```
data.head()
```

[5]:
```
    geo     date  tv_impression  radio_impression  print_impression  \
0  Geo0  1/25/21         280668                 0                 0
1  Geo0   2/1/21         366206            182108             19825
2  Geo0   2/8/21         197565            230170                 0
3  Geo0  2/15/21         140990             66643                 0
4  Geo0  2/22/21         399116            164991                 0

   search_impression  social_impression  competitor_sales_control  \
0             470611             108010                 -1.338765
1             527702             252506                  0.893645
2             393618             184061                 -0.284549
3             326034             201729                 -1.034740
4             381982             153973                 -0.319276

   sentiment_score_control   tv_spend  radio_spend  print_spend  search_spend  \
0                 0.115581  2058.0608      0.00000      0.00000     3667.3965
1                 0.944224  2685.2874   1755.74540    147.31808     4112.2974
2                -1.290579  1448.6895   2219.12230      0.00000     3067.4023
3                -1.084514  1033.8406    642.52057      0.00000     2540.7310
4                -0.017503  2926.6072   1590.71640      0.00000     2976.7249

   social_spend  population        sales
0      841.6044   136670.94  39198.55690
1     1967.5044   136670.94  41497.96063
2     1434.1870   136670.94  41579.08885
3     1571.8545   136670.94  56492.86151
4     1199.7440   136670.94  71039.82718
```

0.3.2 2. Null values identification

[6]:
```
sns.heatmap(data.isnull())
plt.title("Null values identification")
plt.show()
```



[7]:
```
print(f"Number of null values: {data.isnull().sum().sum()}")
```

```
Number of null values: 0
```

0.3.3 3. Duplicates detection

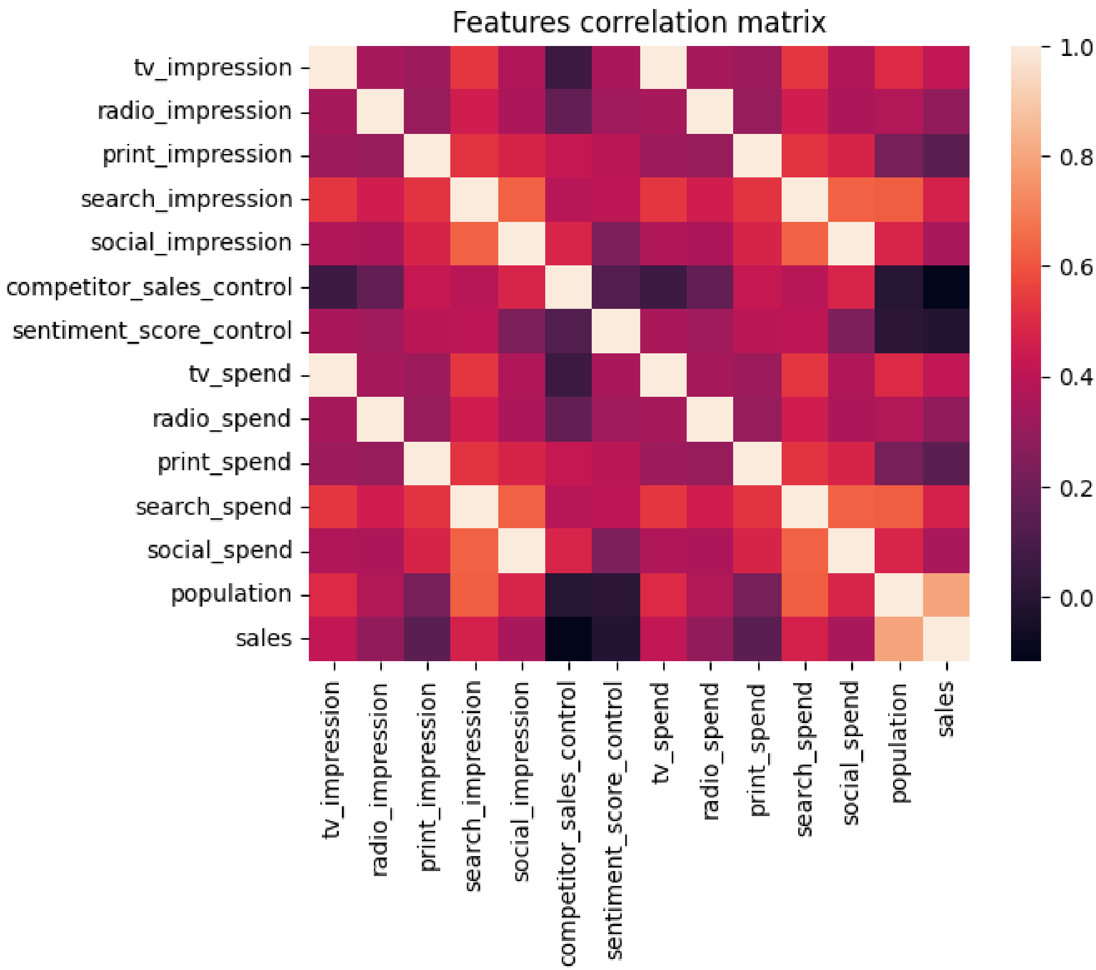
[8]:
```
duplicates = data[data.duplicated()]
print(f"Number of duplicates records: {len(duplicates)}")
```

```
Number of duplicates records: 0
```

0.3.4 1. Pearson Correlation Heatmap

Visualizes linear relationships between numerical variables using Pearson's method.

