

The variation of fundamental constants and the role of $A = 5$ and $A = 8$ nuclei on primordial nucleosynthesis

Alain Coc*

*Centre de Spectrométrie Nucléaire et de Spectrométrie de Masse (CSNSM),
IN2P3-CNRS and Université Paris Sud 11, UMR 8609, Bât. 104, 91405 Orsay Campus (France)*

Pierre Descouvemont†

*Physique Nucléaire Théorique et Physique Mathématique, C.P. 229,
Université Libre de Bruxelles (ULB), B-1050 Brussels, Belgium*

Keith A. Olive‡

*William I. Fine Theoretical Physics Institute, School of Physics and Astronomy,
University of Minnesota, Minneapolis, MN 55455 (USA)*

Jean-Philippe Uzan§ and Elisabeth Vangioni¶

*Institut d'Astrophysique de Paris, UMR-7095 du CNRS,
Université Pierre et Marie Curie, 98 bis bd Arago, 75014 Paris (France)*

We investigate the effect of a variation of fundamental constants on primordial element production in big bang nucleosynthesis (BBN). We focus on the effect of a possible change in the nucleon-nucleon interaction on nuclear reaction rates involving the $A = 5$ (${}^5\text{Li}$ and ${}^5\text{He}$) and $A = 8$ (${}^8\text{Be}$) unstable nuclei and complement earlier work on its effect on the binding energy of deuterium. The reaction rates for ${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$ and ${}^3\text{H}(\text{d},\text{n}){}^4\text{He}$ are dominated by the properties of broad analog resonances in ${}^5\text{He}$ and ${}^5\text{Li}$ compound nuclei respectively. While the triple alpha process ${}^4\text{He}(\alpha\alpha,\gamma){}^{12}\text{C}$ is normally not effective in BBN, its rate is very sensitive to the position of the “Hoyle state” and could in principle be drastically affected if ${}^8\text{Be}$ were stable during BBN. The nuclear properties (resonance energies in ${}^5\text{He}$ and ${}^5\text{Li}$ nuclei, and the binding energies of ${}^8\text{Be}$ and D) are all computed in a consistent way using a microscopic cluster model. The $\text{n}(\text{p},\gamma)\text{d}$, ${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$, ${}^3\text{H}(\text{d},\text{n}){}^4\text{He}$ and ${}^4\text{He}(\alpha\alpha,\gamma){}^{12}\text{C}$ reaction rates are subsequently calculated as a function of the nucleon-nucleon interaction that can be related to the fundamental constants. We found that the effect of the variation of constants on the ${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$, ${}^3\text{H}(\text{d},\text{n}){}^4\text{He}$ and ${}^4\text{He}(\alpha\alpha,\gamma){}^{12}\text{C}$ reaction rates is not sufficient to induce a significant effect on BBN, even if ${}^8\text{Be}$ was stable. In particular, no significant production of carbon by the triple alpha reaction is found when compared to standard BBN. We also update our previous analysis on the effect of a variation of constants on the $\text{n}(\text{p},\gamma)\text{d}$ reaction rate.

PACS numbers:

June 4, 2018

I. INTRODUCTION

Constraints on the possible variation of fundamental constants are an efficient method of testing the equivalence principle [1, 2], which underpins metric theories of

gravity and in particular general relativity. These constraints are derived from a wide variety of physical systems and span a large spectrum of redshifts and physical conditions, from the comparison of atomic clocks in the laboratory, the Oklo phenomena, to quasar absorption spectra up to a typical redshift of order $z \sim 2$ and big bang nucleosynthesis (BBN) at a redshift of order $z \sim 10^9$.

Primordial nucleosynthesis is considered a major pillar of the standard cosmological model (see e.g., Refs. [3]). Using inputs from WMAP for the baryon density [4], BBN yields excellent agreement between the theoretical predictions and astrophysical determinations for the abundances of D and ${}^4\text{He}$ [5–9] despite the discrepancy between the theoretical prediction of ${}^7\text{Li}$ and its deter-

*Electronic address: coc@csnsm.in2p3.fr

†Electronic address: pdesc@ulb.ac.be

‡Electronic address: olive@physics.unm.edu

§Electronic address: uzan@iap.fr

¶Electronic address: vangioni@iap.fr

mined abundance in halo stars [8]. Indeed, BBN has been used extensively to constrain deviations from the standard model framework, and in particular from general relativity, see e.g., Ref. [10].

The effects of the variation of fundamental constants on BBN predictions is difficult to model because of the intricate structure of QCD and its role in low energy nuclear reactions and because one cannot restrict the analysis to a single constant. One can, however, proceed in a two step approach: first by determining the dependencies of the light element abundances on the BBN parameters and then by relating those parameters to the fundamental constants (see Section 3.8 of Ref. [1] for an up-to-date overview). While early works have mostly focused on a single parameter such as the fine structure constant [11], the Higgs vacuum expectation value (vev) [12–14] or the QCD scale [15], in many theories which allow for the variation of fundamental parameters, often, the variation of several parameters are correlated in a model dependent way [16, 17].

The variation of a fundamental parameter such as the fine structure constant will affect the BBN analysis through the proton-to-neutron mass difference and the neutron lifetime [11] as well as the deuterium binding energy [18] and the binding energies of other light nuclei such as tritium, helium-3 and 4, lithium-6 and 7, and beryllium-7 [19]. These effects can in principle be used to probe the coupled variations of several parameters [20–22].

Following our previous work [22–24], we allow for a variation of all fundamental constants and in order to reduce the arbitrariness, we focus on scenarios in which the variations of the different constants are correlated. In effectively all unification models of non-gravitational interactions, a variation in the fine structure constant is associated with the variation in other gauge couplings [16, 17]. Any variation of the strong gauge coupling α_s will induce a variation in the QCD scale, Λ_{QCD} , as can be seen from the expression

$$\Lambda_{\text{QCD}} = \mu \left(\frac{m_c m_b m_t}{\mu^3} \right)^{\frac{2}{27}} \exp \left[-\frac{2\pi}{9\alpha_s(\mu)} \right] \quad (1.1)$$

valid for a renormalization scale $\mu > m_t$. In this expression $m_{c,b,t}$ are the masses of the charm, bottom and top quarks. Since the masses of the quarks are proportional to the product, $h v$, of a Yukawa coupling h and Higgs vacuum expectation value v , any variation of the Yukawa couplings will also induce a variation of Λ_{QCD} . These variations can be related by

$$\frac{\Delta \Lambda}{\Lambda} = R \frac{\Delta \alpha}{\alpha} + \frac{2}{27} \left(3 \frac{\Delta v}{v} + \frac{\Delta h_c}{h_c} + \frac{\Delta h_b}{h_b} + \frac{\Delta h_t}{h_t} \right). \quad (1.2)$$

The coefficient R is determined by the particular grand unified theory and particle content of the theory which control both the value of $\alpha(M_{\text{GUT}}) = \alpha_s(M_{\text{GUT}})$ and the low energy relation between α and α_s , leading to a considerable model dependence in its value [25, 26].

Here we shall assume a typical value, $R \sim 36$ [17, 27]. Furthermore, in theories in which the electroweak scale is derived by dimensional transmutation, changes in the Yukawa couplings (particularly the top Yukawa, h_t) lead to exponentially large changes in the Higgs vev. In such cases, the Higgs expectation value is related to the Planck mass, M_{P} , by

$$v \sim M_{\text{P}} \exp \left(-\frac{2\pi c}{\alpha_t} \right) \quad (1.3)$$

where c is a constant of order unity, and $\alpha_t = h_t^2/4\pi$. Thus we can write,

$$\frac{\Delta v}{v} \equiv S \frac{\Delta h}{h}, \quad (1.4)$$

and, as in Ref. [22], we take $S \sim 240$, though there is considerable model-dependence in this value as well. For example, in supersymmetric models, S can be related to the sensitivity of the Z gauge boson mass to the top Yukawa, and may take values anywhere from about 80 to 500 [28]. This dependence gets translated into a variation in all low energy particle masses [12]. In addition, in many string theories, all gauge and Yukawa couplings are determined by the expectation value of a dilaton. Therefore, once we allow α to vary, virtually all masses and couplings are expected to vary as well, typically much more strongly than the variation induced by the Coulomb interaction alone.

The use of coupled variations has led to significantly improved constraints in a wide range of environments ranging from big bang nucleosynthesis [17–20, 22, 29–32], the Oklo reactor [33], meteoritic data [21, 33], the microwave background [31, 34], stellar evolution [23] and atomic clocks [24]. Concerning BBN, the effect of coupled variations has mostly focused on the binding energy, B_D , of deuterium [18, 22, 29] (see also Ref. [35] for related investigations). The importance of B_D is easily understood by the fact that the equilibrium abundance of deuterium and the reaction rate $p(n, \gamma)\text{D}$ both depend exponentially on B_D and on the fact that deuterium is in a shallow bound state. Indeed, in Ref. [22], we found that even a relatively small variation in the gauge or Yukawa couplings of order of a few $\times 10^{-5}$ had a significant effect on the light element abundances. In particular, using [22]

$$\frac{\Delta B_D}{B_D} = -13(1+S) \frac{\Delta h}{h} + 18R \frac{\Delta \alpha}{\alpha}, \quad (1.5)$$

a variation in the Yukawa couplings of 2×10^{-5} induces a relative variation in B_D of about 4%. By decreasing B_D , nucleosynthesis begins later at a lower temperature ultimately suppressing the ${}^7\text{Li}$ abundance.

It is well known in principle, that the mass gaps at $A=5$ and $A=8$, prevent the nucleosynthetic chain from extending beyond ${}^4\text{He}$. Although some ${}^6\text{Li}$ and ${}^7\text{Li}$ is produced, their abundances remain far below that of the lighter elements, while B, Be, and CNO isotopes are produced in even smaller amounts. The presence of these

gaps is caused by the instability of ${}^5\text{He}$, ${}^5\text{Li}$ and ${}^8\text{Be}$ with respect to particle emission: their lifetimes are as low as a few 10^{-22} s for ${}^5\text{He}$ and ${}^5\text{Li}$ and $\approx 10^{-16}$ s for ${}^8\text{Be}$. More precisely, ${}^5\text{He}$, ${}^5\text{Li}$ and ${}^8\text{Be}$ are respectively unbound by 0.798, 1.69 and 0.092 MeV with respect to neutron, proton and α particle emission¹. Variations of constants will affect the energy levels of the unbound ${}^5\text{He}$, ${}^5\text{Li}$ and ${}^8\text{Be}$ nuclei [18, 30] and hence, the resonance energies whose contributions dominate the reaction rates. In addition, since ${}^8\text{Be}$ is only slightly unbound, one can expect that for even a small change in the nuclear potential, it could become bound and may thus severely impact the results of standard BBN (SBBN), in a similar way that a bound dineutron impacts BBN abundances [36]. It has been suspected that stable ${}^8\text{Be}$ would trigger the production of heavy elements in BBN, in particular that there would be significant leakage of the nucleosynthetic chain into carbon. Indeed, as we have seen previously [23], changes in the nuclear potential strongly affects the triple alpha process and as a result, strongly affects the nuclear abundances in stars.

