

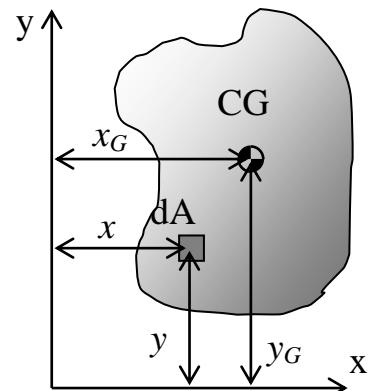
1. FIGURAS PLANAS

- 1.1. Momento estático** (M_S ou S ou Q)
 relativo ao eixo x:
$$M_{sx} = S_x = \int_A y \, dA$$

 relativo ao eixo y:
$$M_{sy} = S_y = \int_A x \, dA$$

1.2. Posição do centro de gravidade

$$x_G = \frac{M_{sy}}{A} = \frac{\sum A_i x_i}{\sum A_i} \quad y_G = \frac{M_{sx}}{A} = \frac{\sum A_i y_i}{\sum A_i}$$

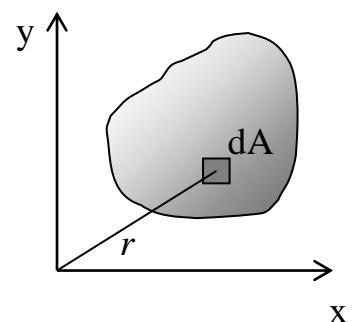


1.3. Momento de inércia relativo a um eixo

$$I_x = J_x = \int_A y^2 \, dA \quad I_y = J_y = \int_A x^2 \, dA$$

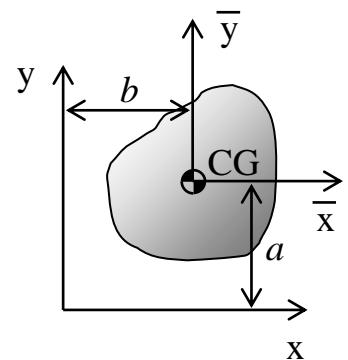
1.4. Raio de giração com relação a um eixo

$$i_{\bar{x}} = \sqrt{\frac{I_{\bar{x}}}{A}} \quad i_{\bar{y}} = \sqrt{\frac{I_{\bar{y}}}{A}}$$



1.5. Produto de inércia (momento centrífugo de inércia)

$$\text{relativo ao par de eixos } xy \quad I_{xy} = J_{xy} = \int_A x y \, dA$$

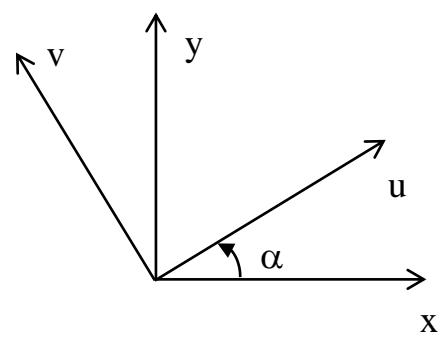


1.6. Momento polar de inércia com relação ao ponto O

$$I_P = J_P = \int_A r^2 \, dA = \int_A (x^2 + y^2) \, dA = I_x + I_y$$

1.7. Translação de eixos – Teorema de Steiner - Teorema dos eixos paralelos

$$I_x = I_{\bar{x}} + a^2 A \quad I_y = I_{\bar{y}} + b^2 A \\ I_{xy} = I_{\bar{xy}} + ab A$$



1.8. Rotação de eixos

$$I_u = I_x \cos^2 \alpha + I_y \sin^2 \alpha - I_{xy} \sin 2\alpha$$

$$I_{uv} = I_{xy} \cos 2\alpha + \frac{I_x - I_y}{2} \sin 2\alpha$$

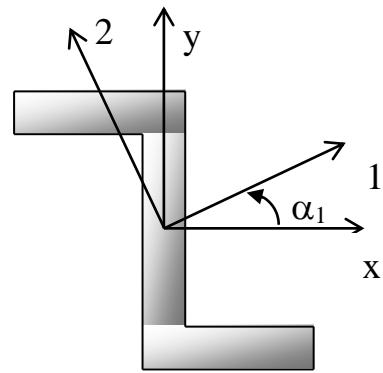
1.9. Eixos e momentos principais de inércia

$$\tan 2\alpha_p = \frac{2I_{xy}}{I_y - I_x}$$

α_p fixa o par sem dizer qual o máx. e o min.

Grandezas $I_1 = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$

$$I_2 = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

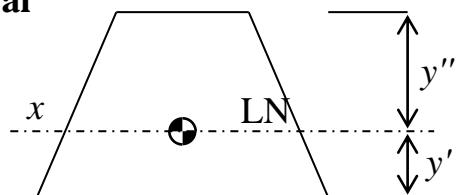


Direção do eixo principal relativamente ao qual temos:

$$I_{\max} \rightarrow \tan \alpha_1 = \frac{I_x - I_1}{I_{xy}} \quad I_{\min} \rightarrow \alpha_2 = \alpha_1 + \pi/2$$

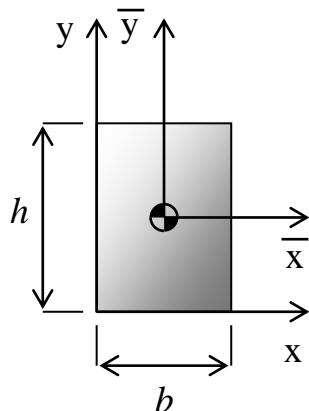
1.10. Módulo de resistência relativo a um eixo x central

fibra inferior $\rightarrow W_x' = \frac{I}{y'}$ fibra superior $\rightarrow W_x'' = \frac{I}{y''}$



