Algorithm Complexity II

Big O, Ω,Θ Notations

ivanovitch.silva@ufrn.br
@ivanovitchm

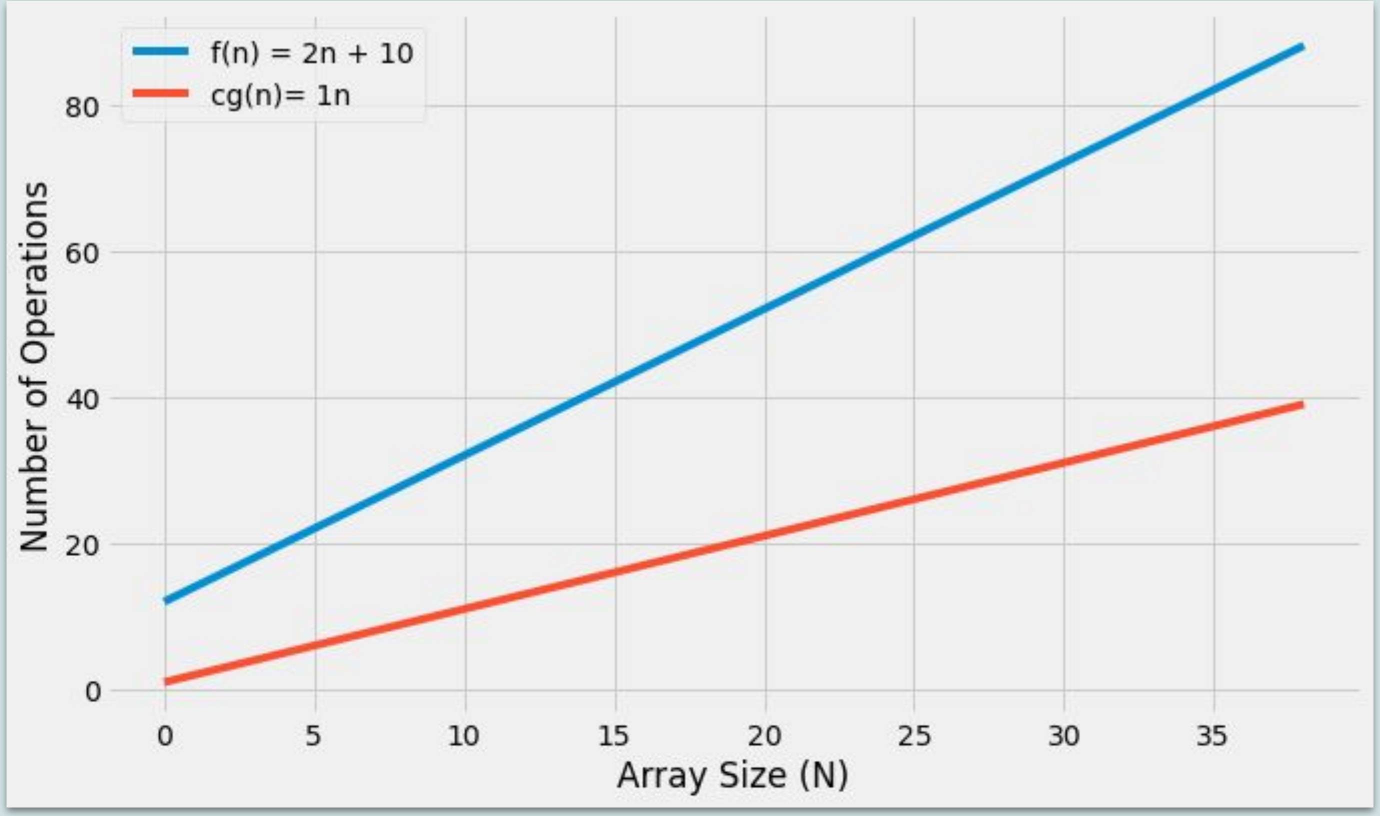
Big O

$$f(n) = O(g(n))$$
$$if \ \exists c, n_0 \ \ \forall n \geq n_0$$
$$f(n) \leq cg(n)$$

Meaning: f(n) is O(g(n)) if there exist two constants **c** and **n_0** such that for every n greater than or equal to n_0, f(n) is smaller than or equal to cg(n).

$$f(n) = O(g(n))$$
$$if \;\; \exists c, n_0 \;\; \forall n \geq n_0$$
$$f(n) \leq cg(n)$$

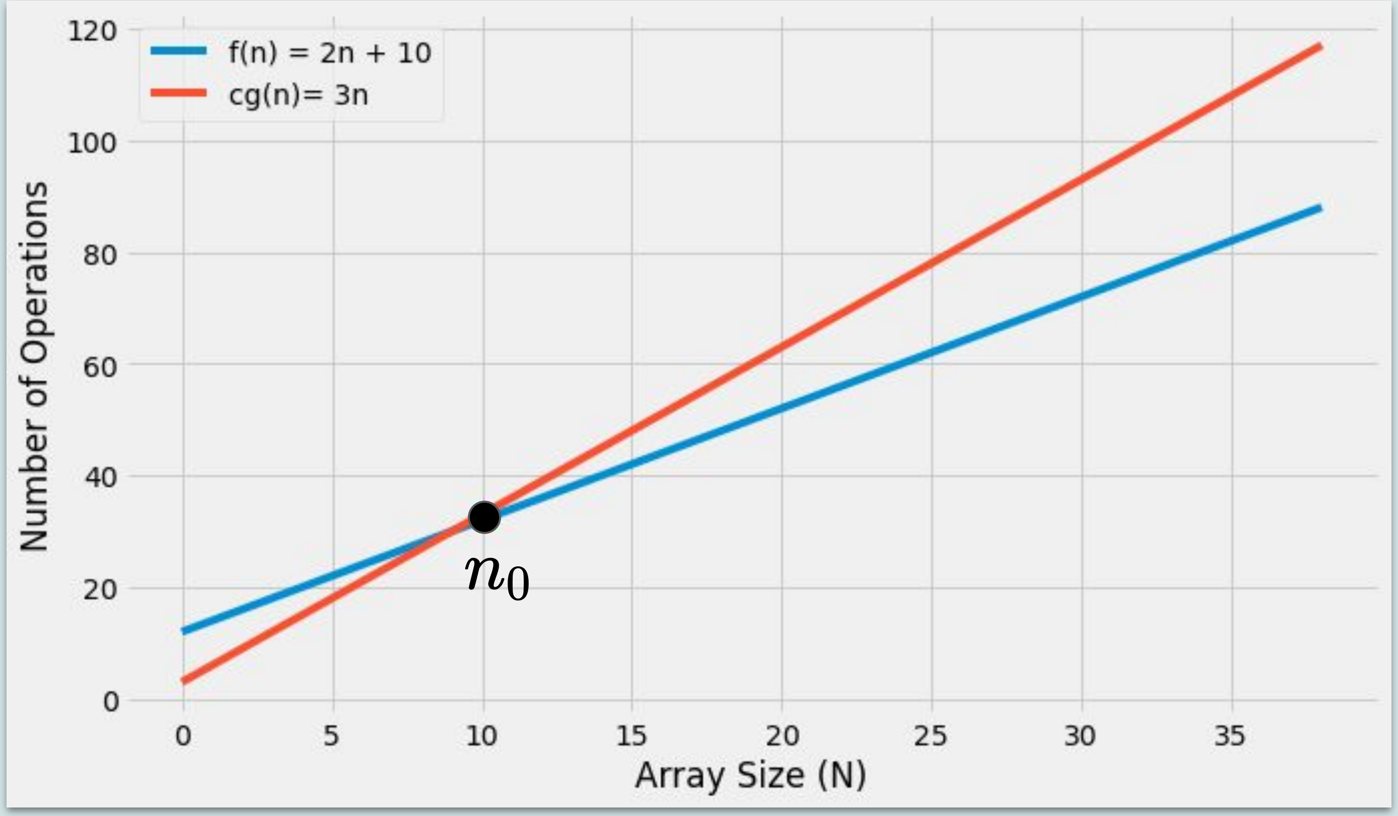
$$g(n) = n$$
$$c = 1$$
$$cg(n) = 1n$$



$$f(n) = O(g(n))$$
$$if \;\; \exists c, n_0 \;\; \forall n \geq n_0$$
$$f(n) \leq cg(n)$$