[9]:
```python
correlations = data.select_dtypes(include=['number']).corr()

sns.heatmap(correlations)
plt.title("Features correlation matrix")
plt.show()
```



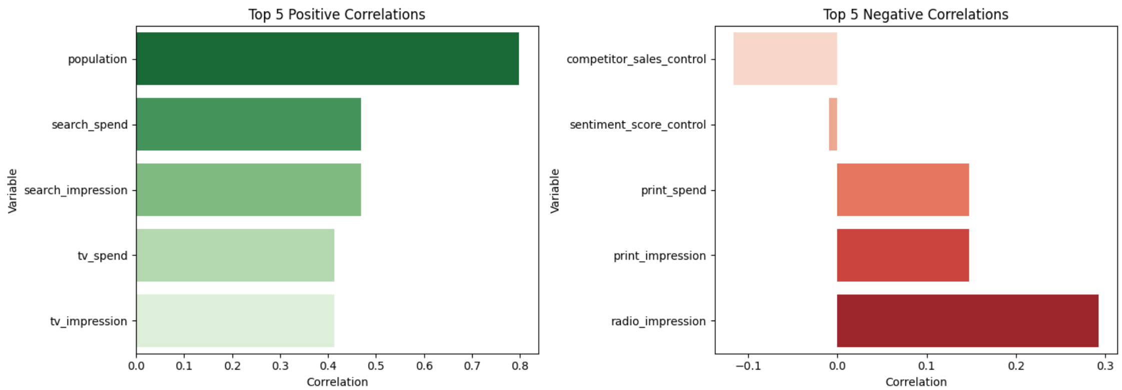
0.3.5 2. Correlation with Target (Sales)

Highlights which variables are most strongly correlated (positively and negatively) with sales.

[10]:
```python
# Compute correlations with 'sales'
numeric_cols = data.select_dtypes(include=['number'])
correlations = numeric_cols.corr()['sales'].drop('sales')

# Top 5 positive and negative correlations
top_positive = correlations.sort_values(ascending=False).head(5).reset_index()
top_negative = correlations.sort_values(ascending=True).head(5).reset_index()
```

```python
# Rename columns
top_positive.columns = ['variable', 'correlation']
top_negative.columns = ['variable', 'correlation']

# Create figure
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(14, 5))

# Plot positive correlations
sns.barplot(
    data=top_positive,
    x='correlation', y='variable',
    ax=axes[0], palette='Greens_r'
)
axes[0].set_title("Top 5 Positive Correlations")
axes[0].set_xlabel("Correlation")
axes[0].set_ylabel("Variable")

# Plot negative correlations
sns.barplot(
    data=top_negative,
    x='correlation', y='variable',
    ax=axes[1], palette='Reds'
)
axes[1].set_title("Top 5 Negative Correlations")
axes[1].set_xlabel("Correlation")
axes[1].set_ylabel("Variable")

plt.tight_layout()
plt.show()
```

```
C:\Users\Rafa Rincon\AppData\Local\Temp\ipykernel_18392\1830579637.py:17:
FutureWarning:

Passing `palette` without assigning `hue` is deprecated and will be removed in
v0.14.0. Assign the `y` variable to `hue` and set `legend=False` for the same
effect.

  sns.barplot(
C:\Users\Rafa Rincon\AppData\Local\Temp\ipykernel_18392\1830579637.py:27:
FutureWarning:

Passing `palette` without assigning `hue` is deprecated and will be removed in
v0.14.0. Assign the `y` variable to `hue` and set `legend=False` for the same
effect.

  sns.barplot(
```



0.4 Feature Engineering

New features are derived from existing data to enhance model performance and capture relevant signals.

0.4.1 1. Geographic Feature Encoding

Converts the 'geo' categorical variable into binary (dummy) variables for use in regression models.

[11]:
```
# Convert the 'geo' categorical variable into dummy/indicator variables (drop␣
↪the first to avoid multicollinearity)
geo_dummies = pd.get_dummies(data['geo'], drop_first=True)

# Ensure all dummy variables are of integer type
geo_dummies = geo_dummies.astype(int)

# Append the dummy variables to the original dataset
data = pd.concat([data, geo_dummies], axis=1)
```

0.4.2 2. Temporal Feature Transformation

Extracts seasonality and calendar-related components from the date column.

