• Boxplot:

$$-Q_1 = N \times 0.25$$

$$-Q_2 = N \times 0.5$$

$$-Q_3 = N \times 0.75$$

$$-IQR = Q_3 - Q_1$$

$$-Bounds = [Q_1 - 1.5 \times IQR, Q_3 + 1.5 \times IQR]$$

• Pearson =
$$\frac{\Sigma(y_{1i} - \bar{y}_1)(y_{2i} - \bar{y}_2)}{\sqrt{\Sigma(y_{1i} - \bar{y}_1)^2 \times (y_{2i} - \bar{y}_2)^2}}$$

• Spearman: Assign ranks and apply Pearson formula. Example: $[20, 10, 20, 30, 20] \rightarrow [3, 1, 3, 5, 3]$

• Information Gain: $IG(y_{out}|y_i) = E(y_{out}) - E(y_{out}|y_i)$

• Entropy: $E(y) = -\sum P(x_i) \log(P(x_i))$

• Normalization:

- MinMax:
$$\frac{y_i - min}{max - min}$$

- Standardization:
$$\frac{y_i - \mu}{\sigma}$$

• Binarization:

- Range (equal width): Depends on variable range Example: $y \in [-1, 1] : [0.2, -0.1, 0.6] \rightarrow [1, 0, 1]$

Frequency (equal depth): Depends on variable mean Example: $\bar{y} = 25 : [10, 40, 30, 20] \rightarrow [0, 1, 1, 0]$

			True		
			A	В	\mathbf{C}
:	d	Α	TA	FA	FA
	rec	В	FP	TB	FB
	Ь	С	FC	FC	TC

• Metrics:

- Accuracy =
$$\frac{TP + TN}{total}$$

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$$\frac{TP + TN}{total}$$

- Error rate = $1 - Accuracy = \frac{FP + FN}{total}$

$$- \text{ Recall} = \frac{TP}{TP + FN} \text{ (Sensitivity)}$$

- Fallout =
$$\frac{TN}{N} = \frac{TN}{TN + FP}$$
 (Specificity)

$$- \text{ Precision} = \frac{TP}{TP + FP}$$

$$-F_1 = \frac{TP}{TP + \frac{1}{2}(FP + FN)}$$

• Error:

$$- \text{MSE} = \sum (Z - \hat{Z})^2$$

$$- RMSE = \sqrt{MSE}$$

$$- \text{MAE} = \sum |Z - \hat{Z}|$$

• Decision trees:

- 1. Choose feature with highest IG.
- 2. Split dataset by that feature, create leaves if necessary.
- 3. Repeat until unable to proceed.

Prune: (Given a twig)

1. Count it's leaves labels.

Example: $\#A = 5, \ \#B = 6$

- 2. Remove it's leaves.
- 3. Relabel twig as a leaf.

Example: B(6/11), #B > #A

• Gaussian Distribution:

- Mean:
$$\mu = \frac{1}{n} \sum y_i$$

- Standard Deviation:
$$\sigma = \sqrt{\frac{\sum (y_i - \mu)^2}{n-1}}$$

$$-P(y|\mu,\sigma) = \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot \exp\left(-\frac{1}{2} \left(\frac{y-\mu}{\sigma}\right)^2\right)$$

• Naive Bayes:

- MAP:
$$P(C|x) = \frac{P(C)P(x|C)}{P(x)}$$

$$- ML: P(C|x) = P(x|C)$$

1. Calculate probability for each class.

2. Calculate P(x|C) for each attribute and each class.

• K-Nearest Neighbors:

1. Distances: (for n variables)

– Manhattan:
$$\sum |y_{1i} - y_{2i}|$$

- Euclidean:
$$\sqrt{\sum (y_{1i} - y_{2i})^2}$$

- Euclidean:
$$\sqrt{\sum_{i=1}^{N} (y_{1i} - y_{2i})^2}$$

- Cosine: $\sqrt{\sum_{i=1}^{N} y_{1i}} \sqrt{\sum_{i=1}^{N} y_{2i}^2}$

2. Choose K nearest neighbors.

3. Classify using mean if variable is numeric, or mode if it is categoric.

4. If weighted, divide by weight.

• Regressions:

