Chapter 3

True-False Exercises

TF. In parts (a)–(k) determine whether the statement is true or false, and justify your answer.

False (a) Two equivalent vectors must have the same initial point.

False (b) The vectors (a, b) and $(a, b, 0)$ are equivalent.

False (c) If k is a scalar and $\mathbf{v}$ is a vector, then $\mathbf{v}$ and $k\mathbf{v}$ are parallel if and only if $k \geq 0$.

True (d) The vectors $\mathbf{v} + (\mathbf{u} + \mathbf{w})$ and $(\mathbf{w} + \mathbf{v}) + \mathbf{u}$ are the same.

True (e) If $\mathbf{u} + \mathbf{v} = \mathbf{u} + \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$.

False (f) If a and b are scalars such that $a\mathbf{u} + b\mathbf{v} = \mathbf{0}$, then $\mathbf{u}$ and $\mathbf{v}$ are parallel vectors.

False (g) Collinear vectors with the same length are equal.

True (h) If $(a, b, c) + (x, y, z) = (x, y, z)$, then (a, b, c) must be the zero vector.

False (i) If k and m are scalars and $\mathbf{u}$ and $\mathbf{v}$ are vectors, then

$$(k + m)(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + m\mathbf{v}$$

True (j) If the vectors $\mathbf{v}$ and $\mathbf{w}$ are given, then the vector equation

$$3(2\mathbf{v} - \mathbf{x}) = 5\mathbf{x} - 4\mathbf{w} + \mathbf{v}$$

can be solved for $\mathbf{x}$.

False (k) The linear combinations $a_1\mathbf{v}_1 + a_2\mathbf{v}_2$ and $b_1\mathbf{v}_1 + b_2\mathbf{v}_2$ can only be equal if $a_1 = b_1$ and $a_2 = b_2$.

True-False Exercises

TF. In parts (a)–(g) determine whether the statement is true or false, and justify your answer.

True (a) The vectors $(3, -1, 2)$ and $(0, 0, 0)$ are orthogonal.

True (b) If $\mathbf{u}$ and $\mathbf{v}$ are orthogonal vectors, then for all nonzero scalars k and m, $k\mathbf{u}$ and $m\mathbf{v}$ are orthogonal vectors.

True (c) The orthogonal projection of $\mathbf{u}$ on $\mathbf{a}$ is perpendicular to the vector component of $\mathbf{u}$ orthogonal to $\mathbf{a}$.

True (d) If $\mathbf{a}$ and $\mathbf{b}$ are orthogonal vectors, then for every nonzero vector $\mathbf{u}$, we have

$$\text{proj}_{\mathbf{a}}(\text{proj}_{\mathbf{b}}(\mathbf{u})) = \mathbf{0}$$

True (e) If $\mathbf{a}$ and $\mathbf{u}$ are nonzero vectors, then

$$\text{proj}_{\mathbf{a}}(\text{proj}_{\mathbf{a}}(\mathbf{u})) = \text{proj}_{\mathbf{a}}(\mathbf{u})$$

False (f) If the relationship

$$\text{proj}_{\mathbf{a}}\mathbf{u} = \text{proj}_{\mathbf{a}}\mathbf{v}$$

holds for some nonzero vector $\mathbf{a}$, then $\mathbf{u} = \mathbf{v}$.

False (g) For all vectors $\mathbf{u}$ and $\mathbf{v}$, it is true that

$$\|\mathbf{u} + \mathbf{v}\| = \|\mathbf{u}\| + \|\mathbf{v}\|$$

True (a) If each component of a vector in R^3 is doubled, the norm of that vector is doubled.

True (b) In R^2, the vectors of norm 5 whose initial points are at the origin have terminal points lying on a circle of radius 5 centered at the origin.

False (c) Every vector in R^n has a positive norm.

True (d) If $\mathbf{v}$ is a nonzero vector in R^n, there are exactly two unit vectors that are parallel to $\mathbf{v}$.

True (e) If $\|\mathbf{u}\| = 2$, $\|\mathbf{v}\| = 1$, and $\mathbf{u} \cdot \mathbf{v} = 1$, then the angle between $\mathbf{u}$ and $\mathbf{v}$ is $\pi/3$ radians.

