Optimization and Engineering
https://doi.org/10.1007/s11081-020-09580-1

RESEARCH ARTICLE

A mathematical model for the scheduling and definition of mining cuts in short-term mine planning

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Received: 5 May 2020 / Revised: 10 September 2020 / Accepted: 9 November 2020

Abstract
Short-term open pit planners have to deal with the task of designing a feasible production schedule. This schedule must fulfill processing, mining and operational constraints and, at the same time, maximize the profit or total metal produced. It also must comply with the long-term production schedule and must incorporate new blasthole sampling data. This task is performed with little support of optimization tools, and therefore, there is a risk of generating suboptimal results. Several approaches have been proposed in the literature to deal with these issues, either generating operational mining cuts or obtaining a mining schedule to fulfill the short-term constraints. However, an integrated approach has remained an open challenge. In this paper, we propose an optimization model to tackle the operational and scheduling issues simultaneously. The model defines the mining cut configuration and the production schedule in the short-term. It is based on representative Selective Mining Units (SMUs) as the potential locations of the mining cuts and then each SMU is assigned to one of these locations. We tested the model with a real case study, and it was able to generate mining cuts and an extraction sequence fulfilling mining, processing and operational constraints, as well as access restrictions given by the ramp location in each bench. The mining cut design captured most of the profit, and thus it can be used as a guide for the short-term mine planner. The location of the representative SMUs and the precedence definition both impact the mining cut configuration, and future research could address how to incorporate different operational considerations and strategies on the location of these representatives.

Keywords Short-term mine planning · Scheduling · Mining cuts · Mixed-integer programming

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Published online: 03 January 2021

1 Introduction

Short-term mine planning in open pit mining still faces several challenges to materialize the long-term plan in the mining operation. One of these challenges is the sequencing and scheduling of the mining units in the short-term, in such a way that they meet the production targets and estimated profit inherited from long-term plans, but also attend to additional constraints given by the operation of loading and hauling equipment.

During the short-term scheduling process, new information from blasthole sampling becomes available, which provides a better estimate of the deposit's true grade. Because the blasthole spacing is usually smaller than the block size used in the long-term planning, each block is divided into Selective Mining Units (SMUs) attached with the grade of their nearest blasthole sample. Therefore, there is a better granularity on the grade distribution that can be used by the short-term planner to define the best processing route for each unit, to maximize the metal produced or the economic profit.

However, the loading equipment imposes a new operational constraint related to the selectivity of the extraction: the loading equipment selectivity and blasting space requirements commonly exceed the SMU dimensions. Therefore, even if the grade is known, the loading operation cannot be done on an SMU basis. Thus, to obtain a mineable schedule, the planner defines *mining cuts* as the union of adjacent SMUs that are mined in the same period, sent to the same destination, and offer enough operational space to be extracted by the mining equipment. These cuts are scheduled to fulfill the processing and operational constraints in the short-term plan.

Figure 1 depicts the change of support from blocks to SMUs and then to mining cuts, and the process of deciding their destination and schedule. The process can be abstracted as follows.

Construction of the mining cuts The first challenge of short-term planning is the construction of the mining cuts by aggregating SMUs. This step is a manual procedure based on the planner's sole expertise and aims to generate feasible mining cuts from the operational point of view.

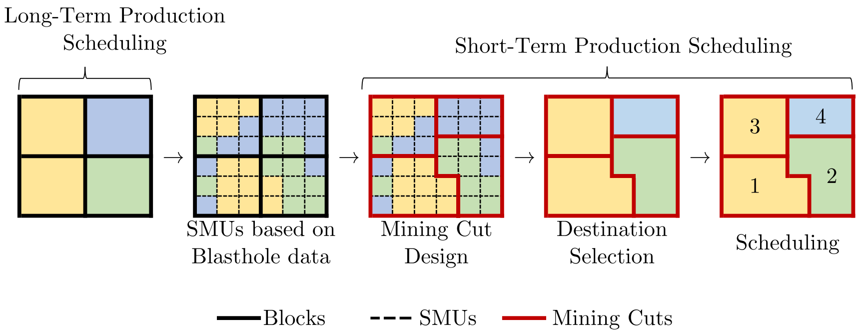


Fig. 1 A conceptual model representing the stages from a long-term plan based on blocks, down to a short-term plan based on mining cuts. Colors represent different destinations

Destination of the mining cuts The planner assigns a destination to each mining cut. As an input, a cut-off grade map defines the destination of each SMU. During this process, although the planner attempts to maintain the guideline given by the cut-off grade map, destination changes of individual SMUs are necessary to obtain feasible mining shapes.

Sequencing and scheduling of mining cuts At this stage, the planner schedules the mining cuts to fulfill the mine and processing production rate, blending constraints, and the access restrictions imposed by the ramp design. The schedule must comply with these considerations daily, weekly, and monthly, and changes according to the mine operation advancement.

Notice that the challenges described above are useful for understanding the complexity of the process and modeling it. However, these stages are not strictly sequential. Indeed, for the construction of the mining cuts, the planner will look at the grades and other attributes of the SMUs, to be able to select the right destination afterward. Also, a given mining cut definition may be unfeasible in terms of production targets at the scheduling stage. In this case, the planner redefines the mining cuts to obtain a feasible schedule. Moreover, an additional step is performed manually to smooth the contours of each cut to account for the loading equipment's movement.

As a result, finding a feasible cut definition and schedule that meets the long-term targets and complies with operational constraints strongly depends on the user's expertise. This task may become in a trial and error process, which is highly time-consuming and suboptimal.

The literature has studied the problem of short-term scheduling using operations research and clustering techniques to assist the mine planner in this time-intensive task. However, considering all the challenges together has not been appropriately addressed so far. Because of that, in this work we tackle these issues at the same time through mathematical programming. For this, we introduce an optimization model that addresses all the challenges described before in a unique optimization problem, i.e., we devise a mathematical model that takes the SMUs, geometrical constraints for the mining cuts and production constraints as an input. As a result, it defines the mining cut configuration, selects their destinations, and schedules the mining cuts for production.

The rest of the paper is organized as follows: Sect. 2 presents the literature review. Section 3 presents the optimization model introduced in this work. Section 4 shows a study case and the discussion related to the results obtained by the model. Finally Sect. 5 contains the conclusions of this work and possible extensions and recommendations.

2 Literature review

In this section, we review the main results in the area. We separate the works related to the cut definitions from the studies about scheduling in short-term mine planning. We conclude with a summary and comments about these results.

2.1 Operational shapes for short-term mine planning

Obtaining operational shapes from the selective mining units in the short-term has been addressed in different ways. One approach related to this matter is the separation between ore and waste in each bench considering the loading equipment selectivity, often called *dig-limit optimization* because the main objective is to define the feasible limits between materials. Norrena and Deutsch (2001) proposed an algorithm based on simulated annealing, where different initial polygons are defined, and each vertex can move to maximize a function of profitability. The algorithm introduced digability as a penalty function to avoid acute angles in the polygons' definition. This work was later tested and extended by Norrena et al. (2002) and Neufeld et al. (2003), introducing grade uncertainty in the profit definition and studying different digability penalization factors. Richmond and Beasley (2004) considered uncertainty in the polygon definition, where an initial shape separates ore and waste, and a local search heuristic looks for better solutions considering risk measures.

Isaaks et al. (2014) proposed a methodology focused on complying with a minimum mining width. The author defined this width as the minimum number of adjacent SMUs that must share the same destination. A simulated annealing algorithm defines the destination of each SMU while complying with the mining width. The algorithm minimizes a loss function to avoid misclassification of each unit. Ruiseco et al. (2016) proposed an optimization model to define the destination of each SMU, maximizing the mining profit and penalizing the aggregation of SMUs with different destinations in a given area. The model is solved using a genetic algorithm since the aggregation penalty is non-linear.

Another optimization model was proposed by Sari and Kumral (2017), where they defined valid "frames" as an aggregation of SMUs sent to the same destination and complying with operational constraints. The optimization model defines the frames associated with each SMU while maximizing the mining revenue and defining the dig-limits. Deutsch (2017) proposed an optimization model and a specialized algorithm based on branch and bound and simulated annealing, to select a set of structuring elements used as the minimum mining width shape given by the operational constraints in the short-term. Vasylchuk and Deutsch (2019) proposed a method based on a floating selection frame, where the operational constraint is introduced as the rectangular size of the frame. The algorithm provided an initial classification based on this moving frame, and an additional step improved the initial solution and fixed the locations that did not satisfy the excavation constraints.