This article investigates in detail the effect of the variation of fundamental constants on the properties of the compound nuclei ${}^5\text{He}$, ${}^5\text{Li}$, ${}^8\text{Be}$ and ${}^{12}\text{C}$ involved in the ${}^3\text{H}(\text{d},\text{n}){}^4\text{He}$, ${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$ and ${}^4\text{He}(\alpha\alpha,\gamma){}^{12}\text{C}$ reactions, and their consequences on reaction rates and BBN abundances. In addition, we consider the particular case of stable ${}^8\text{Be}$.

In Section II, we briefly present the microscopic cluster model used to determine resonance properties. Section III focuses on ${}^8\text{Be}$ and on the CNO production for the cases of both unbound and bound ${}^8\text{Be}$. We compute the C/H ratio as a function of the parameter δ_{NN} . Section IV focuses on the ${}^4\text{He}$ production by the ${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$ and ${}^3\text{H}(\text{d},\text{n}){}^4\text{He}$ reactions. Each of these reactions contains a broad s-wave resonance at low energies, and their reaction rates may depend on the resonance energies. Section V summarizes the BBN constraints on the variation of the nuclear interaction, hence extending our previous analysis [22]. Section VI provides a summary and our conclusions.

II. OUTLINE OF THE NUCLEAR MODEL

We follow the formalism introduced in Ref. [23] in order to model the effect of the variation of the nucleon-nucleon (N-N) interaction. We adopt a phenomenological description of the different nuclei based on a cluster model in which the wave functions are approximated by

clusters of two or three α wave functions. In a microscopic theory, the wave function of a nucleus with nucleon number A , spin J , and total parity π is a solution of a Schrödinger equation with a Hamiltonian given by

$$H = \sum_{i=1}^A T_i + \sum_{i>j=1}^A V_{ij}, \quad (2.1)$$

where T_i is the kinetic energy of nucleon i , and V_{ij} a nucleon-nucleon (N-N) interaction. In general, the potential depends on space, spin and isospin coordinates of nucleons i and j , and can be decomposed as

$$V_{ij} = V_{ij}^C + V_{ij}^N, \quad (2.2)$$

where V^C and V^N represent the Coulomb and nuclear interactions, respectively.

Solving the Schrödinger equation associated with Hamiltonian (2.1) is a difficult problem, in particular for nuclear reactions. We use a microscopic cluster model, where the nucleons are assumed to form "groups", called clusters. This approximation is known as the Resonating Group Method (RGM) [37], and the wave function of a two-cluster system is approximated as

$$\Psi^{JM\pi} = \mathcal{A}\Phi_1\Phi_2g^{J\pi}(\rho)Y_J^M(\Omega), \quad (2.3)$$

where Φ_1 and Φ_2 are the internal wave functions of the clusters (defined in the shell model), and \mathcal{A} is the A -nucleon antisymmetrizer, which accounts for the Pauli principle. In Eq. (2.3), $g^{J\pi}(\rho)$ is the relative function, depending on the relative coordinate ρ , and to be determined from the Schrödinger equation. With these wave functions, any physical quantity, such as spectroscopic properties of the nucleus, or nucleus-nucleus cross sections, can be computed.

Eq. (2.3) is written for two clusters, but the extension to three clusters is feasible [38]. Currently, more sophisticated microscopic models (such as the No Core Shell Model [39] or the variational Monte Carlo (VMC) method [40]) are available for few-nucleon systems. These models use realistic interactions (such as Argonne AV18 [41]) but are quite difficult to apply with nucleus-nucleus collisions. In contrast, the RGM is well adapted to the nuclear spectroscopy and to reactions, but the use of simple cluster wave functions (such as the α particle which is described by four $0s$ orbitals) makes it necessary to adopt effective N-N interactions. We use here the Minnesota potential [42], well adapted to low-mass systems. This central potential reproduces the experimental deuteron energy. It simulates the missing tensor force by an appropriate choice of the central interaction. The Minnesota interaction V_N is described in detail in Ref. [23].

To take into account the variation of the fundamental constants, we introduce the parameters δ_α and δ_{NN} to characterize the change of the strength of the electromagnetic and nucleon-nucleon interactions respectively.

¹ Although ${}^8\text{Li}$ and ${}^8\text{B}$ also contribute to the mass gap due to their short lifetimes (on BBN timescales), they are more deeply unbound, and a far greater change in the fundamental couplings would be needed to affect their stability. We will not consider them further here.

This is implemented by modifying the interaction potential (2.2) so that

$$V_{ij} = (1 + \delta_\alpha)V_{ij}^C + (1 + \delta_{NN})V_{ij}^N. \quad (2.4)$$

Such a modification will affect B_D , the energy levels of ^8Be and ^{12}C simultaneously, as well as the resonant reactions involving $A = 5$ nuclei such as $^3\text{He}(\text{d}, \text{p})^4\text{He}$ and $^3\text{H}(\text{d}, \text{n})^4\text{He}$. If ^8Be becomes bound, one will need to calculate the two reaction rates $^4\text{He}(\alpha, \gamma)^8\text{Be}$ and $^8\text{Be}(\alpha, \gamma)^{12}\text{C}$.

In Ref. [23], we investigated the ^8Be and $^{12}\text{C}(0_2^+)$ energies by scaling the Minnesota interaction. The δ_{NN} parameter then provides a link, for the Minnesota potential, between the deuteron energy B_D and the 2α and 3α resonance energies. Here we extend this idea to the $^3\text{H}(\text{d}, \text{n})^4\text{He}$ and $^3\text{He}(\text{d}, \text{p})^4\text{He}$ reactions. However both reactions are known to be dominated by a low energy $\frac{3}{2}^+$ resonance (at $E_R^{\text{exp}} = 0.048$ MeV for ^5He and $E_R^{\text{exp}} = 0.21$ MeV for ^5Li). From simple angular-momentum couplings it is easy to see that this resonance corresponds to an s wave in the entrance channel, and to a d wave in the exit channel. Consequently the coupling between these channels can be described by a tensor force only. As mentioned earlier, this component is neglected in the Minnesota interaction. However, as our main interest is the variation of the resonance energy, we used single channel $^3\text{He}+\text{d}$ and $^3\text{H}+\text{d}$ approximations, and performed various calculations by modifying δ_{NN} (see Section III). This approximation is justified by the fact that these resonances essentially have a $3+2$ structure.

δ_{NN} is a phenomenological parameter that can be related to the fundamental constants through the dependence of the deuterium binding energy on δ_{NN} . With the Minnesota N-N interaction, we find

$$\Delta B_D/B_D = 5.716 \times \delta_{NN}. \quad (2.5)$$

Note that other forces may provide a slightly different dependence. However, a consistent treatment of B_D and of resonance properties in ^5Li and ^5He requires the same effective interaction, such as the Minnesota potential. Thus we have the possibility of relating δ_{NN} to the gauge and Yukawa couplings if one matches this prediction to a potential model via the σ and ω meson masses [18, 22, 29, 43] or the pion mass, as suggested in Refs. [14, 44, 45]. In Ref. [22], it was concluded that

$$\frac{\Delta B_D}{B_D} = 18 \frac{\Delta \Lambda_{\text{QCD}}}{\Lambda_{\text{QCD}}} - 17 \left(\frac{\Delta v}{v} + \frac{\Delta h_s}{h_s} \right), \quad (2.6)$$

which led to the expression in Eq. (1.5). Eq. (2.5) can then link any constraint on δ_{NN} to the three fundamental constants (h_s, v, Λ) .

III. PRIMORDIAL CNO PRODUCTION AND ^8Be

CNO production in SBBN has been investigated in Ref. [7] and most recently revisited in Coc *et al.* [9]. The direct detection of primordial CNO isotopes seems highly unlikely with the present observational techniques but it is important for other applications. In particular, it may significantly affect the dynamics of population III (Pop. III) stars since hydrogen burning in low mass Pop. III stars proceeds through the slow pp chains until enough carbon is produced, through the triple alpha reaction, to activate the CNO cycle. The minimum value of the initial CNO mass fraction that would affect Pop. III stellar evolution was estimated to be 10^{-10} [46] or even as low as 10^{-12} for less massive stars [47]. This is only two orders of magnitude above the SBBN CNO yields obtained using current nuclear reaction rates. The main difficulty in BBN calculations up to CNO is the extensive network (more than 400 reactions) needed, including n, p, α , but also d, t and ^3He , induced reactions on both stable and radioactive targets.

CNO production (mostly ^{12}C) in SBBN is found to be in the range $\text{CNO}/\text{H} = (0.2 - 3.) \times 10^{-15}$ [9]. In scenarios with varying constants, this number needs to be compared with the $^4\text{He}(\alpha\alpha, \gamma)^{12}\text{C}$ production of ^{12}C , in particular if its rate is dramatically increased by a variation of the ^8Be ground state and Hoyle state position as considered by Ekström *et al.* [23]. Since it is a resonant reaction, its rate is very sensitive to the nuclear interaction and we recall that a 1.5% variation of the N-N interaction ($-0.009 < \delta_{NN} < +0.006$) would induce a change in the rate between ≈ 18 to ≈ 2 orders of magnitude for temperatures between $T = 0.1$ to 1.0 GK (see Fig. 3 in Ref. [23]).

In this section, we investigate the effect of ^8Be on primordial CNO production. We consider two cases in which ^8Be is either unbound (§ III A) or bound (§ III B). We then derive the BBN predictions (§ III C) in each case.

A. Unbound ^8Be

When the N-N interaction is modified by less than 0.75% (i.e. $\delta_{NN} < 7.52 \times 10^{-3}$), ^8Be remains unbound w.r.t. two α -particle emission. We can therefore take the $^4\text{He}(\alpha\alpha, \gamma)^{12}\text{C}$ rate as a function of δ_{NN} as calculated by Ekström *et al.* [23]. We recall that, in the framework of the cluster model using the Minnesota interaction, we obtained that the energy of the ^8Be ground state with respect to the $\alpha + \alpha$ threshold was given by [23]

$$\begin{aligned} -B_8 &\equiv E_R(^8\text{Be}) \\ &= (0.09184 - 12.208 \times \delta_{NN}) \text{ MeV} \end{aligned} \quad (3.1)$$

where B_8 is the ^8Be binding energy with respect to two alpha break-up (with this convention, $B_8 < 0$ for un-

bound ${}^8\text{Be}$). We recall that, within the same model, we obtained

$$E_R({}^{12}\text{C}) = (0.2876 - 20.412 \times \delta_{NN}) \text{ MeV} \quad (3.2)$$

for the dependence of the Hoyle state resonance.