False (f) The expressions $(\mathbf{u} \cdot \mathbf{v}) + \mathbf{w}$ and $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$ are both meaningful and equal to each other.

False (g) If $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$.

(h) If $\mathbf{u} \cdot \mathbf{v} = 0$, then either $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$. False

(i) In R^2, if $\mathbf{u}$ lies in the first quadrant and $\mathbf{v}$ lies in the third quadrant, then $\mathbf{u} \cdot \mathbf{v}$ cannot be positive. True

(j) For all vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in R^n, we have True

$$\|\mathbf{u} + \mathbf{v} + \mathbf{w}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\| + \|\mathbf{w}\|$$

Working with Technology

T1. Let $\mathbf{u}$ be a vector in R^{100} whose ith component is i, and let $\mathbf{v}$ be the vector in R^{100} whose ith component is $1/(i + 1)$. Find the dot product of $\mathbf{u}$ and $\mathbf{v}$.

T2. Find, to the nearest degree, the angles that a diagonal of a box with dimensions 10 cm × 11 cm × 25 cm makes with the edges of the box.

True-False Exercises

TF. In parts (a)–(f) determine whether the statement is true or false, and justify your answer.

True (a) The vector equation of a line can be determined from any point lying on the line and a nonzero vector parallel to the line.

False (b) The vector equation of a plane can be determined from any point lying in the plane and a nonzero vector parallel to the plane.

True (c) The points lying on a line through the origin in R^2 or R^3 are all scalar multiples of any nonzero vector on the line.

True (d) All solution vectors of the linear system $A\mathbf{x} = \mathbf{b}$ are orthogonal to the row vectors of the matrix A if and only if $\mathbf{b} = \mathbf{0}$.

False (e) The general solution of the nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$ can be obtained by adding $\mathbf{b}$ to the general solution of the homogeneous linear system $A\mathbf{x} = \mathbf{0}$.

True (f) If $\mathbf{x}_1$ and $\mathbf{x}_2$ are two solutions of the nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$, then $\mathbf{x}_1 - \mathbf{x}_2$ is a solution of the corresponding homogeneous linear system.

True-False Exercises

TF. In parts (a)–(f) determine whether the statement is true or false, and justify your answer.

True (a) The cross product of two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ is a nonzero vector if and only if $\mathbf{u}$ and $\mathbf{v}$ are not parallel.

True (b) A normal vector to a plane can be obtained by taking the cross product of two nonzero and noncollinear vectors lying in the plane.

False (c) The scalar triple product of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ determines a vector whose length is equal to the volume of the parallelepiped determined by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

True (d) If $\mathbf{u}$ and $\mathbf{v}$ are vectors in 3-space, then $\|\mathbf{v} \times \mathbf{u}\|$ is equal to the area of the parallelogram determined by $\mathbf{u}$ and $\mathbf{v}$.

False (e) For all vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in 3-space, the vectors $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$ and $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$ are the same.

False (f) If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in R^3, where $\mathbf{u}$ is nonzero and $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$.

Chapter 4:

True-False Exercises

TF. In parts (a)–(f) determine whether the statement is true or false, and justify your answer.

True (a) A vector is any element of a vector space.

False (b) A vector space must contain at least two vectors.

False (c) If $\mathbf{u}$ is a vector and k is a scalar such that $k\mathbf{u} = \mathbf{0}$, then it must be true that $k = 0$.

False (d) The set of positive real numbers is a vector space if vector addition and scalar multiplication are the usual operations of addition and multiplication of real numbers.

True (e) In every vector space the vectors $(-1)\mathbf{u}$ and $-\mathbf{u}$ are the same. $1u = u$

False (f) In the vector space $F(-\infty, \infty)$ any function whose graph passes through the origin is a zero vector.

True-False Exercises

TF. In parts (a)–(k) determine whether the statement is true or false, and justify your answer.

True (a) Every subspace of a vector space is itself a vector space.

True (b) Every vector space is a subspace of itself.

