The Ultimatum Game in complex networks

Rafael Koener, 76475

Adrian Quaresma, 77913

1 Introduction

In this report we aimed to obtain the same results studied in "The ultimatum game in complex networks" by .Sinatra, R., J. Iranzo, J. Gomez-Gardenes, L.M. Floria, V. Latora, and Y. Moreno, on how cooperative behaviour emerges in different systems using simulation of the Ultimatum Game in complex networks. Three types of players were considered: (A) who proposes and accepts offers of the same value, (B) those who want to always have the same accumulated value at the end of each "negotiation" and (C) whose proposals and acceptance of offers are independent. The behaviour of populations was analysed using two strategies to update the game's rules, they are natural selection and social penalty.

2 The Model

The model we use associates each player with a node and with these nodes we created a graph. Each node has three parameters: **p**, represents the offer; **q**, represents the acceptance threshold; **f**, represents the payoffs.We studied two graph topologies: **Erdös-Rényi** (ER) and **scale-free network** (SF). We choose these networks because they have different behaviors, while the Erdös-Rényi have his degree distribution that decays exponentially for large k, the scale-free network have a degree distribution that follows a power law of the form $P_k \approx k^{-\gamma}$. We consider $\gamma \approx 3$.

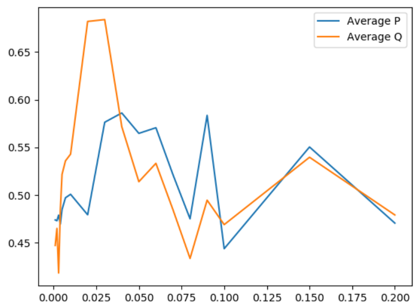

Figure 1: Evolution of average p and q in function of errors introduced when a player imitate other for SF

2.1 Ultimatum Game

In the Ultimate Game, two players are asked to split a certain sum of money. At each time step, each player plays a round robin with all its neighbors. In each round, a randomly chosen player plays the ultimate game twice with each neighbor, first as proposers and then as respondents. If the the offer is greater than the acceptance threshold, respondent accepts the offer and the money will be shared accordingly. However, if the offer is rejected, both players receive nothing.

2.2 Strategies

The strategies studied to update the p and q values for each player, after playing a round of the ultimatum game with each of its neighbours were, as mentioned above, natural selection and social penalty.

- *Natural Selection*: each player i selects one of its neighbours j randomly and compares their fitness f_i, f_j.

  If $f_j > f_i$, then player i uses the strategy of its neighbour j, using its p and q values, with a given probability. This strategy copying happens with a probability proportional to the fitness difference of both players., where d_i and d_j are the degrees of both players.

  $$P_{ij} = \mathrm{f}_j - f_i \frac{}{2*max(d_i, d_j)}$$

  Contrary to this, if $f_j < f_i$, then i keeps its current strategy for the next round.

- *Social Penalty*: The idea of social penalty is removing the player in the entire population with the lowest fitness and all its first neighbours. To these players a new random strategy is given. In natural selection, at each round of the Ultimatum Game, there is a comparison between two players and because of this, the better strategies will begin to spread in the population. While in social penalty the updates of the strategies have global effects on the population, leading to the extinction of certain players and consequently certain strategies.

3 Natural Selection in the Ultimatum Game

In all of the following figures in which the results are shown, it has to be mentioned that for the expected results, it is said that the graphs used were of N=10^4 and each simulation was ran for 2 generations, but the used number of generations used in the graphs obtained in this study was of 10. This was due to not having satisfactory results for a smaller number of rounds in the Ultimatum Game. Both ER and SF graphs used have N = 10^4 nodes.

