REAL-TIME IMPLEMENTATION OF H-infinity AND mu-CONTROLLERS

Article in Applied and Computational Mathematics · January 2015

4 authors:

Petko Petkov
Technical University of Sofia
268 PUBLICATIONS 1,957 CITATIONS

Tsonyo Slavov
Technical University of Sofia
58 PUBLICATIONS 251 CITATIONS

L. Mollov
Technical University of Sofia
5 PUBLICATIONS 30 CITATIONS

Jordan Kralev
Technical University of Sofia
15 PUBLICATIONS 50 CITATIONS

Appl. Comput. Math., V.14, N.1, 2015, pp.87-100

REAL-TIME IMPLEMENTATION OF $\mathcal{H}_\infty$ AND μ–CONTROLLERS

PETKO PETKOV [1], TSONIO SLAVOV[1], LYUBEN MOLLOV[1], JORDAN KRALEV[1]

ABSTRACT. Implementation of $\mathcal{H}_\infty$ and μ–controllers for real-time robust control of multivariable 4th order two input/two output laboratory plant is presented. The control system consists of a Spectrum Digital eZdspTMF28335 development kit with built in Texas Instruments TMS320F28335 Digital Signal Processor (DSP), digital to analogue signal converter, voltage divider, and a test analogue plant. An 8th order discrete-time $\mathcal{H}_\infty$–controller, and a 16th order μ-controller are implemented, with sampling frequency of 100 Hz, by using the DSP. The $\mathcal{H}_\infty$–controller design ensures efficient attenuation of the output disturbances, while the μ-controller also ensures robust performance of the closed-loop system in presence of parametric uncertainty in two plant parameters. Frequency domain analysis of the corresponding discrete-time closed-loop systems is performed. The implementation of high-order robust control laws is facilitated by the usage of technology for automatic code generation. An appropriate software in the MATLAB®/Simulink®environment is developed, which is embedded in DSP by using the Simulink Coder®. Experimental and simulation results are presented, which confirm that both control systems achieve the prescribed performance.

Keywords: Robust Control, $\mathcal{H}_\infty$–Design, μ-Synthesis, Digital Signal Processor, Real-Time Control.

AMS Subject Classification: 93C62.

1. INTRODUCTION

The Robust Control Theory is now a mature discipline that involves powerful methods for analysis and design of control systems in presence of signal and parameter uncertainties [1, 10, 11, 13, 19]. The most frequently used techniques for robust control design are the $\mathcal{H}_\infty$ design and the μ–synthesis [11]. The $\mathcal{H}_\infty$ optimization is usually preferred in robust design because it produces controllers of smaller order, which facilitates their implementation [7, 18]. As far as robust performance is concerned, $\mathcal{H}_\infty$ controllers are known to be more conservative than the μ–controllers when the underlying plant model contains structured or parametric uncertainty. In contrast, the μ–synthesis, which aims at minimization of the structured singular value [19], may ensure robust stability and robust performance in the presence of exogenous disturbances, noises, and different type of uncertainties. The high order of the controller obtained is usually pointed out as a disadvantage of μ–synthesis. However, with the appearing of powerful and cheap processors in the recent years, this peculiarity of the μ–synthesis does not pose a significant difficulty.

In contrast with the theoretical achievements, the practical implementation of robust control laws is still in its beginning. There are few real life applications of high order robust control laws reported in the literature (see for instance [3, 4, 6, 9]). The main obstacle of robust control laws implementation is the difficulties related to the development, testing and verification of the necessary real-time software, which is highly dependent on the type of digital controller platform used. These difficulties are reduced significantly using the recent technologies for automatic code generation and embedding implemented in the MATLAB®/Simulink®program

[1]Department of Systems and Control, Technical University of Sofia, 8 Kliment Ohridski Blvd., Bulgaria
e-mail: php@tu-sofia.bg

Manuscript received 23 October 2014.

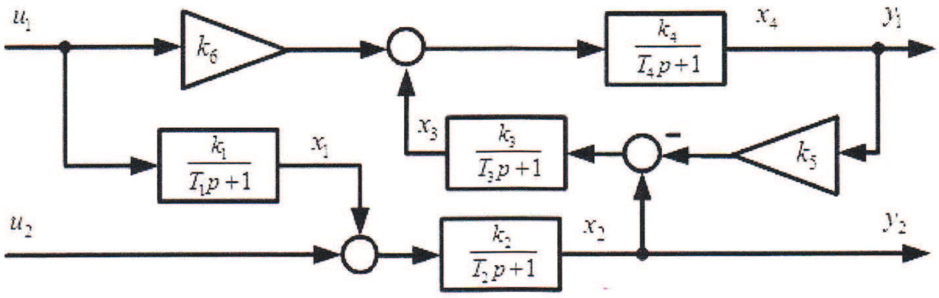

Figure 1. Block-diagram of the plant.

environment [17]. A real-time implementation of linear quadratic control law by using automatic code generation is reported in [12].