The definition of mining cuts based on clustering techniques has been studied as well. Tabesh and Askari-Nasab (2011) implemented a hierarchical clustering algorithm to generate mining cuts based on a similarity index, which depends on distance, rock type, destination, and metal grade, among other block attributes. The resulting clusters are used as an input to a mixed-integer linear program to obtain the short/medium term schedule. Tabesh and Askari-Nasab (2013) presented several hierarchical clustering applications, with different similarity indices and weights of the block attributes. A post-processing procedure improved the solution in terms of cluster size and similarity. Tabesh and Askari-Nasab (2019) presented a methodology to incorporate uncertainty in the clustering definition. They tested several options of similarity indices under geological uncertainty using a *possible worlds* approach, to obtain clusters with good performance on different scenarios.

2.2 Scheduling in short-term mining

Mining scheduling in the short-term has been studied mostly using optimization techniques. This includes heuristics and mixed-integer programming (MIP) models focusing on cost, total metal produced, or target fulfillment as the objective function. Blending, quality, and capacity constraints, and different material processing routes are also typical in this approach. Blom et al. (2019) provide a thorough review of this topic. The rest of this review will focus on operational constraints included in the short-term scheduling.

In regards to the operational considerations for short-term scheduling, several approaches have been proposed. Gholamnejad (2008) focused on accessibility: each block has four possible directions of extraction, which are imposed as constraints in the optimization model. To allow the extraction of each block, at least one of those constraints must be fulfilled. However, the author does not provide a case study to evaluate the performance of the model. Mousavi et al. (2016) presented a similar idea, where each block has five possible directions of extraction: four from the side at the same bench, and one from the upper bench, referred to as *drop-cut*. The model imposes precedence constraints for each direction and enables the extraction of a block if one of them is fulfilled. The authors applied the model to a real case study, but they did not present information on the mining schedule's geometry.

Yavarzadeh et al. (2014) proposed a similar approach with directional constraints. To allow the extraction of a target block, the authors imposed that at least k adjacent blocks must be extracted beforehand. These precedence constraints provided a feasible extraction sequence given an initial mining direction in their case study. Eivazy and Askari-Nasab (2012) used horizontal constraints as well, but with a previous clustering step where the blocks in each bench are grouped into mining cuts using a fuzzy C-means method presented in Askari-Nasab et al. (2010). According to the mining direction, precedence constraints were imposed between these mining cuts to obtain the short-term schedule with a feasible mining width.

Matamoros and Dimitrakopoulos (2016) also presented a MIP model to obtain a short-term schedule, but they introduce operational considerations as two different constraints. The first is the mining direction with horizontal precedence constraints on the same bench, similar to the previous approaches. The second constraint is related to the mining width, which favors the extraction of blocks within a fixed neighborhood on the same bench. Dimitrakopoulos and Ramazan (2004) presented this idea as *inner* and *outer windows*, where failing to extract blocks in these windows is penalized in the objective function. The implementation of these constraints resulted in a schedule with a clear mining direction from the ramp access.

2.3 Summary

Based on the works presented, there are two main approaches to incorporate the operational constraints in the short term-scheduling. The first approach is to do two sequential steps: define the mining cuts or dig-limits and then to use these definitions as input for scheduling. Unfortunately, because the approach is sequential, it may turn out that the mining cut definitions might not be able to fulfill the capacity, quality, or blending constraints for each period. Further on, even if the process yields a feasible schedule, the result may be far from optimal. This issue is especially relevant in the case where several phases are active in the same period, so a feasible cut definition should integrate, simultaneously, all the available production benches to fulfill processing and blending constraints.

The second main approach is incorporating horizontal precedence constraints to ensure the existence of a feasible path from the bench access to each block in each period. However, this type of period-based precedence does not consider the definition of mining cuts, and therefore, there is no control over the destination of adjacent SMUs. Therefore, even when a feasible path from the ramp exists, the schedule might not fulfill the mining cuts operational dimensions.

In this work, we present a novel formulation for the short-term scheduling problem with operational constraints, which aims to maximize the value of the schedule and, at the same time, to generate a feasible mining cut definition and short-term plan.

3 Mathematical model

In this section, we introduce the mathematical model to define the mining cuts and their scheduling.

The model presented selects which SMUs constitute a given mining cut, and assign a destination (processing facility or waste dump) to each cut. The extraction of the cut can span over several periods, but must comply with total extraction capacities as well as capacity and blending constraints at the processing destinations. The extraction of mining cuts is subject to slope precedence constraints to comply with the pitwall stability and precedence constraints within the bench if a mining direction is defined in advance. All these constraints can be modeled considering individual SMUs and using standard techniques from scheduling models in the literature, and therefore we discuss them directly in the formulation (Sect. 3.2).

The model also considers the minimum and maximum size of the mining cuts and a maximum number of active mining cuts which can be used to introduce considerations in terms of the loading equipment fleet. These constraints are direct from the model, so we also delay their discussion to Sect. 3.2.

Finally, the model must abstract the notion of mining cuts. This abstraction is the most novel and complex part of the modeling, and for this reason, we describe it in detail in the following section.

3.1 Modeling mining cuts

In what follows, we will assume that a mining cut is a connected set of SMUs sent to the same destination and extracted over the same periods. We are looking for a *mining cut configuration*, i.e., a partition of the set of SMUs into mining cuts so that the obtained schedule maximizes the economic value. In order to develop the mathematical model, we, therefore, need to address the following aspects: (i) the shape of each mining cut, (ii) the economic value of each mining cut, and (iii) that SMUs belonging to the same cut are sent to the same destination and extracted over the same periods.

Our approach consists in considering a set of SMUs, which are an input for the mathematical model, that we call *representatives*, because, as their name suggests, they represent a potential mining cut. This allows us to separate the problem of defining the mining cut configuration into two parts: first, to select representatives, and second, to assign each SMU to one of the chosen representatives. If the model does not choose a potential representative, it is treated as a regular SMU and must be associated with a selected representative. Figure 2 illustrates this with a small example consisting of 64 SMUs, with eight candidates. The model chose five representatives, therefore generating five mining cuts. The mining cut's shape and connectivity are controlled using precedence arcs directed towards the chosen candidates (we discuss this with more detail later).

Using the above, to model that the SMUs in the same mining cut are sent to the same destination and extracted over the same periods is equivalent to enforcing that they share these decisions with their corresponding representative. That is, if SMU i is associated with representative r, then i and r must share the same destination and extraction periods.

In terms of the economic value of a mining cut, we consider it as the sum of the individual values of SMUs that constitute the cut. Such values may depend on the destination, as the mineral recovery and processing costs depend on the material and the facility where it is processed or stocked. The model selects the best destination based on these differences and the processing capacities of each facility.

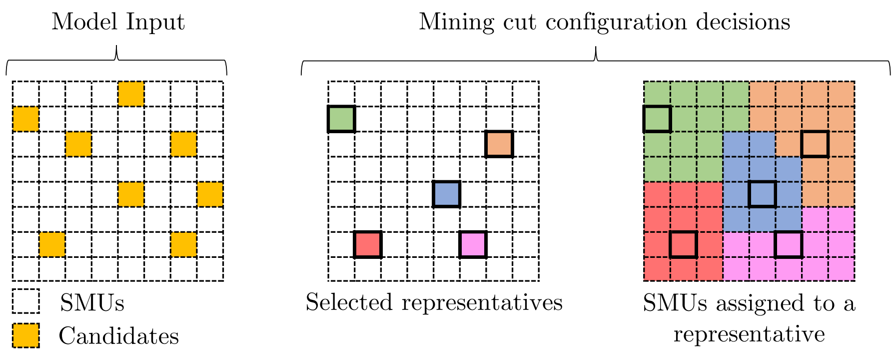


Fig. 2 Mining cuts definition based on the optimization model. Colors represent different mining cuts and their representatives

Finally, to model the shape of the mining cuts, we use precedence arcs directed from the SMUs towards their selected representative. In this way, we make sure that the mining cuts generated are valid (i.e., connected) because transitivity of the precedence arcs ensures a path from every SMU to its representative. Also, we can control the shapes of the mining cuts by using different precedence definitions. However, as the chosen representatives result from the model and are not a fixed input, each SMU has several sets of predecessors (one for each potential representative), and the model only enforces the precedence set corresponding to the chosen representative.