3.1 Networks of players with (p=q)

These are empathetic players as on any given round of the Ultimatum Game, at least one of the two players gets some fitness. Imagine players i and j, if $p_i > p_j$. then j gains a larger fitness than i if $p_i >$ 0.5 and the opposite occurs if $p_i < 0.5$. Given this it is expected that the values of p, and consequently q, to converge to 0.5. Which is clearly visible in both of the following figures (figures 2 and 3). Albeit that on the obtained graph for a SF network the seems to be a large number of players with $p = 0$ there seem to be a tendency to have a convergence to $p = 0.5$

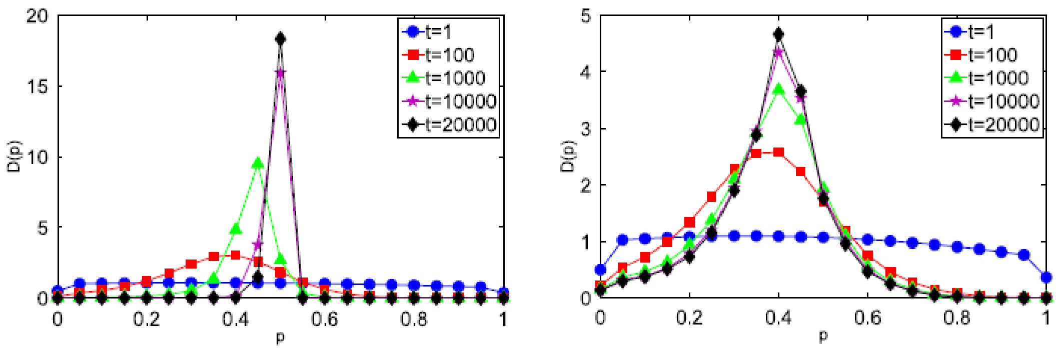

Figure 2: Distribution of offers D(p) for ER (left) and SF (right) networks in the cases p = q

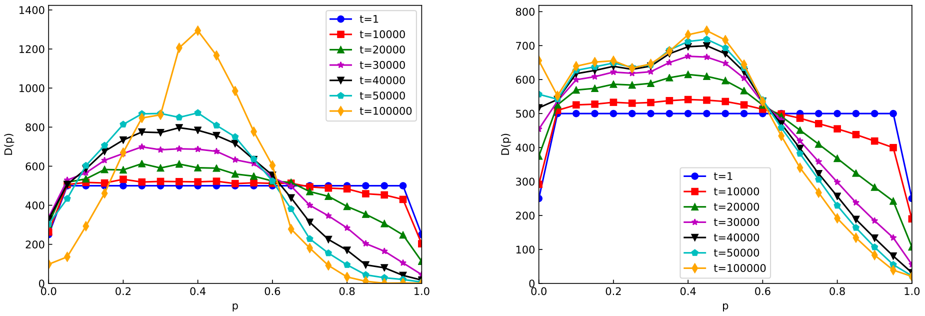

Figure 3: Distribution of offers D(p) for ER (left) and SF (right) networks in the cases p = q obtained.

3.2 Networks of players with (p=1-q)

These players are the mos logical, two players i and j only conclude a deal deal if $p_i + p_j$ is at least 1. If this holds and $p_i > p_j$ the finesses obtained $f_j > f_i$, the contrary applies and if $p_i = p_j$, then $f_i = f_j$. And so it is expected that the smaller offers are quickly extinguished and at later stages of the simulation, the p values tend to converge to, again, 0.5 but starting its values around 0.7. Although the exact shape of the evolution of p values wasn't perfect, we are able to see, in figures 4 and 5 a similar progression of the values for the obtained graphs in this study, especially for the ER network.