In this paper, we present the implementation of $\mathcal{H}_\infty$ and μ–controllers for real-time robust control of a multivariable 4th order two input/two output laboratory plant. The control system consists of a Spectrum Digital eZdspTMF28335 development kit with built in Texas Instruments TMS320F28335 Digital Signal Processor (DSP), digital to analogue signal converter, voltage divider, and a test analogue plant. An 8th order $\mathcal{H}_\infty$ controller, and a 16th order discrete-time μ–controller with sampling frequency of 100 Hz, are implemented by using the DSP. The $\mathcal{H}_\infty$–controller designed ensures efficient attenuation of the output disturbances, while the μ-controller ensures robust performance of the closed-loop system in the presence of parametric uncertainty in two plant parameters. Frequency domain analysis of the corresponding discrete-time closed-loop systems is performed. An appropriate software in the MATLAB®/Simulink®environment is developed, which is embedded in DSP by using the Simulink Coder®. Experimental and simulation results are presented, which confirm that both control systems achieved the prescribed performance.

The paper is organized as follows. In Section 2 we present the plant description which will be used in the subsequent control system design. An $\mathcal{H}_\infty$ controller is designed in Section 3, and a μ–controller is synthesized in Section 4. The implementation of these controllers by using DSP is discussed in Section 5. The experimental results of real-time controllers implementation are presented in Section 6. Finally, in Section 7 we give some conclusions related to the robust controllers implementation.

2. Plant description

The block-diagram of the analog two input/output plant under consideration is shown in Fig.1. The 4th order plant model is described by the state space equations

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{1}$$

where $x(t) = [x_1(t),\ x_2(t),\ x_3(t),\ x_4(t)]^T$ is the state vector, $u(t) = [u_1(t),\ u_2(t)]^T$ is the input vector, $y(t) = [y_1(t),\ y_2(t)]^T$ is the output vector,

$$A = \begin{bmatrix} -\frac{1}{T_1} & 0 & 0 & 0 \\ \frac{k_2}{T_2} & -\frac{1}{T_2} & 0 & 0 \\ 0 & \frac{k_3}{T_3} & -\frac{1}{T_3} & 0 \\ 0 & 0 & \frac{k_4}{T_4} & -\frac{1}{T_3} \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{k_1}{T_1} & 0 \\ 0 & \frac{k_2}{T_2} \\ 0 & 0 \\ \frac{k_4 k_6}{T_4} & 0 \end{bmatrix},$$

Table 1. Nominal plant parameters

Parameter	Value	Parameter	Value
k_1	1.0463	T_1	0.3308
k_2	1.5689	T_2	0.5291
k_3	0.4213	T_3	1.1142
k_4	3.0000	T_4	0.7063
k_5	3.0558		
k_6	1.8970		

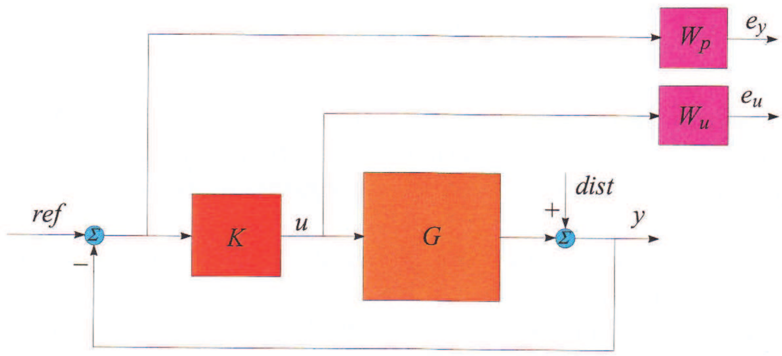

Figure 2. Closed-loop system with performance requirements.

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, D = I_{2\times 2},$$

k_1, $k_2, ...,$ k_6 are coefficients of proportionality, T_1, $T_2, ...,$ T_4 are time constants and $I_{m\times m}$ is the unit $m \times m$ matrix.

The model parameters are estimated by the identification procedure using the MATLAB® function lsqnonlin based on the nonlinear least square method.

The nominal model parameters determined are presented in Table 1.

3. Controller $\mathcal{H}_\infty$ design

The block-diagram of the closed-loop system, which includes the performance and control weighting functions corresponding to the $\mathcal{H}_\infty$ design is shown in Fig.2. The transfer function matrix of the plant is denoted by G and the controller transfer function matrix – by K. The system has a reference vector ref and an output disturbance vector $dist$.

The closed-loop system is described by the equations

$$\begin{aligned} y &= T_o r + S_o d, \\ u &= S_i K(r - d), \end{aligned} \tag{2}$$

where the matrix $S_i = (I + KG)^{-1}$ is the input sensitivity transfer function matrix, $S_o = (I + GK)^{-1}$ is the output sensitivity transfer function matrix, and $T_o = (I + GK)^{-1}GK$ is the output complementary sensitivity function. (Here and further on the reference vector is denoted for brevity by r, and the disturbance vector – by d.)

The weighted closed-loop system outputs e_y and e_u, satisfy the equation

$$\begin{bmatrix} e_y \\ e_u \end{bmatrix} = \Phi(s) \begin{bmatrix} r \\ d \end{bmatrix}, \tag{3}$$

where

$$\Phi(s) = \begin{bmatrix} W_p S_o & -W_p S_o \\ W_u S_i K & -W_u S_i K \end{bmatrix}. \tag{4}$$

The performance criterion in the given case requires the transfer function matrix (4), from the exogenous input signals r and d, to the output signals e_y and e_u, to be small in the sense of $\|\cdot\|_\infty$. The transfer function matrices W_p and W_u are used to reflect the relative importance of the different frequency ranges for which the performance requirements should be fulfilled.

