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Practical performance of an open pit mine scheduling model considering blending and stockpiling

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ABSTRACT

Open pit mine production scheduling (OPMPS) is a decision problem which seeks to maximize net present value (NPV) by determining the extraction time of each block of ore and/or waste in a deposit and the destination to which this block is sent, e.g., a processing plant or waste dump. Spatial precedence constraints are imposed, as are resource capacities. Stockpiles can be used to maintain low-grade ore for future processing, to store extracted material until processing capacity is available, and/or to blend material based on single or multiple block characteristics (i.e., metal grade and/or contaminant).

We adapt an existing integer-linear program to an operational polymetallic (gold and copper) open pit mine, in which the stockpile is used to blend materials based on multiple block characteristics, and call it $(\hat{\mathcal{P}}^{la})$. We observe that the linear programming relaxation of our objective function is unimodal for different grade combinations (metals and contaminants) in the stockpile, which allows us to search systematically for an optimal grade combination while exploiting the linear structure of our optimization model. We compare the schedule of $(\hat{\mathcal{P}}^{la})$ with that produced by $(\mathcal{P}^{ns})$ which does not consider stockpiling, and with $(\tilde{\mathcal{P}}^{la})$, which controls only the metal content in the stockpile and ignores the contaminant level at the mill and in the stockpile. Our proposed solution technique provides schedules for large instances in a few seconds up to a few minutes with significantly different stockpiling and material flow strategies depending on the model. We show that our model improves the NPV of the project while satisfying operational constraints.

1. Introduction

Mining is the extraction of economically valuable minerals or materials from the earth. The open pit mine production scheduling (OPMPS) problem consists of (i) identifying a mineralized zone through exploration, which involves drilling and mapping, (ii) developing an ore body model to numerically represent the mineral deposit by dividing the field into three-dimensional rectangular blocks, (iii) assigning attributes such as grades that are estimated by sampling drill cores, and (iv) using these attributes to estimate the economic value of each block, i.e., the difference between the expected revenue from selling the ore and the associated costs such as those related to mining and processing. Given this data, we seek to maximize the net present value (NPV) of the mining operation by determining the extraction time of each block of ore and/or waste in a deposit and the destination to which this block must be sent, e.g., a processing plant or the waste dump. Solving the OPMPS problem with annual time fidelity is important in that it determines the rate and quality of production involving large cash flows, which can reach hundreds of millions of dollars over the life of the mine.

In order to facilitate solutions for the OPMPS problem, mine planners first determine the ultimate pit limit, which delineates the boundary between ore that is sufficiently economical to extract and that which is not; the problem itself maximizes undiscounted ore value, and balances the stripping ratio, i.e., the ratio of waste to ore, with the cumulative value of blocks within the pit boundaries. The solution of various instances of the ultimate pit limit problem with different ore prices results in a series of nested pits, pushbacks, or phases. Lerchs and Grossmann (1965) provide a tractable approach for solving the ultimate pit limit problem. However, this method only specifies the economic envelope of

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profitable blocks given pit-slope requirements, ignoring the time aspect of the production scheduling problem and the associated operational resource constraints such as blending limits, bounds on production and processing, inventory constraints, and/or constraints regarding cutoff grade, i.e., the grade that differentiates ore and waste in a deposit.

The OPMPS problem is commonly formulated as an integer program with binary variables representing if and when each block is extracted. Some researchers present nonlinear-integer models to solve this problem with stockpiling (OPMPS+S). Their models assume that the material mixes homogeneously in the stockpile and that the grade of material leaving the stockpile is equal to the average grade of all the material within the stockpile, but these models are difficult to solve. Here, we use a linear-integer model to approximate the (OPMPS+S) problem, and instead of computing the average grade in the stockpile, we force the stockpile to have an average grade above a specific limit.

We develop a production schedule for an operational open pit mine with two metal types and a contaminant. Our data show that a considerable number of blocks contain both high metal grade and a high level of contaminant. The processing plant requires us to keep the contaminant level of material below a specific limit. Therefore, blending material in the stockpile based on the metal's grade and contaminant level may result in a higher NPV. In other words, blocks with high metal content and a high contaminant level should be mixed with other material to satisfy the plant requirement; without a good blending strategy, we might lose the value of the aforementioned blocks by sending them to waste. Existing models either do not consider stockpiling as part of the OPMPS problem, or just control one grade in the stockpile.

1.1. Models without a stockpile

A mine schedule can be optimized with respect to maximizing NPV, maximizing metal content, minimizing mining and processing costs, or minimizing the variance of the grade at the mill, inter alia. Using optimization techniques for mine production scheduling dates back to 1969 when Johnson (1969) proposed the first linear model to maximize NPV of an open pit mine. A challenge in solving an OPMPS problem is that its scale can be very large, since there may be more than one million blocks and many time periods. Smith (1999) develops an aggregation-based method to decrease the number of binary variables associated with decisions involving block-time period combinations. In more recent work, Ramazan and Dimitrakopoulos (2004) propose handling the large number of integer variables also by means of aggregation. Maximal groups of blocks are created and a linear program is solved so that any group has a positive value in total and the blocks forming the boundary of the aggregate obey slope-based constraints. Boland et al. (2009) propose an aggregation scheme for solving OPMPS as an integer programming model which assumes a variable cutoff grade and possesses upper bound constraints on production and processing. The authors introduce aggregates of blocks grouped by precedence and use this construct to approximate a solution for the original mixed-integer program. Osanloo et al. (2008) review different models and algorithms for long-term open pit mine production planning. Bley et al. (2010) present an integer program that reduces the number of decision variables using a new block aggregation scheme which considers precedence and resource constraints to group the blocks. The authors show that, in many cases, this scheme leads to considerably decreased computational requirements to obtain an optimal integer solution. Topal and Ramazan (2012) propose a network model to efficiently optimize the production schedule of a large operation. Nelis et al. (2016) present an integer program for the optimal stope design problem which satisfies geomechanical requirements, and avoids unstable and irregular geometries to find feasible stopes.

1.2. Integer-linear and nonlinear models considering a stockpile

Jupp et al. (2013) explain that there are four different reasons for stockpiling before material processing: buffering, blending, storing, and separating grades. Stockpiles can be used as buffers so that extraction and processing can operate independently, which often provides the economic justification for the expense of stockpiling. Moreover, stockpiles can be used for blending material with different grades to reduce grade variation, which, in turn, increases efficiency at the mill. Furthermore, stockpiles can be used to store lower grade material for processing later. Finally, stockpiles can separate different grades of material. In order for the mill to operate efficiently, the mill feed grade should deviate from a predefined grade as little as possible. Some deposits have more than one element of economic interest, and may contain impurities that have to be minimized in the run-of-mine ore. Stockpiles can be used as part of a blending strategy to ensure constant mill feed grade. For some mining operations, ore is produced from several pits to provide feed for the mill. Robinson (2004) discusses grade variation reduction by blending material in the stockpile.

Linear and mixed-integer programming models have been used significantly to optimize open pit production scheduling by focusing on the extraction sequence. Akaike and Dagdelen (1999) propose a model for long-term mine planning which considers a stockpile. The authors use a graph theory-based method that incorporates an infinite number of stockpiles, meaning that every block has its associated stockpile, i.e., there is no blending in the stockpile and the material grade in a stockpile is the same as that of the associated block. Hoerger et al. (1999) describe a mixed integer-linear programming model to optimize mine scheduling for Newmont Mining Corporation's Nevada operations. The authors consider multiple stockpiles, each of which has a specific grade range. When the material is removed from the stockpile, its grade is considered to be the minimum of the associated grade range.

Asad (2005) presents a cutoff grade optimization algorithm for open pit mining operations with stockpiling in a deposit with two minerals. Ramazan and Dimitrakopoulos (2013) consider uncertainty in the geological and economic input data and use a stochastic framework to solve the OPMPS+S problem. The authors do not consider mixing material in the stockpile, meaning that when the material leaves the stockpile, it has the same characteristics as when it enters. Silva et al. (2015) propose a heuristic for stochastic mine production scheduling and apply their approach to a relatively large gold deposit with multiple processing options and a stockpile. Their method finds an initial feasible solution by sequentially solving the stochastic open-pit mine production scheduling problem period-by-period, and then a network-flow algorithm searches for improvements. In their network, the nodes represent blocks, and the goal is to find higher value and lower risk schedules by advancing or postponing the processing of each block.