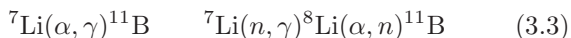
B. Bound ${}^8\text{Be}$

When $\delta_{NN} \gtrsim 7.52 \times 10^{-3}$, ${}^8\text{Be}$ becomes bound and should be considered as a stable isotope during BBN. Hence, we have to calculate two reaction rates: ${}^4\text{He}(\alpha, \gamma){}^8\text{Be}$ and ${}^8\text{Be}(\alpha, \gamma){}^{12}\text{C}$. The calculation of the rate of the second reaction can be achieved using the sharp resonance formula [48] with the varying parameters of the Hoyle state from Ekström *et al.* [23]. For the first reaction, ${}^4\text{He}(\alpha, \gamma){}^8\text{Be}$, we have performed a dedicated calculation using the potential by Buck *et al.* [49] to obtain the astrophysical S -factor displayed in Figure 1 for values of the ${}^8\text{Be}$ binding energy of $B_8 = 10, 50$ and 100 keV. The Buck potential is expressed as a Gaussian, and accurately reproduces the experimental $\alpha + \alpha$ phase shifts up to 20 MeV. The initial 2^+ and final 0^+ wave functions are computed in the potential model, assuming that the ground state is slightly bound. The cross section is then determined from integrals involving the wave functions and the E2 operator (see Ref. [48] for detail). The broad structure corresponds to the well known 2^+ resonance in $\alpha - \alpha$ scattering and it is easily concluded from Figure 1 that the S -factor remains relatively insensitive to a change in δ_{NN} . It was shown in Ref. [50] that describing ${}^8\text{Be}$ as a bound state ($E_R < 0$) or as a low-energy resonance ($E_R > 0$), has a small effect on the $\alpha(\alpha, \gamma){}^8\text{Be}$ capture cross section.

Figure 2 depicts the reaction rate for $B_8 = 10$ and 100 keV relative to the case with $B_8 = 50$. The rate depends very little on the ${}^8\text{Be}$ binding energy for $B_8 > 0$ and the rate changes by less than $\sim 10\%$ for the three values of B_8 considered. As a result, we can safely neglect the difference in the rates once $B_8 > 0$. The reaction rate is essentially given by the radiative capture cross section at the Gamow energy $E_0(T)$, which is proportional to E_γ^5 , where the photon energy is $E_\gamma = E_{\text{cm}} + B_8$ and E_{cm} is the alpha-alpha center-of-mass energy.

C. BBN calculations

CNO is produced at a very low level in SBBN. The chain leading to carbon is dominated by the following reactions:



followed by

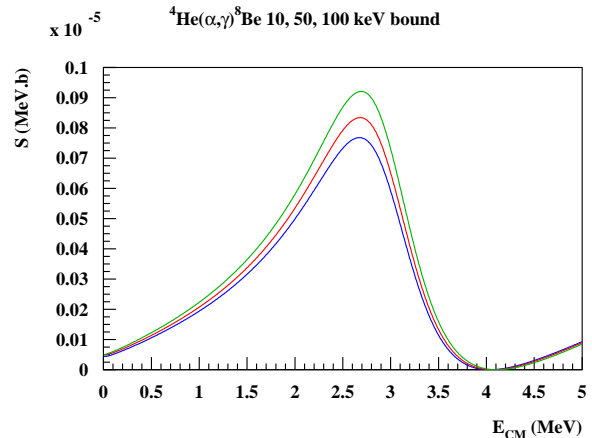
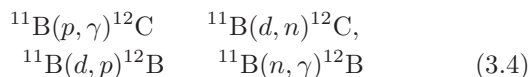


FIG. 1: The astrophysical S -factor for the ${}^4\text{He}(\alpha, \gamma){}^8\text{Be}$ reaction, assuming that ${}^8\text{Be}$ is bound by 10 (blue), 50 (red) and 100 (green) keV, corresponding to $\delta_{NN} = 0.0083, 0.0116, 0.0156$ respectively, from bottom to top.

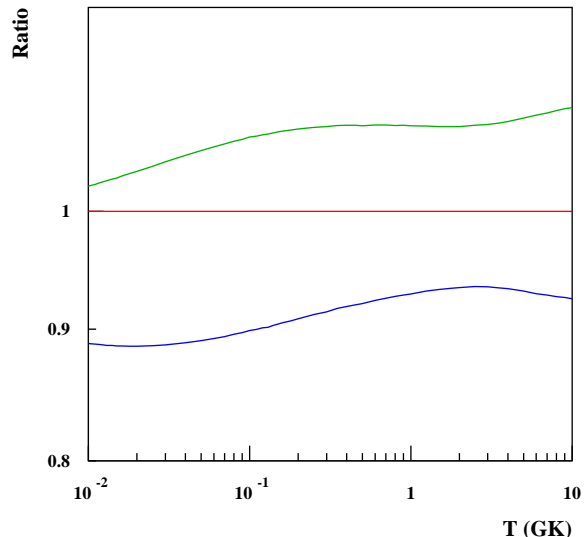


FIG. 2: The relative variation of the ${}^4\text{He}(\alpha, \gamma){}^8\text{Be}$ reaction rate assuming that ${}^8\text{Be}$ is bound by 10 (blue), 50 (red) and 100 keV, relative to the 50 keV rate.

which bridge the gap between the $A \leq 7$ and $A \geq 12$ nuclei [7, 9].

To disentangle standard CNO production through the reactions listed above with the one proceeding through the triple-alpha reaction, we reduced the network to the 15 reactions involved in $A < 8$ nucleosynthesis plus ${}^4\text{He}(\alpha\alpha, \gamma){}^{12}\text{C}$, i.e. we turned off the other reactions, in particular those listed in Eqs. (3.3) and (3.4). As we are investigating a possible enhancement of CNO production we only considered positive δ_{NN} values that lead to a higher triple-alpha reaction rate.

The CNO yield as a function of δ_{NN} is displayed in

Figure 3. The carbon abundance shows a maximum at $\delta_{NN} \approx 0.006$, $C/H \approx 10^{-21}$, which is *six orders of magnitude* below the carbon abundance in SBBN. This can be understood as follows: Figure 4 displays the ${}^4\text{He}(\alpha\alpha, \gamma){}^{12}\text{C}$ rate as a function of δ_{NN} for temperatures relevant to BBN, i.e., from 0.1 to 1 GK. As one can see, the variation of the rate with δ_{NN} is limited at the highest temperatures where BBN production occurs so that the amplification of ${}^{12}\text{C}$ production does not exceed a few orders of magnitudes. Indeed, while stars can process CNO at 0.1 GK over billions of years, in BBN the optimal temperature range for producing CNO is passed through in a matter of minutes. This is not sufficient for ${}^{12}\text{C}$ (CNO) nucleosynthesis in BBN. Furthermore, the baryon density during BBN remains in the range 10^{-5} to 0.1 g/cm^3 between 1.0 and 0.1 GK, substantially lower than in stars (e.g. 30 to 3000 g/cm^3 in Pop. III stars). This makes three-body reactions like ${}^4\text{He}(\alpha\alpha, \gamma){}^{12}\text{C}$ much less efficient compared to two-body reactions. Finally, in stars, ${}^4\text{He}(\alpha\alpha, \gamma){}^{12}\text{C}$ operates during the helium burning phase without significant sources of ${}^7\text{Li}$, d , p and n that allow the processes listed in Eqs. (3.3) and (3.4).

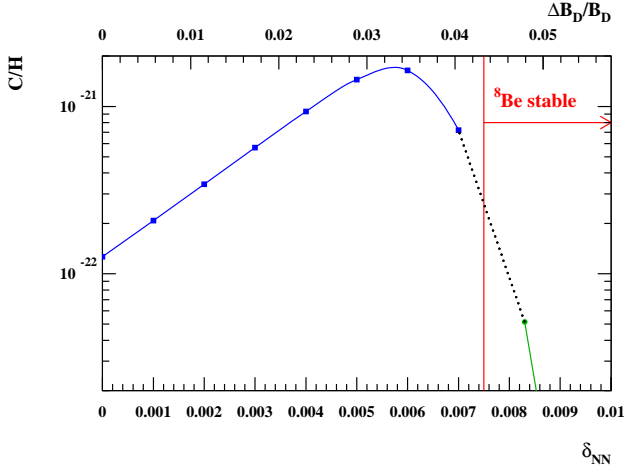


FIG. 3: ${}^{12}\text{C}$ production (in number of atoms relative to H) through the ${}^4\text{He}(\alpha\alpha, \gamma){}^{12}\text{C}$ reaction (left) or via a stable ${}^8\text{Be}$ (right) as a function of the N-N interaction. For clarity, the rates of all other CNO producing reactions are set to zero to study these specific channels. The dotted line just connects the results of the two types of calculations: via ${}^4\text{He}(\alpha\alpha, \gamma){}^{12}\text{C}$ as in [23] and § III A or via ${}^4\text{He}(\alpha, \gamma){}^8\text{Be}^{\text{stable}}(\alpha, \gamma){}^{12}\text{C}$ as in § III B.

The maximum of the ${}^{12}\text{C}$ production as a function of δ_{NN} in Figure 3 reflects the maxima in the ${}^8\text{Be}(\alpha, \gamma){}^{12}\text{C}$ and ${}^4\text{He}(\alpha\alpha, \gamma){}^{12}\text{C}$ rates displayed in Figs. 4 and 5. They are due to the effect of the sharp resonances (in both the ${}^8\text{Be}$ ground state and the ${}^{12}\text{C}$ “Hoyle state” that dominate the cross section).

The contribution of a sharp resonance in the (α, γ)

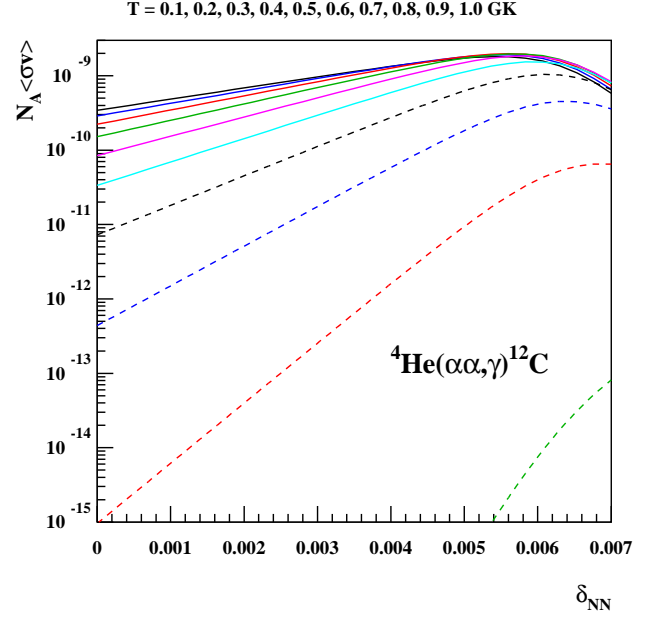


FIG. 4: The ${}^4\text{He}(\alpha\alpha, \gamma){}^{12}\text{C}$ rate as a function of δ_{NN} at constant temperature relevant for BBN from 0.1 GK (lower curve) to 1 GK (upper curve), by 0.1 GK steps.

channel is given by

$$\langle\sigma v\rangle \propto \frac{\Gamma_\alpha(E_R)\Gamma_\gamma(E_R)}{\Gamma(E_R)} \exp\left(-\frac{E_R}{k_B T}\right) \quad (3.5)$$

where Γ_α is the entrance (alpha-)width, Γ_γ the exit (gamma-)width and Γ is the total width ($\Gamma = \Gamma_\alpha + \Gamma_\gamma$ if these are the only open channels). Figure 1 in Ref. [23] displays the ${}^{12}\text{C}$ level scheme: the radiative width, Γ_γ , is associated with the decay to the first ${}^{12}\text{C}$ excited state at 4.44 MeV as the decay to the ground state proceeds only through the much less efficient electron-positron pair emission. The corresponding decay energy is then $E_\gamma(E_R) = 3.21 \text{ MeV} + \Delta E_R({}^{12}\text{C})$. Equation 3.5 shows that for a fixed E_R , i.e. δ_{NN} , the contribution increases with temperature as seen in Figs. 4 and 5. While the radiative width $\Gamma_\gamma(E_R) \propto E_\gamma^{2\ell+1}$ is almost insensitive to $E_R(\delta_{NN})$, $\Gamma_\alpha(E_R)$ is very sensitive to $E_R(\delta_{NN})$ variations because of the Coulomb and centrifugal barriers penetrability, $P_\ell(E)$. The reduced widths γ_x^2 , defined by:

$$\Gamma_x(E) = 2\gamma_x^2 P_\ell(E) \quad (x \neq \gamma), \quad (3.6)$$

are corrected for these effects so that they reflect the nuclear properties only, and are, as a good approximation, independent of δ_{NN} .