$g(n)$ is a upper bound of $f(n)$

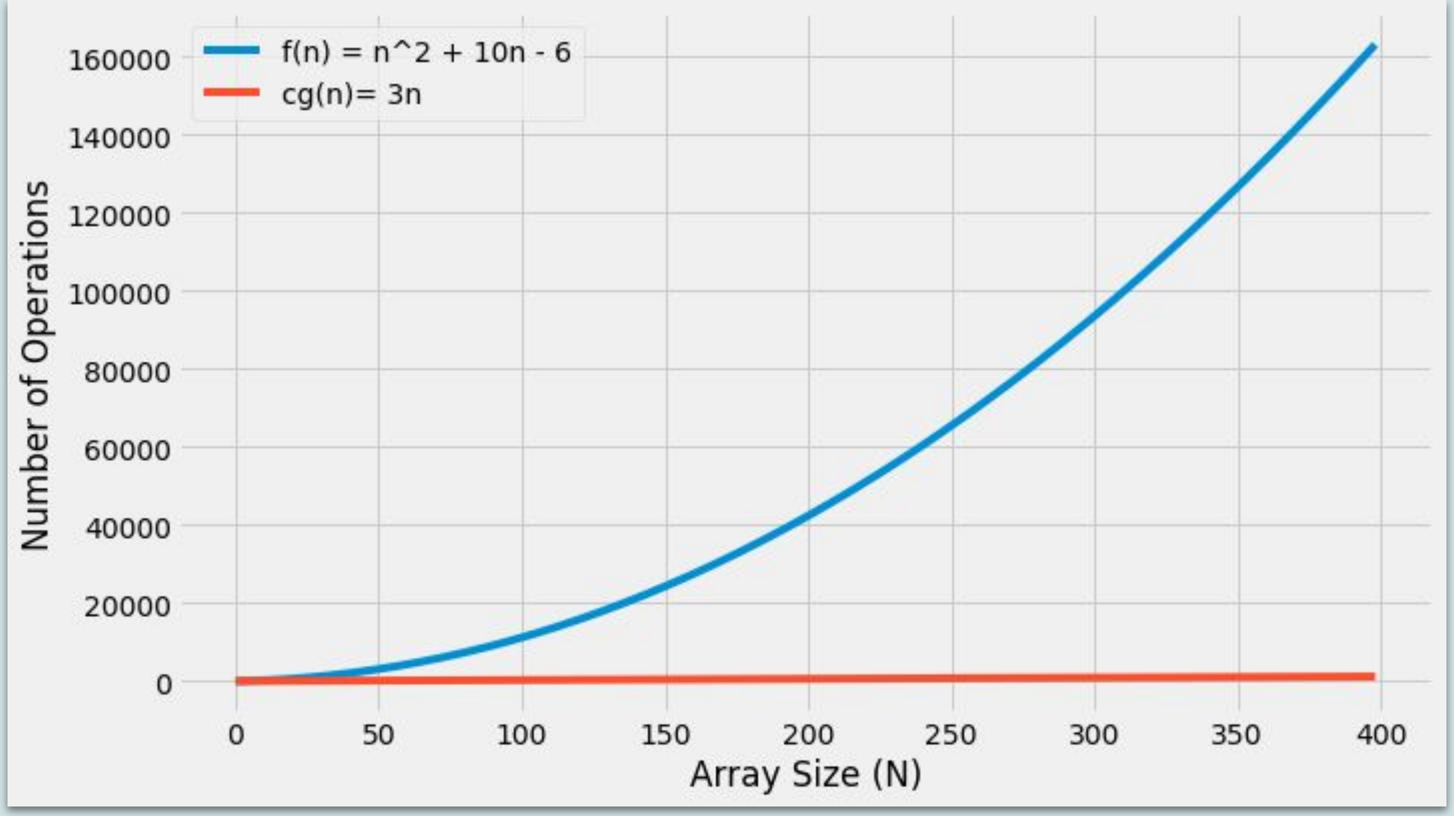
$$g(n) = n$$
$$c = 3$$
$$cg(n) = 3n$$



$$f(n) = O(g(n))$$
$$if\ \ \exists c, n_0\ \ \forall n \geq n_0$$
$$f(n) \leq cg(n)$$