The $\mathcal{H}_\infty$ design is done for the performance weighting function

$$W_p(s) = \begin{bmatrix} 0.9\frac{0.2s+1}{0.3s+10^{-4}} & 0 \\ 0 & 0.92\frac{0.18s+1}{0.30s+10^{-4}} \end{bmatrix}$$

and for the control weighting function

$$W_u(s) = \begin{bmatrix} 0.02\frac{0.005s+1}{0.001s+1} & 0 \\ 0 & 0.02\frac{0.005s+1}{0.001s+1} \end{bmatrix}.$$

The performance weighting transfer functions are chosen as low pass filters to suppress the output disturbance d, and the control weighting transfer functions are chosen as high pass filters with appropriate bandwidth in order to impose constraints on the high frequency spectrum of the control actions [5].

In order to determine a discrete-time $\mathcal{H}_\infty$ controller, the extended open-loop system that includes the plant and weighting functions is discretized with sampling period of 0.01 s, corresponding to sampling frequency of 100 Hz. Using the function `hinfsyn`, an 8th order discrete-time controller is obtained, which reduces the $\mathcal{H}_\infty$ norm of the closed-loop system matrix to the value $\gamma = 0.8690$.

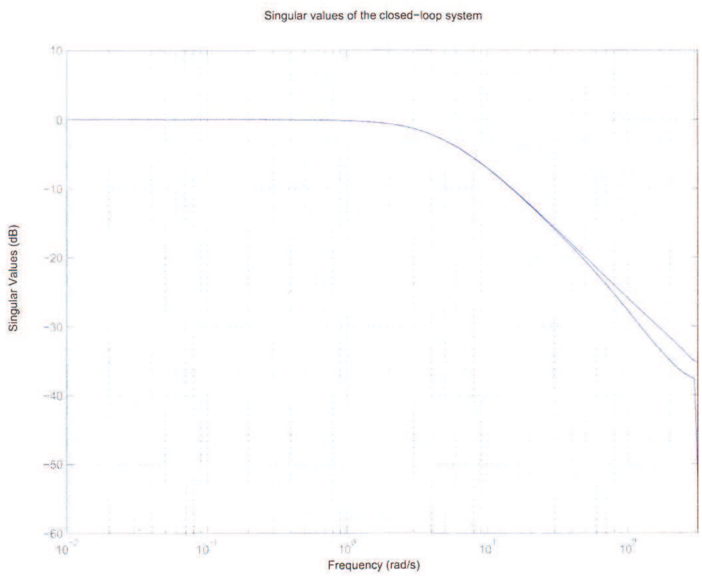

Figure 3. Singular values of the complementary sensitivity function.

The singular values of the complementary sensitivity function T_o plotted in Fig.3 show that the closed-loop system will track accurately references with frequencies up to 5 rad/s.

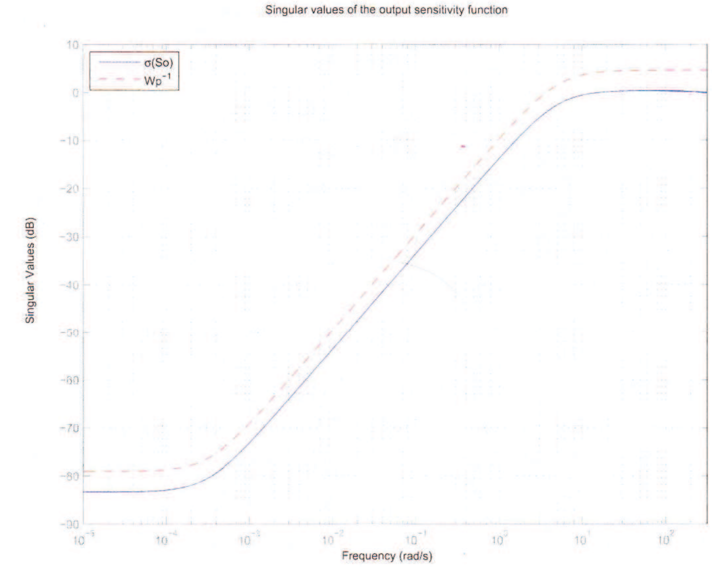

Figure 4. Singular values of the output sensitivity function.

In Fig.4 we show the singular value plot of the output sensitivity function S_o, compared with the singular value plot of the inverse performance weighting function W_p^{-1}. The plot confirms that the system attenuates low frequency output disturbances well (for instance, a sinusoidal disturbance with frequency 0.02 rad/s will be attenuated more than 100 times).

4. CONTROLLER μ–SYNTHESIS

In order to perform a design in the framework of the μ–synthesis, further on it is assumed that the coefficient k_1 is a subject of 25% change around the nominal value, and the parameter T_4 undergoes 40% change around its nominal value. In this way, the plant involves structured (parametric) uncertainty, which is set by using the function `ureal` from Robust Control Toolbox®3 The synthesis of the μ–controller aims to achieve robust stability and robust performance of the closed-loop system in the presence of such uncertainty, and ensures acceptable suppression of the disturbances and noises.

The μ–synthesis is done for performance weighting function

$$W_p(s) = \begin{bmatrix} 0.92\frac{0.15s+1}{0.24s+10^{-4}} & 0 \\ 0 & 0.92\frac{0.18s+1}{0.30s+10^{-4}} \end{bmatrix}$$

and for control weighting function

$$W_u(s) = \begin{bmatrix} 0.02\frac{0.005s+1}{0.001s+1} & 0 \\ 0 & 0.02\frac{0.005s+1}{0.001s+1} \end{bmatrix}.$$

These weighting functions are slightly different from the corresponding weighting functions in the case of $\mathcal{H}_\infty$ design in order to achieve an acceptable tradeoff between robustness and performance of the closed-loop system. The μ–synthesis is done by using the Robust Control Toolbox®3 function `dksyn` [2]. Four iterations are performed that decrease the maximum value of μ to 1.000. The final controller obtained is of the 16th order.