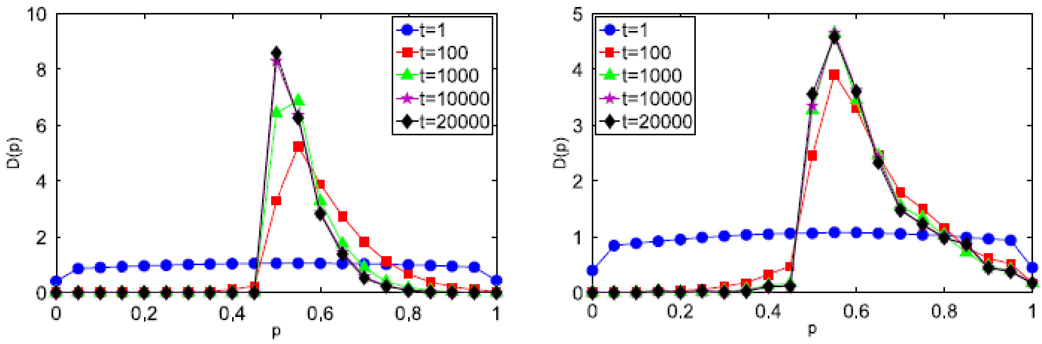

Figure 4: Distribution of offers D(p) for ER (left) and SF (right) networks in the cases p = 1 - q

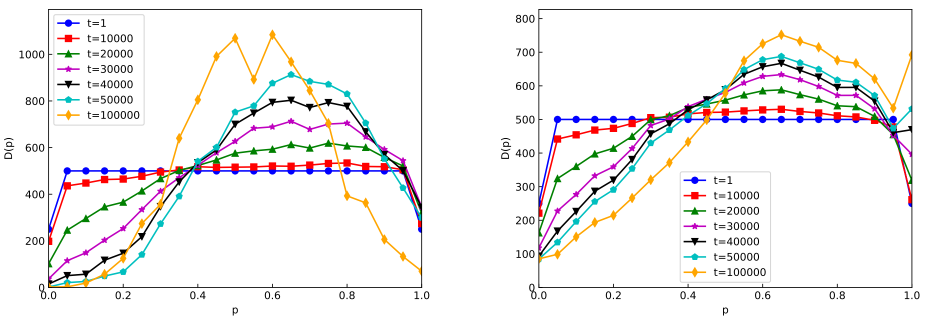

Figure 5: Distribution of offers D(p) for ER (left) and SF (right) networks in the cases p = 1 - q obtained.

3.3 Networks of players with (independent p and q)

Here is when is expected to have some altruism emerge. When it come to the ER network, it is clear to see that the behaviours and evolution of the players strategies are similar to the players with $p = q$ in figure 2, but dislocated to an average p of 0.3.

The larger offers are also the first to disappear for both types of players. This also noticeable on the obtained graph in figure 13.

Now focusing on the SF networks, it is interesting to see that, as happens in both of the other types of players, offers of p larger than 0.5 are is a smaller than those of $p < 0.5$. It is also pleasing to see the distributions for both networks of acceptance rates q, have a maximum of approximately 0.3 and that some players are even considering having an acceptance of 0, this means that some rational behaviour is emerging and will possibly dominate the other strategies ($q = 0$)