$$f(n) = n^2 + 10n - 6$$
$$f(n) = O(n)?$$

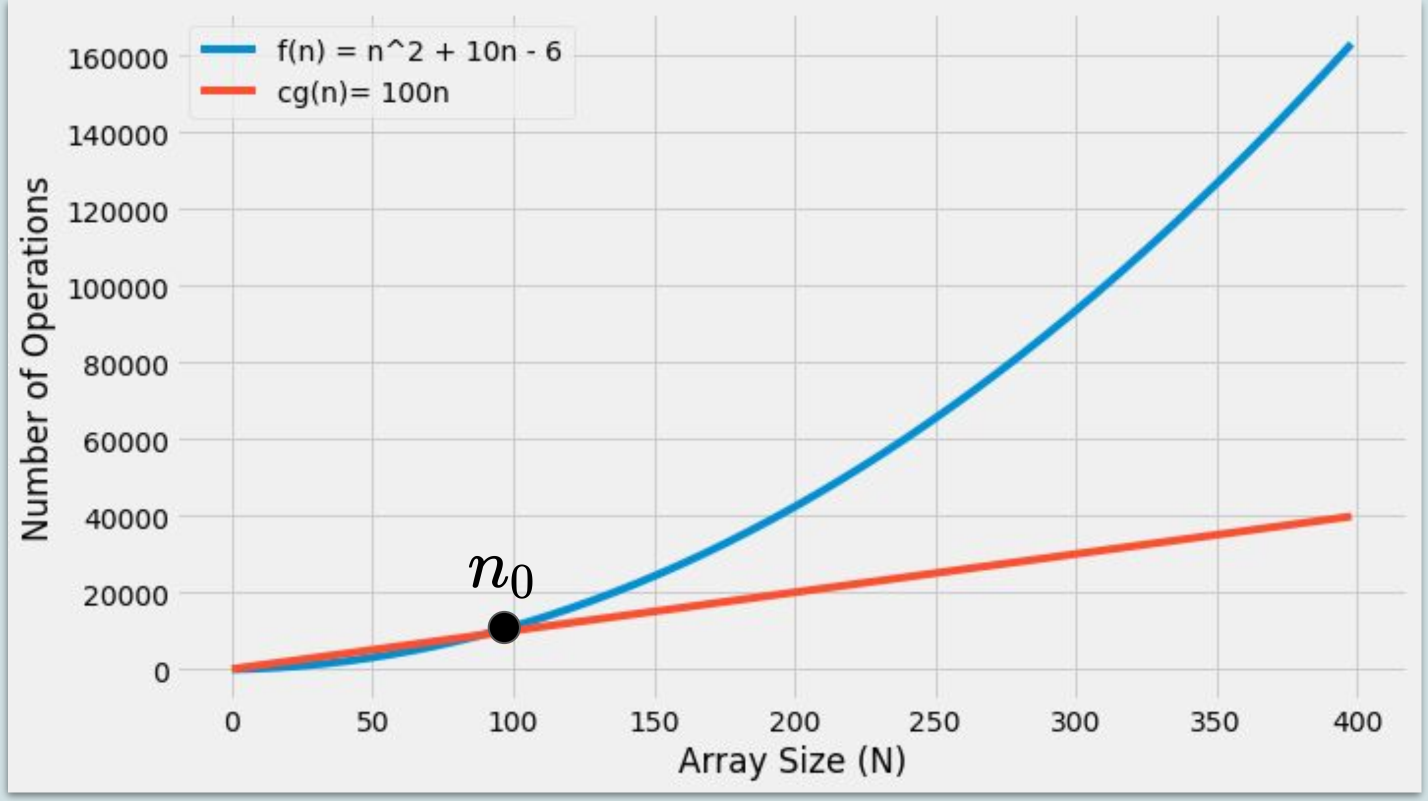
$$g(n) = n$$
$$c = 3$$
$$cg(n) = 3n$$



$$f(n) = O(g(n))$$
$$if \ \ \exists c, n_0 \ \ \forall n \geq n_0$$
$$f(n) \leq cg(n)$$

$$f(n) = n^2 + 10n - 6$$
$$f(n) = O(n)?$$

$$g(n) = n$$
$$c = 100$$
$$cg(n) = 100n$$

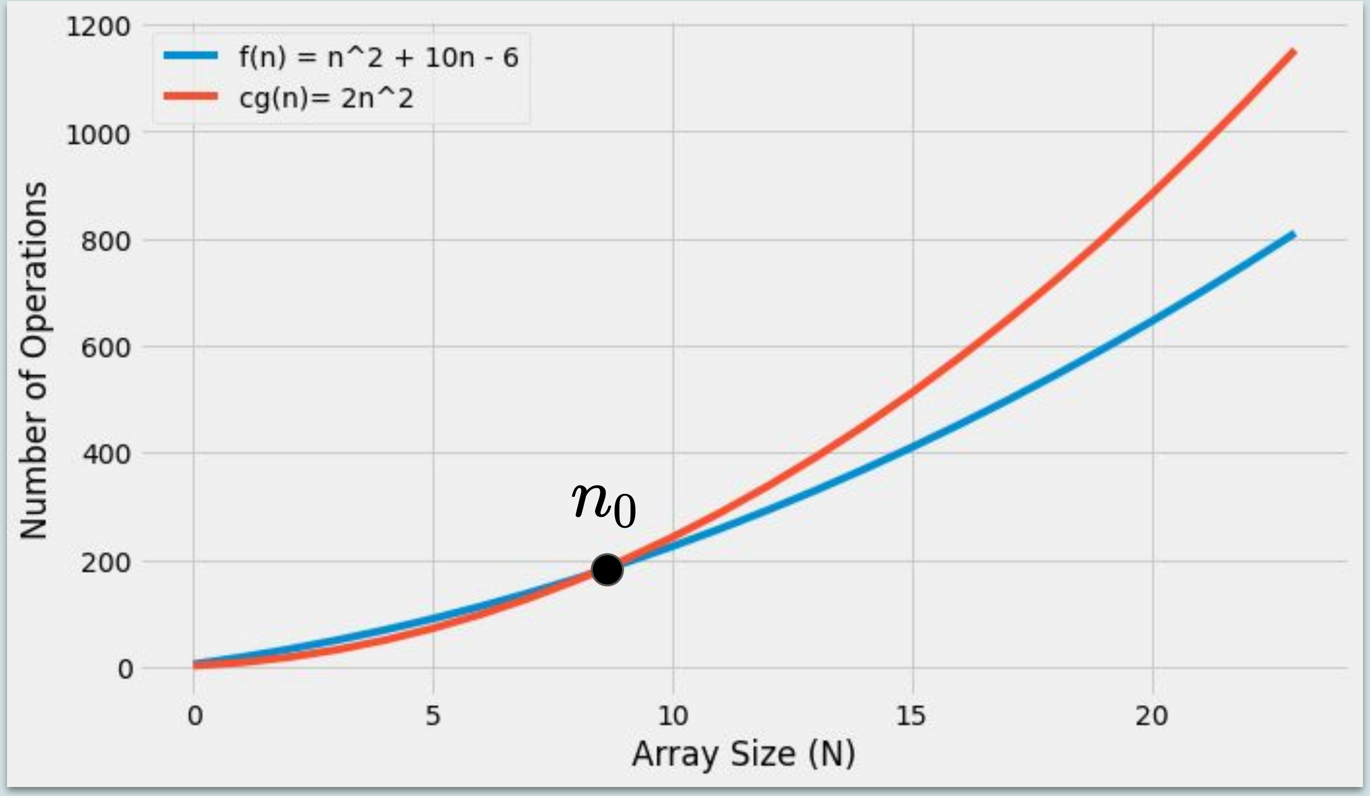


$$f(n) = O(g(n))$$
$$if\ \ \exists c, n_0\ \ \forall n \geq n_0$$
$$f(n) \leq cg(n)$$

