Planning, Learning and Decision Making

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Homework 4. Supervised learning

(a)

$$\pi(1 \mid x) = \sigma(z)$$

$$\pi(a \mid x) = \sigma(a \times z)$$

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$z = w^T \phi(x)$$

$$\pi(a \mid x) = \sigma(az) = \frac{1}{1 + \exp(-az)} = \frac{1}{1 + \exp(-a w^T \phi(x))}$$

$$\hat{L}_N(\pi) = -\frac{1}{N} \sum_{n=1}^{N} \log \frac{1}{1 + \exp(-a_n w^T \phi(x_n))}$$

$$\log(1/x) = -\log(x)$$

$$\Rightarrow \hat{L}_N(\pi) = \frac{1}{N} \sum_{n=1}^{N} \log\left(1 + \exp(-a_n w^T \phi(x_n))\right)$$

$$= \frac{1}{N} \sum_{n=1}^{N} \log\left(\exp(-a_n w^T \phi(x_n)) + 1\right)$$

(b)

$$\frac{d}{dw}\hat{L}_N(\pi) = \frac{d}{dw}\left[\frac{1}{N}\sum_{n=1}^{N}\log\left(\exp(-a_n w^T\phi(x_n)) + 1\right)\right]$$

$$= \frac{1}{N}\sum \frac{d}{dw}\log\left(\exp(\ldots) + 1\right)$$

$$= \frac{1}{N}\sum \frac{1}{\exp()+1} \times \left(-a_n\phi(x_n)\left(\exp(\ldots)\right)\right)$$

$$= \frac{1}{N}\sum \frac{1}{\exp()+1} \times \left(-a_n\phi(x_n)\left(\frac{1}{\pi(a_n|x_n)} - 1\right)\right)$$

$$= \frac{1}{N}\sum \pi(a_n|x_n) \times \left(-a_n\phi(x_n)\left(\frac{1}{\pi(a_n|x_n)} - 1\right)\right)$$

$$= \frac{1}{N}\sum \pi(a_n|x_n) \times \left[\frac{-a_n\phi(x_n)}{\pi(a_n|x_n)} + a_n\phi(x_n)\right]$$

$$= \frac{1}{N}\sum -a_n\phi(x_n) + a_n\phi(x_n)\pi(a_n|x_n)$$

$$= \frac{1}{N}\sum_{n=1}^{N} a_n\phi(x_n)\left(-1 + \pi(a_n|x_n)\right)$$

$$= \frac{1}{N}\sum_{n=1}^{N} a_n\phi(x_n)\left(\pi(a_n|x_n) - 1\right)$$

$$\pi(a_n|x_n) = \frac{1}{1+\exp()}$$

$$1+\exp() = \frac{1}{\pi()}$$

$$\Rightarrow \exp() = \frac{1}{\pi()} - 1$$

(c)

$$\frac{d^2}{dw} \hat{L}_n(\pi) = \frac{d}{dw} \frac{1}{N} \sum_{n=1}^{N} a_n \phi(x_n) (\pi(a_n|x_n) - 1)$$

$$= \frac{d}{dw} \frac{1}{N} \sum_{n=1}^{N} a_n \phi(x_n) \left(\frac{1}{1+\exp()} - 1 \right)$$

$$= \frac{1}{N} \sum_{n=1}^{N} \frac{d}{dw} a_n \phi(x_n) \left(\frac{1}{1+\exp()} - 1 \right)$$

$$= \frac{1}{N} \sum_{n=1}^{N} 0 + a_n \phi(x_n) \left(\frac{1}{1+\exp()} - 1 \right)'$$

$$= \frac{1}{N} \sum_{n=1}^{N} a_n \phi(x_n) \left(\frac{1}{1+\exp()} \right)'$$

$$= \frac{1}{N} \sum_{n=1}^{N} a_n \phi(x_n) \left[\frac{0 - (1+\exp())'}{(1+\exp())^2} \right]$$

$$= \frac{1}{N} \sum_{n=1}^{N} a_n \phi(x_n) \left[\frac{-(-a_n \phi(x_n) \exp())}{(1+\exp())^2} \right]$$

a_n^2

$\Rightarrow (-1)^2 \vee 1^2$

$\Rightarrow a_n^2 = 1$

$$= \frac{1}{N} \sum_{n=1}^{N} a_n \phi(x_n) \left[a_n \phi(x_n) \exp() \times (\pi(a_n|x_n))^2 \right]$$

$$= \frac{1}{N} \sum_{n=1}^{N} a_n \phi(x_n) a_n \phi(x_n) \exp() (\pi(a_n|x_n))^2$$

$$= \frac{1}{N} \sum_{n=1}^{N} \phi(x_n) \phi^T(x_n) \pi(a_n|x_n) \left[\pi(a_n|x_n) \exp(\ldots) \right]$$

$$= \frac{1}{N} \sum_{n=1}^{N} \phi(x_n) \phi^T(x_n) \pi(a_n|x_n) \left[\pi(a_n|x_n) \left(\frac{1}{\pi(a_n|x_n)} - 1 \right) \right]$$

$$= \frac{1}{N} \sum_{n=1}^{N} \phi(x_n) \phi^T(x_n) \pi(a_n|x_n) \left[\frac{\pi(a_n|x_n)}{\pi(a_n|x_n)} - \pi(a_n|x_n) \right]$$

$$= \frac{1}{N} \sum_{n=1}^{N} \phi(x_n) \phi^T(x_n) \pi(a_n|x_n) (1 - \pi(a_n|x_n))$$