[12]:
```
data['date'] = pd.to_datetime(data['date'])
data['year'] = data['date'].dt.year
data['month'] = data['date'].dt.month
data['weekofyear'] = data['date'].dt.isocalendar().week.astype(int)


#data['is_weekend'] = data['date'].dt.dayofweek >= 5
#data['is_weekend'] = data['is_weekend'].astype(int)
data['quarter'] = data['date'].dt.quarter
data['month_sin'] = np.sin(2 * np.pi * data['month'] / 12)
data['month_cos'] = np.cos(2 * np.pi * data['month'] / 12)
```

```
C:\Users\Rafa Rincon\AppData\Local\Temp\ipykernel_18392\1900084449.py:1:
UserWarning: Could not infer format, so each element will be parsed
individually, falling back to `dateutil`. To ensure parsing is consistent and
as-expected, please specify a format.
  data['date'] = pd.to_datetime(data['date'])
```

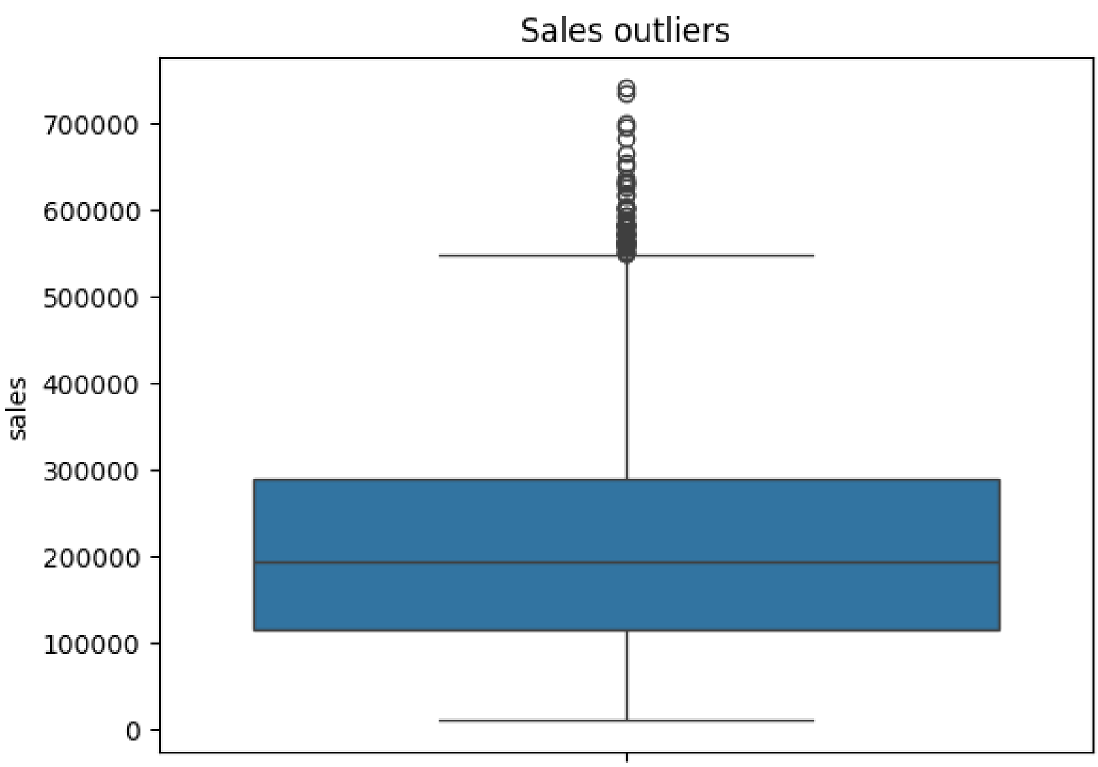
0.5 Seasonality and Time-Series Structure

Analyzes temporal sales patterns, decomposing them into trend, seasonal, and irregular components.

0.5.1 1. Outlier Detection in Sales

Boxplots help identify extreme values or irregular fluctuations in the sales data.