n^2 is a upper bound of *f(n)*

$$f(n) = n^2 + 10n - 6$$
$$f(n) = O(n^2)?$$

$$g(n) = n^2$$
$$c = 2$$
$$cg(n) = 2n^2$$

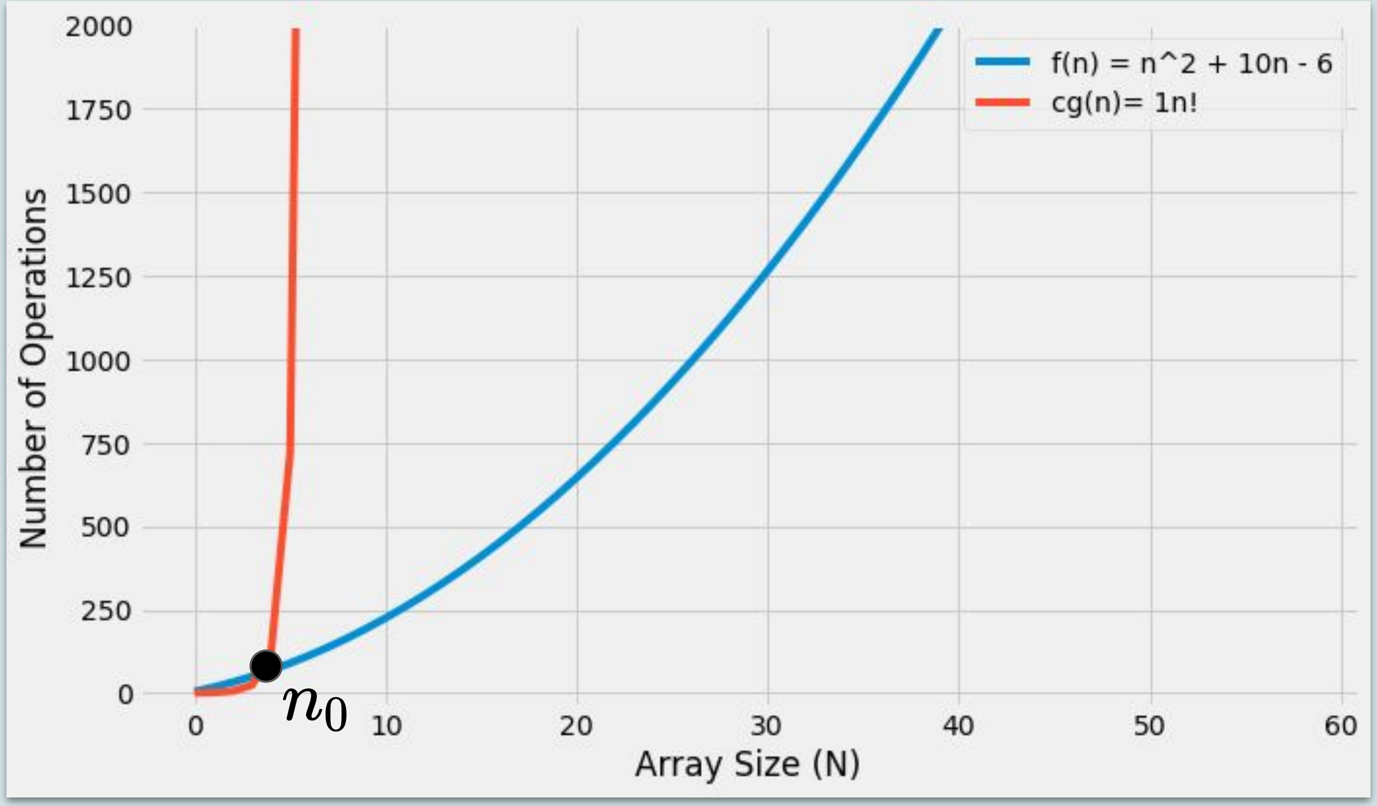


$$f(n) = O(g(n))$$
$$if\ \ \exists c, n_0\ \ \forall n \geq n_0$$
$$f(n) \leq cg(n)$$

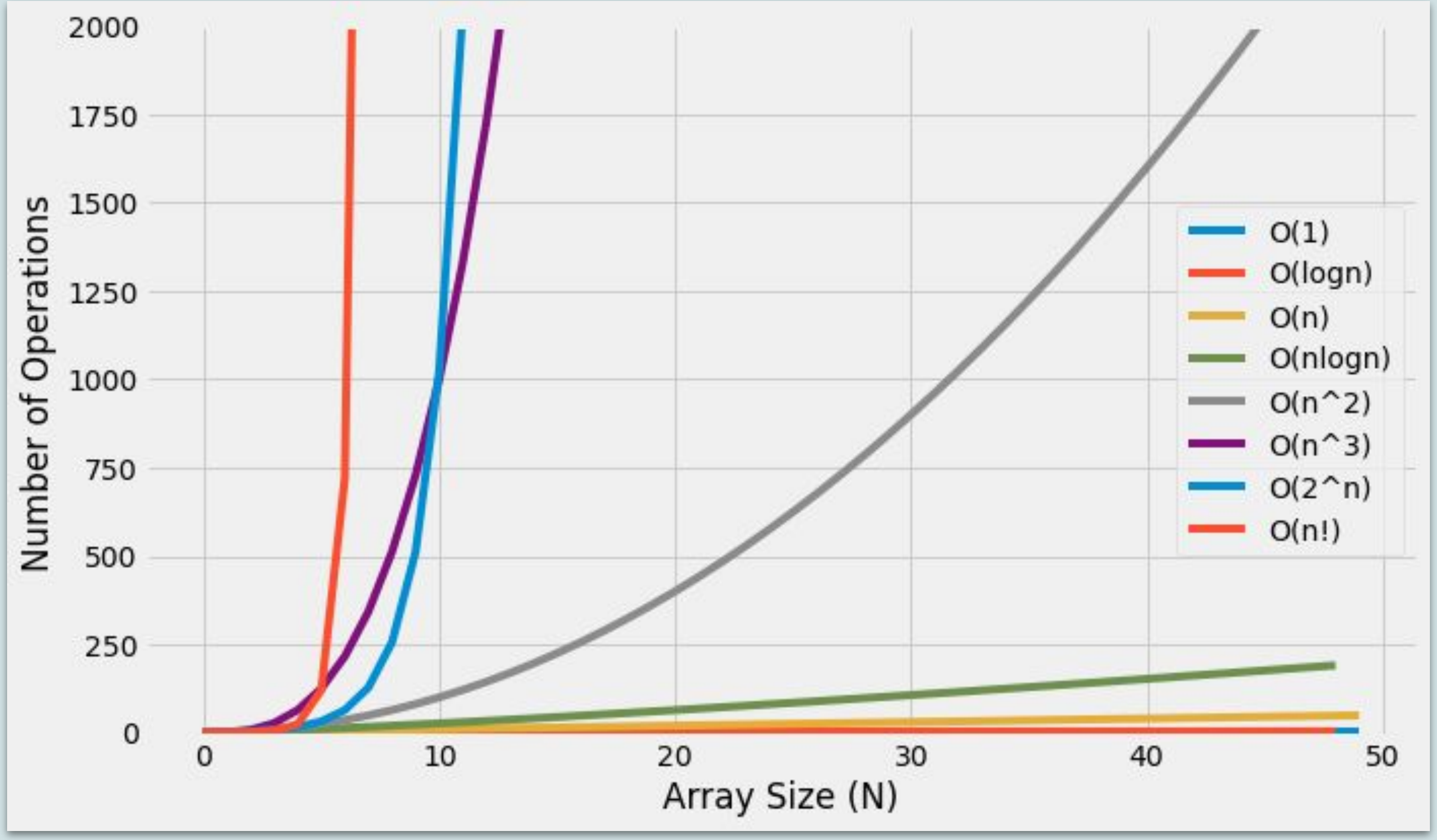
n! is also a upper bound of *f(n)*

$$f(n) = n^2 + 10n - 6$$
$$f(n) = O(n!)?$$

$$g(n) = n!$$
$$c = 1$$
$$cg(n) = 1n!$$



n^2 and $n!$ are both upper bounds of $f(n)=n^2+10n-6$, but the **tight** one is n^2