However, a more realistic assumption is that material in the stockpile is mixed homogeneously. In other words, the characteristics (e.g., grade) of the material change when it enters the stockpile. Bley et al. (2012) propose two different non-linear integer models for stockpiling and assume that the grade of the material removed from the stockpile is the weighted average of the material inside the stockpile. Due to the non-linearity of this model, it cannot be used for solving real-sized instances of the problem. Recently, Moreno et al. (2017) propose different linear integer models to consider stockpiling in open pit mine scheduling, compare the objective function values of these models, and suggest that their

($\mathcal{P}^{la}$) model provides more accurate solutions in which material with a single grade is mixed in the stockpile. However, a deposit usually contains more than one metal and sometimes a contaminant. The use of stockpiling to blend extracted material in these cases has not been treated in previous linear and mixed-integer programming models.

1.3. Existing industrial software

Various industrial software packages facilitate open pit mine planning with stockpiling. Ray (2010) explains that the evolution of such software started in the late 1970s with a focus on operational gold mines in which avoiding any wasteful mining was crucial. He adds that many of the current packages were initiated by existing mining companies or undertaken by universities as research projects. Kaiser et al. (2002) also explain that mine planning and design software packages have existed for decades and that their application has greatly improved the mine design quality, as well as the overall economics throughout the mining process.

In the last few decades, an increasing number of international mining companies have used planning software in which historical information is employed to build large databases and models of existing mines. The first direct benefit of such software is simply the usage and validation of largely unutilized information. The 3D modeling capabilities of mine planning software are important in assessing the environmental impact of new deposits. Available mining software incorporates visualization, modeling, database management, reserve calculation, mine design, and scheduling. Mine planning software can contain a number of different algorithms.

While some mining software such as MineSight Planning Suite (2017) provides solutions that are widely accepted and used in industry and that account for open pit mine scheduling with stockpiling, formal mathematical models employed by said software are not available in the open literature; as such, it is difficult to ascertain whether the associated algorithms produce optimal solutions. To obtain the grade of material leaving the stockpile for the mill, Vulcan (2017) records what has entered it. MineMarket (2017) creates inventory models and catalogs each batch of material as it is delivered to the stockpile and as it leaves it during reclamation, i.e., the process of restoring land that has been mined. Geovia Whittle (2017) considers mixing material in the stockpile, and the grade of the material sent from the stockpile to the mill can be obtained by accumulating the tonnage and metal of material in it.

In this research, we modify the model ($\mathcal{P}^{la}$) proposed by Moreno et al. (2017) to deal with the specifics of the data set, which originates from an operational mine. Then, we solve this model and analyze the differences in metal grade and contaminant level in the stockpile. We show that the capability to blend material with both grade and a contaminant heavily influences the determination of the optimal schedule, especially when the material contains a contaminant. Moreover, we show that the linear programming (LP) relaxation of the objective function value is unimodal with respect to blending criteria in the stockpile, which allows us to search systematically for an optimal grade combination for the LP. Finally, we use an enhanced heuristic to create an integer solution from a corresponding LP solution.

We have organized the remainder of this paper as follows. In Section 2, we describe the data from an operational mine; in Section 3, we explain ($\hat{\mathcal{P}}^{la}$), an adaptation of ($\mathcal{P}^{la}$) to our data set; we also explain two simplified, related models: ($\tilde{\mathcal{P}}^{la}$) and ($\mathcal{P}^{ns}$). In Section 4, we compare the result of our model, ($\hat{\mathcal{P}}^{la}$), with ($\tilde{\mathcal{P}}^{la}$) and ($\mathcal{P}^{ns}$). Section 5 concludes that our model provides a better blending strategy which results in higher NPV than ($\mathcal{P}^{ns}$) while satisfying the arsenic limit constraint at the mill (which the ($\tilde{\mathcal{P}}^{la}$) model does not).

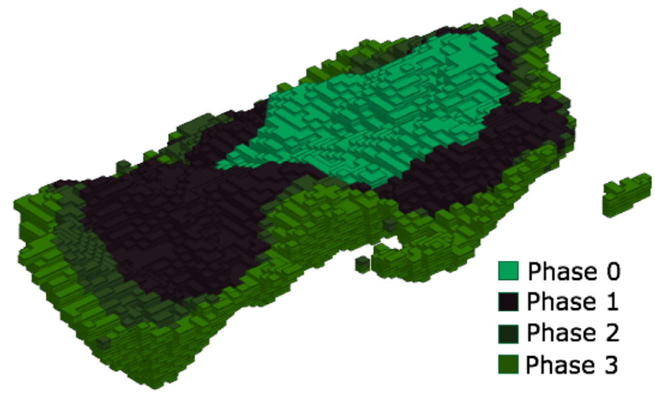


Fig. 1. The pit contains four phases, differentiated by shading. The center light green area is Phase 0, the black region is Phase 1, the dark green region is Phase 2, and the green perimeter region is Phase 3. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

2. Data

Our data set is taken from an operational open pit mine in Southeast Asia and consists of 30,100 blocks, four phases, 56 benches, and 16 time periods. A phase corresponds to a subregion of the pit, and can be obtained by employing the Lerchs–Grossmann (Lerchs and Grossmann, 1965) algorithm. A bench is a ledge that forms a single level of operation to extract both mineral and waste materials. Fig. 1 shows the pit, which includes four phases, differentiated by shading.

Each block may contain two metals, gold and copper, whose selling prices are assumed to be \$20 per gram and \$2.5 per pound, respectively. Copper and gold recovery rates are 60% and 80%, respectively. The mining and processing costs are \$2.5 and \$7.5 per ton, respectively. For the sake of simplicity, we represent the copper and gold contained in each block as a copper-equivalent by jointly considering their selling prices, mining and processing costs, and their recovery rates at the mill.

Blocks also contain arsenic as a contaminant, which should be limited at the mill to 150 particles per million (ppm). In other words, the average grade of arsenic processed in each period should not exceed 150 ppm. Fig. 2 shows the grade distribution of each block. In this data set, 24% of the blocks have a copper-equivalent grade greater than zero and an arsenic level that exceeds the limit at the mill. To maximize NPV, we need to process the blocks containing high copper-equivalent as soon as possible, but Fig. 2 shows that there is a considerable number of blocks whose material includes both high-copper-equivalent grade and a high arsenic level. Here, material blending is very important because, without a good blending strategy, we might send some blocks with high-copper-equivalent grade to the waste dump.

There are 56 benches in the mine and the height of each bench is 12 m. The aforementioned blocks are contained in four predefined phases, computed using the Whittle commercial mining software, resulting in 220 phase-benches to schedule. To extract any block in a given phase-bench, the predecessor phase-benches must be mined completely (see Fig. 3). As a phase-bench is mined, equal proportions must be taken from each block. We emphasize that our approach solves OPMPS+S for this data set at the block level, and predefined phase-benches force precedence constraints between blocks in different bench-phases. Mining capacity is 116.8 million tons per year and processing capacity is 67.89 million tons

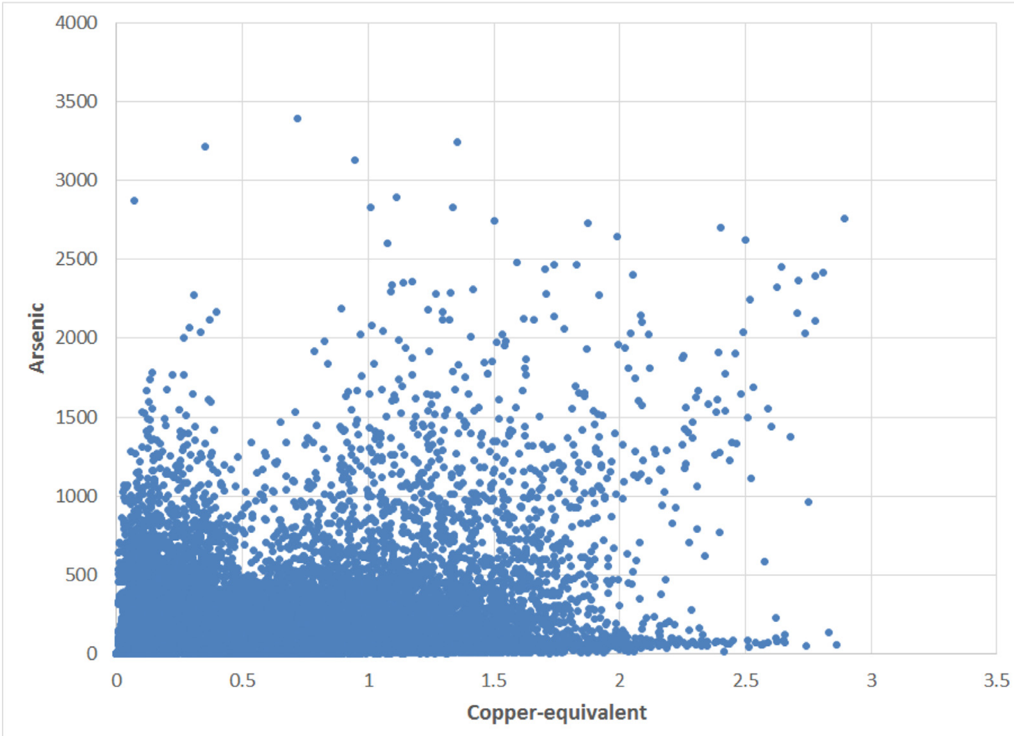


Fig. 2. Distribution of copper-equivalent and arsenic contained in each block in the data set. A considerable number of blocks have both high-copper-equivalent grade and a high arsenic level.

