Engenharia de Computadores e Telemática

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1. (i) $\int_1^{\infty} \frac{\ln x}{x}\,dx = \int_1^{\infty} \frac{1}{x}\ln x\,dx = \left[\lim_{b\to+\infty} \frac{\ln^2 x}{2}\right]_1^b =$

$$= \lim_{b\to+\infty} \frac{\ln^2 b}{2} - \frac{\ln^2 1}{2} = +\infty - 0 = +\infty$$

R: O integral é divergente.

(ii) $\int_{-\infty}^{\infty} f(x)\,dx$, onde $f(x) := \begin{cases} \frac{1}{\sqrt{(1-x)^3}} & \text{se } x \leq -2 \\ e^{-x} & \text{se } x > -2 \end{cases}$

$$\int \frac{1}{\sqrt{(1-x)^3}}\,dx = \int (1-x)^{\frac{3}{2}}\,dx = -\int -(1-x)^{\frac{3}{2}}\,dx = \frac{(1-x)^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2(1-x)^{\frac{5}{2}}}{5}$$

$$\int e^{-x}\,dx = -\int -e^{-x}\,dx = -e^{-x}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \lim_{x\to+\infty} e^{-x} - \lim_{x\to-\infty} \frac{2(1-x)^{\frac{5}{2}}}{5} =$$

$$= e^{-\infty} - \frac{2(+\infty)^{\frac{5}{2}}}{5} = 0 - \infty = -\infty$$

R: O integral é divergente

2 (a)(i)

$$\sum_{n=1}^{+\infty} \frac{2n+(-1)^n}{n^3+3n^2+4}$$

$$\begin{cases} \dfrac{2n+1}{n^3+3n^2+4} & \text{se } n \text{ par} \\[2ex] \dfrac{2n-1}{n^3+3n^2+4} & \text{se } n \text{ ímpar} \end{cases} \qquad \begin{array}{l} \lim \dfrac{2n+1}{n^3+3n^2+4} = 0 \\[2ex] \lim \dfrac{2n-1}{n^3+3n^2+4} = 0 \end{array}$$

Seja $a_n = \dfrac{1}{n^2}$ uma serie de Dirichlet com $\alpha > 1$, então a_n é convergente.

$$L_1 = \lim \frac{\frac{2n+1}{n^3+3n^2+4}}{\frac{1}{n^2}} = \lim \frac{2n^3+n^2}{n^3+3n^2+4} = \lim \frac{\cancel{n^3}\left(2+\frac{1}{n}\right)}{\cancel{n^3}\left(1+\frac{3}{n}+\frac{4}{n^3}\right)} =$$

$$= \lim \frac{2+\frac{1}{n}}{1+\frac{3}{n}+\frac{4}{n^3}} = 2$$

$$L_2 = \lim \frac{\frac{2n-1}{n^3+3n^2+4}}{\frac{1}{n^2}} = \lim \frac{2n^3-n^2}{n^3+3n^2+4} =$$

$$= \lim \frac{\cancel{n^3}\left(2-\frac{1}{n}\right)}{\cancel{n^3}\left(1+\frac{3}{n}+\frac{4}{n^3}\right)} = \lim \frac{2-\frac{1}{n}}{1+\frac{3}{n}+\frac{4}{n^3}} = 2$$

R: Como $L_1 = L_2 = 2 \in \,]0, +\infty[$ temos que $\dfrac{2n+1}{n^3+3n^2+4}$ e $\dfrac{2n-1}{n^3+3n^2+4}$ são da mesma natureza que $\dfrac{1}{n^2}$, pelo critério da comparação. Como $\dfrac{2n+1}{n^3+3n^2+4}$ e $\dfrac{2n-1}{n^3+3n^2+4}$ são convergentes temos que $\displaystyle\sum_{n=1}^{+\infty} \frac{2n+(-1)^n}{n^3+3n^2+4}$ é absolutamente convergente.

(ii) $\sum_{n=1}^{\infty} \frac{n^n}{2^n n!}$

$$\lim \sqrt[n]{\frac{n^n}{2^n n!}} = \lim \frac{n}{2\sqrt[n]{n!}} =$$

$$\lim \frac{(n+1)^{n+1}}{2^{n+1}(n+1)n!}$$

(b) $\sum_{n=1}^{\infty}\left[\frac{(-3)^{n+1}}{2^{2n}} + \frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$

$$\lim \frac{(-3)^{n-1}}{2^{2n}} + \frac{1}{n^2} - \frac{1}{(n+1)^2} =$$

$$= \lim \frac{(-3)^{n-1}}{2^{2n}} + \lim \frac{1}{n^2} - \lim \frac{1}{(n+1)^2} =$$

$$= 0 + 0 + 0 = 0$$

$$\begin{cases} \lim \frac{(-3)}{2^{2n}} = -\frac{3}{+\infty} = 0 \\ \lim \frac{3}{2^{2n}} = \frac{3}{+\infty} = 0 \end{cases}, \text{logo } \lim \frac{(-3)^{n-1}}{2^{2n}} = 0$$