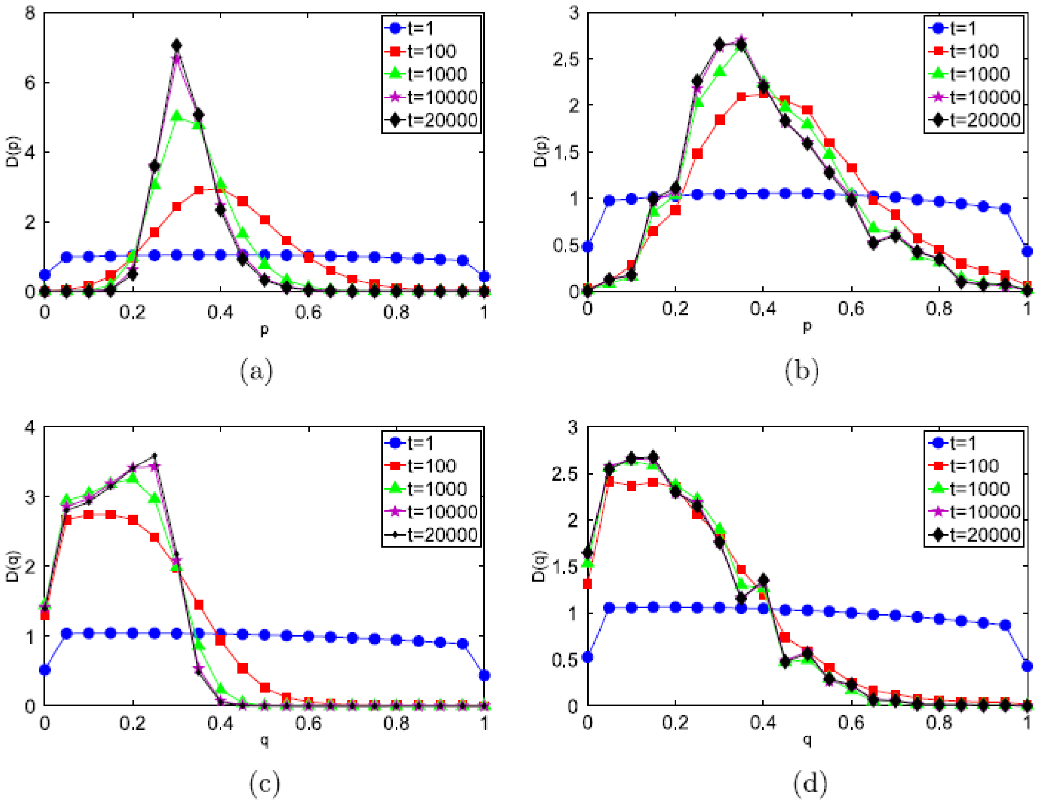

Figure 6: Distributions of offers D(p) ((a) and (b)) and of acceptance thresholds D(q) ((c) and (d)) for ER ((a) and (c)) and SF ((b) and (d)) networks when p and q are independent

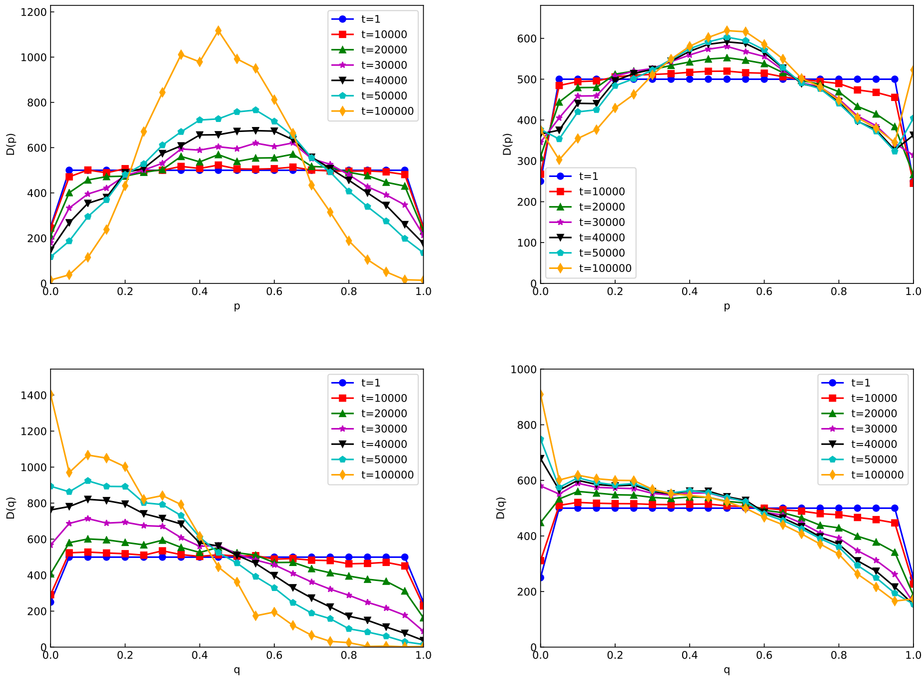

Figure 7: Distributions of offers D(p) (top) and of acceptance thresholds D(q) (bottom) for ER (left) and SF (right) networks when a p and q are independent obtained.