$$O(1) < O(logn) < O(n) < O(nlogn) < O(n^2) < O(2^n) < O(n!)$$

Big Ω

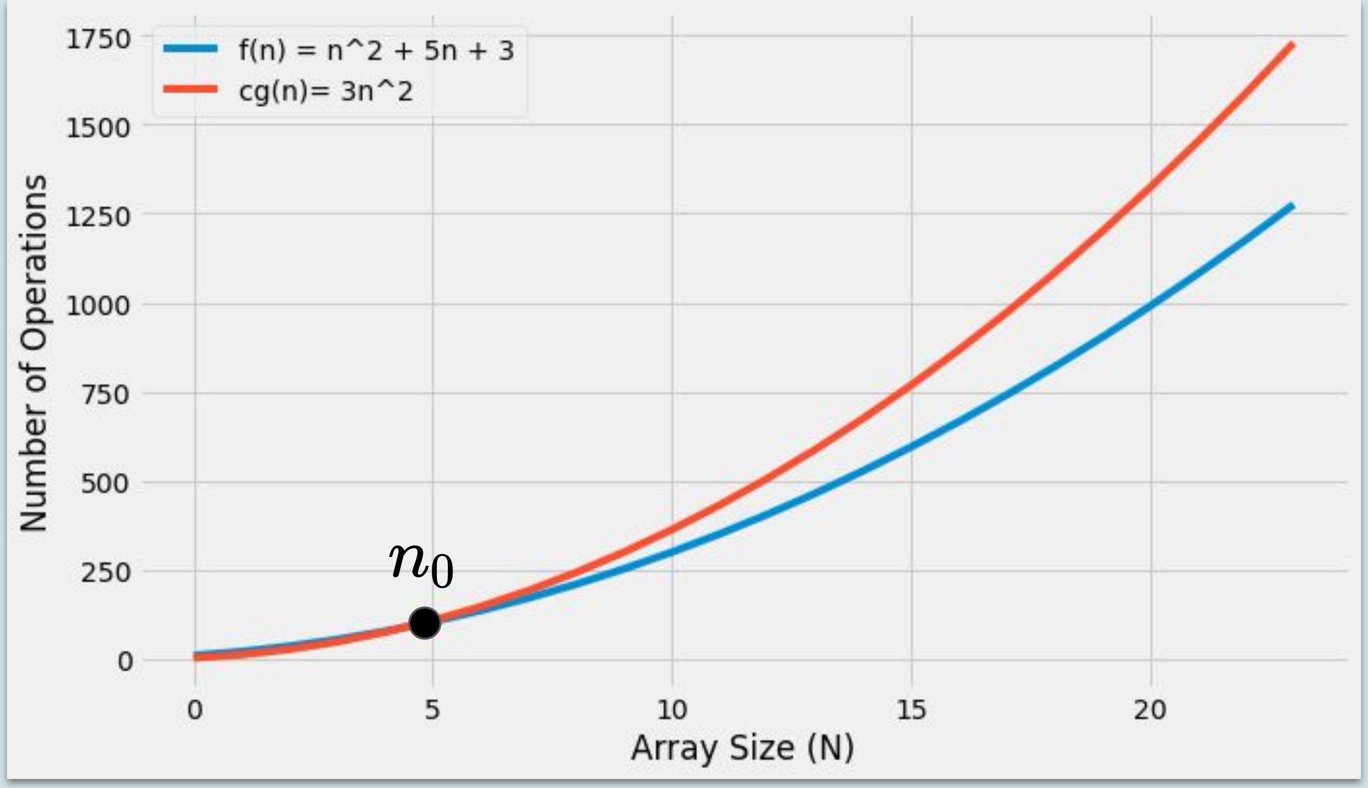
$$f(n) = \Omega(g(n))$$
$$if \;\; \exists c, n_0 \;\; \forall n \geq n_0$$
$$f(n) \geq cg(n)$$

Meaning: f(n) is O(g(n)) if there exist two constants **c** and $\mathbf{n_0}$ such that for every n greater than or equal to n_0, f(n) is **greater than or equal to** cg(n).

$$f(n) = \Omega(g(n))$$
$$if \;\; \exists c, n_0 \;\; \forall n \geq n_0$$
$$f(n) \geq cg(n)$$