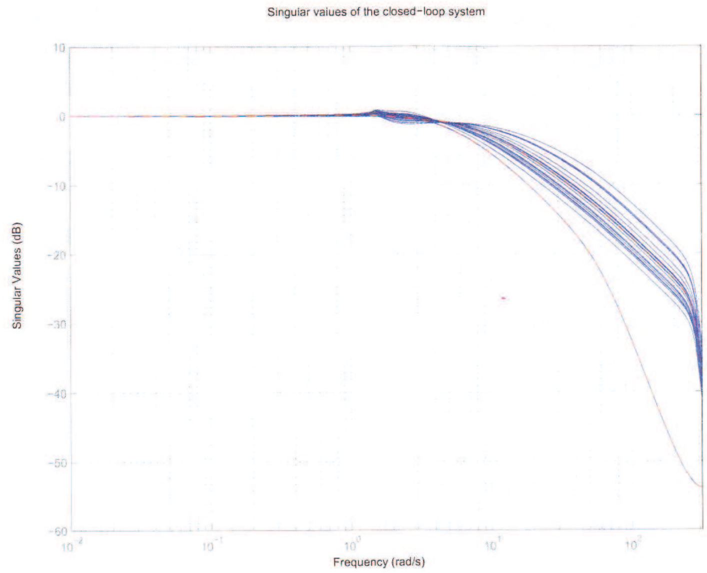

Figure 5. Singular values of the complementary sensitivity function.

The singular values of the complementary sensitivity function T_o, of an uncertain closed-loop system plotted in Fig.5, show that similarly to the case of the $\mathcal{H}_\infty$, controller the closed-loop system will accurately track references with frequencies up to 5 rad/s.

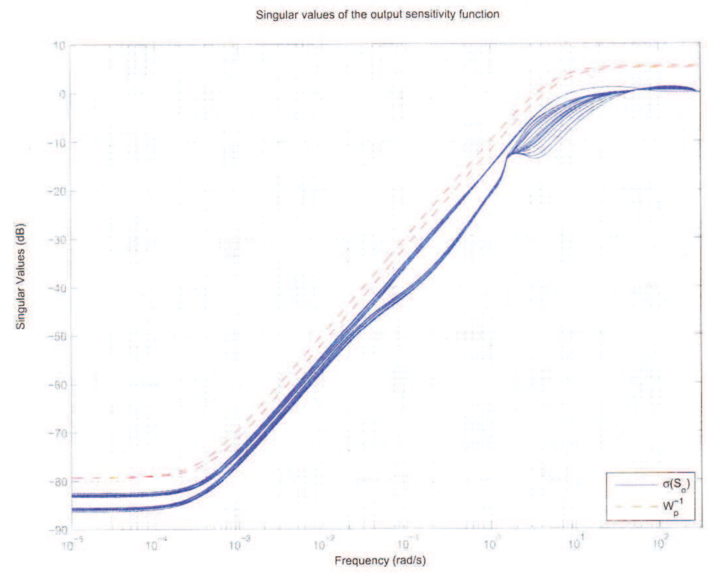

Figure 6. Singular values of the output sensitivity function.

In Fig.6 we show the singular value plot of the output sensitivity function S_o of the uncertain closed-loop system, compared with the singular value plot of the inverse performance weighting function W_p^{-1}. The plot confirms that the system attenuates low frequency output disturbances well.

5. Implementation of the robust controllers

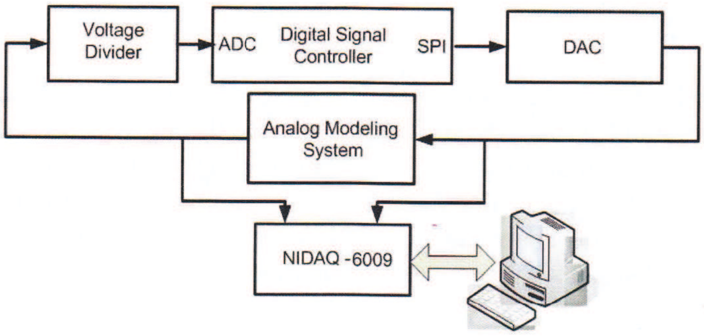

Figure 7. Block-diagram of the control system with DSP.

The block-diagram of the developed control system with DSP is shown in Fig.7. The block "Digital Signal Controller" (DSC) is the Spectrum Digital eZdspTMF28335 development board with an integrated Digital Signal Processor Texas Instruments TMS320F28335 [14]. This controller works at 150 MHz, and may perform single precision (32-bit) computations by using FPU (Floating-Point Unit). It has 68K bytes on-chip RAM, 256K bytes off-chip SRAM memory, 512K bytes on-chip Flash memory, and on-chip 12 bit Analog to Digital (A/D) converter with 16 input channels.

The two input/output plant (1) is modeled on an analog modeling board, consisting of a bus on which functional blocks are operating independently. The bus is powered by a DC power supply. The linear range of input-output signals is ± 10 V. The main linear blocks are adder, differentiator, inverter, integrator, aperiodic unit, and gain unit. The analog system makes it easy to model plants with a time constant up to 1 s, and gain factor up to 10.