Figure 3 shows two examples of the concept of representatives and how precedence arcs model the cuts' shape. In all the figures, the squares represent individual SMUs looked from the top and a few precedence arcs.

The left-side figures present an arbitrary SMU (in black), three representative SMUs (colored red, green, and yellow). For each representative, a different set of predecessors with the same color as the corresponding representative. The model defines which one of these precedence sets is enforced and, as a result, assigns the black SMU to one of the representative SMUs. The set of all precedence arcs directed towards a representative SMU r is P_r. If an arc (i, j) belongs to $\mathcal{P}_r$, it reads as "if SMU i is assigned to representative r, SMU j must also be assigned to representative r and they must share the same destination".

Figure 3 also illustrates two possible ways to define the precedence arcs. Figure 3a presents the case of a *diagonal*-type precedence, which offers more flexibility for the mining cut's shape, but may have some problematic zones depending on the loading equipment selectivity, while Fig. 3b shows a *square*-type precedence which is more constrained. The right-side figures show the geometries of the mining cuts induced by both sets of precedence arcs.

Notice also that, for example, Fig. 3a shows only 4 elements of P_2, 5 of P_1 and 6 of P_3, but these sets consist of many more arcs that are not shown to keep the figure simple.

It is relevant to emphasize that, even though the model considers all the precedence arcs, it only enforces those corresponding to the chosen representative. In the example in Fig. 3a, the model chooses representative one, and only the red precedence arcs are enforced, while in Fig. 3b, representative two is chosen instead. It is also important to note that, as the figure suggests, the precedence arcs do not link the SMU and the representative directly. Instead, they propagate through the bench incorporating different SMUs in the process. This propagation ensures connectivity in the mining cut, with a feasible path between different SMUs belonging to the same cut.

3.2 Optimization model

In this section, we present in detail the optimization model that we propose to integrate the mining cut definition and scheduling problems.

– Sets, indices, and parameters

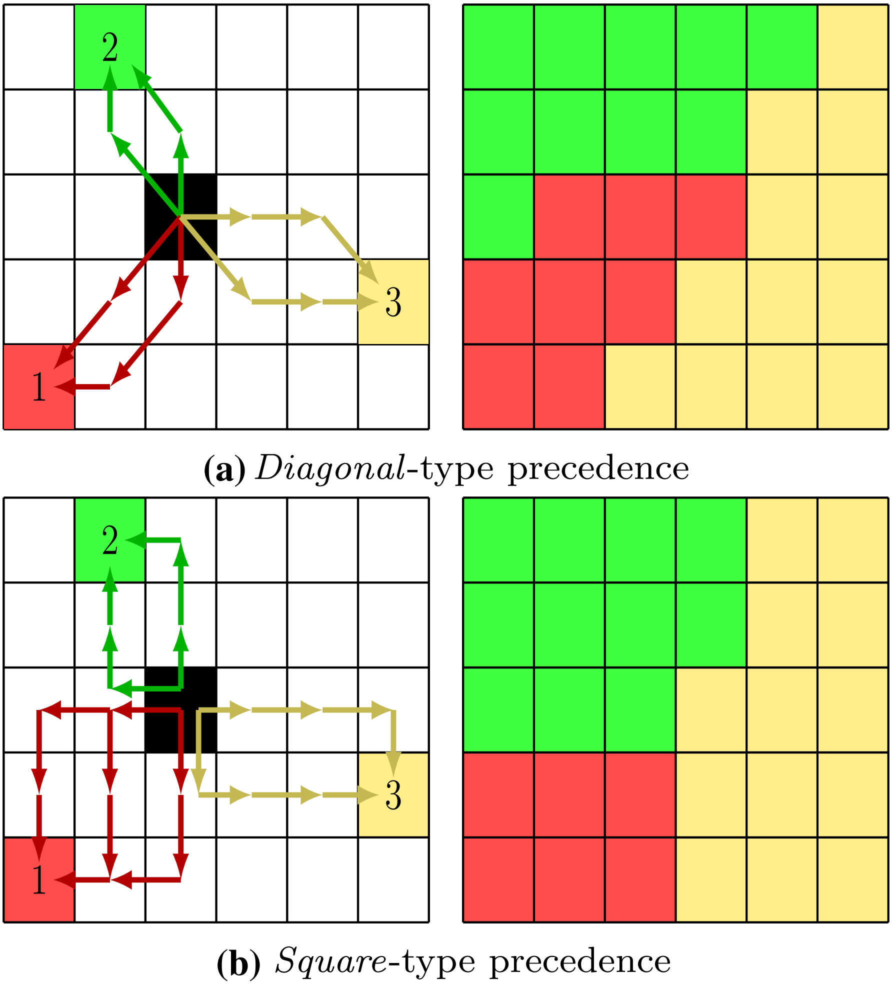


Fig. 3 Left: examples of precedence arcs between a single SMU (black) to three representative SMUs (red, green and yellow). Right: shapes induced by these precedence constraints in the model. Arrows go from successor to predecessor

Sets and indices	
$b, i, j \in \mathcal{B}$	SMUs and set of SMUs.
$t, p \in \mathcal{T}$	Periods and set of time periods.
$r \in \mathcal{R} \subseteq \mathcal{B}$	Representative and set of representative SMUs.
$d \in \mathcal{D}$	Destination and set of possible destinations.
$c \in \mathcal{C}$	Resource and set of operational and processing resources.
$q \in \mathcal{Q}$	Blending attribute and set of blending attributes.

Sets and indices	
$\mathcal{P} \subseteq \mathcal{B} \times \mathcal{B}$	Set of slope precedence arcs between SMUs.
$\mathcal{P}_r \subseteq \mathcal{B} \times \mathcal{B}$	Set of precedence arcs directed to representative SMU $r \in \mathcal{R}$.
$\mathcal{P}_{\mathcal{R}} \subseteq \mathcal{R} \times \mathcal{R}$	Set of precedence arcs between representative SMUs.

Parameters	
v_{btdr}	Profit obtained if SMU b is assigned to representative r and sent to destination d at period t.
α_{cb}	Quantity of resource c consumed by extracting SMU b.
u_c	Upper limit of resource c.
ℓ_c	Lower limit of resource c.
β_{cbd}	Quantity of resource c consumed by processing SMU b at destination d.
u_{cd}	Upper limit of resource c available at destination d.
ℓ_{cd}	Lower limit of resource c available at destination d.
ℓ_r	Minimum cut size associated with representative SMU r.
u_r	Maximum cut size associated with representative SMU r.
g_{qb}	Value of additive attribute q in SMU b.
u_{qd}	Upper limit on the blend of attribute q at destination d.
ℓ_{qd}	Lower limit on the blend of attribute q at destination d.
vt_r	Maximum timespan in the extraction of a mining cut.
N_{active_cuts}	Maximum number of active mining cuts in the same period.
N_{total_reps}	Maximum number of representative SMUs used in the schedule.

– Decision Variables

$$x_{btdr} = \begin{cases} 1 \text{ if SMU } b \text{ is sent to destination } d \text{ at period } t \text{ assigned to representative SMU } r. \\ 0 \text{ otherwise.} \end{cases}$$

$$y_{tr} = \begin{cases} 1 \text{ if an SMU associated with representative } r \text{ is extracted at period} t. \\ 0 \text{ otherwise.} \end{cases}$$

– Objective Function

$$\max \quad \sum_{b \in \mathcal{B}} \sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} \sum_{r \in \mathcal{R}} v_{btdr} x_{btdr} \tag{1}$$

Equation (1) represents the total profit obtained by the schedule in the planning horizon considered. Other objective functions could be used as well: maximize the total metal content extracted in the planning horizon, or minimize the total cost of the schedule changing the parameter v_{btdr} accordingly.