Depending on whether $\Gamma_\alpha \ll \Gamma_\gamma$ or $\Gamma_\gamma \ll \Gamma_\alpha$, the sensitivity of $\langle\sigma v\rangle$ (Eq. 3.5) to E_R or δ_{NN} variations is very different. This is due to the very different energy dependence of Γ_α and Γ_γ , as discussed in detail in [51]. In the latter case, using Eq. (3.5), the sensitivity of the

rate to E_R (δ_{NN}) variations is simply given by :

$$\frac{\partial \ln \langle \sigma v \rangle}{\partial \ln E_R} = -\frac{E_R}{k_B T} \quad (3.7)$$

as the prefactor in Eq. (3.5) is reduced to Γ_γ which is almost constant. Since δE_R and δ_{NN} have opposite signs (Eqs. 3.1 and 3.2), the rate *increases* with δ_{NN} . In the former case, the same factor is reduced to the very energy dependent Γ_α and we have:

$$\langle \sigma v \rangle \propto \gamma_\alpha^2 \exp \left(-\sqrt{\frac{E_G}{E_R}} - \frac{E_R}{k_B T} \right) \quad (3.8)$$

where the penetrability, $P_\ell(E)$, has been approximated by $\exp(-\sqrt{E_G/E})$ with Gamow energy, E_G . It is well known that the exponential in Eq. (3.8) can be well approximated (see e.g. Ref. [48]) by

$$\exp \left[-\left(\frac{E_R - E_0}{\Delta E_0/2} \right)^2 \right] \quad (3.9)$$

with

$$\begin{aligned} E_0 &= \left(\frac{\mu}{2} \right)^{1/3} \left(\frac{\pi e^2 Z_1 Z_2 k T}{\hbar} \right)^{2/3} \\ &= 0.1220 (Z_1^2 Z_2^2 A)^{1/3} T_9^{2/3} \text{ MeV} \end{aligned} \quad (3.10)$$

and

$$\begin{aligned} \Delta E_0 &= 4 (E_0 k T / 3)^{1/2} \\ &= 0.2368 (Z_1^2 Z_2^2 A)^{1/6} T_9^{5/6} \text{ MeV}. \end{aligned} \quad (3.11)$$

that define the Gamow window. Recalling that the reduced width γ^2 only reflects the nuclear structure and is assumed to be constant, it is straightforward to calculate the sensitivity of the rate to E_R (δ_{NN}) variations:

$$\frac{\partial \ln \langle \sigma v \rangle}{\partial \ln E_R} = 4 \left(\frac{E_0(T) - E_R(\delta_{NN})}{\Delta E_0(T)/2} \right) \quad (3.12)$$

Since, for large δ_{NN} and T , we have $E_0 > E_R$, the rate *decreases* with δ_{NN} .

The condition $\Gamma_\alpha = \Gamma_\gamma$ that marks the boundary between these two opposite evolutions in Eq. (3.5) can be found in Fig. A.1 of Ekström *et al.* [23], at $\delta_{NN} \approx 0.006$. It also corresponds to the maximum of the ${}^8\text{Be}(\alpha, \gamma){}^{12}\text{C}$ rate depicted in Fig. 5. The more complicated two step, three body ${}^4\text{He}(\alpha\alpha, \gamma){}^{12}\text{C}$ reaction shows a similar dependence.

To summarize, for $\delta_{NN} \lesssim 0.006$, the rate decreases (increases) as a function of E_R (δ_{NN}) because of the dominating exponential factor, $\exp(-E_R/kT)$, while for $\delta_{NN} \gtrsim 0.006$, it increases (decreases) because of the penetrability. This evolution is followed by the ${}^{12}\text{C}$ production displayed in Fig. 3.

For $\delta_{NN} \geq 0.00752$, when ${}^8\text{Be}$ is bound, ${}^{12}\text{C}$ production drops to $C/H \approx 5 \times 10^{-23}$ for $B_8 = 10$. For larger B_8 ,

the abundance drops sharply as seen in Figure 3. For $B_8 = 50$ keV, $C/H \approx 5 \times 10^{-29}$ and is no longer in the range shown in the figure. For $B_8 = 100$ keV, corresponding to $\delta_{NN} = 0.0156$, the Hoyle state is even below threshold and the production is vanishingly small. If ${}^8\text{Be}$ is bound, reactions that normally produce two α -particles could form ${}^8\text{Be}$ instead. We considered the following reactions

$$\begin{array}{ll} {}^7\text{Be}(n, \gamma)2\alpha & {}^7\text{Li}(p, \gamma)2\alpha \\ {}^7\text{Li}(n, \gamma){}^8\text{Li}(\beta^+)2\alpha & {}^7\text{Be}(d, p)2\alpha \\ {}^7\text{Li}(d, n)2\alpha & {}^7\text{Be}(t, np)2\alpha \\ {}^7\text{Be}({}^3\text{He}, 2p)2\alpha & {}^7\text{Be}(n, \gamma)2\alpha \\ {}^7\text{Li}({}^3\text{He}, d)2\alpha & {}^7\text{Be}(t, d)2\alpha \\ {}^7\text{Li}(t, 2n)2\alpha & {}^7\text{Li}({}^3\text{He}, np)2\alpha \end{array}$$

using the same rates as in Ref. [9] but replacing 2α by ${}^8\text{Be}$. The only significant enhancement comes from the ${}^7\text{Li}(d, n)2\alpha$ reaction but even in the most favorable case ($B_8 = 10$ keV), C/H reaches $\approx 10^{-21}$. This is still *six orders of magnitude* below the SBBN yield [9] that proceeds via the reactions listed in Eqs. (3.3) and (3.4).

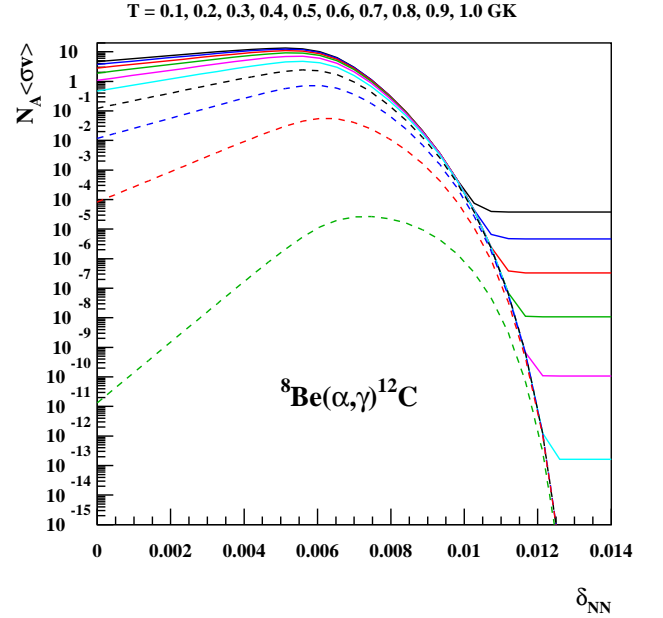


FIG. 5: As in Fig. 4 but for the ${}^8\text{Be}(\alpha, \gamma){}^{12}\text{C}$ rate. Above $\delta_{NN} \approx 0.006$, the contribution of the Hoyle state decreases (see text), leaving only the contribution of the 3^- resonance at $E_R = 2.274$ MeV at the highest δ_{NN} values.

Figure 6 displays the evolution of the ${}^{12}\text{C}$ and ${}^8\text{Be}$ mass fractions as a function of time. They both increase with time until the ${}^4\text{He}(\alpha, \gamma){}^8\text{Be}$ drops with decreasing temperature. Afterwards, equilibrium between two α -particle fusion and ${}^8\text{Be}$ photodissociation prevails as shown by the dotted lines. Indeed, the Be mass fraction is

$$Y_{\text{Be}} = \frac{Y_\alpha^2}{2} \frac{\rho}{R} \propto T^{\frac{3}{2}} \exp(-B_8/k_B T) \quad (3.13)$$

where we have taken the reverse ratio, R , to be proportional to $\exp(B_8/k_B T) \times T^{\frac{3}{2}}$ and the baryon density, $\rho \propto T^3$. For the highest values of B_8 , the ${}^8\text{Be}$ mass fraction increases until, due to the expansion, equilibrium drops out, as shown by the late time behaviour of the upper curve ($B_8 = 100$ keV) in Figure 6.

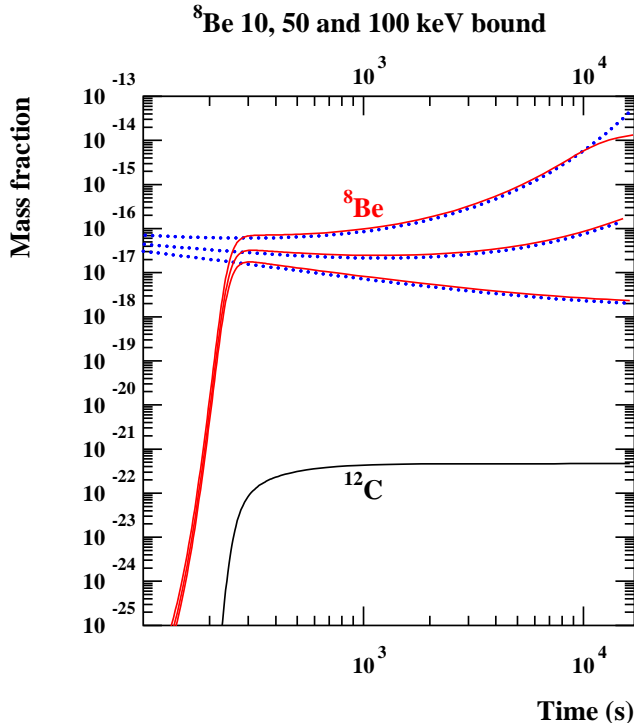


FIG. 6: ${}^{12}\text{C}$ and ${}^8\text{Be}$ mass fractions as a function of time, assuming ${}^8\text{Be}$ is bound by 100, 50 and 10 keV as shown by the upper to lower curves respectively. (Only the ${}^{12}\text{C}$ mass fraction curve, for $B_8 = 10$ keV, is shown; others are far below the scale shown). The dotted lines correspond to the computation at thermal equilibrium.