The block "Voltage Divider" is a specially designed dual channel board, which linearly converts two input analog signals with range $-5 \div 5$ V, into two analog output signals with range $0 \div 3$ V.

The block "DAC" is a digital to analogue converter DAC8734EVM produced by Texas Instruments [15]. The DAC is a 16-bit, quad-channel, and can be configured to outputs 10V, 5V, 0V to 20V, or 0V to 10V. It features a standard high-speed serial peripheral interface (SPI) that operates at clock rates of up to 50MHz to communicate with the DSP.

The block "NIDAQ-6009" is a specialized module for data acquisition NIDAQ-6009 of National Instruments [8]. A specially developed software provides the user interface, connection, and exchange input/output data with a standard PC in real time.

The control algorithm is embedded and runs with frequency 100 Hz on the DSC. The software development environment includes MATLAB®v. 7.11.0.584 (R2010b), Simulink®v. 7.6, Simulink Coder®v. 7.6 [17], Embedded Coder®v. 5.6 [16], Microsoft Visual C++ v. 8.0, and Code Composer Studio®(CCS) v. 3.3. A technology for automatic generation and embedding the code using the Simulink Coder®is implemented. The main advantages of this technology are the relatively easy implementation of complex control algorithms, and the short time to translate the control algorithm from the working simulation environment to the real working environment with physical plants, reducing the overall time for application testing and verification of the developed algorithm.

6. Experimental results

The controllers designed in Sections 3 and 4 are implemented using DSP, and tested experimentally. In this Section, we show the experimental transient responses of the closed-loop system for both controllers compared with the corresponding simulation results. The experiments are done for a zero reference and a sinusoidal disturbance with magnitude 1 V and frequency 1 rad/s which is added to the first output of the system at the 5th second. In order to assess the influence of the processor precision, the experiments are performed using single as well as double precision arithmetic.

6.1. **Experiment with the $\mathcal{H}_\infty$ controller.** The output and control signals from experiments and simulation are shown in Fig.8-11 (here and further on all signals are measured in volts). After the initial deviation at the 5th second, the first output quickly decreases, and oscillations with magnitude 0.12 V are settled, i.e., the disturbance is attenuated approximately 10 times. The steady-state oscillations around the equilibrium point of the second output are negligible. The Figures displaying control signals show a good coincidence between experimental and simulation results.

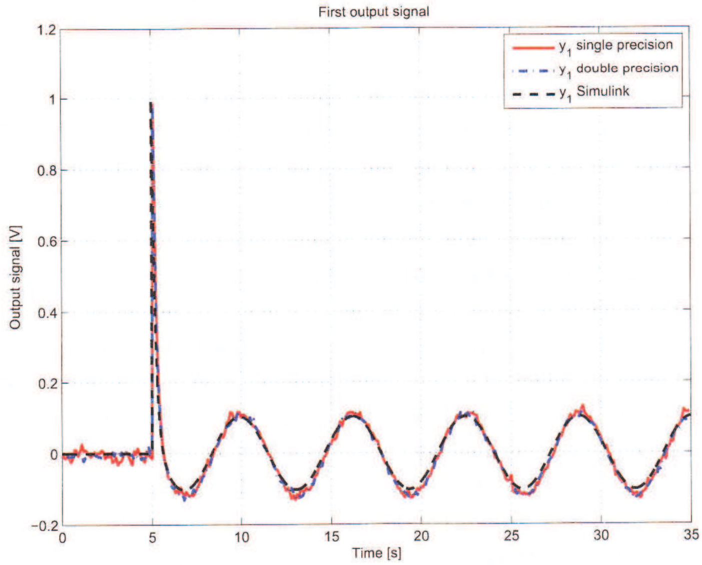

Figure 8. First output signal.

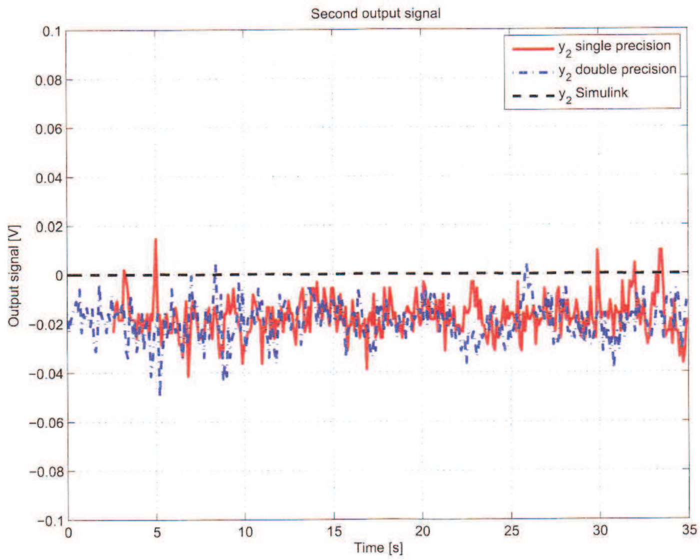

Figure 9. Second output signal.