4 Social Penalty in the Ultimatum Game

In this case, how it is applied social penalty after each round robin of the Ultimate Game, a player has to take care of its payoff and of its neighbors, since the poorest player of the network is replaced together with all its neighbors. Therefore, if a player exploits its neighborhood, he take the risk of being dropped out of the game, since one of its neighbors can be the player with the lowest payoff in the game.

4.1 Networks of players with (p=q)

For ER networks the distribution of p does not vary much, so any strategy can survive in a population of this type of players with homogeneous degree. While in the SF network there is a large portion of the population with a high p, so it is evident that most of the offers are accepted.

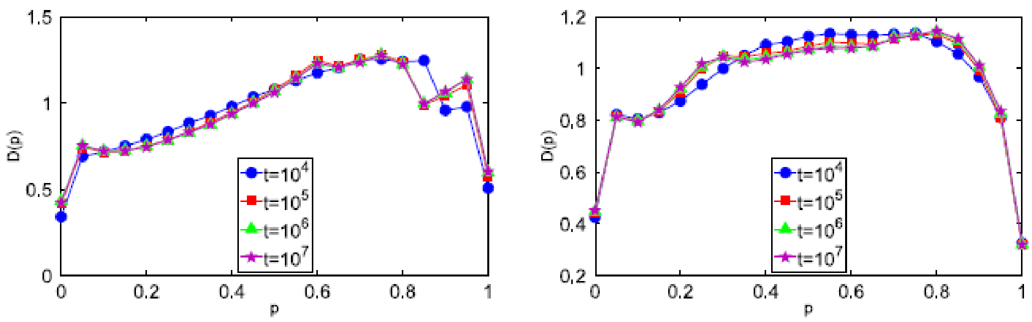

Figure 8: Distribution of offers D(p) for ER (left) and SF (right) networks in the cases p = q

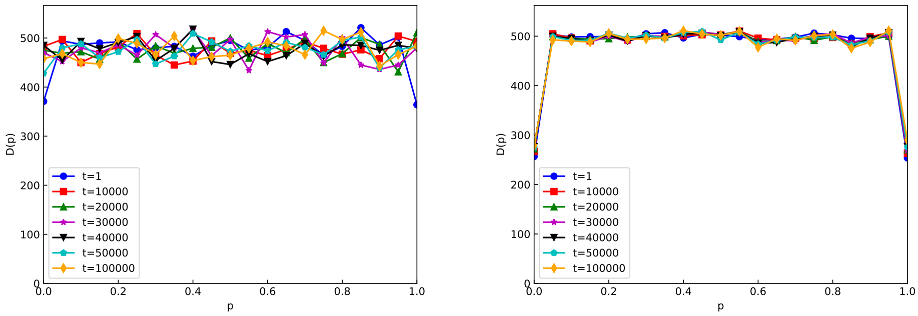

Figure 9: Distribution of offers D(p) for ER (left) and SF (right) networks in the cases p = q obtained.

4.2 Networks of players with (p=1-q)

An interesting fact that we found is that both ER and SF networks have the same average value for the offers. However, the distribution densities are different for the two kind of networks.

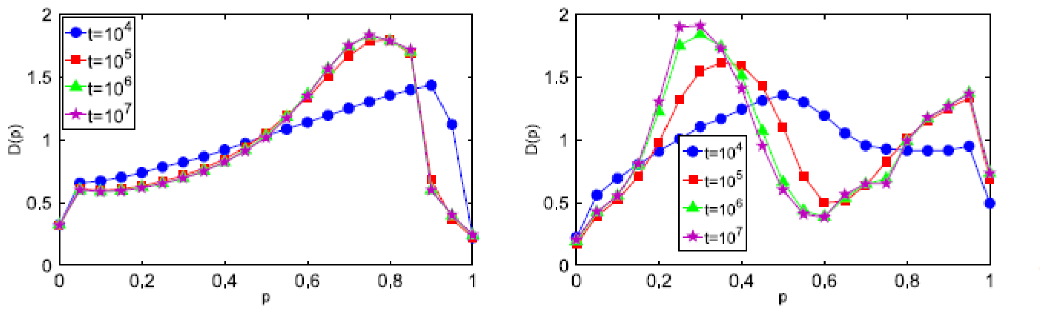

Figure 10: Distribution of offers D(p) for ER (left) and SF (right) networks in the cases p = 1 - q