In conclusion, if one keeps only the first 15 nuclear reactions while adding the possibility of primordial carbon production through 3α -process, one gets a typical abundance of only order $\text{C}/\text{H} \sim 10^{-21}$ which remains six orders of magnitude smaller than the SBBN carbon production, which is of order $\text{C}/\text{H} \sim 10^{-15}$. This explicitly shows that CNO production in BBN cannot be increased through either an increase of the 3α -reaction rate or even through the stabilization of ${}^8\text{Be}$.

IV. REACTIONS INVOLVING $A = 5$ NUCLEI

A. Introduction

In contrast to the CNO elements, the ${}^4\text{He}$, D, ${}^3\text{He}$ and ${}^7\text{Li}$ isotopes are produced in observable quantities in BBN and can thus constrain δ_{NN} . It would be desirable to have the dependence of each of the main SBBN

reactions to the N-N interaction or other fundamental quantities. This was achieved in Ref. [22] for the first two BBN reactions: the $n \leftrightarrow p$ weak interaction and the $p(n, \gamma)d$ bottleneck.

Here, we propose to extend this analysis to the ${}^3\text{H}(d, n){}^4\text{He}$ and ${}^3\text{He}(d, p){}^4\text{He}$ reactions that proceed through the $A = 5$ compound nuclei ${}^5\text{He}$ and ${}^5\text{Li}$. In these two reactions, the rates are dominated by the contribution of a $\frac{3}{2}^+$ resonance whose properties can be calculated within the same microscopic model that we used for ${}^4\text{He}(\alpha\alpha, \gamma){}^{12}\text{C}$, but with ${}^3\text{H}+d$ and ${}^3\text{He}+d$ cluster structures. Unlike the case for ${}^8\text{Be}$, the lifetime of the ${}^5\text{He}$ and ${}^5\text{Li}$ states is extremely short (the width of the ${}^8\text{Be}$ ground state is 6 eV, whereas the widths of the $\frac{3}{2}^+$ resonances in ${}^5\text{He}$ and ${}^5\text{Li}$ are of the order of 1 MeV). Therefore the issue of producing $A = 5$ bound states, or even shifting their ground state energy down to the Gamow window, is not relevant. Even a two step process, like the triple alpha reaction, where ${}^5\text{He}$ or ${}^5\text{Li}$ in thermal equilibrium would capture a subsequent nucleon to form ${}^6\text{Li}$ is completely negligible because they are unbound by ~ 1 MeV compared with the 92 keV of ${}^8\text{Be}$. Hence no significant equilibrium abundance of $A = 5$ nuclei can be reached.

The analysis of the effect of δ_{NN} on the other rates requires additional effort, but should be smaller because important reactions like ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ do not display resonant behaviour and the S -factor does not change much with energy (see e.g. Fig. 1h in Ref. [52]). In order to proceed, we used the following procedure

- We performed single-channel ${}^3\text{H}+d$ and ${}^3\text{He}+d$ calculations of the resonance energy as a function of δ_{NN} .
- We analyzed the experimental cross sections within an R -matrix approach.
- With the energy dependence obtained from the first step, we computed the reaction rate for various δ_{NN} or B_D values.

B. Resonance energy in ${}^5\text{He}$ and ${}^5\text{Li}$

In principle an RGM calculation of the ${}^3\text{H}(d, n){}^4\text{He}$ and ${}^3\text{He}(d, p){}^4\text{He}$ cross sections could be performed as it was done for the 2α and 3α systems. The main difference is that each reaction involves two channels, and their cross section is dominated by a low-energy $\frac{3}{2}^+$ resonance. As mentioned before, a central N-N interaction, such as the Minnesota potential, does not provide a coupling between the ${}^3\text{H}+d$ and $\alpha+n$ (or mirror) channels, since the tensor force is missing. Consequently, the neutron width is strictly zero, and obtaining a realistic S -factor with the Minnesota interaction is not possible. We therefore use the RGM to estimate the sensitivity of the resonance energy as a function of δ_{NN} .

Using the parameterization (2.4) for the nucleon-nucleon interaction, we modify the resonance energy. Both the excitation energies of the $\frac{3}{2}^+$ resonance and of the thresholds vary. We find

$$\Delta E_R = -0.327 \times \delta_{NN} \quad (4.1)$$

for ${}^3\text{H}(\text{d},\text{n}){}^4\text{He}$ and

$$\Delta E_R = -0.453 \times \delta_{NN} \quad (4.2)$$

for ${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$ (units are MeV). These energy dependences are much weaker ($\sim 20\text{--}30$ keV for $|\delta_{NN}| \leq 0.03$) than for ${}^8\text{Be}$ and ${}^{12}\text{C}$ (see Eqs. (3.1) and (3.2)). This is expected for broad resonances which are weakly sensitive to the nuclear interaction [37].

In contrast, Berengut *et al.* [30] find a stronger energy dependence. These authors perform VMC calculations with realistic N-N interactions, which provide better d and ${}^3\text{H}/{}^3\text{He}$ wave functions. However the VMC approach is not well adapted to broad resonances, such as those observed in ${}^5\text{He}$ and ${}^5\text{Li}$. Our cluster model, although using a simpler N-N interaction, is more suited to unbound states since the asymptotic Coulomb behaviour of scattering states is exactly taken into account.

C. R -matrix fits of the ${}^3\text{H}(\text{d},\text{n}){}^4\text{He}$ and ${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$ S -factors

To be consistent with our previous work, we want to reproduce, for $\delta_{NN}=0$, the experimental S -factors obtained (see references in Ref. [52]) by a full R -matrix analysis, but for convenience we restrict ourselves here to the single pole R -matrix approximation which will be shown to be sufficient. For the ${}^3\text{H}(\text{d},\text{n}){}^4\text{He}$ reaction, we use the parameterization of Barker [53], which reproduces the resonance corresponding to the corresponding to the $\frac{3}{2}^+$ state at 16.84 MeV. Because the widths are energy dependent, the maximum of the cross section at $E_R^{exp} = 0.048$ MeV, differs from the minimum of the denominator that appears in the single pole R -matrix prescription:

$$\sigma(E) \propto \frac{(\hbar c)^2}{\mu E} \frac{\Gamma_{in}(E)\Gamma_{out}(E)}{(E_R^* + \Delta E_R^* - E)^2 + \Gamma^2(E)/4} \quad (4.3)$$

where μ the reduced mass, $\Gamma = \Gamma_{in} + \Gamma_{out}$ and we have dropped numerical factors. The astrophysical S -factor displayed in Figure 7 is just $\sigma(E)E \exp \sqrt{(E_G/E)}$ where the exponential factor approximates the Γ_{in} energy dependence due to the penetrability. The widths, Γ_x , are related to the reduced widths γ_x^2 by Eq. (3.6), and $\Delta E_R^*(E_R)$ is given by Eqs. (5.5) in Descouvemont & Baye [54]. Note that ΔE_R^* is the *shift factor* of R -matrix theory, not related to the variation of constants, and hence is different from the $\Delta E_R(\delta_{NN})$, that we will discuss below. Using $E_R^* = 0.091$ MeV, the reduced entrance width $\gamma_d^2 = 2.93$ MeV and the reduced exit width $\gamma_n^2 = 0.0794$ MeV provided by Barker [53], our results

(not a fit) are in perfect agreement with the R -matrix fit [52] of the experimental data shown in Figure 7 that we used in previous work [6, 9, 22, 55].

Barker was also able to reproduce the existing experimental data for the ${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$ cross section using the same parameters for the $\frac{3}{2}^+$ state at 16.87 MeV with $E_R^{exp} = 0.21$ MeV as seen in Figure 8. However, when including modern data as in Descouvemont *et al.* [52] the agreement was found to be poor. Accordingly, we have performed a fit of the S -factor provided by Ref. [52] that gave $E_R^* = 0.35779$ MeV, with $\gamma_{he-3}^2 = 1.0085$ MeV and $\gamma_p^2 = 0.025425$ MeV. Hence, our new calculations of the ${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$ and ${}^3\text{H}(\text{d},\text{n}){}^4\text{He}$ rates coincide with those we have been using in previous papers, when the constants do not vary, showing that the single pole approximation is sufficient.

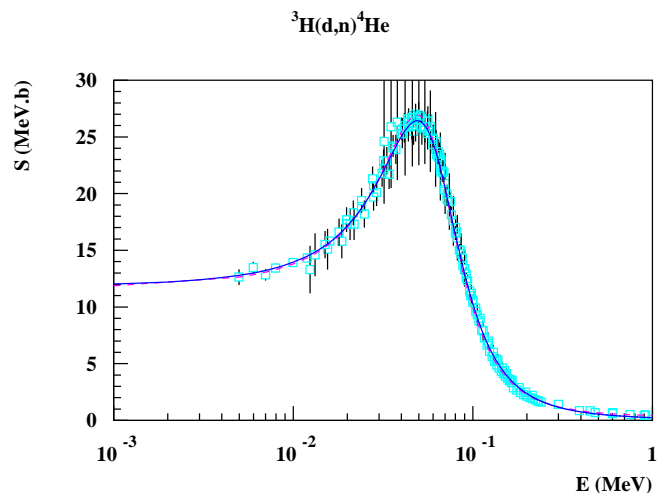


FIG. 7: S -factor for the ${}^3\text{H}(\text{d},\text{n}){}^4\text{He}$ reaction from Ref. [52] (dashed) and our calculation using the parameters from Barker [53] (solid). See Ref. [52] for the references to experimental data.

D. Reaction rates

The reaction rates are shown in Figure 9. As expected from Eqs. (4.1) and (4.2) they are only slightly affected by variations of δ_{NN} (less than 5%). From the sensitivity study of Ref. [55], we deduce that ${}^3\text{H}(\text{d},\text{n}){}^4\text{He}$ rate variations have no effect on BBN while ${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$ rate variations induce only very small ($\leq 4\%$) changes in the ${}^7\text{Li}$ and ${}^3\text{He}$ abundances. Because the change is so small, we can make a linear approximation to the sensitivity as shown in Table I displaying $(\delta Y/Y)/\delta_{NN}$ values for both reactions.