1.11. Momentos de inércia para figuras elementares

Retângulo



$$x_G = \frac{b}{2}$$

$$y_G = \frac{h}{2}$$

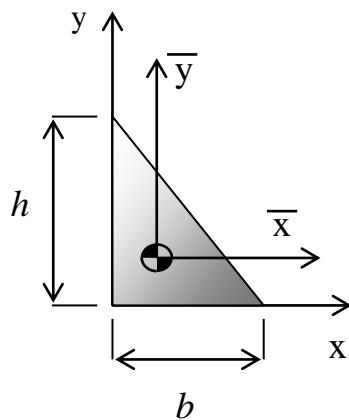
$$A = bh$$

$$I_{\bar{x}} = \frac{bh^3}{12}$$

$$I_{\bar{y}} = \frac{hb^3}{12}$$

$$I_{\bar{x}\bar{y}} = 0$$

Triângulo retângulo



$$x_G = \frac{b}{3}$$

$$y_G = \frac{h}{3}$$

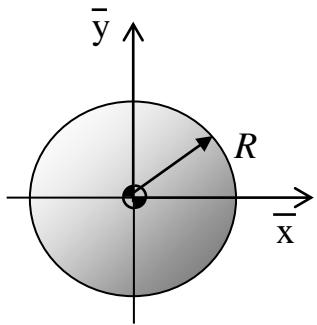
$$A = \frac{bh}{2}$$

$$I_{\bar{x}} = \frac{bh^3}{36}$$

$$I_{\bar{y}} = \frac{hb^3}{36}$$

$$I_{\bar{x}\bar{y}} = \frac{-b^2h^2}{72}$$

Círculo



$$A = \frac{\pi D^2}{4} = \pi R^2$$

$$I_{\bar{x}} = I_{\bar{y}} = \frac{\pi D^4}{64} = \frac{\pi R^4}{4}$$

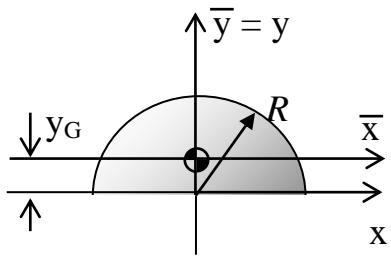
$$I_{\bar{x}\bar{y}} = 0$$

$$W_{\bar{x}} = \frac{\pi D^3}{32}$$

$$I_P = \frac{\pi D^4}{32} = \frac{\pi R^4}{2}$$

$$W_T = \frac{\pi D^3}{16}$$

Semicírculo



$$y_G = \frac{4R}{3\pi}$$

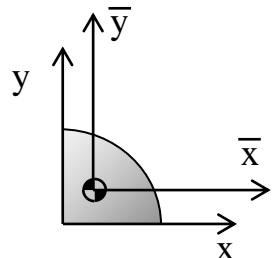
$$A = \frac{\pi R^2}{2} = \frac{\pi D^2}{8}$$

$$I_{\bar{x}} = 0,1098 R^4$$

$$I_y = I_{\bar{y}} = \frac{\pi D^4}{128} = \frac{\pi R^4}{8}$$

$$I_{\bar{x}\bar{y}} = I_{xy} = 0$$

Quarto de círculo



$$x_G = y_G = \frac{4R}{3\pi}$$

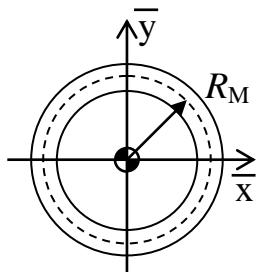
$$A = \frac{\pi R^2}{4} = \frac{\pi D^2}{16}$$

$$I_{\bar{x}} = I_{\bar{y}} = 0,0549 R^4$$

$$I_{\bar{x}\bar{y}} = -0,01647 R^4$$

Tubo de parede fina

$$R_M > 10 t$$



$$A = 2 \pi R_M t$$

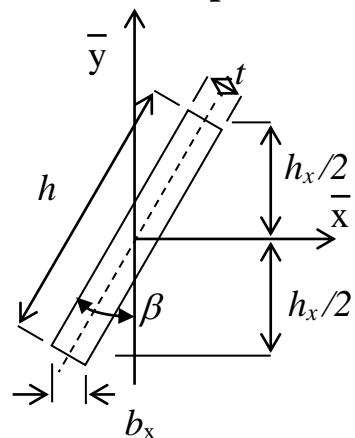
$$W_{\bar{x}} = \pi R_M^2 t$$

$$I_T = 2 \pi R_M^3 t$$

$$W_T = 2 \pi R_M^2 t$$

R_M – raio médio

Perfil de chapa fina

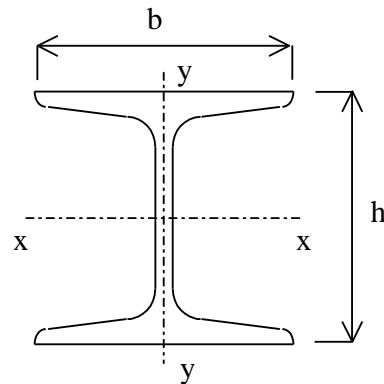


$$b_x = \frac{t}{\cos \beta}$$

$$h_x = h \cos \beta$$

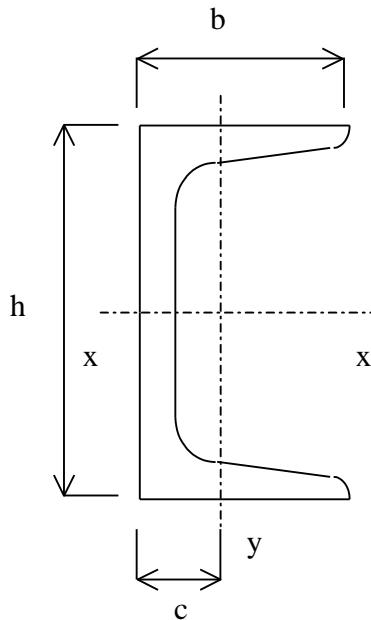
$$I_x = \frac{b_x h_x^3}{12} = \frac{t h^3 \cos^2 \beta}{12}$$