– Constraints

1. Resource Constraints:

$$\ell_c \leq \sum_{b \in \mathcal{B}} \sum_{d \in \mathcal{D}} \sum_{r \in \mathcal{R}} \alpha_{cb} x_{btdr} \leq u_c \qquad \forall t \in \mathcal{T}, c \in \mathcal{C} \tag{2}$$

$$\ell_{cd} \leq \sum_{b \in \mathcal{B}} \sum_{r \in \mathcal{R}} \beta_{cbd} x_{btdr} \leq u_{cd} \qquad \forall t \in \mathcal{T}, c \in \mathcal{C}, d \in \mathcal{D} \tag{3}$$

Equation (2) represents the mining capacity constraint, which imposes upper and lower limits on the total resources $c \in \mathcal{C}$ consumed in the extraction process. Similarly, Eq. (3) imposes upper and lower limits on the resources $c \in \mathcal{C}$ processed in each destination $d \in \mathcal{D}$. This type of resource constraint could limit the total tonnage extracted in each period or the total hours of processing capacity in each destination, for example.

2. Blending Constraints

$$\ell_{qd} \leq \frac{\sum_{b \in \mathcal{B}} \sum_{r \in \mathcal{R}} g_{qb} c_b x_{btdr}}{\sum_{b \in \mathcal{B}} \sum_{r \in \mathcal{R}} c_b x_{btdr}} \leq u_{qd} \quad \forall t \in \mathcal{T}, d \in \mathcal{D}, q \in \mathcal{Q}, c \in \mathcal{C} \tag{4}$$

Destinations may limit the average value of some attribute across all the SMUs processed at each period. Equation (4) imposes upper and lower limits on the blend of attributes $q \in \mathcal{Q}$ for the optimal performance of each destination $d \in \mathcal{D}$.

3. Slope Precedence:

$$\sum_{d \in \mathcal{D}} \sum_{r \in \mathcal{R}} x_{itdr} \leq \sum_{p=1}^{t} \sum_{d \in \mathcal{D}} \sum_{r \in \mathcal{R}} x_{jpdr} \quad \forall (i,j) \in \mathcal{P}, t \in \mathcal{T} \tag{5}$$

Equation (5) represents the slope precedence constraint to maintain the pit-wall stability. Set $\mathcal{P} \subseteq \mathcal{B} \times \mathcal{B}$ contains ordered pairs calculated using a predefined angle given by geomechanical constraints. If a pair $(i,j) \in \mathcal{P}$, SMU j must be extracted before SMU i or during the same period.

4. Representative SMUs Precedence Constraints:

$$\sum_{t \in \mathcal{T}} x_{itdr} \leq \sum_{t \in \mathcal{T}} x_{jtdr} \quad \forall (i,j) \in \mathcal{P}_r, r \in \mathcal{R}, d \in \mathcal{D} \tag{6}$$

Equation (6) imposes a destination-based precedence constraint for every SMU assigned to representative r. Therefore, for every pair $(i,j) \in \mathcal{P}_r$, if SMU i is assigned to representative r, SMU j must also be assigned to representative r and they must share the same destination. Note that this precedence does not impose a temporal order of extraction between SMUs. The extraction sequence is related to the mining cut extraction order, which is imposed with Eq. (8).

5. Mining Cut Size Constraints:

$$\sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} x_{rtdr} \ell_r \leq \sum_{b \in \mathcal{B}} \sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} x_{btdr} \leq \sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} x_{rtdr} u_r \quad \forall r \in \mathcal{R} \tag{7}$$

Equation (7) imposes upper and lower limits for the size of each cut, defined as the number of SMUs $b \in \mathcal{B}$ associated with each representative SMU $r \in \mathcal{R}$. The lower limit represents the minimum feasible operational size according to the mining equipment. The upper limit is optional and could be used to control the shape of the mining cuts.

It is relevant to note the use of variable x_{rtdr} in both bounds. According to the definition, this variable is 1 if SMU r is assigned to representative r. Since the optimization model may not use every representative SMU provided as an input, variable x_{rtdr} serves as an indicator of whether the model chose this representative. In case the representative r is chosen, $x_{rtdr} = 1$ and both bounds are active. On the other hand, if representative r is not chosen, $x_{rtdr} = 0$ and both bounds become null.

6. Representative SMUs Advancement Constraints:

$$\sum_{d \in \mathcal{D}} x_{itdi} \leq \sum_{p=1}^{t} \sum_{d \in \mathcal{D}} x_{jtdj} \quad \forall (i,j) \in \mathcal{P}_{\mathcal{R}}, t \in \mathcal{T} \tag{8}$$

Equation (8) imposes an extraction order for the different representative SMUs. This constraint maintains a feasible extraction advancement starting from each bench's access point according to the long-term design of the pit. This provides a feasible schedule in the short-term and avoids the extraction of mining cuts with no viable path to the ramp in each period.

For this constraint, we also use the indicator variables since the constraints are enforced only between representative SMUs chosen by the model.

7. Mining Cut Scheduling Constraints:

$$\sum_{t \in \mathcal{T}} y_{tr} \leq vt_r \quad \forall r \in \mathcal{R} \tag{9}$$

$$\sum_{r \in \mathcal{R}} y_{tr} \leq N_{active\_cuts} \quad \forall t \in \mathcal{T} \tag{10}$$

Equation (9) limits the number of periods a mining cut can be active. Note that this constraint could be omitted for the case of $vt = 1$ since it can be incorporated into Eq. (6) imposing not only the same destination for the cut but also the same extraction period, which is the most common approach in the short-term.

On the other hand, Eq. (10) limits the maximum number of active mining cuts in each period.

8. Number of Representative SMUs Constraint:

$$\sum_{t \in \mathcal{T}} \sum_{d \in \mathcal{D}} \sum_{r \in \mathcal{R}} x_{rtdr} \leq N_{total\_reps} \tag{11}$$

Equation (11) imposes an upper limit on the total number of representative SMUs used in the schedule. This constraint allows the model to choose the representative SMUs among some potential locations constrained by a maximum number defined by N_{total_reps}.It is worth noting that if the model does not choose a representative, that does not mean that the SMU is not extracted. Equation (13) forces the model to assign every SMU to a representative.

9. Variable Definition Constraints

$$\sum_{d\in\mathcal{D}} x_{btdr} \leq y_{tr} \quad \forall b \in \mathcal{B}, t \in \mathcal{T}, r \in \mathcal{R} \tag{12}$$

$$\sum_{t\in\mathcal{T}} \sum_{d\in\mathcal{D}} \sum_{r\in\mathcal{R}} x_{btdr} = 1 \quad \forall b \in \mathcal{B} \tag{13}$$

$$x_{btdr}, y_{tr} \in \{0, 1\} \quad \forall b \in \mathcal{B}, t \in \mathcal{T}, r \in \mathcal{R}, d \in \mathcal{D} \tag{14}$$

Equation (12) links both decision variables: variable x can be 1 only if the corresponding variable y is 1 as well. Equation (13) imposes that every SMU must be extracted in the planning horizon to comply with the long-term plan. Finally, Eq. (14) imposes that both decision variables are binary.

3.3 Comments

The introduction of representatives may appear as a practical problem because the set $\mathcal{R}$ may significantly impact the results, and it is not clear how to choose it. Because of this, in the case study carried out in Sect. 4, we compare two different approaches to define the set of potential representatives and show that, at least in the example studied, these rules perform well and similarly.

Despite the above, we observe that the model presented herein provides enough flexibility to tackle several short-term problems. For example, there is no conceptual upper limit on the number of representative SMUs defined as an input. A user could start with as many representatives as SMUs and let the model choose among all these possibilities, limited only by Eq. (11). Therefore, a planner could perform some exploratory analysis before settling with the definitive representatives.

The incorporation of slope precedence arcs in the model, while uncommon in the short-term, allows for larger planning horizons, where more than one bench is extracted in the same mining phase. Even when the slope precedence is imposed on an SMU basis, the model could force the extraction of complete mining cuts in the same period, and as a result, the slope precedence is fulfilled on a mining cut level. This generates larger operational spaces when a new bench is opened, which is a common restriction in the short-term to allow the correct operation of the loading equipment.

The separation of the destination-based precedence to control the mining cut's shape and the temporal-based precedence to control the sequence is another advantage. The sequence is controlled by the representative SMUs according to the mining direction provided as an input. However, the model allows the use of Eq. (5) to control the advancement on an SMU basis. This constraint is similar to the advancement constraints described in Sect. 2.2, but also accounts for the shovel selectivity given by the mining cut definition.