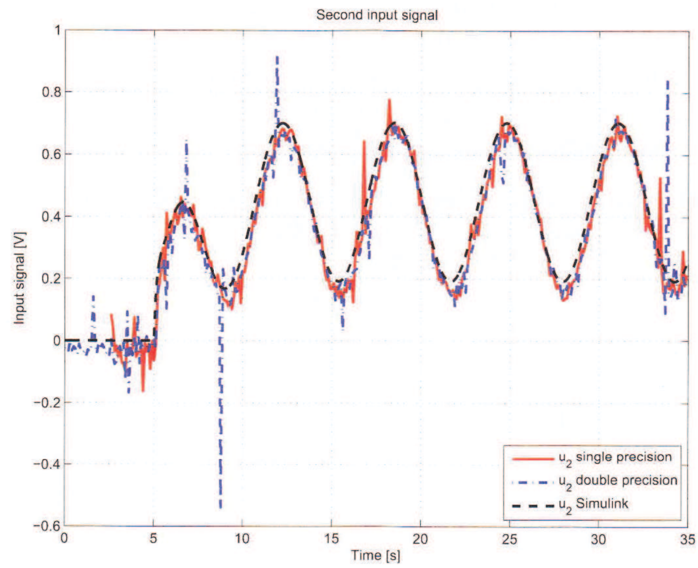

Figure 11. Second control signal.

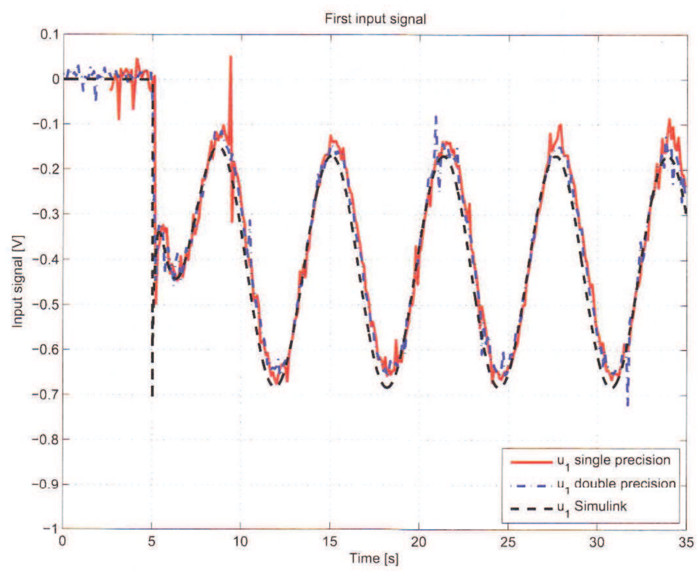

Figure 10. First control signal.

Table 2. Experimental errors – $\mathcal{H}_\infty$ controller

Quantity	Mean value	Variance
$y_1^{single} - y_1^{sim}$	-3.94×10^{-3}	3.37×10^{-3}
$y_2^{single} - y_2^{sim}$	-1.76×10^{-2}	6.39×10^{-5}
$y_1^{double} - y_1^{sim}$	-5.29×10^{-3}	3.18×10^{-3}
$y_2^{double} - y_2^{sim}$	-1.96×10^{-2}	6.66×10^{-5}

To estimate the accuracy of the experimental results, we computed the mean values and the variances of the differences between the experimental values of the variables with the corresponding values obtained by the help of MATLAB®using double precision arithmetic. The results for the $\mathcal{H}_\infty$ controller are shown in Table 2, where the superscripts single and double denote the experimental results obtained by using the respective processor precision, and the superscript sim denotes the results obtained by simulation. Obviously, in the given case the processor floating point errors do not dominate the experimental errors, which result from uncertain parameters, nonlinearities, and noises.

6.2. **Experiment with the μ-controller.** The experiments with the μ-controller are done for the same conditions as in the case of the $\mathcal{H}_\infty$ controller.

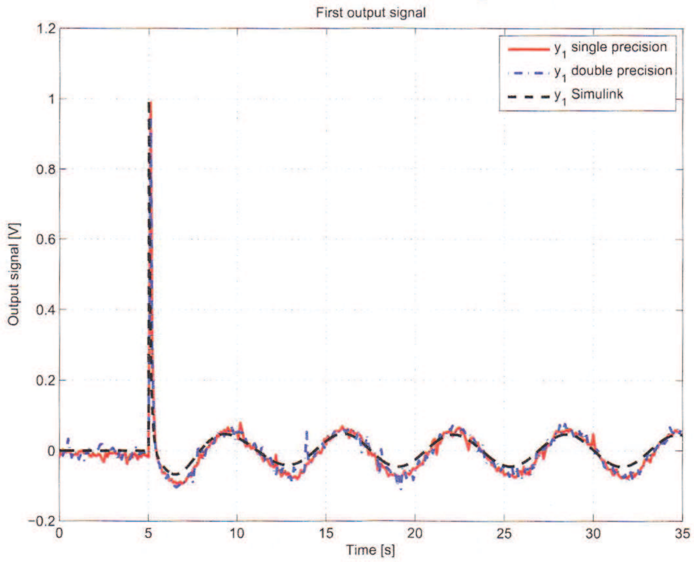

Figure 12. First output signal.