Perfil I - Padrão Americano - CSN



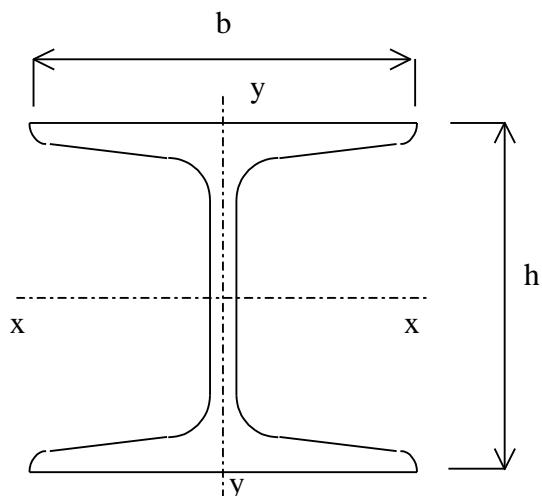
TAMANHO NOMINAL		Larg da Mesa mm	Esp da Alma mm	Area cm ²	Peso kgf/m	I _x cm ⁴	I _y cm ⁴	W _x cm ³	W _y cm ³	r _x cm	r _y cm
pol	mm										
3 x 2 3/8	76,2 x 60,3	59,2	4,32	10,8	8,45	105,1	18,9	27,6	6,41	3,12	1,33
		61,2	6,38	12,3	9,68	112,6	21,3	29,6	6,95	3,02	1,31
		63,7	8,86	14,2	11,20	121,8	24,4	32,0	7,67	2,93	1,31
4 x 2 5/8	101,6 x 66,7	67,6	4,83	14,5	11,4	252	31,7	49,7	9,4	4,17	1,48
		69,2	6,43	16,1	12,7	266	34,3	52,4	9,9	4,06	1,46
		71,0	8,28	18,0	14,1	283	37,6	55,6	10,6	3,96	1,45
		72,9	10,16	19,9	15,6	299	41,2	58,9	11,3	3,87	1,44
5 x 3	127,0 x 76,2	76,2	5,33	18,8	14,8	511	50,2	80,4	13,2	5,21	1,63
		79,7	8,81	23,2	18,2	570	58,6	89,8	14,7	4,95	1,59
		83,4	12,55	28,0	22,0	634	69,1	99,8	16,6	4,76	1,57
6 x 3 3/8	152,4 x 85,7	84,6	5,84	23,6	18,5	919	75,7	120,6	17,9	6,24	1,79
		87,5	8,71	28,0	22,0	1003	84,9	131,7	19,4	5,99	1,74
		90,6	11,81	32,7	25,7	1095	96,2	143,7	21,2	5,79	1,72
8 x 4	203,2 x 101,6	101,6	6,86	34,8	27,3	2400	155	236	30,5	8,30	2,11
		103,6	8,86	38,9	30,5	2540	166	250	32,0	8,08	2,07
		105,9	11,20	43,7	34,3	2700	179	266	33,9	7,86	2,03
		108,3	13,51	48,3	38,0	2860	194	282	35,8	7,69	2,00
10 x 4 5/8	254,0 x 117,5	118,4	7,9	48,1	37,7	5140	282	405	47,7	10,30	2,42
		121,8	11,4	56,9	44,7	5610	312	442	51,3	9,93	2,34
		125,6	15,1	66,4	52,1	6120	348	482	55,4	9,60	2,29
		129,3	18,8	75,9	59,6	6630	389	522	60,1	9,35	2,26
12 x 5 1/4	304,8 x 133,4	133,4	11,7	77,3	60,6	11330	563	743	84,5	12,10	2,70
		136,0	14,4	85,4	67,0	11960	603	785	88,7	11,80	2,66
		139,1	17,4	94,8	74,4	12690	654	833	94,0	11,60	2,63
		142,2	20,6	104,3	81,6	13430	709	881	99,7	11,30	2,61
15 x 5 1/2	381,0 x 139,7	139,7	10,4	86,6	63,3	18580	598	975	85,7	15,20	2,73
		140,8	11,5	84,7	66,5	19070	614	1001	87,3	15,00	2,70
		143,3	14,0	94,2	73,9	20220	653	1061	91,2	14,70	2,63
		145,7	16,5	103,6	81,4	21370	696	1122	95,5	14,40	2,59
18 x 6	457,2 x 152,4	152,4	11,7	103,7	81,4	33460	867	1464	113,7	18,00	2,89
		154,6	13,9	113,8	89,3	35220	912	1541	117,9	17,60	2,83
		156,7	16,0	123,3	96,8	36880	957	1613	122,1	17,30	2,79
		158,8	18,1	132,8	104,3	38540	1004	1686	126,5	17,00	2,75
20 x 7	508,0 x 177,8	177,8	15,2	154,4	121,2	61640	1872	2430	211	20,00	3,48
		179,1	16,6	161,3	126,6	63110	1922	2480	215	19,80	3,45
		181,0	18,4	170,7	134,0	65140	1993	2560	220	19,50	3,42
		182,9	20,3	180,3	141,5	67190	2070	2650	226	19,30	3,39
		184,7	22,2	189,7	148,9	69220	2140	2730	232	19,10	3,36