The model could also be used without the scheduling constraints to obtain mining cuts considering only the best destination according to the profit. In this sense, the model offers a different version of the dig-limit optimization problem, which separates different types of materials accounting for the loading equipment selectivity, but offering a broad range of possible destinations for each SMU.

4 Numerical experiments

In this section, we apply the optimization model described in Sect. 3 in a real case study. We evaluate the short-term schedule, the geometry of the resulting mining cuts, the effect of the representatives' location, the influence of the precedence definition, and computational aspects of the model implementation.

4.1 Case study

The case study corresponds to two benches of a real copper mine from northern Chile. The dimensions of each bench are 90m × 120m, and they contain a total of 864 SMUs of 5m × 5m, with a bench height of 12m. Figure 4 shows the copper grade distribution of both benches. The grade was obtained using conditional simulation to replicate the local variability of the deposit found in the short-term. A description of the conditional simulation algorithm can be found in Emery (2008),

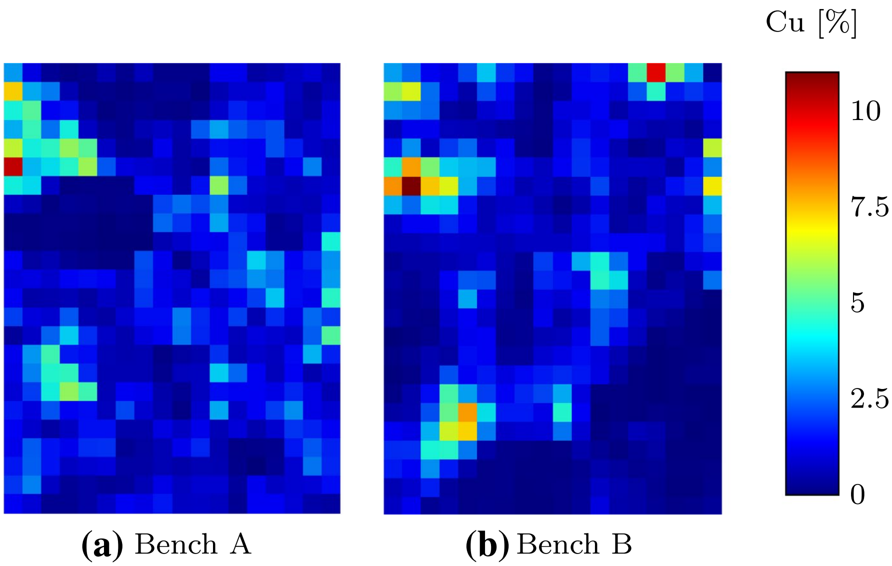


Fig. 4 Copper grade

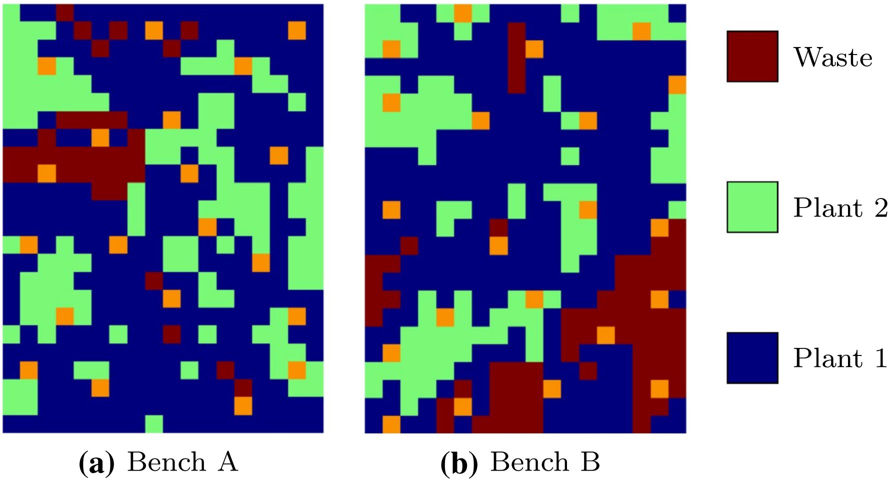


Fig. 5 Best destination based on cut-off grade. Representative SMU locations shown in orange

Table 1 Economical and scheduling parameters for each destination

Parameter	Plant 1	Plant 2	Waste
Processing cost [USD/ton]	$6	$11	$0
Recovery [%]	75	90	0
Selling cost [USD/lb]	$0.15	$0.3	$0
Processing capacity [kton/week]	[0–81.0]	[0–48.6]	[0–145.8]
Blending [Cu%]	–	[0.5 - 4.5]	–
Mining capacity [kton/week]	145.8		
Mining cost [USD/ton]	$1.2		
Copper sale price [USD/lb]	$2.0		

and other applications of conditional simulation in grade control data can be found in Dimitrakopoulos and Jewbali (2013) and Vasylchuk and Deutsch (2018).

Since these benches belong to different phases of the mine, they can be extracted simultaneously, and hence there is no slope precedence in this case study. However, the extraction must begin at the bottom left corner of each bench, where the access is located. The production scheduling in this case study corresponds to a five-week planning horizon.

The mine has two processing facilities, each with different costs and recoveries, with an additional destination corresponding to the discard waste. The economic and scheduling parameters for each destination are shown in Table 1.

It is possible to perform a cut-off grade analysis to select the best destination of each SMU, i.e., the destination that produces the highest profit. Figure 5 illustrates the result of this analysis, and shows that the heterogeneity of the copper grade causes a highly variable destination definition, which is impractical in the operation. This high variability motivated the application of the model in this case study.

The mining cut parameters used are shown in Table 2. The cut-off grade analysis was used as a base to define the set of representatives $\mathcal{R}$, which consists of 41 locations shown in Figure 5. For this case study, these locations were defined manually by a mining engineer. The first representative was placed in the lower-left corner where the ramp access is located. Then, the representatives are defined sequentially, following three main rules:

- Representatives are located in zones where the cut-off grade map shows mostly a single destination. In this way, the representatives try to capture the same destination policy as the cut-off grade map.
- The higher the destination variability, the closer the representatives. This favors the definition of many mining cuts in zones where it is not clear how to define them manually from the cut-off grade map.
- A minimum of three SMUs of separation is maintained between representatives. This avoids the definition of small mining cuts, which could present feasibility issues in the operation.

Finally, all the results were obtained using Gurobi 9.0 in a laptop with 8 Gb of RAM and an Intel Core i5-7200U CPU (2.5 GHz).

4.2 Results

Figure 6 shows the location of representative SMUs used in the schedule and the final mining cuts obtained by the optimization model in both benches. Every SMU in a mining cut shares the same extraction period and destination, and every mining cut fulfills the operational constraints imposed by the model. The model also determines the shape of the mining cuts considering the profit, but also the mining, blending, and capacity constraints imposed in each period. These constraints restrict all the mining cuts extracted in the same period, whether they belong to Bench A or Bench B, to ensure the mine schedule's feasibility.

Figure 7 shows the mining sequence. Given the restriction of extracting complete mining cuts in the same period and the precedence constraint imposed between the representative SMUs, large zones of each bench are extracted in conjunction producing enough operational space for the loading and hauling equipment. The set of mining cuts in each period fulfills both the processing constraints

Table 2 Mining cut definition parameters

Minimum cut size	10 SMU
Maximum cut size	30 SMU
Maximum active cuts per period	10 SMU
Cut extraction span	1 period
Maximum allowable distance	6 SMU