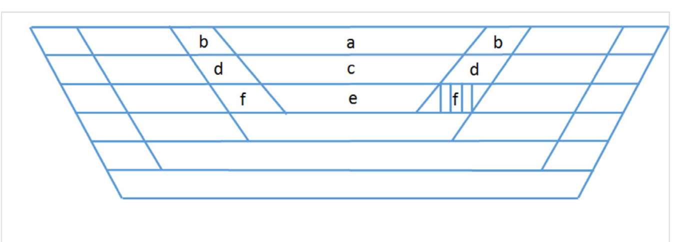


Fig. 3. Blocks are differentiated by vertical lines, and are aggregated in each phase-bench. Before mining any block in phase-bench *f*, both phase-benches *d* and *e* must have been mined.

per year. The cash flow discount factor is 10% per year. For each extracted block, there are three destinations: waste dump, mill, and stockpile.

Fig. 4 shows the distribution of arsenic in each phase-bench and indicates that the upper level of the mine (Phase 0) includes high-arsenic material; this emphasizes the importance of having a reliable blending strategy, especially in the beginning of the mine life, to avoid sending material containing high copper-equivalent grade and high arsenic level to the waste dump. We use the afore-mentioned raw data to populate the optimization models given in Section 3. We define additional data for said models in Section 4.

3. An optimization model considering stockpiling and blending

Bley et al. (2012) propose a nonlinear-integer model to solve the OPMPS+S problem. Although their model most accurately characterizes the nature of the material within the stockpile, their model is not tractable. Moreno et al. (2017) propose a linear-integer model, $(\mathcal{P}^{la})$, which is tractable and provides an objective function value that is very close to that of the nonlinear-integer model proposed by Bley et al. (2012) for the numerical experiments presented. We modify $(\mathcal{P}^{la})$ in such a way that it (i) enforces the precedences between phase-benches, (ii) homogeneously mixes material within the stockpile by controlling both metal content and contaminant level, (iii) satisfies the contaminant level at the mill by blending material from the mine and the stockpile, and (iv) considers the mining and processing resource

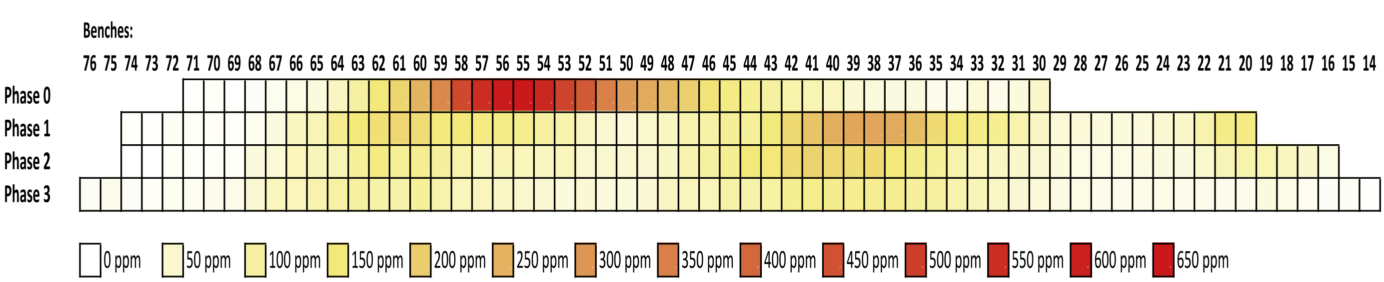


Fig. 4. The arsenic level of each phase-bench is differentiated by color. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 1
Summary of characteristics in $(\hat{\mathcal{P}}^{la})$, $(\tilde{\mathcal{P}}^{la})$ and $(\mathcal{P}^{ns})$. The checkmarks represent whether or not the constraint is present.

Model	Stockpile		Contaminant level at the mill
	Metal content	Contaminant level	
$(\hat{\mathcal{P}}^{la})$	✓	✓	✓
$(\tilde{\mathcal{P}}^{la})$	✓		
$(\mathcal{P}^{ns})$			✓

constraints explicitly. We call this model $(\hat{\mathcal{P}}^{la})$. We compare the schedule of $(\hat{\mathcal{P}}^{la})$ with that provided by two other models, $(\tilde{\mathcal{P}}^{la})$ and $(\mathcal{P}^{ns})$. $(\tilde{\mathcal{P}}^{la})$ is a relaxation of $(\hat{\mathcal{P}}^{la})$ which blends material within the stockpile based on one metal content, and there is no contaminant limit at the mill. We expect $(\tilde{\mathcal{P}}^{la})$ to have a higher NPV than $(\hat{\mathcal{P}}^{la})$, but the schedule is operationally infeasible. In other words, the difference between the NPV of $(\tilde{\mathcal{P}}^{la})$ and $(\hat{\mathcal{P}}^{la})$ represents overestimation of NPV caused by not adequately accounting for contaminant level at the mill. $(\mathcal{P}^{ns})$ does not possess a stockpile, so this model provides a lower NPV than $(\hat{\mathcal{P}}^{la})$. The difference between the NPV of $(\hat{\mathcal{P}}^{la})$ and $(\mathcal{P}^{ns})$ shows the value that a stockpile provides. Table 1 summarizes the differences between these three models.

We next introduce notation, for which our convention is that capital letters represent parameters and lower-case letters refer to variables; the following sections provide the mathematical formulation.

3.1. Notation

Indices and sets:

$b \in \mathcal{B}$:	blocks; $1, \ldots, B$
$n \in \mathcal{N}$:	phase-benches; $1, \ldots, N$
$b \in \tilde{\mathcal{B}}_n$:	all blocks in a phase-bench n that must be mined together
$\hat{n} \in \hat{\mathcal{N}}_n$:	all phase-benches that must be mined directly before phase-bench n
$r \in \mathcal{R}$:	resources {1 = mine, 2 = mill}
$t \in \mathcal{T}$:	time periods; $1, \ldots, T$

Raw parameters:

δ_t :	discount factor for time period t (fraction)
C_b^m :	mining cost per ton of material in block b (\$/ton)
C_b^p :	processing cost per ton of material in block b (\$/ton)
$\bar{C}^p$:	average processing cost per ton of material (\$/ton)
P_b :	profit generated per ton of metal in block b (\$/ton)
$\bar{P}$:	average profit generated per ton of metal (\$/ton)
W_b :	tonnage of block b (ton)
M_b :	metal obtained by completely processing block b (ton)
C^h :	rehandling cost per ton of material (\$/ton)
L_b :	grade of metal in block b (%)
G_b :	grade of contaminant in block b (ppm)
$\hat{G}$:	contaminant limit at the mill (ppm)
R_{rt} :	maximum amount of resource r available in time t (tons/yr)

Derived parameters:

$\bar{L}$:	average grade of metal in the stockpile (%)
$\bar{G}$:	average grade of contaminant in the stockpile (ppm)

Decision variables:

y_{bt}^m :	fraction of block b mined in time period t
y_{bt}^p :	fraction of block b mined in time period t and sent to the mill
y_{bt}^s :	fraction of block b mined in time period t and sent to the stockpile
y_{bt}^w :	fraction of block b mined in time period t and sent to waste
x_{nt} :	1 if all blocks $b \in \tilde{\mathcal{B}}_n$ in phase-bench $n \in \mathcal{N}$ have finished being mined *by* time t; 0 otherwise
i_t^p :	tonnage of ore sent from the stockpile to the mill in time period t
i_t^s :	tonnage of ore remaining in the stockpile at the end of time period t