- Linear:
$$W = (X^T X)^{-1} X^T Z$$

- Ridge:
$$W = (X^T X + \lambda I)^{-1} X^T Z$$

$$\hat{Z} = a(W^T X), \ a \leftarrow \text{activation function}$$

Se $Z \neq \hat{Z} \longrightarrow W' = W + \eta(Z - \hat{Z})X$

• Neural Networks (MLP):

- Forward:
$$x^{[0]} \to z^{[1]} = w^{[1]}x^{[0]} + b^{[1]} \to x^{[1]} = a(z^{[1]}) \to \dots \to z^{[i]} = w^{[i]}x^{[i-1]} + b^{[i]} \to x^{[i]} = a(z^{[i]}) \to E$$

- Backward:

*
$$\delta^{[last]} = \frac{\partial E}{\partial x^{[last]}} \circ \frac{\partial x^{[last]}}{\partial z^{[last]}}$$

*
$$\delta^{[last]} = \frac{\partial E}{\partial x^{[last]}} \circ \frac{\partial x^{[last]}}{\partial z^{[last]}}$$
* $\delta^{[i]} = \left(\frac{\partial z^{[i+1]}}{\partial x^{[i]}}\right)^T \cdot \delta^{[i+1]} \circ \frac{\partial x^{[i]}}{\partial z^{[i]}}$

$$* w^{[i]'} = w^{[i]} - \eta \frac{\partial E}{\partial w^{[i]}}$$

$$* \ \frac{\partial E}{\partial w^{[i]}} = \delta^{[i]} \cdot \left(\frac{\partial z^{[i]}}{\partial w^{[i]}}\right)^T$$

*
$$b^{[i]'} = b^{[i]} - \eta \frac{\partial E}{\partial b^{[i]}}$$

*
$$\frac{\partial E}{\partial b^{[i]}} = \delta^{[i]}$$

tangent

*
$$\frac{\partial z^{[i+1]}}{\partial x^{[i]}} = w^{[i+1]} \qquad \frac{\partial z^{[i]}}{\partial w^{[i]}} = x^{[i-1]} \qquad \frac{\partial z^{[i]}}{\partial b^{[i]}} = 1$$

$\partial x^{[i]}$	$\partial w^{[i]}$	$\partial b^{[\imath]}$
Name	Error function	$\frac{\partial E}{\partial x^{[i]}}$
Squared Error	$\frac{1}{2} \left(x^{[i]} - t \right)^2$	$x^{[i]} - t$
Cross-entropy	$-\sum_{i=1}^{n} t_i \log \left(x_i^{[i]} \right)$	$x^{[i]} - t$
Name	Activation function	$\frac{\partial x^{[i]}}{\partial z^{[i]}}$
Sigmoid	$\frac{1}{1+e^{-x}}$	$\sigma(z^{[i]})(1-\sigma(z^{[i]}))$
Hyperbolic	$\frac{e^x - e^{-x}}{e^x + e^{-x}}$	$1 - \tanh\left(z^{[i]}\right)^2$

 $e^{x} + e^{-x}$

• Gaussian Mixture:

aussian Mixture:
$$- \text{ Covariance Matrix: } \Sigma = \begin{bmatrix} TODO & TODO \\ symmetric & TODO \end{bmatrix}$$

$$- P(y|\mu, \Sigma) = \frac{1}{(2\pi)^{d/2} |\Sigma_k|^{1/2}} \cdot exp\left(-\frac{1}{2}(y-\mu)^T \Sigma^{-1}(u-\mu)\right)$$

• K-Means:

TODO

• EM: TODO