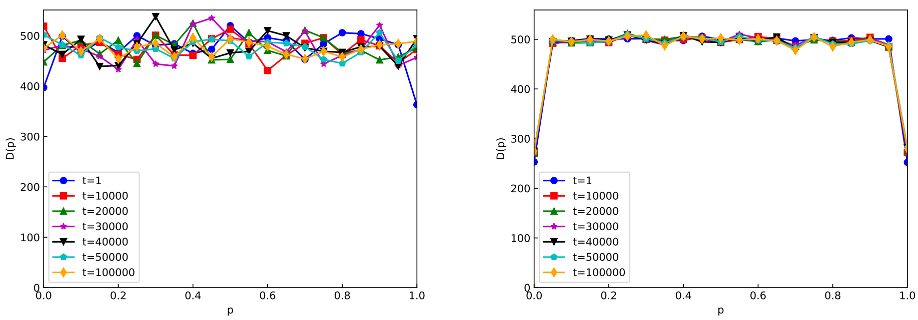

Figure 11: Distribution of offers D(p) for ER (left) and SF (right) networks in the cases p = 1 - q obtained.

4.3 Networks of players with independent p and q

From the ER and SF networks we can see that, both at first have low offers p and high acceptance treshold.

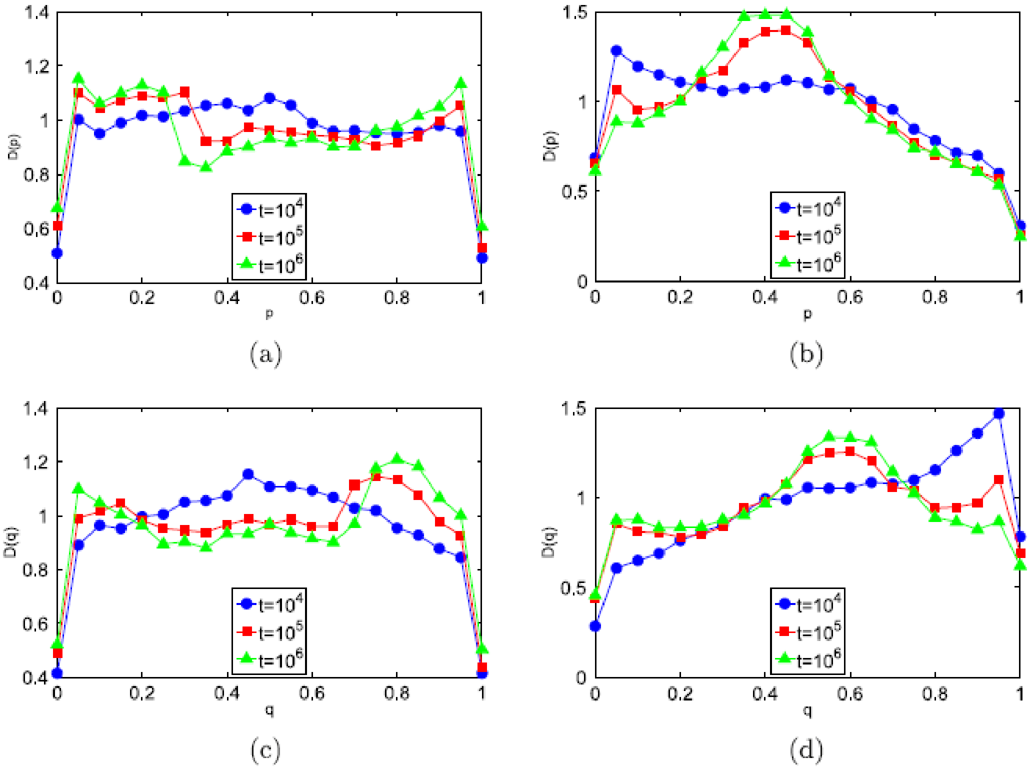

Figure 12: Distributions of offers D(p) ((a) and (b)) and of acceptance thresholds D(q) ((c) and (d)) for ER ((a) and (c)) and SF ((b) and (d)) networks when p and q are independent