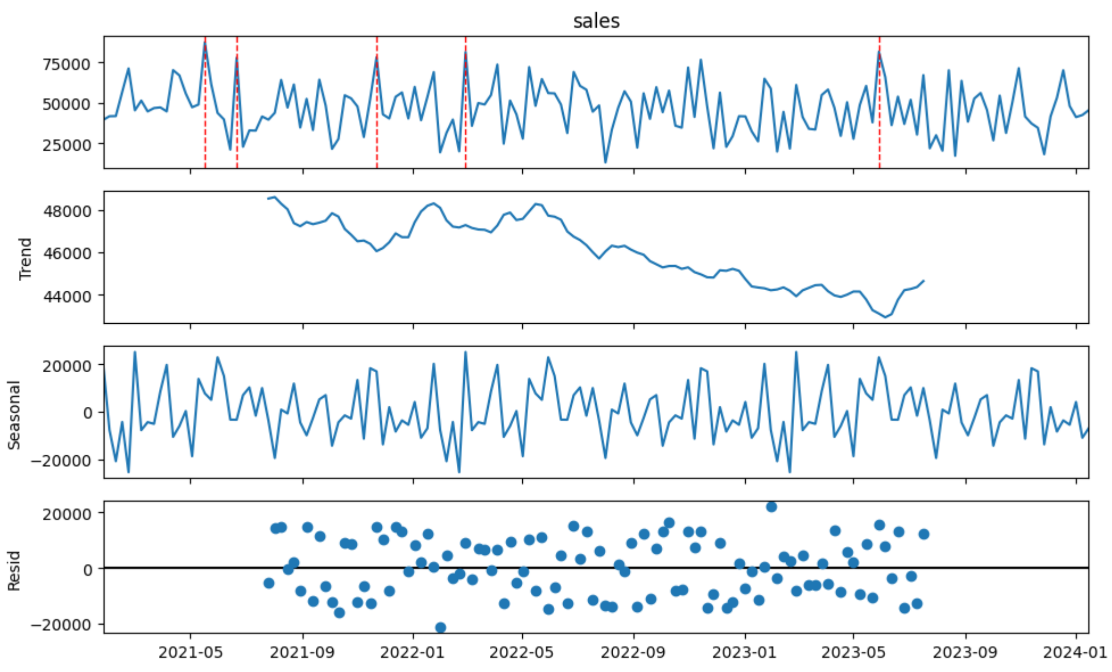
[13]:
```
sns.boxplot(data["sales"])
plt.title("Sales outliers")
plt.show()
```



0.5.2 2. Time-Series Decomposition

Uses additive decomposition to separate sales into seasonal, trend, and residual components.

[14]:
```python
# Filter data by a specific geography and aggregate sales by date
geo_df = data[data['geo'] == 'Geo0'].groupby('date')['sales'].sum()
geo_df = geo_df.sort_index()

# Perform seasonal decomposition (weekly frequency, assuming 52 periods per␣
↪year)
result = seasonal_decompose(geo_df, model='additive', period=52)

# Identify top 5 dates with the highest sales (sales peaks)
top_peaks = geo_df.sort_values(ascending=False).head(5).index

# Plot decomposition components and mark sales peaks with red lines
fig = result.plot()
fig.set_size_inches(10, 6)

# Add vertical red dashed lines on the observed component for peak dates
observed_ax = fig.axes[0]
for date in top_peaks:
    observed_ax.axvline(x=date, color='red', linestyle='--', linewidth=1)

plt.tight_layout()
plt.show()
```



0.6 Media Mix Modeling: Channel Effectiveness

This model evaluates how different marketing channels contribute to overall sales.

0.6.1 1. Adstock and Saturation Effects

Applies an adstock transformation to simulate delayed media effects, followed by log transformation for diminishing returns.

[15]:
```
# Define a function to apply the adstock transformation
# This simulates the carryover effect of advertising over time using a decay␣
↪factor
def apply_adstock(series, decay=0.5):
    result = []
    for i, val in enumerate(series):
        if i == 0:
            result.append(val)  # First value remains unchanged
        else:
            # Current value plus a portion of the previous value based on the␣
↪decay factor
            result.append(val + decay * result[i - 1])
    return pd.Series(result, index=series.index)

# List of media spend channels to apply adstock to
spend_channels = ['tv_spend', 'radio_spend', 'print_spend', 'search_spend',␣
↪'social_spend']

# Set decay rate for adstock effect
decay = 0.5

# Apply adstock and log transformation (to model diminishing returns) for each␣
↪channel
for channel in spend_channels:
    adstocked = apply_adstock(data[channel], decay)
    data[f'{channel}_transformed'] = np.log1p(adstocked)  # Log transformation␣
↪for saturation effect
```

0.6.2 2. Regression Coefficient Interpretation

Estimates the marginal effect of each feature on sales using Ordinary Least Squares (OLS) regression.