3.2. Modified L-average bound model $(\hat{\mathcal{P}}^{la})$

Here, we use a modified version of $(\mathcal{P}^{la})$ to explore the effect of blending material while controlling different grades in the stockpile. This model requires the blocks that enter the stockpile to have an average metal grade of at least $\bar{L}$ and an average contaminant grade of at most $\bar{G}$. The model is as follows:

$$(\hat{\mathcal{P}}^{la}) : \max \sum_{t\in\mathcal{T}} \delta_t \left[\left(\sum_{b\in\mathcal{B}} P_b M_b y_{bt}^p + \bar{P}\bar{L} i_t^p \right) - \left(\sum_{b\in\mathcal{B}} C_b^p W_b y_{bt}^p + \bar{C}^p i_t^p \right) - \left(\sum_{b\in\mathcal{B}} C_b^m W_b y_{bt}^m \right) - C^h i_t^p \right] \quad (1)$$

$$\text{s.t.} \sum_{t\in\mathcal{T}} y_{bt}^m \le 1 \qquad \forall b \in \mathcal{B} \quad (2)$$

$$y_{bt}^p + y_{bt}^w + y_{bt}^s = y_{bt}^m \qquad \forall b \in \mathcal{B}, t \in \mathcal{T} \quad (3)$$

$$x_{nt} \le \sum_{t' \le t} y_{bt'}^m \qquad \forall b \in \tilde{\mathcal{B}}_n, n \in \mathcal{N}, t \in \mathcal{T} \quad (4)$$

$$\sum_{t' \le t} y_{bt'}^m \le x_{\hat{n}t} \qquad \forall n \in \mathcal{N}, b \in \tilde{\mathcal{B}}_n, \hat{n} \in \hat{\mathcal{N}}_n, t \in \mathcal{T} \quad (5)$$

$$\sum_{b\in\mathcal{B}} W_b y_{bt}^m \le R_{rt} \qquad \forall t \in \mathcal{T}, r \in \mathcal{R} : r = 1 \quad (6)$$

$$\sum_{b\in\mathcal{B}} W_b y_{bt}^p + i_t^p \le R_{rt} \qquad \forall t \in \mathcal{T}, r \in \mathcal{R} : r = 2 \quad (7)$$

$$\sum_{b\in\mathcal{B}} G_b W_b y_{bt}^p + \bar{G} i_t^p \le \hat{G} \left(\sum_{b\in\mathcal{B}} W_b y_{bt}^p + i_t^p \right) \qquad \forall t \in \mathcal{T} \quad (8)$$

$$i_t^p \le i_{t-1}^s \qquad \forall t \in \mathcal{T} : t \ge 2 \quad (9)$$

$$i_t^s = \begin{cases} \sum_{b\in\mathcal{B}} W_b y_{bt}^s & t = 1 \\ i_{t-1}^s - i_t^p + \sum_{b\in\mathcal{B}} W_b y_{bt}^s & t \in \mathcal{T} : t \ge 2 \end{cases} \quad (10)$$

$$\sum_{b\in\mathcal{B}} \sum_{t' \le t} L_b W_b y_{bt'}^s \ge \bar{L} \sum_{b\in\mathcal{B}} \sum_{t' \le t} W_b y_{bt'}^s \qquad \forall t \in \mathcal{T} \quad (11)$$

$$\sum_{b\in\mathcal{B}} \sum_{t' \le t} G_b W_b y_{bt'}^s \le \bar{G} \sum_{b\in\mathcal{B}} \sum_{t' \le t} W_b y_{bt'}^s \qquad \forall t \in \mathcal{T} \quad (12)$$

$$0 \le y_{bt}^m, y_{bt}^p, y_{bt}^w, y_{bt}^s, i_t^p, i_t^s \le 1; \; x_{nt} \in \{0, 1\} \qquad \forall b \in \mathcal{B}, n \in \mathcal{N}, t \in \mathcal{T} \quad (13)$$

The objective function is the sum of the revenues of blocks sent to the mill from the mine or the stockpile – which are inextricably intertwined with grade, and, correspondingly, recovery; we subtract from this term the sum of the processing, mining and rehandling costs, where we assume that mining cost is invariant with respect to the destination to which the block is sent. Profits and costs associated with the extraction of each block may be block-specific, and can be accounted for as the block leaves the deposit; the corresponding profits and costs associated with material drawn from the stockpile must be averaged across all blocks under consideration for extraction (which can be done a priori based on the data). To represent NPV, all terms are multiplied by a discount rate with respect to each time period t.

Constraint (2) ensures that each block is not extracted more than once. Constraint (3) establishes that the amount of material sent to different destinations is equal to the amount of extracted material. Constraint (4) forces all of the blocks in each phase-bench to be mined by time period t, i.e., at time period t or before, if the binary variable associated with that phase-bench is set to one. Constraint (5) enforces mining precedence constraints by ensuring that all phase-bench predecessors are completely mined before the successor phase-bench.

Constraints (6) and (7) represent extraction and processing restrictions, respectively. Constraint (8) requires that the average arsenic level (ppm) of material at the mill in each time period should not exceed $\hat{G}$. Constraint (9) ensures that the amount of material sent from the stockpile to the mill in time period t is at most the amount of material in the stockpile in time period $t-1$. Constraint (10) enforces inventory balance for an initial time period and a general time period t, ensuring that the amount of material in the stockpile during time period t is equal to that of the previous period plus or minus any material added to or subtracted from the stockpile and sent to the mill, respectively. Constraint (11) ensures that blocks in the stockpile in any time period have an average metal grade of at least $\bar{L}$, and Constraint (12) guarantees that blocks in the stockpile in any time period have an average contaminant level of at most $\bar{G}$. While the simpler, and more restricted, problem of fixing minimum and maximum levels of contaminant and metal at the *mill*, rather than in the stockpile, could be considered, we explore the option of performing the mixing in the stockpile; note that because of the costs incurred from handling stockpiled material, the model only chooses this option if it is economically advantageous to do so with respect to the benefits it can derive from mixing to lower the level of contaminant seen at the mill. Moreno et al. (2017) show that constraints (11) and (12) are linear, and that $(\hat{\mathcal{P}}^{la})$ is tractable and provides an objective function value that is very close to that of the nonlinear-integer model proposed by Bley et al. (2012).

3.3. L-*average bound model without an arsenic limit at the mill* $(\tilde{\mathcal{P}}^{la})$

Here, we modify $(\hat{\mathcal{P}}^{la})$ to explore the effect of blending material while controlling just one characteristic (copper-equivalent grade) in the stockpile. This model requires the blocks that enter the stockpile to have an average metal grade of at least $\bar{L}$. We remove the arsenic limit constraint at the mill (8) and in the stockpile (12). The objective function and other constraints are the same as those in $(\hat{\mathcal{P}}^{la})$.

3.4. *No stockpile model* $(\mathcal{P}^{ns})$

Here, we use a model without stockpiling, so we remove constraints (9)–(12), and retain constraints (2) and (4)–(6). The objective function and constraints (3), (7), (8), and (13) are modified as follows:

$$(\mathcal{P}^{ns}): \max \sum_{t\in\mathcal{T}} \delta_t \left[\left(\sum_{b\in\mathcal{B}} P_b M_b y^p_{bt} \right) - \left(\sum_{b\in\mathcal{B}} C^p_b W_b y^p_{bt} \right) - \left(\sum_{b\in\mathcal{B}} C^m_b W_b y^m_{bt} \right) \right] \quad (14)$$

$$\text{s.t.} \qquad (2), (4), (5), (6)$$

$$y^p_{bt} + y^w_{bt} = y^m_{bt} \qquad \forall b \in \mathcal{B}, t \in \mathcal{T} \quad (15)$$

$$\sum_{b\in\mathcal{B}} W_b y^p_{bt} \le R_{rt} \qquad \forall t \in \mathcal{T}, r \in \mathcal{R} : r = 2 \quad (16)$$

$$\sum_{b\in\mathcal{B}} G_b W_b y^p_{bt} \le \hat{G}\left(\sum_{b\in\mathcal{B}} W_b y^p_{bt}\right) \qquad \forall t \in \mathcal{T} \quad (17)$$

$$0 \le y^m_{bt}, y^p_{bt}, y^w_{bt} \le 1; \; x_{nt} \in \{0,1\} \qquad \forall b \in \mathcal{B}, n \in \mathcal{N}, t \in \mathcal{T} \quad (18)$$

4. Results: benefits of stockpiling and blending

We compare the schedule produced by solving $(\hat{\mathcal{P}}^{la})$ to that provided by $(\tilde{\mathcal{P}}^{la})$ and $(\mathcal{P}^{ns})$. Both the consideration of a production schedule in tandem with average grade (which $(\hat{\mathcal{P}}^{la})$ and $(\tilde{\mathcal{P}}^{la})$ do) and a clairvoyant approach to examining the entire time horizon (which all our optimization models in this paper do) are important in mine planning. We concede that there are simpler methods for computing average grade (Ayu and Cardew-Hall, 2002), and that one can use a type of "sliding time window" heuristic to determine a corresponding production schedule (Cullenbine et al., 2011; Pochet and Wolsey, 2006), but not necessarily within the context of creating an *optimal* production schedule and matching strategy for average grade within a stockpile.