$$f(n) = 2n^2 + 5n + 3$$
$$f(n) = \Omega(n^2)?$$

$$g(n) = n^2$$
$$c = 3$$
$$cg(n) = 3n^2$$



$$f(n) = \Omega(g(n))$$

$$if \;\; \exists c, n_0 \;\; \forall n \geq n_0$$

$$f(n) \geq cg(n)$$

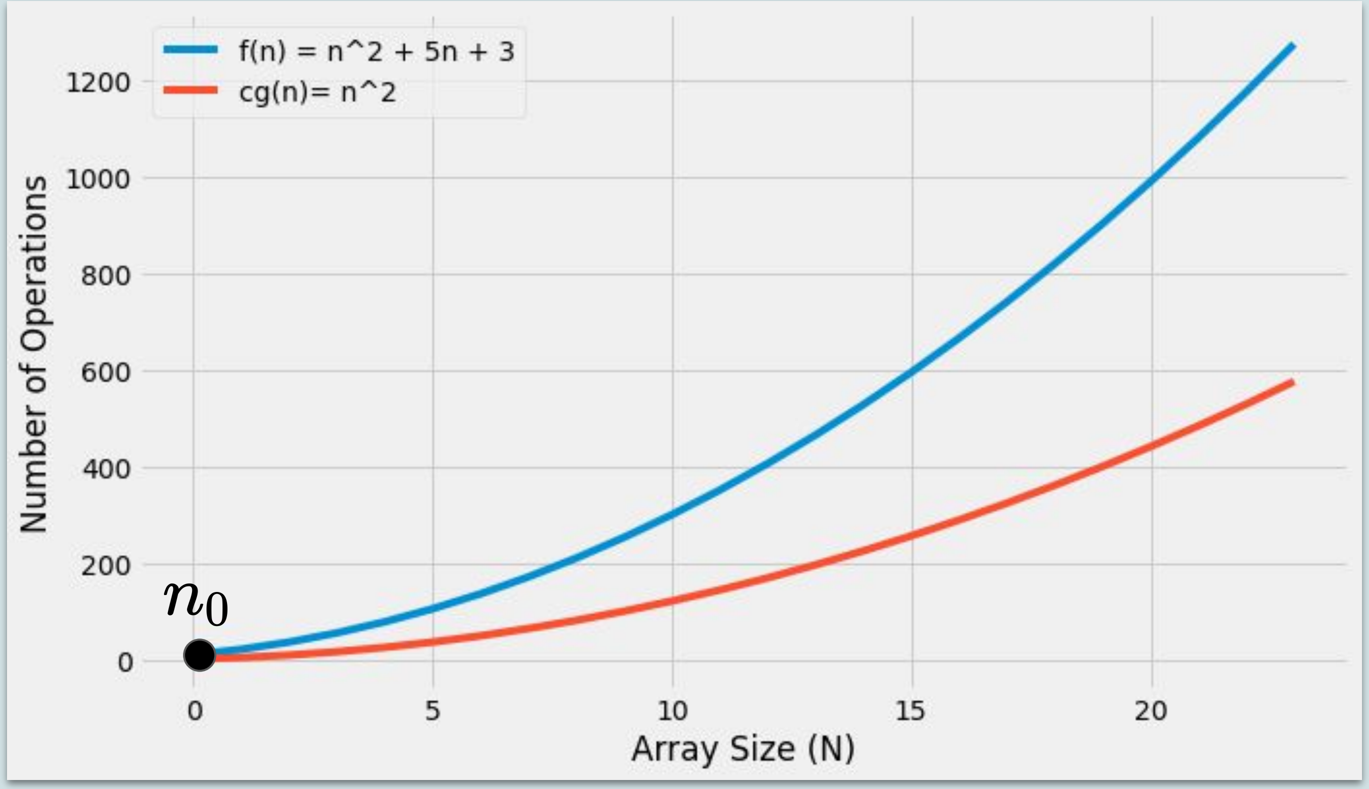
$$f(n) = 2n^2 + 5n + 3$$

$$f(n) = \Omega(n^2)?$$

$$g(n) = n^2$$

$$c = 1$$

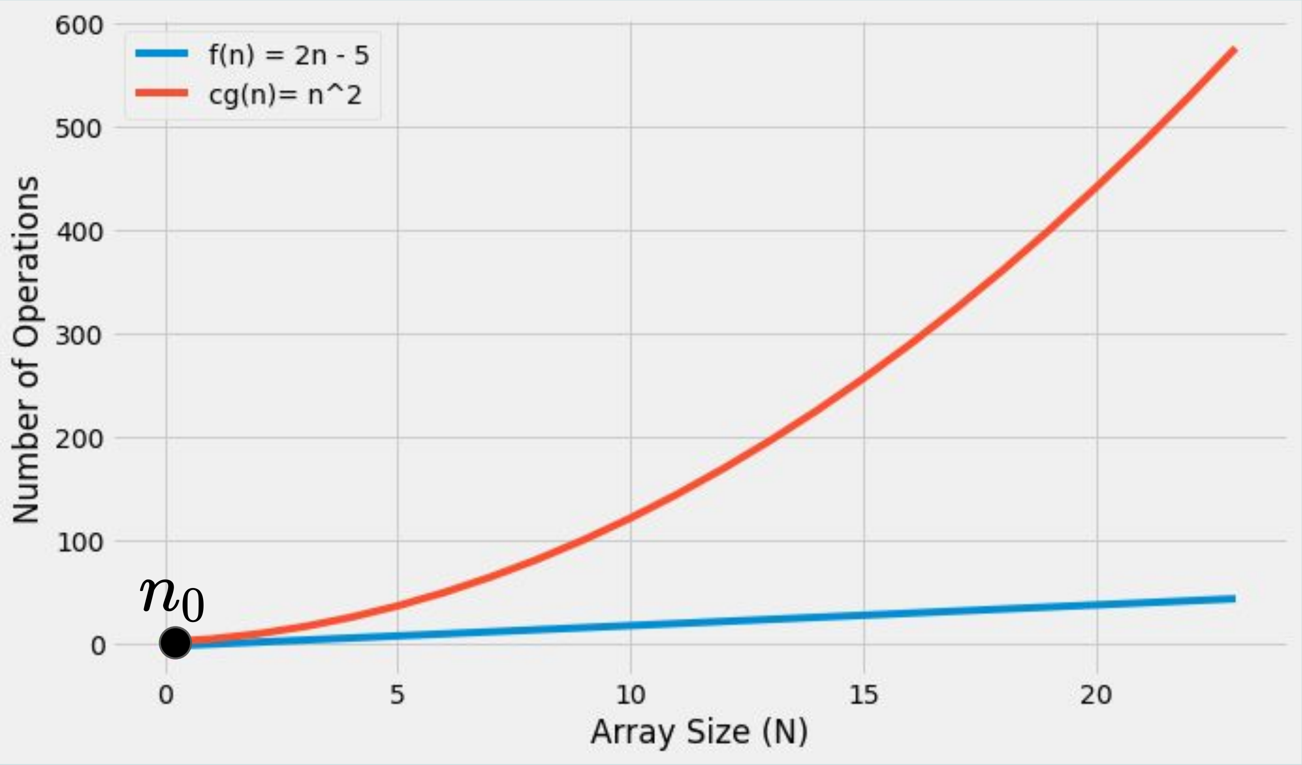
$$cg(n) = 1n^2$$



$$f(n) = \Omega(g(n))$$
$$if \;\; \exists c, n_0 \;\; \forall n \geq n_0$$
$$f(n) \geq cg(n)$$

$$f(n) = 2n - 5$$
$$f(n) = \Omega(n^2)?$$

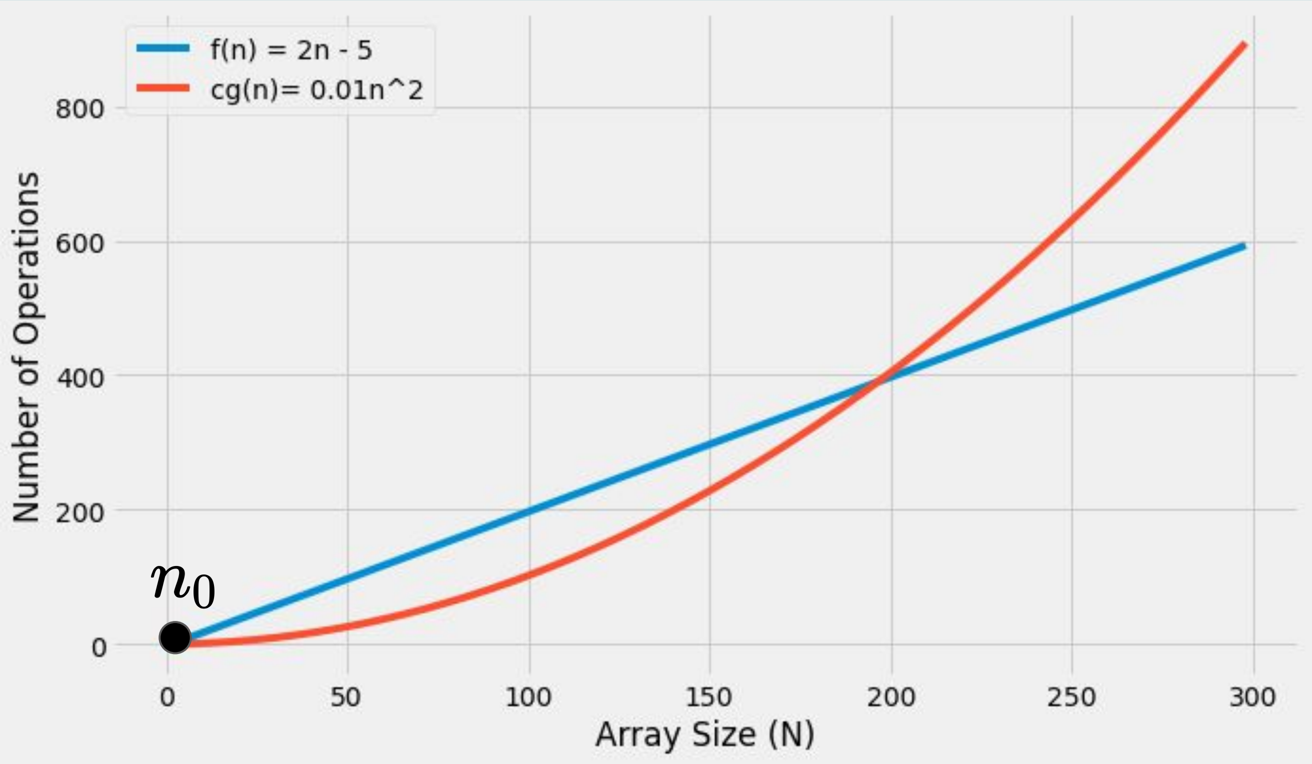
$$g(n) = n^2$$
$$c = 1$$
$$cg(n) = 1n^2$$



$$f(n) = \Omega(g(n))$$
$$if\ \ \exists c, n_0\ \ \forall n \geq n_0$$
$$f(n) \geq cg(n)$$