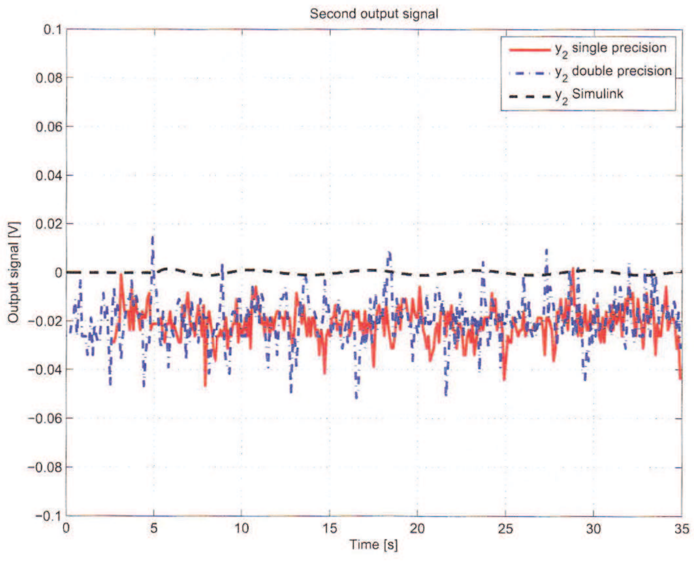

Figure 13. Second output signal.

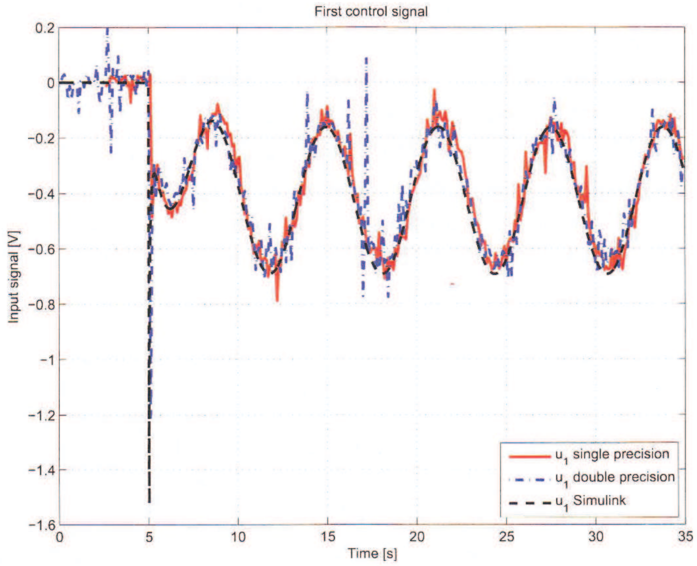

Figure 14. First control signal.

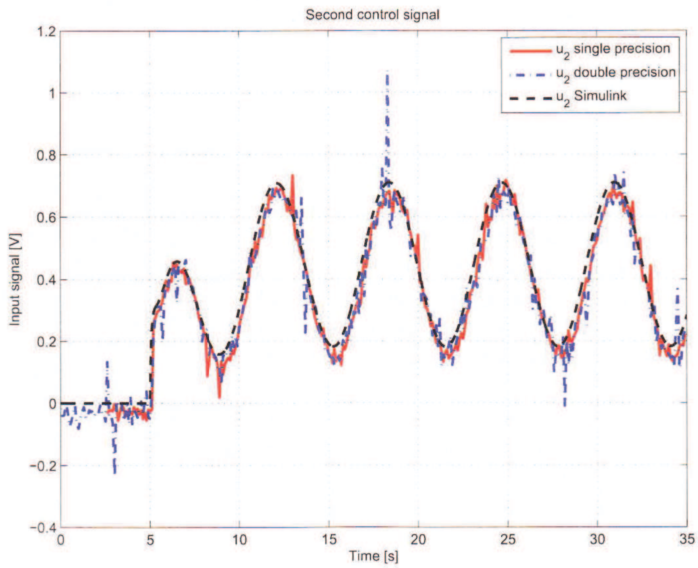

Figure 15. Second control signal.

The output and control signals from simulation and experiments are shown in Fig.12-15. It is seen from the Figures displaying the output signals that the experimental signals coincide well with the output signals obtained by simulation. It may be observed that the magnitude of the steady-state error in the first output is smaller than the error in case of the $\mathcal{H}_\infty$ controller.

Table 3. Experimental errors – μ – controller

Quantity	Mean value	Variance
$y_1^{single} - y_1^{sim}$	-7.81×10^{-3}	3.29×10^{-3}
$y_2^{single} - y_2^{sim}$	-2.10×10^{-2}	4.95×10^{-5}
$y_1^{double} - y_1^{sim}$	-5.08×10^{-3}	3.05×10^{-3}
$y_2^{double} - y_2^{sim}$	-1.93×10^{-2}	1.03×10^{-4}

As in the case of the $\mathcal{H}_\infty$ controller, the plots of control signals show the presence of some small deviations from simulation results. This is explained by the properties of the transfer function matrix KS_o which amplifies the noises in the control signals.

The mean values and the variances of the experimental errors in the case of using the μ–controller are shown in Table 3. It is seen from the Table that the errors corresponding to the usage of the double precision processor arithmetic are slightly smaller than the errors corresponding to the usage of single precision arithmetic. This indicates that in the case of implementing the μ–controller, the errors due to uncertain parameters, nonlinearities, and noises have a smaller effect than the processor floating point errors.

7. Conclusions

This paper presents a system for robust real-time control of a fourth order plant using a Digital System Processor. A specialized software for automatic generation of the control code is developed that facilitates the implementation of high order control algorithms for multivariable plants. The main conclusion is that the existing technologies allow us to easily implement high order controllers that ensures robust stability and robust performance of the closed-loop system. The experimental results confirm the performance of the closed-loop system, and demonstrate the superiority of the μ-controllers over the $\mathcal{H}_\infty$ controllers. The control code may be easily modified for usage with other plants.