[16]:
```
features_adstock = [
    "tv_spend_transformed", "radio_spend_transformed",␣
↪"print_spend_transformed",
    "search_spend_transformed", "social_spend_transformed",
    'competitor_sales_control', 'sentiment_score_control', 'population',
    'month_sin', 'month_cos'
]

# Add geographic dummy variables (geo_*) to the list of features
geo_features = geo_dummies.columns.tolist()
```

```
features_with_geo = features_adstock + geo_features
```

[17]:
```
# Add a constant (intercept) term to the feature set
X_with_geo = sm.add_constant(data[features_with_geo])

# Define the target variable (sales)
y_with_geo = data['sales']

# Fit an Ordinary Least Squares (OLS) regression model using the selected␣
↪features
model_with_geo = sm.OLS(y_with_geo, X_with_geo).fit()

# Display the summary of regression results
print(model_with_geo.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  sales   R-squared:                       0.696
Model:                            OLS   Adj. R-squared:                  0.694
Method:                 Least Squares   F-statistic:                     295.2
Date:                Mon, 16 Jun 2025   Prob (F-statistic):               0.00
Time:                        11:06:03   Log-Likelihood:                -78201.
No. Observations:                6240   AIC:                         1.565e+05
Df Residuals:                    6191   BIC:                         1.568e+05
Df Model:                          48
Covariance Type:            nonrobust
================================================================================
============
                               coef    std err          t      P>|t|      [0.025
0.975]
--------------------------------------------------------------------------------
------------
const                    -1.526e+05   3.08e+04     -4.960      0.000   -2.13e+05
-9.23e+04
tv_spend_transformed      4379.7523   1877.944      2.332      0.020     698.330
8061.175
radio_spend_transformed   2549.3814   1227.115      2.078      0.038     143.810
4954.953
print_spend_transformed   1553.4649    547.205      2.839      0.005     480.752
2626.177
search_spend_transformed  7195.1300   3424.370      2.101      0.036     482.176
1.39e+04
social_spend_transformed  2964.4336   2012.665      1.473      0.141    -981.089
6909.956
competitor_sales_control -1.517e+04    882.308    -17.194      0.000   -1.69e+04
-1.34e+04
sentiment_score_control  -4354.0644    879.537     -4.950      0.000   -6078.263
-2629.866
```

population	0.3489	0.015	23.651	0.000	0.320	0.378
month_sin	-4076.6565	1380.674	-2.953	0.003	-6783.257	-1370.056
month_cos	1775.7455	1330.578	1.335	0.182	-832.649	4384.140
Geo1	-7430.5528	7241.948	-1.026	0.305	-2.16e+04	6766.180
Geo10	1.509e+04	6295.585	2.396	0.017	2745.811	2.74e+04
Geo11	-3.007e+04	5323.410	-5.649	0.000	-4.05e+04	-1.96e+04
Geo12	-9183.6249	5509.839	-1.667	0.096	-2e+04	1617.573
Geo13	-5776.0835	5992.854	-0.964	0.335	-1.75e+04	5971.992
Geo14	-2.647e+04	5345.552	-4.951	0.000	-3.69e+04	-1.6e+04
Geo15	1.283e+04	5312.333	2.415	0.016	2412.962	2.32e+04
Geo16	-6194.2840	6324.056	-0.979	0.327	-1.86e+04	6203.062
Geo17	-2.007e+04	5364.338	-3.741	0.000	-3.06e+04	-9554.135
Geo18	-2.96e+04	5415.389	-5.466	0.000	-4.02e+04	-1.9e+04
Geo19	-2.002e+04	6109.157	-3.278	0.001	-3.2e+04	-8047.415
Geo2	-3.12e+04	5690.988	-5.483	0.000	-4.24e+04	-2e+04
Geo20	-1.307e+04	5528.165	-2.364	0.018	-2.39e+04	-2231.386
Geo21	-3.822e+04	5511.291	-6.935	0.000	-4.9e+04	-2.74e+04
Geo22	7690.0288	6124.062	1.256	0.209	-4315.259	1.97e+04
Geo23	-1.494e+04	6227.989	-2.399	0.016	-2.71e+04	-2729.040
Geo24	1894.3192	7599.960	0.249	0.803	-1.3e+04	1.68e+04
Geo25	9408.5695	5365.957	1.753	0.080	-1110.569	1.99e+04
Geo26	-7649.6668	6865.332	-1.114	0.265	-2.11e+04	5808.767
Geo27	-7482.6548	5529.120	-1.353	0.176	-1.83e+04	3356.340
Geo28	7049.2178	5308.324	1.328	0.184	-3356.941	1.75e+04

```
Geo29                     1.188e+04   6042.941      1.966      0.049      33.547
2.37e+04
Geo3                     -4606.0132   7071.983     -0.651      0.515   -1.85e+04
9257.528
Geo30                     7080.0069   6838.591      1.035      0.301   -6326.006
2.05e+04
Geo31                     3710.7586   5656.443      0.656      0.512   -7377.834
1.48e+04
Geo32                    -2.056e+04   5310.595     -3.871      0.000    -3.1e+04
-1.01e+04
Geo33                    -7233.0453   5458.467     -1.325      0.185   -1.79e+04
3467.446
Geo34                    -5059.8442   5405.985     -0.936      0.349   -1.57e+04
5537.763
Geo35                    -2.481e+04   5310.746     -4.671      0.000   -3.52e+04
-1.44e+04
Geo36                     4.657e+04   6166.777      7.551      0.000    3.45e+04
5.87e+04
Geo37                     3020.6015   6563.431      0.460      0.645   -9846.003
1.59e+04
Geo38                     3477.4263   7016.740      0.496      0.620   -1.03e+04
1.72e+04
Geo39                     1.026e+05   5423.319     18.909      0.000    9.19e+04
1.13e+05
Geo4                      8606.5773   6474.249      1.329      0.184   -4085.200
2.13e+04
Geo5                     -1.203e+04   6473.701     -1.858      0.063   -2.47e+04
665.616
Geo6                     -1475.8298   7128.859     -0.207      0.836   -1.55e+04
1.25e+04
Geo7                      4.445e+04   5315.113      8.362      0.000     3.4e+04
5.49e+04
Geo8                      3900.6397   6645.157      0.587      0.557   -9126.175
1.69e+04
Geo9                      4.031e+04   5695.099      7.079      0.000    2.91e+04
5.15e+04
==============================================================================
Omnibus:                      221.064   Durbin-Watson:                   1.988
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              646.214
Skew:                          -0.003   Prob(JB):                   4.75e-141
Kurtosis:                       4.577   Cond. No.                    3.99e+21
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The smallest eigenvalue is 1.38e-28. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.
```