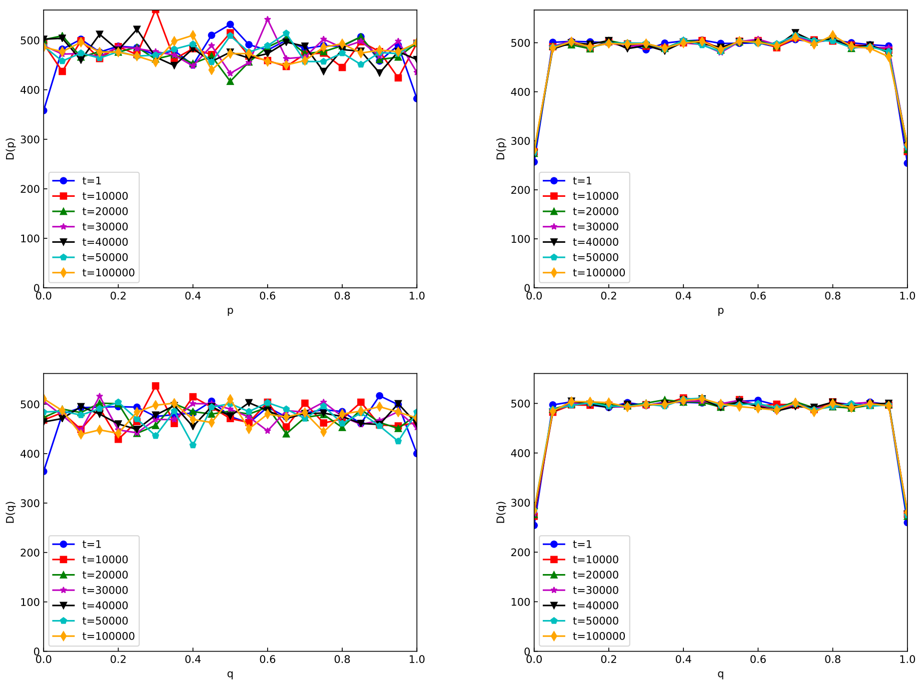

Figure 13: Distributions of offers D(p) (top) and of acceptance thresholds D(q) (bottom) for ER (left) and SF (right) networks when a p and q are independent obtained.

5 Discussion

It was tried to obtained the same results obtained in the original paper, and on some points we were able to get some interesting and somewhat similar results for the natural selection, but when it comes to social penalty, we were not able to produce similar results, with great disappointment. This can be due to the fact that our encoding of the removal of the worst performing player in the network and its first neighbours is not executed correctly.

The Ultimatum Game was explored in different configurations, in regard to the behaviour of players in it.

Firstly we explored empathetic players (who offer the same amount as they want to receive), logical players (who wish to obtain a fixed amount as proposer and as responder) and players with independent values for offers and proposiitons. The existence of hubs in the SF networks can explain the selection of fitness strategies to have asymptotic values in the strategies values.

References

[1] Sinatra, R., J. Iranzo, J. Gomez-Gardenes, L.M. Floria, V. Latora, and Y. Moreno, The ultimatum game in complex networks. (Journal of Statistical Mechanics: Theory and Experiment, 2009.)

[2] Page, K.M., M.A. Nowak, and K. Sigmund, The spatial ultimatum game. (Proceedings of the Royal Society of London B: Biological Sciences, 2000. 267(1458))

[3] Code available at https://github.com/RafaelKoener/Ultimatum-Game