False (c) Every subset of a vector space V that contains the zero vector in V is a subspace of V.

False (d) The kernel of a matrix transformation $T_A: R^n \to R^m$ is a subspace of R^m.

False (e) The solution set of a consistent linear system $A\mathbf{x} = \mathbf{b}$ of m equations in n unknowns is a subspace of R^n.

True (f) The span of any finite set of vectors in a vector space is closed under addition and scalar multiplication.

True (g) The intersection of any two subspaces of a vector space V is a subspace of V.

False (h) The union of any two subspaces of a vector space V is a subspace of V.

False (i) Two subsets of a vector space V that span the same subspace of V must be equal.

True (j) The set of upper triangular $n \times n$ matrices is a subspace of the vector space of all $n \times n$ matrices.

False (k) The polynomials $x - 1$, $(x - 1)^2$, and $(x - 1)^3$ span P_3.

Working with Technology

False (a) A set containing a single vector is linearly independent.

True (b) The set of vectors $\{\mathbf{v}, k\mathbf{v}\}$ is linearly dependent for every scalar k.

False (c) Every linearly dependent set contains the zero vector.

True (d) If the set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent, then $\{k\mathbf{v}_1, k\mathbf{v}_2, k\mathbf{v}_3\}$ is also linearly independent for every nonzero scalar k.

True (e) If $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are linearly dependent nonzero vectors, then at least one vector $\mathbf{v}_k$ is a unique linear combination of $\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}$.

False (f) The set of 2×2 matrices that contain exactly two 1's and two 0's is a linearly independent set in M_{22}.

True (g) The three polynomials $(x - 1)(x + 2)$, $x(x + 2)$, and $x(x - 1)$ are linearly independent.

False (h) The functions f_1 and f_2 are linearly dependent if there is a real number x such that $k_1 f_1(x) + k_2 f_2(x) = 0$ for some scalars k_1 and k_2.

True-False Exercises

TF. In parts (a)–(e) determine whether the statement is true or false, and justify your answer.

False (a) If $V = \text{span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$, then $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is a basis for V.

False (b) Every linearly independent subset of a vector space V is a basis for V.

True (c) If $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ is a basis for a vector space V, then every vector in V can be expressed as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$.

True (d) The coordinate vector of a vector $\mathbf{x}$ in R^n relative to the standard basis for R^n is $\mathbf{x}$.

False (e) Every basis of P_4 contains at least one polynomial of degree 3 or less.

True-False Exercises

TF. In parts (a)–(k) determine whether the statement is true or false, and justify your answer.

True (a) The zero vector space has dimension zero.

True (b) There is a set of 17 linearly independent vectors in R^{17}.

False (c) There is a set of 11 vectors that span R^{17}.

True (d) Every linearly independent set of five vectors in R^5 is a basis for R^5.

True (e) Every set of five vectors that spans R^5 is a basis for R^5.

True (f) Every set of vectors that spans R^n contains a basis for R^n.

True (g) Every linearly independent set of vectors in R^n is contained in some basis for R^n.

True (h) There is a basis for M_{22} consisting of invertible matrices.

True (i) If A has size $n \times n$ and $I_n, A, A^2, \ldots, A^{n^2}$ are distinct matrices, then $\{I_n, A, A^2, \ldots, A^{n^2}\}$ is a linearly dependent set.

False (j) There are at least two distinct three-dimensional subspaces of P_2.

(k) There are only three distinct two-dimensional subspaces of P_2.

True-False Exercises

TF. In parts (a)–(f) determine whether the statement is true or false, and justify your answer.

True (a) If B_1 and B_2 are bases for a vector space V, then there exists a transition matrix from B_1 to B_2.

True (b) Transition matrices are invertible.

True (c) If B is a basis for a vector space R^n, then $P_{B \to B}$ is the identity matrix.

True (d) If $P_{B_1 \to B_2}$ is a diagonal matrix, then each vector in B_2 is a scalar multiple of some vector in B_1.

False (e) If each vector in B_2 is a scalar multiple of some vector in B_1, then $P_{B_1 \to B_2}$ is a diagonal matrix.