0.6.3 3. ROI and Marginal Contribution Analysis

Calculates Return on Investment (ROI) and average contribution to sales by media channel.

[18]:
```python
# Define the list of original media spend channels
spend_channels = ['tv_spend', 'radio_spend', 'print_spend', 'search_spend',␣
↪'social_spend']

# Retrieve model coefficients (from the geo-inclusive model or use␣
↪model_adstock if geo is not included)
coeffs = model_with_geo.params

# Calculate contribution and ROI for each media channel
contrib_values = []
roi_values = []

for ch in spend_channels:
    ch_trans = f'{ch}_transformed'  # Transformed version of the spend variable
    coef = coeffs[ch_trans]  # Coefficient from the regression model
    avg_trans = data[ch_trans].mean()  # Average transformed spend
    avg_spend = data[ch].mean()  # Average actual spend

    # Contribution is the coefficient times the average transformed value
    contribution = coef * avg_trans

    # ROI is contribution divided by the average actual spend
    roi = contribution / avg_spend if avg_spend != 0 else None

    # Store results
    contrib_values.append(contribution)
    roi_values.append(roi)

# Create a DataFrame to summarize contributions and ROIs by channel
df_contrib_roi_geo = pd.DataFrame({
    'channel': spend_channels,
    'contribution': contrib_values,
    'roi': roi_values
})
```

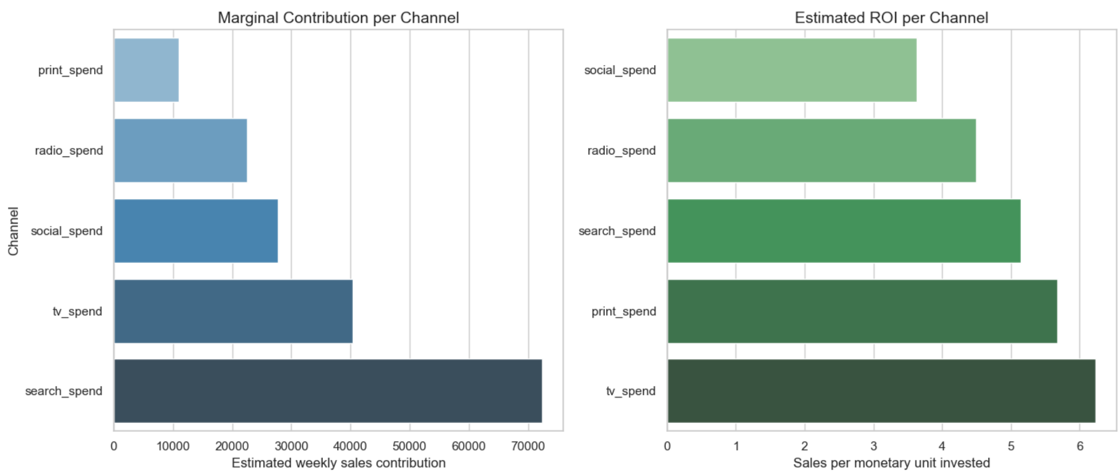
[19]:
```python
df_contrib_roi_geo
```

[19]:

	channel	contribution	roi
0	tv_spend	40426.675355	6.228439
1	radio_spend	22575.263459	4.497711
2	print_spend	10995.905744	5.680310
3	search_spend	72374.227808	5.140147
4	social_spend	27812.133260	3.635978

