Ficha 2015

14 de fevereiro de 2022 15:56

I

$m = 2kg$ $\qquad \vec{r_0} = -1\vec{e_x} + 2\vec{e_y}\ (m)$

$\vec{F}(t) = -t\vec{e_x} + 2\vec{e_y}$

a) $\vec{F} = m\vec{a} \Leftrightarrow \vec{a} = \dfrac{-t\vec{e_x} + 2\vec{e_y}}{2} = -\dfrac{t}{2}\vec{e_x} + \vec{e_y}$

e $\vec{r}(t) = \vec{r_0}(t) + \int_0^t \vec{v}(t)\,dt = \vec{r_0}(t) + \int_0^t \vec{v_0} + \int_0^t \vec{a}(t)\,dt\,dt$

Como o corpo inicialmente está em repouso então $\vec{v_0} = \vec{0}$

logo $\vec{r}(t) = -1\vec{e_x} + 2\vec{e_y} + \int_0^t\int_0^t -\frac{t}{2}\vec{e_x} + \vec{e_y}\,dt = -1\vec{e_x} + 2\vec{e_y} + \int_0^t -\frac{t^2}{4}\vec{e_x} + t\vec{e_y}\,dt =$

$= -1\vec{e_x} + 2\vec{e_y} - \dfrac{t^3}{12}\vec{e_x} + \dfrac{t^2}{2}\vec{e_y} = \left(-1 - \dfrac{t^3}{12}\right)\vec{e_x} + \left(2 + \dfrac{t^2}{2}\right)\vec{e_y}\ (m)$

assim, $\vec{r}(3) = \left(-1 - \dfrac{27}{12}\right)\vec{e_x} + \left(2 + \dfrac{9}{2}\right)\vec{e_y} = \left(-\dfrac{39}{12}\right)\vec{e_x} + \left(\dfrac{13}{2}\right)\vec{e_y}$

b) $\vec{a_t} = a_t\vec{u_t} = \dfrac{d|v|}{dt}\vec{u_t} = \dfrac{d\left|\sqrt{\left(\frac{t^2}{4}\right)^2 + t^2}\right|}{dt}\vec{u_t} =$

$= \dfrac{2\left(\frac{t^2}{4}\right)\cdot 2t + 2t}{2\sqrt{\frac{t^4}{16} + t^2}} = \dfrac{\frac{t^3}{2} + t}{\sqrt{\frac{t^4}{16} + t^2}}\vec{u_t}$ logo $\vec{a_t}(4) = \dfrac{\frac{64}{2} + 4}{\sqrt{\frac{256}{16} + 16}} =$

$$= \frac{36}{\sqrt{32}} = \frac{36}{4\sqrt{2}} = \frac{9\sqrt{2}}{2} = 4,5 \cdot 1,41 = 6,3\ \vec{u}_t\ (m/s^2)$$

c) $W_{FR} = \Delta E_c$, $E_{ci} = 0\ J$ $(v = 0\ m/s)$

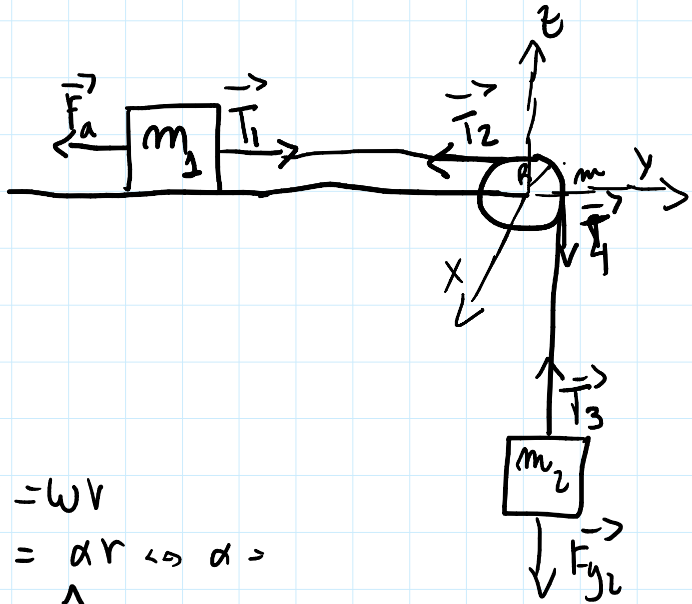
$$E_{cf} = \frac{1}{2} m v_f^2 = \frac{1}{2} \cdot 2 \cdot (v(2))^2 = (v(2))^2 =$$

$$= \left(\sqrt{\left(\frac{t^2}{4}\right)^2 + t^2} \right)^2 \Bigg|_{t=2} = \left(\frac{2^2}{4}\right)^2 + 2^2 = 5\ J$$

logo $W_F = 5\ J$

a, b, c, d

II



$$\begin{cases} m_1 \vec{a_1} = \vec{F_a} + \vec{T_1} \\ m_2 \vec{a_2} = \vec{F_{g_2}} + \vec{T_3} \\ \underbrace{\vec{\tau}_{t_2} + \vec{\tau}_{t_4} = I \vec{\alpha}}_{\text{eq movimento para os corpos}} \end{cases}$$

$v = \omega r$

$a = \alpha r \Leftrightarrow \alpha =$

$\vec{a_1} = a\,\hat{e}_y$ e $\vec{T_1} = T_a\,\hat{e}_y$ e $\vec{T_3} = T_b\,\hat{e}_z$ e $g > 0$

$\vec{a_2} = -a\,\hat{e}_z$ $\quad \vec{T_2} = -T_a\,\hat{e}_y$ $\quad \vec{T_4} = -T_b\,\hat{e}_z$ e

$$\Rightarrow \begin{cases} m_1 a\,\hat{e}_y = -(\mu m_1 g)\,\hat{e}_y + (T_a\,\hat{e}_y) \\ m_2 (-a)\,\hat{e}_z = -(m_2 g)\,\hat{e}_z + T_b\,\hat{e}_z \end{cases} \Leftrightarrow$$

$$\Rightarrow \begin{cases} m_2(-a)\hat{e}_z = -(m_2 g)\hat{e}_z + T_b \hat{e}_z \\ (R\hat{e}_z) \times (-T_a \hat{e}_y) + (R\hat{e}_y) \times (-T_b \hat{e}_z) = I\vec{\alpha} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} (m_1 a)\hat{e}_y = (T_a - \mu_{m_1} g)\hat{e}_y \\ (m_2 a)\hat{e}_z = (m_2 g - T_b)\hat{e}_z \\ R T_a \hat{e}_x - R T_b \hat{e}_x = I \alpha \hat{e}_x \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} m_1 a = T_a - \mu m_1 g \\ m_2 a = m_2 g - T_b \\ R T_a - R T_b = I \frac{a}{R} \hat{e}_x \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} T_a = m_1 a + \mu m_1 g \\ T_b = m_2(g-a) \\ R(m_1 a + \mu_{m_1} g) - R(m_2(g-a)) = \frac{1}{2} m R a \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \underline{\qquad\qquad} \\ \underline{\qquad\qquad} \\ m_1 a + \mu m_1 g - m_2 g + m_2 a = \frac{1}{2} m a \end{cases} \Leftrightarrow$$

$$\Leftrightarrow a\left(\frac{1}{2}m - m_1 - m_2\right) = \mu m_1 g - m_2 g$$

$$\Leftrightarrow \boxed{a = \frac{\mu m_1 g - m_2 g}{\frac{1}{2}m - m_1 - m_2}}$$

III