Our results are significantly different from those of Berengut *et al.* [30]: first, we use a more elaborate parameterization of the cross-section, second, our calculation of the resonance energy shift is less sensitive to the

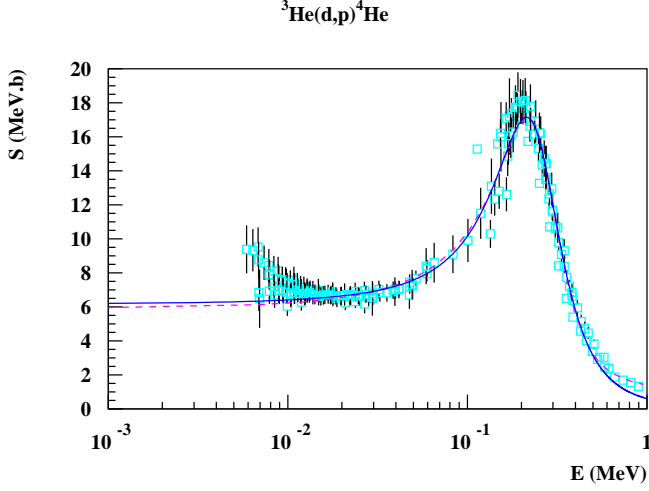


FIG. 8: As in Fig. 7 but for ${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$ and our calculation with parameters fit to ref. [52] (dashed). Deviations with experimental data at very low energy are due to screening.

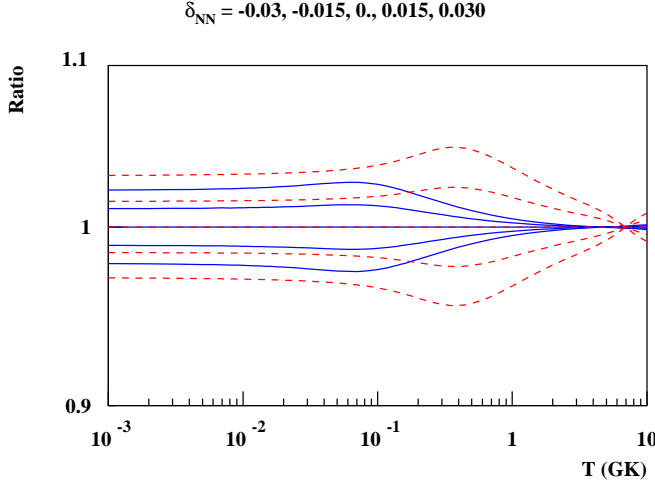


FIG. 9: Relative variation of the ${}^3\text{H}(\text{d},\text{n}){}^4\text{He}$ (solid line) and ${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$ (dashed line) rates for $\delta_{NN} = -0.030, -0.015, 0.015$ and 0.030 .

variation of constants.

While, for the nominal values of the constants, both parameterizations reproduce the experimental data very well, their comparison (our Eq. (4.3), and Eq. (9) Ref.[30]) shows important differences. Only the partial width energy dependence in the entrance channel ($\Gamma_{in} = \Gamma_d$) is considered in Ref. [30], neglecting the outgoing and total energy dependences. To be consistent, the latter must indeed depend on energy at least through Γ_{in} . The partial width ($\Gamma_{out} = \Gamma_p$ or Γ_n) energy dependence in the exit channel is small (large outgoing energy) and can be neglected. But, in addition, as shown in Eq. (4.3), the total width ($\Gamma = \Gamma_d + \Gamma_p$ or $\Gamma_d + \Gamma_n$) energy dependence (essentially from Γ_d) must

TABLE I: Abundance sensitivity, $\partial \log Y / \delta_{NN}$, to a variation of the N-N interaction at WMAP baryon density. Blank entries correspond to negligible values.

Reaction	Y_p	D/H	${}^3\text{He}/\text{H}$	${}^7\text{Li}/\text{H}$
${}^3\text{H}(\text{d},\text{n}){}^4\text{He}$				-0.015
${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$		-0.027	-1.14	-1.10

be included. We also include in our calculations the energy shift $\Delta E_R^*(E_R)$. As can be seen in Figure 10, the variation of the S-factor with ΔE_R (now the supplementary shift due to variation of constants i.e. $\neq \Delta E_R^*(E_R)$) is very different from ours when following Berengut *et al.* [30] prescription. (To emphasize the different behaviour, we have explored, in that figure a wider range of ΔE_R values ± 135 keV.)

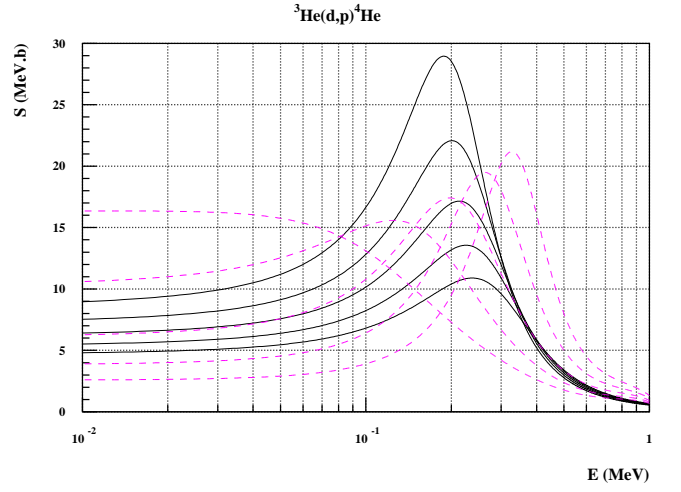


FIG. 10: S-factor for the ${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$ reaction and $\delta_{NN} = -0.30, -0.15, 0., 0.15$ and 0.30 according to our calculation (solid black) or following the parameterization of Ref. [30] with the same energy shifts (dashed pink). To emphasize the effect we use larger variations, by a factor of 10, than in Fig. 9.

These large differences reflect the differences in the parameterization of the cross section. This shows that the expression of the cross section provided by Cyburt [56] and used by Berengut *et al.* [30] is excellent for a parameterization of the experimental data, it is by no means adapted to a study of the influence of nuclear parameters on the cross-section.

The calculation of ΔE_R as a function of δ_{NN} is obtained by the difference between the energy of the $\frac{3}{2}^+$ states and of the two clusters emission thresholds in the entrance channel, both depending on the N-N-interaction. Berengut *et al.* [30] assume that these levels follow the dependence of the ${}^5\text{Li}$ and ${}^5\text{He}$ ground states but these $\frac{3}{2}^+$ state have indeed a ${}^3\text{He} \otimes \text{d}$ or $\text{t} \otimes \text{d}$ structure so that our model is more appropriate (see Ref. [57]).

Furthermore, we note that significant changes in element abundance's occur when the variation in the quark mass, $\delta m_q/m_q$ (their parameter of variation), is of order 1 %. We recall that $\delta m_q/m_q$ can be related to a change in a Yukawa coupling through $(1+S)\delta h/h$. Using Eqs. (1.5) and (2.5) we can further relate $\delta m_q/m_q$ to δ_{NN} (ignoring the contribution from $R\delta\alpha/\alpha$) as

$$\delta m_q/m_q \approx -0.4\delta_{NN}. \quad (4.4)$$

Thus for $\delta_{NN} < 0.01$, we are essentially restricting $\delta m_q/m_q < 0.004$ and would expect a significantly smaller effect on the abundances than seen in Ref. [30].

V. BBN CONSTRAINTS

A. Observational constraints

Deuterium, a very fragile isotope, is systematically destroyed after BBN. Its most primitive abundance is determined from the observation of clouds at high redshift, along the line of sight of distant quasars. Very few observations of these cosmological clouds are available [58] and a weighted mean of this data yields a D/H abundance of

$$\text{D/H} = (3.02 \pm 0.23) \times 10^{-5}. \quad (5.1)$$

In contrast, after BBN, ^4He is produced by stars. Its primordial abundance is deduced from observations in HII (ionized hydrogen) regions of low metallicity blue compact galaxies. The primordial ^4He abundance Y_p (mass fraction) is given by the extrapolation to zero metallicity but is affected by systematic uncertainties such as plasma temperature and underlying stellar absorption [59]. Using the Markov Chain-Monte Carlo methods described in Aver *et al.* [60] and data compiled in Izotov *et al.* [61], Aver *et al.* [62] found

$$Y_p = 0.2534 \pm 0.0083. \quad (5.2)$$

Given the uncertainty, this value is consistent with the BBN prediction.

^3He on the other hand, is both produced and destroyed in stars so that the evolution of its abundance as a function is subject to large uncertainties and has only been observed in our Galaxy [63],

$$^3\text{He}/\text{H} = (1.1 \pm 0.2) \times 10^{-5}. \quad (5.3)$$

Consequently, the baryometric status of ^3He is not firmly established [64].

The primordial lithium abundance is deduced from observations of low metallicity stars in the halo of our Galaxy where the lithium abundance is almost independent of metallicity, displaying a plateau, the so-called Spite plateau [65]. This interpretation assumes that lithium has not been depleted on the surface of these stars, so that the presently observed abundance is presumably primordial. The small scatter of values around

the Spite plateau is an indication that depletion may not have been very effective.

Astronomical observations of these metal poor halo stars [66] have led to a relative primordial abundance of

$$\text{Li/H} = (1.23^{+0.34}_{-0.16}) \times 10^{-10}. \quad (5.4)$$

A more recent analysis by Sbordone *et al.* [67] gives

$$\text{Li/H} = (1.58 \pm 0.31) \times 10^{-10}. \quad (5.5)$$

For a recent review of the latest Li observations and their different astrophysical aspects, see Ref. [68].

B. Revisiting SBBN with variable couplings

The results of the former sections can be implemented in a BBN code in order to compute the primordial abundances of the light elements as a function of δ_{NN} . We can rephrase our analysis of Ref. [22] in terms of δ_{NN} by using Eqs. (1.5) and (2.5) which yields the constraints

$$-0.7\% < \delta_{NN} < +0.5\%, \quad (5.6)$$

assuming $R = 36$ and $S = 240$. To test the importance of the variations in the $A = 5$ rates, we first include only those variations in $^3\text{He}(\text{d,p})^4\text{He}$ and $^3\text{H}(\text{d,n})^4\text{He}$. Figure 11 compares the BBN predictions for different values of δ_{NN} up to 30%. We emphasize that a 30% variation in δ_{NN} is unrealistic since it corresponds to a 175% variation on B_D . As one can see, the curves for ^4He and D/H are nearly horizontal, and the effect on ^7Li is insufficient to solve the lithium problem.

Next, we allow all rates depending on δ_{NN} to vary. Thus, Figure 12 updates Figure 4 of Ref. [22] assuming again $S = 240$ and $R = 36$. In this case, we see that the D and ^4He abundances can provide constraints on the variations of h , compatible with zero, while the ^7Li abundance can only be reconciled with observations for $\delta h/h \approx +3 \times 10^{-5}$. We now find

$$-2 \times 10^{-6} < \frac{\delta h}{h} < 8 \times 10^{-6}. \quad (5.7)$$

Recalling that with $S = 240$ and $R = 36$, one has

$$\delta_{NN} \sim -321 \frac{\delta h}{h}, \quad (5.8)$$

which gives

$$-0.0025 < \delta_{NN} < 0.0006. \quad (5.9)$$

Thus, we see that variations in δ_{NN} as large as that needed to reconcile ^7Li induce an excess of D/H and a deficit of ^4He (even when the large uncertainties in Y_p are taken into account).

