TOPOLOGICAL ENTROPY CALCULATION PROBLEMS

1. Test Flow

We have as test flows various time-dependent vortex drivers (blinking vortices) on $\mathbb{R}^2$: One case is for vortices with opposite direction (cf. $\sigma_1\sigma_2^{-1}$) and the second case vortices of the same direction (cf. $\sigma_1\sigma_2$). (This creates already two different flows and then our plan was to create further test cases by moving the centre of one vortex further/closer from the other.)

Having calculated entropies for both cases we found that the value for the flow based on $\sigma_1\sigma_2^{-1}$ was lower than that based on $\sigma_1\sigma_2$.

In an attempt to investigate this we simplified our system down to the simple case of one vortex $\mathbf{v} = v_\phi(r)\mathbf{e}_\phi$. Specifically the velocity profile is given by:

$$\begin{aligned} v_x &= yf(t)\exp((-x^2 - y^2)/2) \\ v_y &= -xf(t)\exp((-x^2 - y^2)/2) \end{aligned} \tag{1}$$

where $f(t)$ is given by

$$f(t) = \begin{cases} -0.075\cos t + 0.075 & 0 < t \text{ and } t < \pi \\ 0.15 & \pi < t \text{ and } t < 11\pi \\ -0.075\cos t + 0.075 & 11\pi < t \text{ and } t < 12\pi \end{cases} \tag{2}$$

Here with trajectories staying on constant radii, material lines grow linearly in time.

2. Topological Entropy Results

Using the `braidlab` routines to calculate the topological entropy of the flow, we expected zero entropy for this flow.

Problem 1 With sufficient numbers of tracing particles the flow is identified in the routines as having positive topological entropy (using `entropy(b)`). A minimal example test case where this is seen is Case 3 in Table 1, but it is a generic feature.

Problem 2

Using the `tntype(b)` method we have an identification as reducible with 0 entropy – Case 1 of Table 1.

Set of particles	`entropy(b)`	`tntype(b)`
(1) (0,0.2), (-0.9,0.3), (0.4,-0.7), (-0.3,-0.4)	0 (f.t.c)	reducible, 0
(2) (0,0.2), (-0.9,0.3), (0.4,-0.7), (-0.3,-0.4), (0.8,0.6), (0.1,0), (-0.6,-0.3), (0.5,0.7), (-0.1,-0.6)	0.4421	reducible, 0
(3) (0,0.5), (-0.1,-0.9), (1,0.67), (-0.6,-0.2), (0.3,0.3)	0.5435	pA, 0.5435

Table 1: Entropy values for specific particle sets.

We noted that the function `tntype_helper.m` states in its description that it fails for certain braids. Perhaps this explains Problem 2? However that still leaves us with the problem of the positive entropies, Problem 1.

For reference the `MATLAB` commands used in Case 3 are below, calling on the flow function attached.

```
n=5;tmax=12*pi;npts=3000; XY=zeros(npts,2,n);
[t,xy]= ode45(@drive_simple, linspace(0, tmax, npts)',[0,0.5]);
XY(:,1,1)=xy(:,1); XY(:,2,1)=xy(:,2);
[t,xy]= ode45(@drive_simple, linspace(0, tmax, npts)',[-0.1,-0.9]);
XY(:,1,2)=xy(:,1); XY(:,2,2)=xy(:,2);
[t,xy]= ode45(@drive_simple, linspace(0, tmax, npts)',[1,0.67]);
XY(:,1,3)=xy(:,1); XY(:,2,3)=xy(:,2);
[t,xy]= ode45(@drive_simple, linspace(0, tmax, npts)',[-0.6,-0.2]);
XY(:,1,4)=xy(:,1); XY(:,2,4)=xy(:,2);
[t,xy]= ode45(@drive_simple, linspace(0, tmax, npts)',[0.3,0.3]);
XY(:,1,5)=xy(:,1); XY(:,2,5)=xy(:,2);
b=braid(XY)
entropy(b)
```