4.1. *Obtaining values for derived parameters of* $(\hat{\mathcal{P}}^{la})$ *and* $(\tilde{\mathcal{P}}^{la})$

The $(\hat{\mathcal{P}}^{la})$ model sends material to the stockpile according to a predefined average grade of copper-equivalent, $\bar{L}$, and an arsenic level, $\bar{G}$. Changing both $\bar{L}$ and $\bar{G}$ alters the schedule and NPV for the OPMPS+S problem.

Table 2
LP relaxation of the objective function (NPV (M$)) for $(\hat{\mathcal{P}}^{la})$ for different combinations of copper-equivalent grade and arsenic level in the stockpile.

		Copper-equivalent grade (%)							
		0.80	0.85	0.9	0.95	1	1.1	1.2	1.3
Arsenic level (ppm)	500	9344	9334	9320	9303	9284	9246	9215	9187
	600	9382	9377	9370	9359	9349	9318	9284	9250
	700	9397	9398	9398	9394	9388	9365	9343	9312
	800	9384	9399	9405	9406	9406	9395	9374	9353
	900	9347	9375	9393	9403	9407	9407	9396	9377
	1000	9308	9338	9366	9384	9397	9403	9403	9392
	1100	9279	9304	9331	9357	9377	9396	9396	9392
	1200	9257	9278	9301	9325	9349	9380	9390	9386
	1400	9224	9241	9259	9277	9297	9337	9365	9374
	1600	9200	9215	9230	9245	9260	9293	9328	9350
	1800	9182	9195	9208	9221	9235	9262	9291	9320
	2000	9169	9180	9019	9203	9215	9238	9263	9289
	2200	9158	9168	9178	9188	9199	9220	9241	9264

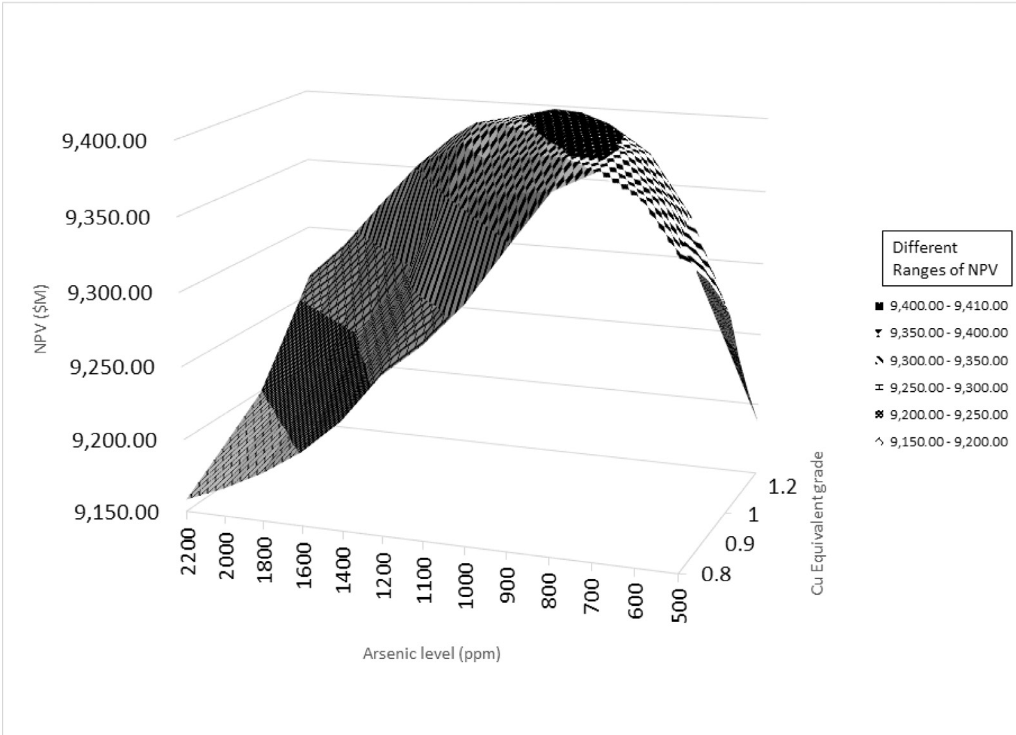


Fig. 5. LP relaxation of the objective function (NPV (M$)) for different combinations of copper-equivalent grade and arsenic level in the stockpile. We observe that the objective function of the LP relaxation is unimodal.

To select these parameters, we consider eight different values for $\bar{L}$ and thirteen different values for $\bar{G}$, shown in Table 2. We use OMP (Rivera et al., 2016) to solve the LP relaxation of $(\hat{\mathcal{P}}^{la})$ to find the highest NPV among all combinations. We emphasize that fixing $\bar{L}$ and $\bar{G}$ means both a linear model and one whose structure we can exploit such that we can use OMP as the solution algorithm, which, when combined with the modified version of the TopoSort heuristic (Chicoisne et al., 2012), is able to obtain a near-optimal solution to $(\hat{\mathcal{P}}^{la})$, even for large instances. We remark that a problem of this size cannot be solved using general integer programming solvers, nor can a corresponding nonlinear-integer model be solved in a reasonable amount of time (Bley et al., 2012).

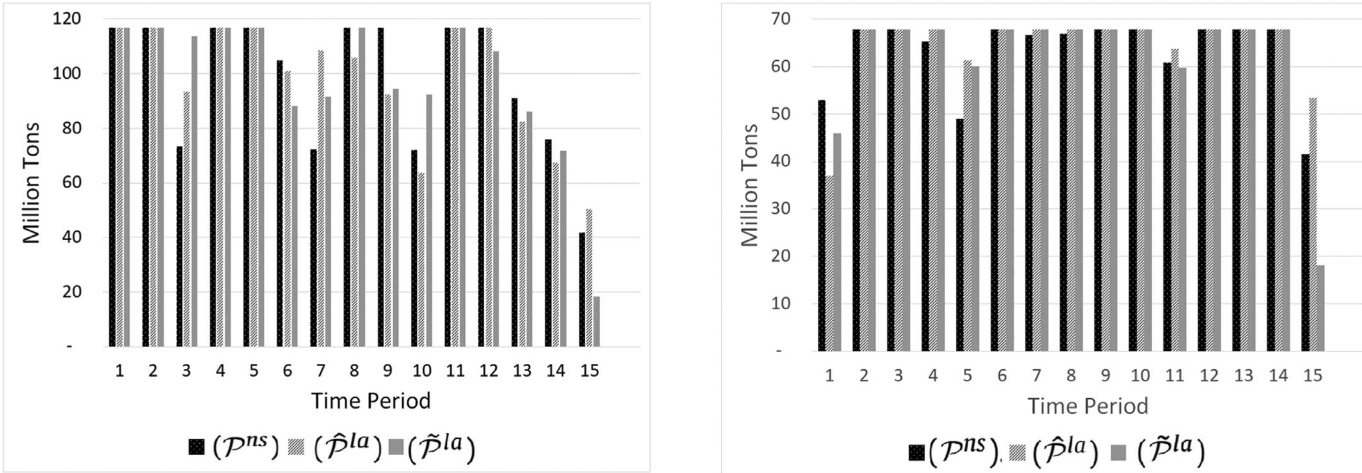
Our numerical experiments show that the objective function of the relaxed problem is unimodal in $\bar{L}$ and $\bar{G}$. Table 2 displays the resulting NPV. Fig. 5 shows the surface plot of the LP relaxation of NPV for different combinations of copper-equivalent grade and arsenic levels in the stockpile. Table 2 and Fig. 5 show that a highest LP relaxation value of the NPV is associated with a copper-equivalent of 1% and an arsenic level of 900 ppm, meaning that we should set the optimal average copper-equivalent grade greater than or equal to 1% and average arsenic level less than or equal to 900 ppm for the material sent from the stockpile to the mill. The value of $\bar{L}$ in the $(\tilde{\mathcal{P}}^{la})$ model is 1% and obtained in the same way as the corresponding value of $\bar{L}$ in $(\hat{\mathcal{P}}^{la})$.