It is interesting to note that the latter problem can be reconciled by adding a relativistic degree of freedom. Indeed, it has recently been pointed out that extra relativistic degrees of freedom may be required from the analysis

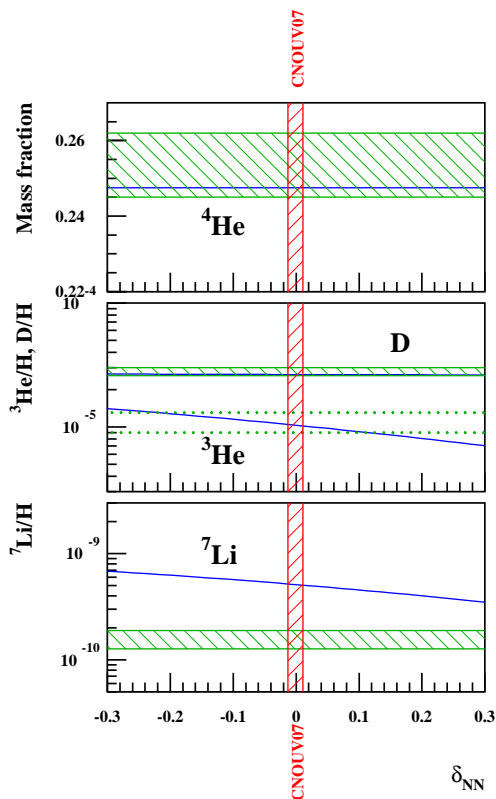


FIG. 11: Effect of the variation of the N-N interaction induced solely by the modification of the nuclear rates of ${}^3\text{He}(\text{d},\text{p}){}^4\text{He}$ and ${}^3\text{H}(\text{d},\text{n}){}^4\text{He}$ on the primordial abundances of the light element compared to the constraints obtained in Ref. [22].

of CMB data [71] and BBN data [72]. As a consequence, we also show in Figure 12, the resulting abundances when number of relativistic species is equivalent to four neutrino families. As one can see, even at large δ_{NN} , the helium abundance is concordant with observations. Interestingly, although the constraint from ${}^4\text{He}$ on $\delta h/h$ for positive values is relaxed, the constraint from D/H is tightened and becomes more severe for $N_\nu = 4$. Therefore, to reconcile D and ${}^7\text{Li}$ abundances we must rely on subsequent destruction of D/H to match the quasar absorption system data. In fact, high D/H is a general consequence of lowering ${}^7\text{Li}$ from SBBN values [73]. For $\delta h/h < 0$, while the constraint from D/H is relaxed, the constraint from ${}^4\text{He}$ is strengthened and a similar limit is obtained.

VI. DISCUSSION

This article investigated the influence of the variation of the fundamental constants on the predictions of BBN and extended our previous analysis [22]. Through our de-

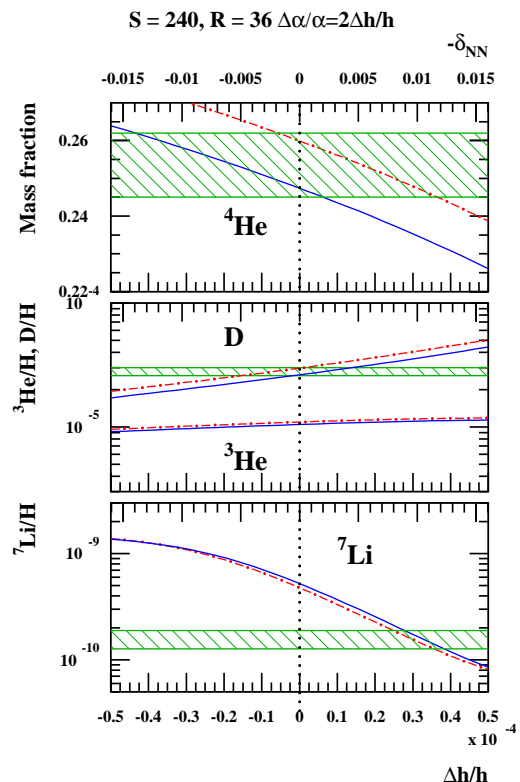


FIG. 12: Update Figure 4 of Ref. [22] assuming $S = 240$ and $R = 36$ (solid blue line), using new rates for ${}^3\text{He}(\alpha,\gamma){}^7\text{Li}$ [69] and ${}^1\text{H}(\text{n},\gamma)\text{D}$ [70] and the Ω_b value from WMAP7 [4]. The top axis is $-\delta_{NN}$ from Eq. (5.8) (mind the sign) and the dashed red line assumes $N_\nu = 4$.

tailed modeling of the cross-sections we have shown that although the variation of the nucleon-nucleon potential can greatly affect the triple- α process in stars, its effect on BBN and the production of heavier elements such as CNO is minimal at best. At the temperatures, densities and timescales associated with BBN, the changes in the ${}^4\text{He}(\alpha\alpha,\gamma){}^{12}\text{C}$ and ${}^8\text{Be}(\alpha,\gamma){}^{12}\text{C}$ reaction rates are not sufficient to produce more than $\text{C}/\text{H} \sim 10^{-21}$, and is therefore typically 6 orders of magnitude smaller than standard model abundances. This conclusion holds even when including the possibility that ${}^8\text{Be}$ can be bound. Using the variation of the fundamental constants provides a physically motivated and well-defined model to allow for stable ${}^8\text{Be}$.

We have also extended previous analysis by including effects involving ${}^5\text{He}$ and ${}^5\text{Li}$. This allowed us to revisit the constraints obtained in Ref. [22] and in particular to show that the effect on the cross-sections remain small compared to the induced variation of B_D . This analysis demonstrates the robustness of our previous analysis and places the understanding of the effect of a variation of

fundamental constants on BBN on a safer ground.

Our analysis can be compared with Ref. [30] who reached the conclusion that such variations may increase the variation of ${}^7\text{Li}$ and exacerbate the lithium problem. While formally correct, our results show that this required large variation of δ_{NN} is incompatible with the BBN constraints. Note also that Ref. [74] assumed an independent variation of the energies of the resonances while our work considers the variation of the energies of these resonance that arises from the same physical origin, so that their amplitudes are correlated.

Finally, we have extended our analysis to include the possibility of an extra relativistic degree of freedom. Because of the different dependencies, $Y(\delta_{NN}, N_\nu)$ and

$D/H(\delta_{NN}, N_\nu)$, the limits on δ_{NN} or $\delta h/h$ are not relaxed for $N_\nu = 4$. The only possibility to reconcile ${}^7\text{Li}$ in this context is a variation of $\delta_{NN} \sim -0.01$, along with the post BBN destruction of D/H .

Acknowledgments

The work of KA was supported in part by DOE grant DE-FG02-94ER-40823 at the University of Minnesota. This work was also supported by the PICS CNRS/USA and the French ANR VACOUL.

-
- [1] J.-P. Uzan, Living Rev. Rel. **14** (2011) 2 [arXiv:1009.5514 [astro-ph.CO]].
 - [2] J.-P. Uzan, Space Sci. Rev. **148** (2010) 249, [arXiv:0907.3081 [gr-qc]]; J.-P. Uzan, Rev. Mod. Phys. **75** (2003) 403 [hep-ph/0205340].
 - [3] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, (John Wiley & Sons, 1972); P. Peter and J.-P. Uzan, *Primordial cosmology*, (Oxford University Press, Oxford, 2009).
 - [4] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **192** (2011) 18, [arXiv:1001.4538 [astro-ph.CO]].
 - [5] T. P. Walker, *et al.*, Astrophys. J. **376** (1991) 51; K. A. Olive, G. Steigman and T. P. Walker, Phys. Rept. **333** (2000) 389, [arXiv:astro-ph/9905320]; R. H. Cyburt, B. D. Fields and K. A. Olive, New Astron. **6** (2001) 215, [arXiv:astro-ph/0102179]; R. H. Cyburt, B. D. Fields and K. A. Olive, Astropart. Phys. **17** (2002) 87, [arXiv:astro-ph/0105397]; A. Coc, *et al.*, Phys. Rev. D **65** (2002) 043510, [arXiv:astro-ph/0111077]; R. H. Cyburt, B. D. Fields and K. A. Olive, Phys. Lett. B **567** (2003) 227; A. Cuoco, *et al.*, Int. J. Mod. Phys. A **19** (2004) 4431, [arXiv:astro-ph/0307213]; R. H. Cyburt, Phys. Rev. D **70** (2004) 023505, [arXiv:astro-ph/0401091]; P. D. Serpico, *et al.*, JCAP **0412** (2004) 010, [arXiv:astro-ph/0408076]; B. D. Fields and S. Sarkar, in K. Nakamura *et al.*, J. Phys. G **37** (2010) 075021.
 - [6] A. Coc, *et al.*, Astrophys. J. **600** (2004) 544, [arXiv:astro-ph/0309480].
 - [7] F. Iocco, G. Mangano, G. Miele, O. Pisanti, P.D. Serpico, Phys. Rev. D **75** (2007) 73041.
 - [8] R. H. Cyburt, B. D. Fields and K. A. Olive, JCAP **0811** (2008) 012, [arXiv:0808.2818 [astro-ph]].
 - [9] A. Coc, *et al.*, ApJ **744** (2012) 158.
 - [10] T. Damour, and B. Pichon, Phys. Rev. D **59** (1999) 123502, [astro-ph/9807176]; A. Coc, *et al.*, Phys. Rev. D **73** (2006) 083525, [astro-ph/0601299]; A. Coc, *et al.*, Phys. Rev. D **79** (2009) 103512.
 - [11] E. W. Kolb, M. J. Perry and T. P. Walker, Phys. Rev. D **33** (1986) 869; L. Bergstrom, S. Iguri and H. Rubinstein, Phys. Rev. D **60** (1999) 045005; K. Ichikawa and M. Kawasaki, Phys. Rev. D **65** (2002) 123511; K.M. Nollett and R.E. Lopez, Phys. Rev. D **66** (2002) 063507; R. H. Cyburt *et al.*, Astropart. Phys. **23**, 313 (2005).
 - [12] V.V. Dixit and M. Sher, Phys. Rev. D **37** (1988) 1097.
 - [13] R. J. Scherrer and D. N. Spergel, Phys. Rev. D **47** (1993) 4774; B. Li and M. C. Chu, Phys. Rev. D **73** (2006) 023509.
 - [14] J. J. Yoo and R. J. Scherrer, Phys. Rev. D **67** (2003) 043517;
 - [15] J. P. Kneller and G. C. McLaughlin, Phys. Rev. D **68** (2003) 103508.
 - [16] J. R. Ellis, S. Kalara, K. A. Olive, C. Wetterich, Phys. Lett. B **228**, 264 (1989).
 - [17] B. A. Campbell and K. A. Olive, Phys. Lett. B **345** (1995) 429.
 - [18] V. V. Flambaum and E. V. Shuryak, Phys. Rev. D **65** (2002) 103503, [arXiv:hep-ph/0201303].
 - [19] T. Dent, S. Stern, C. Wetterich, Phys. Rev. D **76** (2007) 063513, [arXiv:0705.0696 [astro-ph]].
 - [20] K. Ichikawa and M. Kawasaki, Phys. Rev. D **69**, 123506 (2004); C. M. Muller, G. Schafer and C. Wetterich, Phys. Rev. D **70** (2004) 083504; S. J. Landau, M. E. Mosquera, H. Vucetich, Astrophys. J. **637** (2006) 38, [astro-ph/0411150];
 - [21] T. Dent, S. Stern, C. Wetterich, Phys. Rev. D **78** (2008) 103518, [0808.0702 [hep-ph]]; T. Dent, S. Stern, C. Wetterich, Phys. Rev. D **79** (2009) 083533, [0812.4130 [hep-ph]].
 - [22] A. Coc, *et al.*, Phys. Rev. D **76** (2007) 023511 (2007), [astro-ph/0610733].
 - [23] S. Ekström, *et al.*, Astron. Astrophys. **514** (2010) A62, [arXiv:0911.2420]; A. Coc, *et al.*, Mem. Soc. Astron. Ital. **80** (2009) 658.
 - [24] F. Luo, K. Olive, and J.-P. Uzan, Phys. Rev. D **84** (2011) 096004, [arXiv:1107.4154 [pdf, ps, other]].
 - [25] T. Dent and M. Fairbairn, Nucl. Phys. B **653** (2003) 256; T. Dent, Nucl. Phys. B **677** (2004) 471.
 - [26] M. Dine, *et al.*, Phys. Rev. D **67** (2003) 015009.
 - [27] P. Langacker, G. Segre and M. J. Strassler, Phys. Lett. B **528** (2002) 121.
 - [28] J. R. Ellis, K. A. Olive and Y. Santoso, New J. Phys. **4** (2002) 32, [arXiv:hep-ph/0202110].
 - [29] V. F. Dmitriev and V. V. Flambaum Phys. Rev. D **67** (2003) 063513, [astro-ph/0209409]; V. F. Dmitriev, V. V. Flambaum and J. K. Webb, Phys. Rev. D **69** (2004) 063506; V. V. Flambaum and E. V. Shuryak, Phys. Rev. D **67** (2003) 083507, [arXiv:hep-ph/0212043].