Perfil U - Padrão Americano - CSN



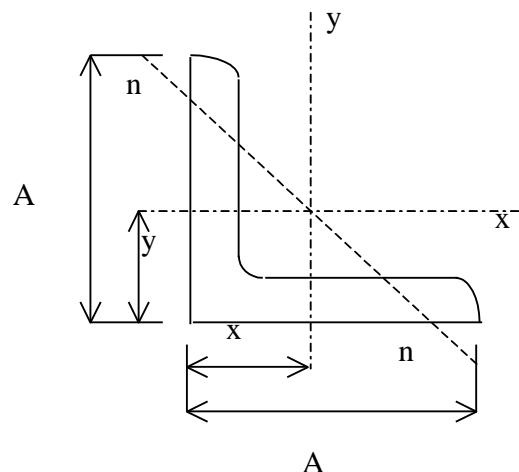
TAMANHO NOMINAL		Larg da aba mm	Esp da alma mm	Area cm ²	Peso kgf/m	c cm	I _x cm ⁴	I _y cm ⁴	W _x cm ³	W _y cm ³	r _x cm	r _y cm
pol	mm											
3 x 1 1/2	76,2 x 38,1	35,8	4,32	7,78	6,11	1,11	68,9	8,2	18,1	3,32	2,98	1,03
		38,0	6,55	9,48	7,44	1,11	77,2	10,3	20,3	3,82	2,85	1,04
		40,5	9,04	11,40	8,93	1,16	86,3	12,7	22,7	4,39	2,75	1,06
4 x 1 5/8	101,6 x 41,3	40,1	4,57	10,1	7,95	1,16	159,5	13,1	31,4	4,61	3,97	1,14
		41,8	6,27	11,9	8,30	1,15	174,4	15,5	34,3	5,10	3,84	1,14
		43,7	8,13	13,7	10,80	1,17	190,6	18,0	37,5	5,61	3,73	1,15
6 x 2	152,4 x 50,8	48,8	5,08	15,5	12,2	1,30	546	28,8	71,7	8,16	5,94	1,36
		51,7	7,98	19,9	15,6	1,27	632	36,0	82,9	9,24	5,63	1,34
		54,8	11,10	24,7	19,4	1,31	724	43,9	95,0	10,50	5,42	1,33
		57,9	14,20	29,4	23,1	1,38	815	52,4	107,0	11,90	5,27	1,33
8 x 2 1/4	203,2 x 57,2	57,4	5,59	21,8	17,1	1,45	1356	54,9	133,4	12,8	7,89	1,59
		59,5	7,70	26,1	20,5	1,41	1503	63,6	147,9	14,0	7,60	1,56
		61,8	10,00	30,8	24,2	1,40	1667	72,9	164,0	15,3	7,35	1,54
		64,2	12,40	35,6	27,9	1,44	1830	82,5	180,1	16,6	7,17	1,52
		66,5	14,70	40,3	31,6	1,49	1990	92,6	196,2	17,9	7,03	1,52
10 x 2 5/8	254,0 x 66,7	66,0	6,10	29,0	22,7	1,61	2800	95,1	221	19,0	9,84	1,81
		69,6	9,63	37,9	29,8	1,54	3290	117,0	259	21,6	9,31	1,76
		73,3	13,40	47,4	37,2	1,57	3800	139,7	299	24,3	8,95	1,72
		77,0	17,10	56,9	44,7	1,65	4310	164,2	339	27,1	8,70	1,70
		80,8	20,80	66,4	52,1	1,76	4820	191,7	379	30,4	8,52	1,70
12 x 3	304,8 x 76,2	74,7	7,11	39,1	30,7	1,77	5370	161,1	352	28,3	11,70	2,03
		77,4	9,83	47,4	37,2	1,71	6010	186,1	394	30,9	11,30	1,98
		80,5	13,00	56,9	44,7	1,71	6750	214,0	443	33,7	10,90	1,94
		83,6	16,10	66,4	52,1	1,76	7480	242,0	491	36,7	10,60	1,91
		86,7	19,20	75,9	59,6	1,83	8210	273,0	539	39,8	10,40	1,90
15 x 3 3/8	381,0 x 85,7	86,4	10,20	64,2	50,4	2,00	13100	338	688	51,0	14,30	2,30
		86,9	10,70	66,4	52,1	1,99	13360	347	701	51,8	14,20	2,29
		89,4	13,20	75,8	59,5	1,98	14510	387	762	55,2	13,80	2,25
		91,9	15,70	85,3	67,0	1,99	15650	421	822	58,5	13,50	2,22
		94,4	18,20	94,8	74,4	2,03	16800	460	882	62,0	13,30	2,20
		96,9	20,70	104,3	81,9	2,21	17950	498	942	66,5	13,10	2,18

Perfil H - Padrão Americano - CSN



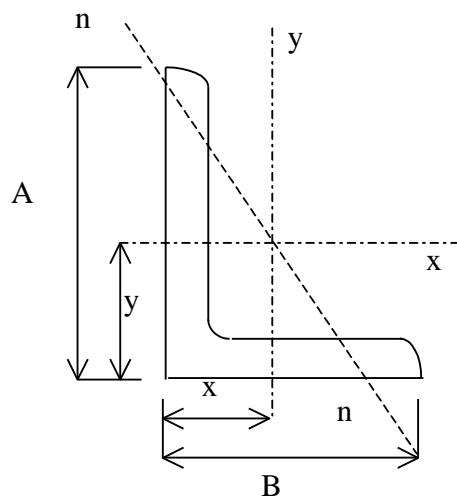
TAMANHO NOMINAL pol mm		Larg da mesa mm	Esp da alma mm	Area cm^2	Peso kgf/m	I_x cm^4	I_y cm^4	W_x cm^3	W_y cm^3	r_x cm	r_y cm
4 x 4	101,6 x 101,6	101,6	7,95	26,1	20,5	449	146	88	28,8	4,15	2,38
5 x 5	127,0 x 127,0	127,0	7,95	35,6	27,9	997	321	157	50,6	5,29	3,01
6 X 6	152,4 X 152,4	150,8 154,0	7,95 11,13	47,3 52,1	37,1 40,9	1958 2050	621 664	257 269	81,5 87,1	6,43 6,27	3,63 3,57

Cantoneiras de Abas Iguais - Padrão Americano - CSN



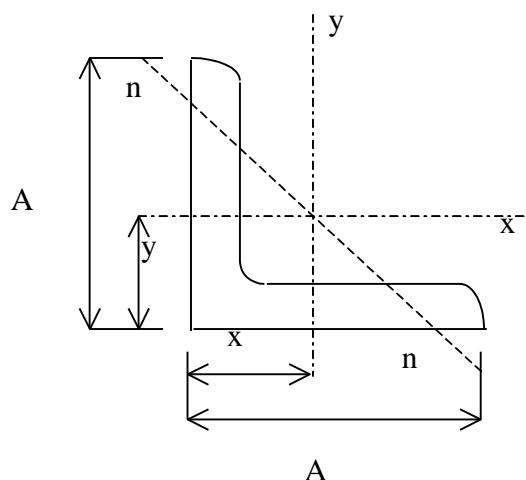
TAMANHO NOMINAL		Esp	Area	Peso	$I_x = I_y$	$W_x = W_y$	$r_x = r_y$	$x = y$	Eixo n r min.
pol	mm	mm	cm^2	kgf/m	cm^4	cm^3	cm	cm	cm
2 1/2 x 2 1/2	63,5 x 63,5	6,35	7,67	6,10	29	6,4	1,96	1,83	1,24
		7,94	9,48	7,44	35	7,9	1,93	1,88	1,24
		9,53	11,16	8,78	41	9,3	1,91	1,93	1,22
3 x 3	76,2 x 76,2	7,94	11,48	9,08	62	11,6	2,34	2,21	1,50
		9,53	13,61	10,70	75	13,6	2,31	2,26	1,47
		12,70	17,74	14,00	91	18,0	2,29	2,36	1,47
4 x 4	101,6 x 101,6	9,53	18,45	14,6	183	24,6	3,12	2,90	2,00
		12,70	24,19	19,1	233	32,8	3,10	3,00	1,98
		15,90	29,73	23,4	279	39,4	3,05	3,12	1,96
5 x 5	127,0 x 127,0	9,53	23,29	18,3	362	39,5	3,94	3,53	2,51
		12,70	30,64	24,1	470	52,5	3,91	3,63	2,49
		15,88	37,80	29,8	566	64,0	3,86	3,76	2,46
		19,05	44,76	35,1	653	73,8	3,81	3,86	2,46
6 x 6	152,4 x 152,4	9,53	28,12	22,2	641	57,4	4,78	4,17	3,02
		12,70	37,09	29,2	828	75,4	4,72	4,27	3,00
		15,88	45,86	36,0	1007	93,5	4,67	4,39	2,97
		19,05	54,44	42,7	1173	109,9	4,65	4,52	2,97
		22,23	62,76	49,3	1327	124,6	4,60	4,62	2,97
8 x 8	203,2 x 203,2	12,70	49,99	39,3	2022	137,8	6,38	5,56	4,01
		15,88	61,98	48,7	2471	168,9	6,32	5,66	4,01
		19,05	73,79	57,9	2899	200,1	6,27	5,79	3,99
		22,23	85,33	67,0	3311	229,6	6,22	5,89	3,96
		25,40	96,75	75,9	3702	259,1	6,20	6,02	3,96