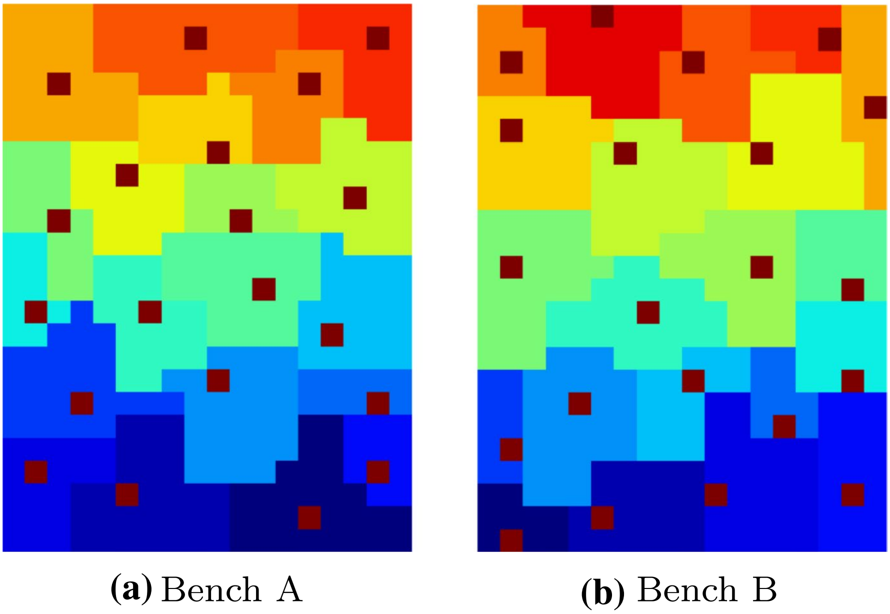


Fig. 6 Representative SMUs (dark red) and resulting mining cuts

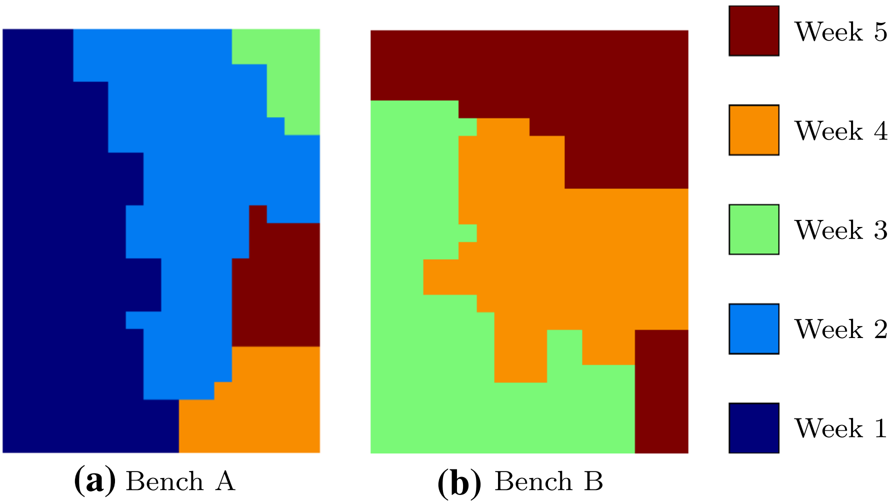


Fig. 7 Extraction period of the SMUs in both benches

for each destination and the mining constraint for the total tonnage extracted. This ensures that there is enough capacity to extract and process the SMUs according to the mining sequence. Figure 8 shows the production plan where each destination fulfills its capacity in each period.

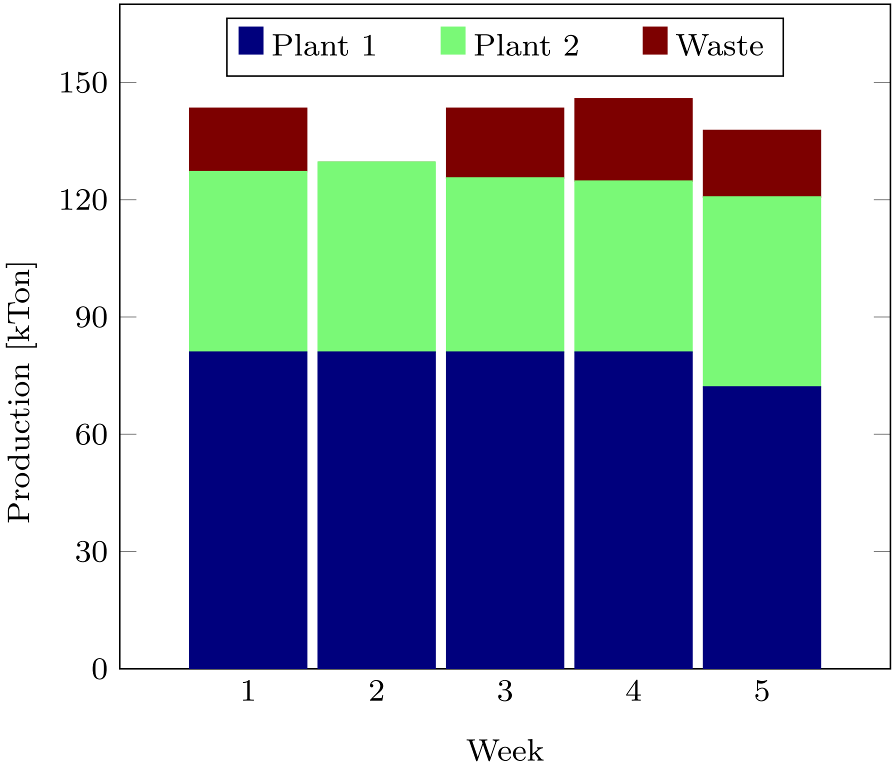


Fig. 8 Production plan

Another product of the mining cut definition from the optimization model is the separation between types of material in each bench, referred to as dig-limit definition in the literature. Figure 9 shows the destination of each SMU and the destination of the mining cuts, which illustrates how the model defines the dig-limits for this case study. The result from the model presents larger zones with the same destination, in contrast to the cut-off grade approach for each SMU. However, the results from both approaches are not entirely different. The optimization model tries to preserve the original classification of each SMU, but fulfilling the minimum cut size and changing some destinations accordingly to comply with all the constraints.

It is interesting to analyze the effect of the scheduling constraints on the mining cuts since usually, their definition resides in operational considerations. Figure 10 shows a comparison between different destination definitions: Fig. 10a shows the cut-off grade analysis, Fig. 10b is obtained by the model with operational and scheduling constraints, while Fig. 10c is also obtained by the model but without capacities, blending, nor extraction advancement constraints. Table 3 shows the tonnage assigned to each destination, with the upper limit considered for the five-week planning horizon.

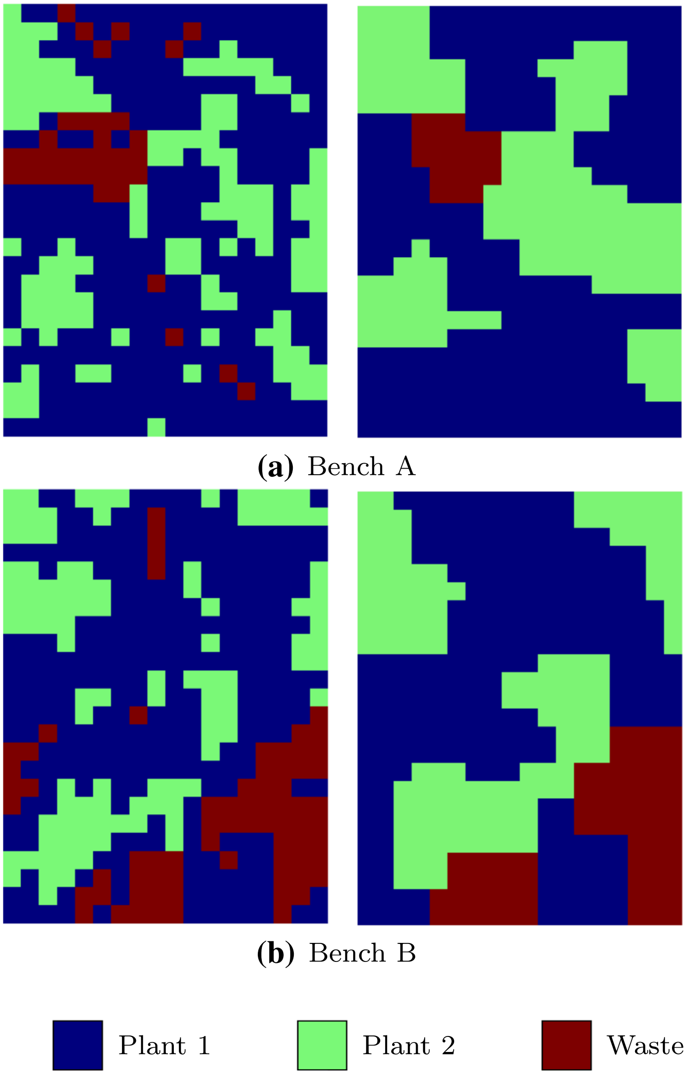
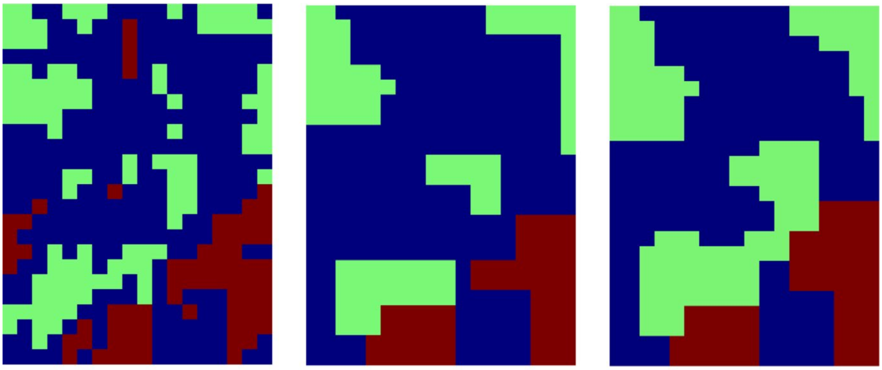