- [30] J. C. Berengut, V. V. Flambaum and V. F. Dmitriev, Phys. Lett. B **683** (2010) 114, [arXiv:0907.2288 [nucl-th]].
- [31] S. J. Landau, *et al.*, Phys. Rev. D **78** (2008) 083527, [arXiv:0809.2033 [astro-ph]].
- [32] P. F. Bedaque, T. Luu, L. Platter, Phys. Rev. C **83** (2011) 045803, [arXiv:1012.3840 [nucl-th]]; M. -K. Cheoun, *et al.*, Phys. Rev. D **84** (2011) 043001, [arXiv:1104.5547 [astro-ph.CO]].
- [33] K. A. Olive, *et al.*, Phys. Rev. D **66** (2002) 045022; K. A. Olive, *et al.*, Phys. Rev. D **69** (2004) 027701.
- [34] K. Ichikawa, T. Kanzaki and M. Kawasaki, Phys. Rev. D **74**, 023515 (2006) [astro-ph/0602577]; M. Nakashima, *et al.*, JCAP **1001** (2010) 030, [arXiv:0910.0742 [astro-ph.CO]]; C. J. A. Martins, *et al.*, Phys. Rev. D **82** (2010) 023532, [arXiv:1001.3418 [astro-ph.CO]].
- [35] F. J. Dyson, Sci. Am. **225** (1971) 51; P. Davies, J. Phys. A **5** (1972) 1296; J. D. Barrow, Phys. Rev. D **35** (1987) 1805; T. Pochet *et al.*, Astron. Astrophys. **243** (1991) 1.
- [36] J. P. Kneller, G. C. McLaughlin, Phys. Rev. D **70** (2004) 043512, [arXiv:astro-ph/0312388].
- [37] K. Wildermuth and Y. C. Tang, Book, Editors K. Wildermuth and P. Kramer, Publishers Vieweg, Braunschweig (1977).
- [38] P. Descouvemont and D. Baye, Phys. Rev. C **36** (1987) 54.
- [39] P. Navrátil, S. Quaglioni, I. Stetcu, and B. R. Barrett, J. Phys. G **36** (2009) 083101.
- [40] S. C. Pieper and R. B. Wiringa, Ann. Rev. Nucl. Part. Sci. **51** (2001) 53.
- [41] R. B. Wiringa, V. G. J. Stoks and R. Schiavilla, Phys. Rev. C **51**, 38 (1995) [arXiv:nucl-th/9408016].
- [42] D. R. Thompson and M. LeMere, and Y. C. Tang, Nucl. Phys. A, **286** (1977) 53.
- [43] T. Damour, and J. F. Donoghue, Phys. Rev. D **78** (2008) 014014.
- [44] E. Epelbaum, U.-G. Meissner, and W. Gloeckle, Nucl. Phys. A **714** (2003) 535.
- [45] S. R. Bean, and M. J. Savage, Nucl. Phys. A **717** (2003) 91.
- [46] S. Cassisi and V. Castellani, ApJS **88** (1993) 509.
- [47] S. Ekström, *et al.*, Astron. Astrophys. **489** (2008) 685.
- [48] C. Angulo *et al.*, Nucl. Phys. A **656** (1999) 3.
- [49] B. Buck, H. Friedrich, and C. Wheatley, Nucl. Phys. **275** (1977) 246.
- [50] D. Baye and P. Descouvemont, Nucl. Phys. A **443** (1985) 302.
- [51] J. R. Newton, C. Iliadis, A. E. Champagne, A. Coc, Y. Parpottas, and C. Ugalde, Phys. Rev. C **75** (2007) 045801.
- [52] P. Descouvemont, *et al.*, ADNDT **88** (2004) 203.
- [53] F. C. Barker, Phys. Rev. C **56** (1997) 2646.
- [54] P. Descouvemont and D. Baye, Rep. Prog. Phys. **73** (2010) 036301.
- [55] A. Coc & E. Vangioni, J. of Phys. Conf. Series, **202** (2010) 012001.
- [56] R. H. Cyburt, Phys. Rev. D **70** (2004) 023505, [arXiv:astro-ph/0401091].
- [57] L. N. Bogdanova, G. M. Haie, V. E. Marksushin, Phys. Rev. C **44** (1991) 1289.
- [58] S. Burles and D. Tytler, Astrophys. J. **499**, 699 (1998) [arXiv:astro-ph/9712108]; S. Burles and D. Tytler, Astrophys. J. **507**, 732 (1998) [arXiv:astro-ph/9712109]; J. M. O'Meara, D. Tytler, D. Kirkman, N. Suzuki, J. X. Prochaska, D. Lubin and A. M. Wolfe, Astrophys. J. **552**, 718 (2001) [arXiv:astro-ph/0011179]; M. Pettini and D. V. Bowen, Astrophys. J. **560**, 41 (2001) [arXiv:astro-ph/0104474]; S. A. Levshakov, M. Dessauges-Zavadsky, S. D'Odorico and P. Molaro, Astrophys. J. **565**, 696 (2002) [astro-ph/0105529]; D. Kirkman, D. Tytler, N. Suzuki, J. M. O'Meara and D. Lubin, Astrophys. J. Suppl. **149**, 1 (2003) [arXiv:astro-ph/0302006]; J. M. O'Meara, S. Burles, J. X. Prochaska, G. E. Prochter, R. A. Bernstein and K. M. Burgess, Astrophys. J. **649**, L61 (2006) [arXiv:astro-ph/0608302]; Pettini, M., Zych, M. Pettini, B. J. Zych, M. T. Murphy, A. Lewis and C. C. Steidel, MNRAS **391**, 1499 (2008) [arXiv:0805.0594 [astro-ph]]; Fumagalli, M., M. Fumagalli, J. M. O'Meara and J. X. Prochaska, [arXiv:1111.2334 [astro-ph.CO]]; R. Srikanand, N. Gupta, P. Petitjean, P. Noterdaeme and C. Ledoux, MNRAS **405**, 1888 (2010) [arXiv:1002.4620 [astro-ph.CO]].
- [59] K. A. Olive, and E. Skillman, New Astronomy, **6**, 119 (2001); K. A. Olive and E. D. Skillman, Astrophys. J. **617**, 29 (2004) [arXiv:astro-ph/0405588].
- [60] E. Aver, K. A. Olive and E. D. Skillman, JCAP **1103**, 043 (2011) [arXiv:1012.2385 [astro-ph.CO]].
- [61] Y. I. Izotov, T. X. Thuan and G. Stasińska, Astrophys. J. **662**, 15 (2007) [arXiv:astro-ph/0702072].
- [62] E. Aver, K. A. Olive and E. D. Skillman, JCAP **1204**, 004 (2012) [arXiv:1112.3713 [astro-ph.CO]].
- [63] T. Bania, R. Rood and D. Balser, Nature **415** (2002) 54.
- [64] E. Vangioni-Flam, *et al.*, Astrophys. J. **585** (2003) 611.
- [65] F. Spite and M. Spite, Astron. Astrophys. **115** (1992) 357.
- [66] S. G. Ryan, T. C. Beers, K. A. Olive, B. D. Fields, and J. E. Norris, Astrophys. J. Lett. **530** (2000) L57, [arXiv:astro-ph/9905211].
- [67] L. Sbordone, *et al.*, Astron. Astrophys. **522** (2010) 26.
- [68] F. Spite and M. Spite, Proc. IAU Symposium No. 268, "Light Elements in the Universe", 9-13 November, Geneva, Switzerland, Eds. C. Charbonnel, M. Tosi, F. Primas & C. Chiappini (Cambridge University Press, 2010) p. 221.
- [69] R. H. Cyburt and B. Davids, Phys. Rev. C **78** (2008) 064614.
- [70] S. Ando, R. H. Cyburt, S. W. Hong, and C. H. Hyun, Phys. Rev. C **74** (2006) 025809.
- [71] J. Hamann, S. Hannestad, G. G. Raffelt, I. Tamborra and Y. Y. Y. Wong, Phys. Rev. Lett. **105**, 181301 (2010) [arXiv:1006.5276 [hep-ph]]; arXiv:1104.2333 [astro-ph.CO]; M. Archidiacono, E. Calabrese and A. Melchiorri, Phys. Rev. D **84**, 123008 (2011) [arXiv:1109.2767 [astro-ph.CO]]; JCAP **1203**, 021 (2012) [arXiv:1110.4271 [astro-ph.CO]].
- [72] Y. I. Izotov and T. X. Thuan, Astrophys. J. **710**, L67 (2010) [arXiv:1001.4440 [astro-ph.CO]]; K. M. Nollett and G. P. Holder, arXiv:1112.2683 [astro-ph.CO]; M. E. Mosquera, and O. Civitaresse, Phys. Rev. C **84** (2011) 065803.
- [73] K. A. Olive, P. Petitjean, E. Vangioni and J. Silk, arXiv:1203.5701 [astro-ph.CO].
- [74] M. -K. Cheoun, *et al.*, Phys. Rev. D, **84** (2011) 043001.