Cantoneiras de Abas Desiguais - Padrão Americano - CSN



TAMANHO NOMINAL		Esp pol	Area mm	Peso kgf/m	y cm	x cm	I_x cm^4	I_y cm^4	W_x cm^3	W_y cm^3	r_x cm	r_y cm	Eixo n r min.
		mm	cm ²	kgf/m	cm	cm							cm
3 1/2 x 2 1/2	88,9 x 63,5	6,35	9,29	7,29	2,82	1,55	75	32	12	7	2,84	1,88	1,37
		7,94	11,48	9,08	2,90	1,63	92	39	15	8	2,82	1,85	1,37
		9,53	13,61	10,70	2,95	1,68	108	46	18	10	2,79	1,83	1,37
4 x 3	101,6 x 76,2	7,94	13,48	10,7	3,20	1,93	141	71	20	12	3,23	2,26	1,65
		9,53	16,00	12,7	3,25	1,98	166	79	25	14	3,20	2,24	1,63
		12,70	20,96	16,5	3,38	2,11	208	100	31	18	3,18	2,18	1,63
4 x 3 1/2	101,6 x 88,9	6,35	11,67	9,1	2,95	2,31	121	87	16	13	3,23	2,72	1,85
		7,94	14,51	11,5	3,00	2,36	158	108	21	16	3,20	2,72	1,85
		9,53	17,22	13,5	3,07	2,44	175	125	25	20	3,18	2,69	1,85
		12,70	22,58	17,7	3,18	2,54	221	158	31	25	3,12	2,64	1,83
5 x 3 1/2	127,0 x 88,9	7,94	16,51	13,0	4,04	2,13	275	112	31	16	4,09	2,62	1,93
		9,53	19,67	15,5	4,09	2,18	325	133	38	20	4,06	2,59	1,93
		12,70	25,80	20,2	4,22	2,31	416	166	49	26	4,01	2,57	1,91
		15,88	31,73	25,0	4,32	2,41	499	200	60	31	3,96	2,51	1,91
		19,05	37,47	29,5	4,45	2,54	578	233	70	36	3,94	2,49	1,91
6 x 4	152,4 x 101,6	9,53	23,28	18,3	4,93	2,39	562	204	54	26	4,90	2,97	2,24
		12,70	30,64	24,1	5,05	2,51	724	262	71	34	4,85	2,92	2,21
		15,88	37,80	29,8	5,16	2,62	878	312	87	41	4,83	2,87	2,18
		19,05	44,76	35,1	5,28	2,74	1019	362	102	49	4,78	2,84	2,18
7 x 4	177,8 x 101,6	12,70	33,86	26,6	6,15	2,34	1111	270	95	34	5,72	2,82	2,21
		15,88	41,86	32,9	6,25	2,44	1348	325	116	43	5,69	2,79	2,18
		19,05	49,60	39,0	6,38	2,57	1573	379	138	49	5,64	2,77	2,18
8 x 4	203,2 x 101,6	12,70	37,09	29,2	7,26	2,18	1602	279	123	36	6,58	2,74	2,18
		15,88	45,86	36,0	7,39	2,31	1951	337	151	43	6,50	2,72	2,18
		19,05	54,44	42,7	7,49	2,41	2284	391	179	51	6,48	2,67	2,16
		22,23	62,76	49,3	7,62	2,54	2596	437	205	58	6,43	2,64	2,16
		25,40	70,95	55,7	7,75	2,67	2895	483	231	64	6,40	2,62	2,16

Cantoneiras de Abas Iguais - 5/8" a 2"



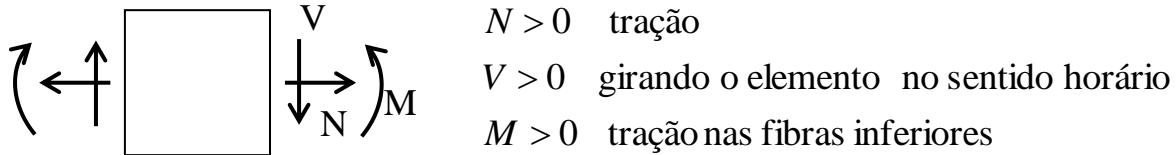
TAMANHO NOMINAL		Esp	Area	Peso	$I_x = I_y$	$W_x = W_y$	$r_x = r_y$	$x = y$	Eixo n r min.
pol	mm	mm	cm ²	kgf/m	cm ⁴	cm ³	cm	cm	cm
5/8 x 5/8	16 x 16	3,17	0,96	0,71	0,20	0,18	0,45	0,51	0,30
3/4 x 3/4	19 x 19	3,17	1,16	0,88	0,37	0,28	0,58	0,58	0,38
7/8 x 7/8	22 x 22	3,17	1,35	1,04	0,58	0,37	0,66	0,66	0,48
1 x 1	25 x 25	3,17	1,48	1,19	0,83	0,49	0,76	0,76	0,51
		4,75	2,19	1,73	1,24	0,65	0,76	0,81	0,48
		6,35	2,83	2,21	1,66	0,98	0,73	0,86	0,48
1 1/4 x 1 1/4	32 x 32	3,17	1,93	1,50	1,66	0,81	0,96	0,91	0,63
		4,75	2,77	2,20	2,49	1,14	0,96	0,96	0,61
		6,35	3,61	2,86	3,32	1,47	0,93	1,01	0,61
1 1/2 x 1 1/2	38 x 38	3,17	2,32	1,83	3,32	1,14	1,19	1,06	0,76
		4,75	3,42	2,68	4,57	1,63	1,16	1,11	0,73
		6,35	4,45	3,48	5,82	2,13	1,14	1,19	0,73
		7,94	5,42	4,26	6,65	2,53	1,11	1,24	0,73
		9,52	7,91	5,94	12,90	4,26	1,29	1,45	0,86
1 3/4 x 1 3/4	44 x 44	3,17	2,70	2,14	5,41	1,63	1,39	1,21	0,88
		4,75	3,99	3,15	7,49	2,29	1,37	1,29	0,88
		6,35	5,22	4,12	9,57	3,11	1,34	1,34	0,86
		7,94	6,45	5,05	11,23	3,77	1,32	1,39	0,86
		9,52	7,91	5,94	12,90	4,26	1,29	1,45	0,86
2 x 2	51 x 51	3,17	3,09	2,46	7,90	2,13	1,60	1,39	1,01
		4,75	4,58	3,63	11,23	3,11	1,57	1,44	0,99
		6,35	6,06	4,76	14,56	4,09	1,54	1,49	0,99
		7,94	7,41	5,83	17,48	4,91	1,52	1,54	0,99
		9,52	8,77	6,99	19,97	5,73	1,49	1,62	0,99