[20]:

```python
# Set visual style
sns.set(style="whitegrid")

# Create figure with two horizontal bar plots
fig, axes = plt.subplots(1, 2, figsize=(14, 6))

# Contribution plot
sns.barplot(
    x="contribution", y="channel", hue="channel", legend=False,
    data =df_contrib_roi_geo.sort_values("contribution", ascending=True),
    palette="Blues_d", ax=axes[0]
)
axes[0].set_title("Marginal Contribution per Channel", fontsize=14)
axes[0].set_xlabel("Estimated weekly sales contribution")
axes[0].set_ylabel("Channel")

# ROI plot
sns.barplot(
    x="roi", y="channel", hue="channel", legend=False,
    data = df_contrib_roi_geo.sort_values("roi", ascending=True),
    palette="Greens_d", ax=axes[1]
)
axes[1].set_title("Estimated ROI per Channel", fontsize=14)
axes[1].set_xlabel("Sales per monetary unit invested")
axes[1].set_ylabel("")  # Hide redundant label

# Adjust layout for clarity
plt.tight_layout()
plt.show()
```



0.6.4 4. Regional and Temporal ROI Analysis

Examines how ROI varies across geographies and time periods to uncover channel performance patterns.

[21]:
```
# Define the list of media spend channels
spend_channels = ['tv_spend', 'radio_spend', 'print_spend', 'search_spend',␣
↪'social_spend']

# Extract global model coefficients
coeffs = model_with_geo.params

# Initialize list to store ROI and contribution results by region and time
geo_time_results = []

# Group the data by geography, year, month, and quarter
for (geo, year, month, quarter), group in data.groupby(['geo', 'year', 'month',␣
↪"quarter"]):
    for ch in spend_channels:
        ch_trans = f'{ch}_transformed'
        coef = coeffs.get(ch_trans, 0)  # Get coefficient; use 0 if not found

        avg_trans = group[ch_trans].mean()  # Mean transformed media spend for␣
↪the group
        avg_spend = group[ch].mean()       # Mean actual media spend for the␣
↪group

        contribution = coef * avg_trans  # Compute estimated contribution to␣
↪sales
        roi = contribution / avg_spend if avg_spend != 0 else None  # Compute␣
↪ROI

        # Store the result for this geo-time-channel combination
        geo_time_results.append({
            'geo': geo,
            'year': year,
            'month': month,
            "quarter": quarter,
            'channel': ch,
            'contribution': contribution,
            'roi': roi
        })

# Create a final DataFrame with the results
df_geo_month_roi = pd.DataFrame(geo_time_results)

# Export results to CSV (optional)
```

```python
df_geo_month_roi.to_csv("../data/ROI_Contribution_geo_year.csv", index=False)

# Display the top results sorted by ROI in descending order
print(df_geo_month_roi.sort_values(by="roi", ascending=False).head())
```

```
        geo  year  month  quarter      channel  contribution          roi
7332   Geo9  2022     12        4  print_spend  10544.370508  2752.638300
752   Geo12  2021      3        1  print_spend   7042.527624  1362.467476
57     Geo0  2021     12        4  print_spend   7497.059152  1122.246918
4952  Geo32  2023      5        2  print_spend   3273.768886   896.540688
4452  Geo30  2021      3        1  print_spend   8517.333353   740.344313
```

0.7 Model Evaluation

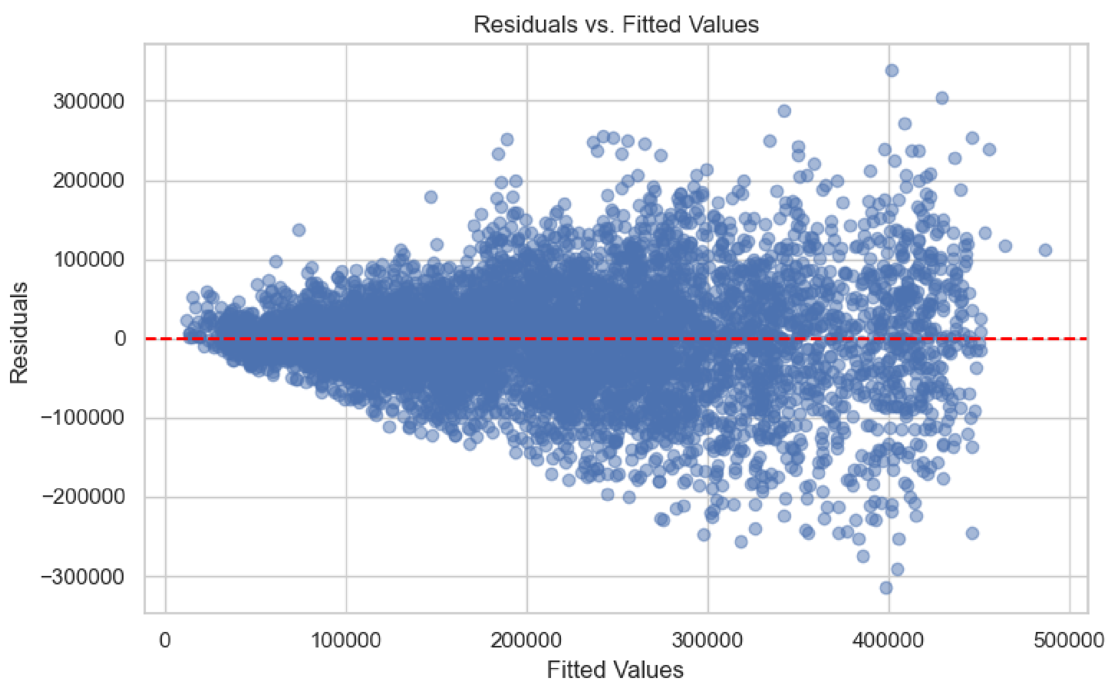
This section includes statistical validation of the regression model using residual diagnostics.