Fig. 9 SMUs destination policy based on cut-off grade (left) and mining cuts (right)

While the destination definition is similar among the three cases, the tonnage assigned for each destination varies between them. The upper limit for ‘Plant 2’ is fulfilled in all cases, but this does not occur for ‘Plant 1’. The cut-off grade policy surpasses the maximum tonnage by 38 kTon, and if the mining cuts are defined only by value and operational space, the limit is surpassed by 63 kTon. Using these mining cuts in a production schedule would require modifying the destination policy to comply with the processing capacity of ‘Plant 1’, or to change the cut definition itself to obtain a feasible plan. The model with scheduling constraints takes this into account during the optimization process, and therefore the



(a) Cut-off grade **(b)** Model without scheduling constraints **(c)** Model with scheduling constraints

Plant 1 Plant 2 Waste

Fig. 10 Effect of the scheduling constraints on the destination policy—Bench B

Table 3 Total tonnage by destination policy

Policy	Plant 1 [kton]	Plant 2 [kton]
Cut-off grade	434.16	179.82
Model without scheduling constraints	459.27	176.58
Model with scheduling constraints	396.09	231.66
Upper limit	405	243

mining cuts already fulfill the scheduling considerations. This result highlights how addressing the problem partially may lead to unfeasible solutions and, therefore, the need for a holistic approach including mining cuts definition and scheduling, like the one proposed in this work.

It is important to note that the geometry presented may introduce some operational issues, where SMUs assigned to one destination are mostly surrounded by SMUs assigned to different one or a different period. This is related to the indirect way of imposing the operational constraints through the precedence definition towards each representative SMUs. These shapes are still allowed and fulfill the precedence arcs, even when they are not fully operationally feasible. A post-processing step could modify the initial cut definition to account for these problematic zones. Tabesh et al. (2014) introduces an example of this shape-refinement step.

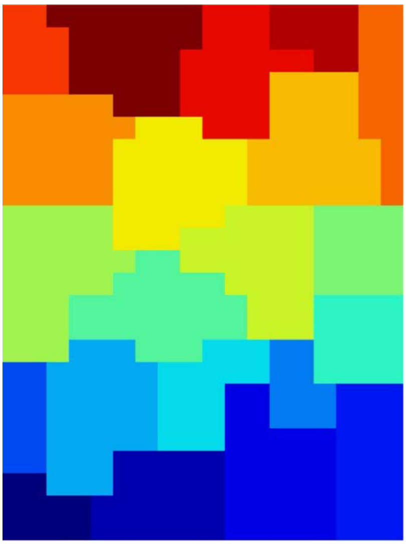
However, these issues should disappear once the mining engineer defines the final shapes of the cuts assisted by a Computer-aided design (CAD) software to smooth their boundaries. To validate this, a short-term mine planner reviewed the model's results to assess their usefulness for a real operation. According to his evaluation, the mining cuts and the schedule comply with the operational considerations related to the size of the loading equipment, which is the primary constraint in the short-term. Therefore the model provides a valuable guide for the final mining cuts design.

Interestingly enough, the analysis provided by the expert planner also indicated that the results of the model are useful to allocate resources associated with the extraction process, such as auxiliary equipment and services since it is known beforehand which sectors are going to be extracted in the next periods. Also, the definition of mining cuts and dig-limits could be useful to produce a better evaluation of the real profit in the long-term, because it allows a better estimation of the operational costs, compared to the current approach based on blocks.

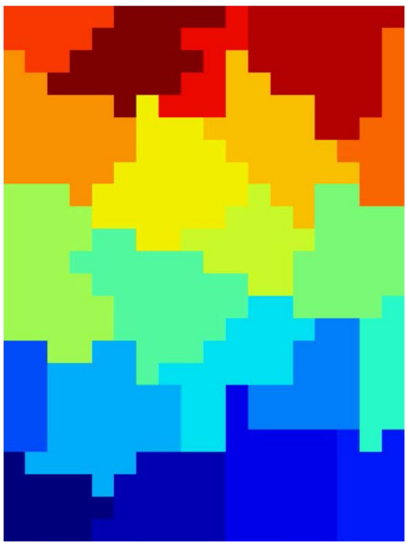
4.3 Impact of the precedence arcs to model mining cut shapes

As indicated before, how the precedence arcs are defined may be used to generate different geometries. Figure 11 shows a comparison of the results obtained by the same instance of the problem, but with different precedence definition (Fig. 3 shows an example of these definitions).

For the two precedence definitions considered, the major differences include the boundaries between mining cuts, where diagonal interfaces are allowed. This could entail a particular operational extraction strategy according to the loading equipment and the mining direction. The shapes of the cuts are also different,



(a) *Square*-type precedence



(b) *Diagonal*-type precedence

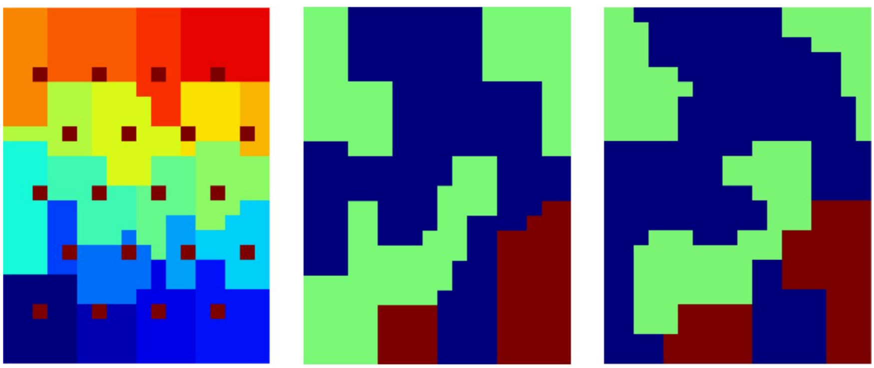
Fig. 11 Mining cuts with different precedence definition presenting more problematic areas with the *Diagonal*-type since each SMU has fewer precedence arcs directed to the representatives. In this way, the precedence definition impacts the cut shapes, which could be used to account for different extraction strategies or present the mine planner with different mining cut definitions options.

4.4 Impact of the initial selection of representatives

As mentioned before, a potential drawback of the approach presented in this paper is the need to define the potential locations of the representative SMUs and its impact on the final schedule. The case study shows that the cut-off grade destination policy serves as a good indicator of the representative's locations and requires less effort from the planner that constructing all the cuts manually.

However, it is interesting to study the case where the cut-off grade analysis is not performed. Instead of that, the representatives are distributed systematically on the bench, which is shown in Fig. 12. Using these representatives leads to the mining cuts shown in Fig. 12a and the destination definition shown in Fig. 12b. As a comparison, Fig. 12c shows the destination definition based on the representative SMUs chosen by a mining engineer (previously shown in Fig. 6b).