2. ESFORÇOS

2.1. Equações de equilíbrio

$$\begin{aligned}\sum \vec{F} &= 0 \\ \sum \vec{M} &= 0\end{aligned}$$

2.2. Convenção de sinais dos esforços internos solicitantes



2.3. Relação entre os esforços internos solicitantes

$$\frac{d^2M}{dx^2} = \frac{dV}{dx} = -p(x)$$

3. TENSÕES E DEFORMAÇÕES

3.1. Tração e Compressão Simples

Tensões

$$\sigma = \frac{N}{A}$$

Deformações

$$\varepsilon = \frac{\Delta L}{L}$$

$$\Delta L = \int \frac{N dx}{EA} \quad \Delta L = \frac{NL}{EA}$$

Lei de Hooke

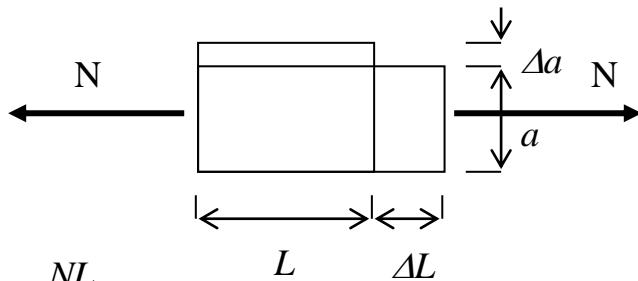
$$\sigma = E \varepsilon$$

$E \rightarrow$ Módulo de elasticidade (longitudinal)

Deformação transversal $\varepsilon_t = \frac{\Delta a}{a}$

$$\varepsilon_t = -\nu \varepsilon$$

$\nu \rightarrow$ Coeficiente de Poisson



3.2. Cisalhamento Puro

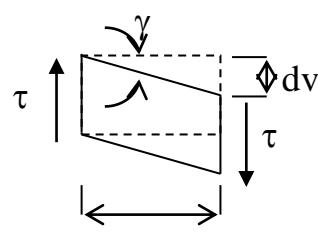
Tensões

$$\tau = \frac{V}{A}$$

Deformações

$$\Delta v_V = \int \frac{V dx}{G A}$$

$$\Delta v_V = \frac{V L}{G A}$$

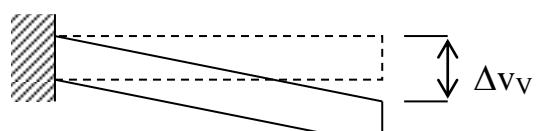


Lei de Hooke

$$\tau = G \gamma$$

$$G = \frac{E}{2(1 + \nu)}$$

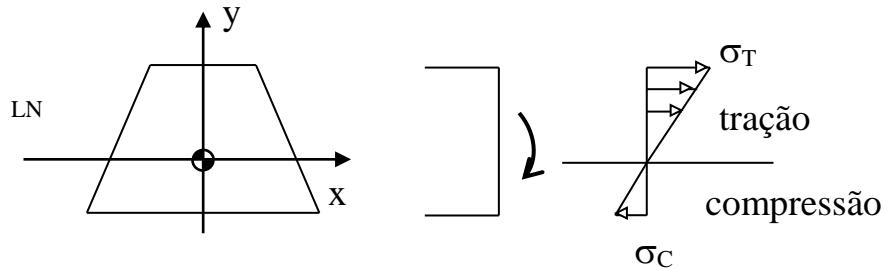
$G \rightarrow$ Módulo de elasticidade transversal