4.2. Solving $(\hat{\mathcal{P}}^{la})$

To obtain an integer-feasible solution for $(\hat{\mathcal{P}}^{la})$, we use a modified version of the TopoSort heuristic, which is a two-step algorithm that creates an integer solution from the LP relaxation values of each unit (i.e., block or bench-phase). In the first step, we define a topological ordering of the units based on the relaxed solution. Since, in the LP relaxation, each unit can be fractionally extracted over several periods, we compute the expected (weighted) extraction time of each unit, and use this value to calculate a topological order, that is, a total ordering of units in which all predecessors of a unit are placed before the given unit in the ordering, and unrelated units are sorted according to their expected extraction times.

In the second step of Toposort, we assign a unique time period to each unit. Given a set of capacity constraints, i.e., mining and processing, per period and a set of resources used by each unit provided that it is sent to a particular destination, TopoSort assigns units with the lowest topological order to be extracted in the first time period as resource capacity allows, and then to successive time periods, increasing by their topological ordering. To define the destination of a unit (and its corresponding resources required) we use the proportion of the unit sent to each destination based on the LP solution. Before making this assignment, we decrease the available plant capacity by the amount of material sent from the stockpile to the mill (i_t^p) based simply on the value from the LP solution.

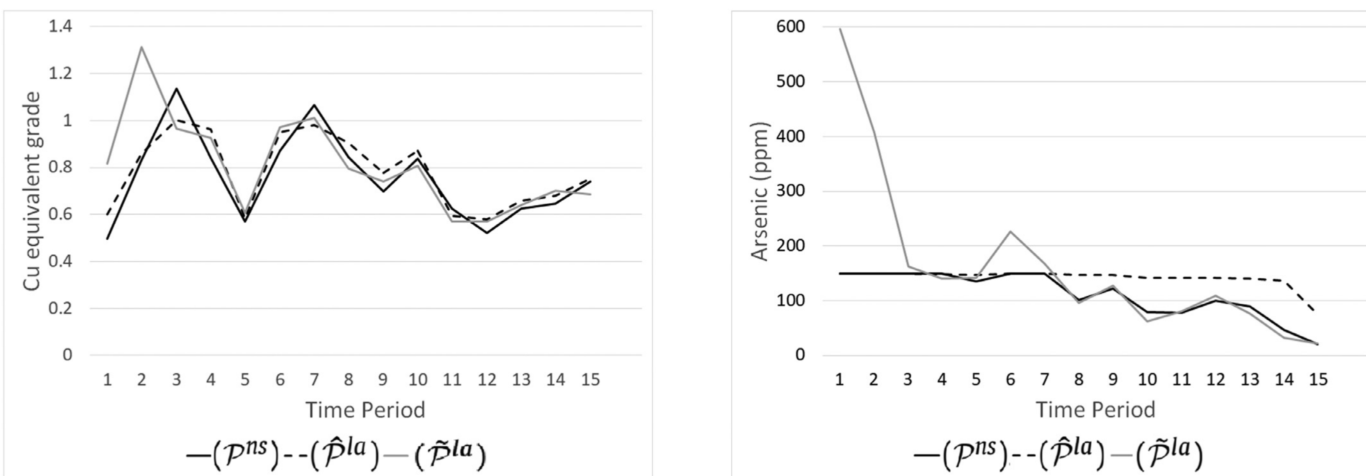
The TopoSort heuristic provides an extraction time by which each bench-phase is completed (i.e., an assignment of values to the x variables) and the destination of its blocks (an assignment of values to the y variables). However, the values of these and other variables from the LP solution can create an infeasibility, for example, by violating the blending constraints, or by exceeding the total material available in the stockpile to be sent to the mill in a given period. In order to obtain a feasible solution, we re-solve $(\hat{\mathcal{P}}^{la})$ by fixing the values of variables x_{nt} according to the solution obtained from TopoSort. This considerably reduces the size of the problem and eliminates the binary variables, allowing us to use a standard linear programming solver. Within a few seconds, this heuristic provides an integer solution with a 2% integrality gap; that is, the relative difference between the objective value of the LP relaxation (an upper bound of the optimal value) and the objective value of the solution found is no more than 2%. We solved the $(\tilde{\mathcal{P}}^{la})$ and $(\mathcal{P}^{ns})$ models in a similar way.



(a) Extracted tonnage from each of the three models

(b) Processed tonnage from each of the three models

Fig. 6. Extracted and processed tonnage comparison between the three models over the life of the mine.



(a) Average copper-equivalent grade

(b) Average arsenic level

Fig. 7. Comparison of average grades at the mill between the three models.

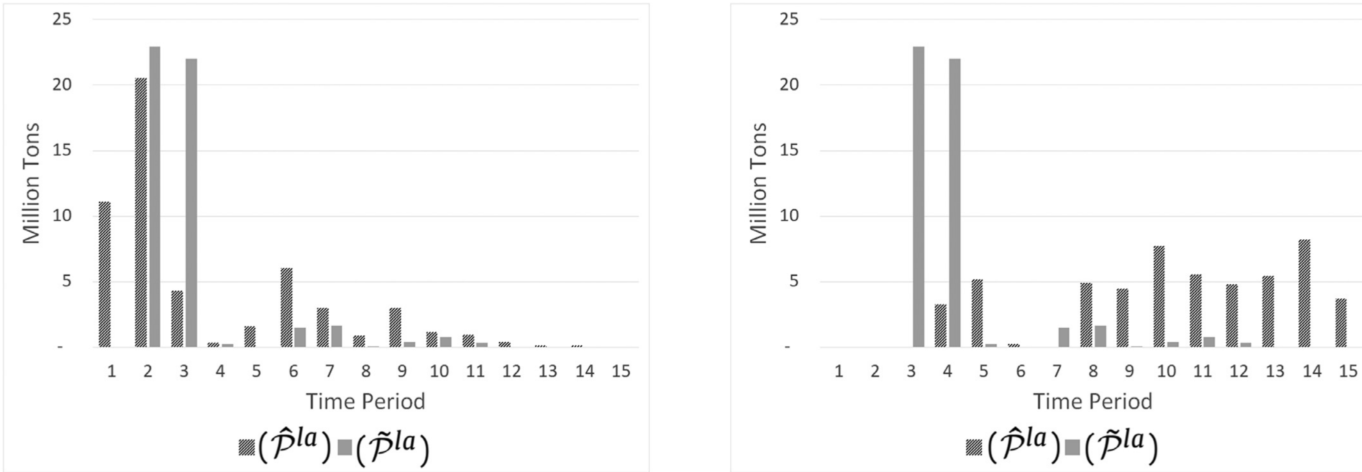
4.3. Comparison of $(\hat{\mathcal{P}}^{la})$ with $(\tilde{\mathcal{P}}^{la})$ and $(\mathcal{P}^{ns})$

We analyze the schedules obtained by solving $(\hat{\mathcal{P}}^{la})$, $(\tilde{\mathcal{P}}^{la})$ and $(\mathcal{P}^{ns})$. Specifically: we compare the tonnage of extracted and processed material, the average grades of the material sent to the different destinations, and the resulting NPV from these schedules.

Fig. 6a compares the extraction schedule of the $(\hat{\mathcal{P}}^{la})$, $(\tilde{\mathcal{P}}^{la})$ and $(\mathcal{P}^{ns})$ models in each time period. Although the amount of extracted material is about the same for all cases, Fig. 6b shows that the $(\tilde{\mathcal{P}}^{la})$ model extracts more material than the other models early in the mine life, and in the last time period it only uses 15% of the mining capacity.