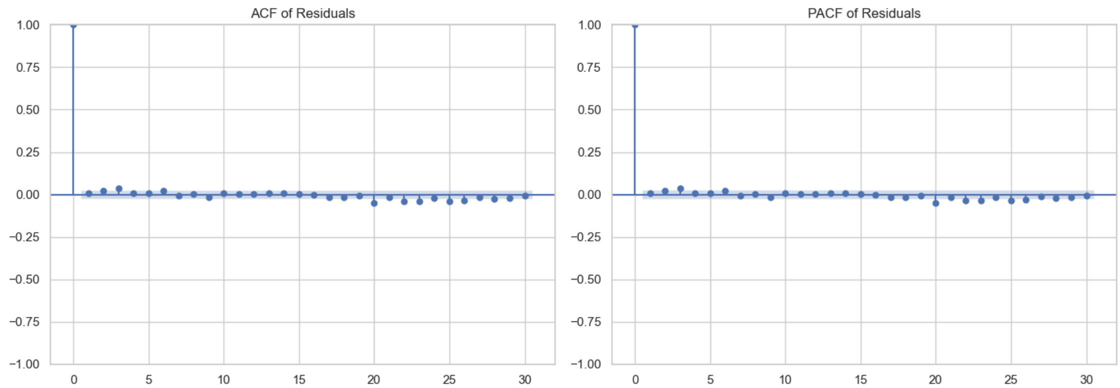
0.7.1 1. Residual Diagnostics

Uses the Ljung-Box and Breusch-Pagan tests to check for autocorrelation and heteroskedasticity in residuals.

[22]:
```python
# Extract model residuals and fitted (predicted) values
residuals = model_with_geo.resid
fitted = model_with_geo.fittedvalues

# Plot residuals vs. fitted values to visually assess homoscedasticity␣
↪(constant variance)
plt.figure(figsize=(8, 5))
plt.scatter(fitted, residuals, alpha=0.5)
plt.axhline(0, color='red', linestyle='--')  # Reference line at zero
plt.title("Residuals vs. Fitted Values")
plt.xlabel("Fitted Values")
plt.ylabel("Residuals")
plt.tight_layout()
plt.show()

# Plot Autocorrelation Function (ACF) and Partial Autocorrelation Function␣
↪(PACF) of residuals
fig, ax = plt.subplots(1, 2, figsize=(14, 5))
plot_acf(residuals, lags=30, ax=ax[0])  # Autocorrelation plot
plot_pacf(residuals, lags=30, ax=ax[1])  # Partial autocorrelation plot
ax[0].set_title("ACF of Residuals")
ax[1].set_title("PACF of Residuals")
plt.tight_layout()
plt.show()

# Perform Ljung-Box test to check for autocorrelation in residuals
ljung_box = acorr_ljungbox(residuals, lags=[12], return_df=True)
```

```
print("Ljung-Box test (lag=12):")
print(ljung_box)

# Perform Breusch-Pagan test to detect heteroskedasticity (non-constant␣
↪variance of residuals)
exog = model_with_geo.model.exog
bp_test = het_breuschpagan(residuals, exog)

# Output Breusch-Pagan test results
print("\nBreusch-Pagan test:")
print(f"Test statistic: {bp_test[0]:.2f}")
print(f"p-value: {bp_test[1]:.4f}")
print(f"Interpretation: {'No heteroskedasticity detected' if bp_test[1] > 0.05␣
↪else 'Heteroskedasticity present'}")
```






```
Ljung-Box test (lag=12):
      lb_stat  lb_pvalue
12  17.907369   0.118532

Breusch-Pagan test:
Test statistic: 1027.96
p-value: 0.0000
Interpretation: Heteroskedasticity present
```