3.3. Flexão Simples

$$\sigma = \frac{M}{I_{LN}} y$$

$$\sigma_{máx} = \frac{M}{W}$$



3.4. Equação Diferencial da Linha Elástica

$$\frac{d^2y}{dx^2} = \frac{d\varphi}{dx} = \pm \frac{M}{EI}$$

3.5. Cisalhamento na Flexão

$$\tau = \frac{V M_s}{b I_{LN}}$$

$$\tau_{máx} = k \frac{V}{A}$$

$$k = 1,5 \quad \text{seção retangular}$$

$$k = \frac{4}{3} \quad \text{seção circular}$$

$$q = \tau b = \frac{V M_s}{I_{LN}}$$

3.6. Flexão Oblíqua Simples

$$\sigma = \sigma_{M_x} + \sigma_{M_y}$$

$$\sigma = \frac{|M_x|}{I_x} y + \frac{|M_y|}{I_y} x$$

3.7. Flexão Normal Composta

$$\sigma = \frac{N}{A} + \frac{|M_x|}{I_x} y$$

3.8. Flexão Oblíqua Composta

$$\sigma = \frac{N}{A} + \frac{|M_x|}{I_x} y + \frac{|M_y|}{I_y} x$$

3.9. Torção Simples - Fórmulas Gerais

Tensões

$$\tau_{máx} = \frac{M_T}{W_T} = \frac{T}{W_T}$$

$$M_T = 71620 \frac{P}{n}$$

$$M_T = kgf.cm$$

$$P = CV$$

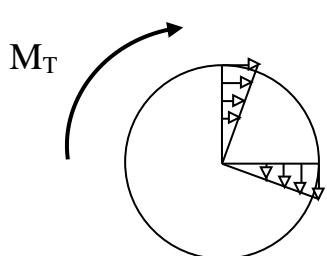
$$n = rpm$$

Deformações

$$\Delta\theta = \int_L \frac{M_T dx}{GI_T}$$

$$\Delta\theta = \frac{M_T L}{GI_T}$$

a) Seção Circular e Coroa de Círculo

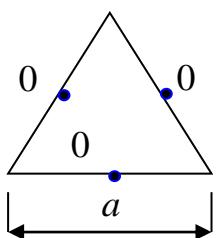


$$I_T = I_P \quad W_T = \frac{I_P}{R}$$

$$\tau = \frac{M_T}{I_P} r$$

b) Seções Maciças em Geral

Triângulo Equilátero

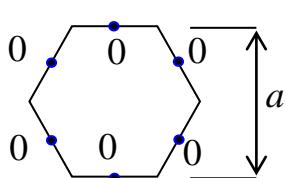


0 – ponto de maior tensão

$$I_T = 0,0217 a^4$$

$$W_T = 0,05 a^3$$

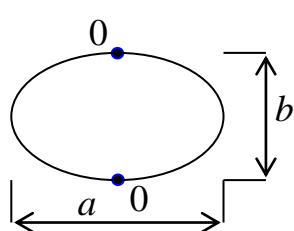
Hexágono regular



$$I_T = 0,115 a^4$$

$$W_T = 0,177 a^3$$

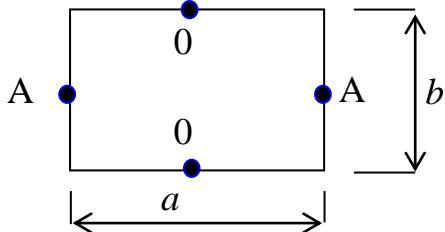
Elipse



$$I_T = \frac{\pi a^3 b^3}{16(a^2 + b^2)}$$

$$W_T = \frac{\pi a b^2}{16}$$

Retângulo



$$I_T = \beta a b^3$$

$$\tau_o = \tau_{\max}$$

$$W_T = \alpha a b^2$$

$$\tau_A = \eta \tau_o$$

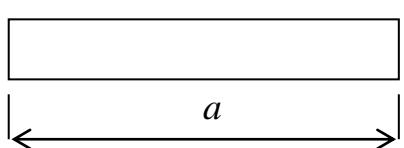
a – lado maior

b – lado menor

a/b	1	1,5	2	3	4	6	8	10	∞
α	0,208	0,231	0,246	0,267	0,282	0,299	0,307	0,313	0,333
β	0,141	0,196	0,229	0,263	0,281	0,299	0,307	0,313	0,333
η	1,000	0,859	0,795	0,753	0,745	0,743	0,742	0,742	0,742

Retângulo de pequena espessura

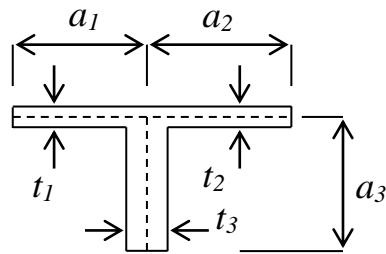
$a > 10 t$



$$\uparrow t$$

$$I_T = \frac{a t^3}{3} \quad W_T = \frac{I_T}{t} = \frac{a t^2}{3}$$

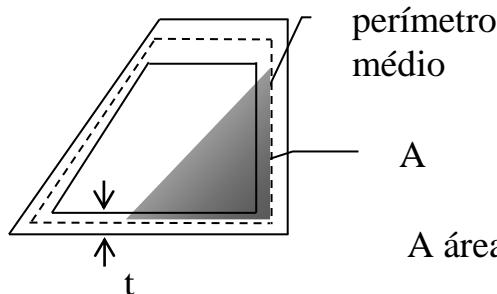
c) Perfis Abertos de Parede Fina



$$I_T = \sum \frac{a_i t_i^3}{3} \quad W_T = \frac{I_T}{t}$$

τ_{\max} ocorrem t_{\max}

d) Tubos de Parede Fina



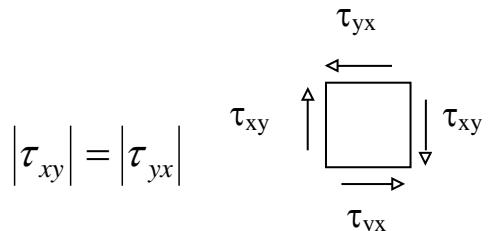
$$I_T = \frac{4 A^2}{\oint \frac{ds}{t}} \quad W_T = 2 A t$$

τ_{\max} ocorrem t_{\min}

A área limitada pelo perímetro médio

4. ESTADO DUPLO DE TENSÕES

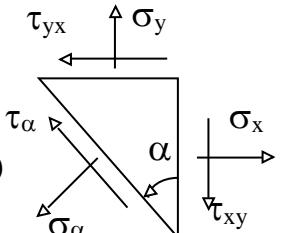
4.1. Condição Fundamental de Equilíbrio



4.2. Rotação de Eixos

$$\sigma_\alpha = \sigma_x \cos^2 \alpha + \sigma_y \sin^2 \alpha - \tau_{xy} \sin 2\alpha \quad \sigma > 0 \quad (\text{tração})$$

$$\tau_\alpha = \frac{\sigma_x - \sigma_y}{2} \sin 2\alpha + \tau_{xy} \cos 2\alpha \quad \tau > 0 \quad (\text{sentido horário})$$



4.3. Tensões Principais

$$\tau = 0 \quad \text{para } \operatorname{tg} 2\alpha_p = \frac{2\tau_{xy}}{\sigma_y - \sigma_x} \quad \alpha_p = \text{fixa o par sem dizer qual o máx ou mín.}$$