$$f(n) = 2n - 5$$
$$f(n) = \Omega(n^2)?$$

$$g(n) = n^2$$
$$c = 0.01$$
$$cg(n) = 0.01n^2$$

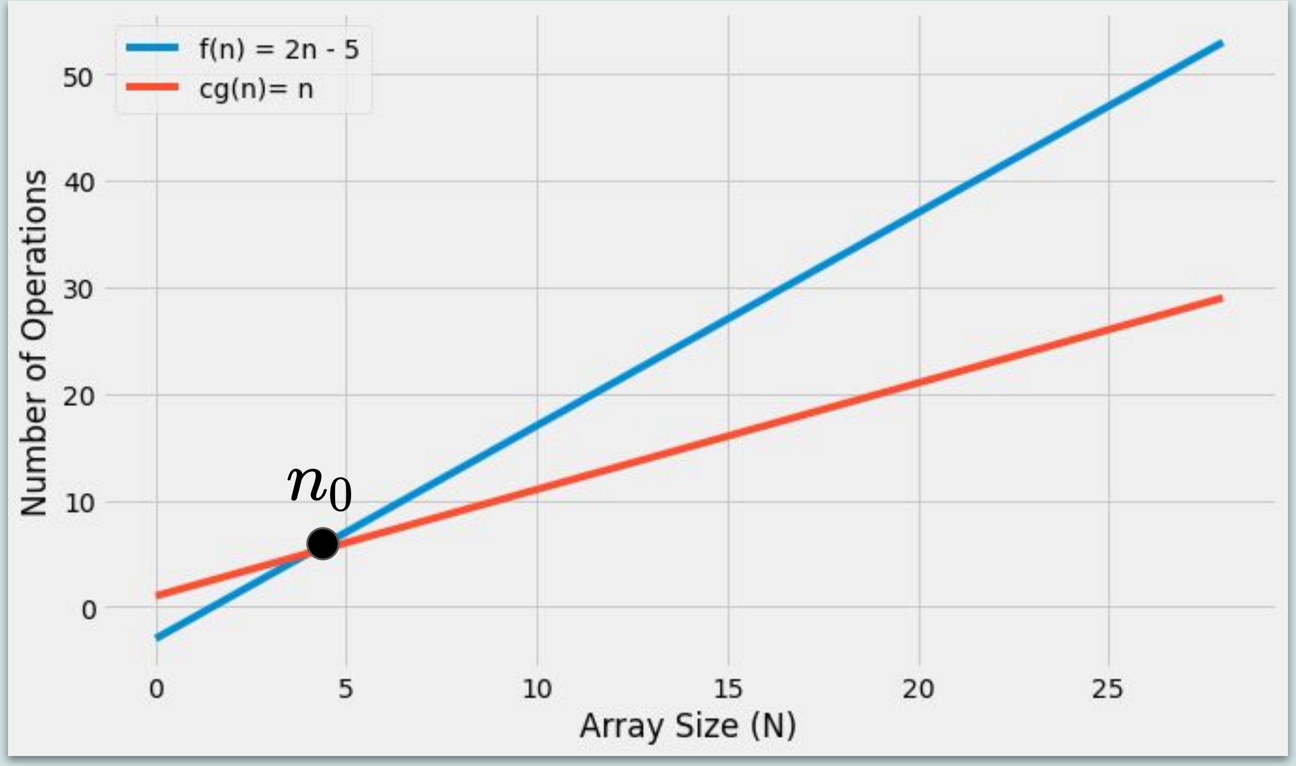


$$f(n) = \Omega(g(n))$$
$$if \;\; \exists c, n_0 \;\; \forall n \geq n_0$$
$$f(n) \geq cg(n)$$

$$f(n) = 2n - 5$$
$$f(n) = \Omega(n)?$$

$$g(n) = n$$
$$c = 1$$
$$cg(n) = n$$

n is a lower bound of **f(n)**



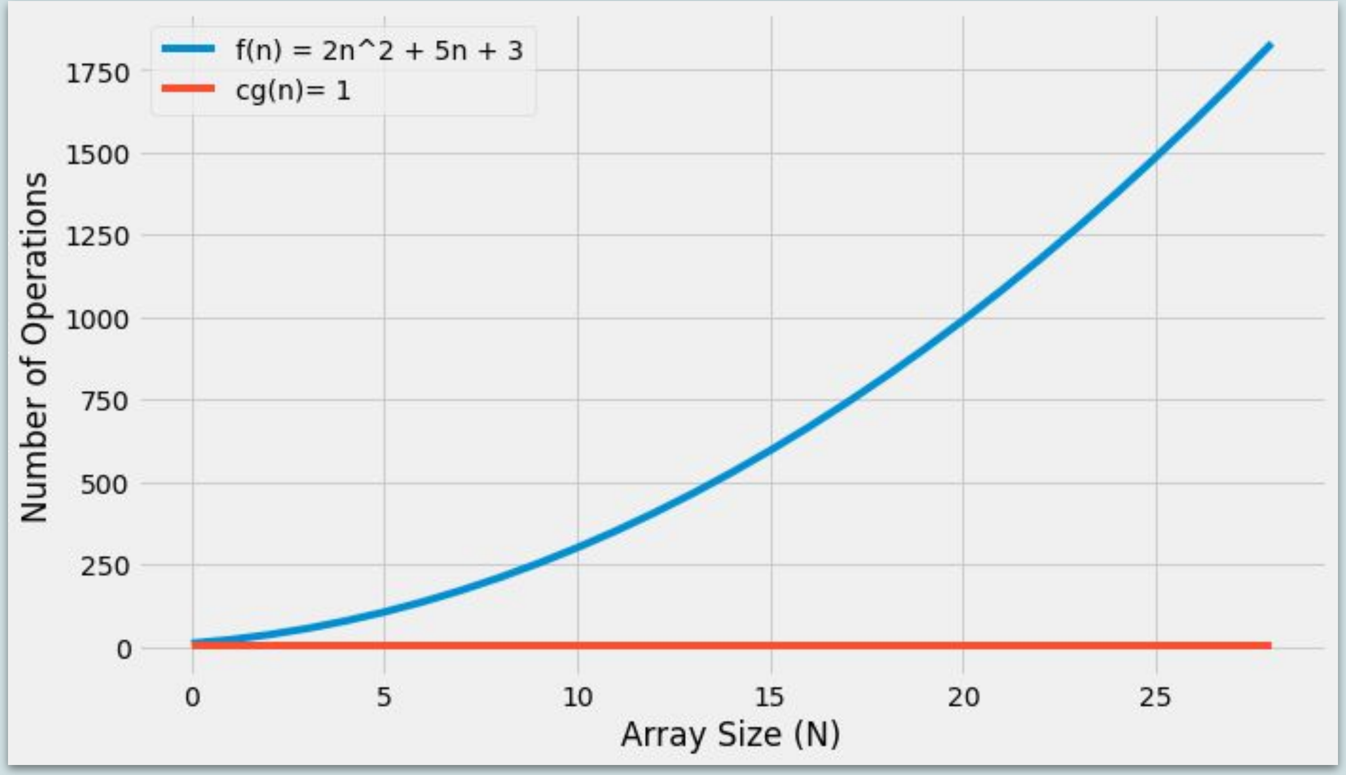
$$f(n) = \Omega(g(n))$$

$$if\ \ \exists c, n_0\ \ \forall n \geq n_0$$

$$f(n) \geq cg(n)$$

$$f(n) = 2n^2 + 5n + 3$$

is $\Omega(1)$, but it is not the tight lower bound.