References

[1] Aliev, F.A., Larin, V.B. *Optimization of Linear Control Systems: Analytical Methods and Computational Algorithms*, Gordon and Breach Science Publishers, Amsterdam, 1998, 261 p.

[2] Balas, G., Chiang, R., Packard, A., Safonov, M. *Robust Control Toolbox® 3 User's Guide*, The MathWorks, Inc., Natick, MA, 2013, *Available from* `http://www.mathworks.com/help/pdf_doc/robust/robust_ug.pdf`

[3] Bautista-Quintero, R., Pont, M.J. Implementation of H-infinity control algorithms for sensor-constrained mechatronic systems using low-cost microcontrollers, *IEEE Transactions on Industrial Informatics*, V.4, N.3, 2008, pp.175-184.

[4] La Civita, M., Papageorgiou, G., Messner, W.C., Kanade, T. Design and flight testing of a high-bandwidth $\mathcal{H}_\infty$ loop shaping controller for a robotic helicopter, *Journal of Guidance, Control, and Dynamics*, V.29, N.2, 2006, pp.485-494.

[5] Gu, D.-W., Petkov, P.Hr., Konstantinov, M.M. *Robust Control Design with MATLAB®, 2nd ed.*. Springer-Verlag, London, 2013, 468 p.

[6] Howlader, A.M., Urasaki, N., Yona, A., Senjyu, T., Saber, A.Y. Design and implement a digital H_∞ robust controller for a MW-class PMSG-based grid-interactive wind energy conversion system, *Energies*, V.6, N.4, 2013, pp.2084-2109.

[7] Karagül, A.E., Demir, O., Özbay, H. Computation of optimal H_∞ controllers and approximations of fractional order systems: A Tutorial review, *Appl. Comput Math.*, V.12, N.3, 2013, pp.261-288.

[8] National Instruments. *NI USB-6009 14-Bit, 48 kS/s Low-Cost Multifunction DAQ*. *Available from:* `http://sine.ni.com/nips/cds/view/p/lang/en/nid/201987` [Accessed 15 March 2013].

[9] Raafat, S.M., Akmeliawati, A.I. Robust H_∞ controller for high precision positioning system, design, analysis, and implementation, *Intelligent Control and Automation*, V.3, N.3, 2012, pp.262-273.

[10] Sánchez-Peña, R.S., Sznaier, M. *Robust Systems. Theory and Applications*, John Wiley & Sons, Inc., New York, 1998, 490 p.

[11] Skogestad, S., Postlethwaite, I. *Multivariable Feedback Control, 2nd ed.*, John Wiley and Sons Ltd, Chichester, UK, 2005, 572 p.

[12] Slavov, T., Mollov, L., Petkov, P. Real-time quadratic control using Digital Signal Processor, *TWMS Journal of Pure and Applied Mathematics*, V.3, N.2, 2012, pp.145-157.

[13] Stoorvogel, A.A., *The $\mathcal{H}_\infty$ Control Problem: A State Space Approach*, Prentice Hall, Englewood Cliffs, NJ, 1992, 275 p.

[14] Spectrum Digital, Inc. *eZdspTMF28335 Technical Reference, 2007, Available from:* `http://c2000.spectrumdigital.com/ezf28335/docs/ezdspf28335c_techref.pdf`

[15] Texas Instruments. *DAC8734 Evaluation Module, Available from:* `http://www.ti.com/tool/dac8734evm`

[16] The MathWorks, Inc. *Embedded Coder.* Natick, MA, 2013, *Available from:* http://www.mathworks.com/products/datasheets/pdf/embedded-coder.pdf

[17] The MathWorks, Inc. *Simulink Coder.* Natick, MA, 2013, *Available from:* http://www.mathworks.com/products/datasheets/pdf/simulink-coder.pdf

[18] Yonchev, A.S., Konstantinov, M.M., Petkov, P.H. Linear perturbation bounds of the continuous-time LMI based H_∞ quadratic stability problem, *Appl. Comput Math.*, V.12, N.2, 2013, pp.133-139.

[19] Zhou, K., Doyle, J.C., Glover, K. *Robust and Optimal Control*, Prentice Hall, Upper Saddle River, NJ, 1996, 596 p.

Petko Petkov - received his M.S. and Ph.D. degrees in control engineering from the Technical University of Sofia in 1971 and 1979. Since 1995, he has been a Professor of Control Theory in the Department of Systems and Control at the Technical University of Sofia. He is coauthoring Computational Methods for Linear Control Systems (Prentice Hall, Hemel Hempstead, U.K., 1991), Perturbation Theory for Matrix Equations (North-Holland, Amsterdam, 2003), and Robust Control Design with MATLAB (Springer, London, 2005, 2013).

Tsonyo Slavov - graduated from the Department of Systems and Control of the Technical University of Sofia in 2002. He received his Ph.D degree in control engineering from the Technical University of Sofia in 2007. Presently, he is an Associated Professor in the Department of Systems and Control at the Technical University of Sofia.

Luben Mollov - graduated from the Department of Systems and Control of the Technical University of Sofia in 2008. Presently, he is an assistant professor in the same department.

Jordan Kralev - has an M.S. degree in Automatics, Information and Control Technologies. Since 03.2013 he is a Ph.D. student in the Department of Systems and Control, Technical University of Sofia. His works are in the field of implementation of complex control structures with programmable platforms.