Valor das tensões principais

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \rightarrow \text{age na face definida por} \rightarrow \operatorname{tg} \alpha_1 = \frac{\sigma_x - \sigma_1}{\tau_{xy}}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \rightarrow \text{age na face definida por} \rightarrow \alpha_2 = \alpha_1 + \pi/2$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \rightarrow \text{age no plano definido por} \rightarrow \alpha_1 + \pi/4$$

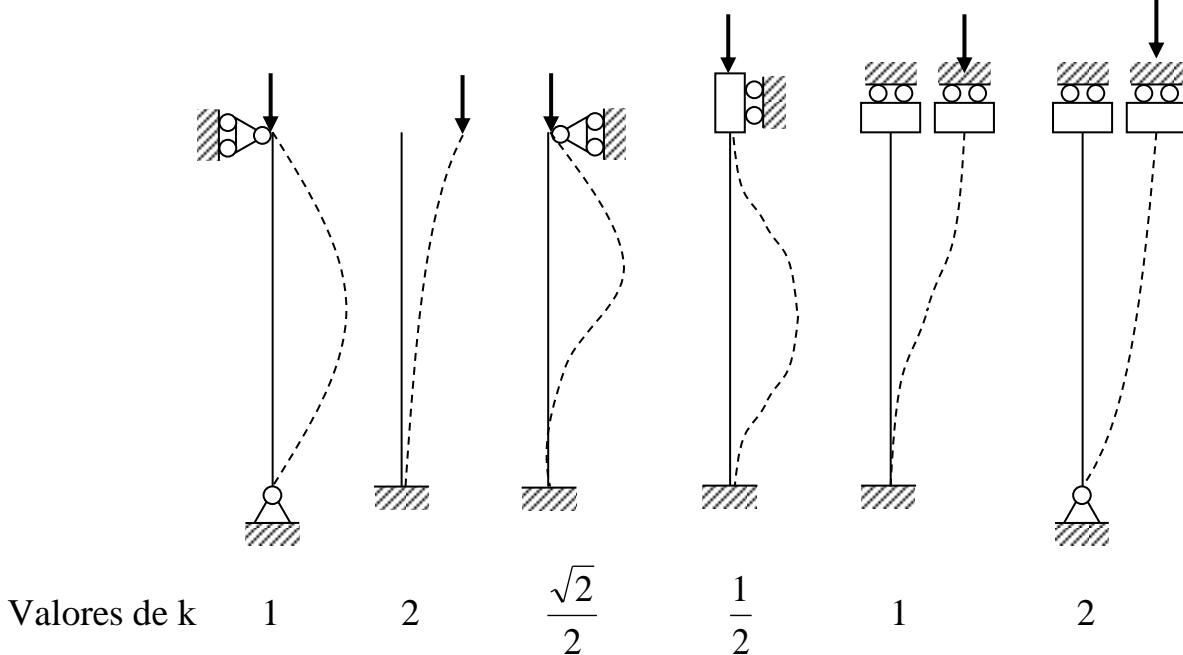
$$\tau_{\min} = -\tau_{\max} \rightarrow \text{age no plano definido por} \rightarrow \alpha_2 + \pi/4$$

5. FLAMBAGEM

5.1. Fórmulas Gerais

$$\lambda = \frac{L_{fl}}{i} \quad L_{fl} = k \cdot L \quad \sigma_{fl} = \frac{P_{fl}}{A} \quad \lambda_{lim} = \sqrt{\frac{\pi^2 E}{\sigma_p}}$$

Comprimentos de flambagem



a) Regime Elástico – Euler $\lambda > \lambda_{lim}$

$$P_{fl} = \frac{\pi^2 EI}{L_{fl}^2} \quad \sigma_{fl} = \frac{\pi^2 E}{\lambda^2}$$

b) Regime Plástico $\lambda < \lambda_{lim}$

b.1) Tetmajer

$$\sigma_{fl} = a - b \cdot \lambda + c \cdot \lambda^2$$

[kgf/cm²]

Material	λ_{lim}	a	b	c
Aço ST 37	105	3100	11,4	0
Aço ST 60	89	3350	6,2	0
Ferro Fundido	80	7760	120	0,53
Madeira	100	293	1,94	0

b.2) Engesser- Shanley -Bleich

$$\sigma_{fl} = a - b \cdot \lambda^2 \quad a = \sigma_e \quad b = \frac{\sigma_e - \sigma_p}{(\lambda_{lim})^2}$$

b.3) Fórmula do AISC

$$\sigma_{fl} = \sigma_e \left[1 - 0,5 \left(\frac{\lambda}{\lambda_{lim}} \right)^2 \right] \quad \lambda_{lim} = \sqrt{\frac{\pi^2 E}{0,5 \sigma_e}}$$

6. CRITÉRIOS DE RESISTÊNCIA

Tensões equivalentes - usar para materiais dúcteis

$$\sigma_{eq} \leq \sigma_e$$

Tensão equivalente σ_e para estado duplo de tensões com $\sigma_z = \tau_{xz} = \tau_{yz} = 0$ e			
Critério	EDT qualquer	EDT com $\sigma_y = 0$	EDT expresso por σ_1 e σ_2
τ_{max}	$\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$		$\sigma_1 - \sigma_2$
Adotar o maior dos valores da coluna em módulo	$\frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$ $\frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$	$\sqrt{\sigma_x^2 + 4\tau_{xy}^2}$	σ_1 σ_2
τ_{oct}	$\sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2}$	$\sqrt{\sigma_x^2 + 3\tau_{xy}^2}$	$\sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$

7. VASOS DE PRESSÃO DE PAREDES FINAS

Tubo cilíndrico fechado

Tensão circumferencial $\sigma_C = \frac{pd}{2e} = \frac{pr}{e}$ p pressão
d diâmetro interno
e espessura da parede

Tensão longitudinal $\sigma_L = \frac{pd}{4e} = \frac{pr}{2e}$

Deformações $\Delta c = \frac{\pi pd^2}{2eE}$ $\Delta d = \frac{pd^2}{2eE}$

8. UNIDADES

$$1 \text{ Pa} = 1 \text{ N/m}^2 \quad 1 \text{ MPa} = 10^6 \text{ Pa} \quad 1 \text{ GPa} = 10^9 \text{ Pa}$$

$$1 \text{ MPa} = 1 \text{ N/mm}^2 \approx 10 \text{ kgf/cm}^2$$

$$1 \text{ rad} = 180^\circ/\pi$$

$$1'' = 1 \text{ pol} = 2,54 \text{ cm}$$