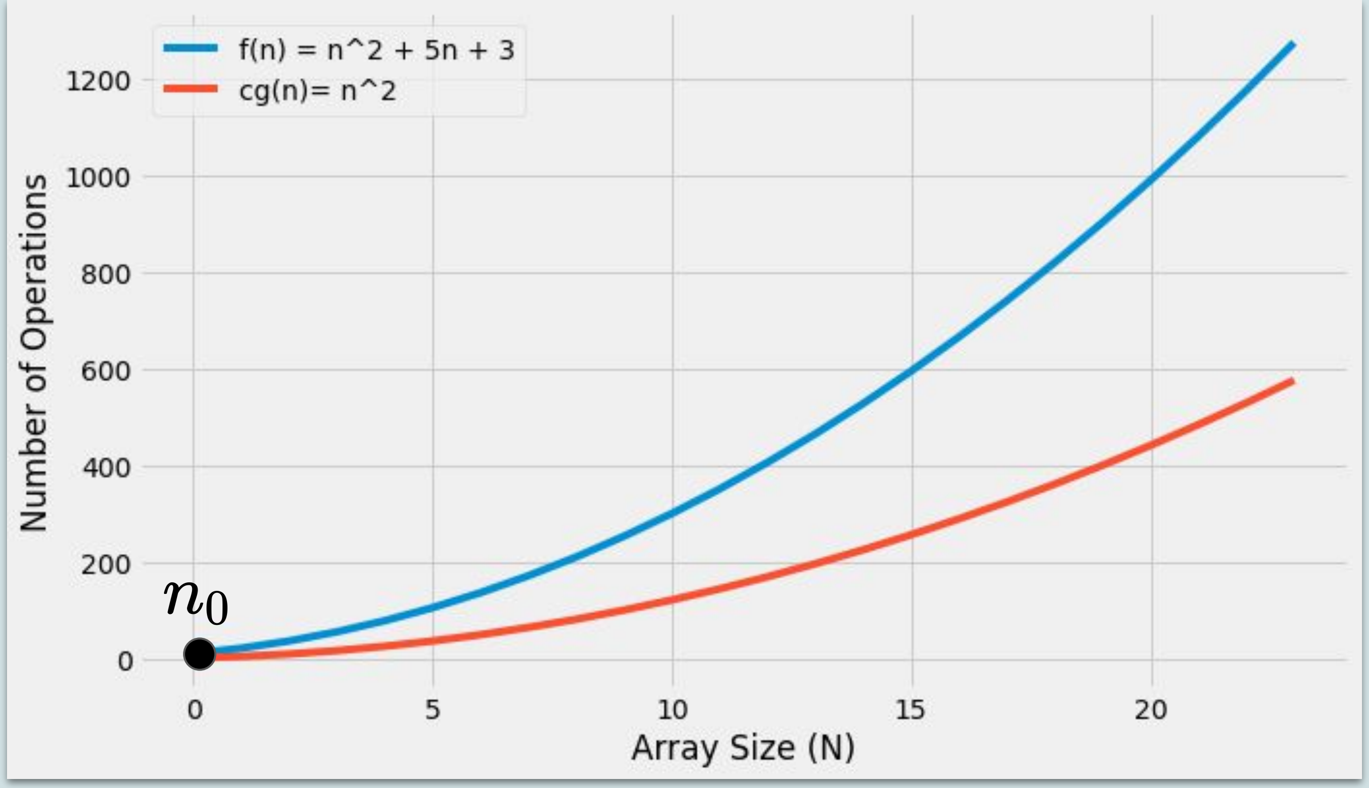
$$f(n) = \Omega(g(n))$$

$$if\ \ \exists c, n_0\ \ \forall n \geq n_0$$

$$f(n) \geq cg(n)$$

$$f(n) = 2n^2 + 5n + 3$$

$g(n) = n^2$ is the tight lower bound.



Big Θ

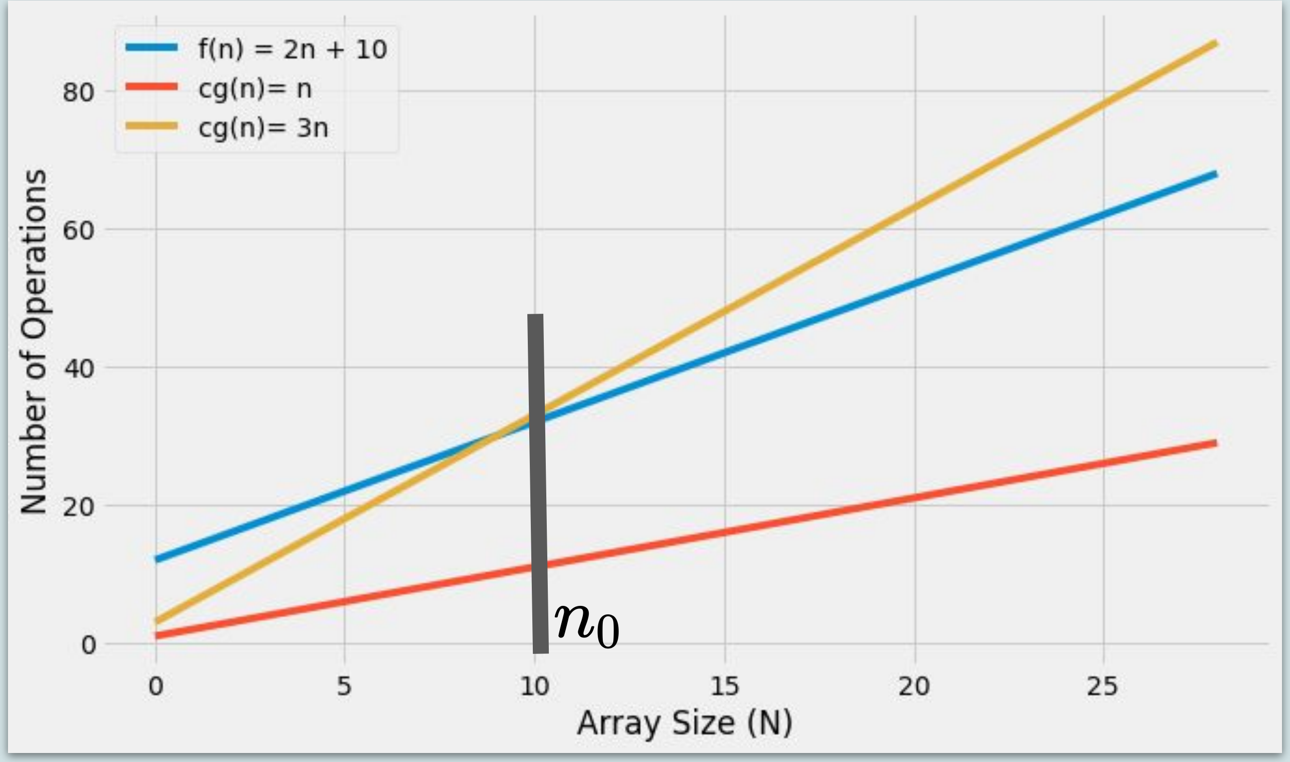
$$f(n) = \Theta(g(n))$$
$$if\ \ \exists c_1, c_2, n_0\ \ \forall n \geq n_0$$
$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

Meaning: f(n) is Θ(g(n)) if there exist three constants **c_1,c_2,** and **n_0** such that for every n greater or equal to to n_0, f(n) is **greater than or equal to** $c_1g(n)$ and **smaller than or equal to** $c_2g(n)$.

$$f(n) = \Theta(g(n))$$
$$if\ \exists c_1, c_2, n_0\ \forall n \geq n_0$$
$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

$$f(n) = 2n + 10$$
$$f(n) = \Theta(n)?$$

$$g(n) = n$$
$$c_1 = 1$$
$$c_2 = 3$$
$$c_1 g(n) = n$$
$$c_2 g(n) = 3n$$



$$f(n) = \Theta(n) \Rightarrow f(n) = O(n)$$

$$f(n) = O(n) \not\Rightarrow f(n) = \Theta(n)$$

$$f(n) = \Theta(n) \Rightarrow f(n) = \Omega(n)$$

$$f(n) = \Omega(n) \not\Rightarrow f(n) = \Theta(n)$$