False (f) If A is a square matrix, then $A = P_{B_1 \to B_2}$ for some bases B_1 and B_2 for R^n.

True-False Exercises

TF. In parts (a)–(j) determine whether the statement is true or false, and justify your answer.

True (a) The span of $\mathbf{v}_1, \ldots, \mathbf{v}_n$ is the column space of the matrix whose column vectors are $\mathbf{v}_1, \ldots, \mathbf{v}_n$.

False (b) The column space of a matrix A is the set of solutions of $A\mathbf{x} = \mathbf{b}$.

False (c) If R is the reduced row echelon form of A, then those column vectors of R that contain the leading 1's form a basis for the column space of A.

False (d) The set of nonzero row vectors of a matrix A is a basis for the row space of A.

False (e) If A and B are $n \times n$ matrices that have the same row space, then A and B have the same column space.

True (f) If E is an $m \times m$ elementary matrix and A is an $m \times n$ matrix, then the null space of EA is the same as the null space of A.

True (g) If E is an $m \times m$ elementary matrix and A is an $m \times n$ matrix, then the row space of EA is the same as the row space of A.

False (h) If E is an $m \times m$ elementary matrix and A is an $m \times n$ matrix, then the column space of EA is the same as the column space of A.

True (i) The system $A\mathbf{x} = \mathbf{b}$ is inconsistent if and only if $\mathbf{b}$ is not in the column space of A.

False (j) There is an invertible matrix A and a singular matrix B such that the row spaces of A and B are the same.

True-False Exercises

TF. In parts (a)–(j) determine whether the statement is true or false, and justify your answer.

False (a) Either the row vectors or the column vectors of a square matrix are linearly independent.

True (b) A matrix with linearly independent row vectors and linearly independent column vectors is square.

False (c) The nullity of a nonzero $m \times n$ matrix is at most m.

False (d) Adding one additional column to a matrix increases its rank by one.

True (e) The nullity of a square matrix with linearly dependent rows is at least one.

False (f) If A is square and $A\mathbf{x} = \mathbf{b}$ is inconsistent for some vector $\mathbf{b}$, then the nullity of A is zero.

False (g) If a matrix A has more rows than columns, then the dimension of the row space is greater than the dimension of the column space.

False (h) If $\operatorname{rank}(A^T) = \operatorname{rank}(A)$, then A is square.

True (i) There is no 3×3 matrix whose row space and null space are both lines in 3-space.

False (j) If V is a subspace of R^n and W is a subspace of V, then $W^\perp$ is a subspace of $V^\perp$.

True-False Exercises

TF. In parts (a)–(g) determine whether the statement is true or false, and justify your answer.

False (a) The image of the unit square under a one-to-one matrix operator is a square.

True (b) A 2×2 invertible matrix operator has the geometric effect of a succession of shears, compressions, expansions, and reflections.

True (c) The image of a line under an invertible matrix operator is a line.

True (d) Every reflection operator on R^2 is its own inverse.

False (e) The matrix $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ represents reflection about a line.

False (f) The matrix $\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ represents a shear.

True (g) The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ represents an expansion.

Chapter 5:

True-False Exercises

TF. In parts (a)–(f) determine whether the statement is true or false, and justify your answer.

False (a) If A is a square matrix and $A\mathbf{x} = \lambda\mathbf{x}$ for some nonzero scalar λ, then $\mathbf{x}$ is an eigenvector of A.

False (b) If λ is an eigenvalue of a matrix A, then the linear system $(\lambda I - A)\mathbf{x} = \mathbf{0}$ has only the trivial solution.

True (c) If the characteristic polynomial of a matrix A is $p(\lambda) = \lambda^2 + 1$, then A is invertible.

False (d) If λ is an eigenvalue of a matrix A, then the eigenspace of A corresponding to λ is the set of eigenvectors of A corresponding to λ.

False (e) The eigenvalues of a matrix A are the same as the eigenvalues of the reduced row echelon form of A.

False (f) If 0 is an eigenvalue of a matrix A, then the set of columns of A is linearly independent.