Fig. 6b compares the processing schedule of three models in each time period. The discount makes it preferable to obtain profit sooner rather than later, which is effected by the efficient use of processing capacity. The figure shows that the $(\mathcal{P}^{ns})$ model has a better utilization of the mill in the first time period, but over the life of the mine, $(\hat{\mathcal{P}}^{la})$ processes 3.4% and 1.7% more material than $(\tilde{\mathcal{P}}^{la})$ and $(\mathcal{P}^{ns})$, respectively. This is due to the better blending strategy of $(\hat{\mathcal{P}}^{la})$ which allows this model to process more material by blending it in the stockpile.

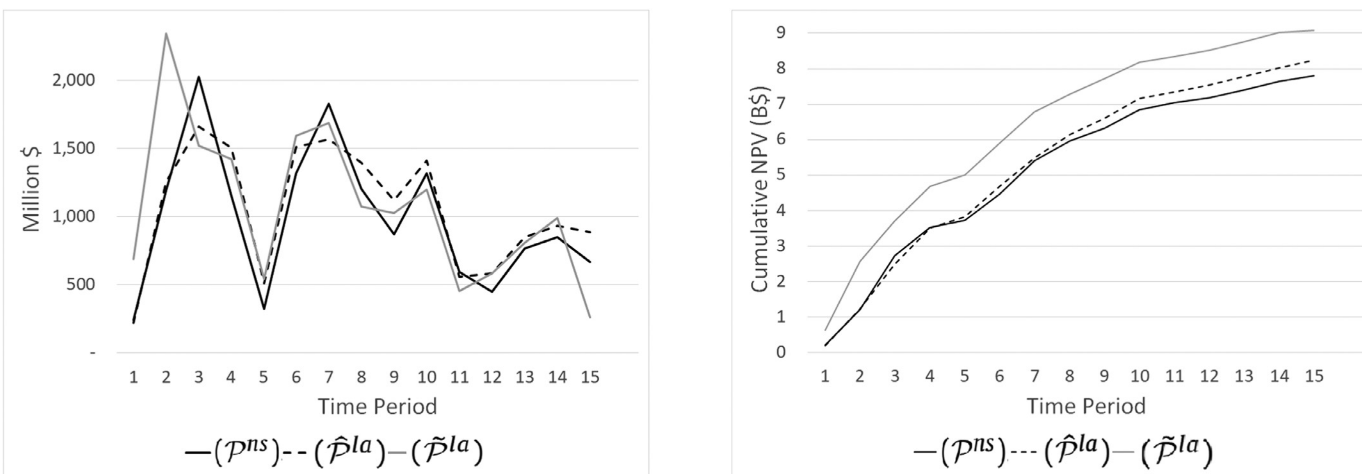
Fig. 7 compares the average copper-equivalent and average arsenic per ton processed at the mill between the these three models in each time period. The high level of arsenic in the mine makes its limit at the mill a binding constraint for at least the first seven time periods, requiring a good blending strategy to satisfy this requirement. Fig. 7a demonstrates that $(\tilde{\mathcal{P}}^{la})$ processes ore with a higher copper-equivalent grade than the other models over the life of the mine. However, Fig. 7b shows that, for this model, the arsenic level fluctuates over the life of the mine and violates the arsenic limit constraint at the mill, rendering the corresponding schedule operationally infeasible. The mill arsenic level for $(\hat{\mathcal{P}}^{la})$ is about the same over the life of the mine, while it decreases in the out-years for $(\mathcal{P}^{ns})$. However, considering that $(\hat{\mathcal{P}}^{la})$ processes more material than $(\mathcal{P}^{ns})$ while not violating the arsenic limit constraint, $(\hat{\mathcal{P}}^{la})$ has a better blending strategy. Despite the fact that we do not place lower and upper bounds on the metal grade at the mill, the grade range from the solution of $(\hat{\mathcal{P}}^{la})$ over the time horizon we explore is about 0.6–1; for $(\tilde{\mathcal{P}}^{la})$ in which we do not constrain the contaminant level at the mill, the range is less than 0.6 to greater than 1.3; and, for the model without stockpiling, $(\mathcal{P}^{ns})$, the range is less than 0.5 to greater than 1.1. We see, therefore,



(a) Tonnage sent from the mine to the stockpile

(b) Tonnage sent from the stockpile to the mill

Fig. 8. Comparison of material movement at the stockpile.



(a) Undiscounted cash flow

(b) Cumulative discounted cash flow

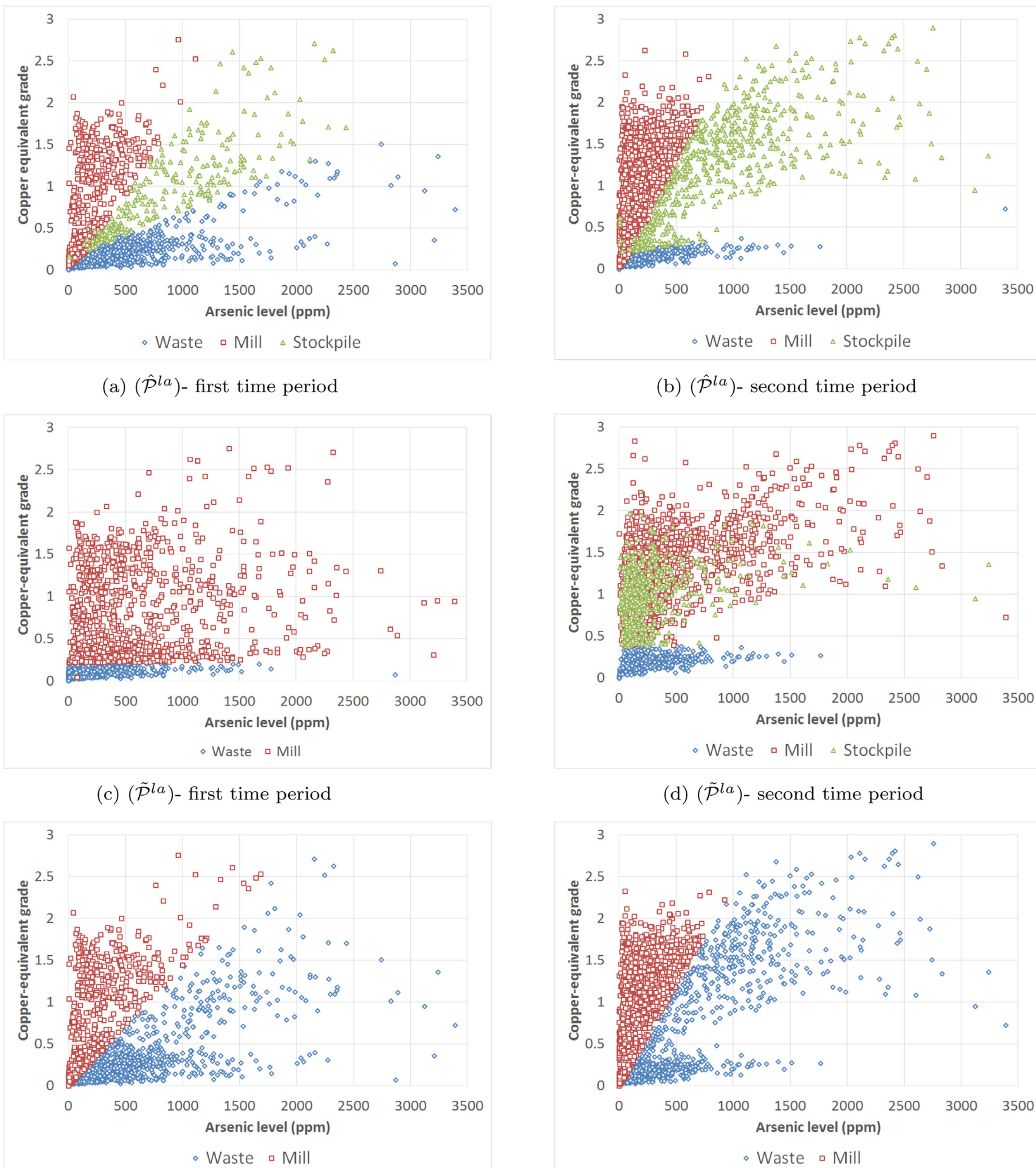
Fig. 9. Comparison of cash flows between the three models.

that both the constraint on the arsenic level at the mill and the presence of a stockpile provide a buffer against particularly large mill grade ranges. A short-term production planning model would include constraints to enforce tighter control of these ranges, and we therefore omit these from our long-term model.