It is clear that the definition of mining cuts and destination policy is not equal. The lack of representative SMUs in some areas forces the model to change the destination of some SMUs to comply with the systematic representatives used in this example. However, the systematic representatives managed to capture similar



(a) Systematic Representative SMUs **(b)** Destination based on systematic representative SMUs **(c)** Destination based on engineer's representative SMUs

Plant 1 Plant 2 Waste

Fig. 12 Effect of representative SMUs location on the mining cut definition and destination policy—Bench B destination definitions in most of the bench, which shows that in this case study, even when a mining engineer does not define the representative location, the model achieves similar results.

In cases where a systematic approach is not satisfactory, the model could also decide the optimal location of the representative SMUs in each bench from a pool of candidate locations using Eq. (11), as previously discussed in Sect. 3. The downside of this approach is the increase in the size of the problem and its solving time. Therefore, better methodologies to obtain the location of the representative SMUs could be a future line of research, where a heuristic could test and improve different configurations, or solve the model efficiently with a large pool of candidate locations.

4.5 Computational aspects

Finally, regarding the optimization process, Table 4 shows the model performance in the three cases described previously: *Square*-type precedence (shown in Fig. 6), *Diagonal*-type precedence (shown in Fig. 11a), and systematic representatives (shown in Fig. 12). The cut-off grade destination policy is used as a reference since it is the maximum potential profit the schedule can achieve if perfect selectivity on the bench is assumed.

The value achieved by the model in this numerical experiment is lower than the cut-off grade profit as expected. However, the difference is relatively small, which shows that the optimization model captures most of the profit on these benches. The larger difference belongs to the systematic representatives' approach, with a reduction of almost 2% in terms of value.

In terms of runtime, however, there are significant differences. Systematic representatives take between three and four times longer than the others, which indicates that this representative definition makes the model harder to solve. In any case, the total execution time is very short and more than acceptable for the application. A mining engineer could take between hours or days to manually define a feasible mining cut configuration depending on the complexity. Therefore, the optimization model could provide a useful guide to shorten this planning time.

Table 4 Optimization results

Case	Value [MUSD]	Difference (%)	Best bound [MUSD]	Optimality gap (%)	Runtime [s]
Cut-off grade	24.18	–	–	–	–
Square-type precedence	23.85	1.36	23.89	0.16	151.0
Diagonal-type precedence	23.75	1.75	23.96	0.87	98.1
Systematic representatives	23.72	1.90	23.81	0.38	435.2

5 Conclusions

In this paper, we addressed the mining cut definition and scheduling for short-term open-pit planning. For this, we introduced an optimization model that defines the mining cuts and the mining sequence, fulfilling operational and scheduling constraints, and maximizing the profit obtained. We then applied the model in a realistic case and evaluate its performance in terms of operational considerations, production plan, and optimization runtime.

The optimization model presented in this work successfully defines the mining cut configuration and the production plan simultaneously. The shapes of the mining cuts obtained are useful. The model provides enough flexibility to tackle different cases in the short-term, related to the mining cut's size and shape, accessibility and advancement through the bench, simultaneous extraction of different phases, and dig-limit definitions for multiple materials.

Future research on this approach could study the best location of the representative SMUs and their influence on the mining cut definition and the production schedule. Algorithms to place the representatives in a good location based on the bench information or heuristics to handle the complete optimization model with a large pool of representatives could tackle more extensive and complex cases. Other extensions may include the incorporation of uncertainty in grade or rock type and a more complex geometallurgical model for the objective function to account for changes in recovery due to the blending of SMUs in the processing plants.

Acknowledgements This work was (partially) funded by the National Agency for Research and Development (ANID) through PIA Project AFB180004 and Scholarship Program Doctorado Nacional 2018 - 2118155.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

References

Askari-Nasab H, Awuah-Offei K, Eivazy H (2010) Large-scale open pit production scheduling using mixed integer linear programming. Int J Min Miner Eng 2(3):185–214. https://doi.org/10.1504/IJMME.2010.037624

Blom M, Pearce A, Stuckey P (2019) Short-term planning for open pit mines: a review. Int J Min Reclam Environ 33(5):318–339. https://doi.org/10.1080/17480930.2018.1448248

Deutsch M (2017) A branch and bound algorithm for open pit grade control polygon optimization. In: Proceedings of the 38th international symposium on the applications of computers and operations research in the mineral industry (APCOM), pp 3.1–3.8

Dimitrakopoulos R, Jewbali A (2013) Joint stochastic optimisation of short and long term mine production planning: method and application in a large operating gold mine. Min Technol 122(2):110–123. https://doi.org/10.1179/1743286313Y.0000000040

Dimitrakopoulos R, Ramazan S (2004) Uncertainty-based production scheduling in open pit mining. Trans Soc Min Metall Explor 316:106–112

Eivazy H, Askari-Nasab H (2012) A mixed integer linear programming model for short-term open pit mine production scheduling. Min Technol 121(2):97–108. https://doi.org/10.1179/1743286312Y.0000000006

Emery X (2008) A turning bands program for conditional co-simulation of cross-correlated Gaussian random fields. Eng Optim 34(12):1850–1862. https://doi.org/10.1016/j.cageo.2007.10.007

Gholamnejad J (2008) A zero-one integer programming model for open pit mining sequences. J South Afr Inst Min Metall 108(12):759–762

Isaaks E, Treloar I, Elenbaas T (2014) Optimum dig lines for open pit grade control. In: Proceedings of the 9th international mining geology conference, Australian institute of mining and metallurgy, pp 425 – 432

Matamoros MEV, Dimitrakopoulos R (2016) Stochastic short-term mine production schedule accounting for fleet allocation, operational considerations and blending restrictions. Eur J Oper Res 255(3):911–921. https://doi.org/10.1016/j.ejor.2016.05.050

Mousavi A, Kozan E, Liu S (2016) Open-pit block sequencing optimization: a mathematical model and solution technique. Eng Optim 48(11):1932–1950. https://doi.org/10.1080/0305215X.2016.1142080

Neufeld C, Norrena K, Deutsch C (2003) Semi-automatic dig limit generation. In: Centre for computational geostatistics report 5–115

Norrena K, Deutsch C (2001) Automatic determination of optimal dig limits accounting for uncertainty and equipment constraints. In: Centre for computational geostatistics Report 3–114

Norrena KP, Neufeld C, Deutsch CV (2002) An update on automatic dig limit determination. In: Centre for computational geostatistics report 4–115

Richmond A, Beasley J (2004) Financially efficient dig-line delineation incorporating equipment constraints and grade uncertainty. Int J Surf Min Reclam Environ 18(2):99–121. https://doi.org/10.1080/13895260412331295376

Ruiseco J, Williams J, Kumral M (2016) Optimizing ore-waste dig-limits as part of operational mine planning through genetic algorithms. Nat Resour Res 25(4):99–121. https://doi.org/10.1007/s11053-016-9296-1

Sari YA, Kumral M (2017) Dig-limits optimization through mixed-integer linear programming in open-pit mines. J Oper Res Soc 69(2):171–182. https://doi.org/10.1057/s41274-017-0201-z

Tabesh M, Askari-Nasab H (2011) Two-stage clustering algorithm for block aggregation in open pit mines. Min Technol 120(3):158–169. https://doi.org/10.1179/1743286311Y.0000000009

Tabesh M, Askari-Nasab H (2013) Automatic creation of mining polygons using hierarchical clustering techniques. J Min Sci 49(3):426–440. https://doi.org/10.1134/S1062739149030106

Tabesh M, Askari-Nasab H (2019) Clustering mining blocks in presence of geological uncertainty. Min Technol 10(1080/25726668):1596425

Tabesh M, Mieth C, Askari-Nasab H (2014) A multi-step approach to long-term open-pit production planning. Int J Min Miner Eng 5(4):273–298. https://doi.org/10.1504/IJMME.2014.066577

Vasylchuk Y, Deutsch C (2018) Improved grade control in open pit mines. Min Technol 127(2):84–91. https://doi.org/10.1080/14749009.2017.1363991

Vasylchuk Y, Deutsch C (2019) Optimization of surface mining dig limits with a practical heuristic algorithm. Min Metall Explor. https://doi.org/10.1007/s42461-019-0072-8

Yavarzadeh S, Abodallheisharif J, Neishabouri A (2014) Modeling of short term production scheduling with the objective grade control. In: Drebenstedt C, Singhal R (eds) Mine planning and equipment selection. Springer, Berlin, pp 379–387. https://doi.org/10.1007/978-3-319-02678-7_37

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