Regarding the use of the stockpile, Fig. 8a shows that $(\hat{\mathcal{P}}^{la})$ sends more material from the mine to the stockpile than $(\tilde{\mathcal{P}}^{la})$, since there is no arsenic limit constraint in the latter model. Fig. 8 compares the material flow to and from the stockpile to the mill and shows that $(\hat{\mathcal{P}}^{la})$ sends material from the mine to the stockpile from the first time period and starts to use the material within the stockpile from the fourth time period onwards, but $(\tilde{\mathcal{P}}^{la})$ sends material to the stockpile from the second time period onwards and uses it from the third time period onwards. In other words, in $(\hat{\mathcal{P}}^{la})$, the material remains in the stockpile for blending purposes to a greater extent. On the other hand, $(\tilde{\mathcal{P}}^{la})$ sends material in the stockpile to the mill primarily in time periods 3 and 4, which means that it does not use the stockpile for blending purposes over the life of the mine. By combining Figs. 7 and 8, we realize that $(\hat{\mathcal{P}}^{la})$ blends the material in the stockpile with the material sent directly from the mine to the mill in such a way that the arsenic limit at the mill is not violated. Fig. 8a and b show that $(\hat{\mathcal{P}}^{la})$ possesses a better stockpile management strategy, because it blends material within the stockpile considering both copper-equivalent grade and arsenic level.

Finally, Fig. 9 compares the cash flow between these three models, showing that in the first two time periods, $(\tilde{\mathcal{P}}^{la})$ provides higher cash flow than the other two models (Fig. 9a), but with an operationally infeasible schedule that does not consider the arsenic limit at the mill. The difference obtained in these two periods is maintained throughout the remainder of the time horizon (Fig. 9b). Also, $(\mathcal{P}^{ns})$ provides a higher cash flow than $(\hat{\mathcal{P}}^{la})$ in the first two time periods, but over the life of the mine, $(\hat{\mathcal{P}}^{la})$ has a higher NPV, showing the benefits of the stockpile and the blending strategy in $(\hat{\mathcal{P}}^{la})$. Note that in the cases we consider, the mine life is independent of stockpiling, because extraction capacity is not a binding constraint; this would not be true in general, but our models are sufficiently flexible to handle variable mine lives depending on the assumptions.

In summary, the schedules produced by these three models extract about the same amount of material, but $(\hat{\mathcal{P}}^{la})$ processes 3.4% and 1.7% more ore than $(\tilde{\mathcal{P}}^{la})$ and $(\mathcal{P}^{ns})$, respectively, with different copper-equivalent grade at the mill in different years, obtaining an NPV that is 5.7% higher than that of $(\mathcal{P}^{ns})$ and



(a) $(\hat{\mathcal{P}}^{la})$- first time period

(b) $(\hat{\mathcal{P}}^{la})$- second time period

(c) $(\tilde{\mathcal{P}}^{la})$- first time period

(d) $(\tilde{\mathcal{P}}^{la})$- second time period

(e) $(\mathcal{P}^{ns})$- first time period

(f) $(\mathcal{P}^{ns})$- second time period

Fig. 10. Destination of extracted material in the three models in the first two time periods.

10.2% lower than that of $(\tilde{\mathcal{P}}^{la})$. Also, $(\hat{\mathcal{P}}^{la})$ presents a better use of the stockpile. Because the main difference in the NPV between $(\hat{\mathcal{P}}^{la})$ and $(\tilde{\mathcal{P}}^{la})$ is obtained at the beginning of the mine life, we provide a deeper analysis of the first two periods.

4.4. Analysis of the first two periods

In order to understand how the schedule obtained by $(\hat{\mathcal{P}}^{la})$ results in higher NPV than $(\mathcal{P}^{ns})$ and why the schedule provided by $(\tilde{\mathcal{P}}^{la})$ is operationally infeasible, we analyze the characteristics of the extracted material, such as tonnage and grades of metal and contaminant, in the first two time periods.

Fig. 7b shows that $(\tilde{\mathcal{P}}^{la})$ processes higher arsenic material at the mill than the other two models, meaning that $(\tilde{\mathcal{P}}^{la})$ exploits its ability to violate the contaminant limit at the mill. Fig. 8a shows that the average copper-equivalent grade of material sent to the plant is greater for $(\tilde{\mathcal{P}}^{la})$. In other words, $(\tilde{\mathcal{P}}^{la})$ processes more valuable material than the other models, resulting in a considerably higher cash flow in the first two time periods.

Fig. 10 shows important differences between the schedules of these three models; each dot represents a block, located by its corresponding grades, and the destination of the block in the schedules. The $(\tilde{\mathcal{P}}^{la})$ model sends to waste all blocks with a copper-equivalent grade less than the economic cutoff grade of 0.21%. In other words, if processing a block does not yield a positive profit, then it is sent to waste. On the contrary, $(\hat{\mathcal{P}}^{la})$ sends to the mill blocks with a copper-equivalent grade below this economic cutoff if they contain a low level of arsenic. In other words, $(\hat{\mathcal{P}}^{la})$ sends low-grade material to the mill, contrary to engineering intuition. This allows these blocks to be blended with material containing a high arsenic and copper-equivalent grade, resulting in $(\hat{\mathcal{P}}^{la})$ just maintaining the maximum arsenic level at the mill while $(\tilde{\mathcal{P}}^{la})$ violates that limit. While material with a high ratio of arsenic to copper is sent to waste by $(\hat{\mathcal{P}}^{la})$, $(\tilde{\mathcal{P}}^{la})$ sends all material above the economic cutoff grade to either the stockpile or to the mill, obtaining higher levels of arsenic at the stockpile, and explaining the violation of the arsenic limit at the mill. Finally, we note that $(\hat{\mathcal{P}}^{la})$ provides a clear delineator regarding the destination of each block based on the ratio of arsenic to copper, whereas $(\tilde{\mathcal{P}}^{la})$ does not, resulting in blocks with similar characteristics sent to different destinations. This may be due to the fact that $(\tilde{\mathcal{P}}^{la})$ does not consider the contaminant level at the mill. Comparing $(\hat{\mathcal{P}}^{la})$ and $(\mathcal{P}^{ns})$ shows the role of the stockpile. In the first time period, $(\hat{\mathcal{P}}^{la})$ and $(\mathcal{P}^{ns})$ extract the same material, but $(\mathcal{P}^{ns})$ sends more material to the mill, resulting in a higher cash flow in that time period; however, $(\hat{\mathcal{P}}^{la})$ sends some material to the stockpile for blending purposes, which results in a higher NPV over the life of the mine (Fig. 9).

5. Conclusion

We use a modified version of $(\mathcal{P}^{la})$ presented by Moreno et al. (2017) to provide long-term planning for an operational open pit mine and compare its schedule with that from two existing models. We observe through our numerical experiments that the LP relaxation of the objective function corresponding to $(\hat{\mathcal{P}}^{la})$ is unimodal regarding blending criteria in the stockpile, which allows us to find the optimal grade combination for the LP. Then, we use TopoSort to create an IP solution from the solution corresponding to the highest LP relaxation value. By comparing the schedule from $(\hat{\mathcal{P}}^{la})$ to those of $(\tilde{\mathcal{P}}^{la})$ and $(\mathcal{P}^{ns})$, we detect that $(\hat{\mathcal{P}}^{la})$ provides a schedule with higher NPV than $(\mathcal{P}^{ns})$. The difference between the NPV of $(\hat{\mathcal{P}}^{la})$ and $(\mathcal{P}^{ns})$ represents the value of the stockpile. Although $(\tilde{\mathcal{P}}^{la})$ provides a higher NPV than $(\hat{\mathcal{P}}^{la})$, it violates the arsenic limit constraint at the mill so its schedule is operationally infeasible. The difference between the NPV of $(\tilde{\mathcal{P}}^{la})$ and $(\hat{\mathcal{P}}^{la})$ represents overestimation of NPV caused by not adequately accounting for contaminants. We show that $(\tilde{\mathcal{P}}^{la})$ sends more material to the processing plant. Also, $(\hat{\mathcal{P}}^{la})$ produces a blending strategy in the stockpile that controls more than one grade. To the best of our knowledge, this research is the first implementation of a MIP model using real data that considers blending and stockpile constraints. Here, we recognize that the economic impact of a suitable blending strategy results in using processing plant capacity more efficiently. A drawback of these stockpiling models is that their instances require longer solution times than those that omit stockpiling. Extensions to this work may include the use of specialized software (Rivera et al., 2016) that has been employed in similar settings (but without stockpiling) to handle very large-scale problems; at the time of this writing, however, this software does not include the option to stockpile.

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